GP503

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Assignment - Computing ACF & CCF of time series signals

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[2]:
```
import numpy as np
import matplotlib.pyplot as plt
```

Code

The Correlation class below implements the following methods-

- **init**(self, algo): Initializes a Correlation object. **algo** = **'acf'** or **'ccf'**.
- **call**(self, x, tau, y): This function takes 2 parameters if algo = **acf** and 3 parameters if algo = **ccf** and one additional parameter **tau**(lag) for which the **acf** or **ccf** needs to be printed.

Assignment 01 functions-

- **acf**(self, x): Returns the normalized acf of a time series signal **x** over integer values in the interval (-100, 100).
- **ccf**(self, x, y): Returns the normalized ccf of time series signal **x** & **y** over integer values in the interval (-100, 100).

Assignment 02 functions-

- **autocorrelate**(self,x): Returns the acf computed using **numpy.correlate**(x, x, 'full') after normalization.
- **crosscorrelate**(self, x, y): Returns the ccf of x & y computed using **numpy.correlate**(x, y, 'full') after normalization.

Plotting function-

- **plot**(self): Plots the values of acf/ccf w.r.t integer values in the interval (-100, 100).

Algorithms

- Let **X** and **Y** be two distinct time series signals of dimensions 1xM and 1xN respectively.

Computng ACF

- We iterate using a variable $\tau \,\epsilon\, [0, M-1]$ such that-

  $\phi(\tau) = \mathbf{X}[\tau{:}]\ \mathbf{X'}[{:}M-\tau]$

- Then we use the property $\phi(\tau) = \phi(-\tau)$ to get the remaining values in the ACF.

Computng CCF

- First iteration with variable $\tau \,\epsilon\, [0, M-1]$, we obtain-

  $\phi(\tau) = \mathbf{X}[\tau{:}N+\tau]\ \mathbf{Y'}[{:}\mathrm{min}(size(\mathbf{X}[\tau{:}N+\tau]), \mathrm{N})]$

- We store all these values in a list (say A).
- Second iteration with variable $i \,\epsilon\, [0, N-1]$, we obtain-

  $\mu(\tau) = \mathbf{Y}[i{:}M+i]\ \mathbf{X'}[{:}\mathrm{min}(size(\mathbf{Y}[i{:}M+i]), \mathrm{M})]$

- We store all these values in a list (say B).
- On reverting B we get $\phi(-\tau)$ where $\tau \,\epsilon\, [0, N-1]$
- We combine A and B and remove one instance of $\phi(0)$ as it is present in both A and B to obtain the final CCF over all the valid integer values of τ i.e. $\tau \,\epsilon\, [-N+1, M-1]$.

Finally we divide all the values obtained by either **ACF** or **CCF** by the maximum value of list to normalize the function as follows-

