i) $P_x = \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$
$T \to \infty$

ii) $P_a = \frac{P_x}{R_a}$ P_a - actual power
R_a - " resistance

iii) Power of DC sig
$A \longrightarrow A^2$

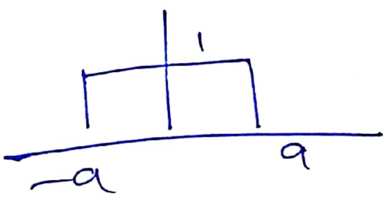
iv) $\left.\begin{array}{l} A \sin \omega_0 t \\ A \cos \omega_0 t \end{array}\right] \longrightarrow \frac{A^2}{2}$

v) $A e^{j\omega_0 t} \longrightarrow A^2$

vi) $P_x = P_1 + P_2 + P_3 + \dots$ [additive nature of power]

vii) $X_{rms} = \sqrt{Power}$

viii) $\frac{\sin at}{\pi t} \xleftrightarrow{FT}$



ix) $\cos \omega_c t \xleftrightarrow{FT} \pi [\delta(\omega - \omega_c) + \delta(\omega + \omega_c)]$

x) $\sin \omega_c t \xleftrightarrow{FT} \frac{\pi}{j} (\delta(\omega - \omega_c) - \delta(\omega + \omega_c))$

xi) $\cos \omega_c t \xleftrightarrow{FT} \frac{1}{2} [\delta(f - f_c) + \delta(f + f_c)]$

xii) $\sin \omega_c t \xleftrightarrow{FT} \frac{1}{2j} [\delta(f - f_c) - \delta(f + f_c)]$

xiii) $x(t) \cos \omega_c t \xleftrightarrow{FT} \frac{1}{2} [X(\omega - \omega_c) + X(\omega + \omega_c)]$

xiv) $x(t) \cos \omega_c t \xleftrightarrow{FT} \frac{1}{2} [X(f - f_c) + X(f + f_c)]$

xv) $x(t) \sin \omega_c t \leftrightarrow \frac{1}{2j} (X(\omega - \omega_c) - X(\omega + \omega_c))$
$(X(f - f_c) - X(f + f_c))$

$\frac{1}{} (X(f - f_c)$

xvii) $\frac{1}{\pi t} \xleftrightarrow{FT} -j\,sgn(\omega)$

xviii) $y(t) = x(t) \cdot p(t) \xleftrightarrow{FT} \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(\omega - k\omega_s)$

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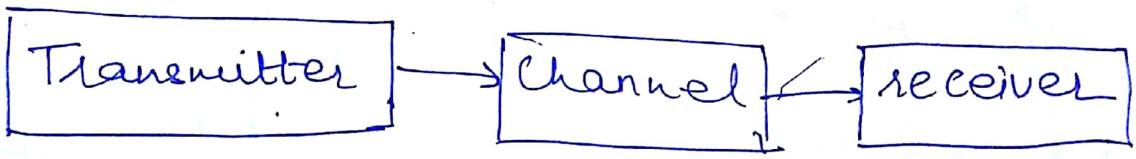
Communication

it is the basic process of exchanging information

→ A communication process consists of three basic building blocks

1. Transmitter
2. channel
3. receiver



Transmitter

→ It is a physical system that transmits the information

Receiver

→ it is a physical system which receives the information

Channel

It is the medium through which information exchanges takes place. Depending upon type of channel used for information exchange, there are two types of communication

(i) Line or wire communication

in this type of communication, a physical channel is created b/w TX and RX through copper wires, coaxial cables, optical fibres etc. before the information exchange can takes place.

ex

Basic telephone, telegraphy etc.

II Radio or wireless communication

In this type of communication no. physical channel is created b/w TX and RX the information exchange takes place through space as channel

ex

Mobile communication, broadcasting, wireless phones, satellite communication.

Modulation

→ Modulation is the process of superimposing the information contents of a baseband the modulating sig by altering the characteristics of a high freq. carrier wave (amplitude, freq., phase.)

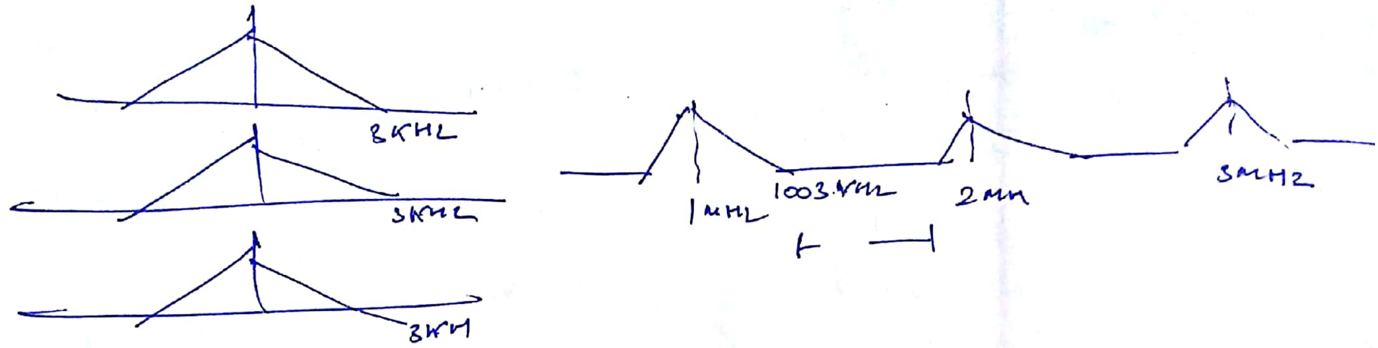
$$c(t) = \underbrace{A_c}_{\text{amplitude}} \cos \overbrace{\underbrace{\omega_c}_{\text{freq}} t}^{\text{phase}}$$

→ Modulation process translates a low freq Baseband s/g into a high freq. band-pass s/g

→ Since a linear system, can not provide the freq. translation so the modulation process can only be generated by a non-linear device

Need of Modulation

① Avoid mixing of sig's



→ the modulation process translates different baseband sig's at different carrier freq'es so that overlapping of spectrums can be avoided

② Allows multiplexing of sig's

→ multiplexing means transmission of two or more sig's simultaneously over same channel.

