Speed and Rotor Flux Estimation of Induction Motors based on Extended Kalman Filter

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***Abstract*—Two problems must be solved in the speed sensorless vector control of induction motor drive: the speed estimation and rotor flux observation. Because of the multiplication terms of state variables, the induction motor model is the non-linear state equations. To estimate the state variables of motor model and gain the rotor flux and speed signals, the paper proposes a method to estimate them using extended kalman filter. Experiment is based on the DSP design system for digital motor control. Software programs carry out extended kalman filter algorithm to estimate the rotor speed and fluxes. The satisfied experimental results prove that extended kalman filter algorithm can real time estimate rotor speed and flux very accurately, and based on which the speed sensorless drive system has good static and dynamic performance.**

Keywords—Induction Motor, Speed Estimation, Speed Sensorless Vector Control, Rotor Flux Observation, Extended Kalman Filter.

I. INTRODUNCTION

Vector control should be orientated by rotor flux vector, and the rotor flux amplitude and phase should be known when calculation. Indirect flux observation is often used in modern vector control to calculate the rotor flux amplitude and phase when voltage, current or speed signals are detected.

Speed closed-loop control requires speed sensor, which increases the cost and complexity of the speed control system. Therefore, during the last 10 years, many scholars at home and abroad had studied speed sensorless vector control, including MRAS, self-tuning and so on[1~6]. However, accuracy of speed estimation and flux observation of this method depends on the accurate measurement of reference model output. Further more, parameters self – adaptation algorithm is too complex, including a large number of differential, integral operations. It is very difficult to discrete through microprocessor. This article described an algorithm about speed and flux estimation which can be easily achieved through software programming.

The state equation of induction motor is nonlinear. According to the system identification theory[7~8], the best way to estimate nonlinear system state is the extended Kalman filter. In this paper, extended Kalman filter was used in the theoretical analysis and experimental research for rotor flux and speed estimation. This algorithm establishes a new state equation. The state variables are the stator current, rotor flux and velocity, input and output variables is stator voltage and current respectively. These five variables are estimated in real-time under the Extended Kalman filtering algorithm.

II. PRICIPLE of VECTOR CONTROL

Induction motor vector control technology was developed by the Germans in the 1970th. It is actually a decoupling control method about motor torque and flux. With vector control technology, induction motors can control the speed through torque and flux like DC motors.

In vector control system, Coordinate transformation and the inverse transformation is a very important concept. One is the transformation from $i_{s\alpha}^*$ and $i_{s\beta}^*$ to i_{sm}^* and i_{st}^*, which is called Park transformation. The other one is the inverse transformation form i_{sm}^* and i_{st}^* to $i_{s\alpha}^*$ and $i_{s\beta}^*$. So the key point of the vector control is to find the amplitude and phase of rotor flux. State equation and mechanical motion equation are shown as following:

$$\dot{i}_{s\alpha} = -\left(\frac{R_s}{\sigma L_s} + \frac{1-\sigma}{\sigma\tau_r}\right)i_{s\alpha} + \frac{L_m}{\sigma L_s L_r \tau_r}\psi_{r\alpha} + \frac{L_m}{L_s L_r \sigma}\omega_r\psi_{r\beta} - \frac{1}{\sigma L_s}u_{s\alpha}$$

$$\dot{i}_{s\beta} = -\left(\frac{R_s}{\sigma L_s} + \frac{1-\sigma}{\sigma\tau_r}\right)i_{s\beta} - \frac{L_m}{L_s L_r \sigma}\omega_r\psi_{r\alpha} + \frac{L_m}{\sigma L_s L_r \tau_r}\psi_{r\beta} - \frac{1}{\sigma L_s}u_{s\beta}$$

$$\dot{\psi}_{r\alpha} = \frac{L_m}{\tau_r}i_{s\alpha} - \frac{1}{\tau_r}\psi_{r\alpha} - \omega_r\psi_{r\beta}$$

$$\dot{\psi}_{r\beta} = \frac{L_m}{\tau_r}i_{s\beta} - \frac{1}{\tau_r}\psi_{r\beta} - \omega_r\psi_{r\alpha}$$

$$\dot{\omega}_r = \frac{P_n^{\,2}L_m}{JL_r}\left(i_{s\beta}\psi_{r\alpha} - i_{s\alpha}\psi_{r\beta}\right) - \frac{P_n}{J}M_l$$

