Foundation of Computer Science

Propositional Logic

The ~~answ~~ statement which is either true or false is called proposition.

Command
Order
Exclamatory

} Not propositions

Propositional logic

A proposition is a declarative statement that is either true or false but not both.

Conjunction
Disjunction
Negation

(1) Implication

If you work hard then you will pass.
p q

$p \rightarrow q$
if p then q

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Truth Table for implication

$p \rightarrow q = \bar{p} + q$ (or) $\neg p \vee q$

$p \leftrightarrow q = \bar{p}\bar{q} + pq$

classmate

Date

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Logical Equivalence

To find the logical connectives b/w the propositions, we need logical equivalence. p is logically equivalence to q if their truth table are same.

$$p \equiv q$$

p	$\neg p$	q	$\neg p \vee q$
T	F	F	F
F	T	T	T
T	F	T	T
F	T	F	T

Distributivity: $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
F	T	T	T	T	T	T	T
T	F	F	F	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

$$\left.\begin{array}{l} p \rightarrow q \\ q \rightarrow p \\ \neg q \rightarrow \neg p \\ \neg p \rightarrow \neg q \end{array}\right\} \text{All 4 possible}$$

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Contrapositive $p \to q$ and $\sim q \to \sim p$
Converse $p \to q$ and $q \to p$
Inverse $p \to q$ and $\sim p \to \sim q$

(*) Contradiction & Tautology

A statement formula which is true regardless the truth value of the statement which replace the variable in it is called a tautology or a logical truth or a universally valid formula.

Tautology ⇒ True value (or) 1   Contradiction ⇒ False value (or) 0

Eg.

p	$\sim p$	$p \vee \sim p$	$p \wedge \sim p$
0	1	1	0
1	0	1	0
		Tautology	Contradiction

$(\overline{p \to q}) \wedge (\bar{q} \to r) \to \bar{p} \vee q$

Q. ① $(p \vee q) \leftrightarrow (q \vee p)$ $(p \wedge q) \to p$
② $q \to (p \vee q)$

Sol^n ①

p	q	$p \wedge q$	$(p \wedge q) \to p$
0	1	0	1
1	0	0	1
0	0	0	1
1	1	1	1

~~Tautology~~
Tautology

(2)

p	q	$p \vee q$	$q \to (p \vee q)$
0	0	0	1
1	0	1	1
0	1	1	1
1	1	1	1

Tautology

(3) $(p \wedge q) \wedge \sim(p \vee q)$

p	q	$(p \wedge q)$	$(p \vee q)$	$\sim(p \vee q)$	$(p \wedge q) \wedge \sim(p \vee q)$
0	0	0	0	1	0
1	0	0	1	0	0
0	1	0	1	0	0
1	1	1	1	0	0

Contradiction

* Contingency
A statement Formula that is neither a tautology nor a contradiction.

$p \to q$
if p then q
p q

$p \leftrightarrow q$
$\Downarrow$
$(p \to q) \wedge (q \to p)$
(or)
iff p then q
$\Downarrow$
$(\bar{p}\bar{q} + pq)$
$\Downarrow$
$(\bar{p}\bar{q} \vee pq)$

p	q	$p \to q$	$q \to p$	$p \leftrightarrow q$ $(p\to q)\wedge(q\to p)$
0	0	1	1	1
1	0	0	1	0
0	1	1	0	0
1	1	1	1	1

① Commutative law:-

$p \vee q = q \vee p$ | $p \wedge q = q \wedge p$

② Associative law

$p \vee (q \vee r) = (p \vee q) \vee r$

$p \wedge (q \wedge r) = (p \wedge q) \wedge r$

③ Distributive law

$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

~~Ind~~ Identity law:-

$A \vee \bar{A} = 1$

$A \wedge \bar{A} = 0$

$1 +$ Expression $= 1$

de Morgan's

$(\overline{A+B}) = \bar{A}\bar{B}$

$(\overline{A.B}) = \bar{A} + \bar{B}$

Q $[p \wedge (p \to q)] \to q$ is a tautology

~~$[p \wedge (\bar{p} \vee q)] \to q$~~

~~$p \wedge \bar{p}$ $[(p \wedge \bar{p}) + (p$~~

$[p \wedge (\sim p \vee q)] \to q$

$[(p \wedge \sim p) \vee (p \wedge q)] \to q$

$(p \wedge q) \to q$

$\sim (p \wedge q) \vee q$

*Predicate Logic :-

A predicate or propositional fn is a fn that takes some variable as argument and return true or false.

Universe of discourse

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Logic - Basis

We can use logic in proofs
↓ If it's true
Theorem

Logic
Valid Invalid

BODMAS → Logic
$1 + 2 \div 3 \times 5 = 0 \rightarrow ①$
$= 5 \rightarrow ②$
can be or cannot be valid

Proof

Proposition

Y 1
N 0

Logic

→ Logics are the basis of all mathematical reasoning. Eg. BODMAS
→ They are used to distinguish b/w valid & invalid statements.
→ They are used to design logic ckts &

computer ckts.

→ It is also used for ~~verifyi~~ constructions of pr. comp. program & verifying their correctness.

Proves

We need to prove when a mathematical statement is true, it is called theorem.

Proves are used for verifying the comp. program & check whether they are producing the correct o/p for all possible i/p values.

