

# CISCE VIRTUAL LEARNING SERIES

## LESSON: MATHEMATICS

### TRIGONOMETRIC IDENTITIES (SESSION 2)

October 19<sup>th</sup>, 2020

#### Response to Questions posed by students during the live Lesson:

S.No.	Questions	Answers
1.	<p>Prove that <math>\sqrt{\frac{1-\cos A}{1+\cos A}} = \frac{\sin A}{1+\cos A}</math>.</p> <p>This sum is proved by taking LHS. How will we prove the sum from the RHS?</p>	<p>Yes you may.</p> <p>Prove that <math>\sqrt{\frac{1-\cos A}{1+\cos A}} = \frac{\sin A}{1+\cos A}</math></p> <p>RHS = <math>\frac{\sin A}{1+\cos A}</math></p> <p>= <math>\sqrt{\frac{\sin^2 A}{(1+\cos A)^2}}</math></p> <p>= <math>\sqrt{\frac{1-\cos^2 A}{(1+\cos A)^2}}</math></p> <p>= <math>\sqrt{\frac{(1+\cos A)(1-\cos A)}{(1+\cos A)(1+\cos A)}}</math></p> <p>= <math>\sqrt{\frac{1-\cos A}{1+\cos A}}</math> LHS</p>
2.	<p><i>Prove that</i></p> <p><math>\sec^4 \theta - \sec^2 \theta = \tan^4 \theta + \tan^2 \theta</math>.</p> <p><i>Can we prove this identity by transferring a term from LHS to RHS or RHS to LHS?</i></p>	<p>When proving an identity, it is advisable to start from one side and prove equal to the other side. Follow the rule for all sums.</p> <p><i>Prove that <math>\sec^4 \theta - \sec^2 \theta = \tan^4 \theta + \tan^2 \theta</math></i></p> <p>LHS = <math>\sec^2 \theta (\sec^2 \theta - 1)</math></p> <p>= <math>(1 + \tan^2 \theta) \tan^2 \theta</math></p> <p>= <math>\tan^2 \theta + \tan^4 \theta</math></p> <p>= <math>\tan^4 \theta + \tan^2 \theta = \text{RHS Proved}</math></p>

S.No.	Questions	Answers
3.	How do we solve a problem like $\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$ , which involves two different angles $A$ and $B$ ?	<p><i>Prove that <math>\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A</math></i></p> <p>LHS= <math>\sin^2 A - \sin^2 B</math></p> <p><math>= 1 - \cos^2 A - (1 - \cos^2 B)</math></p> <p><math>= 1 - \cos^2 A - 1 + \cos^2 B</math></p> <p><math>= \cos^2 B - \cos^2 A</math></p>
4.	As we have algebraic equation, do we have trigonometric equation?	<p>Yes</p> <p>Example (i) <math>\sin x = \cos x</math></p> <p><math>\Rightarrow \tan x = 1 = \tan 45^\circ</math></p> <p><math>\Rightarrow x = 45^\circ</math></p> <p>Example (i) <math>\sin^2 x - 2\sin x + 1 = 0</math></p> <p><math>\Rightarrow (\sin x - 1)^2 = 0</math></p> <p><math>\Rightarrow \sin x - 1 = 0</math></p> <p><math>\Rightarrow \sin x = 1</math></p> <p><math>\Rightarrow \sin x = 1 = \sin 90^\circ</math></p> <p><math>\Rightarrow x = 90^\circ</math></p>
5.	Is it correct to write: $\cos(A + B) = \cos A + \cos B$	<p>No.</p> <p>If we take <math>A=30^\circ</math> and <math>B=60^\circ</math></p> <p><math>LHS=\cos(A + B) = \cos(30^\circ + 60^\circ)</math></p> <p>RHS= <math>\cos A + \cos B</math></p> <p><math>= \cos 90^\circ = 1</math></p> <p><math>= \cos 30^\circ + \cos 60^\circ</math></p> <p><math>= \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2}</math></p> <p>Hence <math>\cos(A + B) \neq \cos A + \cos B</math></p>

S.No.	Questions	Answers
6.	Whenever we use any algebraic identity in the solution, is it important to mention the identity which we have used?	It is better to write them so that you do not go wrong while applying them.
7.	Is it necessary to learn how to use <i>trigonometric tables</i> ?	Yes. It is a part of the ICSE 2021 Scope of Syllabus. Its application is there in Heights and Distances.
8.	If $\tan A + \sin A = m$ and $\tan A - \sin A = n$ , prove that $m^2 - n^2 = 4\sqrt{mn}$	<p>Here <math>m^2 = (\tan A + \sin A)^2 = \tan^2 A + \sin^2 A + 2 \tan A \sin A</math> and <math>n^2 = (\tan A - \sin A)^2 = \tan^2 A + \sin^2 A - 2 \tan A \sin A</math></p> <p><math>\Rightarrow m^2 - n^2 = 4 \tan A \sin A = 4 \cdot \frac{\sin A}{\cos A} \times \sin A = 4 \frac{\sin^2 A}{\cos A}</math></p> <p>and <math>mn = (\tan A + \sin A)(\tan A - \sin A)</math></p> $= \tan^2 A - \sin^2 A = \frac{\sin^2 A}{\cos^2 A} - \sin^2 A = \frac{\sin^2 A - \sin^2 A \cos^2 A}{\cos^2 A} = \frac{\sin^2 A(1 - \cos^2 A)}{\cos^2 A} = \frac{\sin^4 A}{\cos^2 A}$ <p>Therefore <math>\sqrt{mn} = \sqrt{\frac{\sin^4 A}{\cos^2 A}} = \frac{\sin^2 A}{\cos A}</math></p> <p><math>\Rightarrow 4\sqrt{mn} = 4 \frac{\sin^2 A}{\cos A}</math></p> <p><math>m^2 - n^2 = 4\sqrt{mn}</math></p> <p><math>= 4 \frac{\sin^2 A}{\cos A}</math> Hence proved.</p>
9.	Prove: $\frac{2}{1 - 2\cos^2 A} = 2 \frac{\sec^2 A}{\tan^2 A - 1}$	$LHS = \frac{2}{1 - \frac{2}{\sec^2 A}} = \frac{2}{\frac{\sec^2 A - 2}{\sec^2 A}} = \frac{2\sec^2 A}{(1 + \tan^2 A) - 2}$ $= \frac{2\sec^2 A}{1 + \tan^2 A - 2} = \frac{2\sec^2 A}{\tan^2 A - 1} = RHS$
10.	In 2021 Mathematics syllabus are angles other than standard angles included?	Yes

