ICSE

Analysis of Pupil Performance

MATHEMATICS

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This document of the Analysis of Pupils' Performance at the ISC Year 12 and ICSE Year 10 Examination is one of its kind. It has grown and evolved over the years to provide feedback to schools in terms of the strengths and weaknesses of the candidates in handling the examinations.

We commend the work of Mrs. Shilpi Gupta (Deputy Head) and the Research Development and Consultancy Division (RDCD) of the Council who have painstakingly prepared this analysis. We are grateful to the examiners who have contributed through their comments on the performance of the candidates under examination as well as for their suggestions to teachers and students for the effective transaction of the syllabus.

We hope the schools will find this document useful. We invite comments from schools on its utility and quality.

Gerry Arathoon
November 2020   **Chief Executive & Secretary**

The CISCE has been involved in the preparation of the ICSE and ISC Analysis of Pupil Performance documents since the year 1994. Over these years, these documents have facilitated the teaching-learning process by providing subject/ paper wise feedback to teachers regarding performance of students at the ICSE and ISC Examinations. With the aim of ensuring wider accessibility to all stakeholders, from the year 2014, the ICSE and the ISC documents have been made available on the CISCE website www.cisce.org.

The documents for the ICSE and ISC Examination Year 2020 include a detailed qualitative analysis of the performance of students in different subjects. The purpose of this analysis is to provide insights into how candidates have performed in individual questions set in the question paper. This section is based on inputs provided by examiners from examination centers across the country. It comprises of question wise feedback on the performance of candidates in the form of *Comments of Examiners* on the common errors made by candidates along with *Suggestions for Teachers* to rectify/ reduce these errors. The *Marking Scheme* for each question has also been provided to help teachers understand the criteria used for marking. Topics in the question paper that were generally found to be difficult or confusing by candidates, have also been listed down, along with general suggestions for candidates on how to prepare for the examination/ perform better in the examination.

The Analysis of Pupil Performance document for ICSE for the Examination Year 2020 covers the following subjects/papers: English (English Language, Literature in English), History and Civics, Mathematics, Physics, Chemistry, Commercial Studies and Environmental Science.

Subjects covered in the ISC Analysis of Pupil Performance document for the Year 2020 include English (English Language and Literature in English), Hindi, Physics, Chemistry, Mathematics, Computer Science, History, Political Science, Economics, Commerce, Accounts, and Environmental Science.

I would like to acknowledge the contribution of all the ICSE and the ISC examiners who have been an integral part of this exercise, whose valuable inputs have helped put this document together.

I would also like to thank the RDCD team of Dr. M.K. Gandhi, Dr. Manika Sharma, Mrs. Roshni George and Ms. Mansi Guleria, who have done a commendable job in preparing this document.

We hope that this document will enable teachers to guide their students more effectively and comprehensively so that students prepare for the ICSE/ ISC Examinations, with a better understanding of what is required from them.

Shilpi Gupta

November 2020   *Deputy Head - RDCD*

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QUALITATIVE ANALYSIS: MATHEMATICS	**1**

SECTION A (40 Marks)

*Attempt **all** questions from this Section*

Question 1

(a)   Solve the following Quadratic Equation: **[3]**

$x^2 - 7x + 3 = 0$

Give your answer correct to two decimal places.

(b)   Given $A = \begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix}$ **[3]**

If $A^2 = 3I$, where I is the identity matrix of order 2, find x and y.

(c)   Using ruler and compass construct a triangle ABC where AB = 3 cm, BC = 4 cm and   **[4]**
∠ABC = 90º. Hence construct a circle circumscribing the triangle ABC. Measure and
write down the radius of the circle.

Comments of Examiners	Suggestions for Teachers
(a) Some of the common errors made by candidates in this question were: (i) Using incorrect formula for finding roots of the quadratic equation. (ii) Using correct formula but substituting incorrectly, e.g. - 7 was taken as +7 (iii) For finding square root of 37, a number of candidates used division method instead of using Mathematical tables and thereby made calculation errors; some did not go up to the required number of decimal places. Hence, the final answer was incorrect. Many candidates found the square root of 37 only up to two decimal places, even while using tables. (iv) Calculation error such as: 7 - 6.083 = 1.083 instead of 0.917. (v) Error in rounding off, such as 0.4584 rounded to two decimal places was written as 0.45 instead of 0.46. (b) In finding A^2 many candidates squared each of the elements instead of finding the product A x A. Some candidates made mistakes in identifying the 2x2 identity matrix.	▪ *Familiarize students with mathematical tables and their use in finding square roots of numbers to help students to get the correct answer and also to save time.* ▪ *Give sufficient practice to reduce the errors, such as using incorrect formulae and basic calculation errors.* ▪ *Clarify the importance of rounding off numbers.* ▪ *In matrix multiplication explain that the meaning of A^2 means 'A' multiplied by 'A' and not the square of the corresponding elements of 'A', and in an identity matrix, the leading diagonal elements are 1 and all other elements are zero.* ▪ *Give adequate practice on locus, geometrical properties and construction of triangles.* ▪ *Train students to practice construction of geometrical*

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Some candidates did incorrect calculation e.g. x.x was taken as 2x instead of x^2; $3.x + 9 = 0$ was solved and written as $x = -2$

(c) Many candidates constructed 90^0 with a protractor. The most common error was that for perpendicular bisector, candidates drew arcs only on one side of the line to be bisected. Many others did not measure and record the radius of the circle.
Some candidates bisected the *angles* instead of *sides* to draw the circumcircle.

figures, showing all traces of construction, using the ruler and compass only, unless otherwise specified in the question.

■ *Stress upon maintaining the accuracy of measurements.*

MARKING SCHEME

Question 1

(a) $x^2 - 7x + 3 = 0$

$\therefore a = 1, b = -7, c = 3$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 1 \times 3}}{2 \times 1}$

$x = \dfrac{7 \pm \sqrt{49 - 12}}{2}$

$= \dfrac{7 \pm \sqrt{37}}{2}$

$= \dfrac{7 \pm 6.083}{2}$

$x = \dfrac{7 + 6.083}{2}, \quad \dfrac{7 - 6.083}{2}$

$= \dfrac{13.083}{2}, \quad \dfrac{0.917}{2}$

$\therefore x = 6.541, \quad 0.458$

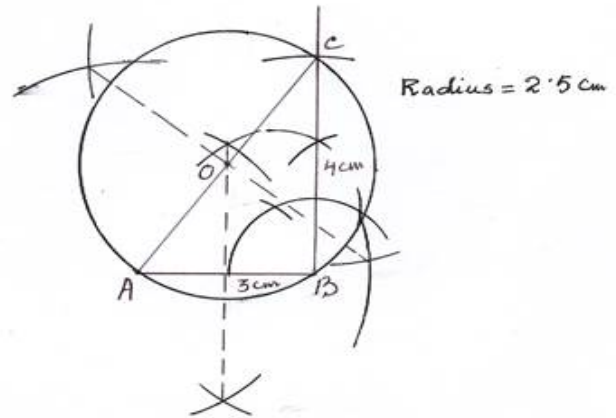$x = 6.54, \ 0.46$ *(All solutions correct to two decimal places)*

(b) $A^2 = 3I$

$\begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix} \begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix} = 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} x^2 + 3y & 3x + 9 \\ xy + 3y & 3y + 9 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

	$\therefore 3x + 9 = 0 \quad OR \quad 3y + 9 = 3$ $\therefore 3x = -9 \qquad 3y = -6$ $\therefore x = -3 \qquad y = -2$ $x = -3, \qquad y = -2$
(c)	Construction of a circle Circumscribing a right ΔABC Constructed using ruler and compass. Radius = 2.5 cm ± 0.2

Question 2

(a)   Use factor theorem to factorise $6x^3 + 17x^2 + 4x - 12$ completely.   **[3]**

(b)   Solve the following inequation and represent the solution set on the number line.   **[3]**

$$\frac{3x}{5} + 2 \; < x + 4 \le \frac{x}{2} + 5, \quad x \in R$$

(c)   Draw a Histogram for the given data, using a graph paper: **[4]**

Weekly Wages (in)	No. of People
3000 – 4000	4
4000 – 5000	9
5000 – 6000	18
6000 – 7000	6
7000 – 8000	7
8000 – 9000	2
9000 – 10000	4
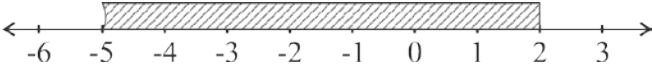
Estimate the mode from the graph.