Logic circuits

OR } Basic gates
AND

NOR } Universal Gates
NAND

EXOR

OR

A	B	Y
0	0	0
1	0	1
0	1	~~0~~1
1	1	1

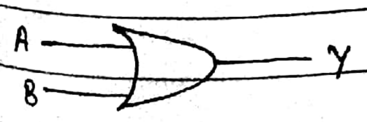
$Y = A + B$

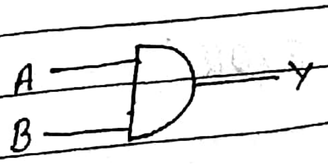
AND

A	B	Y
0	0	0
1	0	0
0	1	0
1	1	1

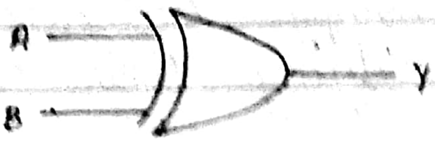
$Y = A.B$

0 - F
1 - T





EXOR



A	B	Y
T	T	F
T	F	T
F	T	T
F	F	F

<u>Propositions</u> logic $\longrightarrow$ Propositional logic

1) Delhi is capital of India - Proposition

Variable that represents proposition are propositional variable / statement variable.

$p \longrightarrow$ Proposition

$\downarrow$

Negation

$\neg p \longrightarrow$ Negation of p

$p, q \longrightarrow r$

Proposition (p, q) — Proposition (r)

→ ~~p~~ $\neg p$

→ $p \vee q$ (p or q)

→ $p \wedge q$ (p & q)

→ $p \oplus q$ (EXOR)

Def 1 : Let p & q be propositions, the conjuction of p and q denoted by $\wedge$ is the proposition of p and q. The conjuction $p \wedge q$ is true when both p & q are true otherwise false.

Def 2: Let p & q be propositions, the disjunction of p & q is $p \vee q$ and it is true when at least one of the two proposition is true otherwise false.

Def 3: Let p & q be propositions. the exclusive OR of p & q is denoted by $p \oplus q$ ~~the~~ the proposition is true when exactly when of p & q is true otherwise false.

Conditional Statement

Def 4: Let p & q be propositions, the conditional statement $p \rightarrow q$ is the proposition 'if p then q'.

The conditional statement $p \rightarrow q$ is false when p is true and q is false and true otherwise.

$p \rightarrow q$

hypothesis conclusion

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

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$p \leftrightarrow q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

If then else

$p \rightarrow q$

3 types of related conditional statement

→ converse $q \rightarrow p$

→ inverse $\neg p \rightarrow \neg q$

→ Contrapositive $\neg q \rightarrow \neg p$

$q \vee \neg p$

p	$\neg p$	q	$\neg q$	$(q \rightarrow p)$ [Converse] $\neg q \vee p$	Inverse $p \vee \neg q$	Contrapositive $\neg q \rightarrow \neg p$
1	0	1	0	1	1	1
1	0	0	1	1	1	0
0	1	1	0	0	0	1
0	1	0	1	1	1	1

Priority

1 $\neg$ → Negation

2 $\wedge$ → And

3 $\vee$ → OR

4 $\rightarrow$ → ~~to~~ Unidirection (Condition)

5 $\leftrightarrow$ → Bidirection

$p \leftrightarrow q$ $\quad p \rightarrow q$ and $q \rightarrow p$

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Logical Ckts

Bits
Logical bits
Binary values < 0, 1

When we combine 0,1 to form a sequence it is called bitstring
Eg - 011000111
length - 9

Bitstring - The sequence of 0 or more bits is bitstring & no. of elements in that string is called length of the string.

Operations

① BITWISE OR
② BITWISE AND
③ BITWISE EXOR

Ex: String 1 00111011011
String 2 10101100101

BITWISE OR 10111111111
" AND 00101000001
" EXOR 10010111110

Logical Equivalence < Tautology, Contradiction

① Length should be same
② O/p should be same

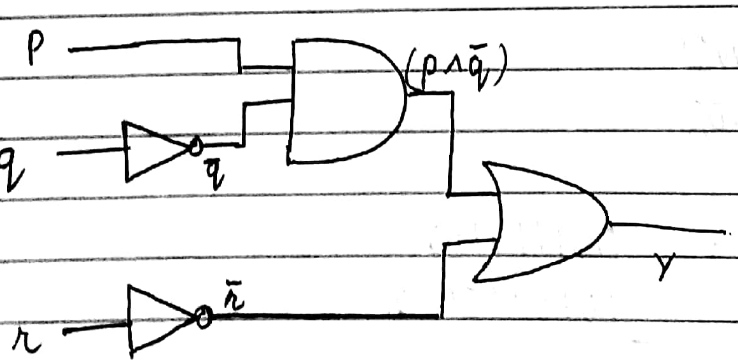
p	$\bar{p}$	$p \vee \bar{p}$	$p \wedge \bar{p}$
T	F	T	F
F	T	T	F
		↑ Tautology	↑ Contradiction

If compound proposition is always true irrespective of the i/p it is called tautology & when it is false it is called contradiction.

Compound proposition that has same truth value in all the possible ~~cat~~ cases then it is called logically equivalent.