where：

R_s、R_r——resistance of stator and rotor；

L_s、L_r——self-inductance of stator and rotor；

L_m——Mutual inductance between stator and rotor；

τ_r——rotor time constant, $\tau_r = L_r / R_r$；

σ——Leakage coefficient, $\sigma = \dfrac{L_s L_r - L_m^{\,2}}{L_s L_r}$；

$u_{s\alpha}$、$u_{s\beta}$——α、β axis component of stator voltage；

ω_r——Rotor electrical angular velocity；

P_n——Number of motor pole pairs；

J——rotary inertia；

M_l——Motor load torque

III. ROTOR FLUX and SPEED ESTTMATION based on EKF

Extended Kalman filter is widely used in non-linear systems for state estimation. In the sensorless vector control system, speed ω_r is considered to be a state variable due to the speed is unknown. According to the motor mechanical equation and the original state equations, a new state equation can be got, as shown in (1). There are five state variable in this equation: $[i_{s\alpha}, i_{s\beta}, \psi_{r\alpha}, \psi_{r\beta}, \omega_r]^T$. The system input and output remain invariable. Assumed that the frequency of speed control loop is much larger relative to the motor mechanical time constant, then the speed in a sampling cycle can be considered as a constant, so mechanical equations of motion can be expressed as: $\dfrac{d\omega_r}{dt} = 0$

$$\begin{bmatrix} \dot{i}_{s\alpha} \\ \dot{i}_{s\beta} \\ \dot{\psi}_{r\alpha} \\ \dot{\psi}_{r\beta} \\ \dot{\omega}_r \end{bmatrix} = \begin{bmatrix} -\xi & 0 & \dfrac{\eta}{\tau_r} & \eta\omega_r & 0 \\ 0 & -\xi & -\eta\omega_r & \dfrac{\eta}{\tau_r} & 0 \\ \dfrac{l_m}{\tau_r} & 0 & -\dfrac{1}{\tau_r} & -\omega_r & 0 \\ 0 & \dfrac{l_m}{\tau_r} & \omega_r & -\dfrac{1}{\tau_r} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \\ \psi_{r\alpha} \\ \psi_{r\beta} \\ \omega_r \end{bmatrix} + \begin{bmatrix} \dfrac{1}{\sigma l_s} & 0 \\ 0 & \dfrac{1}{\sigma l_s} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_{s\alpha} \\ u_{s\beta} \end{bmatrix}$$

(1)

where:

$$\eta = \frac{l_m}{\sigma l_s l_r} \; ; \quad \xi = \frac{R_s}{\sigma l_s} + \frac{l_m^{\,2}}{\sigma l_s l_r \tau_r}$$

Equation (1) is obviously a group of non-linear state equation. Extended Kalman filter algorithm is a recursive state estimation algorithm, so (1) should be discretized. According to discrete formula:

$$\frac{dx(t)}{dt} = \frac{x_{k+1} - x_k}{T} \qquad kT \le t \le (k+1)T$$

We can get discrete form of motor state equations:

$$i_{s\alpha}(k+1) = (1-\xi T)i_{s\alpha}(k) + \frac{\eta}{\tau_r}T\psi_{r\alpha}(k) + \eta T\psi_{r\beta}(k)\omega_r(k) + \frac{1}{\sigma l_s}Tu_{s\alpha}(k)$$

$$i_{s\beta}(k+1) = (1-\xi T)i_{s\beta}(k) - \eta T\psi_{r\alpha}(k)\omega_r(k) + \frac{\eta}{\tau_r}T\psi_{r\beta}(k) + \frac{1}{\sigma l_s}Tu_{s\beta}(k)$$

$$\psi_{r\alpha}(k+1) = -\frac{l_m}{\tau_r}Ti_{s\alpha}(k) + (1-\frac{T}{\tau_r})\psi_{r\alpha}(k) - T\psi_{r\beta}(k)\omega_r(k)$$

$$\psi_{r\beta}(k+1) = -\frac{l_m}{\tau_r}Ti_{s\beta}(k) + T\psi_{r\alpha}(k)\omega_r(k) + (1-\frac{T}{\tau_r})\psi_{r\beta}(k)$$

$$\omega_r(k+1) = \omega_r(k)$$

While calculating the state variable, the actual measurement results including stator current $[i_{s\alpha}, i_{s\beta}]^T$ and stator voltage $[u_{s\alpha}, u_{s\beta}]^T$ should be known. And accurate parameters of stator and rotor should be known too. All these results and parameters have errors inevitable. All these errors will affect the state estimation accuracy of extended Kalman filter, resulting in deterioration of control performance. These uncertainties are attributed to the state noise vector $w(k)$ and measurement noise vector $v(k)$. The state noise vector $w(k)$ represents the error caused from motor parameter variations and discretization, and the measurement noise vector $v(k)$ represents the error caused from measurement error of motor input and output signal. Each noise vector can be demonstrated below:

$$\begin{cases} x(k+1) = f(x(k), u(k), k) + w(k) \\ y(k) = Cx(k) + v(k) \end{cases}$$

