# Lista de Exercícios 01

Gustavo Higuchi

Agosto 2016

## Contents

## Exercício 1

### (a)

$$
\begin{aligned}
\sum_{k=1}^{5}(k_i+1) &= (1+1)+\sum_{k=2}^{5}(k+1) \\
&= 2+(2+1)+\sum_{k=3}^{5}(k+1) \\
&= 5+(3+1)+\sum_{k=4}^{5}(k+1) \\
&= 9+(4+1)+\sum_{k=5}^{5}(k+1) \\
&= 14+(5+1) \\
&= 20
\end{aligned} \tag{1}
$$

### (b)

$$
\begin{aligned}
\sum_{j=0}^{4}(-2)^j &= (-2)^0+\sum_{j=1}^{4}(-2)^j \\
&= -2+(-2)^1+\sum_{j=1}^{4}(-2)^j \\
&= -4+(-2)^2+\sum_{j=2}^{4}(-2)^j \\
&= 0+(-2)^3+\sum_{j=3}^{4}(-2)^j \\
&= -8+(-2)^4+\sum_{j=4}^{4}(-2)^j \\
&= -8+16=8
\end{aligned} \tag{2}
$$

**(c)**

$$
\begin{aligned}
\sum_{t=1}^{100} 3 &= 3 + \sum_{t=2}^{100} 3 \\
&= 6 + \sum_{t=3}^{100} 3 \\
&= 9 + \sum_{t=4}^{100} 3 \\
&\dots \\
&= 297 + \sum_{t=100}^{100} 3 \\
&= 300
\end{aligned} \tag{3}
$$

**(d)**

$$
\begin{aligned}
\sum_{j=0}^{8}(2^{j+1} - 2^j) &= 2^{0+1} + (-2)^0 + \sum_{t=1}^{8}(2^{j+1} - 2^j) \\
&= 2 + 2^{1+1} + (-2)^1 + \sum_{t=1}^{8}(2^{j+1} - 2^j) \\
&= 4 + 2^{2+1} + (-2)^2 + \sum_{t=2}^{8}(2^{j+1} - 2^j) \\
&= 16 + 2^{3+1} + (-2)^3 + \sum_{t=3}^{8}(2^{j+1} - 2^j) \\
&= 28 + 2^{4+1} + (-2)^4 + \sum_{t=4}^{8}(2^{j+1} - 2^j) \\
&= 76 + 2^{5+1} + (-2)^5 + \sum_{t=5}^{8}(2^{j+1} - 2^j) \\
&= 108 + 2^{6+1} + (-2)^6 + \sum_{t=6}^{8}(2^{j+1} - 2^j) \\
&= 300 + 2^{7+1} + (-2)^7 + \sum_{t=7}^{8}(2^{j+1} - 2^j) \\
&= 428 + 2^{8+1} + (-2)^8 + \sum_{t=8}^{8}(2^{j+1} - 2^j) \\
&= 256 + 512 + 428 = 1196
\end{aligned} \tag{4}
$$

**(e)**

$$
\begin{aligned}
\sum_{i=1}^{2}\sum_{j=1}^{3}(i+j) &= 1+1+\sum_{i=1}^{2}\sum_{j=2}^{3}(i+j) \\
&= 2+1+2+\sum_{i=1}^{2}\sum_{j=3}^{3}(i+j) \\
&= 5+1+3+\sum_{i=2}^{2}\sum_{j=1}^{3}(i+j) \\
&= 9+2+1+\sum_{i=2}^{2}\sum_{j=2}^{3}(i+j) \\
&= 12+2+2+\sum_{i=2}^{2}\sum_{j=3}^{3}(i+j) \\
&= 16+2+3=21
\end{aligned}
\tag{5}
$$

