ISyE 320: Simulation and Probabilistic Modeling

Final Report

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A Simulation Study of the Waunakee Roundabout

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1. Executive Summary

As consultants for the town of Waunakee's city planning committee, we intend to create a both a detailed and accurate model of the main street roundabout. This model will provide insights into how traffic is flowing during rush hour as well as suggestions for improvements to the system.

This model will play an instrumental role in the decision making process of the Waunakee City Planning Committee's roundabout expansion. The model will detail the wait times at the roundabout for each queue as well as the total time spent in the system for a typical vehicle. As an assumption, this study does not consider large vehicles such as semi-trucks and trailers are different entities from average sized vehicles. This study includes details on all other assumptions, and how the model was created as well as detailing with statistics the validity of the model. We created the model using the ARENA software program and our assumptions are detailed in section 3 of this report.

The validations and verification of the model will aim to show Waunakee's city planning committee how confident they should be in the predictions made by our model. Forecasts that will ultimately be used as evidence for the need for an expansion of the roundabout. The limitations of our model are discussed clearly in order to accurately state the results of our model. One limitation of our model is that it does not take into account the variance in vehicle types that can arrive to the system. Another is that the data collected was limited to one hour on one weekday, when in reality, the behavior of the system could be more sporadic. However, to understand the system as a whole this limitation is discussed in detail later on in the report, and we believe that it will not impact the validity of our model in any significant way.

An option for Waunakee's city planning committee is to increase the amount of lanes in the roundabout from one lane to two lanes. This study includes a two-system output analysis of our model under these new conditions and compare the data in order to justify or reject the usefulness of the addition of another lane. In this case, adding a second lane does improve performance measures such as average waiting times, number of vehicles in the system and lane utilization. However, the degree of improvement may not be ideal for stakeholders to make the investment to add a secondary lane.

We expect our model to play an important role in making these decisions.

2. Motivation and Problem Definition

The existing system is a roundabout on the Main Street of the town of Waunakee, WI. The roundabout consists of 4 entry points (North Avenue, South Avenue, East Avenue, and West Avenue) through which the vehicles (Customers) can enter. All the customers enter and pass through the same roundabout that consists of one lane (server). Once the customers have used the roundabout, they will exit the roundabout through any of the 3 exit points.

The Waunakee roundabout is used by various companies as a key transportation line to travel between the various cities of Wisconsin. The roundabout gets busy during the rush hours and it is important to manage and reduce the traffic to ensure that personal vehicles and transportation lines for various businesses are not stuck at a bottleneck. Hence, the goal is to try and minimize queue times for all the 4 avenues.

For the project, we collected data from the real system for each of the 4 entry points to get the inter-arrival times and service times. We observed how long it takes for 5 cars to arrive from 5:30 - 5:45 PM in order to determine the average-interarrival time at each of the 4 queues. In order to find the average service time, we measured the amount of time for the roundabout to "service" a car for 25 vehicles starting at each queue.

Following the data collection, we ran Arena's Input Analyzer to verify our speculations on the distributions. Through this we could find the distribution for the interarrival time of each of the 4 entry points at the roundabout as well as the service time of the roundabout. After obtaining the distributions, we used ARENA to create our simulation model, and we simulated the system for two hours in a day for 10 replications. We then calculated the 95% and 99% confidence intervals of the relevant outputs and represented them in a table. The performance measures we recorded were the average process times, average wait times, and average total time in the system for each of the lanes, average roundabout queue time, the average number of entities in the system, and finally, the lane utilization. This allowed us to get a better understanding of the true mean of the output metrics we aim to analyze and improve to make the system more efficient.

In order to further verify our data, we conducted sensitivity analysis and tried to figure out how the assumptions affect our entire simulation study. We then considered 2 alternate models of our system, modified our simulation model according to those changes, and used ARENA to simulate the modified system again. Based on the data from the simulation, we performed output analysis to check if there was any significant advantage in the alternate model that we proposed. Our recommendations and conclusions were then based on our results we obtained from comparisons with the alternate model.

3. Assumptions and Conceptual Model

3.1 Model Description and Assumptions

The primary entities involved in the system are the vehicles driving on the roundabout and the lane of the roundabout that the vehicles would make use of to pass the roundabout. The vehicles are the system customers and the lanes of the roundabout are the system servers. The true system's queueing discipline follows a First-In-First-Out (FIFO) policy, unless there is any overtaking or jockeying, which would then result in Shortest-Processing-Time-First (SPT) policy being a better representation of the roundabouts queuing discipline.

The roundabout consists of various fixed quantities, like the speed-limit for the vehicles entering the roundabout, the number of entry lanes in the roundabout (4), the number of exit lanes in the roundabout (3), and the number of servers in the roundabout (1). The quantities that will however be varying for each customer is the arrival-time of each customer, the average speed of each vehicle while they are using the roundabout and the size and type of each vehicle that would affect the amount of space that they occupy.

For our project, we are making a few assumptions with regards to the system. We are not considering any other possible entities (pedestrians, bicycles) that would be making use of the system. During data collection, we did not observe any setbacks like car accidents or car breakdowns and hence we will not consider the occurrence of any such incidents. The size-type of the vehicle would affect the amount of space that they occupy in the roundabout, however, on average, we will be assuming that around 5 vehicles can use a lane at a time. Semi-trucks and trailers usually take longer to use the roundabout than other vehicles but there is no efficient way to model that, and hence we assume that those data points are anomalies. The simulated model's queuing discipline will be following a First-In-First-Out (FIFO) policy, assuming that there will not be any overtaking.

3.2 Simulated Model

To conduct a simulation study on our real world system, we constructed an ARENA model that hopes to capture the details of the system, simplified by the assumptions stated above.

In the simulation, entities can arrive through 4 entry points (North, South, East, and West), through various distributions. They all enter the same roundabout, a process module which has one server, as the roundabout has one lane that can host a set of vehicles at any given time. We are also assuming that all traffic moves in one direction here (either clockwise or counter-clockwise), as the Waunakee roundabout exhibits the same characteristic. As stated earlier, the assumption made with the process module is that each server or lane can serve up to 5 entities before entities start waiting in a queue. That is, the server capacity for each server is set to 5. Once the entities use the roundabout, they leave through one of three exits, where a record module records the count of each entity passing through it. An aspect of our data collection records the probability of an entity to exit through any of the exits (detailed in the Appendix). An image of the implemented Arena model can be found in the Appendix to help visualize the roundabout in a simulated manner.

The Waunakee roundabout exists as a non-terminating system as entities can enter, utilize servers, and exit the system with no natural end. Therefore, we would only be interested in the steady-state analysis of the simulation, to measure the state independent of initial conditions. However, to recognize through observation that the system has a high amount of traffic during certain periods of time: the rush hours. Since there may not be a definite steady state, we expect cycles in the system, that allow us to estimate state-state behaviour.

For our analysis, we use the replication-deletion method to analyze the steady state, where the warm-up period is calculated using Welch's Method. As per the details of Welch's Method, our simulation also includes a Read-Write module (Advanced Process Module) that writes the waiting time for each data point in every replication in a Comma-Separated Value (.csv) file, where the ensemble averages were calculated and plotted. The plot for the ensemble averages can be found in the Appendix (*Figure 3.2*). This plot helped us find an appropriate warm-up period for our model, which was calculated to be approximately 13% of an hour or 7.8 minutes. We want to note that the cut-off line for the warm-up period was extended to be cautious.

4. Data Collection and Input Modeling

4.1 Data Collection and Initial Interpretation

In order to develop a sound working model for the project, we collected data from the real system. After an initial visit to the roundabout we are studying in Waunakee, we notice there are different times during the day that are busier than others. In order to control this, we observed the system at what we expect to be the busiest times such as 5:30 PM on weekdays.

With regards to the queueing system itself, we collected data for each of the roads, or queues, that feed into the roundabout – the South Century avenue, North Century avenue, East Main Street, and West Main Street – to determine inter-arrival times. Our initial inspection of the system also suggested that the arrival rates of the vehicles would be too large for us to collect precise data. Our solution to this problem was to observe how long it takes for 5 cars to arrive from 5:30 - 5:45 PM in order to determine an average inter-arrival time at each queue.