Assumed that $w(k)$ and $v(k)$ are all static and Gauss distribution. In the extended Kalman filter algorithm, what is really concerned with is not the noise vector $w(k)$ and $v(k)$, but the covariance matrix of noise vector, which is shown below:

$$\operatorname{cov}(w) = E\{ww^T\} = Q \; ; \quad \operatorname{cov}(v) = E\{vv^T\} = R$$

where:

E{ } represents the mathematical expectation.

As was assumed that $\dfrac{d\omega_r}{dt} = 0$ to simplify the mechanical motion equation, this may ignore the problems associated with the load and it is a rough assumption of motor model equations that drops the accuracy. The introduction of the state noise vector $w(k)$, these effects can be looked as noise of motor model, and the model equation is more accurate.

Motor discrete state equation is acquired considered the state noise and measurement noise. All five states of motor can be estimated based on this non-linear equation. Extended Kalman filter estimation is a random observer for non-linear system. Its aim is to estimate state of system accurately when there are model and measurement noise in the system. Structure of extended Kalman filter is shown in Figure 1.

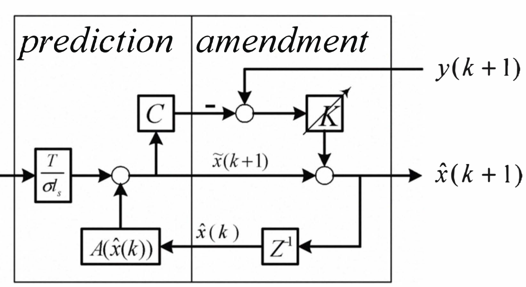


Fig 1. Structure of extended Kalman filter

A. Firstly assumed that system state estimation is $\hat{x}(k)$ at moment k , and predict the state $\hat{x}(k+1)$ at moment $k+1$ based on $\hat{x}(k)$. Because the noise vector can not determine in advance, value of $\tilde{x}(k+1)$ only can be predicted by deterministic equations, which is shown below:

$$\tilde{i}_{s\alpha}(k+1) = (1-\xi T)\hat{i}_{s\alpha}(k) + \frac{\eta}{\tau_r}T\hat{\psi}_{r\alpha}(k) + \eta T\hat{\psi}_{r\beta}(k)\hat{\omega}_r(k) + \frac{1}{\sigma l_s}Tu_{s\alpha}(k)$$

$$\tilde{i}_{s\beta}(k+1) = (1-\xi T)\hat{i}_{s\beta}(k) - \eta T\hat{\psi}_{r\alpha}(k)\hat{\omega}_r(k) + \frac{\eta}{\tau_r}T\hat{\psi}_{r\beta}(k) + \frac{1}{\sigma l_s}Tu_{s\beta}(k)$$

$$\tilde{\psi}_{r\alpha}(k+1) = -\frac{l_m}{\tau_r}T\hat{i}_{s\alpha}(k) + (1-\frac{T}{\tau_r})\hat{\psi}_{r\alpha}(k) - T\hat{\psi}_{r\beta}(k)\hat{\omega}_r(k)$$

$$\tilde{\psi}_{r\beta}(k+1) = -\frac{l_m}{\tau_r}T\hat{i}_{s\beta}(k) + T\hat{\psi}_{r\alpha}(k)\hat{\omega}_r(k) - \frac{1}{\tau_r}T\hat{\psi}_{r\beta}(k)$$

$$\tilde{\omega}_r(k+1) = \hat{\omega}_r(k)$$

B. Calculating covariance matrix, $P(k+1 \mid k)$, of predictive error

$$P(k+1 \mid k) = A(k) * P(k \mid k) * A^T(k) + Q$$

where:

$$A(k) = \left.\frac{\partial f[x(k),k]}{\partial x(k)}\right|_{x(k)=\hat{x}(k)}$$

$$= \begin{bmatrix} 1-\xi T & 0 & \frac{\eta}{\tau_r}T & \eta T\hat{\omega}_r(k) & \eta T\hat{\psi}_{r\beta}(k) \\ 0 & 1-\xi T & -\eta T\hat{\omega}_r(k) & -\frac{\eta}{\tau_r}T & -\eta T\hat{\psi}_{r\beta}(k) \\ -\frac{l_m}{\tau_r}T & 0 & 1-\frac{T}{\tau_r} & -T\hat{\omega}_r(k) & -T\hat{\psi}_{r\beta}(k) \\ 0 & -\frac{l_m}{\tau_r}T & T\hat{\omega}_r(k) & -\frac{1}{\tau_r}T & T\hat{\psi}_{r\alpha}(k) \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