Equivalent '$\equiv$'

Q1.) $(p \wedge \bar{q}) \vee \bar{r}$



De ~~Mog~~ Morgan's Law

1.) $(\overline{p \wedge q}) = \bar{p} \vee \bar{q}$
2.) $(\overline{p \vee q}) = \bar{p} \wedge \bar{q}$

* Properties of Logical Equivalence Law

1.) Identity Law

$P \land T \equiv P$ $\quad P \lor F \equiv P$

$P \text{ AND } 1 = P$

$P \land T \equiv P$

$P \lor F \equiv P$

(1) Domination Law

$P \lor T \equiv T$

$P \land F \equiv F$

(3) Idempotent Law

$p \lor p \equiv p$

$p \land p \equiv p$

(6) Associative Law

$(p \lor q) \lor r = p \lor (q \lor r)$

$(p \land q) \land r = p \land (q \land r)$

(8) de Morgan's Law

$\overline{p \land q} = \bar{p} \lor \bar{q}$

(4) Double negation

$\bar{\bar{P}} = P$

(5) Commutative Law

$p \lor q = q \lor p$

$p \land q = q \land p$

(7) Distributive Law

$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$

(9) Absorption Law

$p \land (p \lor q) \equiv p \lor (p \land q)$

$= p$

$\overline{\cancel{p} \lor (\cancel{p} \land \cancel{q})}$

$= \cancel{\bar{p}} \land \cancel{(\bar{p} \land \bar{q})}$

$=$

0 $\quad p \quad \lor \quad T \searrow p$

1 $\qquad\qquad\quad F \nearrow$

(10) Negation Law

$p \land \bar{p} \equiv F$

$p \lor \bar{p} \equiv T$

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Predicates

$\underbrace{P(x)}_{\text{Function of } x} = \underbrace{x}_{\text{Variable}}+1 = 2$

$P(x)[\underbrace{x}_{\text{Subject}} \underbrace{< 2}_{\text{Predicate}}]$

$P(x)$ "$x < 2$" $\longrightarrow$ Propositional function

$P(x) \Rightarrow$ "$x + 3$"

$P(4) \Rightarrow 7$

$P(2) \Rightarrow 5$

$P(x)$ "$x > 3$"

$P(4)$ $4 > 3$ T

$P(2)$ $2 > 3$ F

$Q(x, y) \Rightarrow$ "$x + y > 3$"

$Q(2, 1)$ F

$Q(1, 4)$ T

n variable predicates are called n-place predicates statement

A statement of the form $P(x_1, x_2, x_3, \ldots x_n)$ is the value of the propositional fn P at the ~~the~~ the ~~one n tub tupple~~ n-tupple $(x_1, x_2 \ldots x_n)$ and P is also called n-place predicate or n-ary predicate.

~~Qualifiers~~ Quantifiers

Predicate → Variable Values → Propositions

Propositional fn → Proposition

Predicates are only defined for ~~true~~ two values i.e. either true or false.

~~Qualifiers~~ Quantifiers are defined for value which may or may not be true (depending on domain)

Quantifier is another method to create proposition from propositional fn. It expresses the extent to which a predicate is true over a range of elements.

There are two types of quantification:
i) Universal quantification
ii) Existential "

- Universal quantification - This tells us that a predicate is true for every element under consideration.

- Existential quantification - This tells us that one or more elements under ~~cond~~ consideration for which the predicate is true.
- The area of logic that deals with the predicates and quantifiers is called as predicate calculus

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Universal Quantification (AND)

$P(x)$ is true for every value in the domain

$\forall x\, P(x)$

↑

Universal quantifier

$\forall \rightarrow$ For all

$n \longrightarrow x$

$P(x_1) \wedge P(x_2) \wedge P(x_3)$ ------ $\wedge P(x_n)$

Q. $P(x) \Rightarrow$ " $x+1 > x$ " -

Truth value of quantification. $\forall x\, P(x)$

Domain consists of all real values.

Ans True

Q. $(x) \Rightarrow$ " $x < 2$ " Domain Real values

$\forall x\, P(x) \Rightarrow$ False

b'coz $P(3)$ it is false

Q. $P(x) \Rightarrow x^2 > 10$

$P(1) \Rightarrow F$ $\qquad$ $\forall x\, P(x) \Rightarrow F$

$P(2) \Rightarrow F$ $\qquad$ b'coz $P(1), p(2), p(3)$ are false

$P(3) \Rightarrow F$

$P(4) \Rightarrow T$

Existential ~~Quat~~ Quantification

$P(x_1) \vee P(x_2) \vee P(x_3)$ ----$\vee P(x_n)$

$\exists x\, P(x)$

↑

There exist

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Q. Let $P(x)$ denote the statement $x > 3$. Find the truth value of $\exists x P(x)$. Domain R

Ans $\exists x P(x)$ True

Q. Let $Q(x) \Rightarrow x = x+1$. $\exists x P(x) \Rightarrow$ False

$\left.\begin{array}{l}\forall x P(x) \\ \exists x P(x)\end{array}\right\} \rightarrow$ highest priority

$$\forall x P(x) \land Q(x) \equiv (\forall x P(x)) \land Q(x)$$

Negating Quantifiers:

(1) $\overline{\forall x P(x)} \equiv \exists x \overline{P(x)}$ $[\overline{\forall x} \ \overline{P(x)}]$

(2) $\overline{\exists x P(x)} \equiv \forall x \overline{P(x)}$

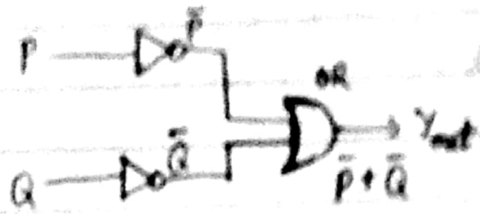
$\downarrow$ De Morgan's law for quantifiers