Our data collection also involved observing the service times for each vehicle that utilizes the roundabout. Our initial expectation was that the service times would be normally distributed because of the assumption that most vehicles would be taking the 2nd exit as this leads to more populated neighbourhoods in Waunakee. In order to find the average service time, we measured the amount of time for the roundabout to "service" a car for 25 vehicles starting at each queue. The data collected can be observed in the Appendix section of our report (*Section 9.4.1, Table 4.1 and Table 4.2)*.

As an observation from our data, we speculate that the inter-arrival time from South Century Ave. is smaller than from the other queues because more vehicles are coming back from working in Madison around 5:30 PM.

For our inter-arrival times from all our queues, the data appears to be discrete, the fact that we are accounting for 5 vehicles per data point indicates that our data is actually continuous. We also know that our inter-arrival times are strictly non-negative due to the nature of the random variable itself. Finally, we cannot observe a noticeable upper-bound for our data. Based on these factors, we expect the inter-arrival rate to follow an Exponential distribution (it is likely that we are observing a Poisson process), with possibilities of the distribution following Lognormal or Weibull trends.

Similar to our inter-arrival times, we can say that the service rate is also continuous, positive and not bounded. Again we speculate that the service rate could also follow an Exponential, Gamma or Weibull distribution.

4.2 Input Analysis

To verify our speculations, we ran Arena's Input Analyzer and obtained the results found in the Appendix (*Section 9.4.2)*. While we were correct to expect continuous distributions for our data, our results from the input analysis suggested that:

- Vehicles arriving from North Century Avenue follow a Lognormal distribution with the expression 3 + LOGN(3.18, 2.78)

- Vehicles arriving from South Century Avenue follow a Beta distribution with the expression 2 + 4 * BETA(0.773, 0.908)

- Vehicles arriving from East Main Street follow a Beta distribution with the expression 3.08 + 3.84 * BETA(1.1, 1.42)

- Vehicles arriving from West Main Street follow a Weibull distribution with the expression 3 + WEIB(4.23, 1.93)

- The time it takes for vehicles to use the roundabout follows a Triangular distribution with the expression TRIA(2.5, 4, 10.5)

Details regarding the goodness-of-fit tests, along with the distribution of each input variable can also be found in the appendix (*Section 9.4.2)* as results of the Arena Input Analyzer. From these details, the Chi-Squared tests and the KS test (wherever applicable) indicate a good fit for all of these distributions. For the West Main Street arrivals, we considered the Weibull distribution over the Erlang distribution due to its low Squared Error value, and greater relevance to the actual system we are analyzing. As a whole, we believe that these distributions vary due to the variations in incoming traffic to the system itself.

5. Output Analysis

Using the simulation model constructed and the distributions obtained from our input analysis, we used ARENA to simulate the system for two hours in a day for 10 replications. This was done to mimic a rush hour period on the roundabout.

The complete report for the simulations is submitted along with this report.

The following tables depicts the 95% and 99% confidence intervals for the various outputs we wish to analyze:

95% Confidence Interval				
Performance Measures	Entities From	Sample Mean Estimator	95% CI Lower	95% CI Upper
Average Process Time	North	5.68	5.66	5.71
	South	5.68	5.65	5.71
	East	5.68	5.65	5.70
	West	5.66	5.63	5.69
Average Wait Time	North	0.62	0.58	0.66
	South	0.65	0.60	0.70
	East	0.63	0.59	0.67
	West	0.62	0.58	0.66
Average Total Time	North	6.30	6.25	6.35
	South	6.33	6.27	6.40

	East	6.30	6.25	6.36
	West	6.28	6.22	6.34
Average Roundabout Queue Time		0.63	0.59	0.67
Average Roundabout Number of Vehicles Waiting		0.49	0.46	0.52
First Exit Count		1307.80	1272.80	1342.80
Second Exit Count		2499.20	2463.16	2535.24
Third Exit Count		1436.80	1366.97	1506.63

Table 5.1: 95% Confidence Intervals for Outputs

99% Confidence Interval				
Performance Measures	Entities From	Sample Mean Estimator	99% CI Lower	99% CI Upper
Average Process Time	North	5.68	5.64	5.73
	South	5.68	5.63	5.72
	East	5.68	5.63	5.72
	West	5.66	5.61	5.71
Average Wait Time	North	0.62	0.56	0.68
	South	0.65	0.57	0.73
	East	0.63	0.56	0.69
	West	0.62	0.55	0.69
Average Total Time	North	6.30	6.22	6.39

	South	6.33	6.22	6.44
	East	6.30	6.21	6.39
	West	6.28	6.19	6.37
Average Roundabout Queue Time		0.63	0.56	0.69
Average Roundabout Number of Vehicles Waiting		0.49	0.44	0.54
First Exit Count		1307.80	1257.49	1358.11
Second Exit Count		2499.20	2451.09	2547.31
Third Exit Count		1436.80	1348.03	1525.57

Table 5.2: 99% Confidence Intervals for Outputs

The details for the calculations can be found in the OutputAnalysis.xlsx file submitted with this report. In general, the equation used to find the confidence intervals is:

$$Y_n \pm t_{n-1,\alpha/2} * \sqrt{s^2/n}$$

where $\mathbf{Y_n}$ is the sample mean estimator (taken as the mean of means in all of the replication),
$\mathbf{t_{n-1,\alpha/2}}$ is the one tailed t-distribution value for degrees of freedom = n-1 and confidence level α[1],
$\mathbf{s^2}$ is the sample standard deviation (taken as the standard deviation of means in all of the replications), and
n is the number of data points

Finally, we also have the lane utilization to be 88.41%. We understand that this metric will vary based on the assumptions that we make. In the real-world, the roundabout has several "lanes" but are unidirectional and not a part of the circle. These are ignored to simplify the model.

[1] Obtained from <http://pages.stat.wisc.edu/~ifischer/Intro_Stat/Lecture_Notes/APPENDIX/A5._Statistical_Tables/A5.2_-_t-distribution.pdf>

Graphs regarding the distribution of arrivals from the various avenues and streets, along with the distribution of exits taken are found in the Appendix (*Figures 5.1 and 5.2*).

The confidence intervals dictate that the true mean of the population of outputs lie between the intervals in the tables above, with a 95% and 99% probability respectively.

6. Verification and Validation

6.1 Model Verification

To verify our model, we decided to trace a single entity, a car, through the roundabout system. We have included a slow motion video of our Arena simulation in order to trace the car and get a better understanding. A video excerpt of our simulation can be found on this link, which was used to trace entities in our simulated system

From the slow motion simulation, we can see that an entity enters from one of the four lanes and uses the roundabout dictated by the process module if the system capacity is not met. The entity waits in a queue if more than 5 other entities are utilizing the process module at that moment. It then passes through a decision block which leads the entity to its preferred exit lane. We can conclude that our conceptual model matches our computer simulated model.

6.2 Sensitivity Analysis

Considering that our study took many influential factors as assumptions, we believe that our results would greatly vary if one or more of the factors varied slightly. For example, how would we expect changing the exit lane probabilities to affect our system and our results? Also, how do we expect changes in the assumed capacity of the lanes will affect our model? The latter question is quite important considering that large vehicles would occupy more space in the lanes of the roundabout, limiting the number of total vehicles to use it at any given point.

To tackle this limitation in our model, we ran a sensitivity analysis. This helped us understand how sensitive each of the performance metrics are to the changing parameters. In response to our first question, our results showed that changing the exit probabilities appropriately changed the number of vehicles that took a specific exit by the same changes in probability.

For the second question, our sensitivity analysis showed more lucrative results. In this analysis, we tested how our outputs would vary if we had a lane capacity over a range of 2 to 9. All results of the analysis are available in the appendix (*Table 6.2, Figure 6.2.1-Figure 6.2.6*). As the data suggests, increasing the lane capacity decreases waiting time, total time in the system and lane utilization time. However, the service time is unaffected. We particularly see the greeted rate of change between capacities 2 and 4. When the capacity of the lane is more than 5, most of the metrics level out, eventually reaching 0, with the exception of the lane utilization, which level at around 50% each.

