The augmented matrix can be expressed as,

$$\begin{pmatrix} 2 & -3 & 5 \\ 3 & -4 & 7 \end{pmatrix}$$

Through pivoting, the augmented matrix will become as,

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -3 \\ 3 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

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2 & -3 & 5 \\
3 & -4 & 7
\end{pmatrix}$$

Through pivoting, the augmented matrix will become as,

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On solving above equation the crossing point of the given equations will be,

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Therefore the Center of the Circle is,

$$\mathbf{C} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{0.0.1}$$

0.1 Calculation of radius of the Circle

As per the given data, the area of the Circle is 154 sq.units Let r be the radius of circle,

$$\pi r^2 = 154 \implies r = 7$$
 (0.1.1)

0.2 Deriving equation for Circle in matrix form

The equation of circle in matrix form is,

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \tag{0.2.1}$$

Where

$$\mathbf{V} = \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \mathbf{f} = -47$$
$$\implies \mathbf{x}^T \mathbf{I} \ \mathbf{x} + 2 \begin{pmatrix} -1 \\ 1 \end{pmatrix}^T \mathbf{x} - 47 = 0$$

Therefore, the circle equation can be written as

$$\mathbf{x}^T \mathbf{x} + 2 \begin{pmatrix} -1\\1 \end{pmatrix}^T \mathbf{x} - 47 = 0 \tag{0.2.2}$$

0.3 Deriving equation for Circle in quadratic form

In quadratic form, the expression for circle can be written as,

$$(x - x1)^2 + (y - y1)^2 = r^2$$

$$(x-1)^2 + (y+1)^2 = 7^2$$

$$x^2 + y^2 - 2x + 2y - 47 = 0 (0.3.1)$$

1 Conclusion

- 1. At first, Center of the Circle has been found which is crossing point of the two diameter lines 2x-3y=5 and 3x-4y=7.
- 2. Radius of the center has been calculated from its area 154sq.units.
- 3. Matrix equation for V, U, U^T and f has been derived.
- 4. Finally, the Circle equation has been derived as,

$$\mathbf{x}.\mathbf{x}^T + 2\begin{pmatrix} -1\\1 \end{pmatrix}^T \mathbf{x} - 47 = 0$$