LATEX ASSIGNMENT

ANAND

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EXERCISE 12.3.5

1. Let $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, show that $(aI + bA)^n = a^n I + na^{n-1}bA$, where I is the identity matrix of order 2 and $n \in N$.

2. If $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$, Prove that $A^n = \begin{pmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{pmatrix}, n \in N$.

3. If $A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$, then prove that $A^n = \begin{pmatrix} 1+2n & -4n \\ n & 1-2n \end{pmatrix}$,Where n is any positive integer.

4. If A and B are symmetric matrices prove that $AB - BA$ is a skew symmetric matrix.

5. Show that the matrix $B'AB$ is a symmetric or skew symmetric according as A is symmetric or skew symmetric.

6. Find the value of x, y, z if the matrix $A = \begin{pmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{pmatrix}$ satisfy the equation $A'A = I$.

7. For what values of x : $\begin{pmatrix} 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ x \end{pmatrix} = 0$

8. If $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$, show that $A^2 - 5A + 7I = 0$.

9. Find x, if $\begin{pmatrix} x & -5 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} \begin{pmatrix} x \\ 4 \\ 1 \end{pmatrix} = O$.

market	products		
I	10,000	2,000	18,000
II	6,000	20,000	8,000

Table 1:

10. A manufacturer produces three products x, y, z which he sells in two markets. Annual Sales are indicated below:

    (a) If unit sale Prices of x, y and z are ₹ 2.50, ₹ 1.50 and ₹ 1.00, respectively. Find the total revenue in each market with the help of matrix algebra.

    (b) If the unit costs of the above three commodities are ₹ 2.00, ₹ 1.00 and 50 paise respectively. Find the gross profit.

11. Find the matrix X so that $X\begin{pmatrix}1 & 2 & 3\\4 & 5 & 6\end{pmatrix} = \begin{pmatrix}-7 & -8 & -9\\2 & 4 & 6\end{pmatrix}$

12. If A and B are square matrices of the same order such that $AB = BA$, then prove by induction that $(AB)^n = B^nA^n$. Further prove that $(AB)^n = A^nB^n$ for all $n \in N$.
    Choose the correct answer in the following questions:

13. If $A = \begin{pmatrix}\alpha & \beta\\ \gamma & -\alpha\end{pmatrix}$ is such that $A^2 = I$, then

    (a) $1 + \alpha^2 + \beta\gamma = 0$

    (b) $1 - \alpha^2 + \beta\gamma = 0$

    (c) $1 - \alpha^2 - \beta\gamma = 0$

    (d) $1 + \alpha^2 - \beta\gamma = 0$

14. If the matrix A is both symmetric and skew symmetric, then

    (a) A is a diagonal matrix

    (b) A is a Zero matrix

    (c) A is a Square matrix

    (d) None of these

15. If A is square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to

    (a) A

    (b) $I - A$

    (c) I

    (d) $3A$