6.3 Model Validation

Initially, the model did not take into account the roundabout being occupied by large vehicles (i.e. trucks and trailers). This was because the large vehicles required longer time to go through the roundabout which was not in sync with the data we collected, it was considered to be an anomaly. The large vehicles would reduce the capacity of each of the lanes and would lead to an increase in the waiting times. A suggestion for an alternative model could be to use all of the data for normal cars plus the data for large vehicles.

We wanted to check if our assumptions made change our simulation output greatly or not. To validate our model, we ran our ARENA simulation to exactly take in the same input data we observed from the real-system. Since we observed the service times of the vehicles, overall roundabout queue time and the number of vehicles taking a specific exit as outputs from the original system, we validated our model by observing these metrics. The following tables depict 95% confidence intervals for the same.

95% Confidence Interval				
Performance Measures	Entities From	Sample Mean Estimator	95% CI Lower	95% CI Upper
Average Process Time	North	5.67	5.61	5.71
	South	5.68	5.57	5.79
	East	5.67	5.61	5.72
	West	5.67	5.61	5.73
Average Roundabout Queue Time		0.38	0.35	0.41
First Exit Count		1327.40	1265.00	1385.00
Second Exit Count		2525.90	2466.00	2617.00
Third Exit Count		1432.10	1377.00	1479.00

Although this assumption does not truly reflect the real-world system, it appears that the output of the model matches the system output observed. The average process times collected from our simulation output closely matches the averages of the real system output (of 5.70 seconds), as the mean process times lie in the confidence intervals produced. Additionally, the 95% confidence intervals for the *proportion* of vehicles taking the first, second and third exits are [0.249, 0.272], [0.475,0.515] and [0.2708, 0.291]. As seen in section 9.4.1, all of our observed probabilities lie in their respective intervals. Hence, we can say with some statistical confidence that our simulation outputs match our system outputs.

7. Comparison of Alternative Models

7.1 Alternative Methods

Although our output analysis shows low wait times and queue lengths, we believe that stakeholders may still seek improvements to the model. So, we propose two models that will change our system and perhaps our analysis of the model as a whole.

Alternative Model 1: Increasing the number of servers in the roundabout from 1 to 2, this will allow for more cars to use the roundabout and allow for larger data collection which is more accurate. Conducting a study on this model, will lead to us identifying if adding another lane to the roundabout would hopefully decrease waiting times and lane utilization.

Alternative Model 2: Increase the speed limit of the roundabout to allow for a significant reduction in process times. This would allow for a lower frequency of times when the lane capacity is met and therefore lead to shorter wait times for all queues.

7.2 Output Analysis on Alternative 1

In order to further explore our alternatives, our study conducts an output analysis on Alternative 1 in comparison with our current model. Since this was also modelled in ARENA, we decided to consider this as a non-independent sampling, as we were able to set the seeds of random numbers to be the same for both simulations. This allows for CRN Synchronization and gives us a narrower confidence interval to draw more concrete statistical conclusions. *Figure 7.2* in the Appendix shows an ARENA model for our new alternative. The decision block shown moves entities to different lanes based on which lane has fewer entities being processed at the given moment (NR(Lane 1) <= NR(Lane 2)).

Taking a null hypothesis of $Current\ Model\ -\ Alternative\ Model\ =\ 0$ for all performance measure, our findings are as displayed in the table below:

95% Confidence Interval				
Performance Measures	Entities From	Sample Estimator (Difference)	95% CI Lower	95% CI Upper
Average Process Time	North	0.025	-0.008	0.058
	South	0.018	-0.004	0.040
	East	-0.022	-0.055	0.012
	West	-0.040	-0.071	-0.010
Average Wait Time	North	0.620	0.582	0.659
	South	0.654	0.604	0.704
	East	0.626	0.585	0.667
	West	0.619	0.576	0.662
Average Total Time	North	0.645	0.588	0.702
	South	0.672	0.614	0.730
	East	0.604	0.541	0.667
	West	0.579	0.517	0.641
Average Roundabout Queue Time		0.629	0.588	0.670
Average Roundabout Number of Vehicles Waiting		0.490	0.457	0.523
First Exit Count		-11.500	-39.224	16.224
Second Exit Count		-2.400	-37.642	32.842
Third Exit Count		14.400	-26.170	54.970

Table 7.2: Paired Output Analysis for Alternative 1 and Current Model

Another key performance metric to consider here is the change in server/lane utilization. With the one lane system, our utilization was 88.41%. With the two lane system, we have a scheduled utilization of 48.27% and 40.17%.

From our output analysis, we can make the statistically sound conclusion that adding another lane to the system would reduce the average amount of time any entity spends in our system, as well as reducing the number of entities waiting in our system at any given time. However, the processing time or time to use the roundabout does not change very much since 0, or our null hypothesis, lies in our confidence interval.

7.3 Output Analysis on Alternative 2

To simulate this we referred to the distribution result that our input analysis drew up for the process time, and reduced the minimum time, mean time and maximum time by 1 second. That is, the distribution used for this alternative simulation was TRIANGULAR (1.5, 3, 9.5). We understand that this an added assumption to the model as the process time distribution could completely differ due to the changes made to the speed limit. However, to understand the effects that it may have without having to collect data for this alternative, this was a required assumption to be made. To compare the results of the base model and alternative 2, we are running a non-independent test to draw a 95% confidence interval with CRN Synchronization. As we did for alternative 1, we take the null hypothesis to be $Current\ Model\ -\ Alternative\ Model\ =\ 0$ for all performance measures. The 95% confidence intervals are displayed in the table below:

95% Confidence Interval				
Performance Measures	Entities From	Sample Estimator (Difference)	95% CI Lower	95% CI Upper
Average Process Time	North	0.995	0.938	1.051
	South	1.009	0.974	1.044
	East	1.009	0.964	1.055
	West	0.997	0.961	1.033
Average Wait Time	North	0.511	0.469	0.552
	South	0.551	0.499	0.604
	East	0.531	0.490	0.572
	West	0.517	0.472	0.562

Average Total Time	North	1.505	1.431	1.580
	South	1.561	1.497	1.624
	East	1.540	1.481	1.599
	West	1.514	1.450	1.578
Average Roundabout Queue Time		0.527	0.484	0.570
Average Roundabout Number of Vehicles Waiting		0.411	0.376	0.445
Lane Utilization		15.6	15.1	16.0

Table 7.3: Paired Output Analysis for Alternative 2 and Current Model

As we can see from the table above, the confidence intervals for all the performance metrics contain values strictly greater than 0, which indicates that the true mean for all of these metrics for the new alternative is smaller than those of the current model. This would suggest that we see smaller wait times for all the queues, smaller process times (as expected), smaller total time spent in the system, a smaller total queue times, fewer vehicles waiting in the systems at any given point, and even smaller lane utilizations. Again, this output analysis could greatly differ were we to make a different assumption about the new process time distribution.

8. Recommendations and Conclusions

We can clearly see some useful insights from the data we have collected from both the real world and our model. The city planning committee will be able to run our simulations with slight modifications to simulate different outcomes before actually deciding to change or expand the roundabout in the future. We know that all models are wrong and we have already proved that this model is useful via statistical analysis. The question now becomes, how useful?

Taking a look at our output analysis, we can see that while there is a very short average waiting time regardless of which arrival point, as well as a relatively small number of entities waiting in the system, we could still improve the system. One feasible alternative would be to renovate the roundabout or construct a new lane in order to further reduce wait times. As seen from our two system output analysis, we do see a reduction in waiting times, as well as reduction in individual lane utilization. One goal of the stakeholders would be to reduce the frequency of maintenance required for the roundabout. Adding another lane would certainly achieve this goal, albeit at some initial cost. A cost/benefit analysis, or an NPV analysis, of this alternative would most likely suggest that this new model is not worth modeling as the initial investments required to make such an expansion may not be paid off lucratively considering the margin with which our performance measures improve.