C. calculating Kalman gain matrix $K(k+1)$ at moment $k+1$:

$$K(k+1) = P(k+1 \mid k) * C^T * \{C * P(k+1 \mid k) * C^T + R\}^{-1}$$

D. calculating state estimation $\hat{x}(k+1)$ at moment $k+1$:

$$\hat{x}(k+1) = \tilde{x}(k+1) + K(k+1) * \{y(k+1) - C * \tilde{x}(k+1)\}$$

E. Determine the covariance matrix of the predictive error at moment $k+1$:

$$P(k+1 \mid k+1) = \{I - K(k+1) * C\} * P(k+1 \mid k)$$

Ⅳ. EXPERIMENTAL SYATEM

Based on the DSP(TMS320LF2407A) [9~10] especially used in motor control, speed sensorless vector control system was studied experimentally. Flux and speed estimation based on extended Kalman filter and SVM(Space Vector Modulation) are all implement the through software. System block diagram shown in Figure 2.

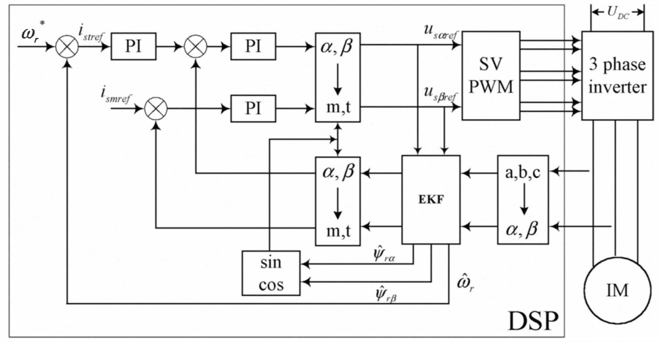


Fig 2. System block diagram of vector control

Vector control system includes hardware circuit and control algorithms. The control algorithm (the dashed part of the block diagram) is the core of the vector control system, which not only uses extended Kalman filter algorithm to estimate the rotor flux and speed of induction motor, but also to transform coordinate and output SVM PWM waveform .

There is a large number of matrix operations in extended Kalman filter algorithm, such as the matrix add, multiply, inverse operation, etc. It is necessary to analyze matrix calculation process and the execution time of instruction. Assumed that X 、 Y are both 5 × 5 matrix and matrix $Z = X \times Y$.

$$\begin{bmatrix} x_{11} & x_{12} & \cdots & x_{15} \\ x_{21} & x_{22} & \cdots & x_{25} \\ & & \cdots & \\ x_{51} & x_{52} & \cdots & x_{55} \end{bmatrix} * \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{15} \\ y_{21} & y_{22} & \cdots & y_{25} \\ & & \cdots & \\ y_{51} & y_{52} & \cdots & y_{55} \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & \cdots & z_{15} \\ z_{21} & z_{22} & \cdots & z_{25} \\ & & \cdots & \\ z_{51} & z_{52} & \cdots & z_{55} \end{bmatrix}$$

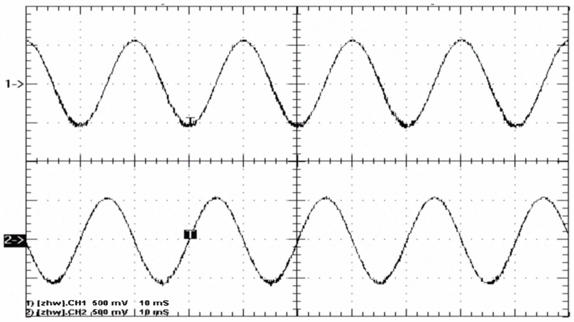
Elements of the matrix should be addressed before operations. Because matrix is stored progressively in DSP F2407A, it is very easy to use the auxiliary registers for indirect addressing of the matrix elements. There are five times multiplication and addition operation while calculating each element of the matrix. This operation is a cycle and each time is so simple that only need to change the operand address. There is a group of assemble language used for cycle: .loop and .endloop. Therefore, matrix multiply can be achieved based on three times nested loop.