$\phi(\tau_i) = \frac{\phi(\tau_i)}{\max_{\tau_j}(\phi(\tau_j))}$

[13]:
```
class Correlation(object):

    def __init__(self, algo):
        self.outputs = None
        self.tau = None
        self.step = 0
        self.algo = algo

    def __call__(self, x, tau, y=[]):
        self.tau = tau
        self.myOps = []
        self.ibOps = []

        self.abscissa = []
```

```python
        if self.algo == 'acf':
            self.myOps = self.acf(x)
            self.ibOps = self.autocorrelate(x)

            if abs(tau) >= x.shape[0]:
                myOut = 0
                ibOut = 0
            else:
                myOut = self.myOps[tau + x.shape[0] -1]
                ibOut = self.ibOps[tau + x.shape[0] -1]

            if 2*x.shape[0]-1 > 200:
                self.myAbscissa = self.myOps[x.shape[0]-101:x.shape[0]+99]
                self.ibAbscissa = self.myOps[x.shape[0]-101:x.shape[0]+99]
                self.ordinates = np.arange(-100, 100)
            else:
                self.myAbscissa = self.myOps
                self.ibAbscissa = self.myOps
                self.ordinates = np.arange(-x.shape[0]+1, x.shape[0])

            print('\n\tACF value (my implementation)    for %d -> %f' %(tau,␣
↪myOut))
            print('\n\tACF value (numpy implementation) for %d -> %f' %(tau,␣
↪ibOut))

            self.plot()

        else:
            if len(y) == 0:
                print('Correlation("ccf") take 3 positional arguments but 2␣
↪were given\n')
            else:
                self.myOps = self.ccf(x, y)
                self.ibOps = self.crosscorrelate(x, y)

                if tau <= -y.shape[0] or tau >= x.shape[0]:
                    myOut1 = 0
                    ibOut1 = 0
                else:
                    myOut1 = self.myOps[tau + y.shape[0] - 1]
                    ibOut1 = self.ibOps[tau + y.shape[0] -1]
                if -tau <= -y.shape[0] or -tau >= x.shape[0]:
                    myOut2 = 0
                    ibOut2 = 0
                else:
                    myOut2 = self.myOps[-tau + y.shape[0] - 1]
                    ibOut2 = self.ibOps[-tau + y.shape[0] -1]
```

```
            if y.shape[0] > 100:
                if x.shape[0] > 100:
                    self.myAbscissa = self.myOps[y.shape[0]-101:y.
↪shape[0]+99]
                    self.ibAbscissa = self.myOps[y.shape[0]-101:y.
↪shape[0]+99]
                    self.ordinates = np.arange(-100, 100)
                else:
                    self.myAbscissa = self.myOps[y.shape[0]-101:]
                    self.ibAbscissa = self.myOps[y.shape[0]-101:]
                    self.ordinates = np.arange(-100, x.shape[0])
            else:
                if x.shape[0] > 100:
                    self.myAbscissa = self.myOps[:y.shape[0]+99]
                    self.ibAbscissa = self.myOps[:y.shape[0]+99]
                    self.ordinates = np.arange(-y.shape[0]+1, 100)
                else:
                    self.myAbscissa = self.myOps
                    self.ibAbscissa = self.myOps
                    self.ordinates = np.arange(-y.shape[0]+1, x.shape[0])

            print('\n\tCCF value (my implementation)    for %d    -> %f'␣
↪%(tau, myOut1))
            print('\n\tCCF value (numpy implementation) for %d -> %f'␣
↪%(tau, ibOut1))
            print('\n\tCCF value (my implementation)    for %d    -> %f'␣
↪%(-tau, myOut2))
            print('\n\tCCF value (numpy implementation) for %d -> %f'␣
↪%(-tau, ibOut2))

            self.plot()

    def acf(self, x):
        out = np.zeros(x.shape[0])
        for i in range(x.shape[0]):
            out[i] = x[i:] @ x[:x.shape[0] - i]

        out = np.flip(out).tolist()[:-1] + out.tolist()
        # print(out)
        return np.array(out) / max(out)

    def ccf(self, x, y):
        out = np.zeros(x.shape[0])
        out_pre = np.zeros(y.shape[0])
        for i in range(x.shape[0]):
```

```
            j = y.shape[0] + i
            k = min(x[i:j].shape[0], y.shape[0])
            out[i] = x[i:j] @ y[:k]

        for i in range(y.shape[0]):
            j = x.shape[0] + i
            k = min(y[i:j].shape[0], x.shape[0])
            out_pre[i] = y[i:j] @ x[:k]

        out = np.flip(out_pre).tolist()[:-1] + out.tolist()
        # print(out)
        return np.array(out) / max(out)

    def autocorrelate(self, x):
        out = np.correlate(x, x, 'full')
        # print(out)
        return out / max(out)

    def crosscorrelate(self, x, y):
        out = np.correlate(x, y, 'full')
        # print(out)
        return out / max(out)

    def plot(self):
        if self.algo == 'acf':
            msg = r'$\phi(\tau)$ (ACF values)'
        else:
            msg = r'$\phi(\tau)$ (CCF values)'

        line1, = plt.plot(self.ordinates, self.myAbscissa, 'y-')
        line2, = plt.plot(self.ordinates, self.ibAbscissa, 'r:', ms=10)
        plt.ylabel(msg)
        plt.xlabel(r'$\tau$ (Time Lag)')
        plt.legend((line1, line2), ('Implemented function', 'Inbuilt function'))

        plt.show()
```

Input format-

1. Run the file in the terminal using `python3 assign1.py`
2. Enter **0** for providing input using a **.txt file** or, **1** for providing input in terminal.

Terminal input format-

1. Enter three space seperated integers **least__value max__value size** for series A.
2. Enter an integer specifying the value of **tau**.
3. Enter the algo type **acf** or **ccf**.
   - If algo type **ccf**, enter three space seperated integers **least__value max__value size** for series B.