→ Once the overlapping of spectrum has been avoided so now they can be simultaneously transmitted over same channel

③ Reduces height of Antenna

$$\lambda = \frac{c}{f}$$

Practical Antenna height $= \frac{\lambda}{4}$

eg

(1) if $f_1 = 15\,KHZ$

$$\frac{\lambda}{4} = \frac{3 \times 10^8}{4 \times 15 \times 10^3} = 5\,Km$$

(2) $f_2 = 15\,MHZ$

$$\frac{\lambda}{4} = \frac{3 \times 10^8}{4 \times 15 \times 10^6} = 5\,m$$

The height of antenna required for transmission and reception of radio waves is a function of freq. used. the minimum height of antenna is given as $\lambda/4$ so freq. of transmission must be increased to have practical height of antenna

④ increases Range of communication

At low freq., the Radiation is poor and sig gets highly attenuated therefore baseband sig's can not be transmitted directly over longer distances. the modulation effectively increases the freq of sig to be radiated and thus increases the distance over which sig can be transmitted faithfully

⑤ improves quality of reception

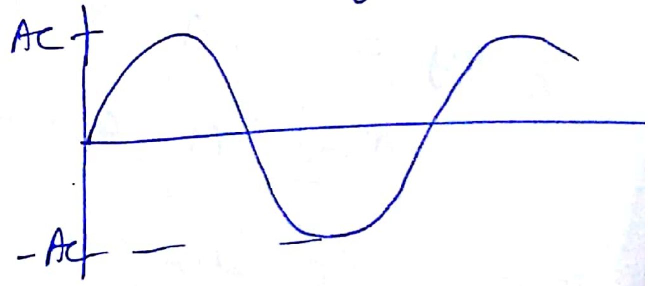
The sig communication using modulation technique such as FM & PCM reduces the effect of Noise to great extent.

→ reduction in noise improves quality of reception

Amplitude Modulation

→ In AM, the amplitude of high freq. carrier sig is varied in accordance with instantaneous value of modulating sig keeping freq & phase constant

→ AM translates a low freq Baseband sig into a high freq Narrow band sig



(1) amplitude
(2) Magnitude
(3) Instantaneous Value

Amplitude - The amplitude of a sig is a vector quantity having magnitude as well as direction

Magnitude - Magnitude of a sig is always +ve hence it is a scalar quantity having no direction associated

Instantaneous value

The value of sig at any instant of time is known as instantaneous value
It can be +ve as well as -ve
i.e. vector quantity

Equation for AM

Let $m(t)$ = Arbitrary baseband modulating signal having max freq ω_m

$c(t) = A_c \cos\omega_c t$

$X_{AM}(t) = A\cos\omega_c t$

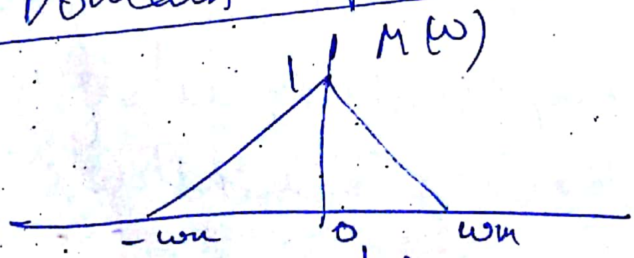
$A = A_c + m(t)$

$X_{AM}(t) = (A_c + m(t))\cos\omega_c t$

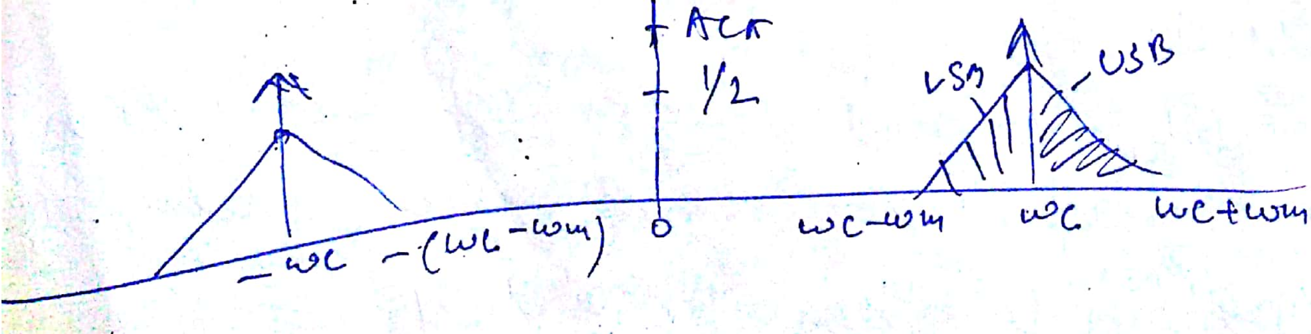
$X_{AM}(t) = A_c\cos\omega_c t + m(t)\cos\omega_c t$

$$X_{AM}(\omega) = A_c\pi[\delta(\omega-\omega_c) + \delta(\omega+\omega_c)] + \frac{1}{2}[M(\omega-\omega_c) + M(\omega+\omega_c)]$$

Freq. Domain representation



$BW = 2\omega_m$
or $2f_m$



$$\frac{\omega_H}{BW} = \frac{\omega_c + \omega_m}{2\omega_m} = \frac{1}{2} + \frac{\omega_c}{2\omega_m} \begin{matrix} \text{MHz} \\ \text{kHz} \end{matrix}$$

$\frac{\omega_H}{BW} >> 1$
so it is Narrowband s/s

Sinusoidal AM

$m(t) = A_m \cos\omega_m t$

$c(t) = A_c \cos\omega_c t$

$X_{AM}(t) = A \cos\omega_c t$

$A = A_c + A_m \cos\omega_m t$

$X_{AM}(t) = (A_c + A_m \cos\omega_m t)\cos\omega_c t$

$= A_c(1 + \frac{A_m}{A_c}\cos\omega_m t)\cos\omega_c t$

$\Rightarrow \frac{A_m}{A_c} = m$ = modulation index

$X_{AM}(t) = A_c(1 + m\cos\omega_m t)\cos\omega_c t$

$= A_c\cos\omega_c t + mA_c\cos\omega_m t\cos\omega_c t$

using $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$$X_{AM}(t) = \underbrace{A_c\cos\omega_c t}_{\text{carrier}} + \frac{mA_c}{2}\underbrace{\cos(\omega_c+\omega_m)}_{\text{USB}} + \frac{mA_c}{2}\underbrace{\cos(\omega_c-\omega_m)}_{\text{LSB}}$$

Freq. domain

$$X_{AM}(\omega) = A_c\pi[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)] + \frac{mA_c\pi}{2}[\delta[\omega-(\omega_c+\omega_m)] + \delta(\omega+(\omega_c+\omega_m))] + \frac{mA_c\pi}{2}[\delta(\omega-(\omega_c-\omega_m)) + \delta(\omega+(\omega_c-\omega_m))]$$