Q. $\forall x (x^2 > x)$. $\exists x (x^2 = 2)$

Negation $\Downarrow$ $\exists x (x^2 < x)$

Negation $\Downarrow$ $\forall x (x^2 \neq 2)$

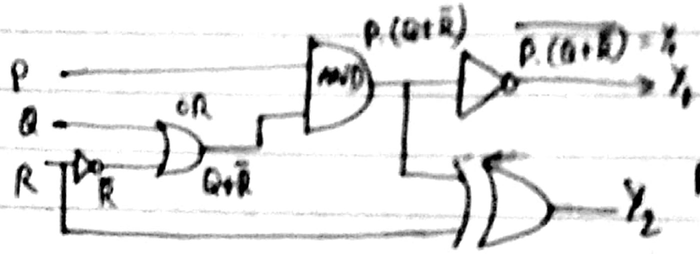
(1)



$\bar{P}+\bar{Q} = \overline{P.Q} = Y_{out}$

P	Q	P.Q	$\overline{P.Q}$
0	0	0	1
1	0	0	1
0	1	0	1
1	1	1	0

(2)



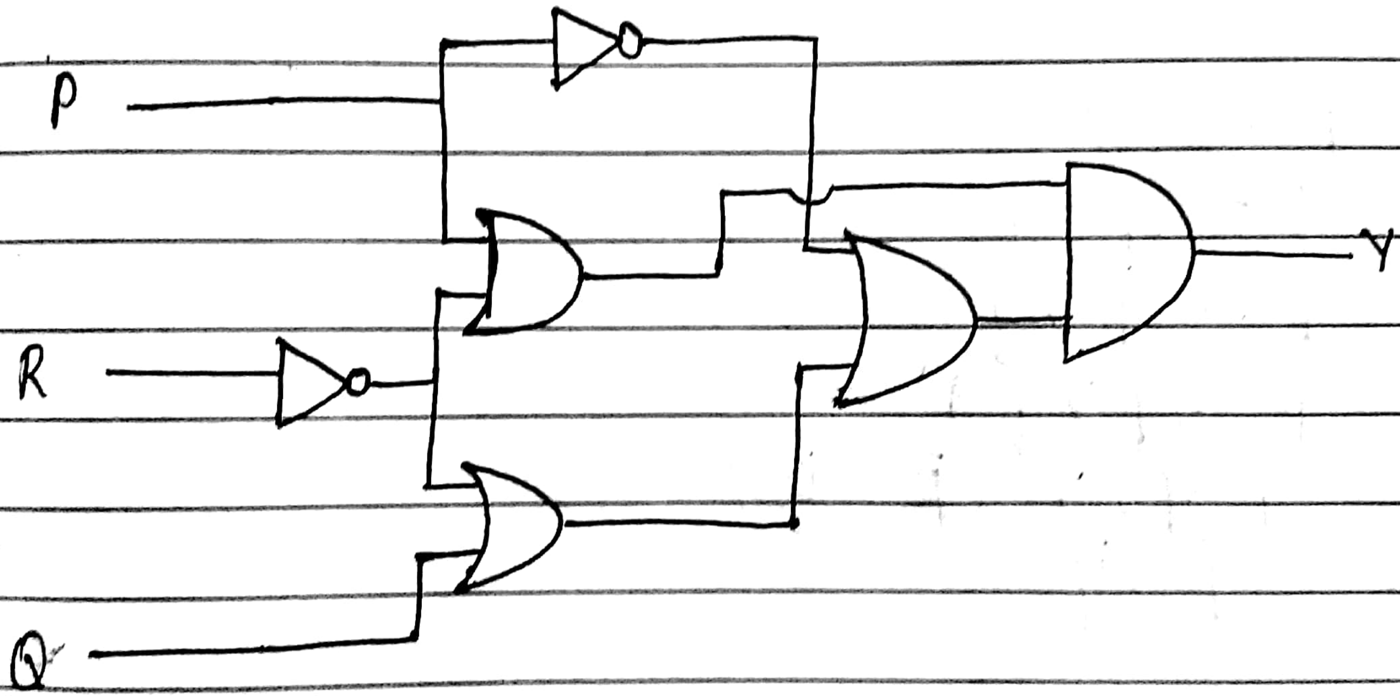
$\bar{P}+(\overline{Q+\bar{R}})$

$\bar{P} + \bar{Q}.R = Y_1$

$P.(Q+\bar{R}) \odot R$

P	Q	R	$\bar{P}$	$\bar{Q}$	$\bar{R}$	$\bar{Q}.R$	Y_1	$P.(Q+\bar{R})$	Y_2
0	0	0	1	1	1	0	1	0	1
1	0	0	0	1	1	0	0	1	0
0	1	0	1	0	1	0	1	0	0
0	0	1	1	1	0	1	1	0	1
1	1	0	0	0	1	0	0	1	1
1	0	1	0	1	0	1	1	0	1
0	1	1	1	0	0	0	1	0	1
1	1	1	0	0	0	0	0	1	0

(3) $(P \vee \bar{R}) \wedge (\bar{P} \vee (Q \vee \bar{R}))$


P
R
Q
Y

Q. $(p \wedge q) \rightarrow (p \vee q)$ is tautology

p	q	$p \wedge q$	$p \vee q$	$p \wedge q \rightarrow p \vee q$
0	0	0	0	1
1	0	0	1	1
0	1	0	1	1
1	1	1	1	1

Q. $\neg(p \vee (\bar{p} \wedge q))$ and $\bar{p} \wedge \bar{q}$

Show that they are logically equivalent

Sol^n- $\cancel{\neg[(p \vee \bar{p}) \wedge (p \vee q)]}$

$$\neg(p \vee (\bar{p} \wedge q))$$

$$= \bar{p} \wedge (\overline{\bar{p} \wedge q})$$

$$= \bar{p} \wedge (\bar{\bar{p}} \vee \bar{q})$$

$$= \bar{p} \wedge (p \vee \bar{q})$$

$$= (\bar{p} \wedge p) \vee (\bar{p} \wedge \bar{q})$$

$$= \bar{p} \wedge \bar{q}$$

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Q. $\overline{\forall x \, P(x) \rightarrow Q(x)}$

