

Lista de Exercícios 01

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Contents

Exercício 1	2
Exercício 2	7
Exercício 4	8
Exercício 5	8
Exercício 6	9
Exercício 7	9
Exercício 8	9

Exercício 1

(a)

$$\begin{aligned}\sum_{k=1}^5 (k_i + 1) &= (1 + 1) + \sum_{k=2}^5 (k + 1) \\ &= 2 + (2 + 1) + \sum_{k=3}^5 (k + 1) \\ &= 5 + (3 + 1) + \sum_{k=4}^5 (k + 1) \\ &= 9 + (4 + 1) + \sum_{k=5}^5 (k + 1) \\ &= 14 + (5 + 1) \\ &= 20\end{aligned}\tag{1}$$

(b)

$$\begin{aligned}\sum_{j=0}^4 (-2)^j &= (-2)^0 + \sum_{j=1}^4 (-2)^j \\ &= -2 + (-2)^1 + \sum_{j=1}^4 (-2)^j \\ &= -4 + (-2)^2 + \sum_{j=2}^4 (-2)^j \\ &= 0 + (-2)^3 + \sum_{j=3}^4 (-2)^j \\ &= -8 + (-2)^4 + \sum_{j=4}^4 (-2)^j \\ &= -8 + 16 = 8\end{aligned}\tag{2}$$

(c)

$$\begin{aligned}\sum_{t=1}^{100} 3 &= 3 + \sum_{t=2}^{100} 3 \\ &= 6 + \sum_{t=3}^{100} 3 \\ &= 9 + \sum_{t=4}^{100} 3 \\ &\dots \\ &= 297 + \sum_{t=100}^{100} 3 \\ &= 300\end{aligned}\tag{3}$$

(d)

$$\begin{aligned}\sum_{j=0}^8 (2^{j+1} - 2^j) &= 2^{0+1} + (-2)^0 + \sum_{t=1}^8 (2^{j+1} - 2^j) \\ &= 2 + 2^{1+1} + (-2)^1 + \sum_{t=1}^8 (2^{j+1} - 2^j) \\ &= 4 + 2^{2+1} + (-2)^2 + \sum_{t=2}^8 (2^{j+1} - 2^j) \\ &= 16 + 2^{3+1} + (-2)^3 + \sum_{t=3}^8 (2^{j+1} - 2^j) \\ &= 28 + 2^{4+1} + (-2)^4 + \sum_{t=4}^8 (2^{j+1} - 2^j) \\ &= 76 + 2^{5+1} + (-2)^5 + \sum_{t=5}^8 (2^{j+1} - 2^j) \\ &= 108 + 2^{6+1} + (-2)^6 + \sum_{t=6}^8 (2^{j+1} - 2^j) \\ &= 300 + 2^{7+1} + (-2)^7 + \sum_{t=7}^8 (2^{j+1} - 2^j) \\ &= 428 + 2^{8+1} + (-2)^8 + \sum_{t=8}^8 (2^{j+1} - 2^j) \\ &= 256 + 512 + 428 = 1196\end{aligned}\tag{4}$$

(e)

$$\begin{aligned}\sum_{i=1}^2 \sum_{j=1}^3 (i+j) &= 1+1 + \sum_{i=1}^2 \sum_{j=2}^3 (i+j) \\ &= 2+1+2 + \sum_{i=1}^2 \sum_{j=3}^3 (i+j) \\ &= 5+1+3 + \sum_{i=2}^2 \sum_{j=1}^3 (i+j) \\ &= 9+2+1 + \sum_{i=2}^2 \sum_{j=2}^3 (i+j) \\ &= 12+2+2 + \sum_{i=2}^2 \sum_{j=3}^3 (i+j) \\ &= 16+2+3 = 21\end{aligned}\tag{5}$$

(f)

$$\begin{aligned}\sum_{i=0}^2 \sum_{j=0}^3 (2i + 3j) &= 0 + 0 + \sum_{i=0}^2 \sum_{j=1}^3 (2i + 3j) \\&= 0 + 3 + \sum_{i=0}^2 \sum_{j=2}^3 (2i + 3j) \\&= 3 + 0 + 6 + \sum_{i=0}^2 \sum_{j=3}^3 (2i + 3j) \\&= 9 + 0 + 9 + \sum_{i=1}^2 \sum_{j=0}^3 (2i + 3j) \\&= 18 + 2 + 0 + \sum_{i=1}^2 \sum_{j=1}^3 (2i + 3j) \\&= 20 + 2 + 3 + \sum_{i=1}^2 \sum_{j=2}^3 (2i + 3j) \\&= 25 + 2 + 6 + \sum_{i=1}^2 \sum_{j=3}^3 (2i + 3j) \\&= 33 + 2 + 9 + \sum_{i=2}^2 \sum_{j=0}^3 (2i + 3j) \\&= 44 + 4 + 0 + \sum_{i=2}^2 \sum_{j=1}^3 (2i + 3j) \\&= 48 + 4 + 3 + \sum_{i=2}^2 \sum_{j=2}^3 (2i + 3j) \\&= 55 + 4 + 6 + \sum_{i=2}^2 \sum_{j=3}^3 (2i + 3j) \\&= 65 + 4 + 9 = 78\end{aligned}\tag{6}$$