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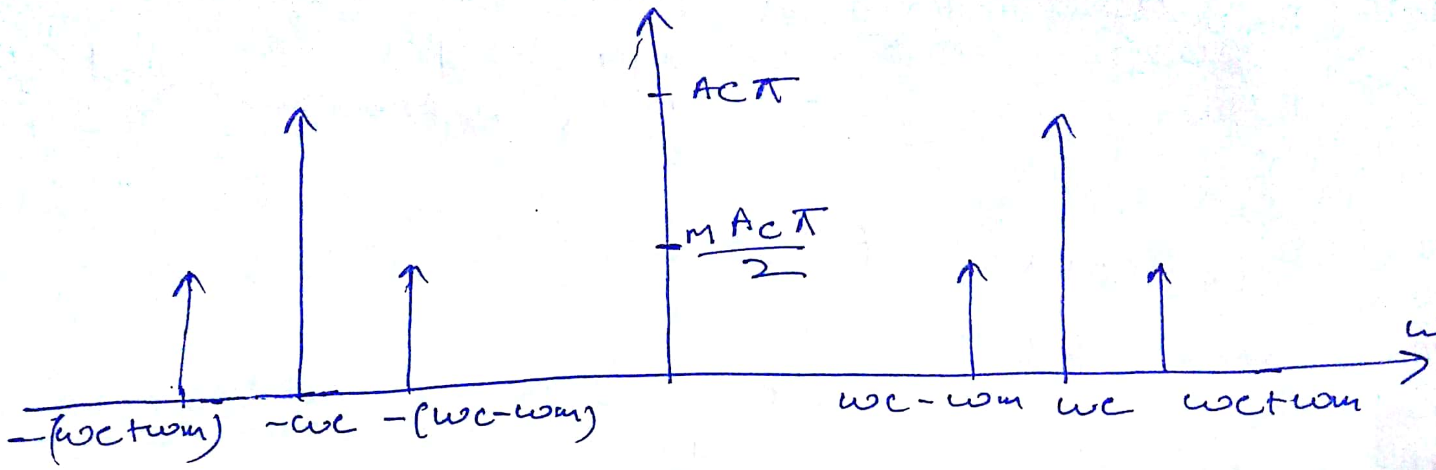

Power Relation in AM

$$X_{AM}(t) = A_c\cos\omega_c t + \frac{mA_c}{2}\cos(\omega_c-\omega_m) + \frac{mA_c}{2}\cos(\omega_c+\omega_m)$$

$$P_t = \frac{A_c^2}{2} + \frac{m^2A_c^2}{8} + \frac{m^2A_c^2}{8}$$

$$P_c = \frac{A_c^2}{2} \text{ (carrier power)}$$

$$P_{SB} = \frac{m^2A_c^2}{4} \text{ (Sideband power)}$$

$$= \frac{m^2}{2}\frac{A_c^2}{2} = \frac{m^2}{2}P_c$$

$$P_t = P_c + \frac{m^2}{2}P_c$$

$$\boxed{P_t = P_c\left(1+\frac{m^2}{2}\right)}$$

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Current relation in AM

$$P = I^2 R$$

for normalised power

$$P_n = I^2$$

$$P_t = I_t^2$$

$$P_c = I_c^2$$

$$I_t^2 = I_c^2 \left(1 + \frac{m^2}{2}\right)$$

$$I_t = I_c \sqrt{\left(1 + \frac{m^2}{2}\right)}$$

Transmission efficiency (η)

Transmission efficiency gives the percentage of usefull power in the total Transmitted power

$$\eta = \frac{\text{useful Power}}{\text{Total Tx. Power}} \times 100\%$$

$$\eta(\%) = \frac{P_c \frac{m^2}{2}}{P_c\left(1 + \frac{m^2}{2}\right)} \times 100\%$$

$$\boxed{\eta(\%) = \frac{m^2}{2+m^2} \times 100\%}$$

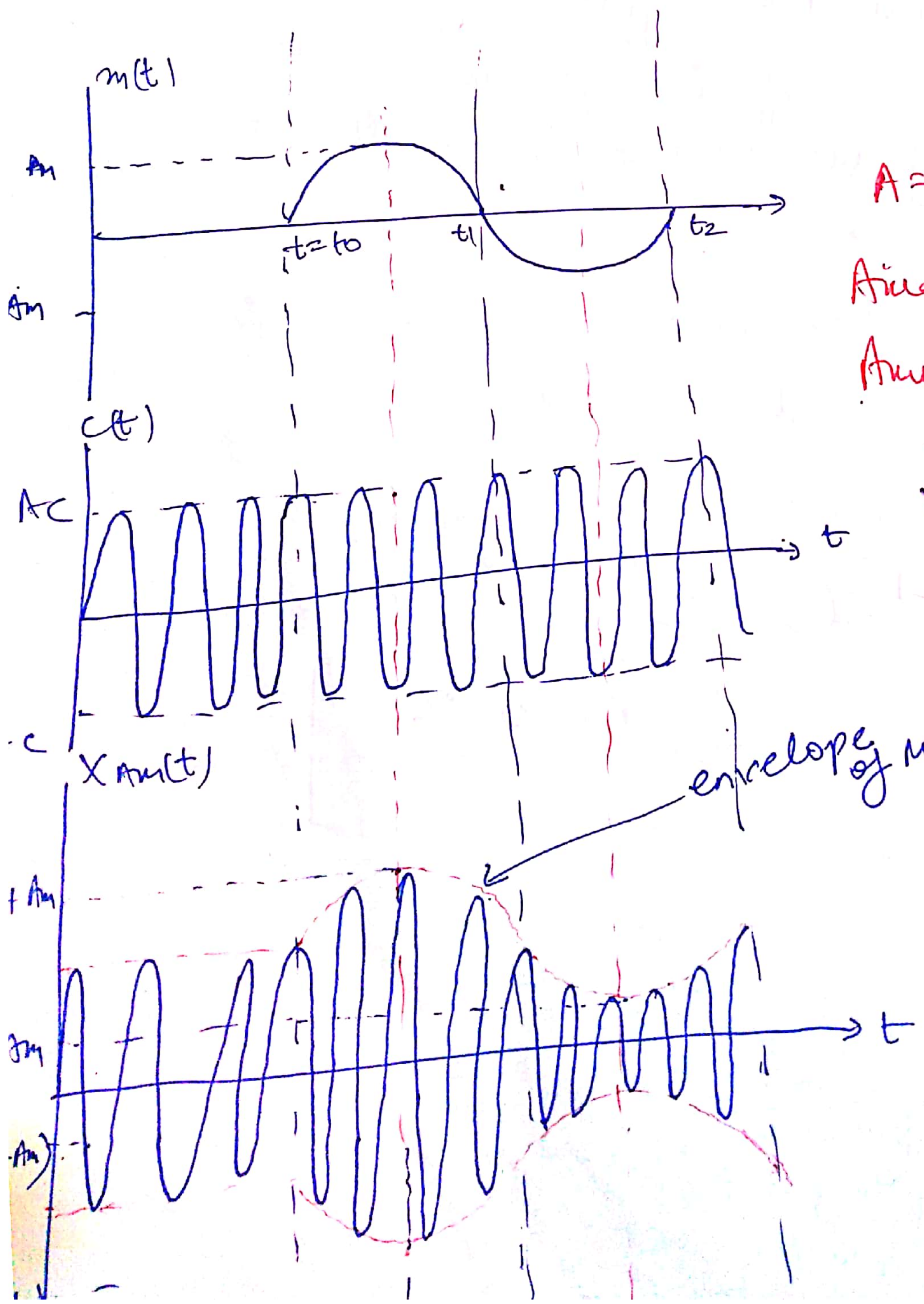
Power efficiency

$$\text{Power efficiency} = \frac{\text{Carrier Power}}{\text{Total Tx. Power}} \times 100\%$$

$$PE = \frac{P_c}{P_c\left(1+\frac{m^2}{2}\right)} \times 100\%$$

$$\boxed{PE = \frac{2}{2+m^2} \times 100\%}$$

Time domain representation of AM sig.



