$\delta_i = C_i + \gamma_i - (C_i + \gamma_i)$

$\delta^* = \min \delta_i$ $\qquad 1/(\delta')^2$

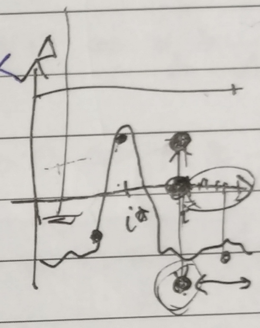
$\hat{u}_{ij}^t = \hat{\gamma}_{ij}^{t-1} + \sqrt{\frac{1.5 \log t}{n_i(t-1)}}$

$u_{ij}^t = \min(u_{ij}^{t-1}, \hat{u}_{ij}^t)$

$O(K$

GLM

Side –



$\star$ $\arg\min \{ i \in [k-1], \; C_j - C_i \geq U_{ij}^t \;\; \forall j > i \}$

$d\sqrt{T}$

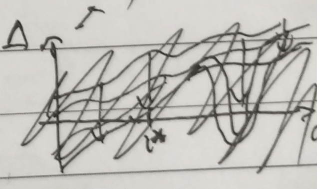
Suppose i is optimal in some t

Can algo get stuck?

$\sqrt{\frac{\log t}{n_i(t)}}$

Algorithm:

$G_t = \{ i \in [K] \mid \forall j \neq i \; : \; C_j - C_i \geq \gamma_{ij}^{UCB}(t) \}$



$P_t = \{ i \in [K] \mid \forall j \neq i \quad C_j - C_i \geq \gamma_{ij}^{LCB}(t) \}$

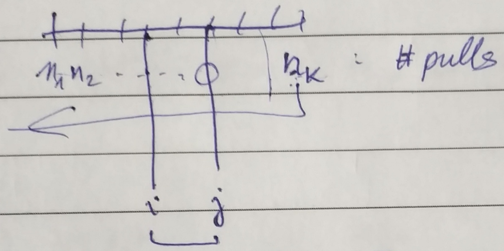
$I_t = \max P_t$

$[K] \setminus P_t$: eliminated arms.

$\sqrt{\frac{2 \log t}{N_i}} + \hat{\gamma}$

$P_t = \{ i \in [K] \mid \hat{\Delta}_i(t) \geq 0 \}$ $\quad \hat{\Delta}_i^{LCB}(t) = \min_{j \neq i} C_j - C_i - \gamma_{ij}^{LCB}(t)$

$I_t = \max(\{ j > \hat{I}_t \mid C_j - C_{\hat{I}_t} - \gamma_{\hat{I}_t j}^{UCB}(t) < 0 \} \cup \{\hat{I}_t\})$ $\quad \Delta^{UCB}(t) = \min_{j \neq i} C_j - C_i - \gamma_{ij}^{UCB}(t)$



$\Delta_{ij} = C_j - C_i - \gamma_{ij}$

$i<j$: γ_{ij} #obs $\cdot\, m_j = \sum_{k \geq j} n_k$

$\boxed{i \text{ optimal} \iff \min_{j\neq i} \Delta_{ij} \geq 0}$

(1) $i = i^*$ $\quad \forall j \neq i$: $C_j - C_i - \gamma_{ij}^{UCB} > 0$

(1.1) $j > i$: $\Delta_{ij} - \frac{1}{\sqrt{m_j}} > 0 \iff m_j \geq \frac{1}{\Delta_{ij}^2}$

(1.2) $j < i$: $\underbrace{\Delta_{ij} - \frac{1}{\sqrt{m_i}} > 0 \iff m_i \geq \frac{1}{\Delta_{ij}^2}}_{m_i \geq \max_{j<i} \frac{1}{\Delta_{ij}^2}}$

j

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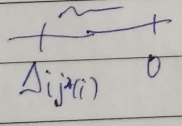
(2) $i \neq i^*$

$i \neq i^*$

$\min_{j\neq i} \Delta_{ij} < 0$

$\arg\min_{j \neq i} \Delta_{ij} =: j^*(i)$



Case 1: $j^*(i) > i$: $\Delta_{ij^*(i)} + \frac{1}{\sqrt{m_{j^*(i)}}} < 0$

(2.1) $m_{j^*(i)} \geq \frac{1}{\Delta_{ij^*(i)}^2}$

Case 2: $j^*(i) < i$: $\Delta_{ij^*(i)} + \frac{1}{\sqrt{m_i}} < 0$

(2.2) $m_i \geq 1/\Delta_{ij^*(i)}^2$.

1.1 $\quad \forall j > i^* \; : \quad m_j \geq \frac{1}{\Delta_{i^*j}^2}$

1.2 $\quad m_{i^*} \geq \max_{j < i^*} \frac{1}{\Delta_{i^*j}^2}$

2.1 $\quad \forall i \neq i^*,$

$\quad$ if $j^*(i) > i \quad : \quad m_{j^*(i)} \geq \frac{1}{\Delta_{ij^*(i)}^2}$

$\quad$ if $j^*(i) < i \quad : \quad m_i \geq \frac{1}{\Delta_{ij^*(i)}^2}$

$$m_j = \sum_{k \geq j} n_k$$

Regret: $\sum_{i \neq i^*} n_i \left(C_i + \gamma_i - (C_{i^*} + \gamma_{i^*}) \right)$