

LAB 3

PART A: Prerequisite for Linear Regression implementation

1. Create an array $x = [1, 1, 2, 3, 4, 3, 4, 6, 4]$ using numpy. Calculate a function $h(x)=t_0+t_1*x$, where $t_0=1.2$ and $t_1=0.5$, for all values of x and plot a graph with x on one axis and $h(x)$ on another axis.

```
In [28]: import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from sklearn.metrics import mean_squared_error
from sklearn.linear_model import LinearRegression
from sklearn.model_selection import train_test_split
plt.style.use("Solarize_Light2")
import random
np.random.seed(9)
np.set_printoptions(suppress=True)
```

```
In [29]: x = [1, 1, 2, 3, 4, 3, 4, 6, 4]
def h(x, t0 = 1.2, t1 = 0.5):
    return t0 + (t1 * x)

print(h(x[5]))
```

2.7

```
In [30]: y = [h(i) for i in x]
print(x)
print(y)
plt.plot(x, y, color='green', label = "line h(x)", marker='o',
markerfacecolor='black', markersize=3)
plt.xlabel('x - axis')
plt.ylabel('y - axis')
plt.title('h(x)')
plt.legend()
plt.show()
```

```
[1, 1, 2, 3, 4, 3, 4, 6, 4]
[1.7, 1.7, 2.2, 2.7, 3.2, 2.7, 3.2, 4.2, 3.2]
```



2. Create two arrays A and B with the following values using numpy array. Let (A_i, B_i) represent a data point with i th element of A and B. $A = [1, 1, 2, 3, 4, 3, 4, 6, 4]$ $B = [2, 1, 0.5, 1, 3, 3, 2, 5, 4]$ Find out the dot product of the vectors. [Hint use numpy `np.dot(a,b)`]

In [31]:

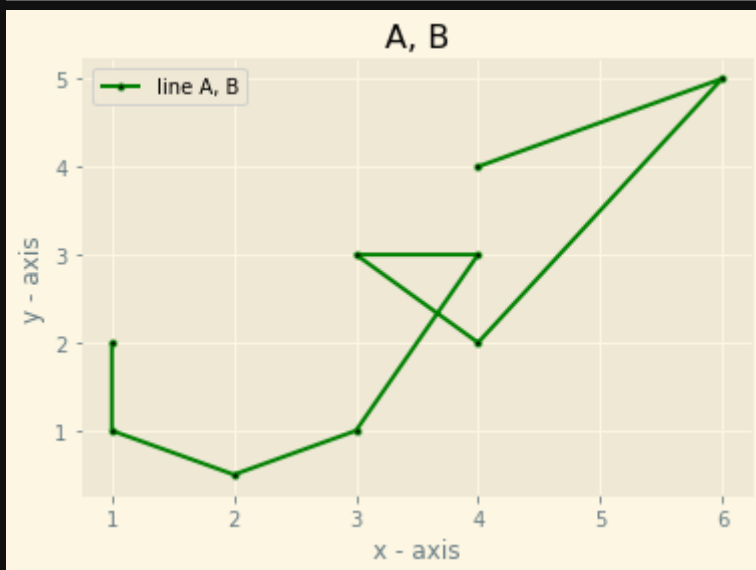
```
A = [1, 1, 2, 3, 4, 3, 4, 6, 4]
B = [2, 1, 0.5, 1, 3, 3, 2, 5, 4]
print("Dot Product of the Vectors: ", np.dot(A, B)) # 2+1+1+3+12+9+8+30+16
= 82
```

Dot Product of the Vectors: 82.0

3. Plot a graph marking the data points (A_i, B_i) with A on the X-axis and B on the Y-axis.

In [32]:

```
plt.plot(A, B, color='green', label = "line A, B", marker='o',
markerfacecolor='black', markersize=3)
plt.xlabel('x - axis')
plt.ylabel('y - axis')
plt.title('A, B')
plt.legend()
plt.show()
```