(g)

$$\begin{aligned}\sum_{i=1}^3 \sum_{j=0}^2 i &= 1 + \sum_{i=1}^3 \sum_{j=1}^2 i \\&= 1 + 1 + \sum_{i=1}^3 \sum_{j=2}^2 i \\&= 2 + 1 + \sum_{i=2}^3 \sum_{j=0}^2 i \\&= 3 + 2 + \sum_{i=2}^3 \sum_{j=1}^2 i \\&= 5 + 2 + \sum_{i=2}^3 \sum_{j=2}^2 i \\&= 7 + 2 + \sum_{i=3}^3 \sum_{j=0}^2 i \\&= 9 + 3 + \sum_{i=3}^3 \sum_{j=1}^2 i \\&= 12 + 3 + \sum_{i=3}^3 \sum_{j=2}^2 i \\&= 15 + 3 = 18\end{aligned}\tag{7}$$

(h)

$$\begin{aligned}\sum_{i=1}^3 \sum_{j=0}^2 j &= 0 + \sum_{i=1}^3 \sum_{j=1}^2 j \\ &= 0 + 1 + \sum_{i=1}^3 \sum_{j=2}^2 j \\ &= 1 + 2 + \sum_{i=2}^3 \sum_{j=0}^2 j \\ &= 3 + 0 + \sum_{i=2}^3 \sum_{j=1}^2 j \\ &= 3 + 1 + \sum_{i=2}^3 \sum_{j=2}^2 j \\ &= 4 + 2 + \sum_{i=3}^3 \sum_{j=0}^2 j \\ &= 6 + 0 + \sum_{i=3}^3 \sum_{j=1}^2 j \\ &= 6 + 1 + \sum_{i=3}^3 \sum_{j=2}^2 j \\ &= 7 + 2 = 9\end{aligned}\tag{8}$$

Exercício 2

Soma telescópica 1. Para qualquer $n \in \mathbb{R}$, a seguinte equação é verdadeira.

$$\sum_{j=1}^n (a_j - a_{j-1}) = a_n - a_0\tag{9}$$

Proof. Para $n = 1$, isso é verdade?

$$\sum_{j=1}^n (a_j - a_{j-1}) = a_1 - a_0\tag{10}$$

Sim! É verdade. E para $n = 2$?

$$\begin{aligned}\sum_{j=1}^n (a_j - a_{j-1}) &= (a_1 - a_0) + (a_2 - a_1) \\ &= a_2 - a_0\end{aligned}\tag{11}$$

Também! Será que para um $n = k$ dá certo também?

$$\begin{aligned}\sum_{j=1}^n (a_j - a_{j-1}) &= a_n - a_0 \\ &= (a_1 - a_0) + (a_2 - a_1) + (a_3 - a_2) + \dots + (a_{k-1} - a_{k-2}) + (a_k - a_{k-1}) \\ &= (a_k - a_0)\end{aligned}\tag{12}$$

Dá! Então, se para $n = k$ dá certo, para $n = k + 1$ também dá.

$$\begin{aligned}\sum_{j=1}^n (a_j - a_{j-1}) &= a_n - a_0 \\ &= (a_1 - a_0) + (a_2 - a_1) + (a_3 - a_2) + \dots + (a_{k-1} - a_{k-2}) + (a_k - a_{k-1}) + (a_{k+1} - a_k) \\ &= (a_{k+1} - a_0)\end{aligned}\tag{13}$$

Sucesso! Provado por indução!

□

Exercício 4

(a)

$$\log_2 1024 = 10\tag{14}$$

(b)

$$\log_{10} 0.0001 = -3\tag{15}$$

(c)

$$\log_{49} 7 = \frac{1}{2}\tag{16}$$

(d)

$$\log_{32} \frac{1}{4} = -\frac{2}{5}\tag{17}$$

Exercício 5

(a)

$$\log_5 125 = 3\tag{18}$$

(b)

$$\log_{81} 3 = \frac{1}{3} \quad (19)$$

(c)

$$\log_e \left(\frac{1}{e^3} \right) = 3 \quad (20)$$

(d)

$$\log_c \sqrt{c} = \frac{1}{2} \quad (21)$$

Exercício 6

(a)

$$\log_2 x = 2y \quad (22)$$

(b)

$$\log_8 x = \frac{1}{2}y \quad (23)$$

(c)

$$\log_{16} x = \frac{1}{4}y \quad (24)$$

Exercício 7

$$\begin{aligned} a^{\log_b c} &= c^{\log_b a} \\ \log_b a^{\log_b c} &= \log_b c^{\log_b a} \end{aligned} \quad (25)$$

Pela propriedade $\log a^b = b \log a$, temos

$$\log_b c * \log_b a = \log_b a * \log_b c \quad (26)$$

Exercício 8

(a)

$$\begin{aligned} 10^x &= 5 \\ \log_{10} 10^x &= \log_{10} 5 \\ x &= \log_{10} 5 \end{aligned} \quad (27)$$

(b)

$$\begin{aligned}e^x &= 8 \\ \log_e e^x &= \log_e 8 \\ x &= \log_e 8\end{aligned}\tag{28}$$

(c)

$$\begin{aligned}\left(\frac{1}{2}\right)^x &= \frac{1}{100} \\ \log_{\frac{1}{2}} \left(\frac{1}{2}\right)^x &= \log_{\frac{1}{2}} \frac{1}{100} \\ x &= \log_{\frac{1}{2}} \frac{1}{100}\end{aligned}\tag{29}$$

(d)

$$\begin{aligned}\left(\frac{1}{2}\right)^x &= \frac{1}{100} \\ \log_{\frac{1}{2}} \left(\frac{1}{2}\right)^x &= \log_{\frac{1}{2}} \frac{1}{100} \\ x &= \log_{\frac{1}{2}} \frac{1}{100}\end{aligned}\tag{30}$$