Comments of Examiners	**Suggestions for Teachers**
(a) Many candidates did not use factor theorem to identify the first factor as specified in the question. Some made mistakes in finding out the quotient while dividing the given polynomial by the factor found. Many candidates did not write the answer in the product form, e.g. $(x+2)(3x-2)(2x+3)$. Some candidates were unable to factorize the quadratic quotient obtained by dividing the polynomial.	▪ *Instruct students to use factor theorem to find the first factor. The final answer should be expressed in the product form and not factors separated by commas.* ▪ *Give adequate practice in division of polynomials by binomials.* ▪ *Advise candidates to read the question carefully to avoid missing out the conditions given in the question e.g., x belongs to N or W or Z or R.*
(b) Many candidates made errors in transposing like terms on the same side. e.g. $\frac{3x}{5} - x < 2 - 4$ Some candidates made various types of errors in simplification. e.g. $x + 4 \le \frac{x}{2} + 5$ was written as $2x + 4 \le x + 5$. $-2x < 10$ on solving was written as $x < -5$ instead of $x \ge 5$ The solution set was not represented in set builder form. Some candidates did not write the solution set. A large number of candidates made errors in representation of solution set on the number line. e.g., Real number solution was represented incorrectly by using dots. Some candidates failed to put extra numbers on each side of the solution for indicating the continuity of the number line.	▪ *Give sufficient drill to reduce basic mistakes of solving inequations, such as, transposing sides and algebraic simplification.* ▪ *Train students to write the solution set.* ▪ *Explain thoroughly the rules of plotting graphs to students, i.e., use of correct scale, when kink is put on the axis, reading of values from graph.* ▪ *Give sufficient practice in locating and finding mode from the graph.*
(c) Numerous errors made by the candidates while plotting the histogram: ▪ Many candidates did not show the kink. ▪ The three guidelines to locate mode were not drawn correctly. ▪ Class interval was represented below each bar as 3000 – 4000, 4000 – 5000, etc. ▪ A few candidates did not join the end points of highest bar with corresponding end points of preceding and succeeding bars to locate mode. ▪ The chosen scale was not used correctly. ▪ Some calculated cumulative frequency and then plotted the bars of increasing heights.	

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Question 2

(a)	$f(x) = 6x^3 + 17x^2 + 4x - 12$

$f(x) = 6x^3 + 17x^2 + 4x - 12$

$f(-2) = 6(-2)^3 + 17(-2)^2 + 4(-2) - 12$

$= 6 \times -8 + 17 \times 4 - 8 - 12$

$= -48 + 68 - 8 - 12$

$= -68 + 68 = 0$

$(x + 2)$ is factor

$6x^2 + 5x - 6$

$6x^2 + 9x - 4x - 6$

$3x(2x + 3) - 2(2x + 3)$

$(3x - 2)(2x + 3)$

$\therefore$ Factors are $(x + 2)(3x - 2)(2x + 3)$

Correct substitution

$$\begin{array}{r} 6x^2 + 5x - 6 \\ x + 2 \overline{)\, 6x^3 + 17x^2 + 4x - 12} \\ 6x^3 + 12x^2 \\ \hline 5x^2 + 4x \\ 5x^2 + 10x \\ \hline -6x - 12 \\ -6x - 12 \\ \hline 0 \end{array}$$

(b)

$\frac{3x}{5} - x < 4 - 2$ 　　　 $x - \frac{x}{2} \leq 5 - 4$ (Any one transformation of x terms on one side)

$\frac{3x - 5x}{5} < 2$ 　　　 $\frac{2x - x}{2} \leq 5 - 4$

$-2x < 10$ 　　　 $\frac{x}{2} \leq 1$

$\therefore 2x > -10$ 　　　 $x \leq 2$

$x > -5$

Solution: $\{x: -5 < x \leq 2, x \in R\}$

(c)	Correct axis, kink, Histogram drawn taking proper scales

Mode = 5400 (±100) with 3 lines drawn for locating mode

Mode 5400 (±100)
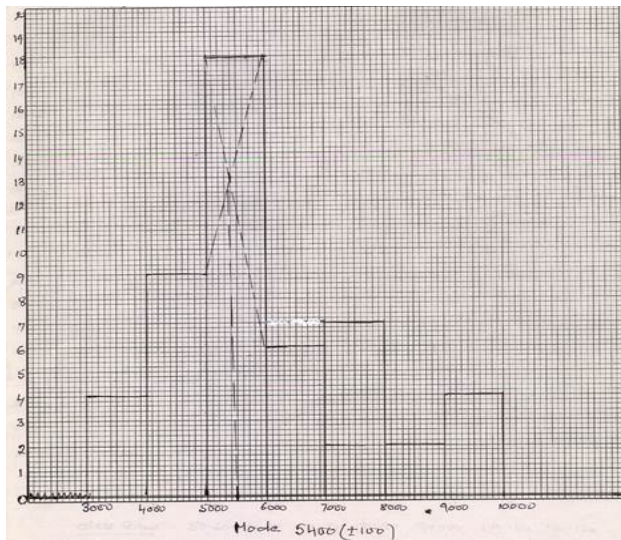
Question 3

(a)   In the figure given below, O is the centre of the circle and AB is a diameter.   **[3]**
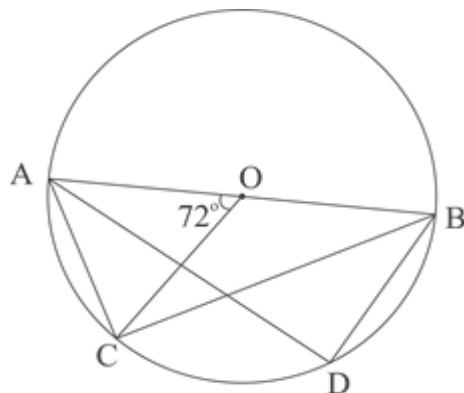
If AC = BD and ∠AOC = 72°. Find:

(i)   ∠ABC

(ii)   ∠BAD

(iii)   ∠ABD

(b)   Prove that: **[3]**

$$\frac{\sin A}{1 + \cot A} - \frac{\cos A}{1 + \tan A} = \sin A - \cos A$$

(c) In what ratio is the line joining P (5, 3) and Q (–5, 3) divided by the y-axis? Also find the coordinates of the point of intersection. **[4]**
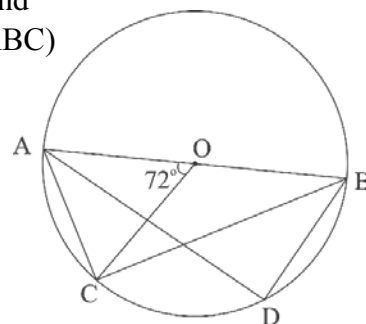
Comments of Examiners	Suggestions for Teachers

(a) Only a few candidates could solve this question correctly giving suitable reasons.
 	
  (i) Many candidates were able to find ∠ABC but could not find the other two angles.
  (ii) Properties of circles like, angle in a semicircle, equal arc subtends equal angle at circumference, etc. were not applied correctly.
  (iii) A few candidates could not use the properties of isosceles triangle and congruency of triangles correctly.
  Some candidates made calculation mistakes. Appropriate reasons supporting the answers were also missing in many scripts.
 	
(b) Some candidates failed to express tan A and cot A in terms of sin A and cos A correctly. Errors were also made in writing the numerator and denominator of resulting trigonometric expressions.
 	
e.g., $\{Sin^2A/(1 + Cos\ A)\} - \{Cos^2A/ (1+SinA)\}$
 	
Some took the LCM correctly but went wrong in simplifying the expression. A few candidates could not express
 	
$Sin^2A - Cos^2A = (Sin\ A + Cos\ A)(Sin\ A - Cos\ A)$.
 	
