COMPUTATIONAL HEAT TRANSFER

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190487

- Heat transfer(or heat) is thermal energy in transit due to a spatial temperature difference.
- **First Law of Thermodynamics:**
  - It states that energy can neither be created nor be destroyed, it can only be transformed from one form to another.
  - Heat is a form of energy.
- Heat transfer is categorized into three different method:
  - **Conduction:** When a temperature gradient exists in a stationary medium, which may be a solid or a fluid, we use the term conduction to refer to the heat transfer that will occur across the medium due to interaction between the particles.
  - **Convection:** The term convection refers to heat transfer that will occur between a surface and a moving fluid when they are at different temperatures. This is a combined effect of conduction as well as fluid motion.
  - **Radiation:** All surfaces of finite temperature emit energy in the form of electromagnetic waves. Hence, in the absence of an intervening medium, there is net heat transfer by radiation between two surfaces at different temperatures

- For heat conduction, the rate equation is known as Fourier's Law and given by;

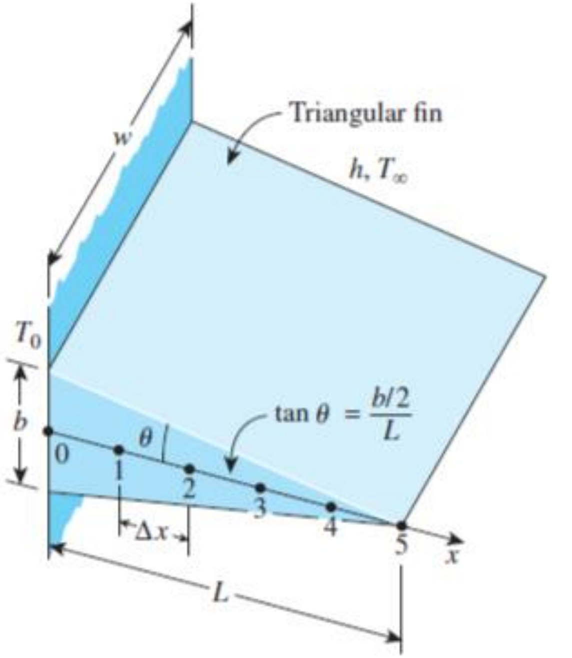
$$q''_x = -\,kdT/dx$$

- As the system of triangular fin is in steady-state, the Fourier's equation gets to :

$$dT/dx = (T_2 - T_1)/L$$

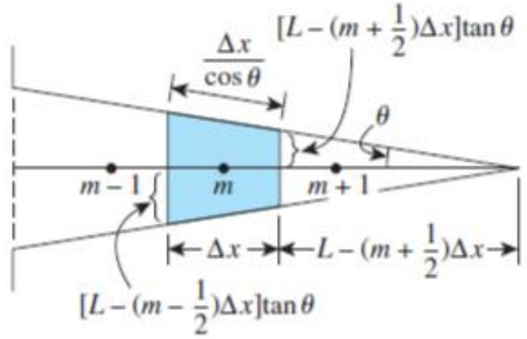
- After applying finite difference method for one dimensional heat conduction, where the fin can be assumed to be made up of certain number of nodes and writing the steady-state energy balances on each surface of the node:

$$\sum_{\text{All sides}} \dot{Q} = 0 \rightarrow kA_{\text{left}}\frac{T_{m-1}-T_m}{\Delta x} + kA_{\text{right}}\frac{T_{m+1}-T_m}{\Delta x} + hA_{\text{conv}}(T_\infty - T_m) = 0$$



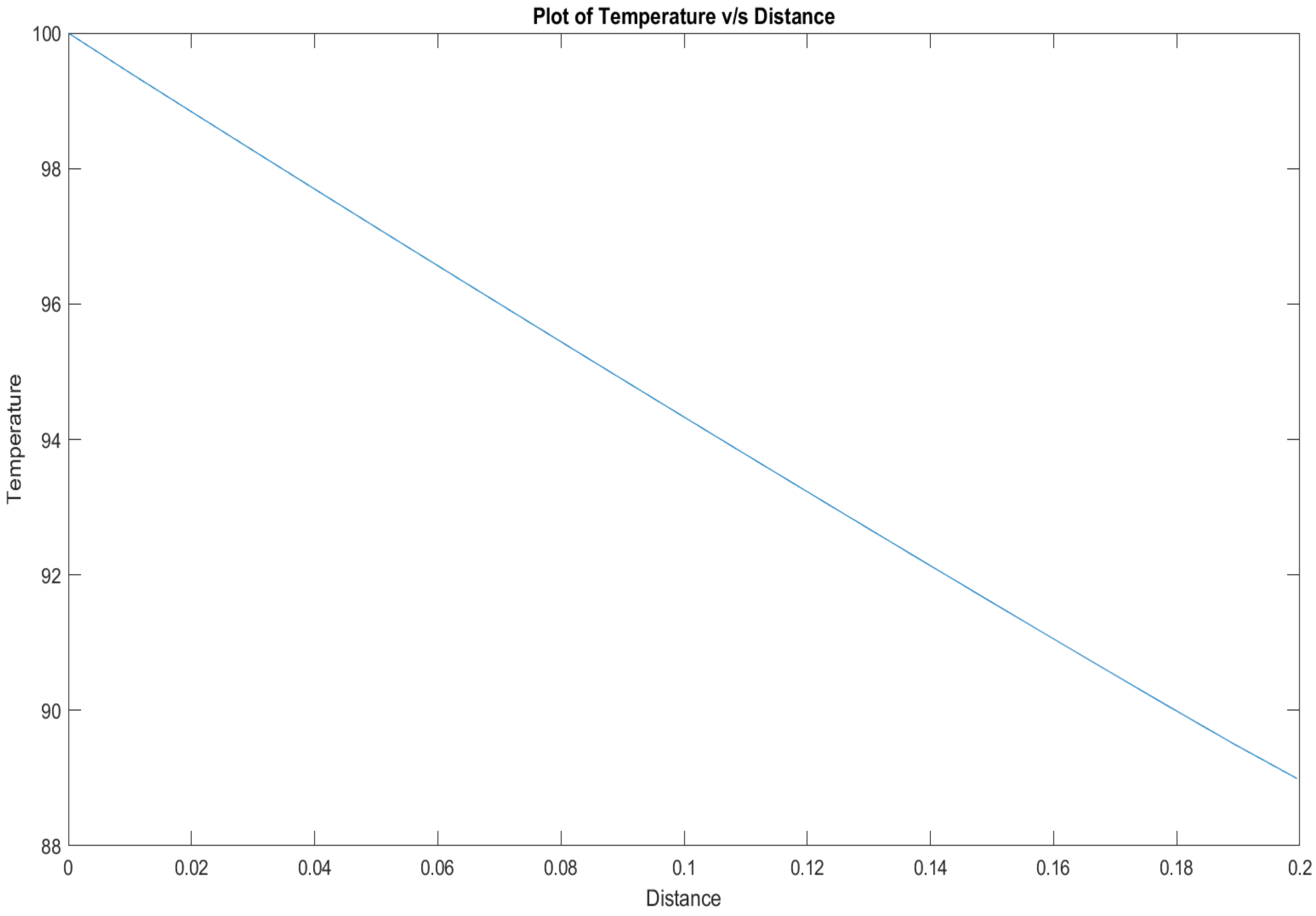
- For the boundary nodes, writing the energy balance with an element half the size of intermediate nodes,

