Lista de Exercícios 01

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Agosto 2016

Contents

Exercício 1

(a)

$$\sum_{k=1}^{5}(k_i + 1) = (1 + 1) + \sum_{k=2}^{5}(k + 1)$$
$$= 2 + (2 + 1) + \sum_{k=3}^{5}(k + 1)$$
$$= 5 + (3 + 1) + \sum_{k=4}^{5}(k + 1) \tag{1}$$
$$= 9 + (4 + 1) + \sum_{k=5}^{5}(k + 1)$$
$$= 14 + (5 + 1)$$
$$= 20$$

(b)

$$\sum_{j=0}^{4}(-2)^j = (-2)^0 + \sum_{j=1}^{4}(-2)^j$$
$$= -2 + (-2)^1 + \sum_{j=1}^{4}(-2)^j$$
$$= -4 + (-2)^2 + \sum_{j=2}^{4}(-2)^j$$
$$= 0 + (-2)^3 + \sum_{j=3}^{4}(-2)^j \tag{2}$$
$$= -8 + (-2)^4 + \sum_{j=4}^{4}(-2)^j$$
$$= -8 + 16 = 8$$

(c)

$$\sum_{t=1}^{100} 3 = 3 + \sum_{t=2}^{100} 3$$

$$= 6 + \sum_{t=3}^{100} 3$$

$$= 9 + \sum_{t=4}^{100} 3 \tag{3}$$

$$...$$

$$= 297 + \sum_{t=100}^{100} 3$$

$$= 300$$

(d)

$$\sum_{j=0}^{8} (2^{j+1} - 2^j) = 2^{0+1} + (-2)^0 + \sum_{t=1}^{8} (2^{j+1} - 2^j)$$

$$= 2 + 2^{1+1} + (-2)^1 + \sum_{t=1}^{8} (2^{j+1} - 2^j)$$

$$= 4 + 2^{2+1} + (-2)^2 + \sum_{t=2}^{8} (2^{j+1} - 2^j)$$

$$= 16 + 2^{3+1} + (-2)^3 + \sum_{t=3}^{8} (2^{j+1} - 2^j)$$

$$= 28 + 2^{4+1} + (-2)^4 + \sum_{t=4}^{8} (2^{j+1} - 2^j)$$

$$= 76 + 2^{5+1} + (-2)^5 + \sum_{t=5}^{8} (2^{j+1} - 2^j) \tag{4}$$

$$= 108 + 2^{6+1} + (-2)^6 + \sum_{t=6}^{8} (2^{j+1} - 2^j)$$

$$= 300 + 2^{7+1} + (-2)^7 + \sum_{t=7}^{8} (2^{j+1} - 2^j)$$

$$= 428 + 2^{8+1} + (-2)^8 + \sum_{t=8}^{8} (2^{j+1} - 2^j)$$

$$= 256 + 512 + 428 = 1196$$

(e)

$$\sum_{i=1}^{2}\sum_{j=1}^{3}(i+j) = 1 + 1 + \sum_{i=1}^{2}\sum_{j=2}^{3}(i+j)$$

$$= 2 + 1 + 2 + \sum_{i=1}^{2}\sum_{j=3}^{3}(i+j)$$

$$= 5 + 1 + 3 + \sum_{i=2}^{2}\sum_{j=1}^{3}(i+j)$$

$$= 9 + 2 + 1 + \sum_{i=2}^{2}\sum_{j=2}^{3}(i+j) \tag{5}$$

$$= 12 + 2 + 2 + \sum_{i=2}^{2}\sum_{j=3}^{3}(i+j)$$

$$= 16 + 2 + 3 = 21$$

(f)

$$\sum_{i=0}^{2}\sum_{j=0}^{3}(2i+3j) = 0 + 0 + \sum_{i=0}^{2}\sum_{j=1}^{3}(2i+3j)$$

$$= 0 + 3 + \sum_{i=0}^{2}\sum_{j=2}^{3}(2i+3j)$$

$$= 3 + 0 + 6 + \sum_{i=0}^{2}\sum_{j=3}^{3}(2i+3j)$$

$$= 9 + 0 + 9 + \sum_{i=1}^{2}\sum_{j=0}^{3}(2i+3j)$$

$$= 18 + 2 + 0 + \sum_{i=1}^{2}\sum_{j=1}^{3}(2i+3j)$$

$$= 20 + 2 + 3 + \sum_{i=1}^{2}\sum_{j=2}^{3}(2i+3j) \qquad (6)$$

$$= 25 + 2 + 6 + \sum_{i=1}^{2}\sum_{j=3}^{3}(2i+3j)$$

$$= 33 + 2 + 9 + \sum_{i=2}^{2}\sum_{j=0}^{3}(2i+3j)$$

$$= 44 + 4 + 0 + \sum_{i=2}^{2}\sum_{j=1}^{3}(2i+3j)$$

$$= 48 + 4 + 3 + \sum_{i=2}^{2}\sum_{j=2}^{3}(2i+3j)$$

$$= 55 + 4 + 6 + \sum_{i=2}^{2}\sum_{j=3}^{3}(2i+3j)$$

$$= 65 + 4 + 9 = 78$$

(g)