Input file format-

Multiple inputs can be provided at once and plots of all can be viewed in a sequential manner. Sample input format shown below-

```
0 20 10
5
acf
0 100 500
65
ccf
0 50 75
```

> Line 1 specifies the **least__value max__value size** for series A for first input. Line 2 is the value of **tau**. Line 3 is the algo type. If **acf** then first input ends here.
>
> Line 4 specifies the **least__value max__value size** for series A for secod input. Line 5 is the value of **tau**. Line 6 is the algo type. If **ccf** then the next line should contain **least__value max__value size** for series B.

Sample input file (input.txt)

```
-500 500 5000
4000
acf
-500 500 5000
-4000
acf
-500 500 5000
4000
ccf
-500 500 5000
-500 500 5000
-4000
ccf
-500 500 5000
```

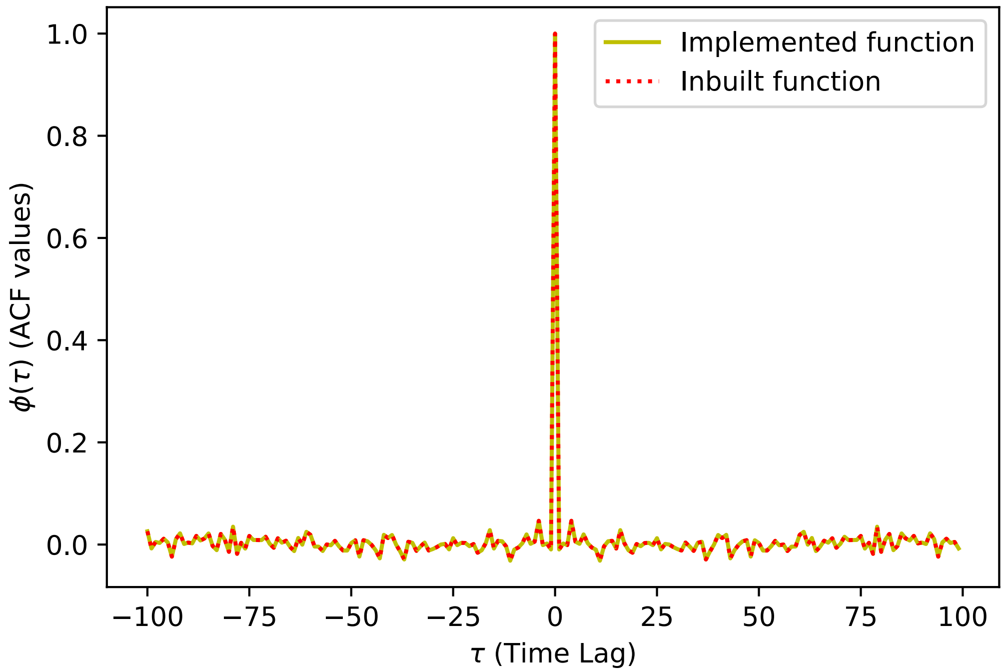
After we run the code for this sample input we get the plots as shown below-

[14]:
```
if __name__ == '__main__':
    option = bool(int(input('Choose one of the following options \n\t 0 -> Use␣
↪a text file to provide input\n\t 1 -> Provide input in terminal\n')))
    np.random.seed(99)
    if option:
        first_samp_i, last_samp_i, num_samp_i = [int(i) for i in input("Enter␣
↪LeastValue  MaximumValue  Size for sequence A:\n").strip().split()]
        data_i = np.random.randint(first_samp_i, last_samp_i, num_samp_i)
        # print(data_i)
        tau = int(input('Enter a value for lag(tau):\n'))
        algo = str(input("Enter the type of Correlation, 'acf' or 'ccf'\n"))
        if algo == 'ccf':
            first_samp_j, last_samp_j, num_samp_j = [int(i) for i in␣
↪input("Enter LeastValue  MaximumValue  Size for sequence B:\n").strip().
↪split()]
            data_j = np.random.randint(first_samp_j, last_samp_j, num_samp_j)
            # print(data_j)
            corr = Correlation('ccf')
            corr(data_i, tau, data_j)
        elif algo == 'acf':
            corr = Correlation('acf')
            corr(data_i, tau)
        else:
            print('UndefinedParameter passed. Valid values are "acf" and␣
↪"ccf"\n')
    else:
        f = str(input('Provide location of the input file\n').strip())
        file = open(f, 'r')
        line = file.readline()
        while line:
            first_samp_i, last_samp_i, num_samp_i = [int(i) for i in line.
↪strip().split()]
            data_i = np.random.randint(first_samp_i, last_samp_i, num_samp_i)
            tau = int(file.readline().strip())
            algo = str(file.readline().strip())
            if algo == 'ccf':
                first_samp_j, last_samp_j, num_samp_j = [int(i) for i in file.
↪readline().strip().split()]
                data_j = np.random.randint(first_samp_j, last_samp_j,␣
↪num_samp_j)
                # print(data_j)
                corr = Correlation('ccf')
                corr(data_i, tau, data_j)
            elif algo == 'acf':
                corr = Correlation('acf')
                corr(data_i, tau)
```

```
        else:
            print('UndefinedParameter passed. Valid values are "acf" and␣
↪"ccf"\n')
        line = file.readline()
```