(c) Several candidates did not take the point on y - axis as (0, y). Some candidates could not apply the section formula correctly. Many applied mid-point formula to get the coordinates of the required point. Others found the ratio correctly but did not write it in ratio form. Simple calculation errors were common. Some candidates obtained the coordinates but did not
	▪ *Emphasise on giving reasons supporting each answer while solving geometry problems.*
	▪ *Give adequate practice in properties of circles to enable students to solve problems based on circles. Importance of giving reasons must be made clear to all students. Drill students in naming angles correctly.*
	▪ *Discuss theorems repeatedly in classes and drill the students to apply theorems and properties correctly in problems.*
	▪ *Help students understand the application of a specific theorem to a given question, correctly.*
	▪ *Train students to solve trigonometry identities by different methods. Advise students that while solving an identity, they must never work with both sides taken together. Proof must be from left hand side to right hand side or the reverse.*
	▪ *Teach students that a point on the x-axis has its ordinate = 0 and a point on y-axis has its abscissa = 0. Give sufficient practice on problems involving section formula.*

write them within the brackets. Multiplication of (m + n) by 0 was taken as (m + n) instead of '0'.

(a) $\angle AOC = 72^o$ (given)

(i) $\angle ABC = \dfrac{\angle AOC}{2} = \dfrac{72}{2}$ (Angle subtended at centre in double the angle made on circumference)

$= 36^o$

(ii) $\angle BAD = \angle ABC$ ($\because$ AC = BD, equal arcs subtend equal angles / or $\triangle BAD \cong \triangle ABC$)

$= 36^o$

(iii) $\angle ABD = 180^o - (36 + 90)$

$= 180 - 126$

$= 54$ ($\angle D = 90^o$ angle on a semi-circle and angles of a triangle adds upto 180°

(OR any correct approach)

(b) $\dfrac{\sin A}{1 + \dfrac{\cos A}{\sin A}} - \dfrac{\cos A}{1 + \dfrac{\sin A}{\cos A}}$

$\dfrac{\sin^2 A}{\sin A + \cos A} - \dfrac{\cos^2 A}{\cos A + \sin A} = \dfrac{\sin^2 A - \cos^2 A}{\sin A + \cos A}$

$\dfrac{(\sin A + \cos A)\ (\sin A - \cos A)}{(\sin A + \cos A)} = \sin A - \cos A$

(c) Let P (0, y) divide the join of (5, 3) and (–5, 3) in the ratio $m : n$

$x = \dfrac{mx_2 + nx_1}{m + n}$

$\Rightarrow 0 = \dfrac{m(-5) + n(5)}{m + n}$

$\Rightarrow -5m + 5n = 0 \Rightarrow 5m = 5n$

$\dfrac{m}{n} = 1$

$m : n = 1 : 1$

$\therefore y = \dfrac{3 + 3}{2} = 3$

$\therefore P(0, 3)$

Question 4

(a) A solid spherical ball of radius 6 cm is melted and recast into 64 identical spherical marbles. Find the radius of each marble. **[3]**

(b) Each of the letters of the word 'AUTHORIZES' is written on identical circular discs and put in a bag. They are well shuffled. If a disc is drawn at random from the bag, what is the probability that the letter is: **[3]**

    (i) a vowel

    (ii) one of the first 9 letters of the English alphabet which appears in the given word

    (iii) one of the last 9 letters of the English alphabet which appears in the given word?

(c) Mr. Bedi visits the market and buys the following articles: **[4]**

    Medicines costing ₹ 950, GST @ 5%

    A pair of shoes costing ₹ 3000, GST @ 18%

    A Laptop bag costing ₹ 1000 with a discount of 30%, GST @ 18%.

    (i) Calculate the total amount of GST paid.

    (ii) The total bill amount including GST paid by Mr. Bedi.

Comments of Examiners	Suggestions for Teachers

(a) A number of candidates used incorrect formulae to calculate volume of a sphere e.g. $4/3\ \pi r^2$ or $2/3\ \pi r^3$. Some candidates made calculation errors. Several candidates simplified and came to the result $r^3 = \{216\}/\{64\}$ but could not find the value of r by taking cube root. Some used a longer calculation method and calculated the radius as cube root of 3.375 but could not calculate further.
 	
(b) (i) Some candidates wrote the number of vowels as '4' instead of '5'. Most candidates did not write the answer in the simplest form.
 	
  (ii) A few candidates used incorrect form for finding probability (total outcomes / number of favourable outcomes). Most candidates did not write the answer in the simplest form.
 	
  (iii) Total outcome of event was incorrect, or favourable outcomes were incorrect.
	▪ *Give sufficient practice in problems based on volume and surface area of the three different solids, cylinder, cone, and sphere.*
	▪ *Drill on concepts of square root and cube root of numbers by factor method.*
	▪ *Give practice in calculation of combination of solids by taking pi and other terms common and then cancelling/simplifying to save time and to minimise calculation errors.*
	▪ *Instruct students to list all the outcomes as well as the outcomes favourable for the events.*
	▪ *Instruct students to read the questions carefully to find the number of favorable outcomes and the total number of outcomes. In probability,*

(c) The concept of finding GST as simple percentage was not clear to some candidates.

  (i) Several candidates missed out the discount of 30% on the laptop bag and directly calculated the tax. Hence, they got incorrect value of GST.

  (ii) Many candidates calculated the total amount of tax and total bill amount correctly, but the answer was not expressed correctly to two places of decimal, for example: Rs 713.50 and Rs 5363.50 were expressed as Rs 713.5 and Rs 5363.5.

    Some candidates made errors in calculation.

emphasis must be given to express the answers in the simplest form.

- *Train students to identify vowels and consonants in the given word, its analysis and procedure to find out the favorable outcome/s of the probability sum.*
- *Drill extensively on the concept of G.S.T and its calculation under given conditions.*
- *Instruct students to always express the bill amount in rupees correct to two decimal places.*
- *Give adequate practice to students in finding discounted price.*

MARKING SCHEME

Question 4

(a)	Volume of sphere = Volume of 64 marbles

$$\frac{4}{3} \times \pi \times r^3 = 64 \times \frac{4}{3} \times \pi \times R^3$$

$$\therefore \frac{4}{3}\pi 6^3 = 64 \times \frac{4}{3} \times \pi \times R^3$$

$$\therefore R^3 = \frac{3^3}{64}$$

$$\therefore R = \frac{6}{4} \ or \ R = \frac{3}{2} \ \text{ or } \ 1\,½ \ \text{or} 1.5$$

(b)	$\{A, U, T, H, O, R, I, Z, E, S\}$ 10 letters

	(i)	Vowels are $\{A, E, I, O, U\}$ 5 letters
		$\therefore$ prob. is $= \frac{5}{10} = \frac{1}{2}$

	(ii)	$\{A, E, H, I\}$
		$\therefore$ prob. is $= \frac{4}{10} = \frac{2}{5}$

	(iii)	$\{R, S, T, U, Z\}$

		$\therefore$ prob. is $= \dfrac{5}{10} = \dfrac{1}{2}$
(c)	(i)	Medicine: GST $= \dfrac{5}{100} \times 950 = ₹ 47.50$
	(ii)	Shoes: GST $= \dfrac{18}{100} \times 3000 = ₹ 540$
	(iii)	Laptop Bag: Discounted price $= 1000 - \dfrac{30}{100} \times 1000 = ₹ 700$ $\therefore$ GST $= \dfrac{18}{100} \times 700 = ₹ 126$ $\therefore$ Total GST $= ₹ 47.50 + ₹ 540 + ₹ 126 = ₹ 713.50$ $\therefore$ Total Bill $= 950 + 3000 + 700 + 713.50 = ₹ 5363.50$

SECTION B (40 Marks)

*Attempt **any four** questions from this Section*

Question 5

(a) A company with 500 shares of nominal value ₹ 120 declares an annual dividend of 15%. Calculate: **[3]**

    (i) the total amount of dividend paid by the company.

    (ii) annual income of Mr. Sharma who holds 80 shares of the company.
If the return percent of Mr. Sharma from his shares is 10%, find the market value of each share.