Ⅴ. EXPERIMENTAL RESULTS

The induction motor starting process and the steady state operation are both recorded in the experimental system to test the dynamic and static performance.

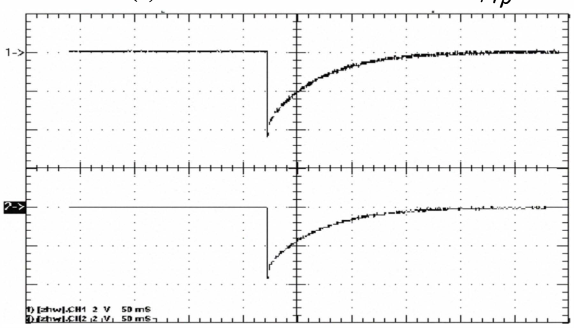
Experimental motor parameters are as follows:
Pn=2.2KW,Un=380V,In=4.5A,n=1440rpm,np=2,
Rs=0.3955Ω,Rr=0.6399Ω,Ls=Lr=917.41mh，Lm=798.92mh

When the speed was set at 50Hz, the flux observation value and the speed estimation were recorded by oscilloscope (Tek TDS210). Both waveforms are shown in Figure 3.



(a) top：flux observation waveform $\psi_{r\alpha}$

(b) bottom：flux observation waveform $\psi_{r\beta}$



(c) top：speed estimation waveform

(d) bottom：speed measure waveform

Figure 3. Flux and speed waveform of 50Hz

Flux waveform was recorded in the stable operation of the motor. It is almost sine wave and its magnitude is obviously constant. There is 1/4 cycle difference between the upper and lower waveforms. It is verified that the estimation of rotor flux by the extended Kalman filter is very accurate. Experimental results and theoretical analysis is exactly the same.

As can be easily observed from Fig.3(c)(d), the estimation speed and the measure one when motor starting are very close. It shown that the system has good dynamic performance based on extended Kalman filter.

Ⅵ. CONCLUSION

In this paper, extended Kalman filter was used to estimate the state variables of induction motor. And the steps of flux and speed estimation were analyzed in detail. Based on DSP development system,[9~10] hardware system of vector control was designed, flux and speed estimation was implemented through extended Kalman filter. The experimental results are satisfactory. According to the recorded waveform, changes of rotor flux is consistent with theoretical analysis, and the observation speed and measure speed of the motor in starting process and the steady-state operation is also consistent. All these verify that estimation of flux and speed by extended Kalman filter algorithm is accurate, and the speed sensorless system has good static and dynamic performance.

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REFERENCES

[1] Li Zhen and Longya Xu, "Sensorless Field Orientation Control of Induction Machines Based on a Mutual MRAS Scheme", [J].IEEE Trans. on Ind. Electron, 1998,vol.45,(5):824-830.

[2] Patrick L.Jansen, Robert D.Lorenz,and Donald W.Novoty, "Obersever-Based Direct Field Orientation:Analysis and Comparison of Alternative Methods",[J].IEEE Trans. Ind. Applicat., 1994,vol.30, (4):945-953.

[3] Colin Dchauder, "Adaptive Speed Identification for Vector Control of Induction Motor without Rotational Transducers", [M].Conf,Rec. IEEE IAS Annual Meeting, 1989:493-499.

[4] Hisao Kubota, and Kouki Matsuse, "Speed Sensorless Field-Oriented Control of Induction Motor with Rotor Resistance Adaptation",[J].IEEE Trans. Ind. Appl., 1994,vol.30, (5):1219-1224.

[5] Cardenas R, Pena R, Member. MRAS observer for doubly fed induction machines [J]. IEEE Trans on Industrial Electronics, 2004, vol.19 (2), 467~468.

[6] ZHAO X in, WANGM ing2yu, L IU Shu2xi ,"Rotor Resistance Identification in Vector Control System of Asynchronous Motor Based on EKF"，[J]. Motors and control applications,2009,(3),18~23

[7] Wu Guang Yu,Fan Qing YiJiang Fu Xing, "System identification and adaptive control",[M]. Harbin， Harbin Institute of Technology Press,1987.

[8] Jiang Jing Ping,"Real-time computer control system"，[M].Hangzhou：Zhejiang University Publishing House，1992.

[9] Texas Instruments Incorporated, "TMS320LF/LC240xA DSP Controllers Reference Guide: System and Peripherals", 2005.

[10] Texas Instruments Incorporated, "TMS320F/C24x DSP Controllers Reference Guide: CPU, and Instruction Set", 1999.