Random Variables

An introduction

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Random Variables

- a numerical description of the outcome of a statistical experiment [1]
- often denoted by capital Roman letters such as **X, Y, Z**

Random Variables

- a numerical description of the outcome of a statistical experiment [1]
- often denoted by capital Roman letters such as **X, Y, Z**
- let **Y** denote the number of heads from tossing two coins
  - **Y** can take the values 0, 1, or 2
  - this means we can have no head, one head, or two heads

Definition

Random Variable

A random variable **X** is a measurable function $X : \Omega \to E$ from a sample space Ω as a set of possible outcomes to a measurable space **E**.

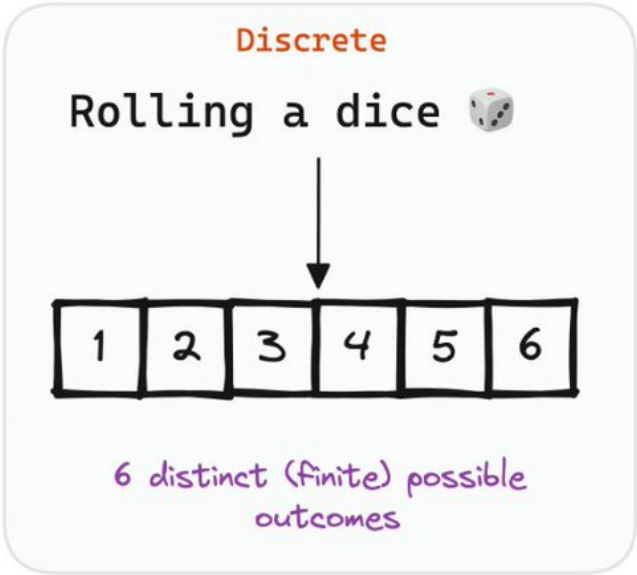
The probability that X takes on a value in a measurable set $S \subseteq E$:

$$P(X \in S) = P(\omega \in \Omega \mid X(\omega) \in S)$$

Types of random variables: Discrete

Discrete random variables

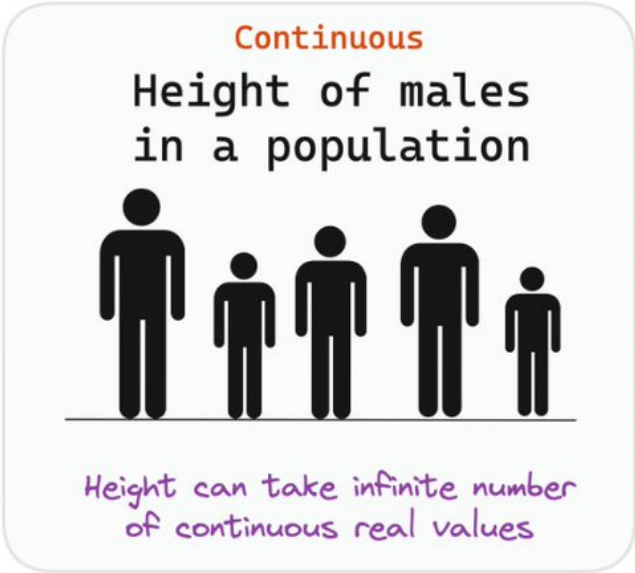
- takes only a countable number of distinct values
- usually (not necessarily) counts
- examples: the Friday night attendance at a cinema, the number of defective light bulbs in a box of ten



Types of random variables: Continuous

Continuous random variables

- takes an infinite number of possible values
- usually measurements
- defined over intervals
- probability of observing any single value equals zero
- examples: height, weight, the amount of sugar in an orange



Problem

Suppose a variable X can take the values 1, 2, 3, or 4. The probabilities associated with each outcome are described below.

Outcome	1	2	3	4
Probability	0.1	0.3	0.4	0.2

Table: Probability distribution $f_X(x)$

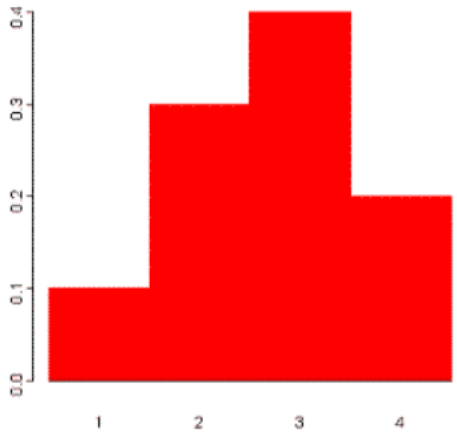


Figure: Probability histogram for $f_X(x)$

Computing the CDF $F_X(x)$

- $F_X(x) = P(X \leq x) = 0 \; \forall x \leq 0$
- $F_X(1) = P(X \leq 1) = 0.1$
- $F_X(2) = P(X \leq 2) = 0.1 + 0.3 = 0.4$
- $F_X(3) = P(X \leq 3) = 0.1 + 0.3 + 0.4 = 0.8$
- $F_X(4) = P(X \leq 4) = 0.1 + 0.3 + 0.4 + 0.2 = 1$
- $F_X(x) = P(X \leq x) = 1 \; \forall x \geq 5$

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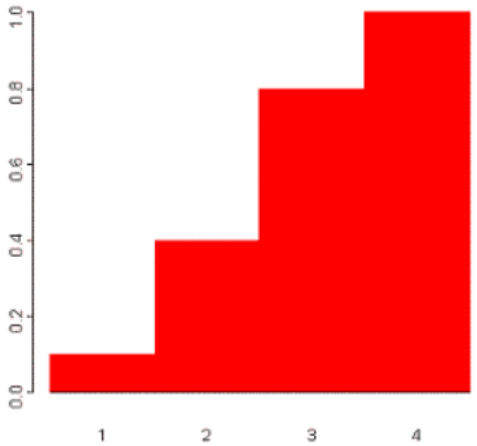

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References

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