**(f)**

$$
\begin{aligned}
\sum_{i=0}^{2}\sum_{j=0}^{3}(2i+3j) &= 0+0+\sum_{i=0}^{2}\sum_{j=1}^{3}(2i+3j) \\
&= 0+3+\sum_{i=0}^{2}\sum_{j=2}^{3}(2i+3j) \\
&= 3+0+6+\sum_{i=0}^{2}\sum_{j=3}^{3}(2i+3j) \\
&= 9+0+9+\sum_{i=1}^{2}\sum_{j=0}^{3}(2i+3j) \\
&= 18+2+0+\sum_{i=1}^{2}\sum_{j=1}^{3}(2i+3j) \\
&= 20+2+3+\sum_{i=1}^{2}\sum_{j=2}^{3}(2i+3j) \\
&= 25+2+6+\sum_{i=1}^{2}\sum_{j=3}^{3}(2i+3j) \\
&= 33+2+9+\sum_{i=2}^{2}\sum_{j=0}^{3}(2i+3j) \\
&= 44+4+0+\sum_{i=2}^{2}\sum_{j=1}^{3}(2i+3j) \\
&= 48+4+3+\sum_{i=2}^{2}\sum_{j=2}^{3}(2i+3j) \\
&= 55+4+6+\sum_{i=2}^{2}\sum_{j=3}^{3}(2i+3j) \\
&= 65+4+9 = 78
\end{aligned} \tag{6}
$$

**(g)**

$$\begin{aligned}
\sum_{i=1}^{3}\sum_{j=0}^{2} i &= 1 + \sum_{i=1}^{3}\sum_{j=1}^{2} i \\
&= 1 + 1 + \sum_{i=1}^{3}\sum_{j=2}^{2} i \\
&= 2 + 1 + \sum_{i=2}^{3}\sum_{j=0}^{2} i \\
&= 3 + 2 + \sum_{i=2}^{3}\sum_{j=1}^{2} i \\
&= 5 + 2 + \sum_{i=2}^{3}\sum_{j=2}^{2} i \\
&= 7 + 2 + \sum_{i=3}^{3}\sum_{j=0}^{2} i \\
&= 9 + 3 + \sum_{i=3}^{3}\sum_{j=1}^{2} i \\
&= 12 + 3 + \sum_{i=3}^{3}\sum_{j=2}^{2} i \\
&= 15 + 3 = 18
\end{aligned} \tag{7}$$

**(h)**

$$\begin{aligned}
\sum_{i=1}^{3}\sum_{j=0}^{2} j &= 0 + \sum_{i=1}^{3}\sum_{j=1}^{2} j \\
&= 0 + 1 + \sum_{i=1}^{3}\sum_{j=2}^{2} j \\
&= 1 + 2 + \sum_{i=2}^{3}\sum_{j=0}^{2} j \\
&= 3 + 0 + \sum_{i=2}^{3}\sum_{j=1}^{2} j \\
&= 3 + 1 + \sum_{i=2}^{3}\sum_{j=2}^{2} j \\
&= 4 + 2 + \sum_{i=3}^{3}\sum_{j=0}^{2} j \\
&= 6 + 0 + \sum_{i=3}^{3}\sum_{j=1}^{2} j \\
&= 6 + 1 + \sum_{i=3}^{3}\sum_{j=2}^{2} j \\
&= 7 + 2 = 9
\end{aligned} \tag{8}$$

# Exercício 2

**Soma telescópica 1.** *Para qualquer $n \in \mathbb{R}$, a seguinte equação é verdadeira.*

$$\sum_{j=1}^{n}(a_j - a_{j-1}) = a_n - a_0 \tag{9}$$

*Proof.* Para n = 1, isso é verdade?

$$\sum_{j=1}^{n}(a_j - a_{j-1}) = a_1 - a_0 \tag{10}$$

Sim! É verdade. E para n = 2?

$$\begin{aligned}
\sum_{j=1}^{n}(a_j - a_{j-1}) &= (a_1 - a_0) + (a_2 - a_1) \\
&= a_2 - a_0
\end{aligned} \tag{11}$$

Também! Será que para um n = k dá certo também?

$$
\begin{aligned}
\sum_{j=1}^{n}(a_j - a_{j-1}) = a_n - a_0 \\
&= (a_1 - a_0) + (a_2 - a_1) + (a_3 - a_2) + .. + (a_{k-1} - a_{k-2}) + (a_k - a_{k-1}) \\
&= (a_k - a_0)
\end{aligned} \tag{12}
$$

Dá! Então, se para n = k dá certo, para n = k + 1 também dá.