Another alternative is to increase the speed limit of the roundabout to reduce the overall process time for vehicles to use the roundabout. As we the two system output analysis indicated that this alternative is better for reducing wait times and having better performance measures in general. However, increasing the speed limit here may not be the best goal when considering the safety of passengers driving the vehicles as accidents may be prone to occur more frequently at high speeds. This could be more apparent in the winter seasons when the roads are covered with snow and taking turns may be more difficult. The tradeoff of the results and their consequences must be weighed before considering this alternative. From the team's perspective, increasing the speed limit for the roundabout would not have as much of a positive impact on the overall system since the current system still performed well in our output analysis.

One limitation of our model is the fact that it only takes into account data taken during rush hour. It does not take into account the rarer events such as community festivals, holidays, etc. Also, when the model is tweaked to model a modification to the system that is desired by the city planning committee the model will make predictions with less confidence. Additionally, we make certain assumptions (such as the number of vehicles that can use the roundabout at once) that could greatly affect our overall recommendation. So, the project next steps should include making more accurate assumptions. Additionally, as stated before, we could obtain more detailed data to help us come to more accurate conclusions.

In conclusion, the alternative models may not be necessary based on what the city planning committee deems important. At the moment, it is fair to say that the roundabout system is working efficiently. We hope that the model we have provided is detailed enough to efficiently and effectively expand the roundabout when the time comes.

9. Appendix and Bibliography

9.3.2 Simulated Model

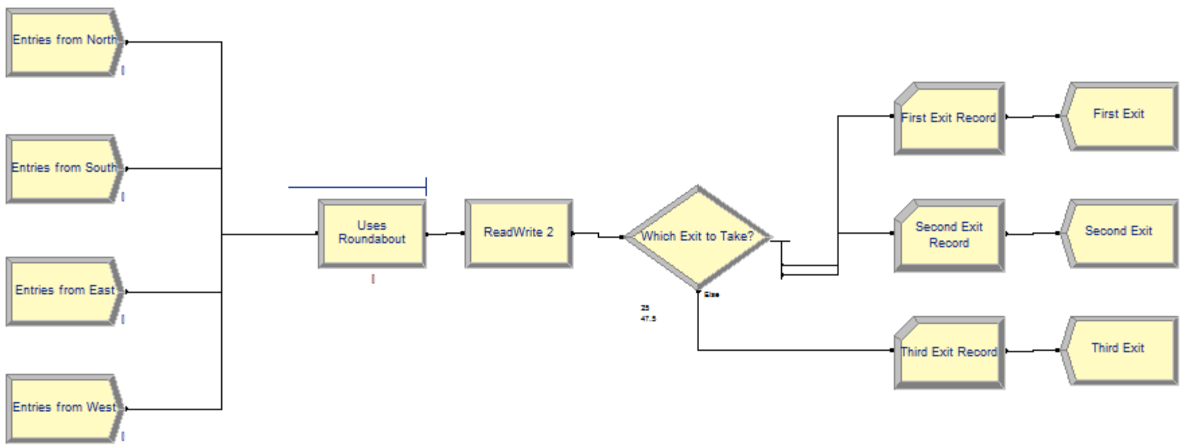

Figure 3.1: Visualization of our Simulated Model

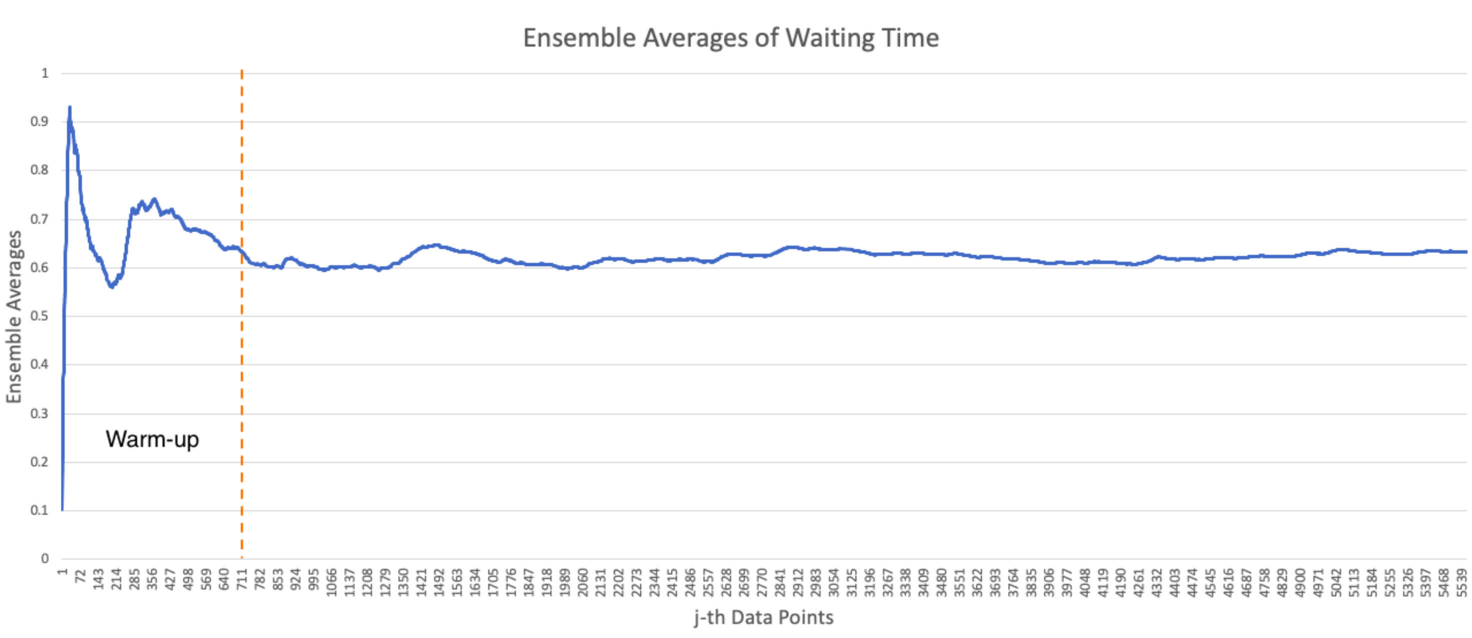

Figure 3.2: Ensemble Averages of Waiting Times

9.4.1 Data Collection

Trial	1	2	3	4	5	6	7	8	9	10	Average
N	25	36	28	17	22	48	33	27	40	24	30
S	12	18	13	27	23	19	11	19	21	29	19.2
E	17	19	28	23	29	24	33	19	21	25	23.8
W	34	23	27	18	33	51	47	32	44	29	33.8

On average, vehicles arrive from North Avenue every 6.00 seconds,
Vehicles arrive from the South Avenue every 3.84 seconds,
Vehicles arrive from the East Street every 4.76 seconds,
Vehicles arrive from the West Street every 6.76 seconds.

Table 4.1: Inter-arrival times of the vehicles (Time until 5 arrivals)

Trial	1	2	3	4	5	6	7	8	9	10	Average
N	3	4	7	4	5	5	6	4	5	4	4.7
S	9	7	6	5	4	7	7	4	8	5	6.2
E	5	8	6	9	4	6	4	7	5	3	5.7
W	4	7	5	8	5	8	3	9	10	7	6.6