$A = A_c + A_m \cos \omega_m t$

$A_{max} = A_c + A_m$

$A_{min} = A_c - A_m$

$|A + m(t)| > 0$

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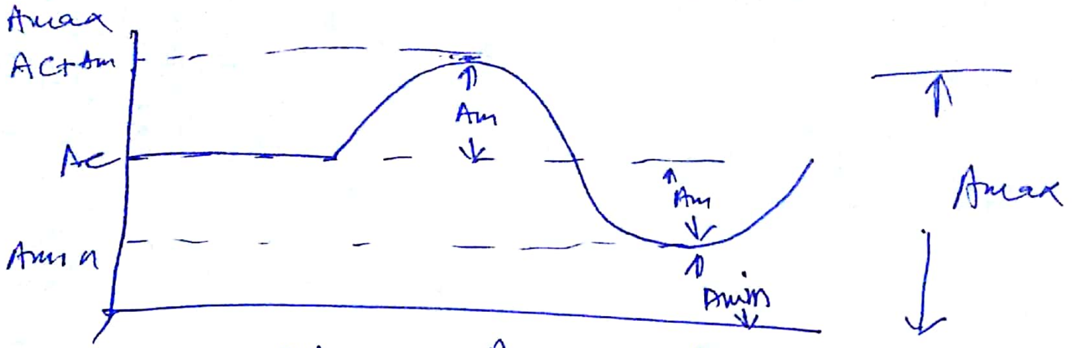

$$2A_m = A_{max} - A_{min}$$

$$\boxed{A_m = \frac{A_{max} - A_{min}}{2}} \quad \text{—(i)}$$

$$A_c = A_{max} - A_m$$

$$= A_{max} - \frac{A_{max} - A_{min}}{2}$$

$$\boxed{A_c = \frac{A_{max} + A_{min}}{2}} \quad \text{—(ii)}$$

$$\boxed{m = \frac{A_m}{A_c} = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}}$$

Concept of Modulation Index

1. it gives the depth to which modulation has occured ~~when~~
ii) when $m=0$, it represents no modulation
iii) when $m=1$, Max. modulation has occured
iv) For $m>1$, overlapping of envelople take place. This is known as "Over modulation"

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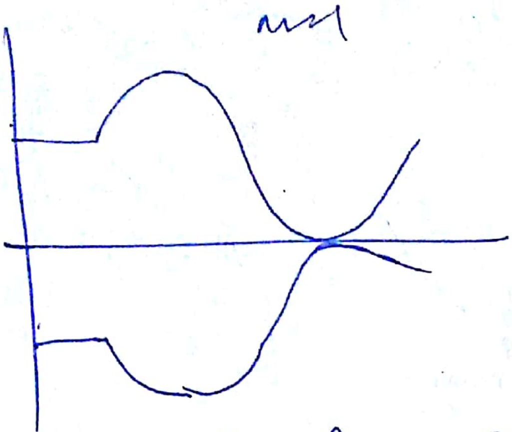
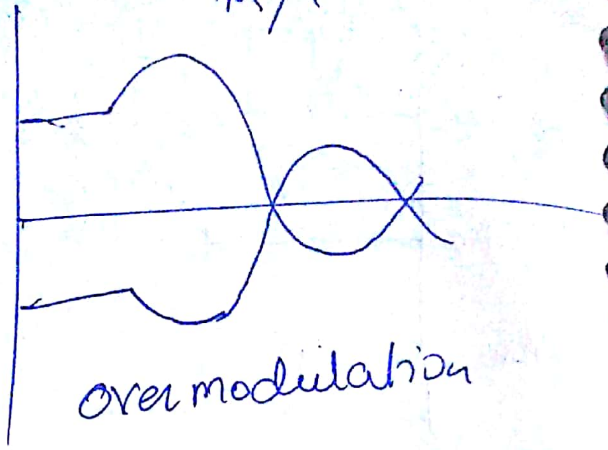
$m = 1$ 

$A_{min} = A_c - A_m = 0$

$m > 1$ 

over modulation

⟹ in overmodulation, The s/g still can be detected but with the help of synchronous demodulation only

Q1 in an amplitude modulated system, if the total power is 600W and $P_c = 400$ W the $m = ?$

$$600 = 400\left(1 + \frac{m^2}{2}\right)$$

$$0.5 = \frac{m^2}{2} \Rightarrow m = 1$$

$$\boxed{P_t = 1.5 P_c}$$

Q2 For an AM wave, Max voltage was found to be 10V and min voltage was found to be 5V the modulation index will be

$$m = \frac{10-5}{15} = 1/3 = 0.33$$

Q3 For an AM s/g, BW is 10KHz and highest freq. component present is 705 KHz The carrier freq. used is

$f_c = 700$ KHz

In AM, the modulation envelope has a peak value which is double the unmodulated carrier value. $m = ?$

$\underline{m=1}$

Critical Condition

$2A_c = A_c + A_m$

$A_m = A_c$

$m = 100\%$

Q a given AM broadcast station transmits an average carrier power o/p of 40KW and $m = 0.707$ for sine wave modulation what is max amp. of o/p. if antenna is represented by 50Ω Resistive load.

$$P_a = \frac{A^2}{2R} \Rightarrow 2R\,40K = A_c^2$$

$$A_c = \sqrt{2 \times 40 \times 50 \times 10^3}$$

$$A_c = 2000\ V$$

$$A_m = mA_c = 2000 \times 0.707$$

$$= 1414\ V$$

$$\boxed{A_{max} = A_c + A_m = 3414\ V}$$

Q a carrier voltage has a peak amplitude of 10V at a freq of 1 MHz. a sinusoidal signal of 1KHz varies the amplitude of RF wave b/w 7.5 & 12.5 V.

$A_m = 2.5$

$20(1 + 0.5\cos\omega_m t)\cos 2\pi \times 10^6$

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Simultaneous modulation by several sine waves

$$m_1(t) = A_{m_1} \cos \omega_{m_1} t$$

$$m_2(t) = A_{m_2} \cos \omega_{m_2} t$$

Let $A_{m_2} > A_{m_1}$

$\omega_{m_2} > \omega_{m_1}$

$$m(t) = m_1(t) + m_2(t)$$

$$m(t) = A_{m_1} \cos \omega_{m_1} t + A_{m_2} \cos \omega_{m_2} t$$

$$c(t) = A_c \cos \omega_c t$$

$$X_{AM}(t) = A \cos \omega_c t$$

$$\Rightarrow A = A_c + A_{m_1} \cos \omega_{m_1} t + A_{m_2} \cos \omega_{m_2} t$$

$$X_{AM}(t) = [A_c + A_{m_1} \cos \omega_{m_1} t + A_{m_2} \cos \omega_{m_2} t] \cos \omega_c t$$

$$= A_c [1 + m_1 \cos \omega_{m_1} t + m_2 \cos \omega_{m_2} t) \cos \omega_c t$$