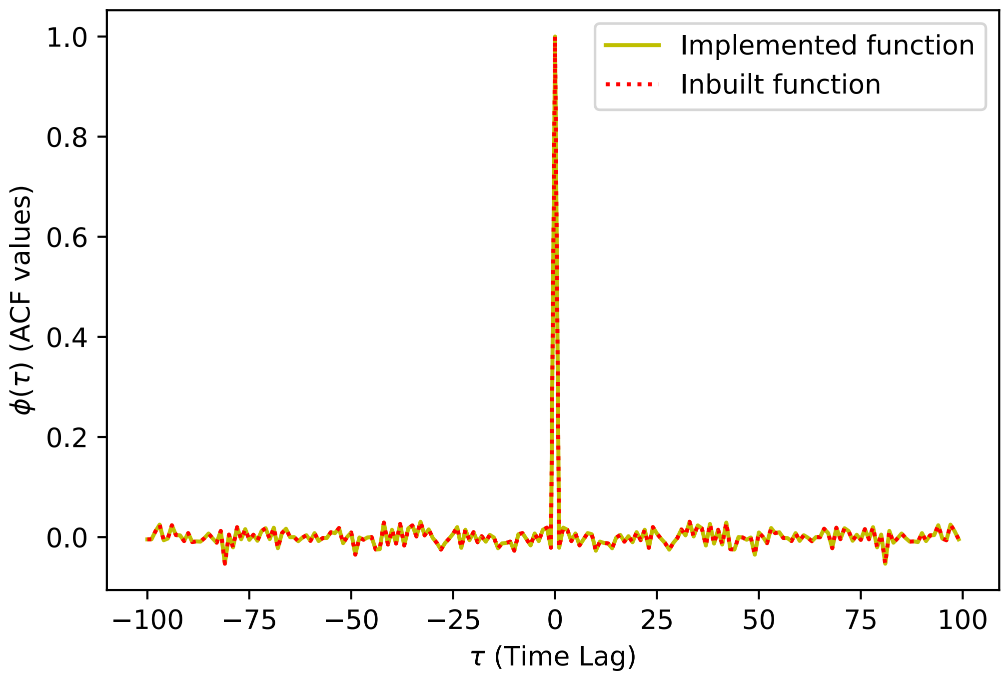
```
ACF value (my implementation)    for 4000 -> -0.002125

ACF value (numpy implementation) for 4000 -> -0.002125
```