$$kA_{\text{left}}\frac{T_4 - T_5}{\Delta x} + hA_{\text{conv}}\,(T_\infty - T_5) = 0$$



- Finally solving all the equations and plotting the temperatures at the nodes.

Relevant Data:
L = 20cm
b = 4 cm
T0 = 373 K
Tinf = 298 K
h = 15 W/m2K
K = 180 W/mK
No of nodes = 20



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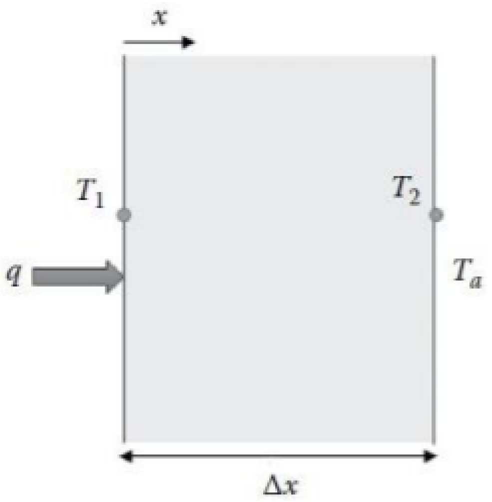
- Heat transfer by Radiation is given by Stefan-Boltzmann law:

$$\left.\frac{q_x}{A}\right|_{x=\Delta x} = \sigma\left(T_2^4 - T_a^4\right)\Big|_{x=\Delta x} \quad \left(\sigma = 5.676\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)$$

- Initially, an estimate of 100 K is taken for T2 for solving the differential equation obtained by equating the heat transfer due to conduction at T1 and the radiative heat transfer in the slab. The convective heat transfer is being neglected between the slab and surrounding is negligible.

$$q_x = -kA\frac{dT}{dx} = \sigma\left(T_2^4 - T_a^4\right) \Longrightarrow \frac{dT}{dx}$$