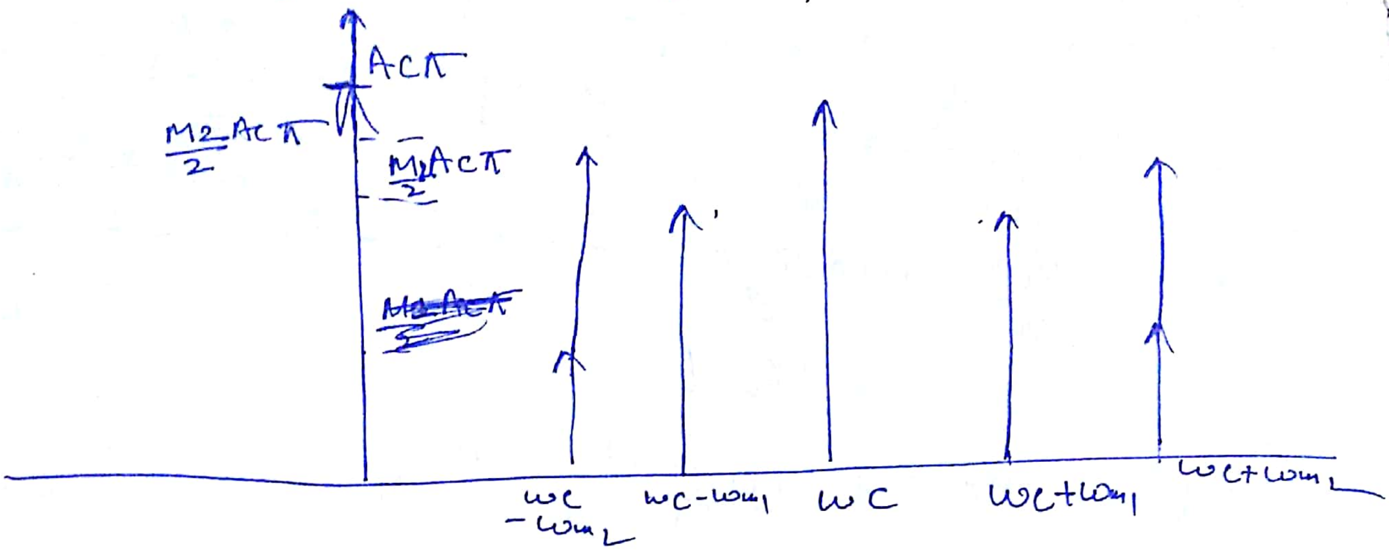
$$m_1 = \frac{A_{m_1}}{A_c} \qquad m_2 = \frac{A_{m_2}}{A_c}$$

$\therefore m_2 > m_1$

$$X_{AM}(t) = A_c \cos \omega_c t + m_1 A_c \cos \omega_c t \cos \omega_{m_1} t + m_2 A_c \cos \omega_c t \cos \omega_{m_2} t$$

$$X_{AM}(t) = A_c \underbrace{\cos \omega_c t}_{\text{carrier}} + \frac{m_1 A_c}{2} \cos (\underbrace{\omega_c + \omega_{m_1}}_{USB1}) t + \frac{m_1 A_c}{2} \cos (\underbrace{\omega_c - \omega_{m_1}}_{LSB1}) t + \frac{m_2 A_c}{2} \cos (\underbrace{\omega_c + \omega_{m_2}}_{USB2}) t + \frac{m_2 A_c}{2} \cos (\underbrace{\omega_c - \omega_{m_2}}_{LSB2}) t$$

Frequency domain Representation



$$BW = \omega_c + \omega_{m_2} - (\omega_c - \omega_{m_2})$$
$$= 2\omega_{m_2}$$

For $\omega_{m_1}, \omega_{m_2}, \omega_{m_3} - - - - -$

$$BW = 2 \cdot Max[\omega_{m_1}, \omega_{m_2}, \omega_{m_3}, - - -]$$

1. When a carrier signal is simultaneously modulated by several sine waves then for each addition of modulating sig a pair of side band gets added in the resultant AM sig. That is why AM is also known as "linear modulation"

2. The BW of resultant modulated sig is given by
$$BW = 2 \cdot Max[\omega_{m_1}, \omega_{m_2}, \omega_{m_3}, - - -]$$

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Power Relation

$$\boxed{P_t = P_c\left(1 + \frac{M_T^2}{2}\right)}$$

$$P_t = \frac{A_c^2}{2} + \frac{M_1^2 A_c^2}{8} + \frac{M_1^2 A_c^2}{8} + \frac{M_2^2 A_c^2}{8} + \frac{M_2^2 A_c^2}{8}$$

$$P_t = \frac{A_c^2}{2} + \frac{M_1^2 A_c^2}{4} + \frac{M_2^2 A_c^2}{4}$$

$$P_c = \frac{A_c^2}{2}$$

$$P_{SB1} = \frac{M_1^2 A_c^2}{4} = \frac{M_1^2}{2}\frac{A_c^2}{2} = \frac{M_1^2}{2} P_c$$

$$P_{SB2} = \frac{M_2^2}{2} P_c$$

$$P_t = P_c\left(1 + \frac{M_1^2 + M_2^2}{2}\right]$$

$$\boxed{P_t = P_c\left(1 + \frac{M_T^2}{2}\right)}$$

$$\boxed{M_T^2 = M_1^2 + M_2^2 + M_3^2 + \cdots}$$

to increa...

Q6 $e(t) = 50(1 + 0.89\cos 5000t + 0.30\sin 9000t)\cos(6\times10^6 t)$

$\omega_c = 6\times10^6$

$\omega_{m1} = 5000$

$\omega_{m2} = 9000$

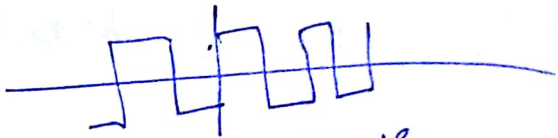
$\omega_c \pm \omega_{m1} = 6000\times10^3 \pm 5\times10^3$

$= 6005\times10^3, 5995\times10^3$

$\omega_c \pm \omega_{m2} = 6009\times10^3, 5991\times10^3$

(8) d

(12) $\frac{1}{100\times10^{-6}}$ 10^4 Hz $=$ 10KHZ



For square wave symm.
odd & half wave symm.
odd harmonics
1,3,5,7 odd exist only

$f_1 = 10$KHZ. 100 ± 10

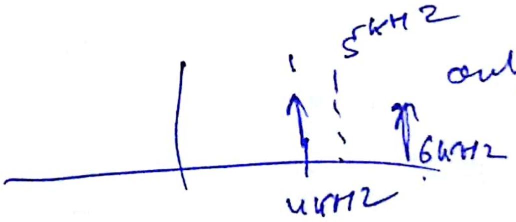
$f_2 = 30$KHZ 100 ± 30

$f_3 = 50$KHZ

(13) $T_s = 100\,\mu sec$

$f_s = 10$KHZ

$x(t) \longrightarrow 4$KHZ



(C)

$Y(\omega) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} X(\omega - k\omega_s)$

$y(t) = 5\times10^{-6}\times10^{4}\times10\cos(2\pi\times10^3 t)$

(C)