(b) The mean of the following data is 16. Calculate the value of f. **[3]**

Marks	5	10	15	20	25
No. of Students	3	7	f	9	6

(c) The 4th, 6th and the last term of a geometric progression are 10, 40 and 640 respectively. If the common ratio is positive, find the first term, common ratio and the number of terms of the series. **[4]**

Comments of Examiners	**Suggestions for Teachers**
(a) Common errors made by many candidates were in finding annual income and the market value of each share.	▪ *Lay stress on problems based on commercial mathematics.*
	▪ *Discuss terms like NV, MV, dividend, etc.*
(i) Some candidates found the dividend on only one share instead of finding the total amount of dividend paid by the company.	▪ *Give adequate practice on sums based on Shares and Dividend, for conceptual clarity.*
(ii) Several candidates made mistakes in finding the dividend on 80 shares of Mr. Sharma. The concept of *return percent* being not very clear. Many candidates could not find the market value of each share of the company. A few candidates used incorrect formulae to find market value of each share. Some made errors in calculation.	▪ *Explain the concept of return percent / yield percent thoroughly and give sufficient drill on sums based on return percent.*
	▪ *Ask students not to change the data from the given form to some other form. Familiarise them with all types of distribution.*
(b) The question was on calculation of mean of ungrouped frequency distribution. Some candidates converted it to grouped frequency distribution which did not tally with the given data and hence went wrong with the sum.	▪ *Give students repeated practice to prevent them from making the basic conceptual errors.*
	▪ *Instruct students to read the question carefully and analyse the given conditions before solving the problem.*
Many candidates wrote incorrect Class mark of the given distribution which led to incorrect value of $\sum f x$. A few candidates wrote $\sum f$ as 25f instead of 25+f and $\sum fx$ as 430f instead of 415 +15f. Several candidates made errors in multiplying $(25 + f)$ by 16 and hence got the incorrect answers. Some made mistakes in applying the formula for mean and solving the equation to find 'f'.	▪ *Clarify the concepts of common difference and common ratio thoroughly.*
	▪ *Give repeated practice in finding the nth term and sum of n terms of both A.P., and G.P.*
(c) Simple calculation errors were observed. Some candidates took the terms as of A. P. instead of G. P. The values of 'a' and 'r' were correctly found by many candidates, but they could not find the value of 'n' correctly.	

MARKING SCHEME

Question 5

(a)	(i)	Total amount of dividend paid by the company $= 5000 \times 120 \times \dfrac{15}{100} = ₹\,90,000$
	(ii)	Annual income of Mr. Sharma having 80 shares $= 80 \times 120 \times \dfrac{15}{100} = ₹\,1,440$

Market value of 1 share × return % = Nominal value of 1 share × Dividend %

$$x \times \dfrac{10}{100} = 120 \times \dfrac{15}{100} \quad \therefore x = ₹\,180$$

Market value of 1 share = ₹ 180

(b)

Marks	No. of students	fx
5	3	15
10	7	70
15	f	$15\,f$
20	9	180
25	6	150

$fx = 415 + 15f$

$\Sigma f = f + 25$

$M = \dfrac{\Sigma fx}{\Sigma f} \quad \therefore 16 = \dfrac{15f + 415}{f + 25}$

$\Rightarrow f = 15$

(c)

$T_4 = ar^3 = 10 \qquad T_6 = ar^5 = 40 \qquad T_n = ar^{n-1} = 640$

$\dfrac{ar^5}{ar^3} = \dfrac{40}{10}$

$r^2 = 4 \qquad r = 2 \qquad\qquad$ (only positive value of r)

$\therefore a \times 8 = 10 \qquad a = \dfrac{10}{8} = \dfrac{5}{4}$

$\dfrac{5}{4} \times 2^{n-1} = 640$

$2^{n-1} = \dfrac{640 \times 4}{5} \qquad\qquad 2^{n-1} = 2^9$
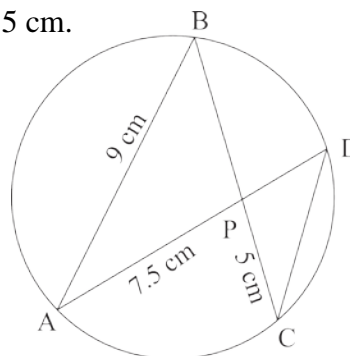
$\therefore n = 10$

Question 6

(a) If $A = \begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix}$ **[3]**

Find $A^2 - 2AB + B^2$

(b) In the given figure AB = 9 cm, PA = 7.5 cm and PC = 5 cm.   **[3]**

Chords AD and BC intersect at P.

    (i)   Prove that ΔPAB ∼ ΔPCD

    (ii)   Find the length of CD.

    (iii) Find area of ΔPAB: area of ΔPCD

(c) From the top of a cliff, the angle of depression of the top and bottom of a tower are   **[4]**
observed to be 45° and 60° respectively. If the height of the tower is 20 m.

Find:

    (i)   the height of the cliff

    (ii) the distance between the cliff and the tower.

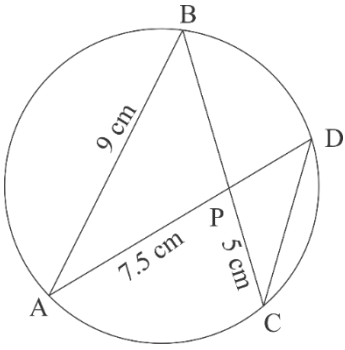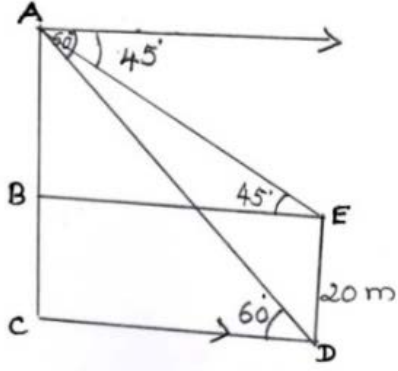| Comments of Examiners	Suggestions for Teachers
(a) Matrix multiplication was a very common area of error, *e.g.* for finding square of 'A' and square of 'B', candidates found the square of each corresponding element instead of finding A x A and B x B. Many candidates found the product A x B by multiplying the corresponding elements of each Matrix. Some candidates made mistakes in simplifying the expression $A^2 - 2AB + B^2$. (b) (i) Many candidates assumed that AB is parallel to CD, hence they could not identify the correct pairs of equal angles to prove Δ PAB ~ Δ PCD (ii) Some candidates calculated the length of CD correctly. A few could not write the corresponding ratio of sides of similar triangles and hence, made mistakes in finding the length of CD.	▪ *Emphasise on showing each and every step of product of matrices. Stress upon finding square of a matrix.* ▪ *Give repeated practice on basic operations with matrix addition, subtraction, and multiplication of matrices. Advise students to be careful while adding two or more terms with +/- sign.* ▪ *Sufficient practice on 'similarity' and getting the proportionality of corresponding sides, etc. should be given regularly.* ▪ *Give practice on properties of circle with emphasis on writing correct reasons for the working.*

<table>
<tr><td>(iii)</td><td>Many candidates did not get the ratio of area of ΔPAB: area of ΔPCD correctly. Some candidates wrote the final answer in the fractional form 9/4 instead of writing it in the ratio form 9:4 as asked in the question.</td><td rowspan="3">

Advise students to express answers to sums in the simplest form.
Give practice on sums based on proving similarity of triangles.
Instruct students to express the ratio of the area of the triangles correctly. Also reminded them not to leave the answer in the fractional form.
Train students to draw diagrams for all problems in geometry and trigonometry. In problems of heights and distances, it is necessary to draw diagrams.
Give adequate practice to students on the values of standard angles of trigonometric ratios used in related sums.
Familiarise students with angle of elevation, and angle of depression and give frequent practice.
Advise students to rationalize the denominator, wherever it is possible, to avoid calculation errors.
Teach students easier methods of simplifying the problem to avoid long complicated calculations.

