Maths Questions (GMAIL)

Sol(i) $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ find rank.

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$R_4 \to R_4 - 6R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_3 \to R_3 - R_4$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 3 & -2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_2 \leftrightarrow R_4$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -11 & 5 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_4 \to R_4 + R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -11 & 5 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank = 3

No. of Non zero rows in Echlon form.

Sol^{n} $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

λ form $\rightarrow$ $\begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix}$

$(2-\lambda)(2-\lambda) - [1]$

$(2-\lambda)^2 - 1 = 0$

$4 + \lambda^2 - 4\lambda - 1 = 0$

$\lambda^2 - 4\lambda + 3 = 0$

$\lambda^2 - 4\lambda + 4 - 1 = 0$

$\lambda^2 - 2\lambda + 3 = 0$

$\lambda^2 - 3\lambda + \lambda + 3 = 0$

$4 + \lambda^2 - 4\lambda - 1 = 0$

$\lambda^2 - 4\lambda + 3 = 0$

$\lambda^2 - \lambda - 3\lambda + 3 = 0$

$\lambda(\lambda - 1) - 3(\lambda - 1) = 0$

$(\lambda - 3)(\lambda - 1) = 0$

$\boxed{\lambda = 3 \text{ and } \lambda = 1}$

$\rightarrow$ These are the Eigen values

For Eigen Vector

for $\lambda = 1$

$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$x - y = 0$

$-x + y = 0$

$\boxed{x = y}$ $\Rightarrow \begin{bmatrix} k \\ k \end{bmatrix}$

Eigen Vector.

for $\lambda = 3$

$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$-x - y = 0$

$-x - y = 0$

$\boxed{x = y}$ $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

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Sol 4) Gauss - Siedel Method. (Three Iterations).

$$\Rightarrow \begin{cases} 3x - 0.1y - 0.2z = 7.85 \\ 0.1x + 7y - 0.3z = -19.3 \\ 0.3x - 0.2y + 10z = 71.4 \end{cases}$$

with initial values.

$x(0) = 0, \quad y(0) = 0, \quad z(0) = 0$

$$x = \frac{7.85 + 0.2z + 0.1y}{3}$$

$$y = \frac{-19.3 + 0.3z - 0.1x}{7}$$

$$z = \frac{71.4 + 0.2y - 0.3x}{10}$$

1st Iteration:

$$x = \frac{7.85}{3} = 2.616$$

$$y = \frac{-19.3 - 0.1(2.616)}{7}$$

$$y = 19.561$$

$$z = \frac{71.4 - 0.3(2.616) - 0.2(19.561)}{10}$$

$$x = 6.69$$

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2nd Iteration:

$(x, y, z) = 2.616, 19.561, 6.670$

$x = \frac{7.85 + 0.1(19.561) + 0.2(6.67)}{3}$

$x = 7.716$

$y = \frac{-19.3 + 0.1(7.716) + 0.3(6.670)}{7}$

$y = -2.36$

$z = \frac{71.7 - 0.3(7.716) - 0.2(-2.36)}{10}$

$z = 6.956$

3rd Iteration

$x = 3.002$

$y = -2.455$

$z = 7.099$

Ans

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Ans) Consistent → System of Eq whose a sol^n exists.

Inconsistent → " " " a sol^n doesn't exist.

$x + 3y + 2z = 0$

$2x - y + 3z = 0$

$3x - 5y + 4z = 0$

$x + 17y + 4z = 0$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{bmatrix}$$

$R_4 \to R_4 - R_1$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{bmatrix}$$

$R_4 \to R_4 + R_3$

$R_3 \to R_3 + 2R_2$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = \rho(A|B) \neq$ No. of unknowns

$2 = 2 \neq 3$

Inconsistent

Ans 8) Same method as (Sol 4).

Ans 9) Matrix operation is used in Image Processing in Linear Transformations particularly when applying Filters to images.

Ans 10) Linear transformations, represented by matrices are imp. in Computer vision as the manipulate spatial properties of image in a systematic wa