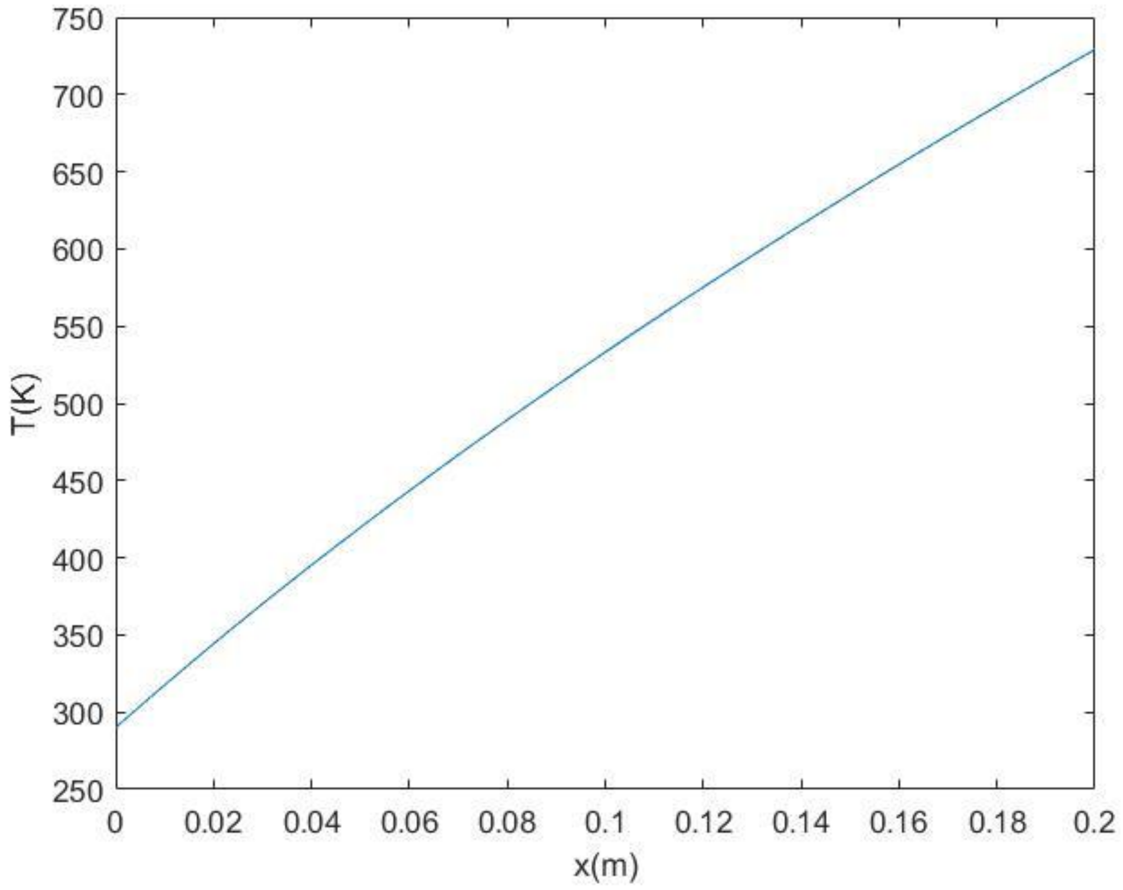
- The solution of the differential equation is found using MATLAB function ode45.
- The iterations are continued till the value of T2 converges. The temperature profile and converged value of T2 are obtained.



Relevant Data:
k = 30(1+0.002T)
T1 = 290 K
Ta = 1273 K
Δx = 0.2 m

Final Temperature(T2): 729.1673 K

Temperature Profile in slab:



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- In transient conduction, the temperature at any point is a function of time.
- The 2-D heat transfer equation is given by

$$(1/\alpha) \cdot (\partial T/\partial t) = \partial^2 T/\partial x^2 + \partial^2 T/\partial y^2$$

- The finite difference approximation is expressed as:

$$(\partial T/\partial t)_{m,n} \approx (T^{p+1}_{m,n} - T^{p}_{m,n})/\Delta t$$

Relevant Data:
T0 = 293 K
Heat flux = 200 kW/m2
Δx = 50 mm
t = 2 min
k = 401 W/mK
α = 117 *10^-6 m2/s
Time step = ½ max value

And it can be used in two ways(assuming $\Delta x = \Delta y$, one-dimensional conduction in x-direction and the slab to be semi-infinite.):

1. **Explicit Method:** The explicit method equation can be formed by substituting finite difference approximation in heat transfer equation and evaluating the temperatures at the previous time.
2. **Implicit Method:** The implicit equation for a two-dimensional system can be derived by approximating the time derivative using finite difference approximation and evaluating all other temperatures at the new time.

- Energy balances for explicit and implicit methods:

$$T^{p+1}_0 = 2Fo((q''\Delta x/k) + T^p_1) + (1-2Fo)T^p_0$$

Explicit Form

$$(1+2Fo)T^{p+1}_0 - 2Fo\, T^{p+1}_1 = ((2\alpha q''\Delta t/k\Delta x) + T^p_0$$

Implicit Form

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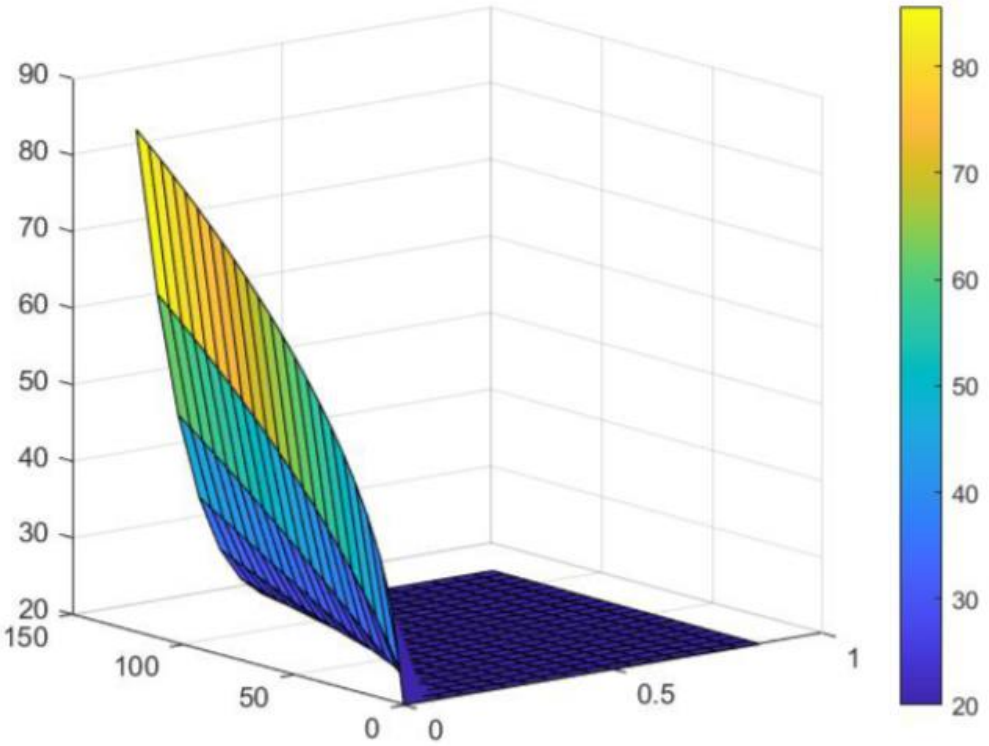
- Taking the boundary conditions(Fo) for explicit and implicit methods to be 1/2 and 1/4 respectively.

