IME639A Course Project Report

Scheduling Aircraft Landings

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Introduction:

Upon entering within the radar range (radar horizon) of air traffic control (ATC) at an airport, a plane requires ATC to assign it a landing time, sometimes known as the broadcast time and also, if more than one runway is in use, assign it a runway on which to land.
The landing time must lie within a specified time window, bounded by an earliest time and a latest time, these times being different for different planes. The earliest time represents the earliest a plane can land if it flies at its maximum airspeed. The latest time represents the latest a plane can land if flies at its most fuel-efficient airspeed whilst also holding (circling) for the maximum allowable time.
Each plane has a most economical, preferred speed referred to as the cruise speed. A plane is said to be assigned its preferred time, or target time, if it is required to fly in to land at its cruise speed. If ATC requires the plane to either slow down, hold or speed up, a cost will be incurred. This cost will grow as the difference between the assigned landing time and the target landing time grows.
The time between a particular plane landing, and the landing of any successive plane, must be greater than a specified minimum (the separation time) which is dependent upon the planes involved.

Problem Statement:

Given a dataset of aircrafts which includes:

1. The targeted landing time.
2. The earliest and the latest landing time for each flight.
3. Separation time needed between two flights.
4. The costs incurred on landing before or after time.
5. Total no. of aircrafts.
6. Total no. of Runways.

Find the optimal schedule for all aircrafts landing with the minimal costs incurred.

Solution Approach:

Decision Variables:

1. x_i: Actual landing time for flight i in static case.
2. X_i: Actual landing time for flight i in dynamic case.
3. δ_{ij}: Binary variable to determine whether flight i lands before flight j or not.
4. q_{ij}: Binary variable to determine whether flight i and j lands on the same runway or not.
5. y_{ir}: Binary variable to determine whether flight i lands on runway r or not.

Objective Function:

Minimize the cost occurred, i.e.,

$$\textbf{Minimize:} \quad \sum_{i=0}^{n-1}(\boldsymbol{g_i z\_n_i} + \boldsymbol{h_i z\_p_i}) + \sum_{i=0}^{n-1}\boldsymbol{D_i(x, X)}$$

Parameters:

1. n : Total no. of flights to be scheduled
2. R : Total no. of runways.
3. E_i : Earliest allowed time for flight i.
4. L_i : Latest allowed time for flight i.
5. T_i : Targeted time for flight i.
6. g_i : Penalty incurred for early landing for flight i.
7. h_i : Penalty incurred for late landing for flight i.
8. s_{ij} : Separation time between flight I and flight j.
9. z_p_i : maximum of 0 and deviation in landing after Targeted time, $\max(0, x_i - T_i)$
10. z_n_i : maximum of 0 and deviation in landing before Targeted time, $\max(0, T_i - x_i)$
11. Δ_i : Maximum perturbation in scheduling flight i from previous decision.
12. M : Big number.
13. Displacement:

$$D_i(X, x) = g_i \max(0, x_i - X_i), \quad \text{if } x_i < T_i$$
$$= h_i \max(0, X_i - x_i), \quad \text{if } x_i > T_i$$
$$= g_i \max(0, x_i - X_i) + h_i \max(0, X_i - x_i) \text{ if } x_i = T_i$$

Constraints:

1. Either flight i land before flight j or not.

$$\delta_{ij} + \delta_{ij} = 1, \qquad i, j = 1,2, \ldots \ldots n-1, j \neq i$$

2. Flight i lands after the earliest landing time.

$$x_i \geq E_i, \qquad i = 0,1,2, \ldots \ldots n-1$$

3. Flight i lands before the latest landing time.

$$x_i \leq L_i \ , \qquad i = 0,1,2, \dots \dots n-1$$

4. Maximum deviation in time for landing after targeted time.

$$z\_p_i \geq x_i - T_i \ , \qquad i = 0,1,2, \dots \dots n-1$$

5. Maximum deviation in time for landing before targeted time.

$$z\_n_i \geq T_i - x_i \ , \qquad i = 0,1,2, \dots \dots n-1$$

6. Landing time between two flights should be at least the separation time if landing on the same runway, otherwise the constraint is redundant.