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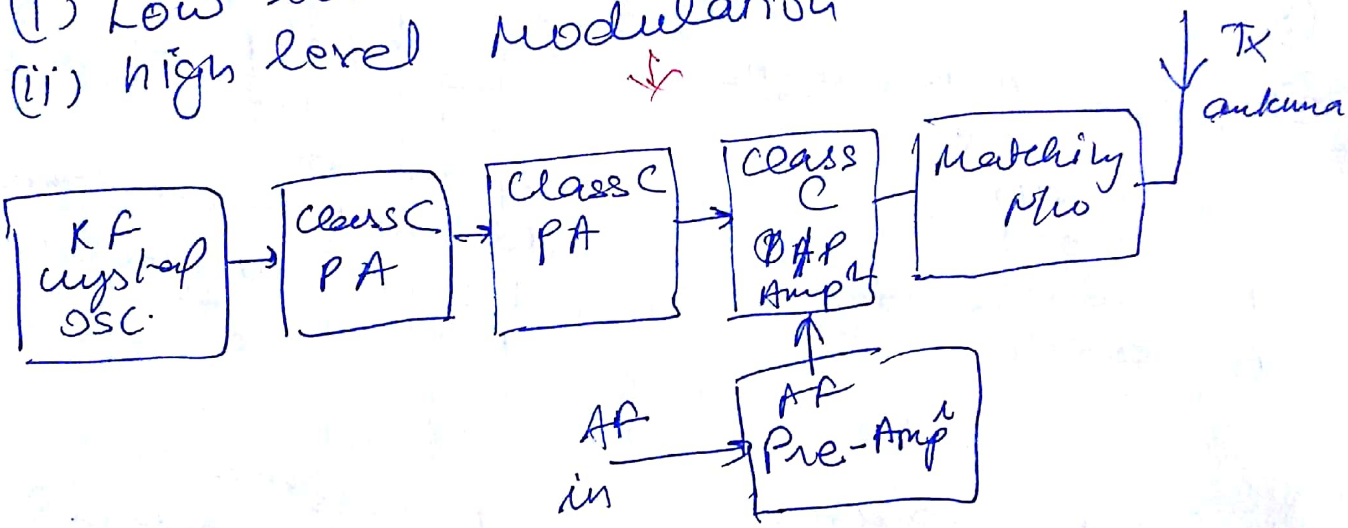
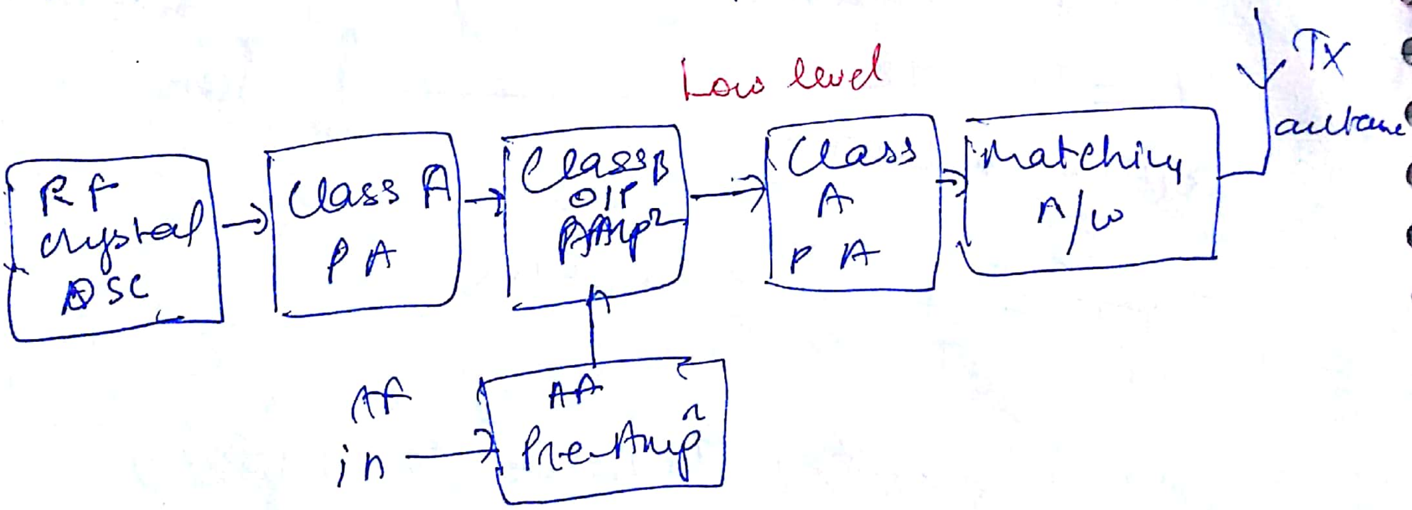
Generation of AM sig

Depending upon the carrier power level at which the modulation takes place there are two methods for generations of AM wave

(i) Low level Modulation
(ii) high level Modulation



Low level



→ Since Modulation is translation of freq. which can only be generated by a non-linear device. That is why class A power Ampr can not be used for generation of modulation

(2) Once a sig. is modulated only a linear PA can be used for Amplification purpose bcoz a non-linear power Ampl will try to distort the information

Comparision b/w low level & high level modulation

SNO	Parameter	Low level modulation	High level modulation
1	Pt. at which mod. take place	modu. takes place in initial stages of Amplification	mod. takes place in final stages of Amplification
2	Power level handing	mod. circuitry has to handle low power	mod. circuitry has to handle high power
3	Complexity	mod. ckty is simple as it has to handle low power	mod. ckty is quite complex
4	Range of Transmission	In this, Range of Transmission is small.	S/g can be transmitted over longer distance
5	Application	This is used only for lab's purposes	This is used in Practical AM system

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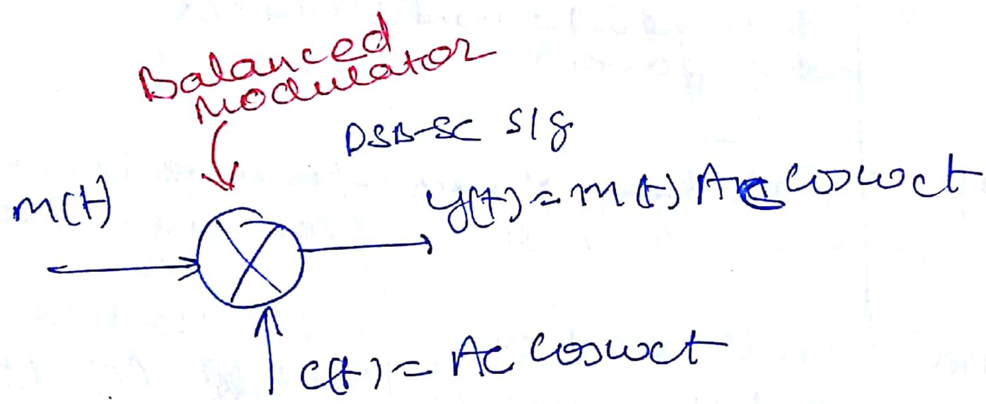
DSB-SC Double side band - suppress carrier