$= \overline{\forall x \, P(x)} + Q(x)$

$= \forall x \, P(x) \cdot \overline{Q(x)}$

$= \forall x \, P(x) \wedge \overline{Q(x)}$

$\overline{\forall x \, P(X) \longrightarrow Q(X)}$

$\exists x \; \overline{P(X) \longrightarrow Q(X)}$

$\exists x \; \overline{\overline{P(X)} \vee Q(X)}$

$\exists x \; P(X) \wedge \overline{Q(X)}$

Rules of Inference

$\boxed{P \quad Q \quad R} = n$

I/p's $\rightarrow 2^n$

An argument in ~~is~~ ~~a~~ propositional logic is a sequence of propositions. All but the final proposition in an argument are called premises & the final proposition is called conclusion. An argument is valid if the truth of all its premises implies that the conclusion is true.

1
2] $\rightarrow$ premises
3
4 $\rightarrow$ Conclusion

Hypothesis
1
2
$\therefore$ 3 $\rightarrow$ conclusion

Rule of Inference	Tautology	Name
1. p $p \rightarrow q$ $\therefore q$	$p \wedge (p \rightarrow q) \rightarrow q$	Modus Ponens
2. $p \rightarrow q \; (\Rightarrow \bar{q} \rightarrow \bar{p})$ $\bar{q}$ $\therefore \bar{p}$	$(p \rightarrow q) \wedge \bar{q} \longrightarrow \bar{p}$	Modus Tollens

Rule of Inference	Tautology	Name
(3) $p \to q$ $q \to r$ $\therefore p \to r$	$(p \to q) \wedge (q \to r)$	Hypothetical Syllogism
(4) $p \vee q$ $\bar{p}$ $\therefore q$	$(p \vee q) \wedge \bar{p}$	Disjunctive Syllogism
(5) p $\therefore p \vee q$	$p \to (p \vee q)$	Addition
(6) $p \wedge q$ $\therefore p$ $\therefore q$	$(p \wedge q) \to p$ $(p \wedge q) \to q$	Simplification
(7) Resolut $p \vee q$ $\bar{p} \vee r$ $\therefore q \vee r$	$(p \vee q) \wedge (\bar{p} \vee r) \to q \vee r$	Resolution

If all the statments in arguments are true ~~accept~~ except the final one then conclusion will be true.

$(P_1) \wedge (P_2) \wedge (P_3) --- (P_n) \longrightarrow q$ is tautology.

Rules of inference can be used as building blocks to construct more complicated valid argument forms.

The tautology $(p \wedge (p \to q)) \longrightarrow q$ is basic

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building block.

Q. Show that the premises, ~~th~~ "if you send me e-mail msg then I will finish writing the program."
"If you do not send me an e-mail then I will go to sleep early.
If I go to sleep early then I will wake up feeling refreshed."
leads to conclusion
"If I don't finish writing the program then I will wake up feeling refreshed."

Sol^n:- $p \rightarrow$ You send me an email
$q \rightarrow$ I will finish writing the program
$r \rightarrow$ I will go to sleep early
$s \rightarrow$ I will wake up feeling refreshed

$p \rightarrow q$
$\bar{p} \rightarrow r$
$r \rightarrow s$

① $p \rightarrow q$ $\longrightarrow$ Premise
② $\bar{q} \rightarrow \bar{p}$ contrapositive of ①
③ $\bar{p} \rightarrow q$ $\longrightarrow$ premise
④ $\bar{q} \rightarrow r$ (H S)
⑤ $r \rightarrow s$ premise
$\bar{q} \rightarrow s$ conclusion

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Proof

Premise
↓ Inference
Conclusion

Theorem : It is sth that is to be proven to be true

Proof
→ Direct Proof
→ Proof by contradiction
→ Proof by ~~ct~~ contraposition

Lemma - We take it as +ve (already proved statement) (less useful proof)
Corollary - Extension of proof

~~Conjectures~~ Lemma
A less imp. theorem that is useful in the proof ~~to prove other~~ of other results is called lemma.
Theorems can be proved using a series of lemma.

Corollary
A corollary is a statement that can be established directly from a theorem

A conjecture is a statement that is being proposed to be a true statement usually on the basis of same partial evidences.

$p \to q$
$\bar{q} \to \bar{p}$

Q Direct proof is a cond^n al statement where $p \to q$ is constructed.
Step 1 Assumption : p is true
Subsequent steps are constructed using rule of inference with the final step of proving q to be true.

Def 1: Integer ~~m~~ n is even and if ∃ an int k such that
n = even
1.) $n = 2K$ 2.) n = odd
$n = 2k+1$

$k = 1, 2, 3 \dots$

Q Give a direct proof of the theorem
If n is odd then n^2 is odd.

