SIMUTECH PROJECT

# COMPUTATIONAL HEAT TRANSFER ASSIGNMENT 3

-- Manvendra Singh, 190487

## Theory:

The heat transfer equation can be written as

$$(1/\alpha) \cdot (\partial T/\partial t) = \partial^2 T/\partial x^2 + \partial^2 T/\partial y^2 \qquad \text{~(1)}$$

And we define $t = p\Delta t$, where p is in an integer.

The finite difference approximation is expressed as :

$$(\partial T/\partial t)_{m,n} \approx (T_{m,n}^{p+1} - T_{m,n}^{p})/\Delta t \qquad \text{~(2)}$$

### The Explicit Method

The explicit method equation can be formed by substituting equation(2) in equation(1), and evaluating the temperatures at the previous(p) time, while assuming $\Delta x = \Delta y$,

$$T_{m,n}^{p+1} = Fo(T_{m+1,n}^{p} + T_{m-1,n}^{p} + T_{m,n+1}^{p} + T_{m,n-1}^{p}) + (1-4Fo)T_{m,n}^{p}$$

while $Fo = \alpha\Delta t/(\Delta x^2)$

For a one dimensional heat flow, the finite-difference method will be reduced to

$$T_{m}^{p+1} = Fo(T_{m+1}^{p} + T_{m-1}^{p}) + (1-2Fo)T_{m}^{p} \qquad \text{~(3)}$$

The stability criterion for a one dimensional interior node is $Fo \leq 1/2$.

### The Implicit Method

The implicit equation for a two-dimensional system can be derived by approximating the time derivative using equation 2 and evaluating all other temperatures at the new time (p+1), instead of the previous (p), and assuming $\Delta x = \Delta y$.

$$T_{m,n}^{p} = (1 + 4Fo)T_{m,n}^{p+1} - Fo(T_{m+1,n}^{p+1} + T_{m-1,n}^{p+1} + T_{m,n+1}^{p+1} + T_{m,n-1}^{p+1})$$

while $Fo = \alpha \Delta t/(\Delta x^2)$

And for the one-dimensional flow, it can be written as:

$$T_{m}^{p} = (1 + 2Fo)T_{m}^{p+1} - Fo(T_{m+1}^{p+1} + T_{m-1}^{p+1}) \qquad \text{~(4)}$$

## Calculations:

Given:

$k = 401\ W/m.K$

$\alpha = 117 \times 10^{-6} m^2/s$

$q" = 2 \times 10^5\ W/m^2$

$\Delta x = 50mm$

$Initial\ T = 20\,^oC$

$t = 120\ s$

$x = 150mm$

Assuming one-dimensional conduction in x, and slab is semi-infinite, i.e., $T(x \to \infty) = T$

The surface node temperature is given by $T_0$ and the interior node at $x = 150mm$ is given by $T_3$.

1. **Explicit form:**

Applying energy balance to obtain an explicit finite difference equation for the surface node:

$$T_0^{p+1} = 2Fo((q''\Delta x/k) + T_1^p) + (1-2Fo)T_0^p$$

Taking max value of Fo = ½,

$$\Delta t_{max} = Fo(\Delta x)^2/\alpha \approx 11\ s$$

We have to take half of this value, $\Delta t$ = 5.5 s

p = t/ $\Delta t$ = 120/5.5 $\approx$22

Also $q''\Delta x/k$ = 24.94 $^oC$

So the equation becomes:

$$T_0^{p+1} = (37.41 + T_1^p + T_0^p)/2$$

and for interior nodes $T_m^{p+1} = (T_{m+1}^p + T_{m-1}^p)/4 + T_m^p/4$ from eq(3)

2. **Implicit Form:**

Applying energy balance to obtain an implicit finite difference equation for the surface node:

$$(1+2Fo)T_0^{p+1} - 2Fo\,T_1^{p+1} = ((2\alpha q''\Delta t/k\Delta x) + T_0^p$$

Choosing Fo = ¼, $\Delta t$ = 5.5 s

After putting all values, the equation becomes,

$$3T_0^{p+1} - T_1^{p+1} = 25.68 + 2T_0^p$$

The equation for interior nodes can be written from equation 4

$$-T_{m-1}^{p+1} + 6T_m^{p+1} - T_{m+1}^{p+1} = 4T_m^p$$

From explicit method, we can see that $T_{15} = 20^oC$ even after 2 mins.