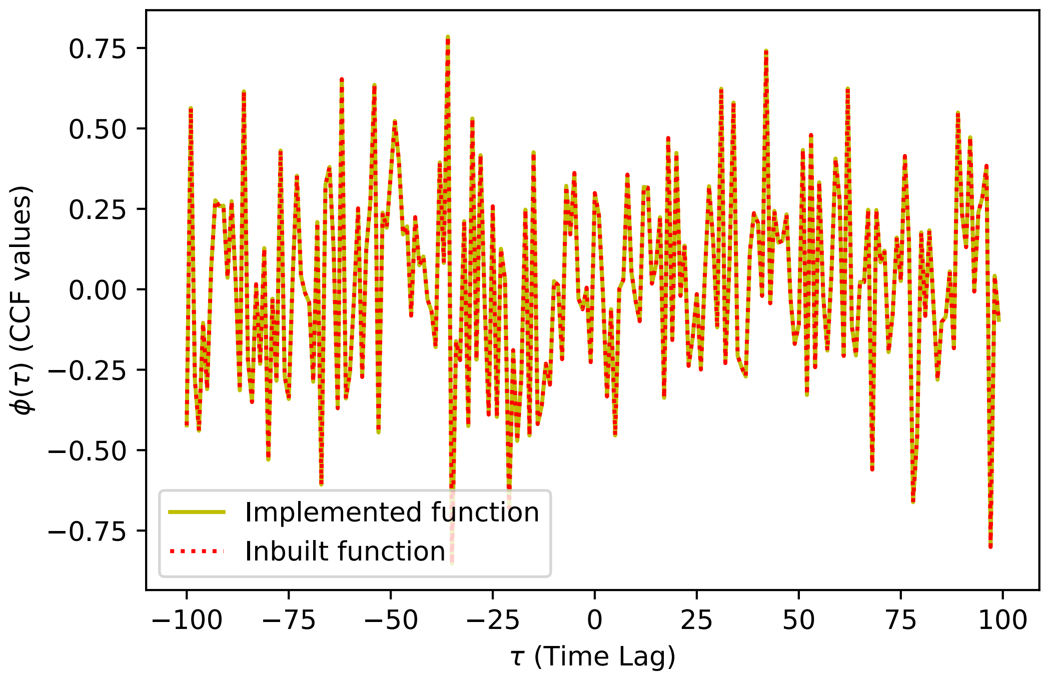



```
ACF value (my implementation)    for -4000 -> 0.000945

ACF value (numpy implementation) for -4000 -> 0.000945
```




```
CCF value (my implementation)    for 4000    -> 0.112322
CCF value (numpy implementation) for 4000 -> 0.112322
CCF value (my implementation)    for -4000    -> -0.324825
CCF value (numpy implementation) for -4000 -> -0.324825
```

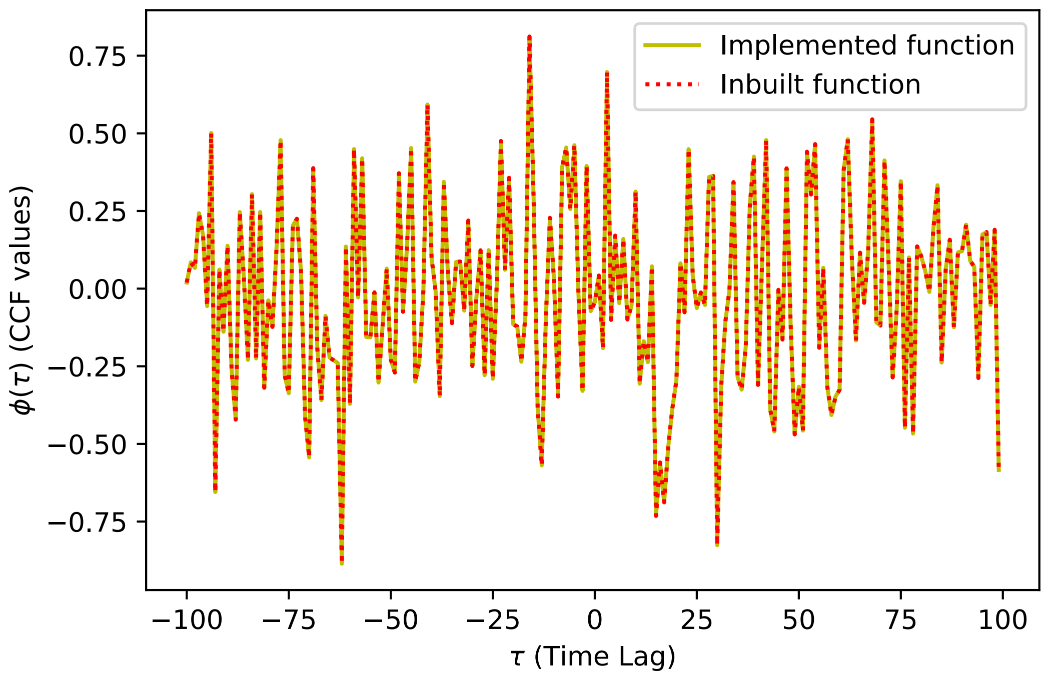



```
CCF value (my implementation)    for -4000    -> 0.018558

CCF value (numpy implementation) for -4000 -> 0.018558

CCF value (my implementation)    for 4000    -> 0.117224

CCF value (numpy implementation) for 4000 -> 0.117224
```



Implemented function
Inbuilt function
$\phi(\tau)$ (CCF values)
τ (Time Lag)
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−100
−75
−50
−25
0
25
50
75
100


```
[ ]:
```