Sol^n- n → odd
$n = 2k+1$
$n^2 = (2k+1)^2$
$= 4k^2 + 4k + 1$
$= 2\underbrace{k(2k+2)}_{A} + 1$
$= 2A + 1 \Rightarrow$ odd

Q Prove by contraposition
There are no hypothesis in contraposition

Q. Prove that if n is a integer and $3n+2$ is odd then n is odd.

Sol^n- $3n+2 \to$ odd
n → odd
Contrapositive
n → even

$\Rightarrow n = 2K$

$3n+2 = 6k + 2$

$= 2\underbrace{(3k+1)}$

$= \cancel{2a}\, 2A$

$3n+2 \rightarrow$ even

$\bar{q} \longrightarrow \bar{p}$ true

Prove by contradiction

Q. Prove that $\sqrt{2}$ is irrational by contradiction.

Sol^n - Let $\sqrt{2}$ be rational

$$\sqrt{2} = \frac{p}{q} \qquad q \neq 0$$

Squaring

$$2 = \frac{p^2}{q^2}$$

$$p^2 = 2q^2$$

Let p = even

$$2q^2 = (2n)^2$$

$$2q^2 = 4n^2$$

$$q^2 = 2n^2$$

$$\frac{q^2}{n^2} = 2$$

Unit - 2

Sets

$A = \{\underbrace{1, 2, 3}_{\uparrow}\}$ Finite

collectⁿ of elements
↓
objects in a set

Cardinality - No. of elements present in a set.

Vowels = {a, e, i, o, u}
X = {1, A, Sneha, Jaipur}

'C' →

Set builder form

$X = \{x \mid \underbrace{x > 0}_{+ve} \wedge x \text{ is an odd no.} \wedge x < 100$

$x \; Z^+$

ϕ → empty set / Null set

Singleton Set → contains only one element
Eg. $S = \{1\}$

⇒ Equal Sets
$A = \{1, 2, 3\}$ $B = \{4, 5, 6\}$
A & B are not equal sets
$A = \{1, 2, 3\}$ $B = \{1, 2, 3\}$ $C = \{1, 1, 2, 2, 2, 3, 3, 3\}$
$A = B$ $A = C$ (~~Repeti~~ Repetition ~~doesn't~~ has no meaning)

Venn Diagram

$A = \{1, 2, 3\}$

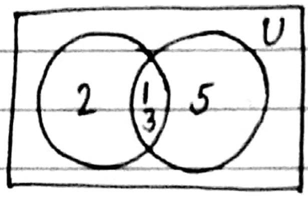
→ Union $(\cup)$

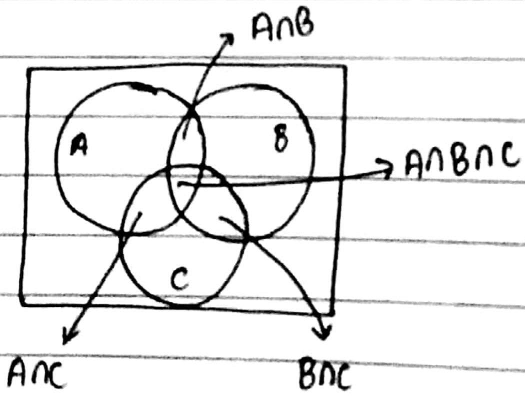
→ Intersection $(\cap)$

$A = \{1, 2, 3\}$ $\quad B = \{1, 3, 5\}$

$A \cup B = \{1, 2, 3, 5\}$

$A \cap B = \{1, 3\}$





Subset:

$A = \{1, 2, 3, 4\}$

$B = \{1, 2\}$

$C = \{1, 2, 3, 4\}$ $\quad D = \{\ \}$

$B \subset A$ $\quad C \subseteq A$ $\quad D \subset A$

No. of subsets $= 2^n$ — ϕ, A

$\phi = \{\ \}$

$= \{\phi, \{\phi\}\}$

Proper Subset

~~No of proper subset~~ :

$A \subset B$ (if $A = B$ then A is proper subset of B)

Power Set

A set containing all the

~~No of~~ possible subsets of a set is called power set.

$A = \{0, 1, 2\}$

$P\{0, 1, 2\} = \{\phi \{0\}, \{1\}, \{2\}, \{0, 1\}, \{1, 2\}, \{0, 2\}, \{0, 1, 2\}\}$

No of Power set $= 2^n$

Ordered Sets

$A = \{0, 1, 2, 3\}$

$\quad a_1 \; a_2 \; a_3 \; a_4$

$\{a, b\} \neq \{b, a\}$

$A = \{a_1, a_2, a_3, \ldots a_n\}$

$B = \{b_1, b_2, b_3, \ldots b_n\}$

$A = B$ (if $a_1 = b_1$, $a_2 = b_2 \ldots a_n = b_n$)

Cartesian product

$A \times B$

$A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$

Set Operations

Union → $A \cup B = \{x / x \in A \vee x \in B\}$

Intersection → $A \cap B = \{x / x \in A \wedge x \in B\}$

For n elements

$A_1 \cup A_2 \cup \ldots \cup A_n = \bigcup_{i=1}^{n} A_i$

$A_1 \cap A_2 \cap \ldots \cap A_n = \bigcap_{i=1}^{n} A_i$

$A' = U - A$ (Complement of A)

Disjoint sets $A \cap B = \phi$

$\overline{A \cup B} = \bar{A} \cap \bar{B}$

LHS

Let

$x \in \overline{A \cup B}$

$x \notin A \cup B$

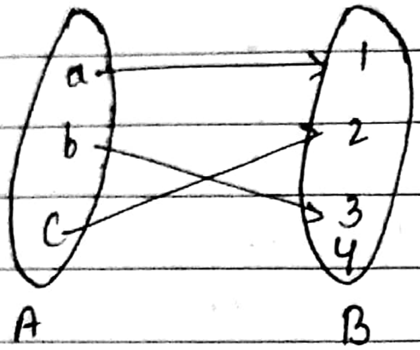
$x \notin A$ And $x \notin B$ $\Rightarrow$ $x \in \bar{A}$, $x \in \bar{B}$

