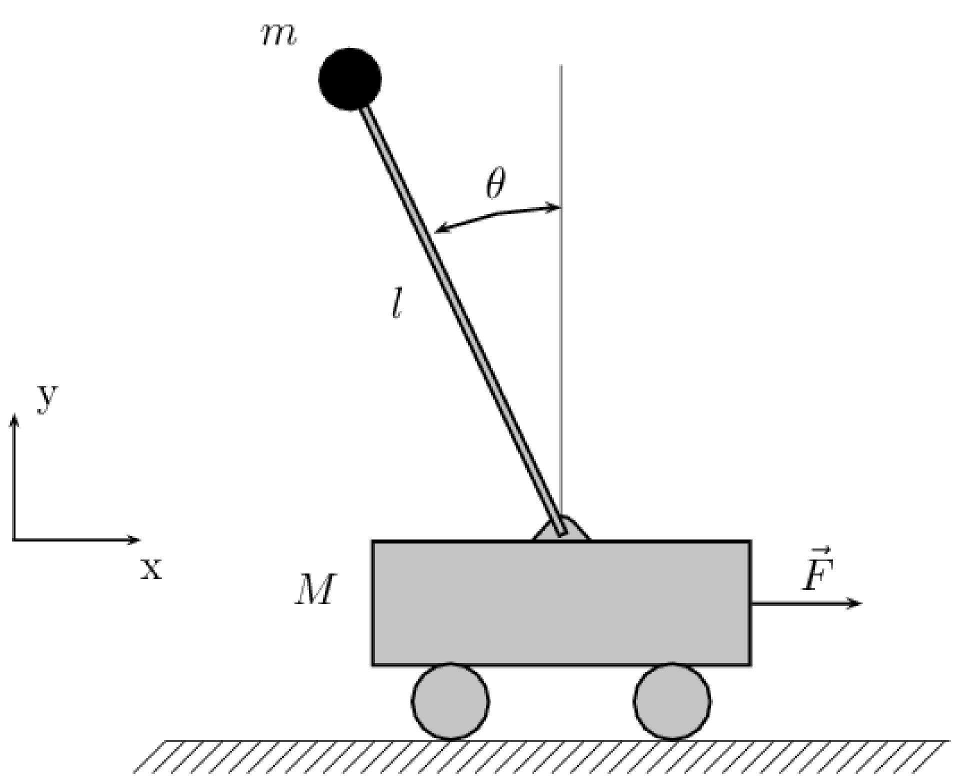
Inverted Pendulum Control System

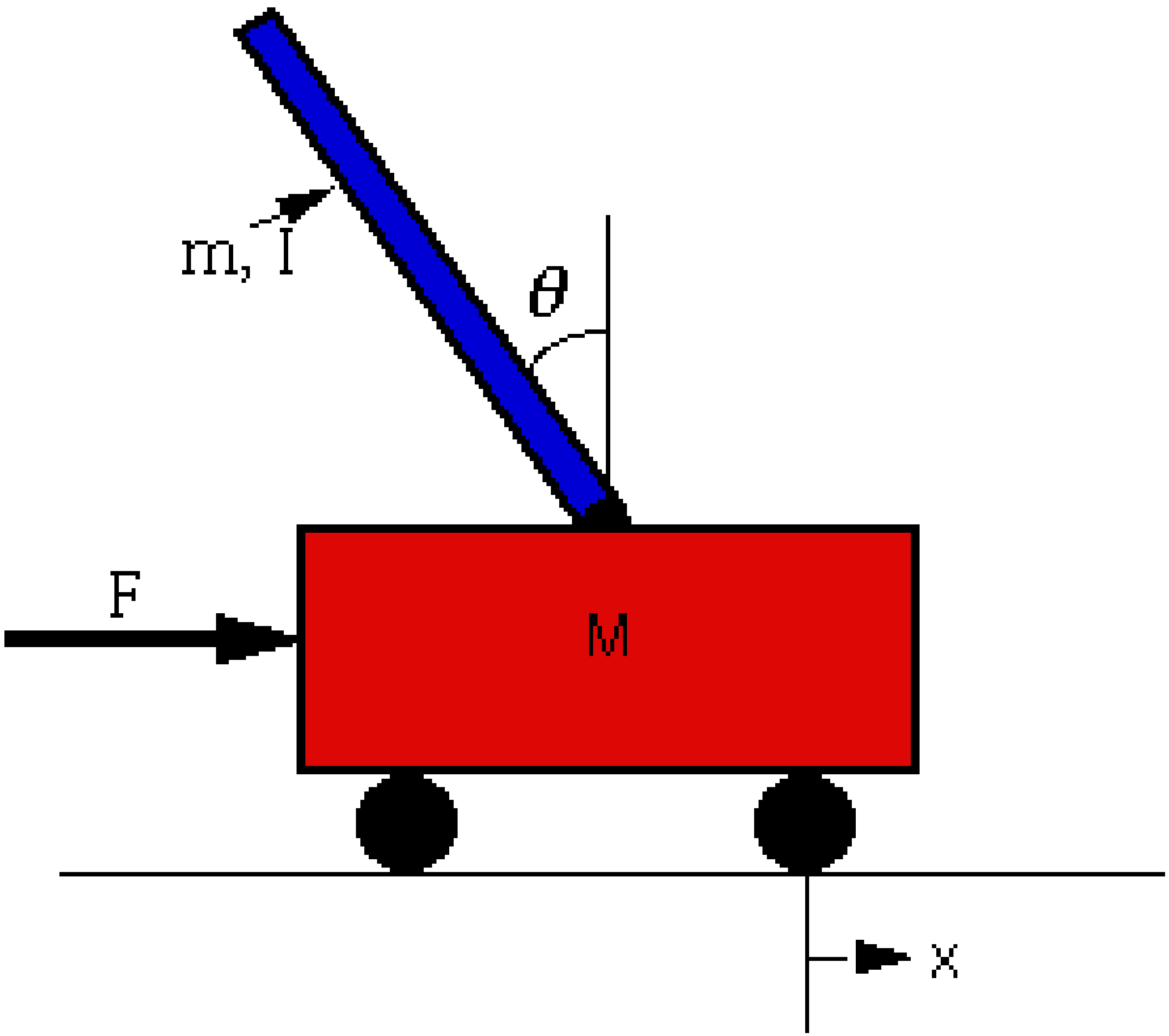


Krishnan Pandalai
Manav Kataria

Outline

- What is an Inverted Pendulum?
- Real-life Examples
- Mathematical Modeling
- Need for a Controller
- Requirement Specifications
- Implementing a Controller
- Conclusion

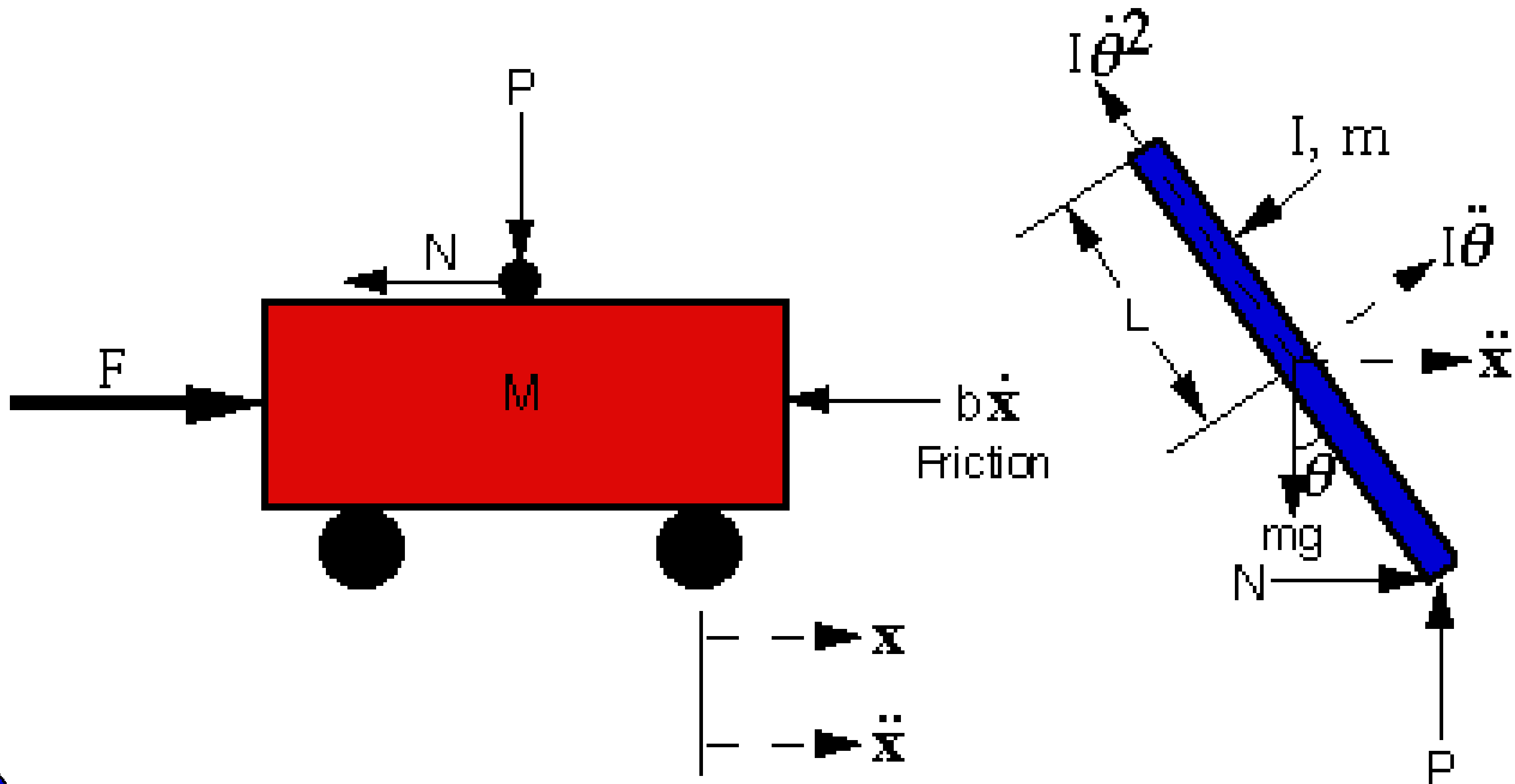
What is an Inverted Pendulum?

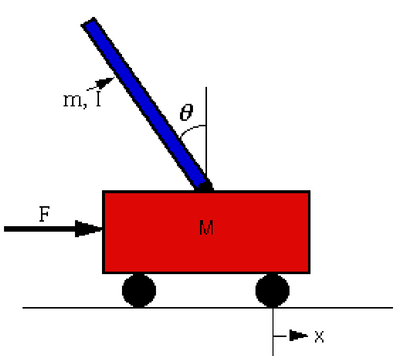


Real Examples

- Rockets and Missiles
- Heavy Cranes lifting containers in shipyards
- Self balancing Robots
- Future Transport Systems like:
  - Segways
  - Jetpacks!

Mathematical Modeling





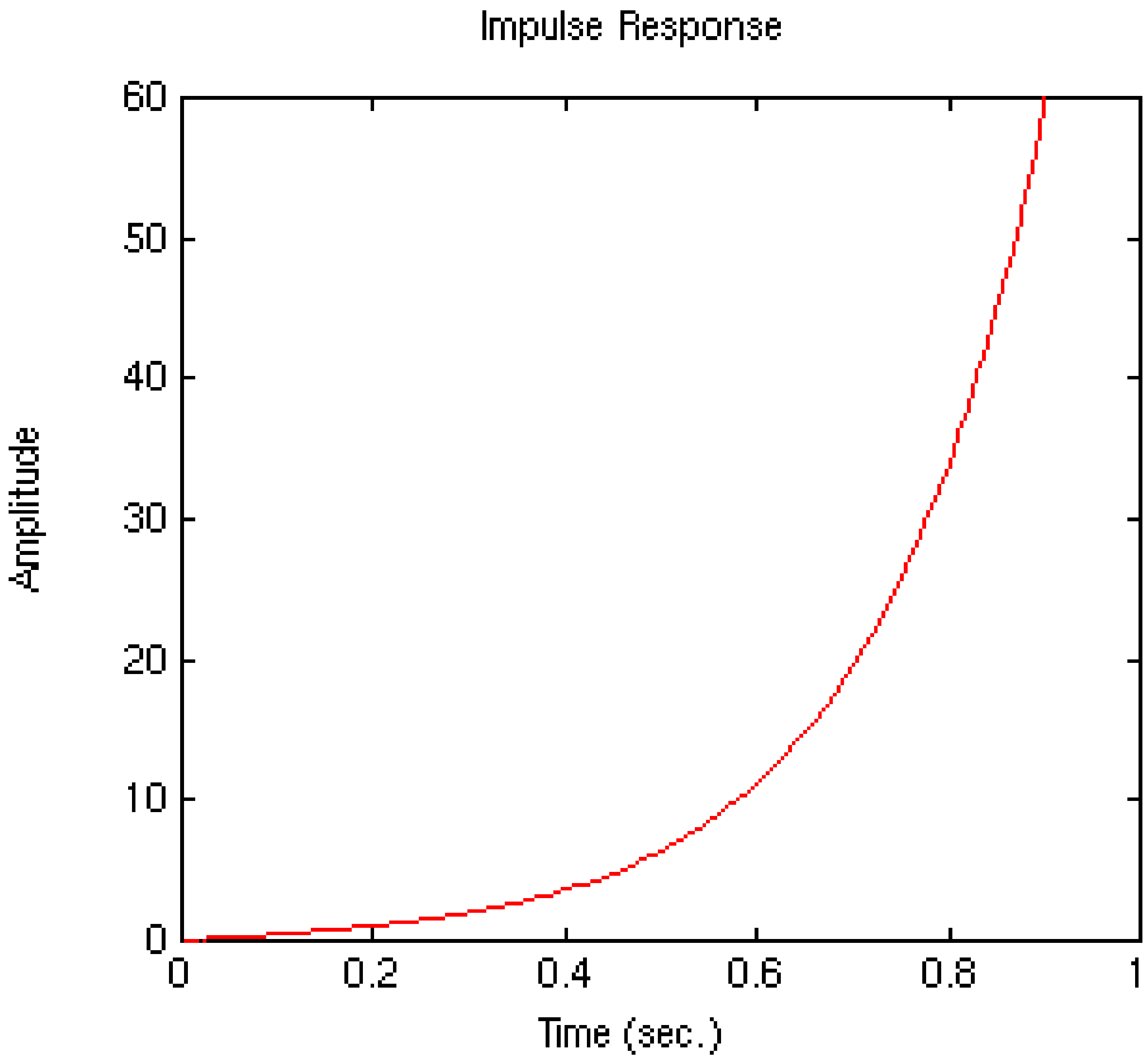
Mathematical Modeling

(1) $$(\mathbf{I}+\mathbf{ml}^2)\ddot{\theta}+\mathbf{mglsin}\,\theta=\mathbf{-ml\ddot{x}cos}\,\theta$$

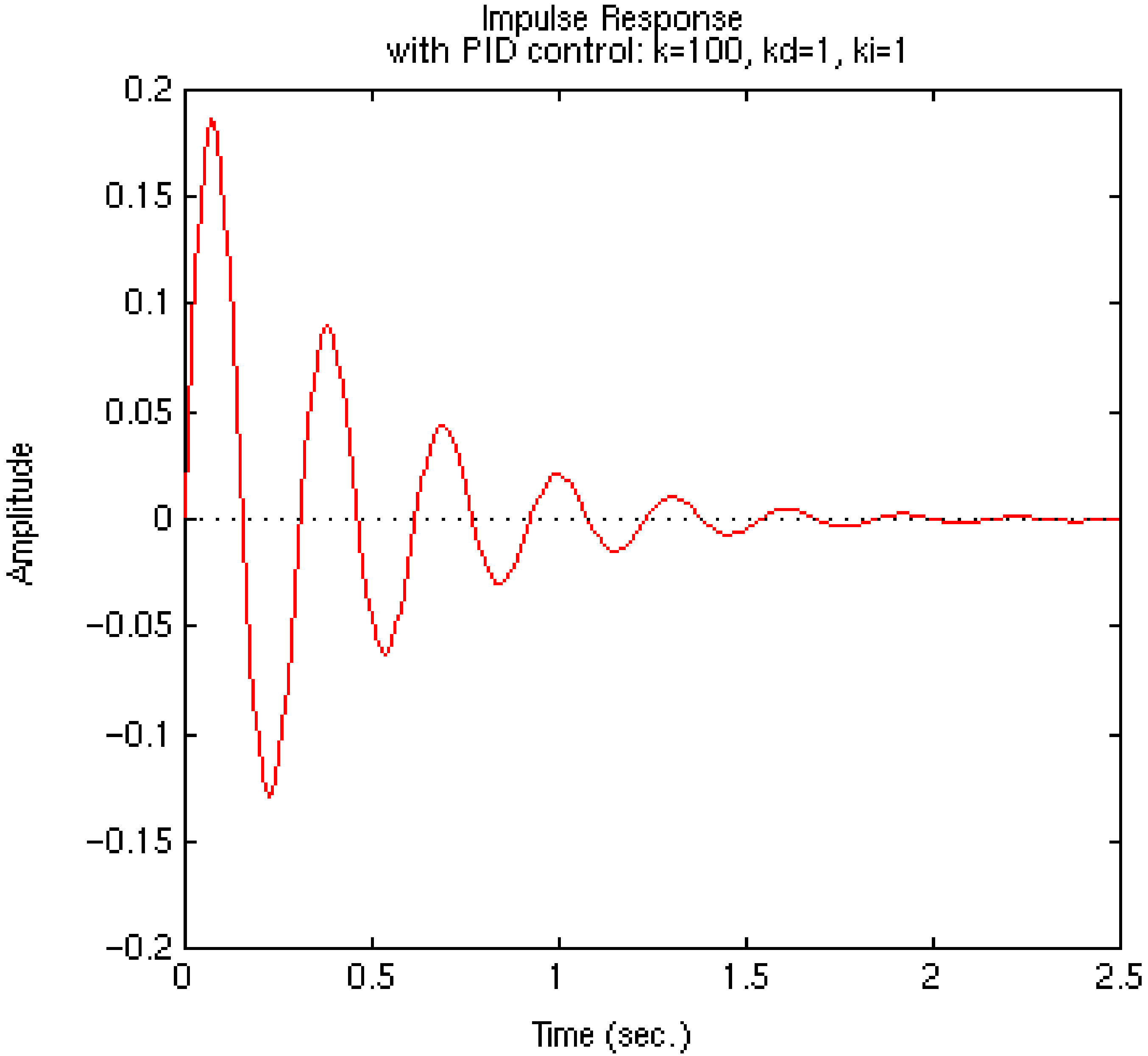
(2) $$(\mathbf{M}+\mathbf{m})\ddot{\mathbf{x}}+\mathbf{b}\dot{\mathbf{x}}+\mathbf{ml}\ddot{\theta}\mathbf{cos}\,\theta-\mathbf{ml}\dot{\theta}^2\mathbf{sin}\,\theta=\mathbf{F}$$

TF: $$\frac{\Phi(s)}{\mathbf{U}(s)}=\frac{\frac{\mathbf{ml}}{\mathbf{q}}s}{s^3+\frac{\mathbf{b}(\mathbf{I}+\mathbf{ml}^2)}{\mathbf{q}}s^2-\frac{(\mathbf{M}+\mathbf{m})\mathbf{mgl}}{\mathbf{q}}s-\frac{\mathbf{bmgl}}{\mathbf{q}}}$$

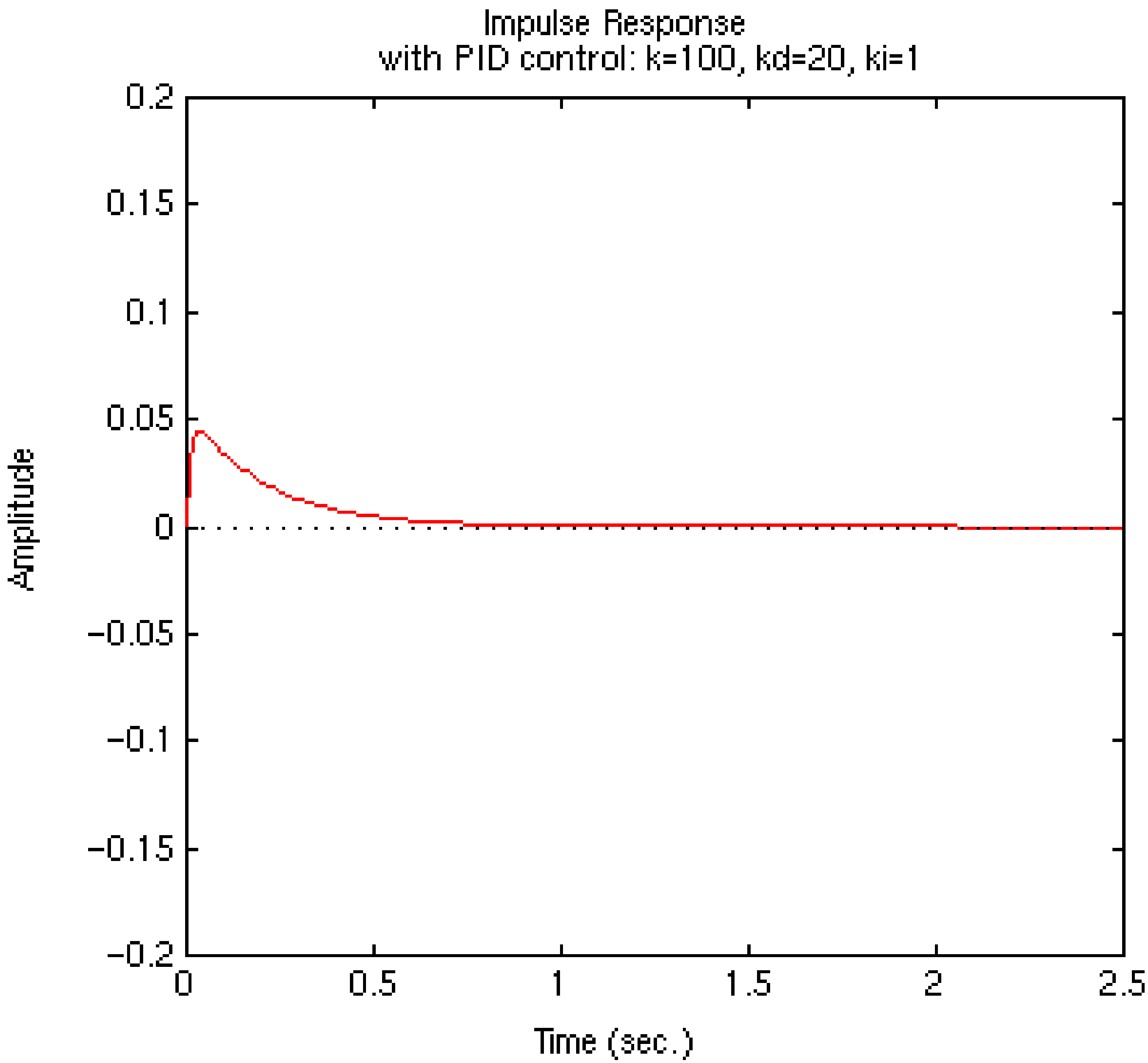
Open Loop Impulse Response



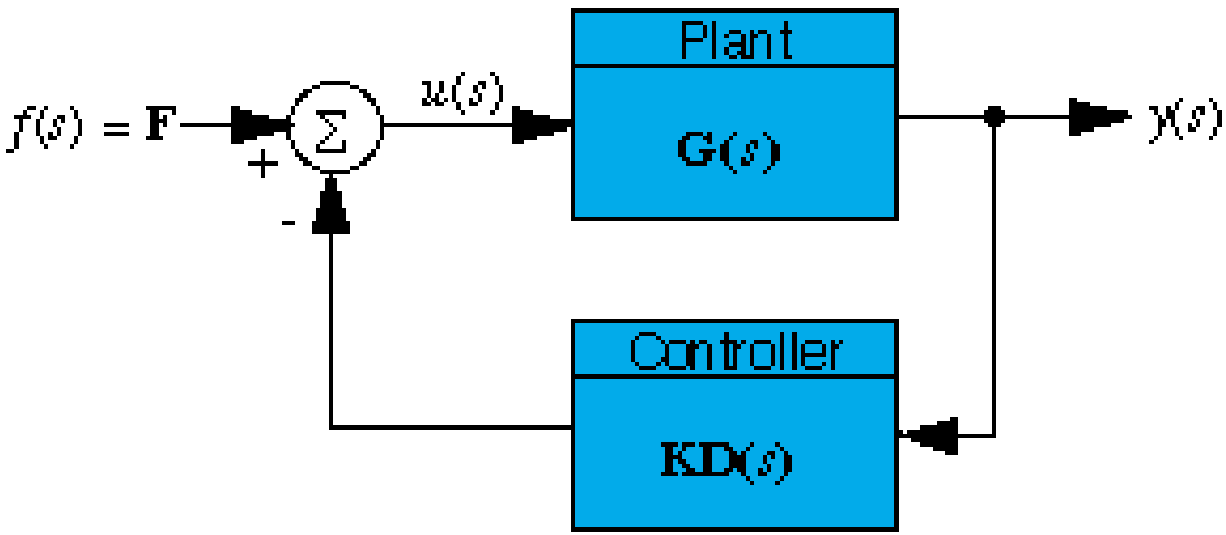
PI Controller



PID Controller



Closed Loop Transfer Function



Thank you!

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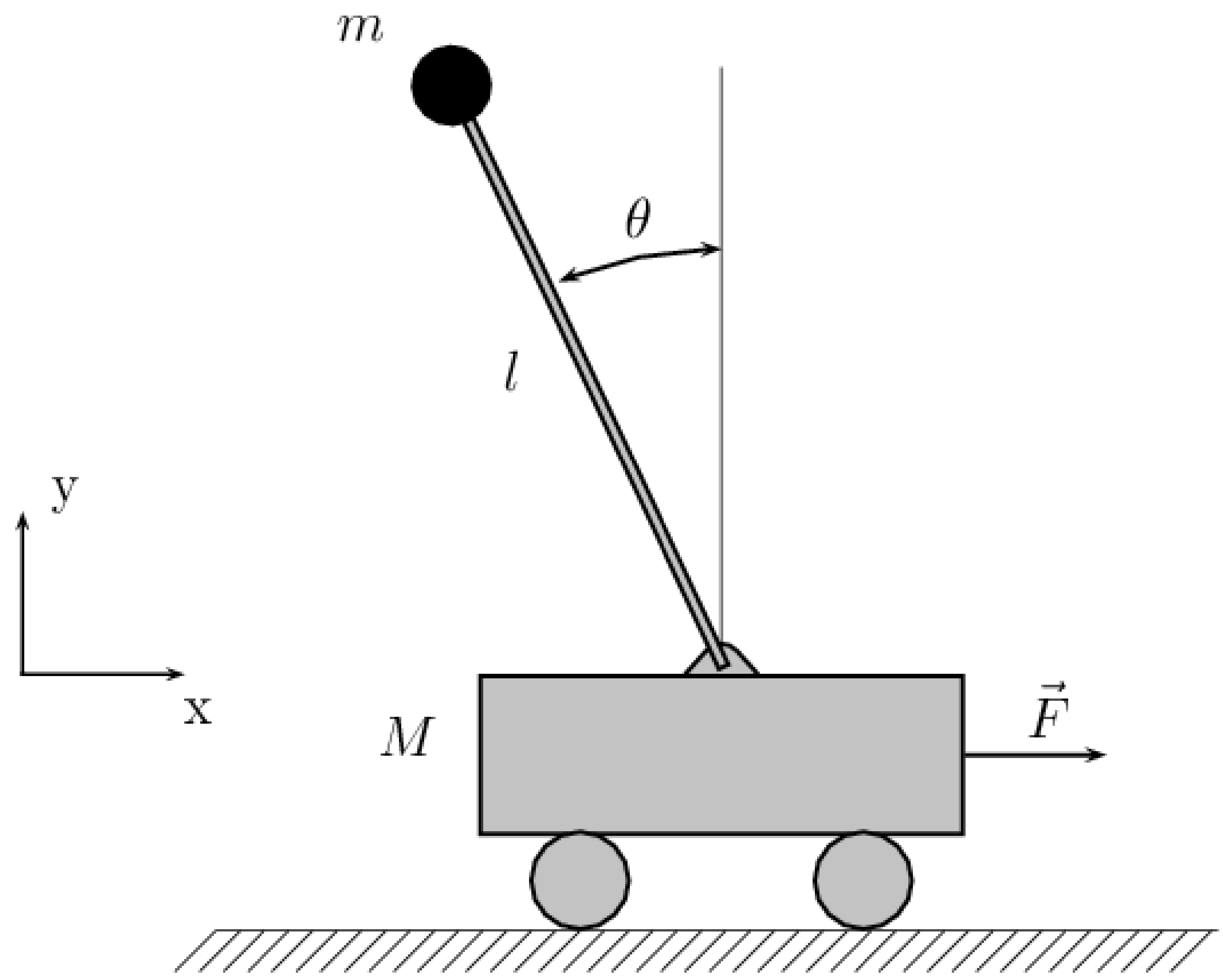
Q & A?...

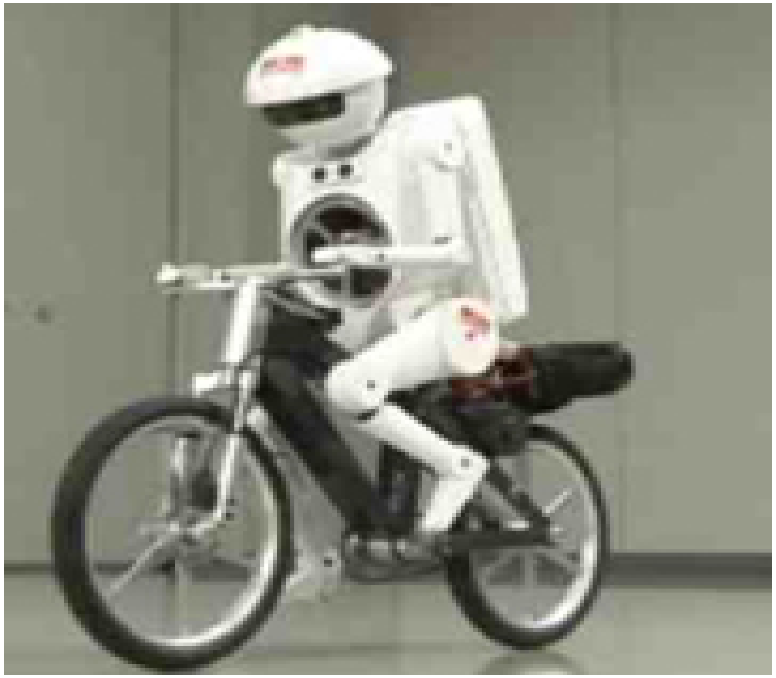
Backup Slides ...

What is an Inverted Pendulum?



Real Examples

Real Examples



Real Examples

Real Examples

Modeling - Equations

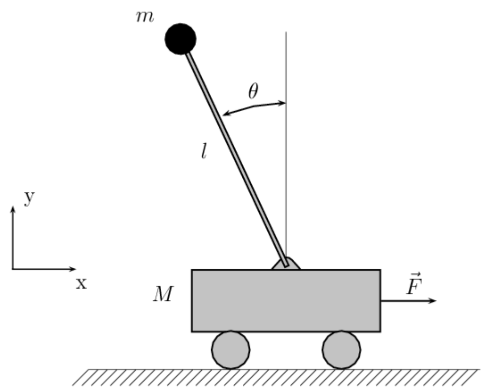
$$M\ddot{x}+b\dot{x}+N=F$$

$$N=m\ddot{x}+ml\ddot{\theta}\cos\theta-ml\dot{\theta}^2\sin\theta$$

$$(M+m)\ddot{x}+b\dot{x}+ml\ddot{\theta}\cos\theta-ml\dot{\theta}^2\sin\theta=F$$

$$(I+ml^2)\ddot{\theta}+mgl\sin\theta=-ml\ddot{x}\cos\theta$$

Inverted Pendulum Control System



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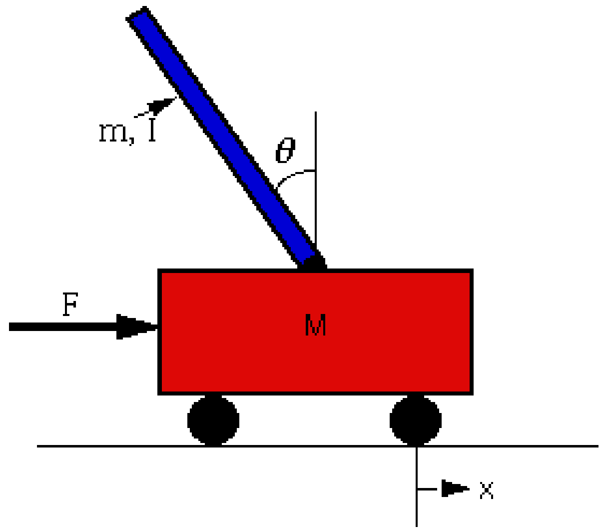
What is an Inverted Pendulum?



An inverted pendulum is a pendulum which has its mass above its pivot point.

Whereas a normal pendulum is stable when hanging downwards, an inverted pendulum is inherently unstable, and must be actively balanced in order to remain upright by moving the pivot point horizontally as part of a feedback system.

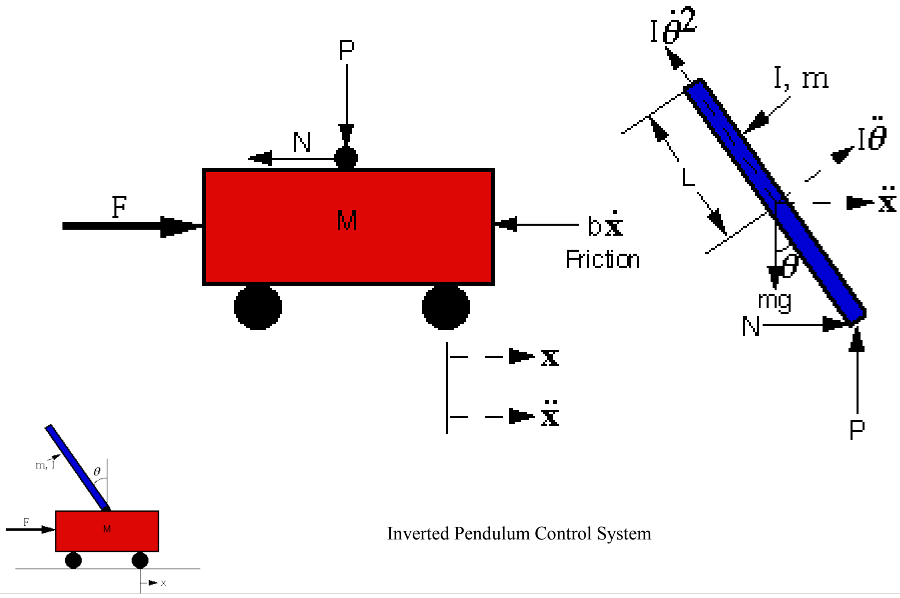
Real Examples

- Rockets and Missiles
- Heavy Cranes lifting containers in shipyards
- Self balancing Robots
- Future Transport Systems like:
  - Segways
  - Jetpacks!

The inverted pendulum is related to rocket or missile guidance, where thrust is actuated at the bottom of a tall vehicle.

The Segway is a two-wheeled, self-balancing electric vehicle

Mathematical Modeling



Inverted Pendulum Control System

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Mathematical Modeling

(1) $$(I+ml^2)\ddot{\theta}+mgl\sin\theta=-ml\ddot{x}\cos\theta$$

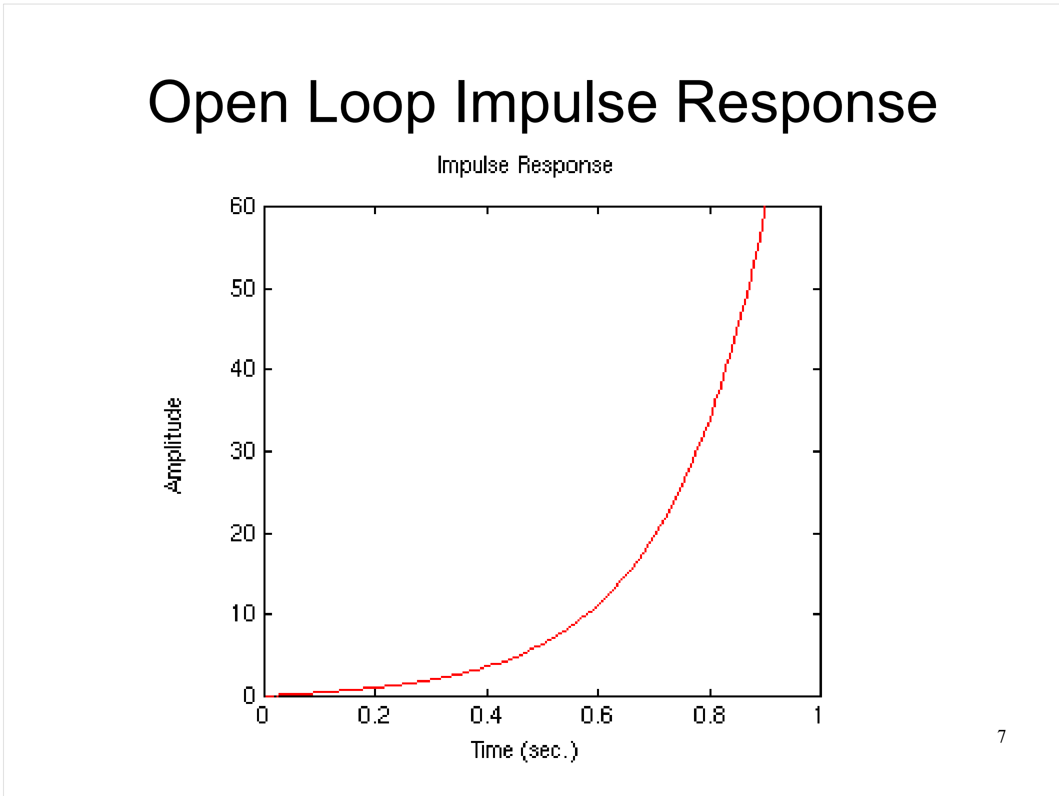
(2) $$(M+m)\ddot{x}+b\dot{x}+ml\ddot{\theta}\cos\theta-ml\dot{\theta}^2\sin\theta=F$$

TF: $$\frac{\Phi(s)}{U(s)}=\frac{\frac{ml}{q}s}{s^3+\frac{b(I+ml^2)}{q}s^2-\frac{(M+m)mgl}{q}s-\frac{bmgl}{q}}$$

Using Newton's laws of Motion and Forces we obtain a a set of **differential equations** governing the system.

These equations are obtained by equating
* Horizontal Forces – Cart, and
* Forces acting Perpendicular to the Pendulum

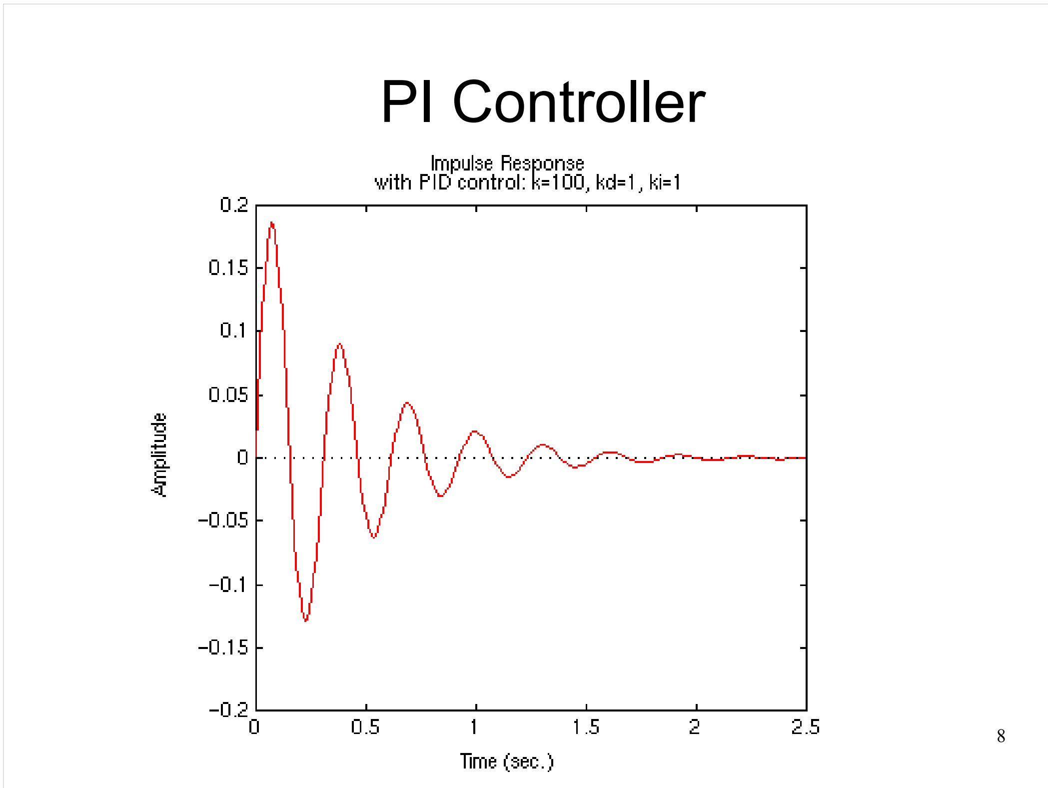
The transfer function of the system indicates that this is a 3rd Order System (Degree of s in the denominator)



Observe the system's **velocity response** to an **impulse force** applied to the cart
It is **not stable** in open loop.
Settling Time = Infinity; SSE = Infinite

In order to bring the Pendulum under control, by “control” I mean back to vertical position after the impulse force has been applied (and following these performance specifications):

* within 5 seconds of **Settling-Time**, and
* with < 3 degree of **Steady State Error**

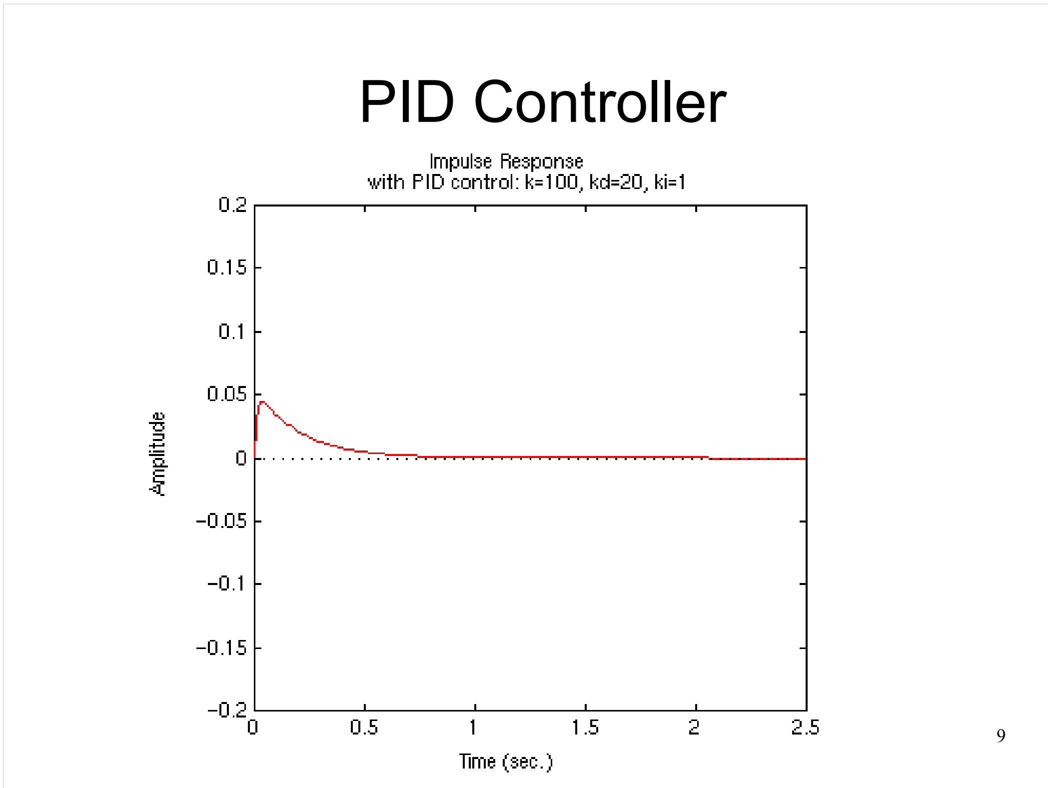


We use a PID controller:

With just **Proportional-Integral** controller

settling time = 2 seconds (acceptable)
steady-state error = 0
overshoot = very high

Since the steady-state error has already been reduced to zero, no more integral control is needed. You can remove the integral gain constant to see for yourself that the small integral control is needed. The overshoot is too high, so that must be fixed.



We use a PID controller:

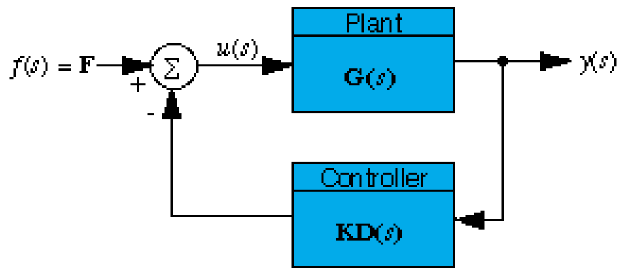
With just **Proportional-Integral-Derivative** controller

settling time = 2 seconds (acceptable)

steady-state error = 0

overshoot = controlled by derivative controller

Closed Loop Transfer Function



Inverted Pendulum Control System 10

Conclusion:

By using a PID Controller:

The overshoot has been reduced so that the pendulum does not move more than 0.05 radians away from the vertical. **All design criteria have been met**, so no further iteration is needed.

Design Not Feasible for a **Actual Physical System**:

The cart moves in the negative direction with a constant velocity. So although the PID controller stabilizes the angle of the pendulum,

Thank you!
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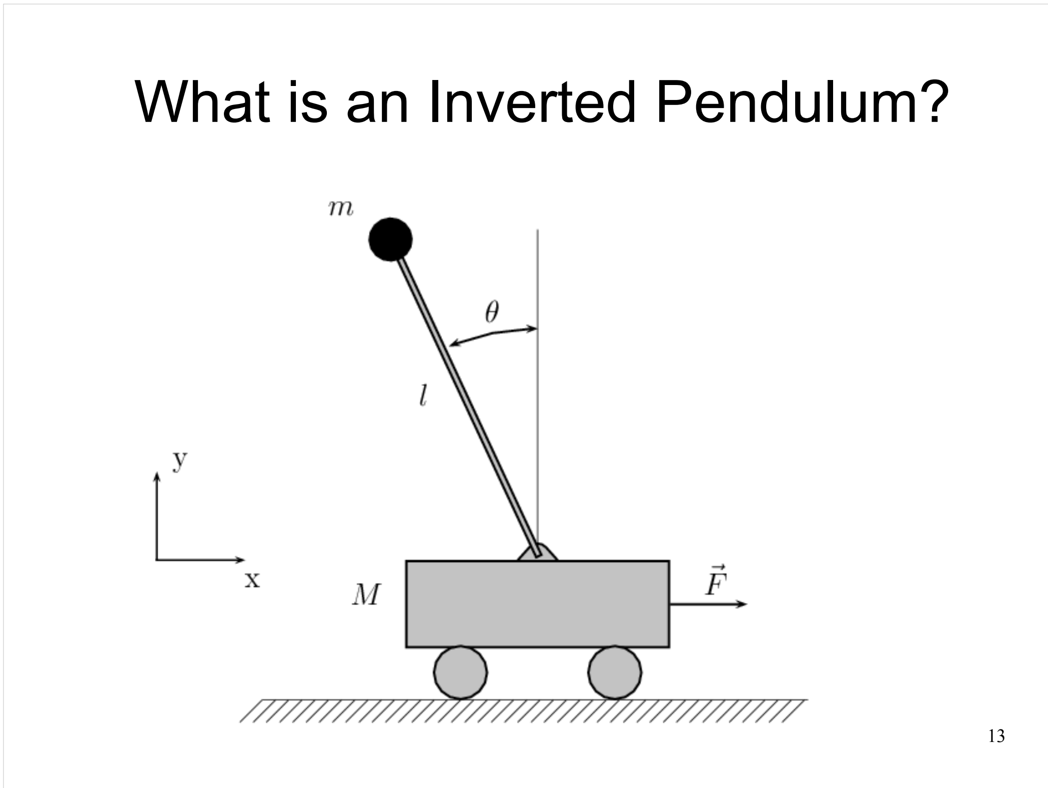
Q & A?...

Backup Slides ...



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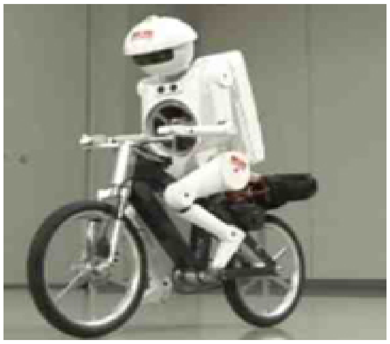
Real Examples

Inverted Pendulum Control System 14

Missles and Rockets

Real Examples



Self Balancing Robots

Real Examples

Inverted Pendulum Control System 16

Future Transport Vehicles - Segways

Real Examples

Inverted Pendulum Control System 17

Or even ...
... “heavier bodies with an ambition to fly!” :-)

Modeling - Equations

$$M\ddot{x}+b\dot{x}+N=F$$

$$N=m\ddot{x}+ml\ddot{\theta}\cos\theta-ml\dot{\theta}^2\sin\theta$$

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