S.No.	Questions	Answers
11.	<p>Prove that</p> <p>(Sec A-Cosec A) (1+tan A+cot A) =tan A . Sec A - cot A. Cosec A</p> <p>Can we prove both sides to a common answer, and not a direct proof?</p>	<p>Yes, in the following way:</p> $LHS = \left( \frac{1}{\cos A} - \frac{1}{\sin A} \right) \left( 1 + \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} \right)$ $= \left( \frac{\sin A - \cos A}{\sin A \cos A} \right) \times \left( \frac{\cos A \cdot \sin A + \sin^2 A + \cos^2 A}{\cos A \cdot \sin A} \right)$ $= \frac{(\sin A - \cos A)(\sin^2 A + \sin A \cos A + \cos^2 A)}{\sin^2 A \cdot \cos^2 A} =$ $\frac{\sin^3 A - \cos^3 A}{\sin^2 A \cdot \cos^2 A}$ $RHS = \frac{\sin A}{\cos A} \cdot \frac{1}{\cos A} - \frac{\cos A}{\sin A} \cdot \frac{1}{\sin A}$ $= \frac{\sin A}{\cos^2 A} - \frac{\cos A}{\sin^2 A} = \frac{\sin^3 A - \cos^3 A}{\sin^2 A \cdot \cos^2 A}$ <p>Hence LHS=RHS</p>
12.	<p>Prove that:</p> $\frac{1+\tan A}{\sin A} + \frac{1+\cot A}{\cos A} = 2(\sec A + \operatorname{cosec} A)$	$LHS = \frac{1 + \tan A}{\sin A} + \frac{1 + \cot A}{\cos A}$ $= \left( \frac{1}{\sin A} + \frac{\frac{\sin A}{\cos A}}{\sin A} \right) + \left( \frac{1}{\cos A} + \frac{\frac{\cos A}{\sin A}}{\cos A} \right)$ $= \left( \operatorname{cosec} A + \frac{\sin A}{\cos A} \times \frac{1}{\sin A} \right) + \left( \sec A + \frac{\cos A}{\sin A} \times \frac{1}{\cos A} \right)$ $= (\operatorname{cosec} A + \sec A) + (\sec A + \operatorname{cosec} A)$ $= 2(\sec A + \operatorname{cosec} A)$ $= RHS$
13.	<p>Can we solve the trigonometric identities using right angled triangle and applying the relation among perpendicular, base, hypotenuse?</p>	<p>Yes, you may but for complex sums this method may become complicated. It is more convenient to prove by using other standard identities.</p>

S.No.	Questions	Answers
14.	Prove that $\frac{1-\sin A}{1+\sin A} = (\sec A - \tan A)^2$	$ \begin{aligned} LHS &= \frac{1-\sin A}{1+\sin A} = \frac{1-\sin A}{1+\sin A} \times \frac{1-\sin A}{1-\sin A} \\ &= \frac{(1-\sin A)^2}{(1)^2-(\sin A)^2} \\ &= \frac{(1-\sin A)^2}{1-\sin^2 A} \\ &= \frac{(1-\sin A)^2}{\cos^2 A} \\ &= \left(\frac{1-\sin A}{\cos A}\right)^2 \\ &= \left(\frac{1}{\cos A} - \frac{\sin A}{\cos A}\right)^2 \\ &= (\sec A - \tan A)^2 \\ &= RHS \end{aligned} $
15.	In board examination do we get marks for every step?	Marks are given for necessary correct relevant steps.
16.	Prove that $2(\sin^6 x + \cos^6 x) - 3(\sin^4 x + \cos^4 x) + 1 = 0$	$ \begin{aligned} LHS &= 2(\sin^6 x + \cos^6 x) - 3(\sin^4 x + \cos^4 x) + 1 \\ &= 2\{(\sin^2 A)^3 + (\cos^2 A)^3\} - 3\{(\sin^2 A)^2 + (\cos^2 A)^2\} + 1 \\ &= 2\{(\sin^2 A + \cos^2 A)^3 - 3\sin^2 A \cdot \cos^2 A(\sin^2 A + \cos^2 A)\} \\ &\quad - 3\{(\sin^2 A + \cos^2 A)^2 - 2\sin^2 A \cos^2 A\} + 1 \\ &= 2(1 - 3\sin^2 A \cdot \cos^2 A) - 3(1 - 2\sin^2 A \cdot \cos^2 A) + 1 \\ &= 2 - 6\sin^2 A \cdot \cos^2 A - 3 + 6\sin^2 A \cdot \cos^2 A + 1 \\ &= 0 = RHS \end{aligned} $
17.	Is it necessary to write the basic trigonometric identities which we are using, beside the sum?	The sum will not be marked wrong if you do not write the standard identities used.