**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: Pickup after 60 mins of drop-off, 10 mins is buffer after every car is dropped so it leaves 50 mins to complete task.

P(X>50) = 1 - P(X<50) = 1 - stats.norm.cdf(50,loc = 45,scale=8) = 0.2676

B . 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44. Ans: False. As P(38<X<44) > P(X>44) . P(38<X<44) = stats.norm.cdf(44, loc = 38, scale = 6) - stats.norm.cdf(38, loc = 38, scale = 6) = 0.3413 & P(X>44) = 1 - stats.norm.cdf(44, loc = 38, scale = 6)
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: True. No of employees below 30 = stats.norm.cdf(30, loc = 38, scale = 6) \* 400 = 36

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: *X*1 + *X*2 ~ N(μ+μ, σ2+σ2 ) = N(2μ, 2σ2 ), will be normal distribution and its parameters will be 2μ and 2σ2.

Whereas, 2*X*1 ~ N(2μ, (2)^2(σ2)) = N(2μ, 4σ2), will be normal distribution only and its parameters will be 2μ and 4σ2.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: D. (48.5,151.5). (a<X<b) = stats.norm.interval(0.99, loc = 100, scale = 20)

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: P = 0.95,

loc1 = 5million $ = 5000000\*45 = 22.5cr, scale1 = 3000000\*45 = 13.5cr

Loc2 = 7000000\*45 = 31.5cr, scale2 = 4000000\*45 = 18cr

Range for division 1 = stats.norm.interval(0.95, loc = 22.5, scale = 13.5) = (-3.95cr, 48.95cr)

Range for division 2 = stats.norm.interval(0.95, loc = 31.5, scale = 18) = (-3.77cr, 66.77cr)

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans: 5th percentile of profit = P(X)>0.95

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: Probability of loss means profit is less than 0.

For 1st  division - P(Loss) = P(X<0) = stats.norm.cdf(0, loc = 5, scale = 3) = 0.0477

For 2nd division - P(Loss) = P(X<0) = stats.norm.cdf(0, loc = 7, scale = 4) = 0.0400

So 1st division has more probability of making a loss in a year.