Big M Method

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What is Big M Method ?

- The Big M method is a method of solving linear programming problems.
- It is a variation of the simplex method designed for solving problems typically encompassing "greater-than" constraints as well as "less-than" constraints
- The "Big M" refers to a large number associated with the artificial variables, represented by the letter M.

Why Big M Method ?

- The Big-M method introduces artificial variables (dummy helpers) to create an initial BFS.
- "Big M" = a very large positive number (penalty).
- Ensures artificial variables disappear from the final solution (if a feasible solution exists).
- Detects infeasibility → If artificial variables remain > 0 at optimum, then the original problem has no feasible solution.

Concept of Artificial Variables

- If constraints are ≤ type → slack variables give BFS directly.
- Required when constraints are of type ≥ (no direct BFS available).
- Definition: Dummy variables added to constraints to obtain an initial basic feasible solution (BFS).

Role of Big M

- M is a very large positive number
- Assign a very large penalty (M) to artificial variables in the objective function.
- Big M is a technique that penalizes artificial variables with a huge cost, ensuring they disappear from the final solution

Steps In The Big-M Method

- Add artificial variables in the model to obtain a feasible solution.
- Added only to the '>' type or the '=' constraints
- Assign penalty cost M to artificial variables
- The transformed problem is then solved using simplex method
- Remove artificial variables from basis

Mathematical Formulation

- Max $Z = c1x1 + c2x2 \ldots - M(A1 + A2 \ldots)$
- Subject to:
    - Constraints with slack/surplus/artificial variables
    - $xi \geq 0$

Important Points To Remember

Solve the modified LPP by simplex method, until any one of the three cases may arise.

- If no artificial variable appears in the basis and the optimality conditions are satisfied
- If at least one artificial variable in the basis at zero level and the optimality condition is satisfied
- If at least one artificial variable appears in the basis at positive level and the optimality condition is satisfied, then the original problem has no feasible solution.

Example

Maximize $Z = x_1 + 5x_2$

Subject to

$4x_1 + 4x_2 \leq 6$

$x_1 + 3x_2 \geq 2$ $x_1, x_2 \geq 0$

Solution :

Introducing slack & surplus variables :

$4x_1 + 4x_2 + S_1 = 6$

$x_1 + 3x_2 - S_2 = 2$

Example

- Now if we let x_1 & x_2 equal to zero in the initial solution, we will have $S_1=6$, $S_2=-2$, which is not possible because a surplus variable cannot be negative. Therefore, we need artificial variables.
- Introducing an artificial variable, say A1.

Maximize $Z = x_1 + 5x_2 + 0S_1 + 0S_2 - M(A1)$

Subject to

$4x_1 + 4x_2 + S_1 = 6$

$x_1 + 3x_2 - S_2 + A1 = 2$

$x_1, x_2, S_1, S_2, A1 \geq 0$

Example

		C_j	1	5	0	0	-M	
B	C_B	X_B	X_1	X_2	S_1	S_2	A_1	MinRatio
S_1	0	6	4	4	1	0	0	$^3/_2$
A_1	-M	2	1	3	0	-1	1	$^2/_3$
		Z_j	-M	-3M	0	M	-M	
		$C_j - Z_j$	M+1	3M+5	0	-M	0	

Entering = X_2, Departing = A_1, Key Element = 3

R_2(new) = R_2(old) / 3 = R_2(old) x 1/3

R_1(new) = R_1(old) - 4 R_2(new)

Example

		C_j	1	5	0	0	-M	
B	C_B	X_B	X_1	X_2	S_1	S_2	A_1	MinRatio
S_1	0	$^{10}/_3$	$^{8}/_3$	0	1	$^{4}/_3$	$^{-4}/_3$	$^{5}/_2$
X_2	5	$^{2}/_3$	$^{1}/_3$	1	0	$^{-1}/_3$	$^{1}/_3$	---
		Z_j	5/3	5	0	-5/3	5/3	
		$C_j - Z_j$	-2/3	0	0	5/3	-M-5/3	

Entering = S_2, Departing = S_1, Key Element = 4/3

$R_1(new) = R_1(old) / 4/3 = R_1(old) \times 3/4$

$R_2(new) = R_2(old) + 1/3\ R_1(new)$

Example

		C_j	1	5	0	0	-M	
B	C_B	X_B	X_1	X_2	S_1	S_2	A_1	MinRatio
S_2	0	$^5/_2$	2	0	$^3/_4$	1	-1	
X_2	5	$^3/_2$	1	1	$^1/_4$	0	0	
		Z_j	5	5	5/4	0	0	
		$C_j - Z_j$	-4	0	-5/4	0	-M	

Optimum Solution is arrived at with value of variables as :

$X_1 = 0$

$X_2 = {}^3/_2$

Maximise $Z = {}^{15}/_2$

Applications of Big M Method in CS

- Resource Allocation in Cloud Computing
  - Cloud systems need to assign CPU, memory, and bandwidth to different tasks.
- Operating Systems – Job Scheduling
  - When scheduling jobs, some tasks have minimum execution requirements (≥ constraints).
- Network Flow Optimization
  - constraints like at least X bandwidth on a path or minimum packets to a node appear.

Applications of Big M Method in CS

- Database Query Optimization
  - Cost-based query optimizers choose execution plans based on constraints (e.g., memory, CPU).
- Machine Learning & AI – Model Training Constraints
  - In optimization-based ML (like linear regression with constraints or SVM formulations), Big M helps convert inequality constraints into solvable forms.

Drawbacks of Big M Method

- Choice of M is Arbitrary
- M is assumed to be a “very large number.”
- If it’s too small, artificial variables may not leave the solution → wrong result.
- If it’s too large, numerical instability occurs during calculations.
- Computers cannot handle extremely large values of M
- Adding artificial variables makes the simplex tableau bigger.

Drawbacks of Big M Method

- More iterations are often required, which slows down computation.
- Not Practical for Large Problems
- For large-scale linear programming problems (like in ML or big-data optimization), Big M can be inefficient compared to other methods.
- Better Alternatives Exist : Two-Phase Simplex Method

Thank you