CS321: Computer Networks

Error Detection and Correction

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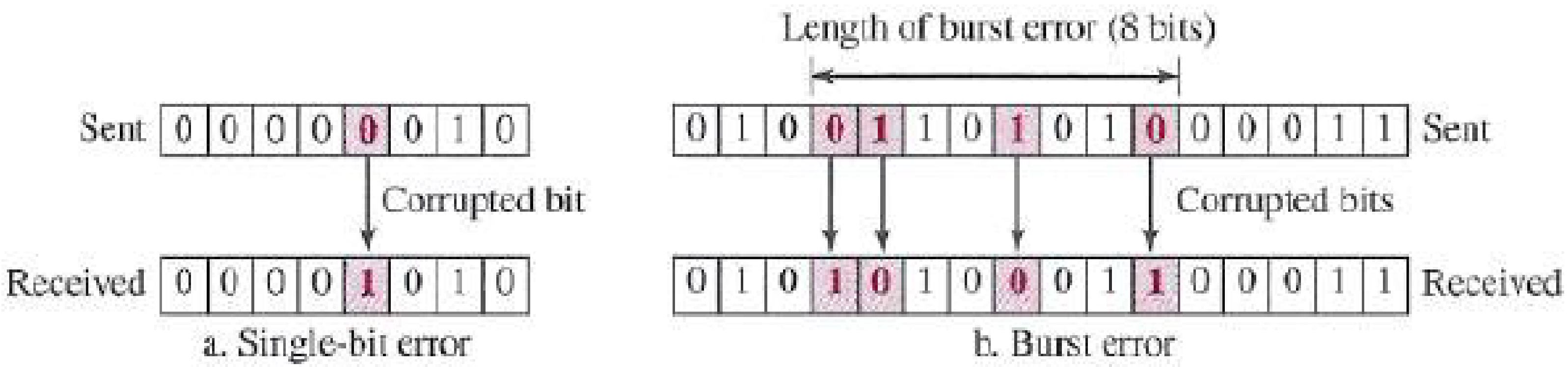
Error Detection and Correction

- Objective: System must guarantee that the data received are identical to the data transmitted
- Methods:
  1. If a frame is corrupted between the two nodes, it needs to be corrected
  2. Drop the frame and let the upper layer (e.g. Transport) protocol to handle it by retransmission

Types of Error



- Single bit error
- Burst error / multibit error

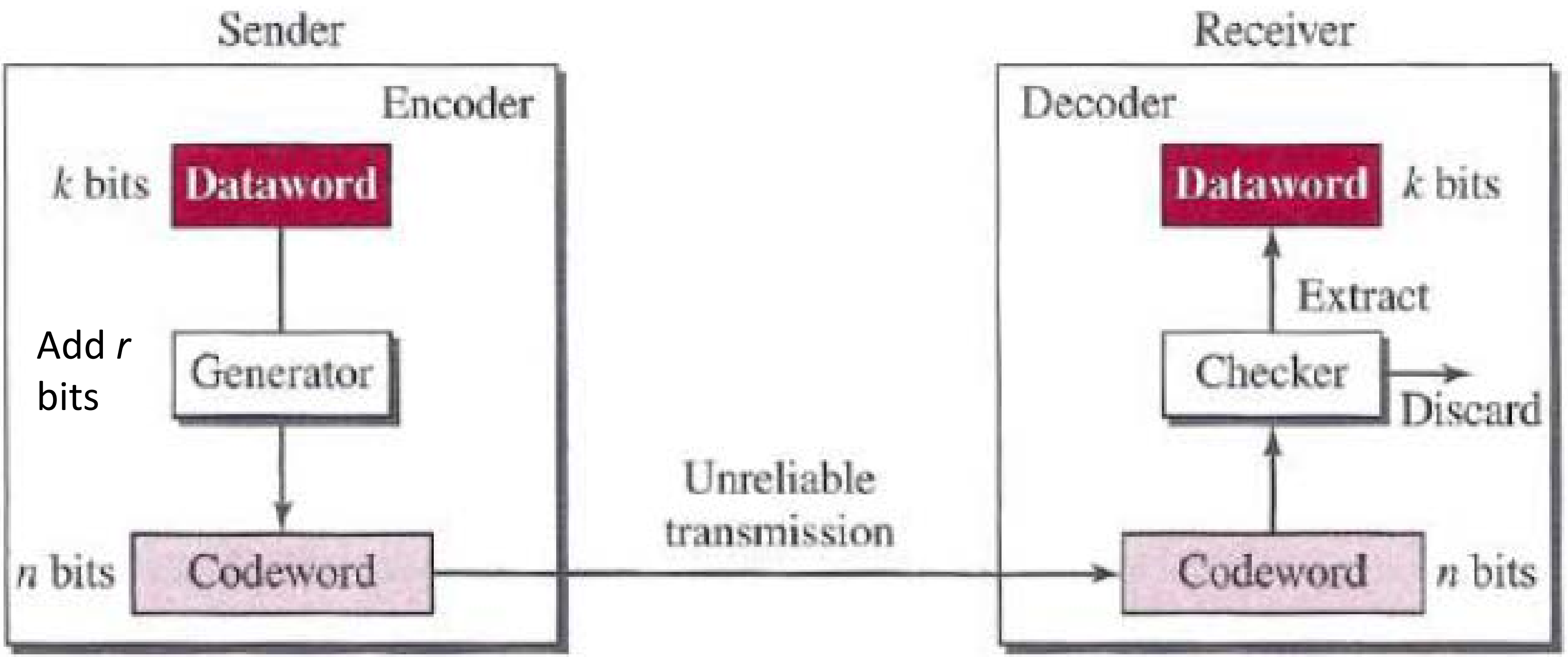
- Reason: noise in the channel

Detection and Correction

- Central idea: redundancy
  - put some extra bit with our data
  - Achieved by coding scheme
    - Block coding
    - Convolution coding

- Error Detection : looking to see if any error has occurred
- Error Correction: trying to recover the corruption
  - Need to know exact number of bits that are corrupted
  - Needs the position of those bits

Block Coding



- How the extra *r* bits are chosen or calculated?
- How can errors be detected?
  - Finds the existence of invalid codeword

Example

- Let us assume that k = 2 and n = 3. Table shows the list of datawords and codewords. Later, we will see how to derive a codeword from a dataword.