$$\sum_{i=1}^{3}\sum_{j=0}^{2} i = 1 + \sum_{i=1}^{3}\sum_{j=1}^{2} i$$

$$= 1 + 1 + \sum_{i=1}^{3}\sum_{j=2}^{2} i$$

$$= 2 + 1 + \sum_{i=2}^{3}\sum_{j=0}^{2} i$$

$$= 3 + 2 + \sum_{i=2}^{3}\sum_{j=1}^{2} i$$

$$= 5 + 2 + \sum_{i=2}^{3}\sum_{j=2}^{2} i$$

$$= 7 + 2 + \sum_{i=3}^{3}\sum_{j=0}^{2} i$$

$$= 9 + 3 + \sum_{i=3}^{3}\sum_{j=1}^{2} i$$

$$= 12 + 3 + \sum_{i=3}^{3}\sum_{j=2}^{2} i$$

$$= 15 + 3 = 18$$

(7)

(h)

$$\sum_{i=1}^{3}\sum_{j=0}^{2} j = 0 + \sum_{i=1}^{3}\sum_{j=1}^{2} j$$

$$= 0 + 1 + \sum_{i=1}^{3}\sum_{j=2}^{2} j$$

$$= 1 + 2 + \sum_{i=2}^{3}\sum_{j=0}^{2} j$$

$$= 3 + 0 + \sum_{i=2}^{3}\sum_{j=1}^{2} j$$

$$= 3 + 1 + \sum_{i=2}^{3}\sum_{j=2}^{2} j \tag{8}$$

$$= 4 + 2 + \sum_{i=3}^{3}\sum_{j=0}^{2} j$$

$$= 6 + 0 + \sum_{i=3}^{3}\sum_{j=1}^{2} j$$

$$= 6 + 1 + \sum_{i=3}^{3}\sum_{j=2}^{2} j$$

$$= 7 + 2 = 9$$

Exercício 2

Soma telescópica 1. *Para qualquer $n \in \mathbb{R}$, a seguinte equação é verdadeira.*

$$\sum_{j=1}^{n}(a_j - a_{j-1}) = a_n - a_0 \tag{9}$$

Será?

Proof. Para n = 1, isso é verdade?

$$\sum_{j=1}^{n}(a_j - a_{j-1}) = a_1 - a_0 \tag{10}$$

Sim! É verdade. E para n = 2?

$$\sum_{j=1}^{n}(a_j - a_{j-1}) = (a_1 - a_0) + (a_2 - a_1)$$

$$= a_2 - a_0 \tag{11}$$

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Também! Será que para um n = k dá certo também?

$$\sum_{j=1}^{n}(a_j - a_{j-1}) = a_n - a_0$$
$$= (a_1 - a_0) + (a_2 - a_1) + (a_3 - a_2) + .. + (a_{k-1} - a_{k-2}) + (a_k - a_{k-1})$$
$$= (a_k - a_0) \tag{12}$$

Dá! Então, se para n = k dá certo, para n = k + 1 também dá.

$$\sum_{j=1}^{n}(a_j - a_{j-1}) = a_n - a_0$$
$$= (a_1 - a_0) + (a_2 - a_1) + (a_3 - a_2) + .. + (a_{k-1} - a_{k-2}) + (a_k - a_{k-1}) + (a_{k+1} - a_k)$$
$$= (a_{k+1} - a_0) \tag{13}$$

Sucesso! Provado por indução! □

Exercício 4

(a)
$$\log_2 1024 = 10 \tag{14}$$

(b)
$$\log_{10} 0.0001 = -3 \tag{15}$$

(c)
$$\log_{49} 7 = \frac{1}{2} \tag{16}$$

(d)
$$\log_{32} \frac{1}{4} = -\frac{2}{5} \tag{17}$$

Exercício 5

(a)
$$\log_5 125 = 3 \tag{18}$$

(b)

$$\log_{81} 3 = \frac{1}{3} \tag{19}$$

(c)

$$\log_e\left(\frac{1}{e^3}\right) = 3 \tag{20}$$

(d)

$$\log_c \sqrt{c} = \frac{1}{2} \tag{21}$$

Exercício 6

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(a)

$$\log_2 x = 2y \tag{22}$$

(b)

$$\log_8 x = \frac{1}{2}y \tag{23}$$

(c)

$$\log_{16} x = \frac{1}{4}y \tag{24}$$

Exercício 8

(a)

$$\begin{aligned}
10^x &= 5 \\
\log_{10} 10^x &= \log_{10} 5 \\
x &= \log_{10} 5
\end{aligned} \tag{25}$$

(b)

$$\begin{aligned}
e^x &= 8 \\
\log_e e^x &= \log_e 8 \\
x &= \log_e 8
\end{aligned} \tag{26}$$

(c)

$$\left(\frac{1}{2}\right)^x = \frac{1}{100}$$
$$\log_{\frac{1}{2}}\left(\frac{1}{2}\right)^x = \log_{\frac{1}{2}}\frac{1}{100} \tag{27}$$
$$x = \log_{\frac{1}{2}}\frac{1}{100}$$