Table 4.2: Service times of vehicles from each queue

Of the first 40 vehicles that entered the system, we noticed that 10 of the vehicles took the first exit, 19 took the second exit, and 11 took the third exit.
From this we assume that approximately probabilities of:

$$P(1^{st}\ exit) = \frac{10}{40} = 0.25$$

$$P(2^{nd}\ exit) = \frac{19}{40} = 0.475$$

$$P(3^{rd}\ exit) = \frac{11}{40} = 0.275$$

9.4.2 Input Analysis

Inter-arrival Rates From North Century Avenue

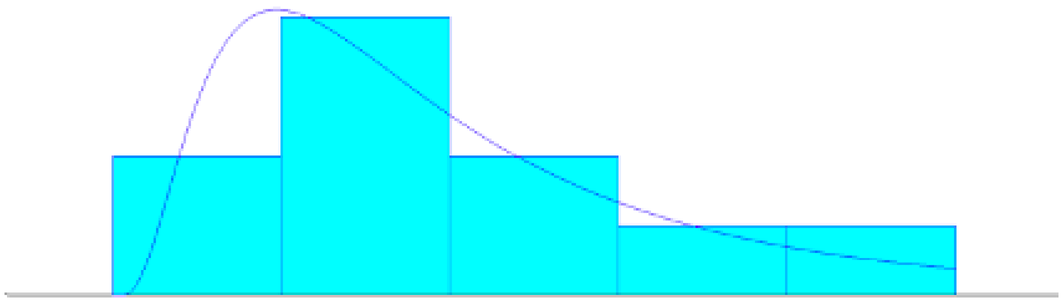

Figure 4.1: North Century Avenue Inter-arrival Rates Distribution

Data Summary	Distribution Summary	Histogram Summary
Number of Data Points = 10 Min Data Value = 3.4 Max Data Value = 9.6 Sample Mean = 6 Sample Std Dev = 1.86	Distribution: Lognormal Expression: 3 + LOGN(3.18, 2.78) Square Error: 0.006841	Histogram Range = 3 to 10 Number of Intervals = 5

Kolmogorov-Smirnov Test
Test Statistic = 0.152
Corresponding p-value > 0.15

Function Sq Error

Lognormal 0.00684
Gamma 0.0073
Erlang 0.0106
Weibull 0.0125
Triangular 0.0185
Beta 0.0323
Normal 0.0373
Uniform 0.06
Exponential 0.0622

Inter-arrival Rates From South Century Avenue

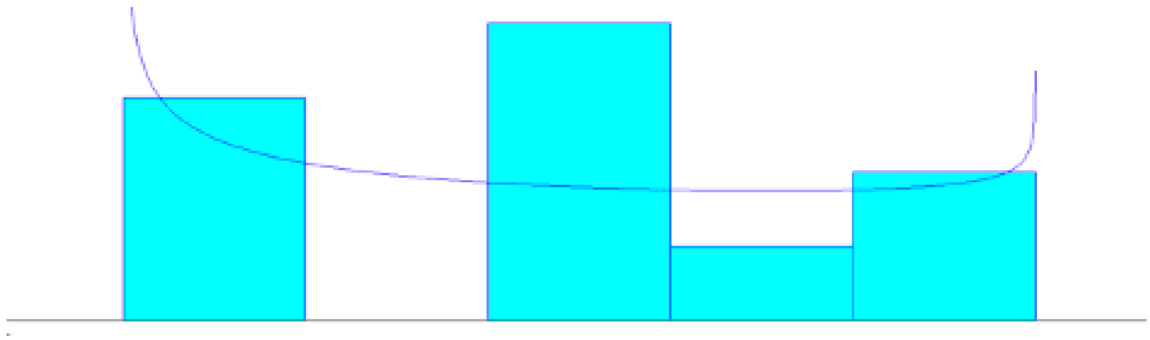

Figure 4.2: South Century Avenue Inter-arrival Rates Distribution

Data Summary	Distribution Summary	Histogram Summary
Number of Data Points = 10 Min Data Value = 2.2 Max Data Value = 5.8 Sample Mean = 3.84 Sample Std Dev = 1.22	Distribution: Beta Expression: 2 + 4 * BETA(0.773, 0.908) Square Error: 0.093258	Histogram Range = 2 to 6 Number of Intervals = 5

Kolmogorov-Smirnov Test
Test Statistic = 0.171
Corresponding p-value > 0.15

Function	Sq Error
Beta	0.0933
Uniform	0.1
Normal	0.123
Weibull	0.137
Exponential	0.138
Triangular	0.142
Gamma	0.144
Erlang	0.15
Lognormal	0.162

Inter-arrival Rates From East Main Street

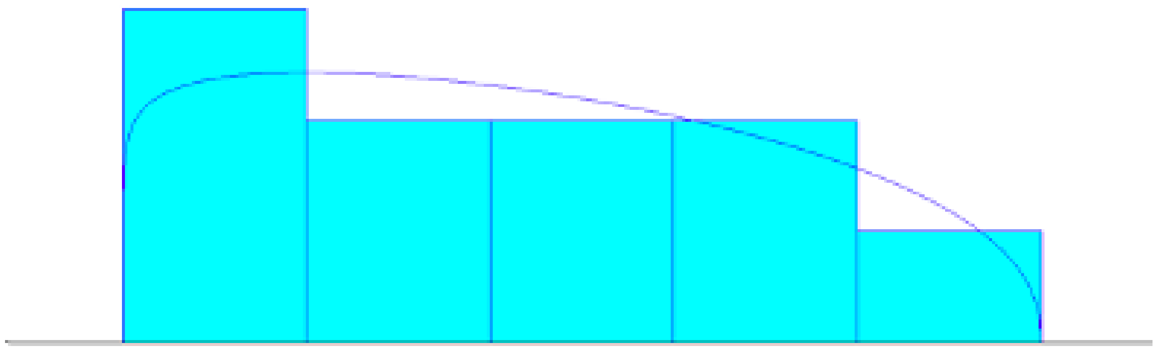

Figure 4.3: East Main Street Inter-arrival Rates Distribution

Data Summary	Distribution Summary	Histogram Summary
Number of Data Points = 10 Min Data Value = 3.4 Max Data Value = 6.6 Sample Mean = 4.76 Sample Std Dev = 1.01	Distribution: Beta Expression: 3.08 + 3.84 * BETA(1.1, 1.42) Square Error: 0.007196	Histogram Range = 3.08 to 6.92 Number of Intervals = 5

Kolmogorov-Smirnov Test
  Test Statistic = 0.117
  Corresponding p-value > 0.15

Function	Sq Error
Beta	0.0072
Uniform	0.02
Triangular	0.0205
Exponential	0.0214
Weibull	0.037
Gamma	0.0412
Normal	0.0452
Lognormal	0.0464
Erlang	0.0546

Inter-arrival Rates From West Main Street

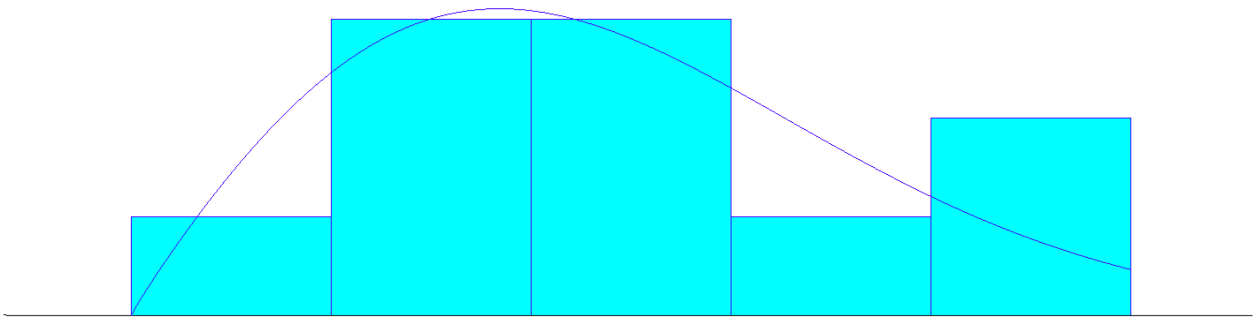

Figure 4.4: West Main Street Inter-arrival Rates Distribution

Data Summary	Distribution Summary	Histogram Summary
Number of Data Points = 10 Min Data Value = 3.6 Max Data Value = 10.2 Sample Mean = 6.76 Sample Std Dev = 2.12	Distribution: Weibull Expression: 3 + WEIB(4.23, 1.93) Square Error: 0.021933	Histogram Range = 3 to 10.9 Number of Intervals = 5

Kolmogorov-Smirnov Test
Test Statistic = 0.144
Corresponding p-value > 0.15

Function	Sq Error
Erlang	0.0218
Weibull	0.0219
Gamma	0.0232
Beta	0.0291
Triangular	0.0296
Normal	0.0298
Lognormal	0.032
Uniform	0.04
Exponential	0.106

*While the input analyser initially suggested Erlang-k, we considered a Weibull distribution due to its low Sq. Error value, and greater relevance to the actual system we are analyzing

