Lista de Exercícios 01

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Contents

Exercício 1

(a)

$$\begin{aligned}
\sum_{k=1}^{5}(k_i+1) &= (1+1)+\sum_{k=2}^{5}(k+1)\\
&= 2+(2+1)+\sum_{k=3}^{5}(k+1)\\
&= 5+(3+1)+\sum_{k=4}^{5}(k+1)\\
&= 9+(4+1)+\sum_{k=5}^{5}(k+1)\\
&= 14+(5+1)\\
&= 20
\end{aligned} \tag{1}$$

(b)

$$\begin{aligned}
\sum_{j=0}^{4}(-2)^j &= (-2)^0+\sum_{j=1}^{4}(-2)^j\\
&= -2+(-2)^1+\sum_{j=1}^{4}(-2)^j\\
&= -4+(-2)^2+\sum_{j=2}^{4}(-2)^j\\
&= 0+(-2)^3+\sum_{j=3}^{4}(-2)^j\\
&= -8+(-2)^4+\sum_{j=4}^{4}(-2)^j\\
&= -8+16=8
\end{aligned} \tag{2}$$

(c)

$$\begin{aligned}
\sum_{t=1}^{100} 3 &= 3 + \sum_{t=2}^{100} 3 \\
&= 6 + \sum_{t=3}^{100} 3 \\
&= 9 + \sum_{t=4}^{100} 3 \\
&\dots \\
&= 297 + \sum_{t=100}^{100} 3 \\
&= 300
\end{aligned} \tag{3}$$

(d)

$$\begin{aligned}
\sum_{j=0}^{8}(2^{j+1} - 2^j) &= 2^{0+1} + (-2)^0 + \sum_{t=1}^{8}(2^{j+1} - 2^j) \\
&= 2 + 2^{1+1} + (-2)^1 + \sum_{t=1}^{8}(2^{j+1} - 2^j) \\
&= 4 + 2^{2+1} + (-2)^2 + \sum_{t=2}^{8}(2^{j+1} - 2^j) \\
&= 16 + 2^{3+1} + (-2)^3 + \sum_{t=3}^{8}(2^{j+1} - 2^j) \\
&= 28 + 2^{4+1} + (-2)^4 + \sum_{t=4}^{8}(2^{j+1} - 2^j) \\
&= 76 + 2^{5+1} + (-2)^5 + \sum_{t=5}^{8}(2^{j+1} - 2^j) \\
&= 108 + 2^{6+1} + (-2)^6 + \sum_{t=6}^{8}(2^{j+1} - 2^j) \\
&= 300 + 2^{7+1} + (-2)^7 + \sum_{t=7}^{8}(2^{j+1} - 2^j) \\
&= 428 + 2^{8+1} + (-2)^8 + \sum_{t=8}^{8}(2^{j+1} - 2^j) \\
&= 256 + 512 + 428 = 1196
\end{aligned} \tag{4}$$

(e)

$$\begin{aligned}
\sum_{i=1}^{2}\sum_{j=1}^{3}(i+j) &= 1+1+\sum_{i=1}^{2}\sum_{j=2}^{3}(i+j) \\
&= 2+1+2+\sum_{i=1}^{2}\sum_{j=3}^{3}(i+j) \\
&= 5+1+3+\sum_{i=2}^{2}\sum_{j=1}^{3}(i+j) \\
&= 9+2+1+\sum_{i=2}^{2}\sum_{j=2}^{3}(i+j) \\
&= 12+2+2+\sum_{i=2}^{2}\sum_{j=3}^{3}(i+j) \\
&= 16+2+3=21
\end{aligned} \tag{5}$$

(f)

$$
\begin{aligned}
\sum_{i=0}^{2}\sum_{j=0}^{3}(2i+3j) &= 0+0+\sum_{i=0}^{2}\sum_{j=1}^{3}(2i+3j)\\
&= 0+3+\sum_{i=0}^{2}\sum_{j=2}^{3}(2i+3j)\\
&= 3+0+6+\sum_{i=0}^{2}\sum_{j=3}^{3}(2i+3j)\\
&= 9+0+9+\sum_{i=1}^{2}\sum_{j=0}^{3}(2i+3j)\\
&= 18+2+0+\sum_{i=1}^{2}\sum_{j=1}^{3}(2i+3j)\\
&= 20+2+3+\sum_{i=1}^{2}\sum_{j=2}^{3}(2i+3j)\\
&= 25+2+6+\sum_{i=1}^{2}\sum_{j=3}^{3}(2i+3j)\\
&= 33+2+9+\sum_{i=2}^{2}\sum_{j=0}^{3}(2i+3j)\\
&= 44+4+0+\sum_{i=2}^{2}\sum_{j=1}^{3}(2i+3j)\\
&= 48+4+3+\sum_{i=2}^{2}\sum_{j=2}^{3}(2i+3j)\\
&= 55+4+6+\sum_{i=2}^{2}\sum_{j=3}^{3}(2i+3j)\\
&= 65+4+9 = 78
\end{aligned}
\tag{6}
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(g)

$$\begin{aligned}
\sum_{i=1}^{3}\sum_{j=0}^{2} i &= 1 + \sum_{i=1}^{3}\sum_{j=1}^{2} i \\
&= 1 + 1 + \sum_{i=1}^{3}\sum_{j=2}^{2} i \\
&= 2 + 1 + \sum_{i=2}^{3}\sum_{j=0}^{2} i \\
&= 3 + 2 + \sum_{i=2}^{3}\sum_{j=1}^{2} i \\
&= 5 + 2 + \sum_{i=2}^{3}\sum_{j=2}^{2} i \\
&= 7 + 2 + \sum_{i=3}^{3}\sum_{j=0}^{2} i \\
&= 9 + 3 + \sum_{i=3}^{3}\sum_{j=1}^{2} i \\
&= 12 + 3 + \sum_{i=3}^{3}\sum_{j=2}^{2} i \\
&= 15 + 3 = 18
\end{aligned} \tag{7}$$

(h)

$$\begin{aligned}
\sum_{i=1}^{3}\sum_{j=0}^{2} j &= 0 + \sum_{i=1}^{3}\sum_{j=1}^{2} j \\
&= 0 + 1 + \sum_{i=1}^{3}\sum_{j=2}^{2} j \\
&= 1 + 2 + \sum_{i=2}^{3}\sum_{j=0}^{2} j \\
&= 3 + 0 + \sum_{i=2}^{3}\sum_{j=1}^{2} j \\
&= 3 + 1 + \sum_{i=2}^{3}\sum_{j=2}^{2} j \\
&= 4 + 2 + \sum_{i=3}^{3}\sum_{j=0}^{2} j \\
&= 6 + 0 + \sum_{i=3}^{3}\sum_{j=1}^{2} j \\
&= 6 + 1 + \sum_{i=3}^{3}\sum_{j=2}^{2} j \\
&= 7 + 2 = 9
\end{aligned} \tag{8}$$

Exercício 4

(a)

$$\log_2 1024 = 10 \tag{9}$$

(b)

$$\log_{10} 0.0001 = -3 \tag{10}$$

(c)

$$\log_{49} 7 = \frac{1}{2} \tag{11}$$

(d)

$$\log_{32} \frac{1}{4} = -\frac{2}{5} \tag{12}$$

Exercício 5

(a)

$$\log_5 125 = 3 \tag{13}$$

(b)

$$\log_{81} 3 = \frac{1}{3} \tag{14}$$

(c)

$$\log_e(\frac{1}{e^3}) = 3 \tag{15}$$

(d)

$$\log_c \sqrt{c} = \frac{1}{2} \tag{16}$$

Exercício 6

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(a)

$$\log_2 x = 2y \tag{17}$$

(b)

$$\log_8 x = \frac{1}{2}y \tag{18}$$

(c)

$$\log_{16} x = \frac{1}{4}y \tag{19}$$