4. Calculate Mean Square Error (MSE) of A and B with the formulae where n is the number of sample data points.

$$MSE = \frac{1}{n} \sum_{i=1}^n (A^i - B^i)^2$$

```
In [33]: print("mean_squared_error = ", mean_squared_error(A, B)) #using in-built function
sum = 0
for i in range(len(A)):
    sum = sum + ((A[i]-B[i]) * (A[i]-B[i]))
mse = sum/len(A)
print("mean_squared_error = ", mse) #using formula
```

```
mean_squared_error = 1.4722222222222223
mean_squared_error = 1.4722222222222223
```

5. Modify the above equation with the following cost function. Implement as a function with prototype `def compute_cost_function(n,t1,A,B):`

Take $h(x) = t_1 * x$ and $t_1 = 0.5$ Modify the above code iterating for different values of t_1 and calculate $J(t_1)$. Try with $t_1 = 0.1, 0.3, 0.5, 0.7, 0.8$. Plot a graph with t_1 on X-axis and $J(t_1)$ on Y-axis. [hint `sum_squared_error = np.square(np.dot(features, theta) - values).sum()` `cost = sum_squared_error / (2*m)`]

$$J(t_1) = \frac{1}{2n} \sum_{i=1}^n (h(A^i) - B^i)^2$$

```
In [34]: def compute_cost_function(n, t1, A, B):
sum_squared_error = np.square(np.dot(A, t1) - B).sum()
cost = sum_squared_error / (2*n)
return cost
```

```
In [35]: t1 =[0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9]
print("Cost Function = ", compute_cost_function(len(A), t1, A, B))
```

```
Cost Function = 109.20222222222225
```

PART B : Linear Regression Implementation

1. Linear regression with one variable.

a. Generate a new data set from student scores with one feature studytime and output variable average grade = $(G1+G2+G3)/3$

```
In [36]: df_student = pd.read_csv("datasets_52721_99691_student-mat.csv")
df_student
```

```
Out[36]:
```

	school	sex	age	address	famsize	Pstatus	Medu	Fedu	Mjob	Fjob	...	famrel	freetime	goout
0	GP	F	18	U	GT3	A	4	4	at_home	teacher	...	4	3	4
1	GP	F	17	U	GT3	T	1	1	at_home	other	...	5	3	3
2	GP	F	15	U	LE3	T	1	1	at_home	other	...	4	3	2

	school	sex	age	address	famsize	Pstatus	Medu	Fedu	Mjob	Fjob	...	famrel	freetime	goout
3	GP	F	15	U	GT3	T	4	2	health	services	...	3	2	2
4	GP	F	16	U	GT3	T	3	3	other	other	...	4	3	2
...
390	MS	M	20	U	LE3	A	2	2	services	services	...	5	5	4
391	MS	M	17	U	LE3	T	3	1	services	services	...	2	4	5
392	MS	M	21	R	GT3	T	1	1	other	other	...	5	5	3
393	MS	M	18	R	LE3	T	3	2	services	other	...	4	4	1
394	MS	M	19	U	LE3	T	1	1	other	at_home	...	3	2	3

```
In [37]: df_student["average_grade"] = ( df_student["G1"] + df_student["G2"] +
df_student["G3"] ) / 3
```

b. Load the new data set

```
In [38]: df_student
```

```
Out[38]:
```

	school	sex	age	address	famsize	Pstatus	Medu	Fedu	Mjob	Fjob	...	freetime	goout	Dalc	Walc
	0	GP	F	18	U	GT3	A	4	4	at_home	teacher	...	3	4	1
	1	GP	F	17	U	GT3	T	1	1	at_home	other	...	3	3	1
	2	GP	F	15	U	LE3	T	1	1	at_home	other	...	3	2	2
	3	GP	F	15	U	GT3	T	4	2	health	services	...	2	2	1
	4	GP	F	16	U	GT3	T	3	3	other	other	...	3	2	1

	390	MS	M	20	U	LE3	A	2	2	services	services	...	5	4	4
	391	MS	M	17	U	LE3	T	3	1	services	services	...	4	5	3
	392	MS	M	21	R	GT3	T	1	1	other	other	...	5	3	3
	393	MS	M	18	R	LE3	T	3	2	services	other	...	4	1	3
	394	MS	M	19	U	LE3	T	1	1	other	at_home	...	2	3	3

395 rows × 34 columns

```
In [39]: df_student.columns
```

```
Out[39]: Index(['school', 'sex', 'age', 'address', 'famsize', 'Pstatus', 'Medu', 'Fedu',
'Mjob', 'Fjob', 'reason', 'guardian', 'traveltime', 'studytime',
'failures', 'schoolsup', 'famsup', 'paid', 'activities', 'nursery',
'higher', 'internet', 'romantic', 'famrel', 'freetime', 'goout', 'Dalc',
'Walc', 'health', 'absences', 'G1', 'G2', 'G3', 'average_grade'],
dtype='object')
```

```
In [40]: X = df_student[['studytime']]
y = df_student[['average_grade']]
X
```

```
Out[40]: studytime
```

	studytime
0	2
1	2
2	2
3	3
4	2
...	...
390	2
391	1
392	1
393	1
394	1

c. Plot data

```
In [41]: plt.scatter(X, y, c="blue")
plt.xlabel("studytime")
plt.ylabel("average_grade")
plt.show()
```



d. Implement linear regression using inbuilt package python Scikit

```
In [42]: y
```

	average_grade
0	5.666667
1	5.333333
2	8.333333
3	14.666667
4	8.666667
...	...
390	9.000000

average_grade

391 15.333333

392 8.333333

393 11.000000

394 8.666667

```
In [43]: X_train, X_test, y_train, y_test = train_test_split(X, y, test_size = 0.25,
shuffle = False)
regressor = LinearRegression()
regressor.fit(X_train, y_train)
y_pred = regressor.predict(X_test)
y_pred = [i[0] for i in y_pred]
print(y_pred[0:10])
print(y_test[0:10])
```

```
[10.647813657332208, 10.647813657332208, 11.400656227665518, 10.27139237216555, 10.6478
13657332208, 10.27139237216555, 11.024234942498863, 11.400656227665518, 10.647813657332
208, 10.647813657332208]
average_grade
296 6.333333
297 8.666667
298 13.666667
299 15.666667
300 11.000000
301 10.666667
302 13.666667
303 17.333333
304 14.000000
305 12.666667
```

e. Implement gradient descent algorithm with the function prototype `def gradient_descent(alpha, x, y, max_iter=1500)`: where `alpha` is the learning rate, `x` is the input feature vector. `y` is the target. Subject the feature vector to normalisation step if needed. Convergence criteria: when no: of iterations exceed `max_iter`.

[hint `sum_squared_error = np.square(np.dot(features, theta) - values).sum()`
`cost = sum_squared_error / (2*m)`]

```
In [44]: x = df_student['studytime']
y = df_student['average_grade']

x = (x - x.mean()) / x.std()
x = np.c_[np.ones(x.shape[0]), x]
```

```
In [45]: def gradient_descent(x, y, theta, iterations, alpha):
    past_costs = []
    past_thetas = [theta]
    for i in range(iterations):
        prediction = np.dot(x, theta)
        error = prediction - y
        cost = 1/(2*m) * np.dot(error.T, error)
        past_costs.append(cost)
        theta = theta - (alpha * (1/m) * np.dot(x.T, error))
        past_thetas.append(theta)

    return past_thetas, past_costs
```

