LATEX ASSIGNMENT

ANAND

9-09-2023

EXERCISE 12.11.4

1. Show that the line joining the origin to the point $(2, 1, 1)$ is perpendicular to the line determined by the points $(3, 5, -1), (4, 3, -1)$.

2. If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular to Both of these are $m_1 n_2 - m_2 n_1, n_1 l_2 - n_2 l_2 - n_2 l_1, l_1 m_2 - l_2 m_1$.

3. Find the angle between the lines whose direction ratios are a, b, c and $b-c$, $c-a$, $a - b$.

4. Find the equation of a line parallel to x-axis and passing through the origin.

5. If the co-ordinates of the points A, B, C, D be $(1, 2, 3)$, $(4, 5, 7)$, $(-4, 3, -6)$ and $(2, 9, 2)$ respectively, then find the angle between the lines AB and CD.

6. If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular, Find the value of k.

7. Find the vector equation of the line passing through $(1, 2, 3)$ and perpendicular to the plane $\overrightarrow{r} \cdot (\hat{i} + 2\hat{j} - 5\hat{k}) + 9 = 0$.

8. Find the equation of the plane passing through (a, b, c) and parallel to the plane $\overrightarrow{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 2$.

9. Find the shortest distance between the lines $\overrightarrow{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$ and $\overrightarrow{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$.

10. Find the co ordinates of the point where the line through $(5, 1, 6)$ and $(3, 4, 1)$ crosses the YZ-plane.

11. Find the co ordinates of the point where the line through $(5, 1, 6)$ and $(3, 4, 1)$ crosses the ZX-plane.

12. Find the co ordinates of the point where the line through $(3, -4, -5)$ and $(2, -3, 1)$ crosses the plane $2x + y + x = 7$.

13. Find the equation of the plane passing through the point $(-1, 3, 2)$ and perpendicular to each of the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.

14. If the points $(1, 1, p)$ and $(-3, 0, 1)$ be equidistant from the plane $\overrightarrow{r} \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13 = 0$, then find the value of p.

15. Find the equation of the plane passing through the line of intersection of the planes $\overrightarrow{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ and $\overrightarrow{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$ and parallel to x-axis.

16. If O be the origin and the coordinates of P be $(1, 2, -3)$, then find the equation of the plane passing through P and perpendicular to OP.

17. Find the equation of the plane which contains the line of intersection of the planes $\overrightarrow{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0, \overrightarrow{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ and which is perpendicular to the plane $\overrightarrow{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$

18. Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the line $\overrightarrow{r} = 2\hat{i} - \hat{j} - 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\overrightarrow{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$.

19. Find the vector equation of the line passing through $(1, 2, 3)$ and parallel to the planes $\overrightarrow{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\overrightarrow{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$

20. Find the vector equations of the line passing through the point $(1, 2, -4)$ and perpendicular to the two lines:

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \text{ and } \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}. \tag{1}$$

21. Prove that if a plane has the intercept a, b, c and is at a distance of p units from the origin, then $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$.

Choose the correct answer in 22 and 23

22. Distance between the two planes: $2x + 3y + 4z = 4$ and $4x + 6y + 8z = 12$ is
    (a) 2 units
    (b) 4 units
    (c) 8 units
    (d) $\frac{2}{\sqrt{29}}$ units

23. The planes: $2x - y + 4z = 5$ and $5x - 2.5y + 10z = 6$ are
    (a) Perpendicular
    (b) Parallel
    (c) Intersect y axis
    (d) Passes through $\left(0, 0, \frac{5}{4}\right)$