</td></tr>
<tr><td>(c) (i)</td><td>Some candidates did not draw a diagram for this question or drew an incorrect diagram. Some used value of tan 60 as 1.732 instead of √3 and made mistakes in dividing 20 by 0. 732.</td></tr>
<tr><td>(ii)</td><td>Candidates made calculation errors and did not arrive at the final answers as 47.32 and 27.32.</td></tr>
</table>

MARKING SCHEME

Question 6

(a)

$$A^2 - 2AB + B^2 = \begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix} - 2\begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix}\begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 0 \\ 15+5 & 1 \end{bmatrix} - 2\begin{bmatrix} -12 & 6 \\ -20+1 & 10 \end{bmatrix} + \begin{bmatrix} 16+2 & -8 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 0 \\ 20 & 1 \end{bmatrix} - \begin{bmatrix} -24 & 12 \\ -38 & 20 \end{bmatrix} + \begin{bmatrix} 18 & -8 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 0 \\ 20 & 1 \end{bmatrix} + \begin{bmatrix} 24 & -12 \\ 38 & -20 \end{bmatrix} + \begin{bmatrix} 18 & -8 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 33 & -12 \\ 58 & -19 \end{bmatrix} + \begin{bmatrix} 18 & -8 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 51 & -20 \\ 54 & -17 \end{bmatrix}$$

(b) (i)

In ΔPAB and ΔPCD

∠B = ∠D (angles in the same segment are equal)

|		
		∠APB = ∠CPD (vertically opposite angles) ∠PAB = ∠PCD (angles in the same segment are equal) \△PAB $\sim$ △PCD (AAA)
	(ii)	Since the 2 triangles are ~ $\therefore \dfrac{AB}{CD} = \dfrac{PA}{PC} = \dfrac{PB}{PD}$ $\therefore \dfrac{9}{CD} = \dfrac{7.5}{5} = \dfrac{PB}{2.5}$ $\therefore CD = \dfrac{9 \times 5}{7.5}$ $CD = 6\ cm$
	(iii)	$\triangle PAB : \triangle PCD = (7.5)^2 : 5^2$ $= 1.5^2 : 1^2$ $= 9 : 4$ (CAO)
(c)		$\tan\ 45° = \dfrac{AB}{BE} \qquad 1 = \dfrac{AB}{BE}$ AB = BE $\tan 60° = \dfrac{AC}{CD}$ $\sqrt{3} = \dfrac{AB+20}{BE}$ $\sqrt{3}\ BE = AB + 20$ $AB\sqrt{3} = AB + 20$ $AB(\sqrt{3} - 1) = 20$ $AB = \dfrac{20\ (\sqrt{3}+1)}{2}$ AB = 27.32
	(i)	Height of cliff = 27.32 + 20 = 47.32 m
	(ii)	Distance between cliff and tower = 27.32 m OR 47.3 m and 27.3 m

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Question 7

(a) Find the value of 'p' if the lines, $5x - 3y + 2 = 0$ and $6x - py + 7 = 0$ are   **[3]**
perpendicular to each other. Hence find the equation of a line passing through
($-2, -1$) and parallel to $6x - py + 7 = 0$.

(b) Using properties of proportion find $x : y$, given: **[3]**

$$\frac{x^2 + 2x}{2x + 4} = \frac{y^2 + 3y}{3y + 9}$$

(c) In the given figure TP and TQ are two tangents to the circle with centre O, touching   **[4]**
at A and C respectively. If $\angle BCQ = 55^\circ$ and $\angle BAP = 60^\circ$, find:

  (i) $\angle OBA$ and $\angle OBC$

  (ii) $\angle AOC$

  (iii) $\angle ATC$
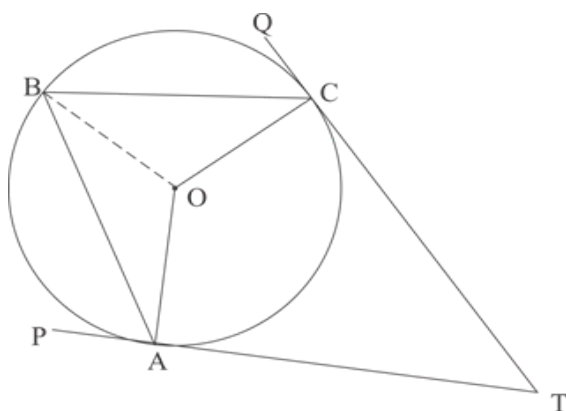
Comments of Examiners	Suggestions for Teachers
(a) Many candidates found the slopes of the two lines as $\frac{3}{5}$ or $-\frac{5}{3}$ for one line and $\frac{p}{6}$ *or* $-\frac{6}{p}$ for the other line. Some candidates found the slopes correctly but made mistakes in applying the condition that for two lines to be perpendicular the product of slopes is -1. Many candidates got incorrect equations of the line parallel to $6\,x - p\,y + 7 = 0$. Some candidates	▪ *Teach the concept of slopes for parallel and perpendicular lines thoroughly and give practice regularly.* ▪ *Train students to find the slope of a line by writing the given equation in the form $y = mx + c$.* ▪ *Advise students to solve the sums based on Ratio and Proportion using properties of proportion*

found the equation correctly but did not give the answer in the simplest form.

(b) Some common errors observed in candidates answers to the question were:

- not reading the instructions given in the question carefully.
- not using componendo and dividendo to work out the sum.
- cross multiplying to solve the sum.
- making errors in signs while applying componendo and dividendo.
- writing $(x^2 - 4)$ as $(x - 2)^2$
- writing the ratio as 2/3 instead of 2:3 as asked for in the question.

(c) Properties of circles like, angle at the centre is double the angle at the circumference, radius tangent perpendicular relation, angle sum property of quadrilateral, tangent secant relation, etc. were not applied correctly.

(i) Many candidates did not identify ∠OBA =30°or ∠OBC = 35° Some used the property of alternate segment theorem and wrote ∠OBA= 60° & ∠OBC = 55°.

(ii) A large number of candidates could not find the value of ∠AOC.

(iii) Some candidates incorrectly calculated the value of ∠OBA. A few candidates solved the question without giving proper reasoning. In some scripts, simple calculation errors were observed.

like, Componendo and Dividendo. to minimise the calculation work.

- *Instruct students to read the question carefully and to answer as per the specifications given in the question.*
- *Insist upon answering the geometry-based problems with logical reasons.*
- *Exhaustive drill of properties of circle theorems and application-based sums is a must.*
- *Teach students to name the angles with three letters, specifically, when there are two or more angles at the same point.*
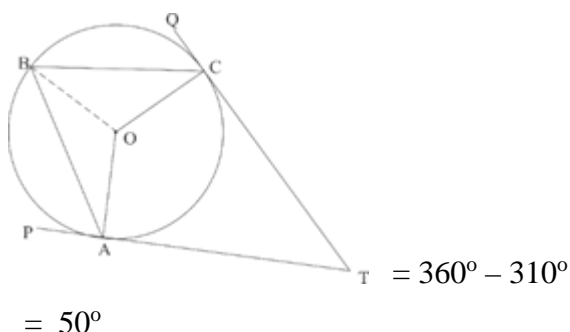- *Teach theorems and their applications in detail and revise them frequently.*

MARKING SCHEME
Question 7

(a) $5x - 3y + 2 = 0$ $\therefore$ slope $= \dfrac{5}{3}$

     $6x - py + 7 = 0$ $\therefore$ slope $= \dfrac{6}{p}$

	∵ they are right angles ∴ $\dfrac{5}{3} \times \dfrac{6}{p} = -1$ i.e. $p = -10$ $6x - py + 7 = 0$ $6x + 10y + 7 = 0,$ slope $= \dfrac{-6}{10} = \dfrac{-3}{5}$ Equation of line with slope $\dfrac{-3}{5}$ and passing through (–2, –1) is $y - (-1) = \dfrac{-3}{5}(x + 2)$ $5y + 5 = -3x - 6$ $3x + 5y + 11 = 0$	
(b)	$\dfrac{x^2 + 2x + 2x + 4}{x^2 + 2x - 2x - 4} = \dfrac{y^2 + 3y + 3y + 9}{y^2 + 3y - 3y - 9}$ (using componendo and dividendo) $\dfrac{x^2 + 4x + 4}{x^2 - 4} = \dfrac{y^2 + 6y + 9}{y^2 - 9}$ $\dfrac{(x+2)^2}{(x+2)\,(x-2)} = \dfrac{(y+3)^2}{(y+3)\,(y-3)}$ $\dfrac{(x+2)}{(x-2)} = \dfrac{(y+3)}{(y-3)}$ $\dfrac{x+2+x-2}{x+2-x+2} = \dfrac{y+3+y-3}{y+3-y+3}$ $\dfrac{2x}{4} = \dfrac{2y}{6}$ $x : y = 2 : 3$	
(c)	(i)	∠BCQ = 55º \ ∠OCB = 90º – 55º = 35º (radius & tangent makes ∠ = 90º) \ ∠OBC = ∠OCB = 35º (** OB = OC) ∠OBA = ∠OAB = 90º – 60º = 30º (** OA = OB & ∠OAP = 90º) ∠OBA = 30º
	(ii)	∠ABC = ∠OBA + ∠OCB = 30º + 35º = 65º \ ∠AOC = 2∠ABC = 2 x 65º = 130º (Angle at the centre is double angle at the ∠AOC = 130º remaining circumference)
	(iii)	∠ATC = 360º – (90 + 90 + ∠AOC) (**angles of quadrilateral adds upto 180º) ∠ATC = 360º – (180º + 130º)

$$= 360° - 310°$$

$$= 50°$$

Question 8

(a) What must be added to the polynomial $2x^3 - 3x^2 - 8x$, so that it leaves a remainder 10 when divided by $2x + 1$? **[3]**

(b) Mr. Sonu has a recurring deposit account and deposits ₹ 750 per month for 2 years. If he gets ₹ 19125 at the time of maturity, find the rate of interest. **[3]**

(c) Use graph paper for this question. **[4]**

Take 1 cm = 1 unit on both x and y axes.