So we can take upto $T_{15}$ for calculating the temperatures.

We need to evaluate 14 interior node equation, and 1 surface node, by evaluating

[A][T] = [B]

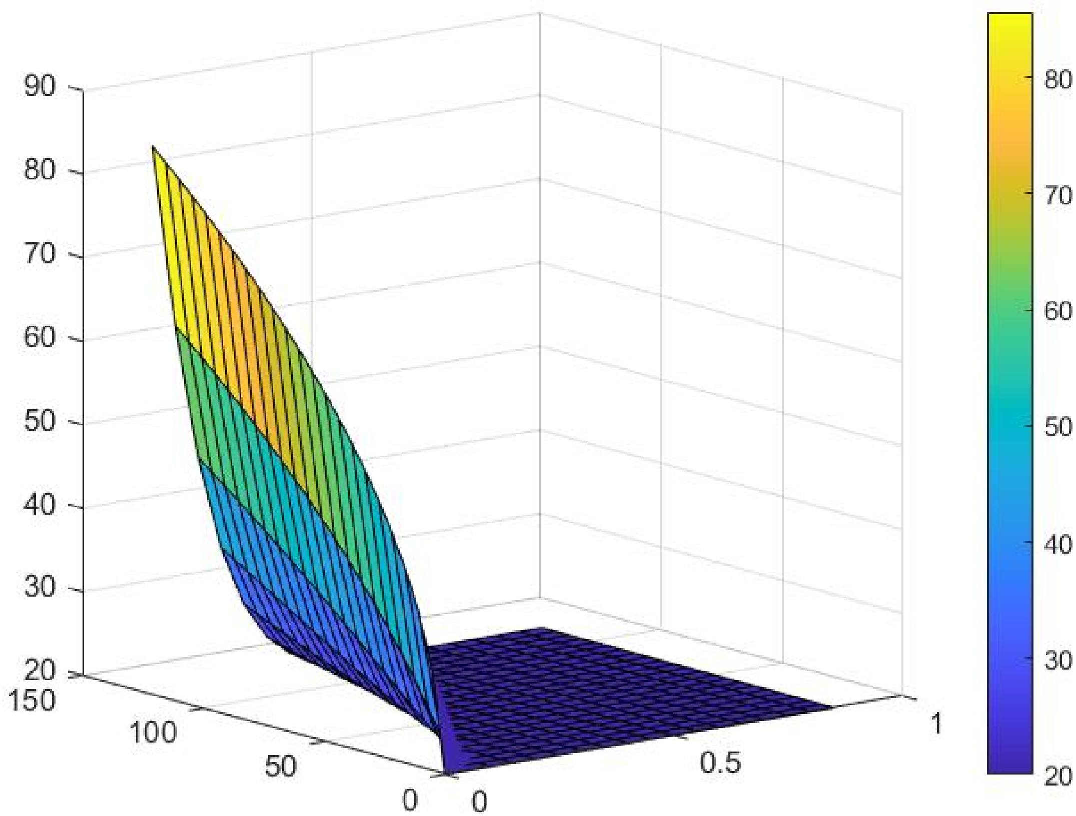
where, A is a tridiagonal matrix of coefficients of temperatures, B is the vector consisting of right hand values.

# RESULT:

### THE EXPLICIT METHOD

The surface temperature after 2 min is : 85.6244 $^oC$

The temperature at x=150mm after 2 min is : 36.1593 $^oC$



**THE IMPLICIT METHOD**

The surface temperature after 2 min is : 86.7704 $^oC$

The temperature at x=150mm after 2 min is : 36.5666 $^oC$