$\Downarrow$

$x \in \bar{A} \cap \bar{B}$

Functions

Functions are a set of instructions



$f : A \to B$

Mapping from A to B

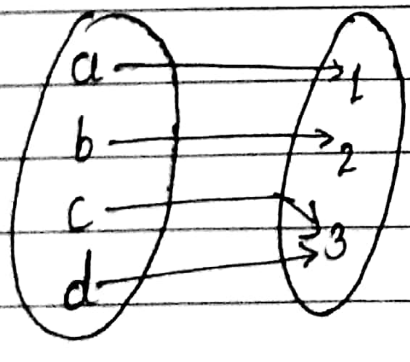
$f(a) = 1$ $\quad$ $f(c) = 2$

$f(b) = 3$

A → Domain $\quad$ B → Co-domain

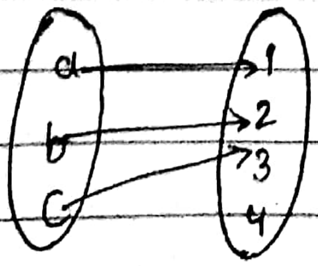
Range = $\{1, 2, 3\}$

If codomain is called image then domain is called pre image



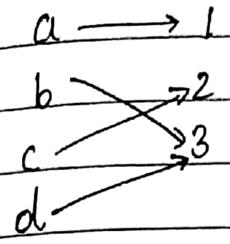
This is not one-one but onto

For a fn to be one-one then every element of domain should have one unique image



One-One → injection

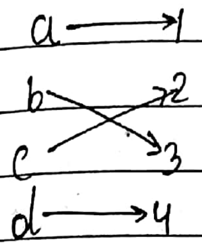
not onto



One to one ✗

onto ✓

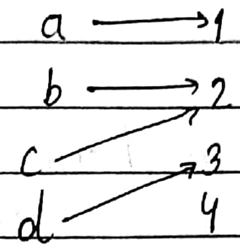
surjection



→ ~~ot~~ onto ✓

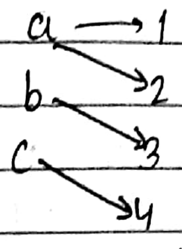
→ one to one ✓

→ One to one correspondance or Bijective



one to one ✗

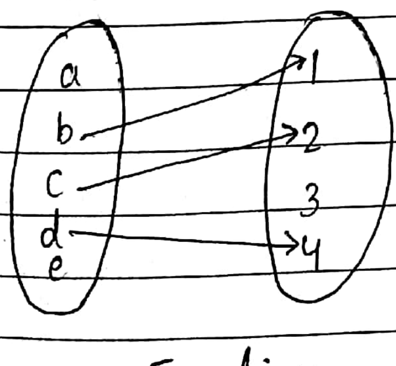
onto ✗



Not a fn

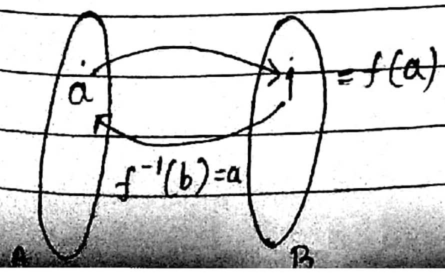
- A fn f is called one to one if & only if $f(A) = f(B) \Rightarrow$ that $A = B$ for all A & B in the domain.

- A fn f is one to one correspondance or bijection if it is both one to one and onto fn.

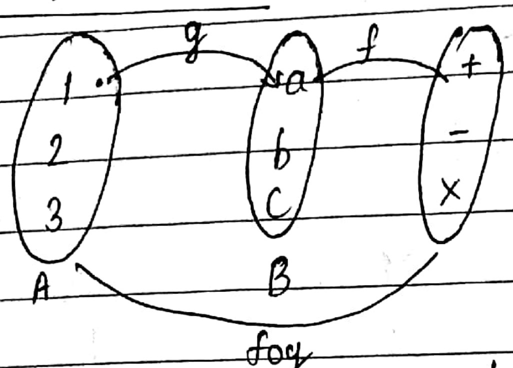


$\{b, c, d\} = \{1, 2, 4\}$

Inverse Function



Composition of functions



Let g be the fn from set A to set B & f be the fn from B to C. Composition of fn is denoted by $fog(a) = f(g(a))$

Q. $f(x) = 2x+3$

$g(x) = 3x+2$

$fog(x) = \cancel{3(}\ 2(3x+2)+3$

$= 6x+7$

$gof(x) = 3(2x+3)+2$

$= 6x+11$

$f(x)$ $f(x)+g(x)$

$g(x)$ $f(x).g(x)$

$f(x) = x^2$ $g(x) = x+2$

$f+g = x^2+x+2$

$fg = x^3+2x^2$

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$A \cap B = A - B$

$A \cup B = A + B - A \cap B$

- $A \cap B \to$ Principle of inclusion & exclusion.

- ~~$A \cap B$~~ $A|B$ — elements only in A not in B.

Proof.