   (i) Plot the following points on your graph sheets:

   A (–4, 0), B (–3, 2), C (0, 4), D (4, 1) and E (7, 3)

   (ii) Reflect the points B, C, D and E on the x-axis and name them as B', C', D' and E' respectively.

   (iii) Join the points A, B, C, D, E, E', D', C', B' and A in order.

   (iv) Name the closed figure formed.

Comments of Examiners	Suggestions for Teachers
(a) A very common error made by candidates in this question was the incorrect substitution of the value of x, which was substituted as ½ instead of – ½. Another common conceptual error made by candidates was of writing $(-1/2)^3$ as equal to 1/8 instead of – 1/8 and $(1/2)^2$ as -1/4 instead of ¼. This error was mostly caused due to not using the bracket. Some candidates substituted correctly but were unable to come to the correct answer k = 7 (b) Simple calculation errors were found, such as, the product of 750 and 24 being expressed as 1800, instead of 18000.	▪ *Give enough practice in problem solving involving algebraic expressions with the use of remainder/factor theorem.* ▪ *Tell students that the term to be added or subtracted must be taken as any constant 'k' or 'a', etc. Enough problems involving such cases must be done.* ▪ *Explain the changes taking place if the brackets are not used in the correct place.*

Some candidates took 'n' as 2 years instead of 24 months.
Mistakes were made in applying the formula for interest. Candidates found it difficult to express interest in terms of 'r'. Hence, many candidates were unable to find rate of interest, r.

(c) Most candidates dealt well with this question. Common errors were observed in taking the scale and plotting the points.
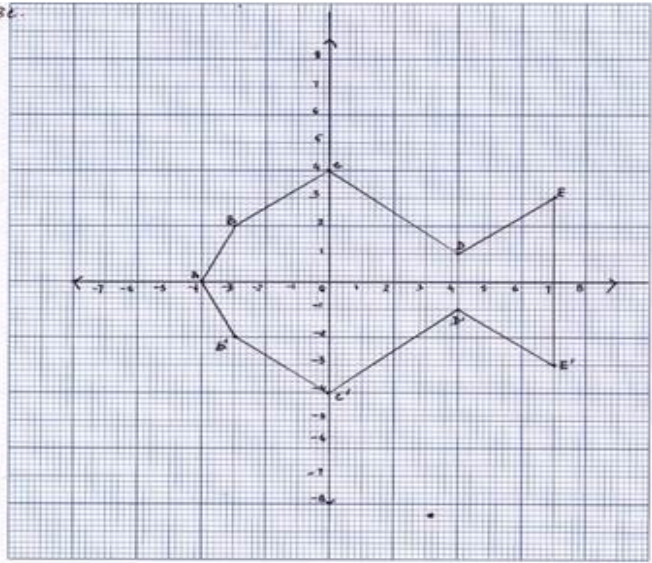
   (i)   A few candidates plotted the points incorrectly. Most mistakes were made in plotting the points (0,4) and (-4, 0).

   (ii)   Some candidates made mistakes in marking the X-axis and the Y-axis. Positive and negative points of the two axes were incorrectly marked.

   (iii)   Some candidates marked the points B', C', D' and E' incorrectly or did not join the points in order.

   (iv)   A few candidates did not name the figure or named it as 'octagon' or 'trapezium', etc.

- *Advise students to read a recurring deposit sum carefully, analyse it and note down what is given and what is required to be found.*
- *Drill students with basic concepts such as, monthly installment, qualifying principal, to find interest, time in months and maturity value.*
- *Instruct students to read the question asking to plot the graph carefully. Ask them to use the scale given in the question.*
- *Advise students to practice plotting numerous points on the graph paper and their reflection on the x-axis, y-axis, origin. Give adequate practice to identify points on the x and y axis. Instruct students to complete the figure formed. Revise the names of basic geometrical figures.*
- *Advise students to read the question carefully and answer all parts.*

MARKING SCHEME

Question 8

(a) $f(x) = 2x^3 - 3x^2 - 8x$

Let k be added to $f(x)$

$\therefore f(x) = 2x^3 - 3x^2 - 8x + k$

$Remainder = f\left(-\dfrac{1}{2}\right) = 10$

$\Rightarrow 2\left(-\dfrac{1}{2}\right)^3 - 3\left(-\dfrac{1}{2}\right)^2 - 8\left(-\dfrac{1}{2}\right) + k = 10$ Putting $x = -\dfrac{1}{2}$

$\Rightarrow -\dfrac{1}{4} - \dfrac{3}{4} + 4 + k = 10$ Equating to 10

$\Rightarrow k + 3 = 10 \Rightarrow k = 7$ *(or any other correct approach)*

(b) $I = \dfrac{750 \times 24 \times 25 \times r \times 1}{2 \times 100 \times 12}$

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<table>
<tr><td rowspan="3">(c)</td><td rowspan="3">(i)</td><td>
$$= \frac{375 \ r}{2}$$

Amount deposit = 750 × 24 = ₹ 18000
Maturity value = ₹ 19125
Interest = ₹ 19125 - 18000 = ₹ 1125

$$1125 \ = \frac{375 \ r}{2}$$

r = 6 %
</td></tr>
</table>

<table>
<tr><td>(c)</td><td>(i)</td><td></td><td>Plotted the points A,B,C,D and E correctly.</td></tr>
<tr><td></td><td>(ii)</td><td colspan="2">Reflected the points B, C, D and E on the x-axis and named them as B', C', D' and E' correctly.</td></tr>
<tr><td></td><td>(iii)</td><td colspan="2">Joined the points A, B, C, D, E, E', D', C', B' and A in order and completed the figure.</td></tr>
<tr><td></td><td>(iv)</td><td colspan="2">Nine-sided polygon or nonagon, polygon fish or kite.</td></tr>
</table>

Question 9

(a) 40 students enter for a game of shot-put competition. The distance thrown (in metres) is **[6]** recorded below:

Distance in m	12 – 13	13 – 14	14 – 15	15 – 16	16 – 17	17 – 18	18 – 19
Number of Students	3	9	12	9	4	2	1

Use a graph paper to draw an ogive for the above distribution.

Use a scale of 2 cm = 1 m on one axis and 2 cm = 5 students on the other axis.

Hence using your graph find:

(i)   the median

(ii) Upper Quartile

(iii) Number of students who cover a distance which is above 16½ m.