Transmission of full AM sig is not advisable because

① Since carrier is also transmitted which does not contain any information

② For m=1, 2/3rd of total transmitted power appears in the carrier which is a complete wastage

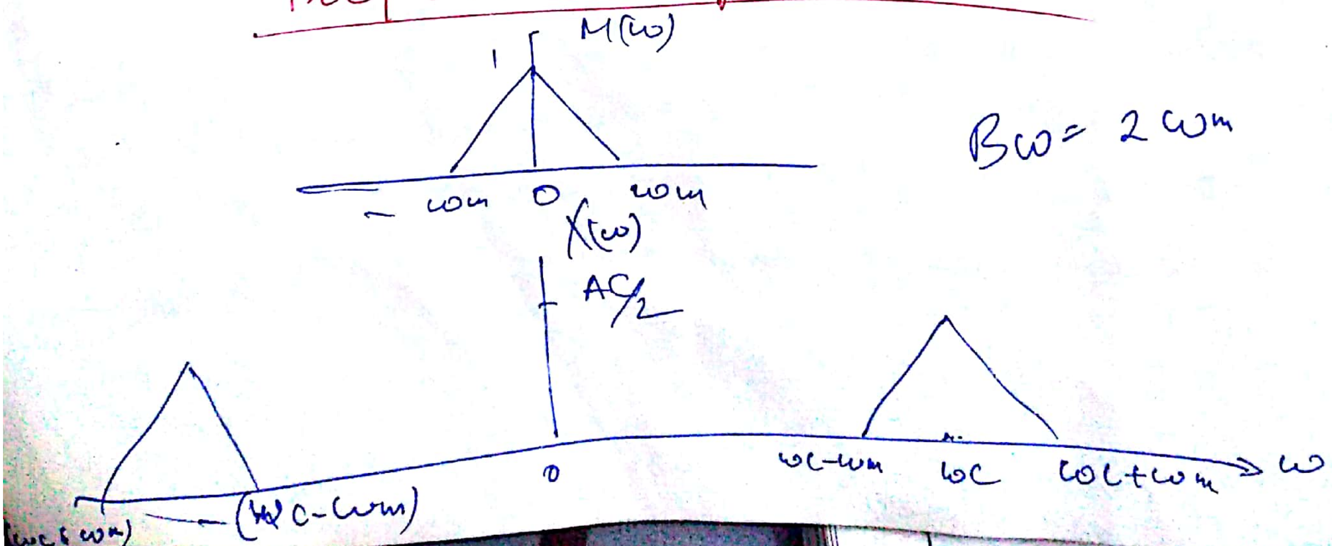
③ So, instead of transmitting full AM sig the carrier is suppressed before transmission and such type of modulation is known as DSB-SC modulation



$$y(t) = m(t)\, A_c \cos\omega_c t$$

$$Y(\omega) = \frac{A_c}{2}\left[M(\omega-\omega_c) + M(\omega+\omega_c)\right]$$

freq domain representation



$BW = 2\omega_m$

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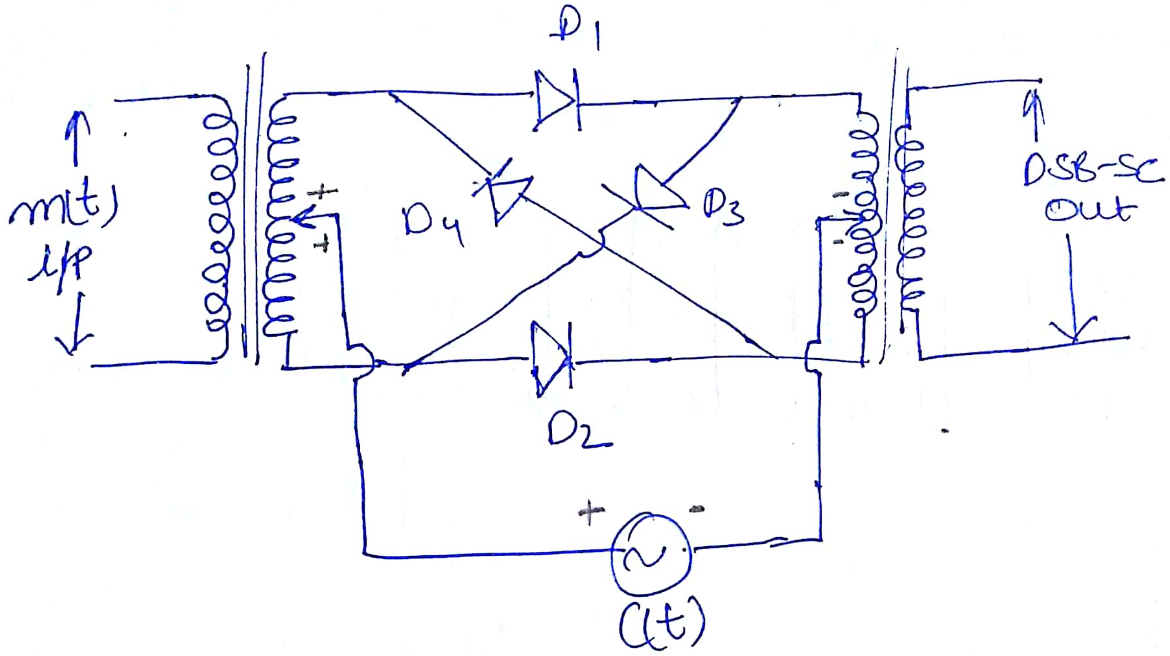
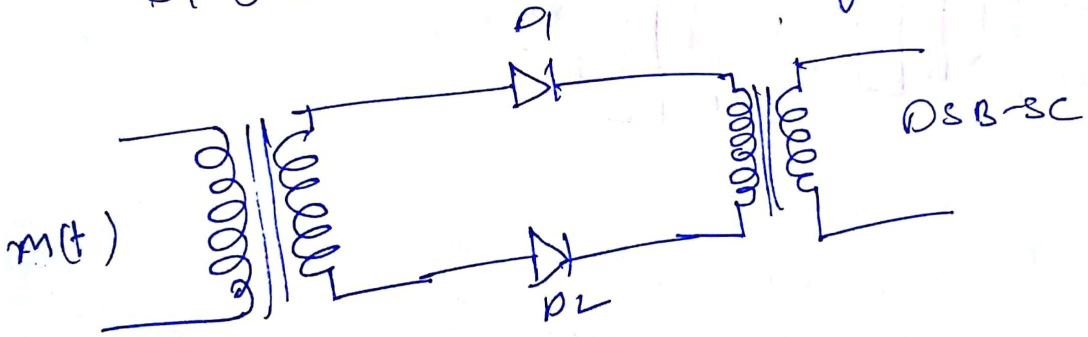
Ring Modulator