THE EXPLICIT METHOD

The surface temperature after 2 min is : 85.6244 $^{\circ}C$

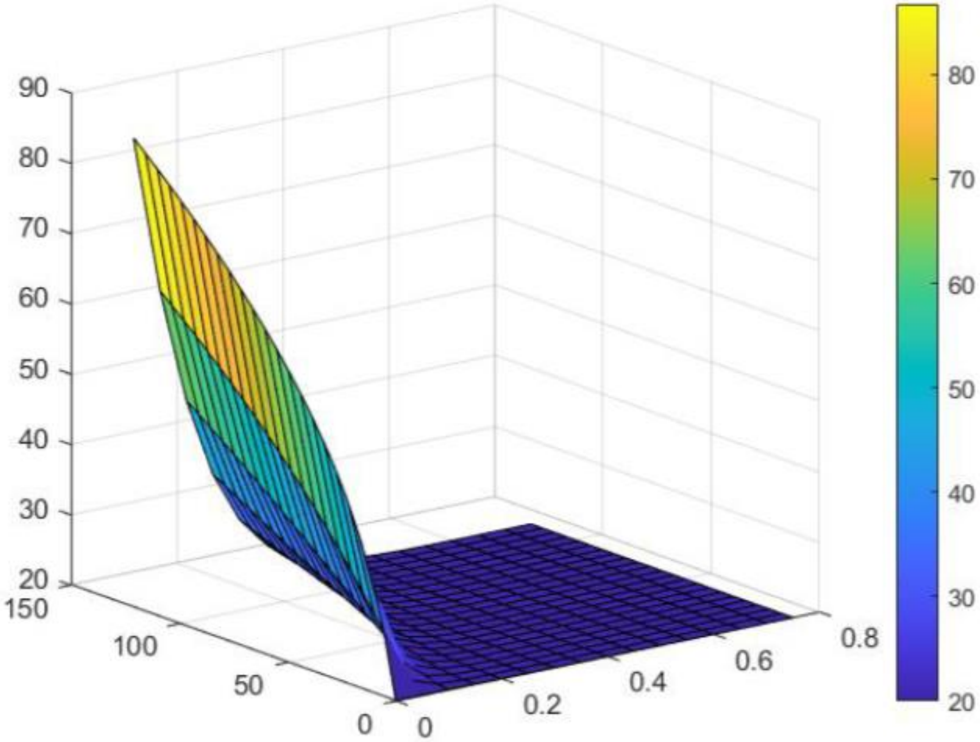
The temperature at x=150mm after 2 min is : 36.1593 $^{\circ}C$



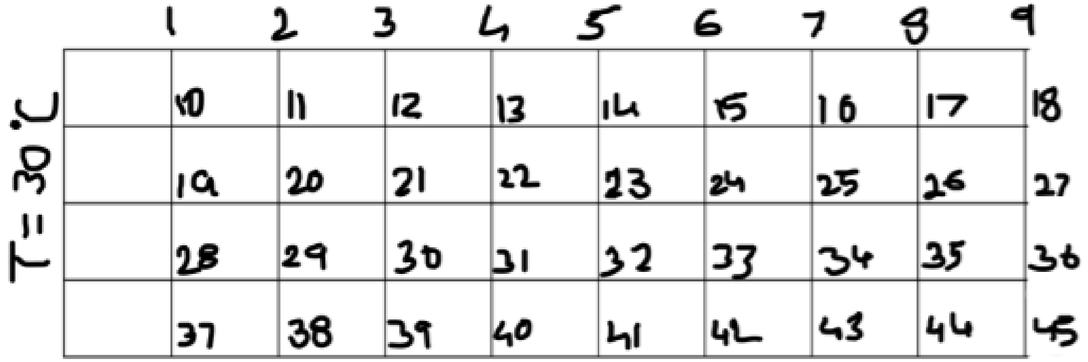
THE IMPLICIT METHOD

The surface temperature after 2 min is : 86.7704 $^{\circ}C$

The temperature at x=150mm after 2 min is : 36.5666 $^{\circ}C$



- The temperature distribution in the fin can be found with dividing it into a grid of 10×5 nodes(5 nodes in the left are neglected as their temperature remains constant). The interior heat transfer takes place due to **conduction** and on the upper and lower sides, heat transfer takes place due to **convection**



Relevant data:
L = 1 m
b = 20 cm
k = 401 W/mK
Tb = 303 K
Tinf = 293 K
h = 10 W/m2K

- The steady-state conduction equation for a node is given by

$$\partial Q_{cond,left}/\partial t + \partial Q_{cond,right}/\partial t + \partial Q_{cond,top}/\partial t + \partial Q_{cond,bottom}/\partial t + \partial E_{gen,element}/\partial t = \Delta E/\Delta t = 0$$