(b)   If $x = \dfrac{\sqrt{2a + 1} + \sqrt{2a - 1}}{\sqrt{2a + 1} - \sqrt{2a - 1}}$, prove that $x^2 - 4ax + 1 = 0$ **[4]**

Comments of Examiners	**Suggestions for Teachers**
(a) The following errors were observed in this question: – Some candidates made mistakes in finding the cumulative frequency. In many cases, the last cumulative frequency did not tally with the total of the given distribution which was given as 40. – A number of candidates did not follow the scale given in the question. – Some plotted the ogive with respect to the lower boundaries instead of upper boundaries. – A few candidates used a ruler to draw the graph instead of a freehand curve. – Kink was not shown on the graph sheet between 0 and 12 to keep uniform gaps between each interval, by some candidates. – In a few cases, perpendicular lines were not dropped to find the values from the ogive. – The values of median, upper quartiles were read incorrectly from the graph, by a number of candidates. (b) Many candidates made errors while applying Componendo and Dividendo, especially in the denominator. Mistakes were made while squaring both the sides. Calculation errors were also common. The final expression was not worked out following all the steps of correct working. Being a proof, some candidates wrote the correct answer from incorrect working.	▪ *Teach students to cross-check the cumulative frequency found.* ▪ *Advise students to be cautious in selecting correct axis and scale.* ▪ *Instruct students to mark the kink when it is required. Ask students to draw Ogive, a cumulative frequency curve as a free hand curve. The points must not be joined with a ruler.* ▪ *Practice needs to be given of drawing the ogive. Instruct students that the graph needs to be plotted with respect to upper boundaries and corresponding cumulative frequency.* ▪ *Sufficient practice is required of reading values from the graph.* ▪ *Practice in problems on properties of ratio and proportion, especially Componendo and Dividendo and squaring / expansion / transformation of algebraic expression must be given.* ▪ *Emphasise the importance of brackets in algebraic expressions.*

Question 9

(a)

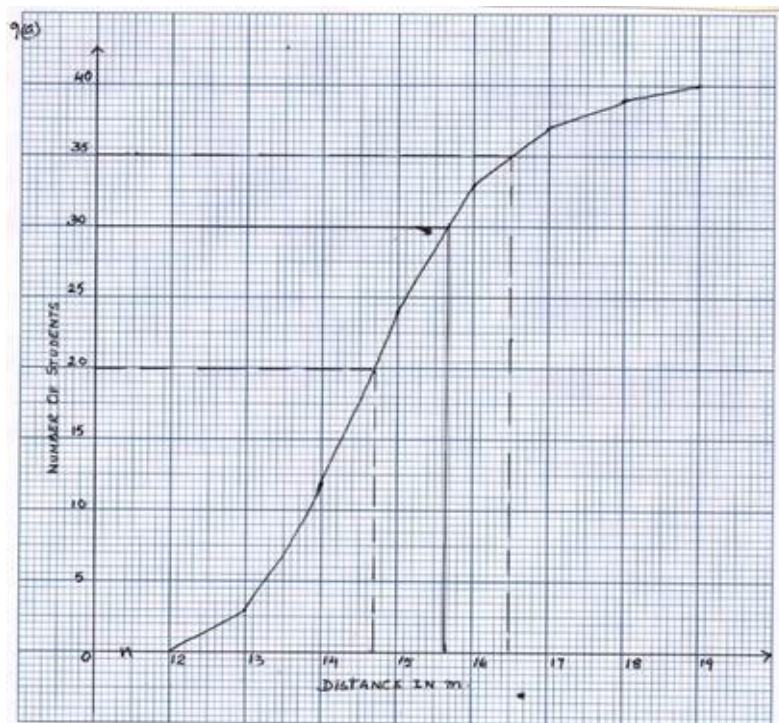| Distance in m	Number of students	Cumulative frequency(c.f.)
12 – 13	3	3
13 – 14	9	12
14 – 15	12	24
15 – 16	9	33
16 – 17	4	37
17 - 18	2	39
18 – 19	1	40

(i) Median = 14.7 m ± 0.2

(ii) Upper Quartile = 15.65 m ± 0.2

(iii) 40 – 35 = 5 students cover a distance above 16½ m.
Ogive: Scale 2 cm = 1m along x-axis and
2 cm = 5 students along y-axis

(b) Using Componendo / Dividendo

$$\frac{\sqrt{2a+1}+\sqrt{2a-1}+\sqrt{2a+1}-\sqrt{2a-1}}{\sqrt{2a+1}+\sqrt{2a-1}-\sqrt{2a+1}+\sqrt{2a-1}}=\frac{x+1}{x-1}$$

$$\frac{2\sqrt{2a+1}}{2\sqrt{2a-1}}=\frac{x+1}{x-1} \text{ on squaring both sides}$$

$$\frac{\left(\sqrt{2a+1}\right)^2}{\left(\sqrt{2a+1}\right)^2}=\frac{(x+1)^2}{(x-1)^2}\Rightarrow\frac{2a+1}{2a-1}=\frac{x^2+2x+1}{x^2-2x+1}$$

$$(2a+1)(x^2-2x+1)=(2a-1)(x^2+2x+1)$$

$$2ax^2-4ax+2a+x^2-2x+1=2ax^2+4ax+2a-x^2-2x-1$$

$$4ax+4ax-x^2-x^2-1-1=0$$

$$8ax-2x^2-2=0 \Rightarrow 4ax-x^2-1=0$$

$$x^2-4ax-1=0 \rightarrow \text{proved}$$

Question 10

(a) If the 6th term of an A.P. is equal to four times its first term and the sum of first six terms is 75, find the first term and the common difference. **[3]**

(b) The difference of two natural numbers is 7 and their product is 450. **[3]**

Find the numbers.

(c) Use ruler and compass for this question. Construct a circle of radius 4.5 cm. **[4]**

Draw a chord. AB = 6 cm.

   (i) Find the locus of points equidistant from A and B.

      Mark the point where it meets the circle as D.

   (ii) Join AD and find the locus of points which are equidistant from AD and AB. Mark the point where it meets the circle as C.

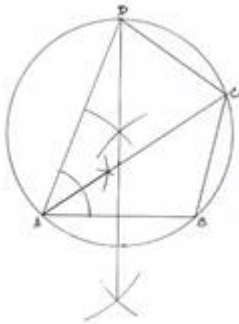   (iii) Join BC and CD. Measure and write down the length of side CD of the quadrilateral ABCD.

Comments of Examiners	Suggestions for Teachers
(a) Some candidates used incorrect formula for finding the 6th term of an arithmetic progression and summation of six terms of an A.P. As per the given condition in the question i.e., 6th term of an A.P.is equal to four times its first term was written as: a + 5d = 4a + d instead of a + 5d = 4a. Some candidates wrote the equations correctly but made mistakes in solving the two equations: a + 5d = 4a and 3(2 a + 5 d) = 75.	▪ Teach the concepts of the two series: Arithmetic Progression (AP) and Geometric Progression (GP) and their differences with different examples. Revise formulae for finding a term, common difference and summation of certain number of terms in Arithmetic Progression frequently.
(b) Some candidates made mistake in taking the two natural numbers according to given condition as x and 7 – x instead of x and x – 7 or x and x + 7.	▪ Ensure that adequate practice of different kinds of word problems is given in class.
Few candidates made mistakes in forming the quadratic equation. Some candidates after solving found the two values of x as - 25 and 18.	▪ Instruct students to read construction-based questions with the conditions, carefully.
According to the question the two numbers are natural numbers. Hence neglecting negative number, the two numbers should have been 25 and 18.	▪ Adequate practice in identifying the locus of points under different geometrical conditions should be given to students.
(c) All the necessary arcs for side bisectors and angle bisectors were not constructed. Concepts of locus theorems were not clear to some candidates.	▪ Instruct candidates to show all traces of construction clearly while working with Geometry Constructions.
(i) Some candidates constructed the circle with radius = 6 cm. Bisector of AB was drawn but point D was not located.	▪ Clarify the concept of loci theorem in detail.
(ii) All the necessary arcs for bisector angle BAD were not shown. Point 'C' was not located on the circumference of the circle.	▪ Give enough practice in drawing perpendicular bisectors of straight lines.
(iii) Many candidates forgot to measure and record the length of CD and wrote = 5cm	

<div align="center">

MARKING SCHEME

</div>

Question 10

(a) $t_6 = a + 5d = 4a \Rightarrow 3a = 5d \dots\dots 1$

$s_6 = \dfrac{6}{2}\{2a + 5d\} = 75 \Rightarrow 2a + 5d = 25 \ \dots\dots 2$

From 1 and 2

$2a + 3a = 25$

	$\Rightarrow a = 5$ $d = 3$	
(b)	Let the numbers be x and $(x + 7)$ $x(x + 7) = 450$ $x^2 + 7x - 450 = 0$ $(x + 25)(x - 18) = 0$ $x = -25$ or 18 The numbers are 18 and 25.	*(Any other correct method)*
(c)	Circle and chord 	

	(i)	Bisector of AB and point D (any side)
	(ii)	Bisector of ∠BAD and point C (any side).
	(iii)	Length of CD = 5 cm (±.3) OR CD = 1.4 (±0.3). If C and D are points below AB.