Service Rates

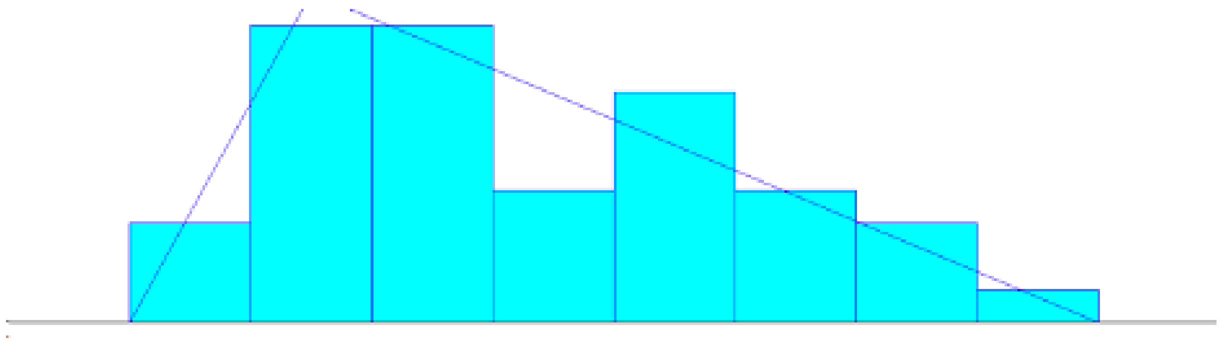

Figure 4.5: West Main Street Inter-arrival Rates Distribution

Data Summary	Distribution Summary	Histogram Summary
Number of Data Points = 40 Min Data Value = 3 Max Data Value = 10 Sample Mean = 5.8 Sample Std Dev = 1.87	Distribution: Triangular Expression: TRIA(2.5, 4, 10.5) Square Error: 0.007570	Histogram Range = 2.5 to 10.5 Number of Intervals = 8

Chi Square Test
  Number of intervals = 5
  Degrees of freedom = 3
  Test Statistic = 2.04
  Corresponding p-value = 0.57

Function Sq Error

Triangular 0.00757
Gamma 0.0107
Weibull 0.0121
Beta 0.0122
Lognormal 0.0124
Erlang 0.0125
Normal 0.0225
Poisson 0.025
Uniform 0.0388
Exponential 0.055

9.5 Output Analysis

Figure 5.1: Simulated Arrival Distribution

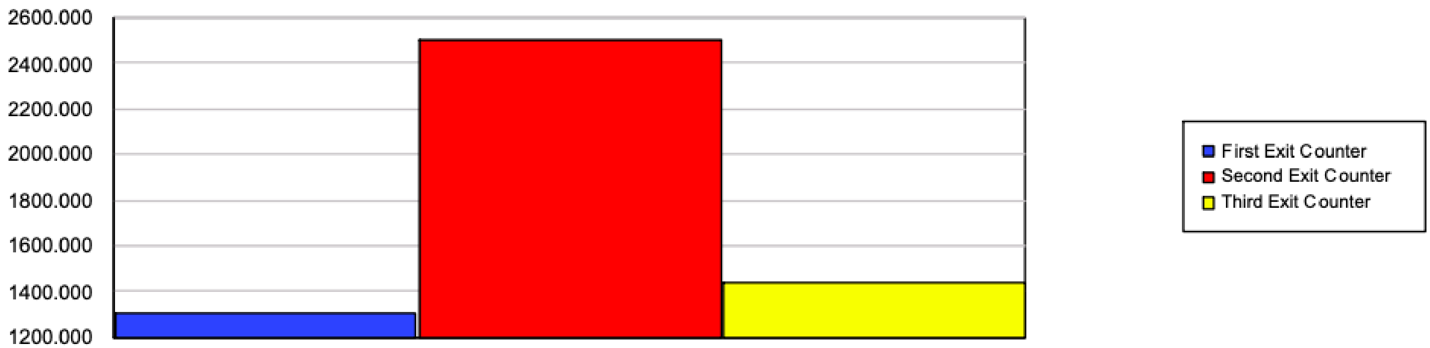

Figure 5.2: Simulated Exit Distribution

9.6.2 Sensitivity Analysis

All data and plots can also be found in the 'Sensitivity Analysis.xlsx' spreadsheet submitted.

Sensitivity Analysis									
	Capacity	2	3	4	5	6	7	8	9
Performance Metric									
Average VA Time	North	5.6355	5.6706	5.6832	5.6788	5.6688	5.6904	5.6913	5.6589
	South	5.686	5.6749	5.678	5.6762	5.6643	5.6549	5.658	5.661
	East	5.671	5.675	5.6883	5.6836	5.6666	5.7037	5.6893	5.6978
	West	5.6235	5.6358	5.6808	5.6599	5.6821	5.6678	5.6794	5.7003
Average Wait Time	North	2093.46	1222.25	366.43	0.6203	0.07336449	0.00857243	0.00028447	0
	South	2087.59	1221.4	367.4	0.6541	0.07151242	0.00882156	0.00053838	0
	East	2096.14	1216.92	366.24	0.6256	0.06447229	0.00857474	0.00036181	0
	West	2087.03	1219.31	366.27	0.6189	0.06795723	0.00908272	0.00035998	0
Average Total Time	North	2099.1	1227.92	372.11	6.3039	5.7421	5.699	5.6916	5.6589
	South	2093.28	1227.07	373.07	6.3329	5.7358	5.6638	5.6585	5.661
	East	2101.81	1222.6	371.93	6.3018	5.7311	5.7123	5.6897	5.6978
	West	2092.66	1224.95	371.95	6.2788	5.75	5.6769	5.6797	5.7003
Average Roundabout Queue Time		2094.1	1221.78	367.22	0.6288	0.06939964	0.00873662	0.00040544	0
Average Roundabout Number of		1632.67	954.04	285.91	0.4899	0.05413497	0.00679147	0.00031644	0

Waiting									
Lane 1 Utilization		100	100	100	88.41	73.69	63.14	55.4	49.14
Lane 2 Utilization		100	100	100	88.41	73.69	63.14	55.4	49.14