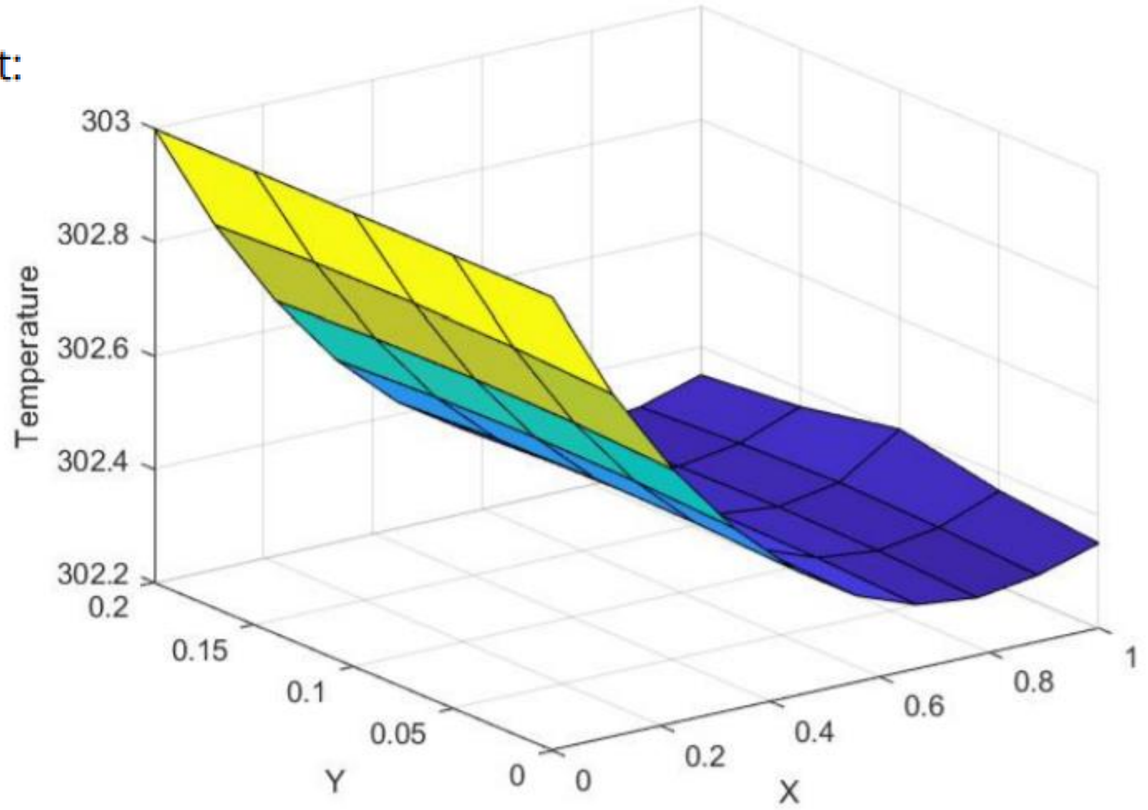
- The energy balances for each nodes are given by:
  - Top Node: $h\Delta x(T_{\infty} - T_{i,j}) + k\Delta x(T_{i+1,j} - T_{i,j})/\Delta y + k\Delta y(T_{i,j-1} - T_{i,j})/2\Delta x + k\Delta y(T_{i,j+1} - T_{i,j})/2\Delta x = 0$
  - Bottom Node: $h\Delta x(T_{\infty} - T_{i,j}) + k\Delta x(T_{i-1,j} - T_{i,j})/\Delta y + k\Delta y(T_{i,j-1} - T_{i,j})/2\Delta x + k\Delta y(T_{i,j+1} - T_{i,j})/2\Delta x = 0$
  - Interior Node: $(T_{i-1,j} - 2T_{i,j} + T_{i+1,j})/(\Delta x)^2 + (T_{i,j-1} - 2T_{i,j} + T_{i,j+1})/(\Delta y)^2 = 0$

- Right Nodes(Except for the corner nodes): The insulated boundary can be taken as a mirror and the node becomes an interior node with the left and right temperatures to be the same.

$$(2T_{i-1,j} - 2T_{i,j})/(\Delta x)^2 + (T_{i,j-1} - 2T_{i,j} + T_{i,j+1})/(\Delta y)^2 = 0$$

- Right Corner Nodes: $(h\Delta x(T_\infty - T_{i,j}) + k\Delta x(T_{i+1,j} - T_{i,j})/\Delta y + k\Delta y(T_{i,j-1} - T_{i,j})/\Delta x)/2 = 0$

2D Temperature plot:



- The unsteady state heat conduction equation is given by:

$$\partial Q_{cond,left}/\partial t + \partial Q_{cond,right}/\partial t + \partial Q_{cond,top}/\partial t + \partial Q_{cond,bottom}/\partial t + \partial E_{gen,element}/\partial t = \Delta E/\Delta t = \rho V_{element} c_p \partial T/\partial t$$

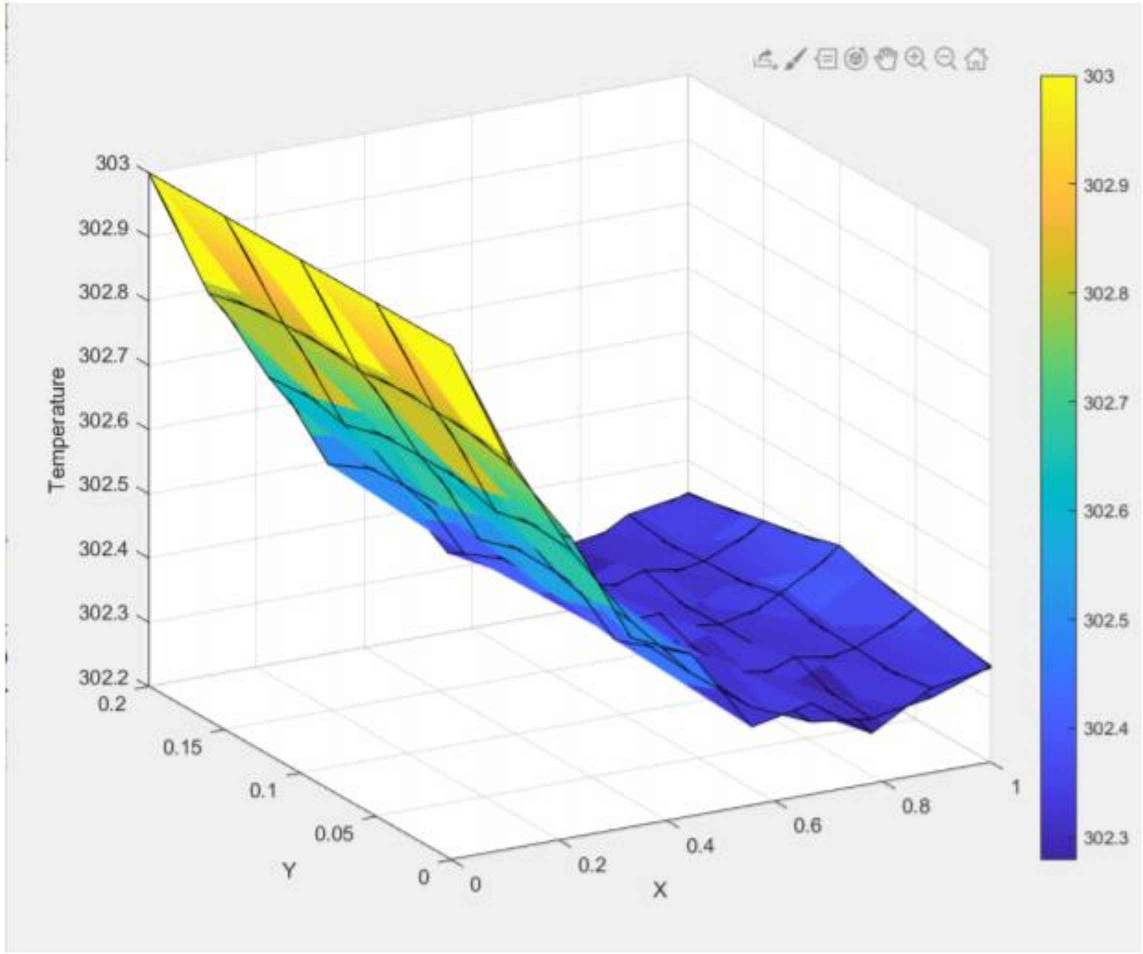
- For different nodes, the energy balances are given by:
  - Top Node: $\partial T_{i,j}/\partial t = h\Delta x(T_{\infty} - T_{i,j}) + k\Delta x(T_{i+1,j} - T_{i,j})/\Delta y + k\Delta y(T_{i,j-1} - T_{i,j})/2\Delta x + k\Delta y(T_{i,j+1} - T_{i,j})/2\Delta x$
  - Bottom Node: $\partial T_{i,j}/\partial t = h\Delta x(T_{\infty} - T_{i,j}) + k\Delta x(T_{i-1,j} - T_{i,j})/\Delta y + k\Delta y(T_{i,j-1} - T_{i,j})/2\Delta x + k\Delta y(T_{i,j+1} - T_{i,j})/2\Delta x$
  - Interior Node: $\partial T_{i,j}/\partial t = (T_{i-1,j} - 2T_{i,j} + T_{i+1,j})/(\Delta x)^2 + (T_{i,j-1} - 2T_{i,j} + T_{i,j+1})/(\Delta y)^2$
  - Right Node(Except for the corner nodes): The insulated boundary can be taken as a mirror and the node becomes an interior node with the left and right temperatures to be the same.

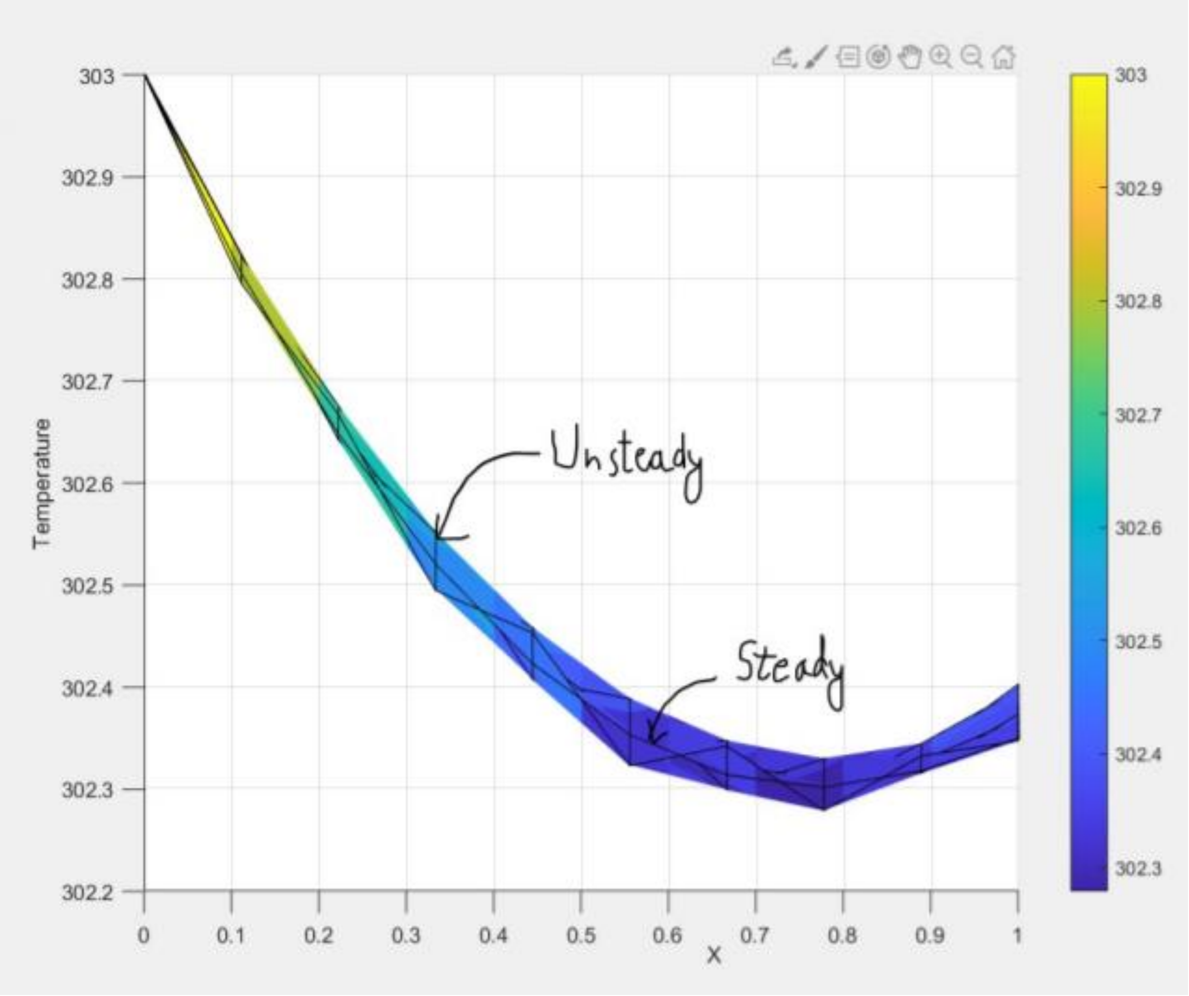
$$\partial T_{i,j}/\partial t = (2T_{i-1,j} - 2T_{i,j})/(\Delta x)^2 + (T_{i,j-1} - 2T_{i,j} + T_{i,j+1})/(\Delta y)^2$$

  - Right Corner Nodes: $\partial T_{i,j}/\partial t = (h\Delta x(T_{\infty} - T_{i,j}) + k\Delta x(T_{i+1,j} - T_{i,j})/\Delta y + k\Delta y(T_{i,j-1} - T_{i,j})/\Delta x)/2$
- From all the above equations we get a matrix equation in the form of F = A*T – C, where F is the differential matrix of temperatures, A is the coefficient matrix of temperatures on the R.H.S and C is the constant matrix.
  - The function F is integrated for 15 seconds using MATLAB function ode45.

Relevant Data :
Ti = Tb, thermal diffusibility = 1m2/s

2D Temperature plot:





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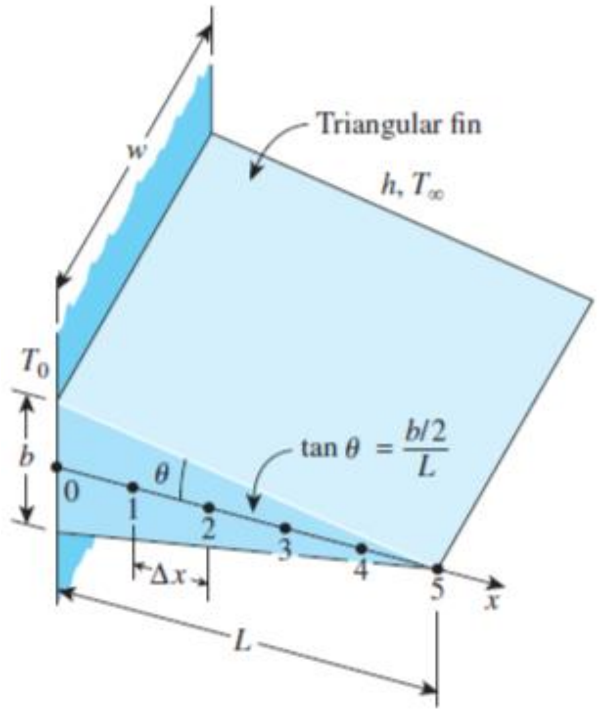
- The temperature distribution in the triangular fin with steady state heat conduction is estimated using 3D finite element analysis technique by taking it to be a mesh made of smaller nodes.
- Made the STL file of the triangular fin.
- Using the PDE toolbox of MATLAB, import the STL file and generate the Mesh Model Plot.
- Using the following inbuilt functions:
  - `thermalProperties:` To define the thermal properties of the fin(thermal conductivity).
  - `thermalBC:` To define the boundary conditions of the fin.
  - `solve:` To solve the model.