$$x_j - x_i \geq s_{ij} q_{ij} - \delta_{ji} M \, , \qquad i, j = 0,1,2, \dots \dots n-1, j \neq i$$

7. Flight i and flight j lands on the same runway.

$$q_{ij} = q_{ji} \, , \qquad i, j = 0,1,2, \dots \dots n-1, j \neq i$$

8. Flight i and flight j lands on the same runway or not and the value of q_{ij} not exceeding that.

$$\sum_r y_{ir} = 1, \qquad i = 0, \dots \dots, n-1$$

$$q_{ij} \geq y_{ir} + y_{jr} - 1 \, , \qquad i, j = 0,1,2, \dots \dots n-1, j \neq i$$

9. Decision Constraint:

$$D_i(x, X) \leq \Delta_i$$

10. Other Constraints:

$$0 \leq x_i, z\_p_i, z\_n_i \leq \infty \, , \qquad \text{i = 0, 1……., n-1}$$

$$\delta_{ij}, q_{ij} \in \{0, 1\}, \qquad i, j = 0, \dots \dots., n-1$$

$$y_{ir} \in \{0, 1\}, \qquad i = 0, \dots \dots, n-1, \ \ r = 0, \dots \dots, R-1$$

The problem is formulated as a **Mixed-Integer Programming problem** and can be solved using **Branch and bound method**.

Proposed Heuristic:

The problem can be considered as a bin packing problem where the order of placement is also to be considered for each bin. The time complexity is $O(n^2)$, n = No. of flights.

The steps involved are:

1. Sort the flights in the increasing order of Appearance time.

2. Starting from the top, for each flight check if it can be accommodated in one the currently used runways (bins). If yes, then accommodate it there.
3. If no, then check free runways. If there is a free runway and not used currently, accommodate the flight there. If no more runways are available, then schedule the flight at the time of landing + separation of the flight earliest in one of the currently used runways and calculate the cost incurred.
4. If no more flights are left then go to step 5, else go to step 2.
5. Calculate the whole schedule again by checking whether the cost of landing a flight late is more than the cost of the previous flight land early. If yes, make the previous flight land early if possible and calculate the new cost.

Result:

For 1st Data set, no. of flights = 10

From MIP solution:

Time Taken: 0.02s Total Cost Incurred: 1320.00 units.

Flight No.	Actual Landing Time
1	161
2	258
3	98
4	106
5	122
6	130
7	138
8	146
9	176
10	184

From Heuristic:

Time Taken: 0.016s Total Cost Incurred: 1890.00 units.

Flight No.	Actual Landing Time
1	167
2	198
3	98
4	113
5	128
6	136
7	144
8	152
9	175
10	183

For 2nd Data set, No. of flights = 50

From MIP Solution:

Time Taken: 0.13s Total Cost Incurred: 1080.00 units.

From Heuristic:

Time Taken: 0.75s Total Cost Incurred: 1920.00 units.

References:

All these files are screenshots that have been taken from research papers:

Static Case Single Runway-

Mixed integer programming model has been developed for static case with single runway. In the static case, all the data is predetermined and does not change over time. The piecewise linear objective is to minimize the penalty associated with diversion of landing times from target times. The constraints include clearance time between landings, earliest landing time, latest landing time for single runway. The formulation is as follows-

n : Total number of flights that are to be scheduled

E_i : Earliest allowable landing time for flight i

L_i : Latest allowable landing time for flight i

T_i : Target landing time for flight i

g_i : Penalty cost per unit time for landing before target time T_i for flight i

h_i : Penalty cost per unit time for landing after target time T_i for flight i

s_{ij} : Separation time between flights i and j, where aircraft i lands before j

x_i : Landing time of flight i

z_p_i : $\max[0, x_i - T_i]$

z_n_i : $\max[0, T_i - x_i]$

δ_{ij} : 1 if flight i lands before j, otherwise 0

M : Big number ($\max L_i - \min E_i$)