Table 6.2: Sensitivity Analysis Data

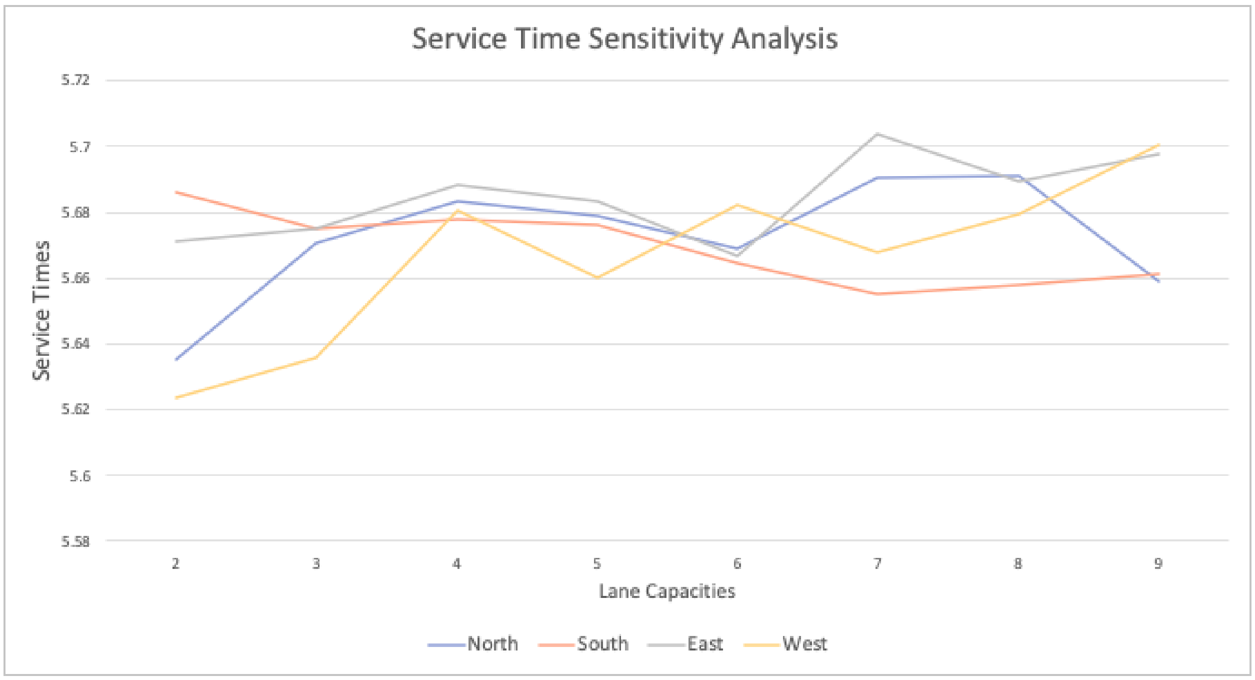

Figure 6.2.1: Service Time Sensitivity Analysis

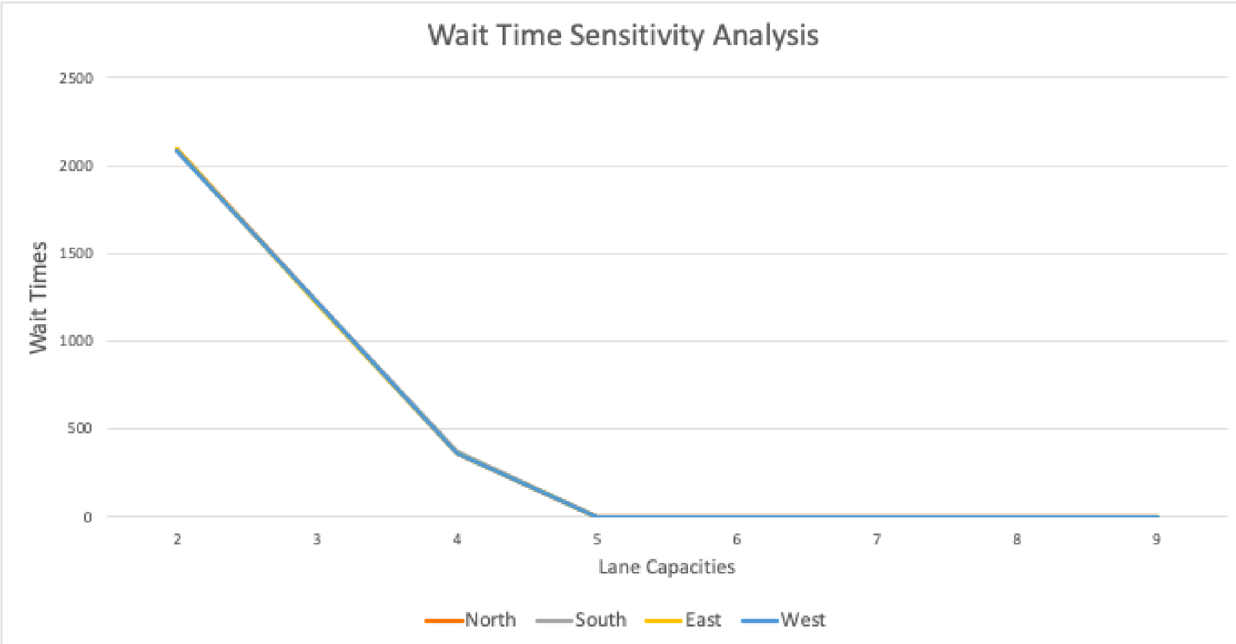

Figure 6.2.2: Wait Time Sensitivity Analysis

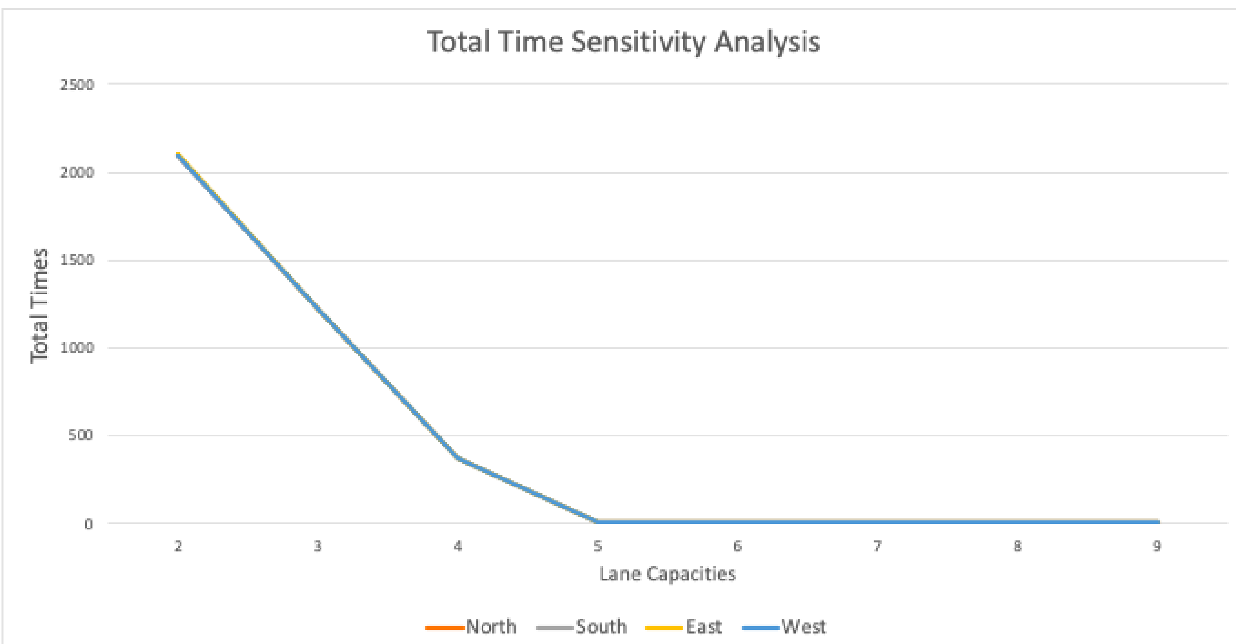

Figure 6.2.3: Total Time Sensitivity Analysis

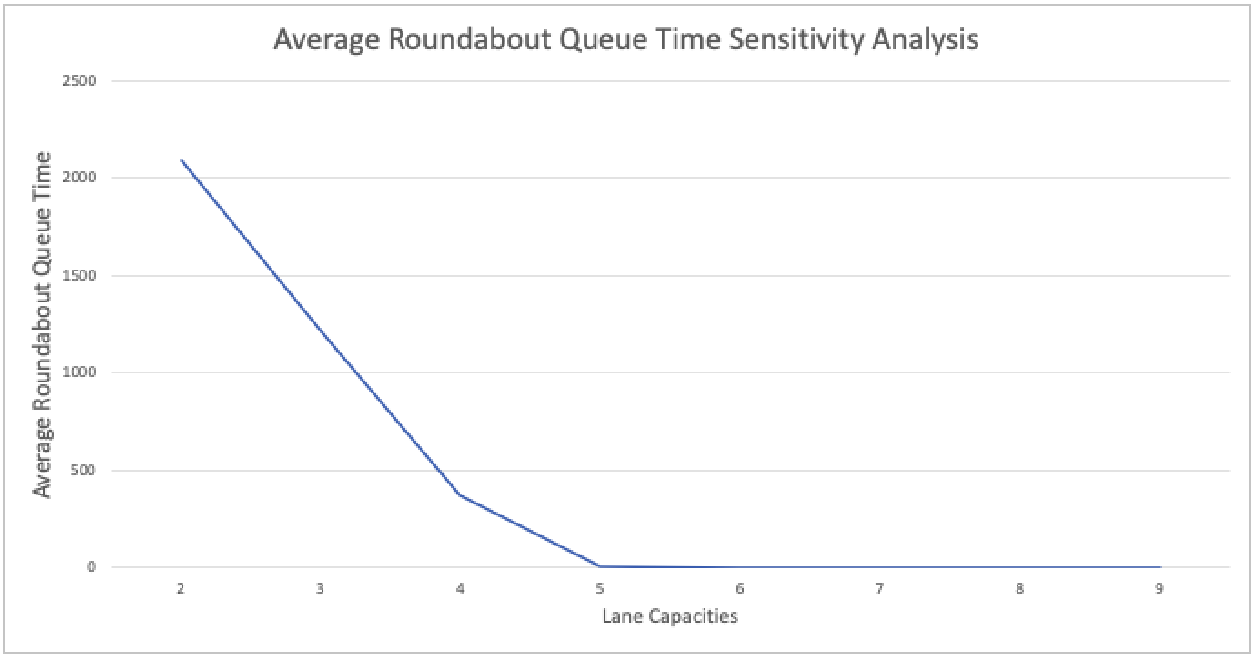

Figure 6.2.4: Average Roundabout Queue Time Sensitivity Analysis

