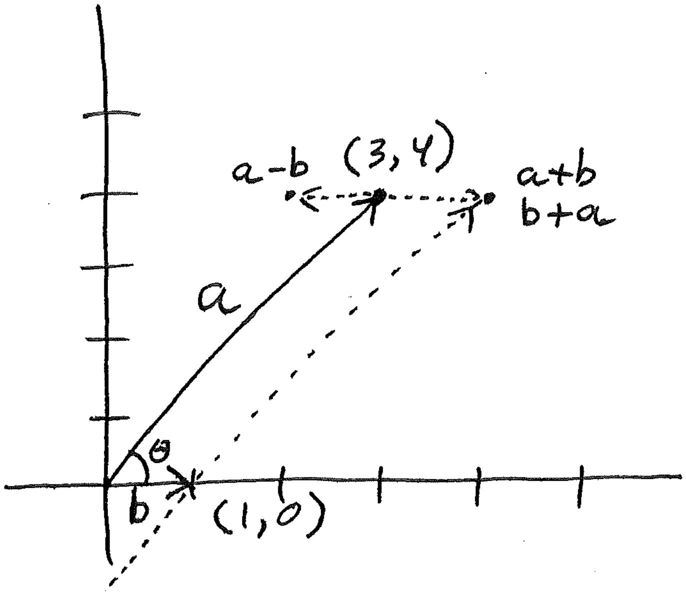
VECTORS



$$a = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Addition:

$$a+b = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$b+a = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$a-b = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$b-a = \begin{bmatrix} -2 \\ -4 \end{bmatrix}$$

Dot Product:

$$a \cdot b = (3 \cdot 1) + (4 \cdot 0) = 3$$

$$= \|a\|\|b\| \cos(\theta) = \sqrt{3^2+4^2}\sqrt{1^2+0^2}\cos(\theta) = 5\cos(\theta)$$

$$\theta = \cos^{-1}(3/5)$$

What if:

1) $a \cdot b = 0$? (minimum)

a) $\|a\| = 0$

b) $\|b\| = 0$

0 vectors!

c) $\cos(\theta) = 0 \Rightarrow \theta = 90°, 270° \Rightarrow$ perpendicular (orthogonal)

2) $a \cdot b = \|a\|\|b\|$ (maximum)

$\cos(\theta) = 1 \Rightarrow \theta = 0°, 180° \Rightarrow \longrightarrow\longrightarrow$ collinear (scalar product)

MATRICES

can be viewed as a collection of vectors

e.g. $\begin{bmatrix} 3 & 1 \\ 4 & 0 \end{bmatrix}$, $\begin{bmatrix} 3 & 1 & 2 \\ 4 & 0 & 2 \end{bmatrix}$, $\begin{bmatrix} 3 & 1 \\ 4 & 2 \\ 5 & 3 \end{bmatrix}$

$\begin{array}{cc} & \\ \end{array}$ a b

Using +, −, scalar multiplication, where can we get to on the 2d plane using only the columns of our matrix? (System of linear equations!)

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$? $\begin{bmatrix} 3 & 1 \\ 4 & 0 \end{bmatrix}$? $\underbrace{\begin{bmatrix} 3 & 6 \\ 4 & 8 \end{bmatrix}}_{\text{Linearly dependent!}}$?

In 3d: $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ ← Linearly dependent, because we can make 3rd column from 1st and 2nd

If a square matrix has linearly independent columns, we say that it is full rank

(Equivalently, a full rank $n \times n$ matrix has columns whose linear combinations cover $\mathbb{R}^n$)

Only full rank matrices have non-zero determinants!

Interpretation: determinant of a $n \times n$ matrix is the volume described by the columns in $\mathbb{R}^n$.

EIGENVECTORS

Matrix M

Vector v

Scalar λ

$$Mv = \lambda v \; ?$$

If so, v is an eigenvector

λ is an eigenvalue

Eigenvalues are values of λ that satisfy $|M - \lambda I| = 0$

determinant

Ex: $M = I$

$$|M - \lambda I| = |(1-\lambda) I| = 0$$

$$\Rightarrow \lambda = 1$$

So 1 is an eigenvalue of the identity matrix.

This should not be surprising.

$$Iv = v.$$