SECOND SEMESTER 2020-21 COURSE HANDOUT

Date: 18.01.2021

In addition to part I (General Handout for all courses appended to the Time table) this portion gives further specific details regarding the course.

Course No : MATH F341

Course Title : INTRODUCTION TO FUNCTIONAL ANALYSIS

Instructor-in-Charge : ASHISH TIWARI

Instructor(s) : Tutorial/Practical Instructors:

- **1. Course Description:** Normed linear spaces, Banach Spaces; Continuous Linear transformations, open mapping theorem, closed graph theorem, uniform boundedness principle, Hahn Banach theorem, Hilbert space theory, dual space, direct sum and orthogonal compliment in Hilbert spaces, function spaces, symmetric and self adjoint linear mapping in Hilbert spaces, finite rank and compact transformations, spectral theory, differential equations and linear transformations
- 2. Scope and Objective of the Course: Functional analysis plays an important role in the applied sciences as well as in mathematics itself. This course is intended to familiarize the student with the basic concepts, principles and methods of functional analysis and its applications in a form suitable for engineers, scientists and applied Mathematicians. Ideas are not always generated by logical process. An engineer may have a feeling for a problem which may lead him in a method of solution but justifying part of that needs analysis. Several concepts of functional analysis were invented as there was need from integral and differential equations. Functional analysis is needed in numerical analysis and differential equations. Modern theory of partial differential equations relies heavily on functional analysis. Theoretical study of numerical solution of partial differential equations is also based on functional analysis.
- **3. Text Books**: Kreyzig E., Introductory Functional Analysis with Applications, Wiley, 2017.

4. Reference Books:

- C. Colin, Numerical Functional Analysis, Oxford University Press, 1982.
- Arch W. Naylor and George R. Sell, Linear operator theory in Engineering & Science: Applied Mathematical Sciences, Springer- Verlag, 1982.
- B. V. Limaye, Functional Analysis, 3rd Edition, New Age International Ltd, 2017.
- M. T. Nair, Functional Analysis: A First Course, Prentice-Hall of India, New Delhi, 2010.

5. Course Plan:

Module No.	Lecture Session	Reference from text book	Learning outcomes	
1.	L1-2. Revision and Self Study	Sec. 1.1-1.6	Review of Metric spaces and introduction of normed spaces	
	L3-4 Normed spaces and Banach spaces and examples	Sec. 2.1-2.2		



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2.	L5-6 Completion of Normed	Sec. 2.3	Completeness and Compactness of normed spaces and applications		
	L7-8 Finite dimensional normed space and compact space	Sec. 2.4-2.5			
3.	L9-11 Linear Operators	Sec. 2.6-2.7	By the end of module, students will understand the importance of operators and functionals defined over normed spaces		
	L12. Linear Functional	Sec. 2.8			
	L13-14 Linear operators and functionals on finite dimensional normed space and dual space	Sec. 2.9-2.10			
4	L15. Inner Product Space and Hilbert space and examples	Sec. 3.1	By the end of the module,		
	L16-17 Properties of IPS	Sec. 3.2-3.3	student will be able to identify the relation		
	L18-19. Orthonormal sets and sequences, series related to orthonormal sets, tototal orthonormal sets	Sec. 3.4-3.6	between metric space, normed space and inner product space		
5	L20. Functionals on Hilbert spaces	Sec. 3.8	Study of functionals and operators defined on Hilbert spaces		
	L21-22 Hilbert adjoint operator and self-adjoint, unitary and normal operators	Sec. 3.9-3.10			
6.	L23-24. Hahn Banach Theorem and its applications	Sec. 4.2-4.4			
	L25-27 Adjoint operators and reflexive spaces, uniform boundedness theorem	Sec. 4.5-4.7	At the end of the module, student will be familiar with the important theorems on normed and Banach spaces		
	L28-29 strong and weak convergence, convergence of sequence of operators and functionals and applications	Sec. 4.8-4.10			
	L30-31 Open mapping theorem and closed graph theorem	Sec. 4.12-4.13			
7	L32-33. Spectral theory in finite dimensional normed spaces	Sec. 7.1-7.4	In this module, student will study the spectral theory of linear operators defined on finite dimensional normed spaces and spectral properties of compact linear operators.		
	L34-35 Compact linear operators on normed spaces	Sec. 8.1-8.2			
	L36-37 Spectral properties of compact linear operators	Sec. 8.3-8.4			
8	L38. Spectral properties of bounded self-adjoint linear operators	Sec. 9.1-9.2	In this module, student will learn about positive operators and projection operators and their properties		
	L39-40. Positive operators, Projection operators	Sec. 9.3-9.6			

6. Evaluation Scheme:

Component	Duration	Weightage (%)	Date & Time	Nature of component (Close Book/ Open Book)
Mid-Semester Test	90 Min.	35%	TBA	Open Book
Comprehensive Examination	2 h	40%	10/5 FN	Open Book
Quizzes/Assignment		25%	Regular (may be conducted during regular class or common hour)	Open Book

- **7. Chamber Consultation Hour**: To be announced in the class.
- **8. Notices:** All notices regarding this course will be displayed on Department of Mathematics Notice Board or on NALANDA or Google Classroom. Normally information will be conveyed in the class.

9. Make-up Policy:

- (i) NO MAKE UP will be given in Class Performance Tests under any circumstances.
- (ii) Make up of other evaluative components (Mid Sem. and Comprehensive Exam) will be granted only in **genuine cases**. **Permission must be taken in advance** except in extreme cases.
- (iii) No MAKE-MAKE-UP will be entertained.

Instructor-in-charge

Course No. MATH F341