Dataword	Codeword
00	000
01	011
10	101
11	110

Assume the sender encodes the dataword 01 as 011 and sends it to the receiver.

Possible options at receiver (assume one bit corruption):

011 => correct

111 => invalid

001 => invalid

010 => invalid

Hamming Distance

- The Hamming distance between two words (of the same size) is the number of differences between the corresponding bits.
- Notation: Hamming distance between two words *x* and *y* as *d(x, y).*
- Calculation*:* apply the XOR operation on the two words and count the number of 1's in the result

- To guarantee the detection of up to *s* errors in all cases, the minimum Hamming distance in a block code must be $d_{min} = s + 1$.

Types of Block Codes

- Linear Block Code
- Non-linear Block Code

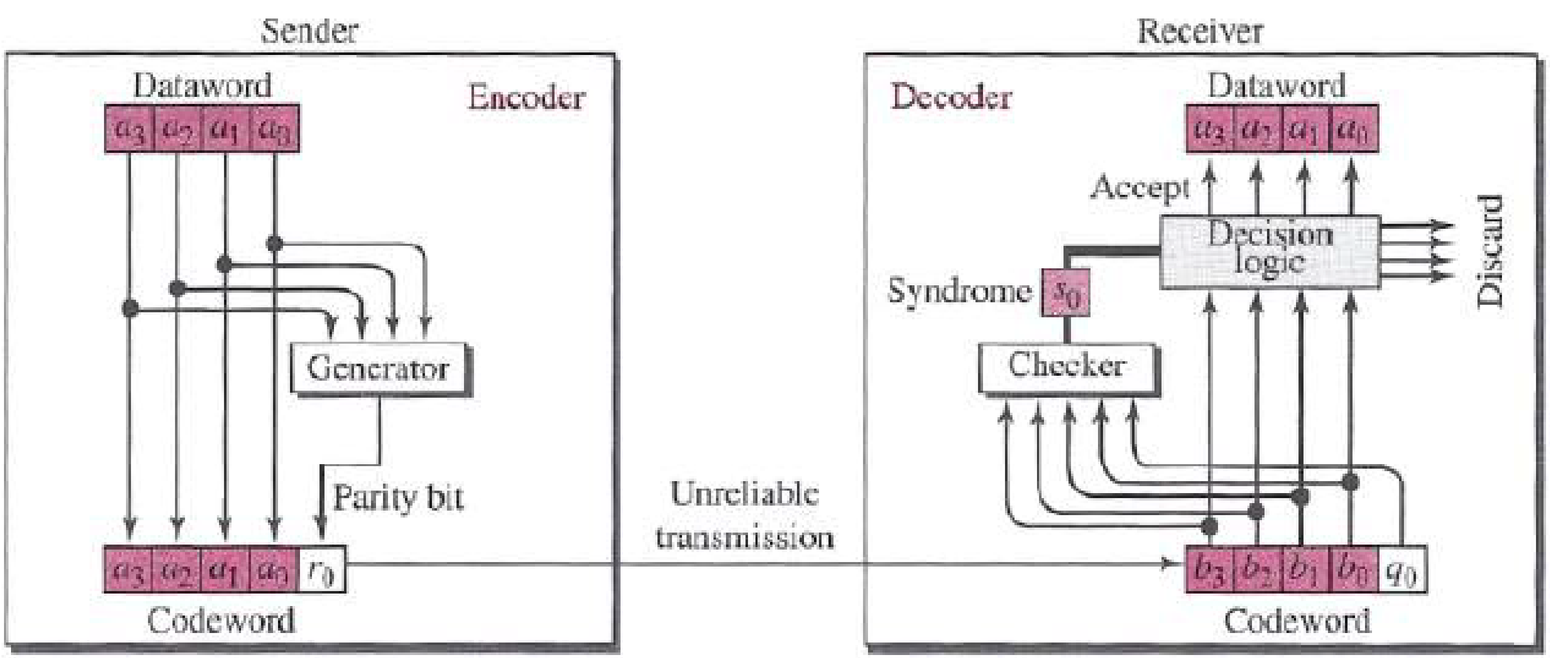
- A linear block code is a code in which the exclusive OR (addition modulo-2) of two valid codewords creates another valid codeword.
- Example:
  - Reed-Solomon codes
  - Hamming codes
  - **Parity Check Code**.

Dataword	Codeword
00	000
01	011
10	101
11	110

- Minimum Hamming distance: number of 1s in the nonzero valid codeword with the smallest number of 1s.

Parity Check Code

Dataword	Codeword	Dataword	Codeword
0000	00000	1000	10001
0001	00011	1001	10010
0010	00101	1010	10100
0011	00110	1011	10111



Parity Check Code

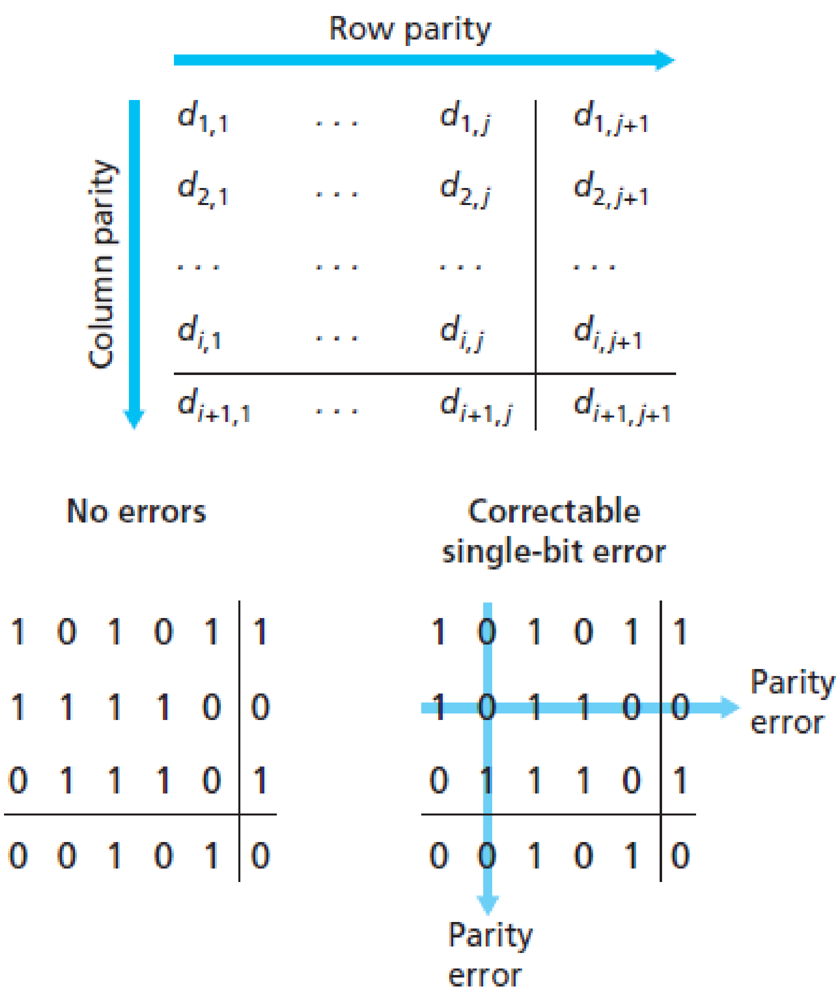
- Modulo arithmetic:
- Generator:

  $r_0 = a_3 + a_2 + a_1 + a_0$ (modulo-2)
- Checker:

  $s_0 = a_3 + a_2 + a_1 + a_0 + q_0$ (modulo-2)

- A parity-check code can detect an odd number of errors.
- what happens if an even number of bit errors occur?
  - a more robust error-detection scheme is needed

Insight into Error-Correction



- two-dimensional parity scheme
- The receiver can thus not only *detect* but can *identify* the bit that has corrupted.
- The ability of the receiver to both detect and correct errors is known as forward error correction (FEC).

Cyclic Redundancy Check (CRC)

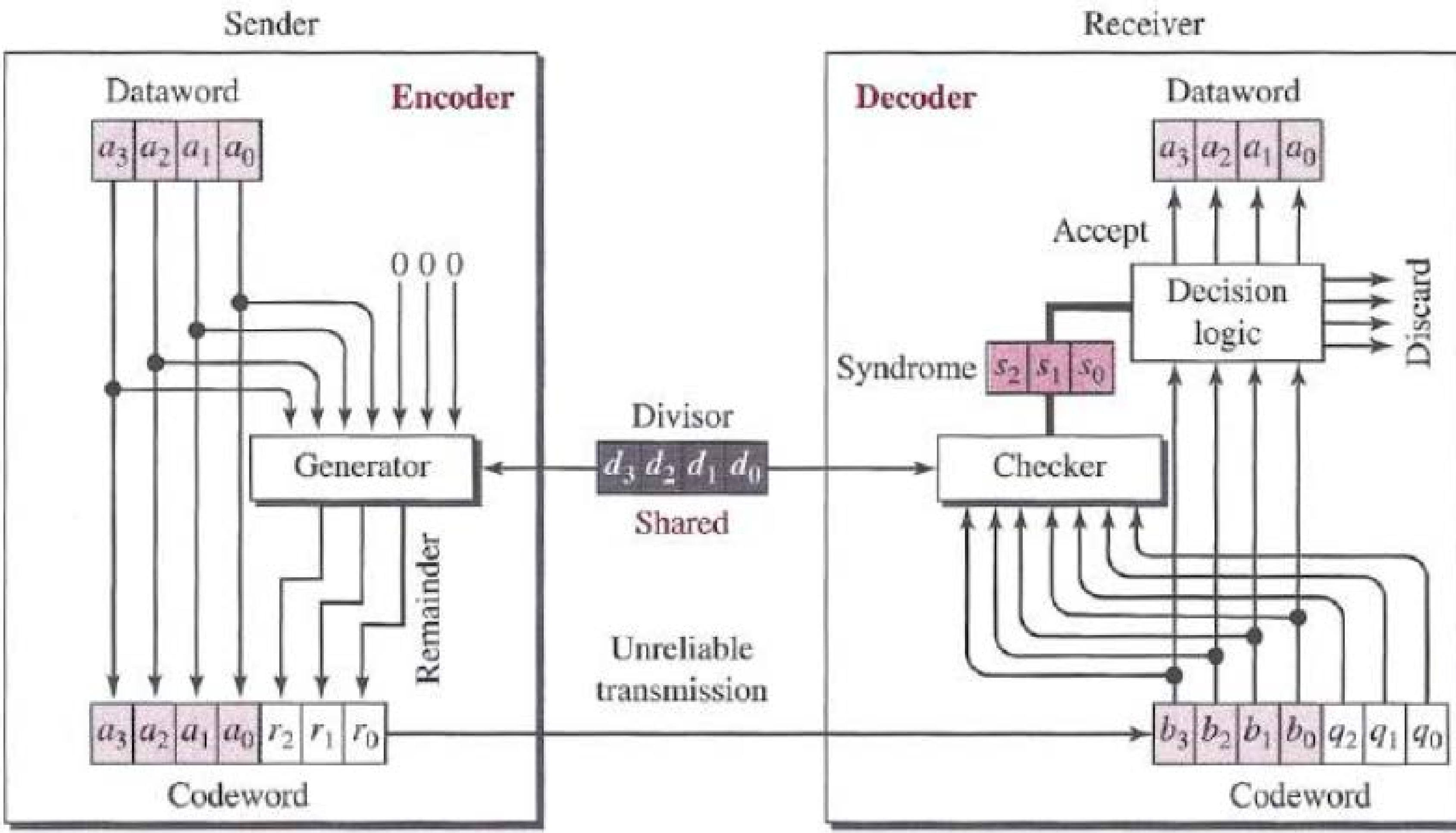
- It is an error-detection technique used widely in today's computer networks (e.g., LAN, WAN)

- It is linear block code

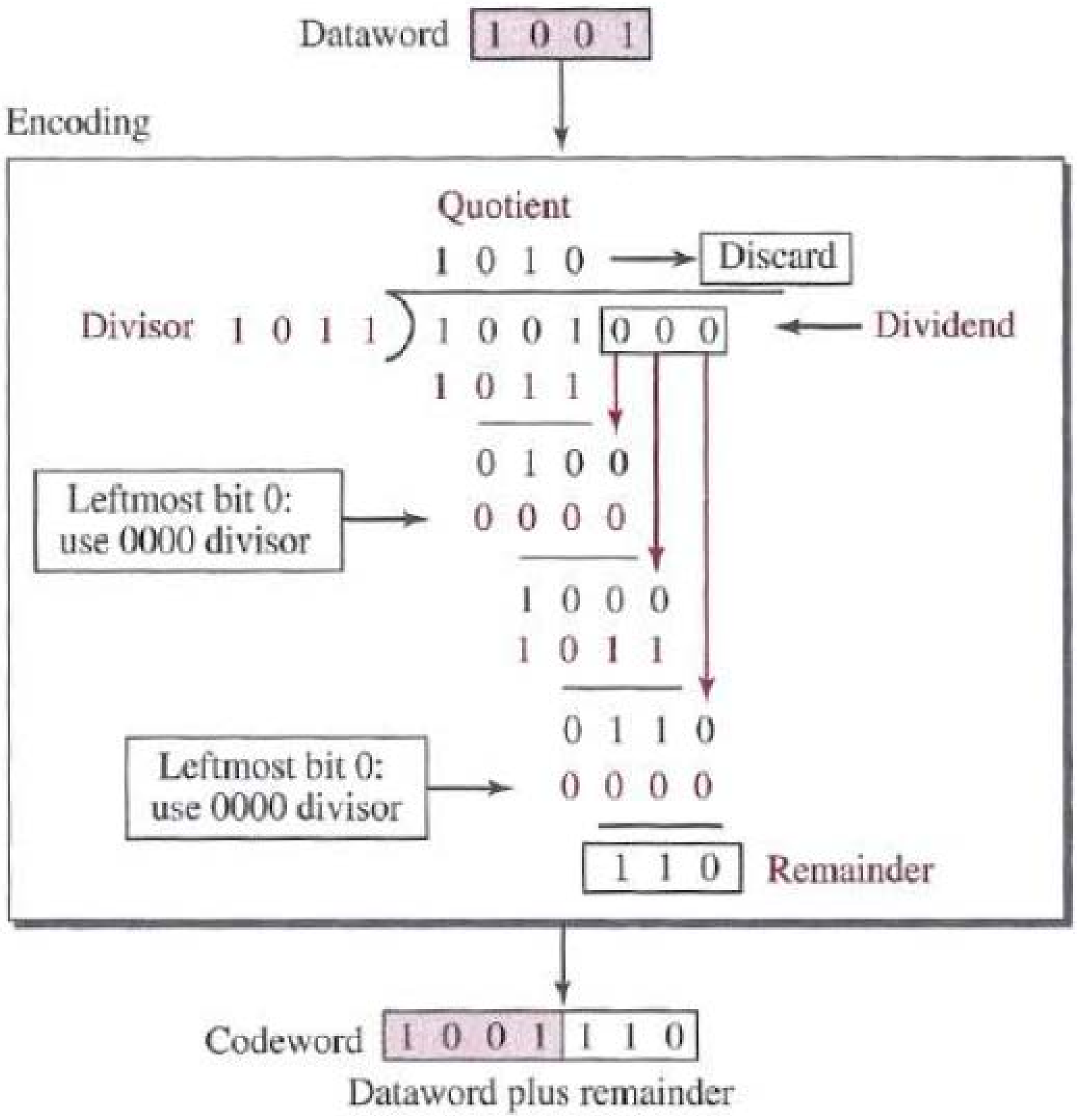
- If a codeword is cyclically shifted (rotated), the result is another codeword.
  - E.g., if **1011000** is a codeword and we cyclically left-shift, then **0110001** is also a codeword.

CRC code with C(7,4)

- C(7,4) => 4 bits dataword, 7 bits codeword



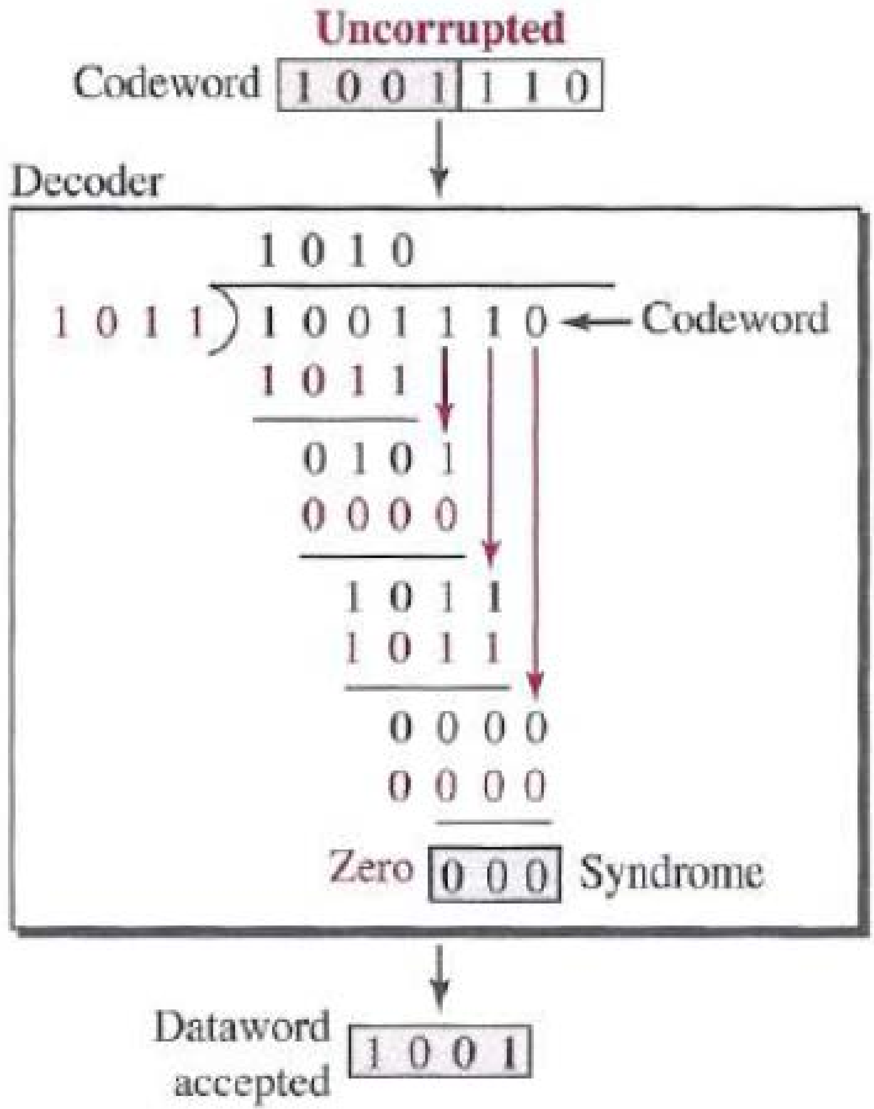
CRC Encoding



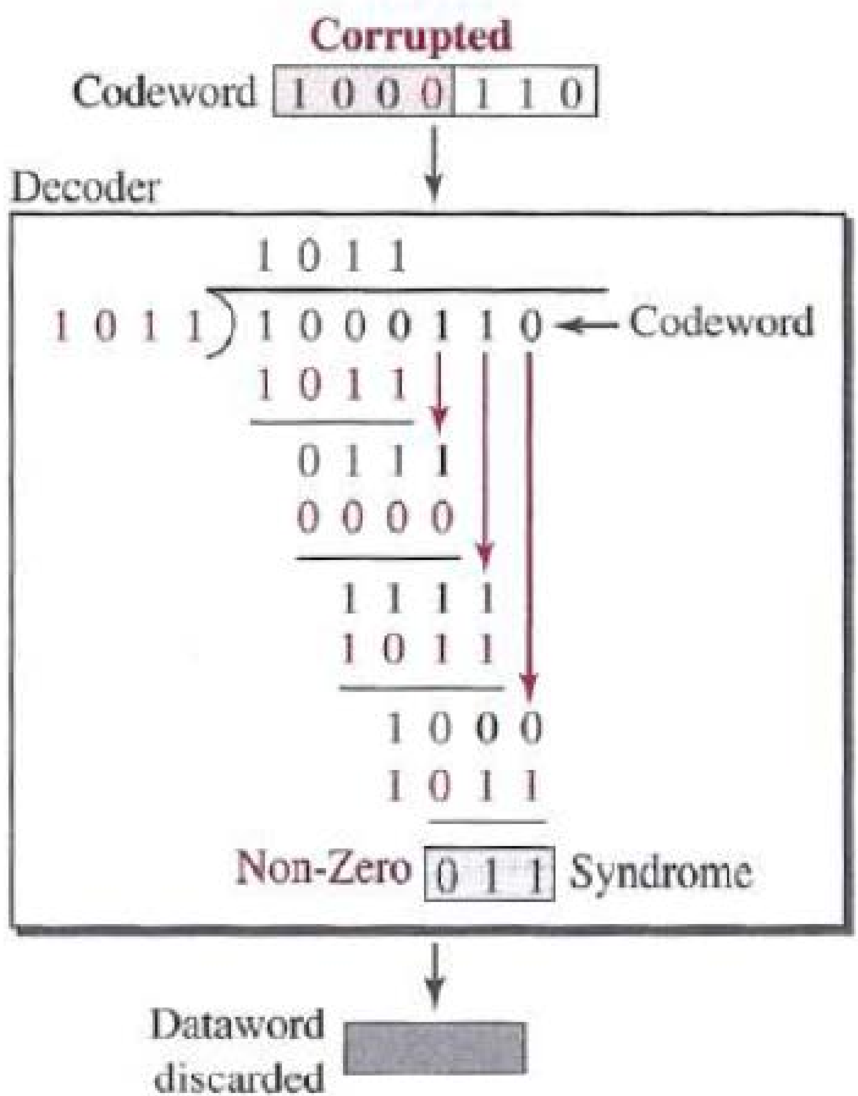
Note:

Multiply: AND
Subtract: XOR

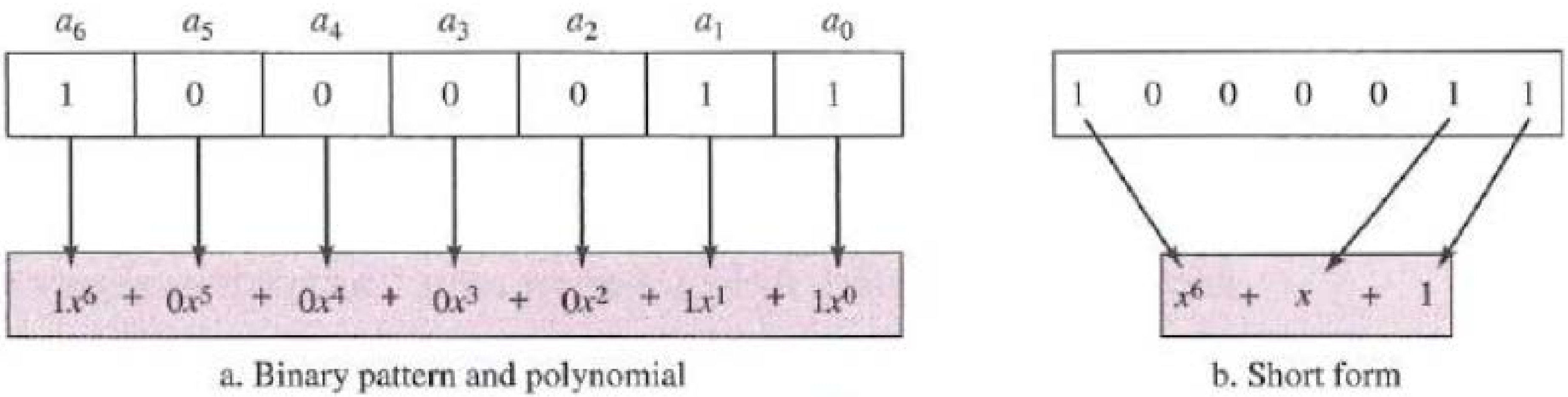
CRC Decoding



CRC Decoding



Polynomial Representation



- The power of each term shows the position of the bit
- The coefficient shows the value of the bit

- The degree of a polynomial is the highest power in the polynomial.

Polynomial Operations

- Adding and Subtracting Polynomials
  - Not same as it is performed in mathematics
  - addition and subtraction are the same
  - adding or subtracting is done by combining terms and deleting pairs of identical terms
  - E.g., $x^5+x^4+x^2 + x^6+x^4+x^2 \Rightarrow x^6+x^5$
- Multiplying or Dividing Terms
  - just add the powers
  - E.g., $x^4 * x^3 \Rightarrow x^7$ $x^7/x^3 \Rightarrow x^4$

Cont…

- Multiplying Two Polynomials
  - is done term by term
  - E.g.,

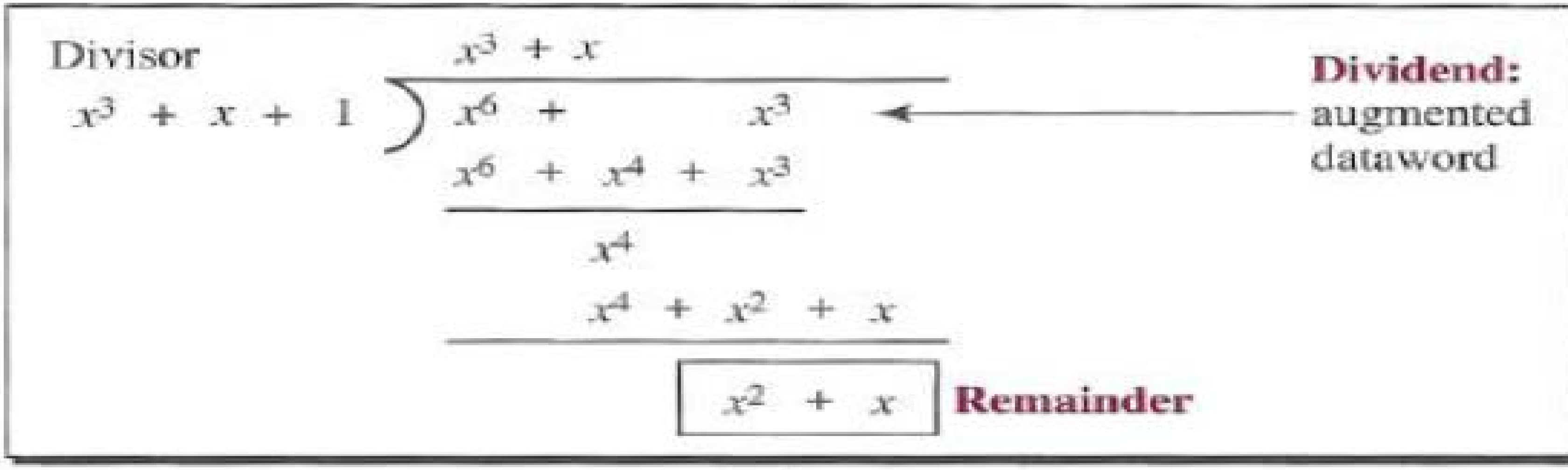
$(x^5+x^3+x^2+x)(x^2+x+1)$

$\Rightarrow(x^7+x^6+x^5)+(x^5+x^4+x^3)+(x^4+x^3+x^2)+(x^3+x^2+x)$

$\Rightarrow x^7+x^6+x^3+x$

- Dividing One Polynomial by Another
  - Division of polynomials is conceptually the same as the binary division we discussed for an encoder.

Cont…

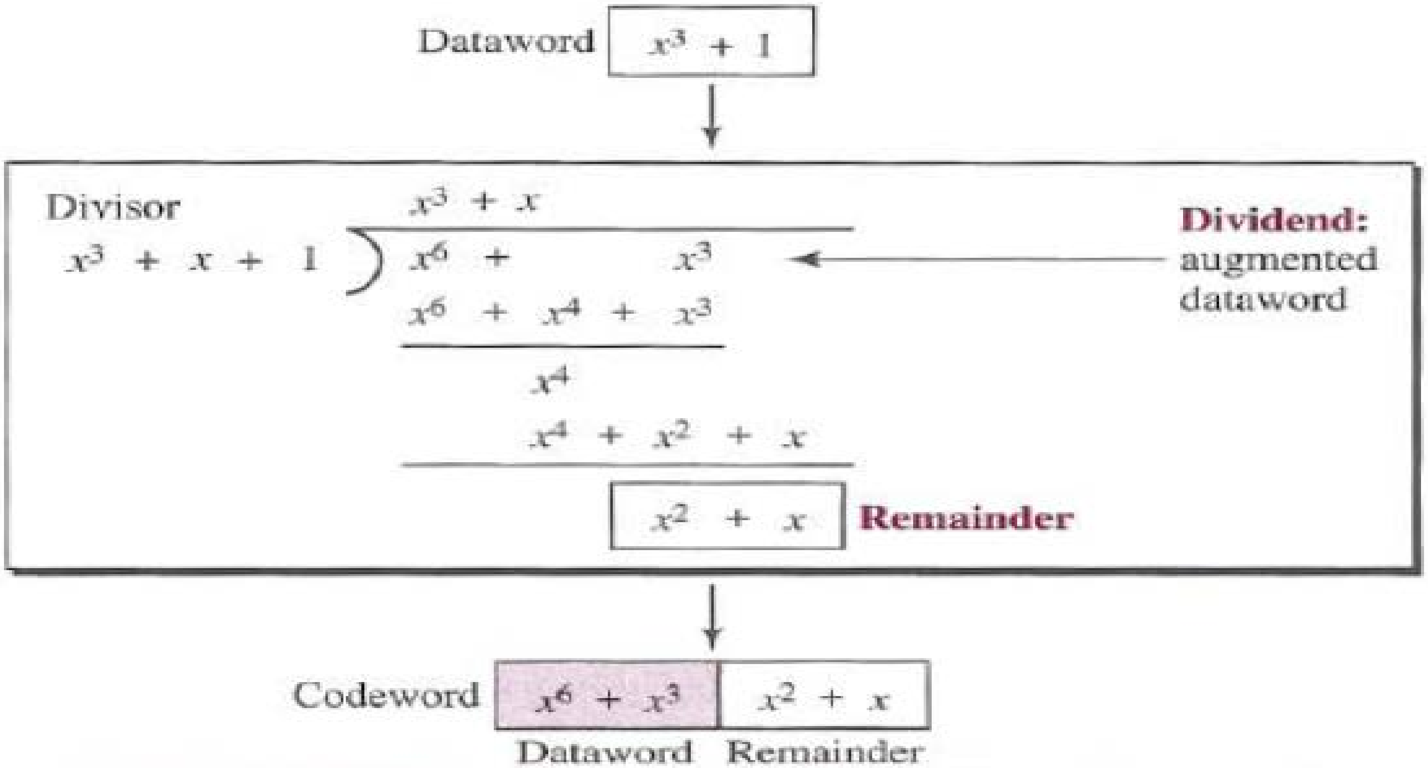


- Shifting

Shifting left 3 bits: 10011 becomes 10011000 $\quad x^4 + x + 1$ becomes $x^7 + x^4 + x^3$

Shifting right 3 bits: 10011 becomes 10 $\quad x^4 + x + 1$ becomes x

Cyclic Code Encoder Using Polynomials



- The divisor in a cyclic code is normally called the *generator polynomial* or simply the *generator.*

Standard Polynomials for CRC

- The divisor in a cyclic code is normally called the *generator polynomial* or simply the *generator*.

Name	Polynomial	Used in
CRC-8	$x^8 + x^2 + x + 1$ 100000111	ATM header
CRC-10	$x^{10} + x^9 + x^5 + x^4 + x^2 + 1$ 11000110101	ATM AAL
CRC-16	$x^{16} + x^{12} + x^5 + 1$ 10001000000100001	HDLC
CRC-32	$x^{32} + x^{26} + x^{23} + x^{22} + x^{16} + x^{12} + x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^2 + x + 1$ 100000100110000010001110110110111	LANs

Divisor Polynomial Selection

- This depends on the expectation we have from the code.
- Let, Dataword: *d(x),* Codeword: *c(x),* Generator: *g(x),* Syndrome: *s(x),* Error: *e(x)*

- If *s(x)* $\neq 0$ --> one or more bits is corrupted
- If *s(x)* == 0 --> either no bit is corrupted or the decoder failed to detect any errors

$$\text{Received codeword} = c(x) + e(x)$$

Cont..