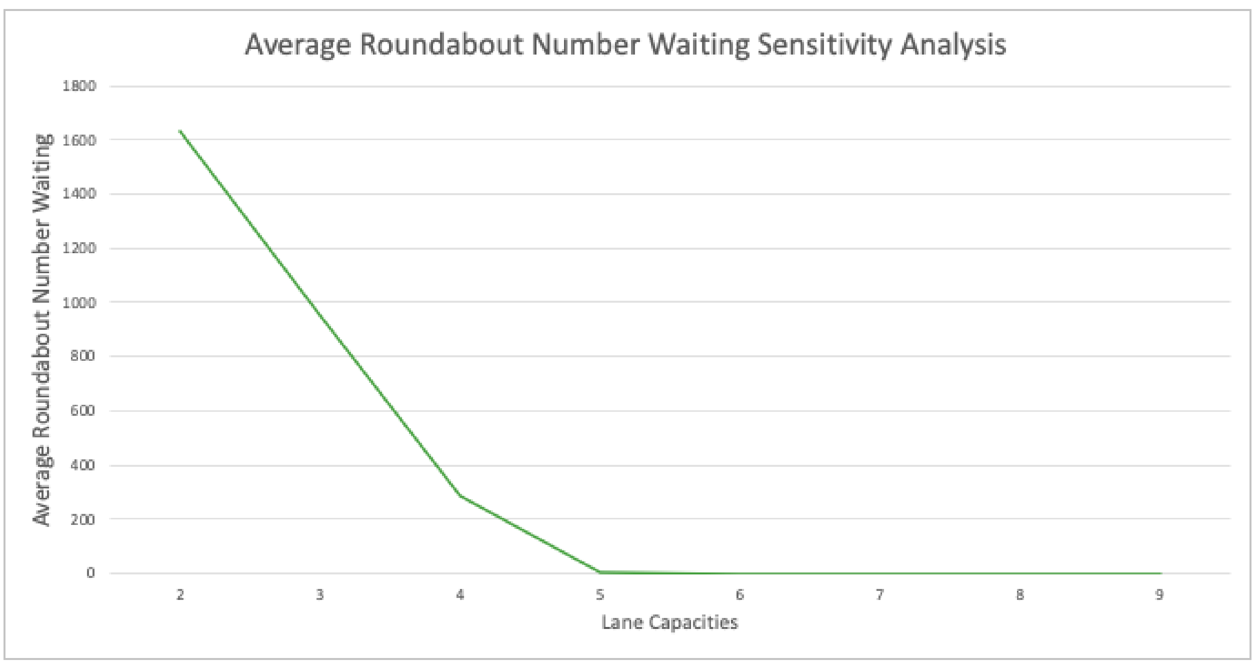

Figure 6.2.5: Average Roundabout Number Waiting Sensitivity Analysis

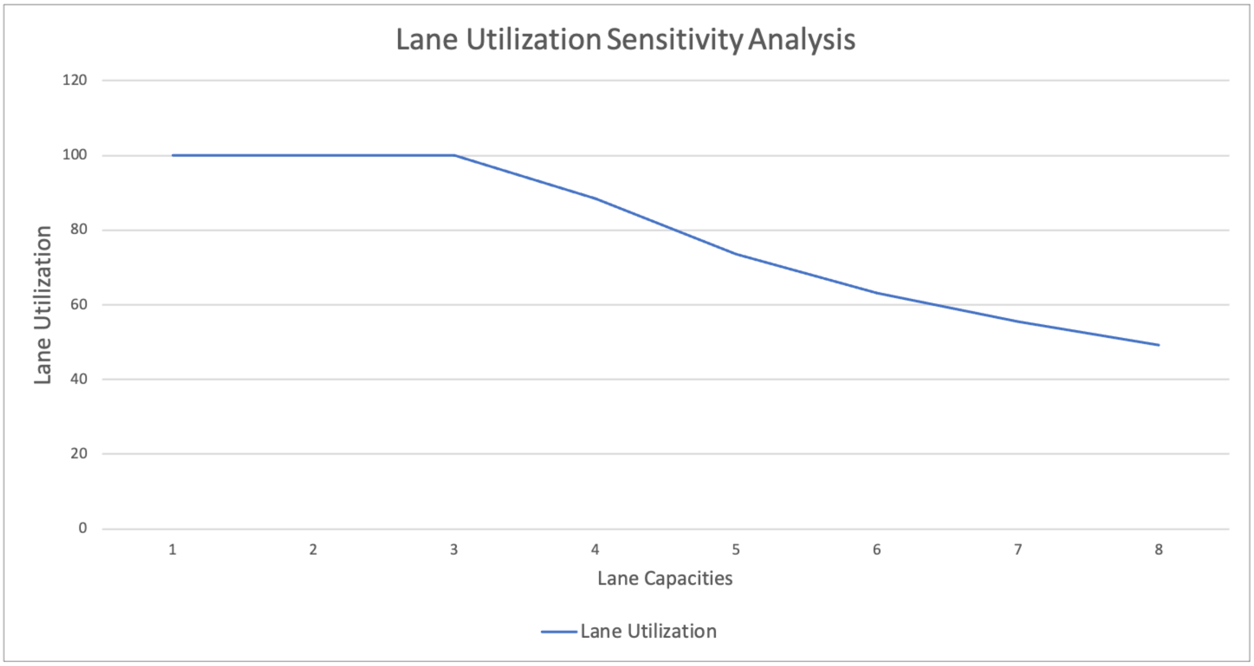

Figure 6.2.6: Lane UtilizationSensitivity Analysis

9.7.2 Two System Output Analysis: Current Model vs. Alternative Model

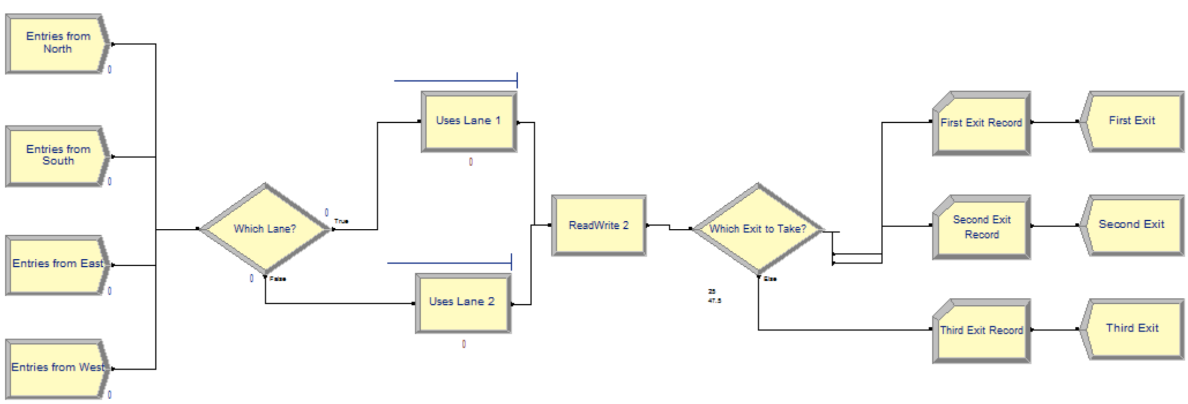

Figure 7.2: ARENA model of the new system with two lanes