Assignment 4

Problem statement:

Consider the problem given in class

0.99 <= |H(e^(jw)| <= 1.01,   0 <= |w| <= 0.19pi

|H(e^(jw)| < 0.01,   0.21pi <= |w| <= pi

Solve it using least squares.

Formulation:

Let the FIR filter length be **L+1** samples, with L even, and suppose we'll initially design it to be centered about the time origin (``zero phase''). Then the frequency response is given on our frequency grid w_k by

$$H(w_k) = \sum_{n=-L/2}^{L/2} h_n e^{-jw_k n}$$

Enforcing *even symmetry* in the impulse response, *i.e.*, $\mathbf{h_n = h_{-n}}$, gives a zero-phase FIR filter that we can later right-shift **L/2** samples to make a causal, *linear phase* filter. In this case, the frequency response reduces to a *sum of cosines which can be represented as:*

$$\underbrace{\begin{bmatrix} H(\omega_0) \\ H(\omega_1) \\ \vdots \\ H(\omega_{N-1}) \end{bmatrix}}_{\underline{d}} = \underbrace{\begin{bmatrix} 1 & 2\cos(\omega_0) & \dots & 2\cos[\omega_0(L/2)] \\ 1 & 2\cos(\omega_1) & \dots & 2\cos[\omega_1(L/2)] \\ \vdots & \vdots & & \vdots \\ 1 & 2\cos(\omega_{N-1}) & \dots & 2\cos[\omega_{N-1}(L/2)] \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_{L/2} \end{bmatrix}}_{\underline{h}}$$

Procedure:

1. Obtain the magnitude response by applying the formulae.
2. Plot the graph by applying the least squares method.
3. Plot the graphs using built-in functions.

Code:

```
wc= 0.19*pi ;

ws= 0.21 *pi;

w=[0:0.06*pi:wc,ws:0.06*pi:pi];


H=zeros(size(w));

H(w<=wc)=1;

a=0;

H(w>=ws)=a;


N=length(w);

A=zeros(N,N);

for i=1:1:N

   for j=1:1:N

       A(i,j)=exp(-complex(0,1)*w(i)*(j-1));

   end

end


A1=(A.')*A;

A2=(A1)\(A.');

h=A2*(H.');

n=[0:1:length(w)-1].';
```

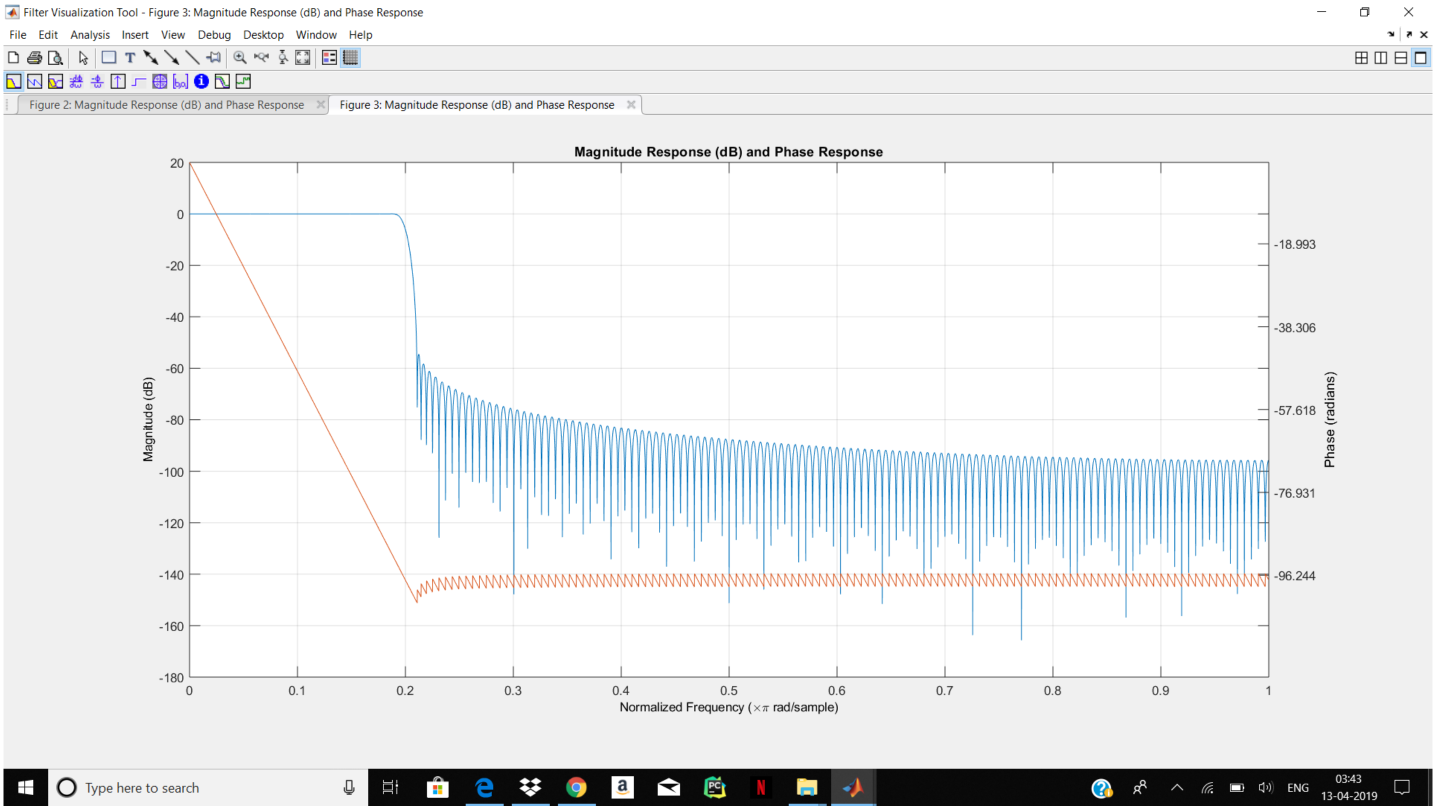
```
y=zeros(size(w));
for w0=1:length(w)
e=exp(-complex(0,1)*w(w0)*n);
xe=h.*e;
y(w0)=sum(xe);
end

figure(1);
plot(w/pi, 20*log(abs(y))/log(10));
title("Magnitude of Frequency Response using Least-Squares");
xlabel("Noramlized Frequency (*pi rad/sample)");
ylabel("Magnitude of Frequency Response");

%Using MATLAB inbuilt function
b = firls(310,[0 0.19 0.21 1],[1 1 a a]);
fvtool(b,1,'OverlayedAnalysis','phase')
```

Result:

Using firls and fvtool



Using user defined code: