GUJARAT TECHNOLOGICAL UNIVERSITY
DIPLOMA ENGINEERING – SEMESTER – 2(NEW) • EXAMINATION – SUMMER - 2018

Subject Code: 3320002   **Date: 23-May-2018**
Subject Name: ADVANCED MATHEMATICS (GROUP-1)
Time: 10:30 AM TO 01:00 PM   **Total Marks: 70**

Instructions:
1. Attempt ALL questions.
2. Make Suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Use of SIMPLE CALCULATOR is permissible. (Scientific/Higher Version not allowed)
5. English version is authentic.

Q.1   Fill in the blanks using appropriate choice from the given options.   **14**

1   If $|\bar{z}| = 16$, then $|z| = $ ....
   (a) 16   (b) 4   (c) 256   (d) 1

૧   જો $|\bar{z}| = 16$ હોય,તો $|z| = $ ....
   (a) 16   (b) 4   (c) 256   (d) 1

2 $i^9 = $ ....
   (a)i (b)-i(c)1(d)-1

૨ $i^9 = $ ....
   (અ) i (બ) -i(ક) 1(ડ) -1

3   If $z = 5 - 2i$ then $\bar{z} = $ _____
   (a) $-5 + 2i$ (b) $-5 - 2i$ (c)$5 + 2i$ (d) none of these

3   જો $z = 5 - 2i$ હોય તો $\bar{z} = $ _____
   (અ) $-5 + 2i$ (બ) $-5 - 2i$ (ક)$5 + 2i$ (ડ) એક પણ નહી

4   If $f(x)= \log (\tan x)$ then $f\left(\frac{\pi}{4}\right)=$ _____

   (a) 1   (b) e   (c) 0   (d) π

૪   જો $f(x)= \log (\tan x)$ તો $f\left(\frac{\pi}{4}\right)=$ _____

   (a) 1   (b) e   (c) 0   (d) π

5 $\lim\limits_{\theta \to 0} \dfrac{\theta}{\tan 3\theta} = $ _____.

   (a)   3   (b) $\dfrac{1}{3}$ (c)   1   (d)   0

ય $\lim\limits_{\theta \to 0} \dfrac{\theta}{\tan 3\theta} = $ _____.

   (a)   3   (b) $\dfrac{1}{3}$ (c)   1   (d)   0

6 $\dfrac{d}{dx}(x^2+2^x+2^2) = $ …………

    (a) 1   (b) $2x+2^x+2^2$ (c) $2x+2^x\log 2$ (d) 0

૬ $\dfrac{d}{dx}(x^2+2^x+2^2) = $ …………

    (a) 1   (b) $2x+2^x+2^2$ (c) $2x+2^x\log 2$ (d) 0

7 If $y = e^x$, then $\dfrac{d^2y}{dx^2} = $ ………

    (a) e^x (b) e^{2x} (c) e^{x^2} (d) e^{-x}

૭ જો $y = e^x$ હોય,તો $\dfrac{d^2y}{dx^2} = $ ………

    (a) e^x (b) e^{2x} (c) e^{x^2} (d) e^{-x}

8 If $x=\cos\theta$, $y=\sin\theta$ then $\dfrac{dy}{dx} = $…………

    (a)$\cot\theta$ $(b)\tan\theta$ $(c)-\cot\theta$ $(d)-\tan\theta$

૮ જો $x=\cos\theta$, $y=\sin\theta$ તો $\dfrac{dy}{dx} = $…………

    (અ)$\cot\theta$ (બ)$\tan\theta$ (ક)$-\cot\theta$ (ડ)$-\tan\theta$

9 $\dfrac{d}{dx}x^x = $ _____ .

    (a) $x-\log x$ (b) $x+\log x$ (c) $x^x(1+\log x)$ (d) $x \cdot x^{x-1}$

૯ $\dfrac{d}{dx}x^x = $ _____ .

    (a) $x-\log x$ (b) $x+\log x$ (c) $x^x(1+\log x)$ (d) $x \cdot x^{x-1}$

10 $\int \dfrac{1}{x^2}dx = $……+c

    (a) $\dfrac{1}{x}$ (b) $\dfrac{-1}{x}$ (c)$\dfrac{-1}{3x^3}$ (d)$\dfrac{1}{3x^3}$

૧૦ $\int \dfrac{1}{x^2}dx = $……+c

    (અ)$\dfrac{1}{x}$ (બ)$\dfrac{-1}{x}$ (ક)$\dfrac{-1}{3x^3}$ (ડ)$\dfrac{1}{3x^3}$

11 $\int \dfrac{1}{x^2+25}dx = \ldots\ldots + c$

(a) $\tan^{-1}\left(\dfrac{x}{5}\right)$ (b) $\dfrac{1}{5}\tan^{-1}\left(\dfrac{x}{5}\right)$ (c) $\dfrac{1}{5}\tan^{-1}\left(\dfrac{5}{x}\right)$ (d) $\tan^{-1}\left(\dfrac{5}{x}\right)$

૧૧ $\int \dfrac{1}{x^2+25}dx = \ldots\ldots + c$

(અ) $\tan^{-1}\left(\dfrac{x}{5}\right)$ (બ) $\dfrac{1}{5}\tan^{-1}\left(\dfrac{x}{5}\right)$ (ક) $\dfrac{1}{5}\tan^{-1}\left(\dfrac{5}{x}\right)$ (ડ) $\tan^{-1}\left(\dfrac{5}{x}\right)$

12 $\int_0^1 e^x\, dx = \ldots\ldots$
(a) e-1 (b)1-e (c) e (d)-e

૧૨ $\int_0^1 e^x\, dx = \ldots\ldots$
(અ)e-1 (બ)1-e (ક) e (ડ)-e

13 The order of a differential equation $\dfrac{d^2y}{dx^2}=\left(3+\dfrac{dy}{dx}\right)^3$ is $\ldots\ldots$

(a) 3   (b) 2   (c) 1   (d) 6

૧૩ વિકલ્ સમીકરણ $\dfrac{d^2y}{dx^2}=\left(3+\dfrac{dy}{dx}\right)^3$ ની કક્ષા $\ldots\ldots$ છે.

(a) 3   (b) 2   (c) 1   (d) 6

14 The integrating factor(I.F.) of $\dfrac{dy}{dx}+\dfrac{2y}{x}=e^x$ is $\ldots\ldots\ldots$

(a) $\dfrac{2}{x^2}$ (b) $\dfrac{2}{x^2}$ (c) x^2 (d) $2x$

૧૪ વિકલ્ સમીકરણ $\dfrac{dy}{dx}+\dfrac{2y}{x}=e^x$ નો સંકલ્યકારક અવયવ (I.F.) $\ldots\ldots\ldots$ છે.

(a) $\dfrac{2}{x^2}$ (b) $\dfrac{2}{x^2}$ (c) x^2 (d) $2x$

Q.2 (a) Attempt any two કોઇપણ બે ના જવાબ આપો.   06

1. Find the modulus and principal argument of $z=\sqrt{3}+i$ and express z into polar form.

૧. $z=\sqrt{3}+i$ નો માનાંક અને કોણાંક શોધો, તથા z ને ધ્રુવિય સ્વરૂપમાં અભિવ્યક્ત કરો

2. Find the square root of $3-4\sqrt{10}i$

૨. $3-4\sqrt{10}i$ નું વર્ગમૂળ શોધો

3. Prove that $(1+\cos\theta+i\sin\theta)^n = 2^n\cos^n\left(\dfrac{\theta}{2}\right)\left[\cos\dfrac{n\theta}{2}+i\sin\dfrac{n\theta}{2}\right]$

3. સાબિત કરો કે $(1+\cos\theta+i\sin\theta)^n = 2^n \cos^n\left(\dfrac{\theta}{2}\right)\left[\cos\dfrac{n\theta}{2}+i\sin\dfrac{n\theta}{2}\right]$

(b) Attempt any two કોઈપણ બે ના જવાબ આપો.　　　　　　　　　　**08**

1. If $f(x) = \log x$, then prove that
   (i) $f(x)+f(y) = f(xy)$ and (ii) $f(x)-f(y) = f(x/y)$

૧. જો $f(x) = \log x$ હોય, તો સાબિત કરોકે
   (i) $f(x)+f(y) = f(xy)$ and (ii) $f(x)-f(y) = f(x/y)$

2. Evaluate : $\displaystyle\lim_{x\to 0}\dfrac{3\sin x - \sin 3x}{x^3}$

૨. $\displaystyle\lim_{x\to 0}\dfrac{3\sin x - \sin 3x}{x^3}$ મેળવો

3. Evaluate : $\displaystyle\lim_{x\to 0}\dfrac{x\log(1+x)}{1-\cos x}$

૩. લક્ષ શોધો $\displaystyle\lim_{x\to 0}\dfrac{x\log(1+x)}{1-\cos x}$

Q.3 **(a)** Attempt any two કોઈપણ બે ના જવાબ આપો.　　　　　　　　　　**06**

1. Differentiate
   e^x with respect to x using first principle of differentiation

૧. e^x નું વિકલન x ની સાપેક્ષ વિકલનનાં પ્રથમ સિધ્ધાંત થી કરો

2. Find $\dfrac{dy}{dx}$ if $y = \log(\sec x + \tan x)$

૨. જો $y = \log(\sec x + \tan x)$ તો $\dfrac{dy}{dx}$ મેળવો

3. Equation of motion of a particle is $s = t^3 - 6t^2 + 8t - 4$ .Then find the velocity and acceleration of the moving particle at t = 3 second.

૩. કણની ગતિનું સમીકરણ $s = t^3 - 6t^2 + 8t - 4$ છે .તો t = 3 સેકન્ડે કણનો વેગ અને પ્રવેગ શોધો.

(b) Attempt any two કોઈપણ બે ના જવાબ આપો.　　　　　　　　　　**08**

1. If $x^3 + y^3 = 3axy$ then find $\dfrac{dy}{dx}$

૧. જો $x^3 + y^3 = 3axy$ તો $\dfrac{dy}{dx}$ શોધો.

2. If $x = \dfrac{1}{2}\left(t + \dfrac{1}{t}\right)$ and $y = \dfrac{1}{2}\left(t - \dfrac{1}{t}\right)$ then find $\dfrac{dy}{dx}$.

૨. જો $x = \dfrac{1}{2}\left(t + \dfrac{1}{t}\right)$ અને $y = \dfrac{1}{2}\left(t - \dfrac{1}{t}\right)$ તો $\dfrac{dy}{dx}$ મેળવો.

3. Find the maximum and minimum values of $f(x) = 3x^3 - 4x^2 - x + 5$

૩. $f(x) = 3x^3 - 4x^2 - x + 5$ નાં મહત્તમ અને ન્યૂનતમ મૂલ્યો શોધો.

Q.4 (a) Attempt any two કોઈપણ બે ના જવાબ આપો. **06**

1. Find $\displaystyle\int \cos x \cdot \sqrt{\sin x}\, dx$

૧. શોધો: $\displaystyle\int \cos x \cdot \sqrt{\sin x}\, dx$

2. Find $\displaystyle\int \dfrac{x}{(x+1)(x+2)}\, dx$

૨. શોધો : $\displaystyle\int \dfrac{x}{(x+1)(x+2)}\, dx$

3. Evaluate: $\displaystyle\int_{0}^{\frac{\pi}{2}} \log \cot x \, dx$

૩. કિંમત શોધો: $\displaystyle\int_{0}^{\frac{\pi}{2}} \log \cot x \, dx$

(b) Attempt any two કોઈપણ બે ના જવાબ આપો. **08**

1. Find $\displaystyle\int x \cdot e^{3x}\, dx$

૧. શોધો : $\displaystyle\int x \cdot e^{3x}\, dx$

2. Evaluate: $\displaystyle\int_{0}^{5} \dfrac{\sqrt[3]{x+2}}{\sqrt[3]{x+2} + \sqrt[3]{7-x}}\, dx$

૨. કિંમત શોધો: $\displaystyle\int_{0}^{5} \dfrac{\sqrt[3]{x+2}}{\sqrt[3]{x+2} + \sqrt[3]{7-x}}\, dx$

3. Find the area bounded by the curve $y = x^2 - 7x + 10$ and x-axis .

3. વક્ર $y = x^2 - 7x + 10$ અને X- અક્ષ વચ્ચે ઘેરાયેલા પ્રદેશનું ક્ષેત્રફળ શોધો

Q.5 **(a)** Attempt any two કોઇપણ બે ના જવાબ આપો. **06**

1. Evaluate : $\lim_{x \to 1} \frac{x^3 - x^2 + x - 1}{x^2 - 1}$

૧. મેળવો : $\lim_{x \to 1} \frac{x^3 - x^2 + x - 1}{x^2 - 1}$

2. Find the order and degree of $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \rho \left(\frac{d^2y}{dx^2}\right)^2$

૨. $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \rho \left(\frac{d^2y}{dx^2}\right)^2$ ની કક્ષા અને પરિમાણ મેળવો.

3. Form the differential equation whose general solution is Y=$A \cos x + B \sin X$

3. જેનો સામાન્ય ઉકેલ Y=$A \cos x + B \sin X$ તે વિકલ સમીકરણ મેળવો.

(b) Attempt any two કોઇપણ બે ના જવાબ આપો. **08**

1. Solve : $\frac{dy}{dx} = \frac{y}{x}$

૧. ઉકેલો : $\frac{dy}{dx} = \frac{y}{x}$

2. Solve $\frac{dy}{dx} = \frac{y}{x} + \text{cosec}\left(\frac{y}{x}\right)$.

૨. $\frac{dy}{dx} = \frac{y}{x} + \text{cosec}\left(\frac{y}{x}\right)$ ઉકેલો.

3. solve : $x \log x \frac{dy}{dx} + y = \log x^2$

3. ઉકેલો: $x \log x \frac{dy}{dx} + y = \log x^2$

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