

# Exercício Introdução

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Agosto 2016

## Questão 1

(a)

$$\begin{aligned}\sum_{k=1}^5 (k_i + 1) &= (1 + 1) + \sum_{k=2}^5 (k + 1) \\ &= 2 + (2 + 1) + \sum_{k=3}^5 (k + 1) \\ &= 5 + (3 + 1) + \sum_{k=4}^5 (k + 1) \\ &= 9 + (4 + 1) + \sum_{k=5}^5 (k + 1) \\ &= 14 + (5 + 1) \\ &= 20\end{aligned}\tag{1}$$

(b)

$$\begin{aligned}\sum_{j=0}^4 (-2)^j &= (-2)^0 + \sum_{j=1}^4 (-2)^j \\ &= -2 + (-2)^1 + \sum_{j=1}^4 (-2)^j \\ &= -4 + (-2)^2 + \sum_{j=2}^4 (-2)^j \\ &= 0 + (-2)^3 + \sum_{j=3}^4 (-2)^j \\ &= -8 + (-2)^4 + \sum_{j=4}^4 (-2)^j \\ &= -8 + 16 = 8\end{aligned}\tag{2}$$

(c)

$$\begin{aligned}\sum_{t=1}^{100} 3 &= 3 + \sum_{t=2}^{100} 3 \\ &= 6 + \sum_{t=3}^{100} 3 \\ &= 9 + \sum_{t=4}^{100} 3 \\ &\quad \dots \\ &= 297 + \sum_{t=100}^{100} 3 \\ &= 300\end{aligned}\tag{3}$$

(d)

$$\begin{aligned}\sum_{j=0}^8 (2^{j+1} - 2^j) &= 2^{0+1} + (-2)^0 + \sum_{t=1}^8 (2^{j+1} - 2^j) \\&= 2 + 2^{1+1} + (-2)^1 + \sum_{t=1}^8 (2^{j+1} - 2^j) \\&= 4 + 2^{2+1} + (-2)^2 + \sum_{t=2}^8 (2^{j+1} - 2^j) \\&= 16 + 2^{3+1} + (-2)^3 + \sum_{t=3}^8 (2^{j+1} - 2^j) \\&= 28 + 2^{4+1} + (-2)^4 + \sum_{t=4}^8 (2^{j+1} - 2^j) \\&= 76 + 2^{5+1} + (-2)^5 + \sum_{t=5}^8 (2^{j+1} - 2^j) \\&= 108 + 2^{6+1} + (-2)^6 + \sum_{t=6}^8 (2^{j+1} - 2^j) \\&= 300 + 2^{7+1} + (-2)^7 + \sum_{t=7}^8 (2^{j+1} - 2^j) \\&= 428 + 2^{8+1} + (-2)^8 + \sum_{t=8}^8 (2^{j+1} - 2^j) \\&= 256 + 512 + 428 = 1196\end{aligned}\tag{4}$$

(e)

$$\begin{aligned}\sum_{i=1}^2 \sum_{j=1}^3 (i+j) &= 1+1 + \sum_{i=1}^2 \sum_{j=2}^3 (i+j) \\ &= 2+1+2 + \sum_{i=1}^2 \sum_{j=3}^3 (i+j) \\ &= 5+1+3 + \sum_{i=2}^2 \sum_{j=1}^3 (i+j) \\ &= 9+2+1 + \sum_{i=2}^2 \sum_{j=2}^3 (i+j) \\ &= 12+2+2 + \sum_{i=2}^2 \sum_{j=3}^3 (i+j) \\ &= 16+2+3 = 21\end{aligned}\tag{5}$$

(f)

$$\begin{aligned}\sum_{i=0}^2 \sum_{j=0}^3 (2i + 3j) &= 0 + 0 + \sum_{i=0}^2 \sum_{j=1}^3 (2i + 3j) \\&= 0 + 3 + \sum_{i=0}^2 \sum_{j=2}^3 (2i + 3j) \\&= 3 + 0 + 6 + \sum_{i=0}^2 \sum_{j=3}^3 (2i + 3j) \\&= 9 + 0 + 9 + \sum_{i=1}^2 \sum_{j=0}^3 (2i + 3j) \\&= 18 + 2 + 0 + \sum_{i=1}^2 \sum_{j=1}^3 (2i + 3j) \\&= 20 + 2 + 3 + \sum_{i=1}^2 \sum_{j=2}^3 (2i + 3j) \\&= 25 + 2 + 6 + \sum_{i=1}^2 \sum_{j=3}^3 (2i + 3j) \\&= 33 + 2 + 9 + \sum_{i=2}^2 \sum_{j=0}^3 (2i + 3j) \\&= 44 + 4 + 0 + \sum_{i=2}^2 \sum_{j=1}^3 (2i + 3j) \\&= 48 + 4 + 3 + \sum_{i=2}^2 \sum_{j=2}^3 (2i + 3j) \\&= 55 + 4 + 6 + \sum_{i=2}^2 \sum_{j=3}^3 (2i + 3j) \\&= 65 + 4 + 9 = 78\end{aligned}\tag{6}$$

(g)

$$\begin{aligned}\sum_{i=1}^3 \sum_{j=0}^2 i &= 1 + \sum_{i=1}^3 \sum_{j=1}^2 i \\&= 1 + 1 + \sum_{i=1}^3 \sum_{j=2}^2 i \\&= 2 + 1 + \sum_{i=2}^3 \sum_{j=0}^2 i \\&= 3 + 2 + \sum_{i=2}^3 \sum_{j=1}^2 i \\&= 5 + 2 + \sum_{i=2}^3 \sum_{j=2}^2 i \\&= 7 + 2 + \sum_{i=3}^3 \sum_{j=0}^2 i \\&= 9 + 3 + \sum_{i=3}^3 \sum_{j=1}^2 i \\&= 12 + 3 + \sum_{i=3}^3 \sum_{j=2}^2 i \\&= 15 + 3 = 18\end{aligned}\tag{7}$$

(h)

$$\begin{aligned}\sum_{i=1}^3 \sum_{j=0}^2 j &= 0 + \sum_{i=1}^3 \sum_{j=1}^2 j \\&= 0 + 1 + \sum_{i=1}^3 \sum_{j=2}^2 j \\&= 1 + 2 + \sum_{i=2}^3 \sum_{j=0}^2 j \\&= 3 + 0 + \sum_{i=2}^3 \sum_{j=1}^2 j \\&= 3 + 1 + \sum_{i=2}^3 \sum_{j=2}^2 j \\&= 4 + 2 + \sum_{i=3}^3 \sum_{j=0}^2 j \\&= 6 + 0 + \sum_{i=3}^3 \sum_{j=1}^2 j \\&= 6 + 1 + \sum_{i=3}^3 \sum_{j=2}^2 j \\&= 7 + 2 = 9\end{aligned}\tag{8}$$