Static Single Runway Model

$$\text{minimize} \sum_{i=0}^{n-1} (g_i z\_n_i + h_i \, z\_p_i)$$

$$\text{such that: } \delta_{ij} + \delta_{ji} = 1, \quad i,j = 0,\ldots,n-1, j \neq i$$

$$x_i \geq E_i, \quad i = 0,..n-1$$

$$x_i \leq L_i, \quad i = 0,..n-1$$

$$z\_p_i \geq x_i - T_i, \quad i = 0,\ldots,n-1$$

$$z\_n_i \geq T_i - x_i, \quad i = 0,\ldots,n-1$$

$$x_j - x_i \geq s_{ij} - \delta_{ji} M, \quad i,j = 0,\ldots,n-1, j \neq i$$

$$0 \leq x_i, z_{p_i}, z_{n_i} \leq \infty, \quad i = 0,\ldots,n-1$$

$$\delta_{ij} \in \{0,1\}, \quad i,j = 0,\ldots,n-1$$

Methodology:- Dynamic Case

[1] Let A_i be the appearance time for flight i. We define a time t^* such that all the flights scheduled to land within t^* of current time t have their landing time frozen.

We define-

$F0(t)$: Set of aircrafts that have not yet appeared, $F0(t) = [i|\, A_i > t]$.

$F1(t)$: Set of aircrafts that have appeared by time t, but have not landed or had their landing times frozen. $F1(t) = [i|\, A_i \leq t \text{ and } X_i > t + t^*]$.

$F2(t)$: Set of aircrafts that have landed by time t, or have had their landing times frozen. $F1(t) = [i|\, A_i \leq t \text{ and } X_i \leq t + t^*]$.

γ : Iteration counter

Z_{disp} : accumulated displacement cost

[2] Set $\gamma = 0$ and $Z_{disp} = 0$. Set the current time to $t_0 = \min[A_i|\, i = 0,..,n-1]$. Solve the static problem with 1 aircraft. Move this aircraft from $F0$ to $F1$.

[3] If aircrafts are still to appear, then go to [4], otherwise go to [5].

[4] Set $\gamma = \gamma + 1$. Set the current time to $t_\gamma = \min[A_i|\, i \epsilon F0(t_{\gamma-1})]$. Solve the dynamic model with aircrafts in $F1(t_\gamma) \cup F2(t_\gamma)$, where aircrafts in $F2(t_\gamma)$ are constrained to land at their previously scheduled time. Add the displacement cost to Z_{disp}.

[5] All the aircrafts in $F1(t_\gamma)$ are scheduled to land. Compute $Z(x)$ for the final schedule of all landings.

The formulation of the dynamic scheduling is as follows-

With all the variables defined for single runway static case, following variables are added in the model-

X_i : Landing time of flight i from previous decision

x_i : Landing time of flight i from new decision

$$D_i(X,x) = \begin{cases} g_i \max[0, X_i - x_i] & if\ X_i < T_i \\ h_i \max[0, x_i - X_i] & if\ X_i > T_i \\ g_i \max[0, X_i - x_i] + h_i \max[0, x_i - X_i] & if\ X_i = T_i \end{cases}$$

Δ_i : Maximum perturbation in scheduling time of flight i from previous decision.

$Z(x)$: Objective from static case

$C(x)$: Constraints from static single runway model

Dynamic Single Runway Model

$$minimize\ Z(x) + \sum_{i^*} D_{i^*}(X,x)$$

$$such\ that: C(x)$$

$$D_i(X,x) \leq \Delta_i, \qquad for\ i^*$$

$*\ flights\ that\ were\ already\ scheduled\ and\ have\ not$

$landed\ or\ had\ their\ landing\ time\ frozen$

Research Papers:

1. **Scheduling aircraft landings - the static case**
https://www.researchgate.net/publication/220413186_Scheduling_Aircraft_Landings-The_Static_Case

2. **Displacement problem and dynamically scheduling aircraft landings:**
https://www.researchgate.net/publication/245281259_Displacement_problem_and_dynamically_scheduling_aircraft_landings