$$
\begin{aligned}
\sum_{j=1}^{n}(a_j - a_{j-1}) = a_n - a_0 \\
&= (a_1 - a_0) + (a_2 - a_1) + (a_3 - a_2) + .. + (a_{k-1} - a_{k-2}) + (a_k - a_{k-1}) + (a_{k+1} - a_k) \\
&= (a_{k+1} - a_0)
\end{aligned} \tag{13}
$$

Sucesso! Provado por indução! $\square$

# Exercício 3

## (a)

$$
\sum_{k=1}^{n}(2k-1) = \sum_{k=1}^{n} k^2 - (k-1)^2 \tag{14}
$$

Se definirmos $a_k = k^2$, a partir da função do exercício anterior, temos

$$
\begin{aligned}
\sum_{k=1}^{n}(2k-1) &= \sum_{k=1}^{n} a_k - a_{k-1} \\
&= a_k - a_1 \\
&= n^2 - (1-1)^2 \\
\sum_{k=1}^{n}(2k-1) &= n^2
\end{aligned} \tag{15}
$$

### (b)

Da equação dada, isolando o $k$:

$$\begin{aligned}
\sum_{k=1}^{n} k &= \frac{\sum_{k=1}^{n} k^2 - (k-1)^2 - 1}{2} \\
&= \frac{\sum_{k=1}^{n} a_k - a_{k-1} - 1}{2} \\
&= \frac{1}{2}\sum_{k=1}^{n}(a_k - a_{k-1} - 1) \\
&= \frac{1}{2} * (a_k - a_{k-1} - 1) \\
&= \frac{n^2 - (1-1)^2 - 1}{2} \\
\sum_{k=1}^{n} k &= \frac{n^2 - 1}{2}
\end{aligned} \tag{16}$$

## Exercício 4

### (a)

$$\log_2 1024 = 10 \tag{17}$$

### (b)

$$\log_{10} 0.0001 = -3 \tag{18}$$

### (c)

$$\log_{49} 7 = \frac{1}{2} \tag{19}$$

### (d)

$$\log_{32} \frac{1}{4} = -\frac{2}{5} \tag{20}$$

## Exercício 5

### (a)

$$\log_5 125 = 3 \tag{21}$$

### (b)

$$\log_{81} 3 = \frac{1}{3} \tag{22}$$

### (c)

$$\log_e(\frac{1}{e^3}) = 3 \tag{23}$$

### (d)

$$\log_c \sqrt{c} = \frac{1}{2} \tag{24}$$

## Exercício 6

### (a)

$$\log_2 x = 2y \tag{25}$$

### (b)

$$\log_8 x = \frac{1}{2}y \tag{26}$$

### (c)

$$\log_{16} x = \frac{1}{4}y \tag{27}$$

## Exercício 7

$$\begin{aligned} a^{\log_b c} &= c^{\log_b a} \\ \log_b a^{\log_b c} &= \log_b c^{\log_b a} \end{aligned} \tag{28}$$

Pela propriedade $\log a^b = b \log a$, temos

$$\log_b c * \log_b a = \log_b a * \log_b c \tag{29}$$

## Exercício 8

### (a)

$$\begin{aligned} 10^x &= 5 \\ \log_{10} 10^x &= \log_{10} 5 \\ x &= \log_{10} 5 \end{aligned} \tag{30}$$

**(b)**

$$\begin{aligned} e^x &= 8 \\ \log_e e^x &= \log_e 8 \\ x &= \log_e 8 \end{aligned} \tag{31}$$

**(c)**

$$\begin{aligned} (\frac{1}{2})^x &= \frac{1}{100} \\ \log_{\frac{1}{2}} (\frac{1}{2})^x &= \log_{\frac{1}{2}} \frac{1}{100} \\ x &= \log_{\frac{1}{2}} \frac{1}{100} \end{aligned} \tag{32}$$

**(d)**

$$\begin{aligned} (\frac{1}{2})^x &= \frac{1}{100} \\ \log_{\frac{1}{2}} (\frac{1}{2})^x &= \log_{\frac{1}{2}} \frac{1}{100} \\ x &= \log_{\frac{1}{2}} \frac{1}{100} \end{aligned} \tag{33}$$