f. Vary learning rate from 0.1 to 0.9 and observe the learned parameter.

```
In [46]: alpha = [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9] #Learning rate
iterations = 1500 #No. of iterations
m = y.size #No. of data points
np.random.seed(123) #Set the seed
theta = np.random.rand(2) #Pick some random values to start with
for i in range(len(alpha)):
    past_thetas, past_costs = gradient_descent(x, y, theta, iterations,
alpha[i])
    theta = past_thetas[-1]
    print("\n\nLearning Rate = ", alpha[i])
    print("Gradient Descent: {:.2f}, {:.2f}".format(theta[0], theta[1]))
    print("Cost = ", past_costs[-1])
```

```
Learning Rate = 0.1
Gradient Descent: 10.68, 0.50
Cost = 6.69239454434365
```

```
Learning Rate = 0.2
Gradient Descent: 10.68, 0.50
Cost = 6.692394544343652
```

```
Learning Rate = 0.3
Gradient Descent: 10.68, 0.50
Cost = 6.69239454434365
```

```
Learning Rate = 0.4
Gradient Descent: 10.68, 0.50
Cost = 6.69239454434365
```

```
Learning Rate = 0.5
Gradient Descent: 10.68, 0.50
Cost = 6.69239454434365
```

```
Learning Rate = 0.6
Gradient Descent: 10.68, 0.50
```

```
Cost = 6.69239454434365
```

```
Learning Rate = 0.7  
Gradient Descent: 10.68, 0.50  
Cost = 6.69239454434365
```

```
Learning Rate = 0.8  
Gradient Descent: 10.68, 0.50  
Cost = 6.69239454434365
```

```
Learning Rate = 0.9  
Gradient Descent: 10.68, 0.50  
Cost = 6.69239454434365
```

In [47]:

```
'''  
def cal_cost(theta,X,y):  
    m = len(y)  
    predictions = X.dot(theta)  
    cost = (1/2*m) * np.sum(np.square(predictions-y))  
    return cost  
  
def gradient_descent(alpha, X, y, iterations):  
    m = len(y)  
    theta = np.random.randn(2,1)  
    cost_history = np.zeros(iterations)  
    theta_history = np.zeros((iterations,2))  
    for it in range(iterations):  
        prediction = np.dot(X, theta)  
        theta = theta - (1/m)*alpha*( X.T.dot((prediction - y)))  
        theta_history[it,:] =theta.T  
        cost_history[it] = cal_cost(theta,X,y)  
  
    return cost_history  
  
X = df_student[['studytime']]  
y = df_student[['average_grade']]  
alpha = [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9]  
X_b = np.c_[np.ones((len(X),1)),X]  
MSE = []  
for learning_rate in alpha:  
    cost_history = gradient_descent(learning_rate, X_b, y, 1500)  
    MSE.append(cost_history[-1])  
for i in range(len(alpha)):  
    print("For Learning Rate = ", alpha[i]," , MSE = ", MSE[i])  
'''
```

Out[47]:

```
'\ndef cal_cost(theta,X,y):\n    m = len(y)\n    predictions = X.dot(theta)\n    cost = (1/2*m) * np.sum(np.square(predictions-y))\n    return cost\n\ndef gradient_descent(alpha, X, y, iterations):\n    m = len(y)\n    theta = np.random.randn(2,1)\n    cost_history = np.zeros(iterations)\n    theta_history = np.zeros((iterations,2))\n    for it in range(iterations):\n        prediction = np.dot(X, theta)\n        theta = theta - (1
```



```

/m)*alpha*( X.T.dot((prediction - y)))\n          theta_history[it,:] =theta.T\n      cost_history[it] = cal_cost(theta,X,y)\n          \n      return cost_history \n\nX = df_student[['studytime\']]\ny = df_student[['average_grade\']]\nalpha = [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9]\nX_b = np.c_[np.ones((len(X),1)),X]\nMSE = []\nfor learning_rate in alpha:\n    cost_history = gradient_descent(learning_rate, X_b, y, 1500)\n    MSE.append(cost_history[-1])\nfor i in range(len(alpha)):\n    print("For Learning Rate

```