~~$A \cup B = A \cup A|B$~~ $(A \cup (A|B))$

$A \cup B = |A \cup B|A|$
$= |A + B|A|$
$= |A| + (B|A) + (A|B) - (A \cap B)$

$A \cup B = |A| + |B| - |A \cap B|$

Q ~~There~~ Total stds — ~~1232~~ 2092
(F) French — 879
(R) Russian — 114, Spanish — 1232
$S \cap F = 103$, ~~$S \cap R$~~ = (S)
$S \cap R = 23$, $F \cap R = 14$

$|S \cap F \cap R| = ?$

$\overline{\overline{w}xy(\overline{x}w+\overline{y}x)}$

$\overline{\overline{w}} + \overline{x} + \overline{y} + (\overline{\overline{x}w + \overline{y}x})$

$\overline{\overline{w}} + \overline{x} + \overline{y} + (\overline{\overline{x}w})(\overline{\overline{y}x})$

$w + \overline{x} + \overline{y} + (\overline{w}+x)(y+\overline{x})$

4/09/17

Permutation and Combination

Counting

(a) Product Rule

$n_1 \longrightarrow$ 1st step

$n_2 \longrightarrow$ 2nd step

Total ways $= n_1 \times n_2$

(b) Sum Rule

n_1, n_2 — To perform a task

Total ways $= n_1 + n_2$

Pigeonhole Principle

$N \rightarrow$ spaces

$K \rightarrow$ elements

if $K > N$

$\frac{K}{N}$

Floor $1.9 \longrightarrow 1$

Ceiling $1.01 \longrightarrow 2$

If $\frac{K}{N} = 2.5$ then we will take ceiling & the value of K/N becomes 3.

If K is a +ve integer and k+1 or more objects are placed into k boxes then there is atleast one box containing two or more objects.

If N objects are placed into K boxes then there is atleast one box containing atleast $\underbrace{\left\lceil \frac{N}{K} \right\rceil}_{\text{ceiling}}$ objects.

Q. Among 100 people ^out of 100 people there are atleast how many persons are there who were born in the same month.

Sol^n- $\left\lceil \frac{100}{12} \right\rceil = 9$

Q. What is the min^m no. of students req^d in a discrete maths class to be sure that 6 will receive the same grade. If there are five possible grade.

Sol^n- $\left\lceil \frac{N}{5} \right\rceil = 6$

$N = 26$

Direct formula for above ques.

$\boxed{N = (r-1)K + 1}$

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$$P(n,r) = {}^nP_r = \frac{n!}{(n-r)!}$$

5 persons

$$\underset{5}{\downarrow} \times \underset{4}{\downarrow} \times \underset{3}{\downarrow} = 60$$

$$5 \times 4 \times 3 \times 2 \times 1 = 120$$

n

$$n(n-1)(n-2)----(n-r+1)\frac{(n-r)!}{(n-r)!}$$

$$= \frac{n!}{(n-r)!}$$

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

$$P(n,r) = C(n,r)\,P(r,r)$$ → r-permutations

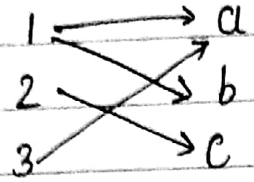
$$C(n,r) = \frac{P(n,r)}{P(r,r)}$$

$$= \frac{\frac{n!}{(n-r)!}}{\frac{r!}{(r-r)!}} = \frac{n!}{r!(n-r)!}$$

Q. How many ways are there to select 5 players from a 10 member tennis team to make a trip to a match at another school?

Sol^n- Ways $= {}^{10}C_5 = \frac{\cancel{10}^{2} \times 9 \times \cancel{8}^{2} \times 7 \times \cancel{6}}{\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2}} = 252$

Relations



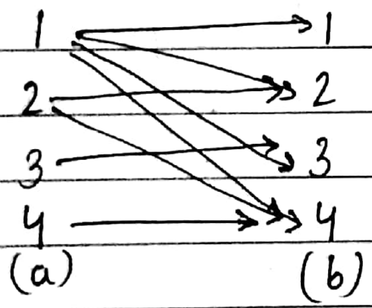
R	a	b	c
1	x	x	
2			x
3	x		

Function as Relation

Q. A is a set

$A = \{1, 2, 3, 4\}$

$R = \{(a,b) \mid a \text{ divides } b,\ a \in A \text{ and } b \in A\}$



$R_1 = (1,1)\ (1,2)\ (1,3)\ (1,4)\ (2,2)$
$(2,4)\ (3,3)\ (4,4)$

Operation on Relations

$R_1 \cap R_2$

$R_1 \cup R_2$

$R_1 - R_2$

$R_2 - R_1$

$R_1 = (1,1)\ (1,2)\ (2,2)\ (2,3)$

$R_2 = (1,2)\ (2,3)\ (3,1)\ (1,3)$

$R_1 - R_2 = (1,1)\ (2,2)$

$R_2 - R_1 = (1,3)\ (3,1)$

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Properties of Relations

① Symmetric

if $(a, a) \in R$

Q. $R_1 = (1,1)\ (1,2)\ (2,1)\ (2,2)\ (3,4)\ (4,1)\ (4,4)$
$R_2 = (1,1)\ (1,2)\ (2,1)$
$R_3 = (1,1)\ (1,2)\ (1,4)\ (2,1)\ (2,2)\ (3,3)\ (4,1)\ (4,4)$
$R_4 = (2,1)\ (3,1)\ (3,2)\ (4,1)\ (4,2)\ (4,3)$
$R_5 = (1,1)\ (1,2)\ (1,3)\ (2,2)\ (2,3)\ (2,4),\ (3,3),\ (3,4)\ (4,4)$

R_5 & R_3 are symmetric

② Reflexive

if $(a, b) \in R$, then $(b, a) \in R$

R_2 & R_3

③ Transitive

if $(a,b) \in R$ $(b, c) \in R$ then $(a, c) \in R$

R_2

Equivalence - When all three properties are satisfied