Relevant Data:
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b = 20 cm
w = 40 cm
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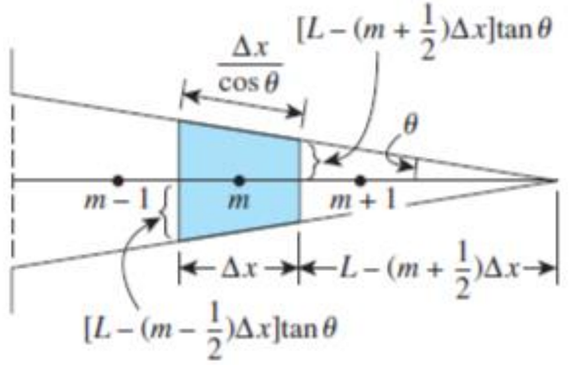
K = 180 W/mK



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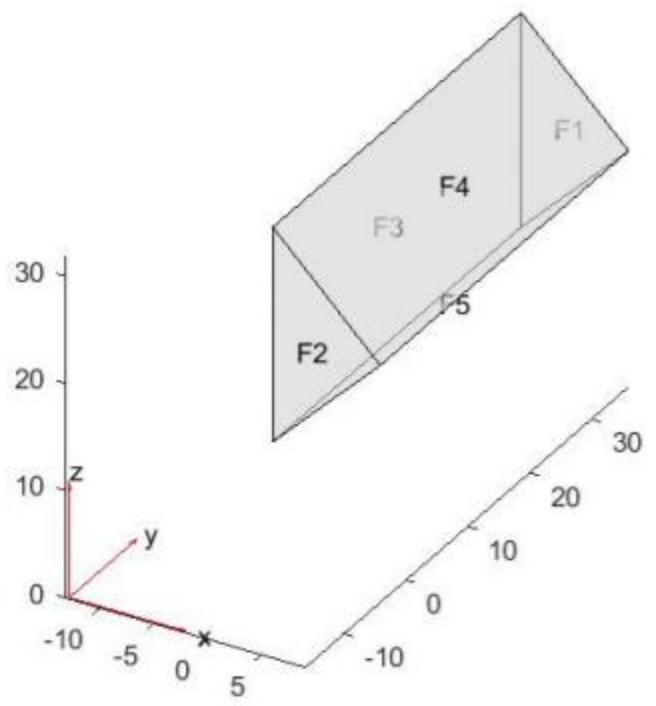
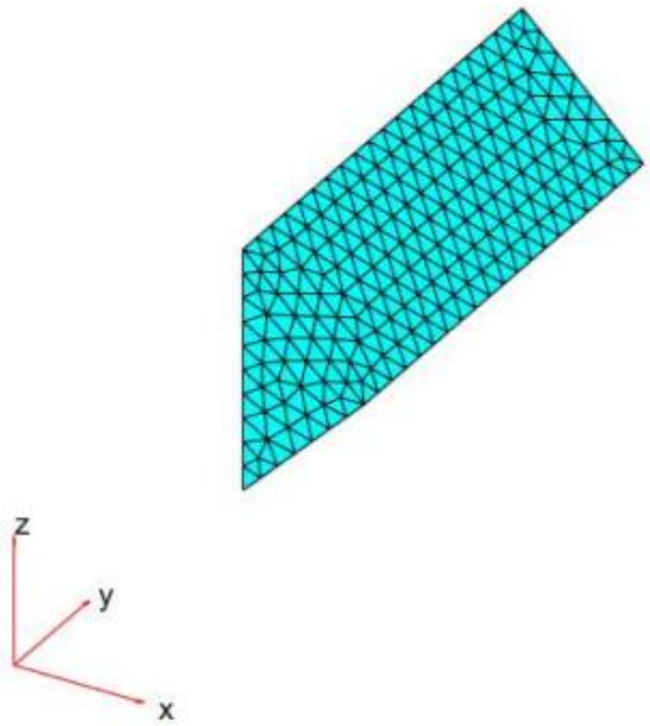
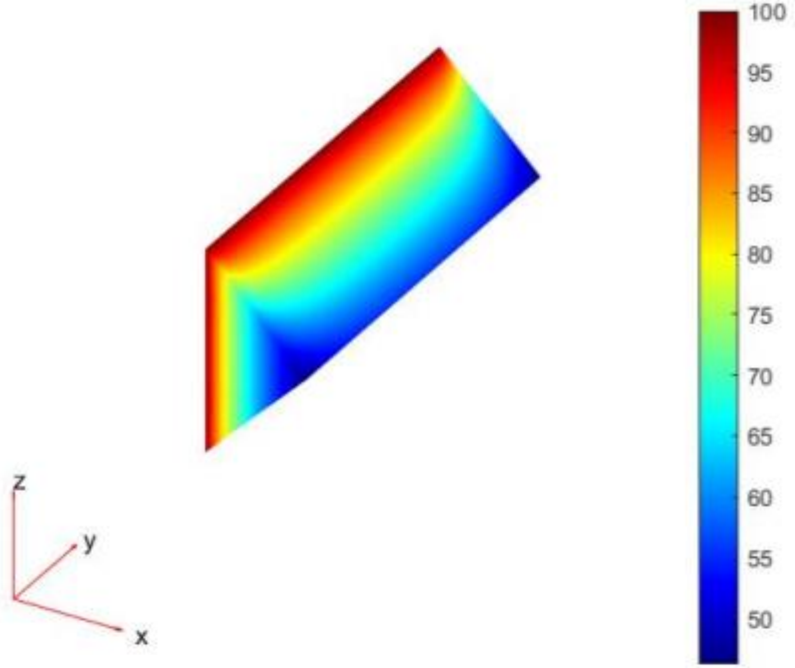


Geometry Plot



Mesh Model Plot



ColorMap Model Plot