Question 11

(a) A model of a high rise building is made to a scale of 1 : 50.   **[3]**

   (i)   If the height of the model is 0.8 m, find the height of the actual building.

   (ii) If the floor area of a flat in the building is 20 m², find the floor area of that in the model.

(b) From a solid wooden cylinder of height 28 cm and diameter 6 cm, two conical cavities   **[3]**
are hollowed out. The diameters of the cones are also of 6 cm and height 10.5 cm.

   Taking $\pi = \frac{22}{7}$ find the volume of the remaining solid.

10.5 cm

28 cm

10.5 cm

6 cm
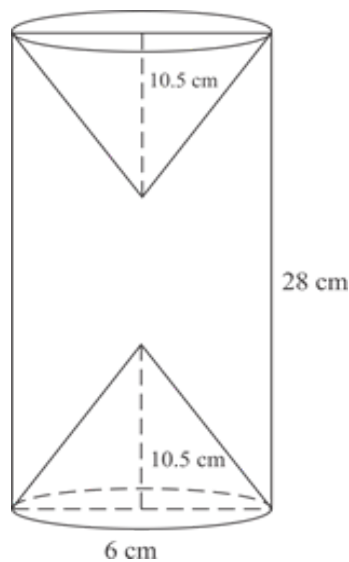
(c) Prove the identity   **[4]**

$$\left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2 = \tan^2\theta$$

Comments of Examiners	**Suggestions for Teachers**
(a) (i) Many candidates made mistakes in applying the scale factor to find the height of the actual building. (ii) Some candidates made mistakes in applying the scale factor to find the ratio of area as square of the scale factor. Several candidates made errors in calculation and conversion of units. Some left the answer of the second part in fractional form as (1/ 125). (b) Some candidates used incorrect formulae for volume of cone and cylinder. The diameter of the cone and cylinder was given in the sum as 6 cm, but some took the radius as '6'. The volume of cone was not multiplied by '2'. Simple calculation errors were very common. The value for π was given as $\dfrac{22}{7}$ but some took it as 3.14. (c) Some candidates wrote the given trigonometric expression as 1- $\tan^2\theta$/ 1- $\cot^2\theta$.	▪ *Clarify the concept of scale factor. Explain the concept of area being proportional to square of sides and volume proportional to cube of sides.* ▪ *Focus on conversions from one system of units to another, like cm to m, m to km, mm to m, cm^2 to m^2, etc.* ▪ *Instruct students not to leave the final answer in a fractional form.* ▪ *Teach students that if each side of the image is k times each side of object then Area of image = k^2 times area of object.* ▪ *Give regular practice with emphasis on manual calculations.* ▪ *Instruct students to use the value given in the question.* ▪ *Give sufficient practice in multiplication and division involving decimals.*

Some candidates failed to express tan A and cot A in terms of sin A and cos A correctly. Others made errors in writing the numerator and denominator of resulting trigonometric expressions correctly. A few candidates were unable to identify
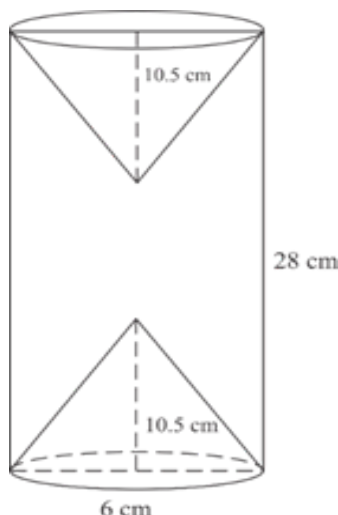
$(\cos\theta - \sin\theta) = - (\sin\theta - \cos\theta)$, hence, could not obtain the correct answer.

- *Train students to calculate volume of a combination of solids by taking π and other like terms common and then simplifying to save time and to avoid calculation errors.*
- *Give ample practice on basic algebraic operations and identities to enable students to simplify trigonometric identities.*
- *Advise students to prove identities with one side at a time instead of working with both sides together.*

MARKING SCHEME

Question 11

(a)	(i)	$\dfrac{1}{50} = \dfrac{0.8}{h}$ $\therefore$ height of the building $= 0.8 \times 50 = 40$ m
	(ii)	$\dfrac{1^2}{50^2} = \dfrac{x}{20}$ $\therefore x = \dfrac{20}{50^2}$ 0.008 m² / 80 cm² = Floor area of the model
(b)		Volume of Remaining Solid = Volume of Cylinder – 2 × Volume of Cone $= \pi r^2 - 2 \times \dfrac{1}{3} \times \pi r^2 \times h$ $= \dfrac{22}{7} \times 3^2 \times 28 - 2 \times \dfrac{1}{3} \times \dfrac{22}{7} \times 3^2 \times 10.5$ $= \dfrac{22}{7} \times 3^2 \left[28 - \dfrac{2}{3} \times 10.5\right]$ $= \dfrac{22}{7} \times 9 \times [28 - 7]$

	$= \dfrac{22}{7} \times 9 \times 21$ $= 66 \times 9$ $= 594 \text{ cm}^3$
(c)	$\left(\dfrac{1 - \tan\theta}{1 - \cot\theta}\right)^2 = \tan^2\theta$ L.H.S. $\left(\dfrac{1 - \tan\theta}{1 - \cot\theta}\right)^2 = \dfrac{\left(1 - \dfrac{\sin\theta}{\cos\theta}\right)^2}{\left(1 - \dfrac{\cos\theta}{\sin\theta}\right)^2}$ $= \left(\dfrac{\dfrac{\cos\theta - \sin\theta}{\cos\theta}}{\dfrac{\sin\theta - \cos\theta}{\sin\theta}}\right)^2$ $= \left(\dfrac{\cos\theta - \sin\theta}{\cos\theta} \times \dfrac{\sin\theta}{-(\cos\theta - \sin\theta)}\right)^2$ $= \left(-\dfrac{\sin\theta}{\cos\theta}\right)^2 = (-\tan\theta)^2 = \tan^2\theta = \text{R.H.S.}$ **Alternative Method:** $LHS = \left(\dfrac{1-\tan\theta}{1-\dfrac{1}{\tan\theta}}\right)^2$ $= \left(\dfrac{1-\tan\theta}{\tan\theta-1}\right)^2 \times tan^2\theta$ $= \left(-\dfrac{1-\tan\theta}{1-\tan\theta}\right)^2 tan^2\theta$ $= tan^2\theta = RHS$

NOTE: For questions having more than one correct answer/solution, alternate correct answers / solutions, apart from those given in the marking scheme, have also been accepted.

GENERAL COMMENTS

Topics found difficult/ confusing by candidates

- Linear inequation
- Matrix multiplication
- Graphs: axis and reading the results from the graph.
- Mensuration formulae of volumes of solids
- Circle Theorems and their application in solving problems
- Rounding off numbers to the desired place
- Coordinate Geometry: applying Section formula
- Size Transformation
- Arithmetic Progression and Geometric Progression
- Shares and Dividend
- Geometrical Constructions and locus
- Trigonometrical Identities and Heights and Distances
- Properties of Ratio and Proportion
- Rate of interest in Banking
- Remainder and Factor Theorem
- Forming correct equation in Quadratic word problem
- Inequation solving and representing solution on number line
- Statistics- Calculation of Central tendency of non-grouped frequency distribution, drawing an ogive
- Solving Geometry problems on circles and similar triangles using the appropriate property

Suggestions for Students

- Read all questions carefully.
- Utilise reading time judiciously to make the right choice of questions. Show all steps of working. All rough work should be done alongside the solution, on the answer script only. Give specific and logical reasons in geometry. Round off the answers correct to the required number **of** decimal places. Convert units from one system of unit to another system of unit carefully.
- Use logarithm table book to find square root or to cross check the value of a square root or other powers/roots.
- Plotting on graphs and selection and use of a suitable scale should be done carefully.
- Follow the given instructions carefully, especially in reflection sums and construction-based problems.
- Revise all calculations to avoid calculation errors. Practice simple methods of calculation.
- Practice number line with due importance to arrows at both the ends.
- Join all the points in the given order, especially in problems of reflection.
- While finding cumulative frequency, verify that the last cumulative frequency tallies with the sum of all frequencies.
- Plot Ogive with respect to upper boundaries and corresponding cumulative frequency.
- Practice basic constructions, such as, construction of standard angles, side bisectors and angle bisectors regularly.