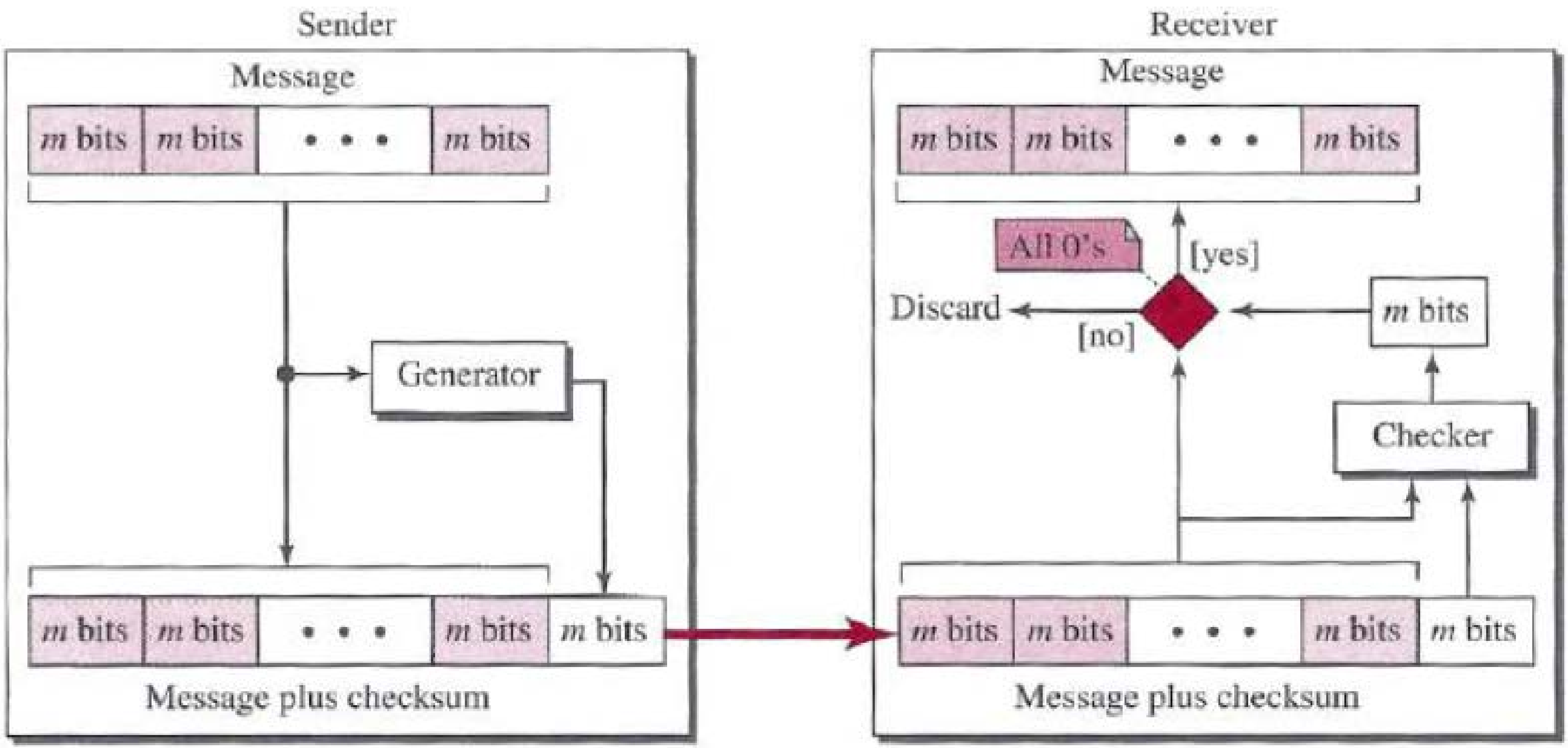
$$\frac{\text{Received codeword}}{g(x)} = \frac{c(x)}{g(x)} + \frac{e(x)}{g(x)}$$

- Those errors that are divisible by *g(x)* are not caught.

- A good polynomial generator needs to have the following characteristics:
  - It should have at least two terms.
  - The coefficient of the term x^0 should be 1.
  - It should not divide $x^t + 1$, for t between 2 and $n - 1$.
  - It should have the factor $x + 1$.

Checksum

- Error detection technique
- Used in Network and Transport Layer



Example

- Want to send a message represented by five 4-bit units (7,11,12,0,6)
- We send (7,11,12,0,6,36)
- Receiver checks (7+11+12+0+6-36)=?

- Problem:
  - Bits required to present the units and checksum
- Solution using One's complement
  - 7,11,12,0,6 require 4 bits
  - Change the 36 (=100100) as follows: (10+0100) = 6; One's complement of 6 is 9
  - Send (7,11,12,0,6,6) OR (7,11,12,0,6,9)

Internet Checksum

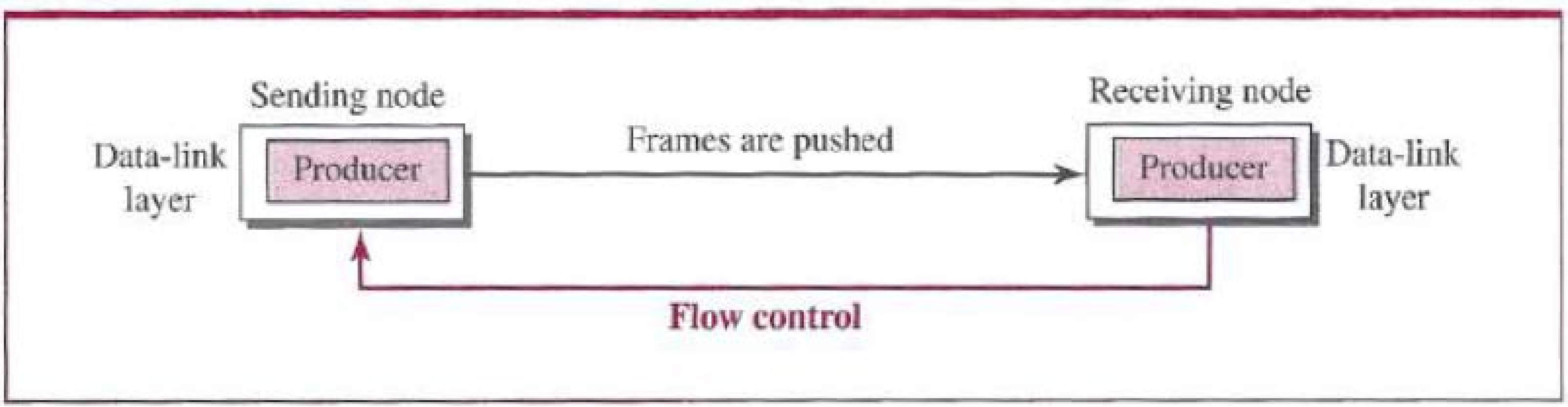
- Used a 16-bit checksum

- Advantage:
  - Fast checking
  - Small number of bits required
- Disadvantage:
  - Not robust
  - e.g., if the values of several words are incremented but the sum and the checksum do not change, the errors are not detected.
  - Solution: weighted checksum proposed by Fletcher and Adler

Error Correction

- Two ways:
  - Forward Error Correction (FEC)
  - Retransmission

Flow Control: production and consumption rates must be balanced



Thanks!