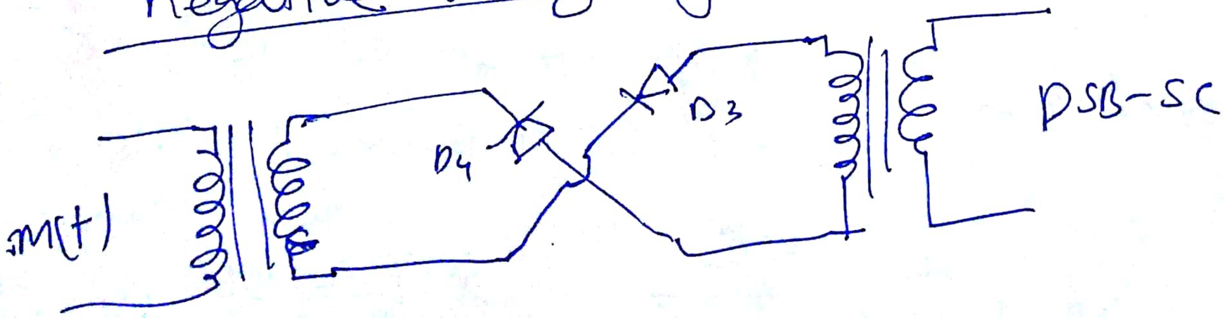


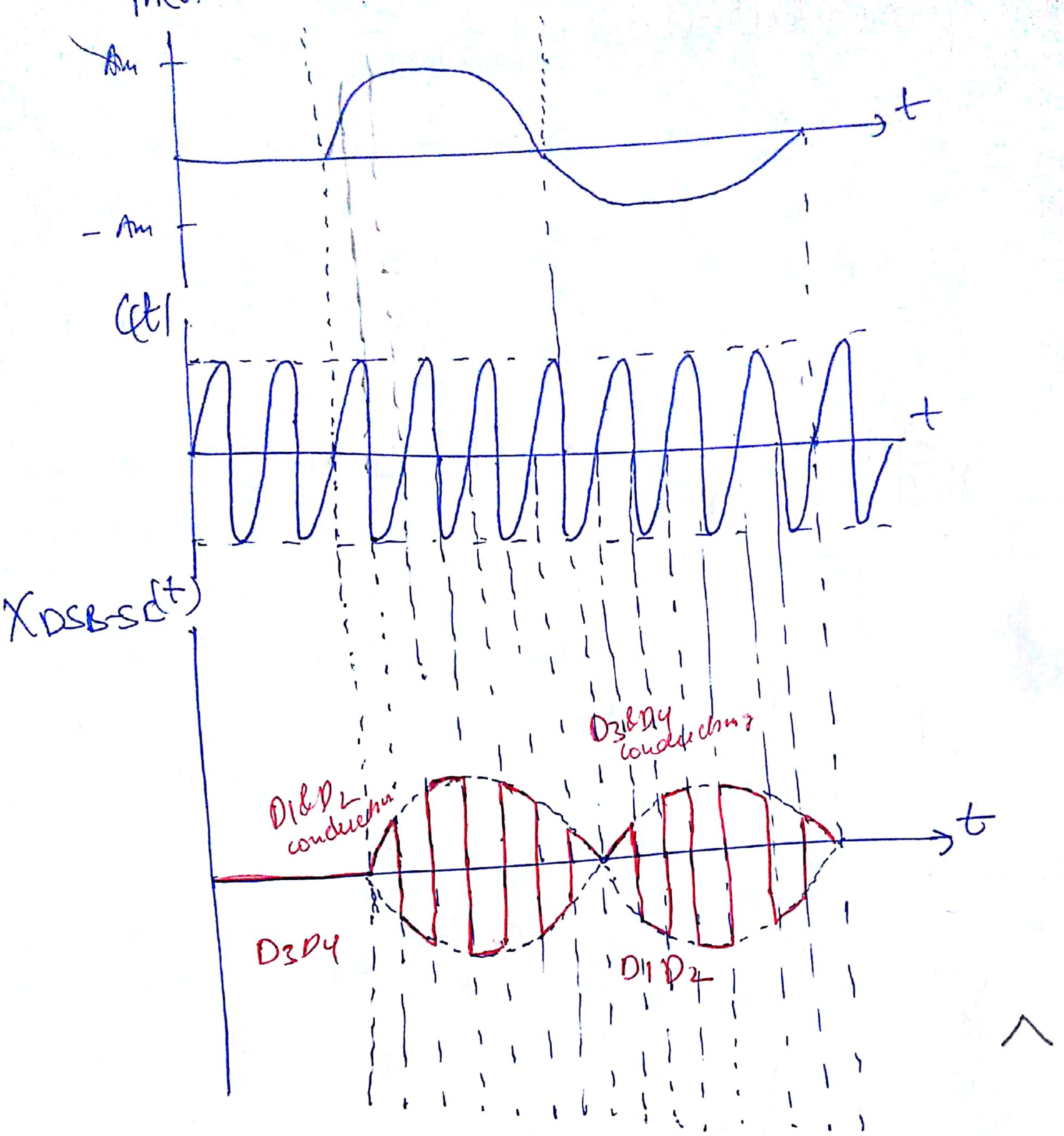
Positive half cycle

D1 & D2 are conducting



Negative half cycle





Power saving in DSB-SC Modulation

$$P_t = P_c + \frac{P m^2}{2}$$

$$\% \text{ Power saving} = \frac{\text{Total Power saved}}{\text{Total Tx. Power}} \times 100\%$$

$$= \frac{P_c}{P_c\left(1 + \frac{m^2}{2}\right)} \times 100\%$$

$$\boxed{\% \text{ Power saving} = \frac{2}{2 + m^2} \times 100\%}$$

For $m=1$; $\rightarrow \frac{2}{2+1} \times 100\%$
$= 66.67\%$

$m=0.5$; $\rightarrow \frac{2}{2.25} \times 100\% = 88.88\%$

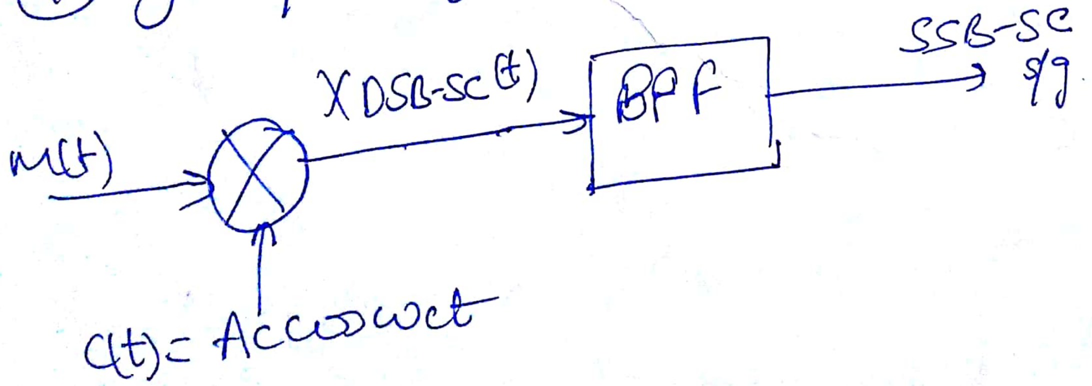
SSB-SC Modulation

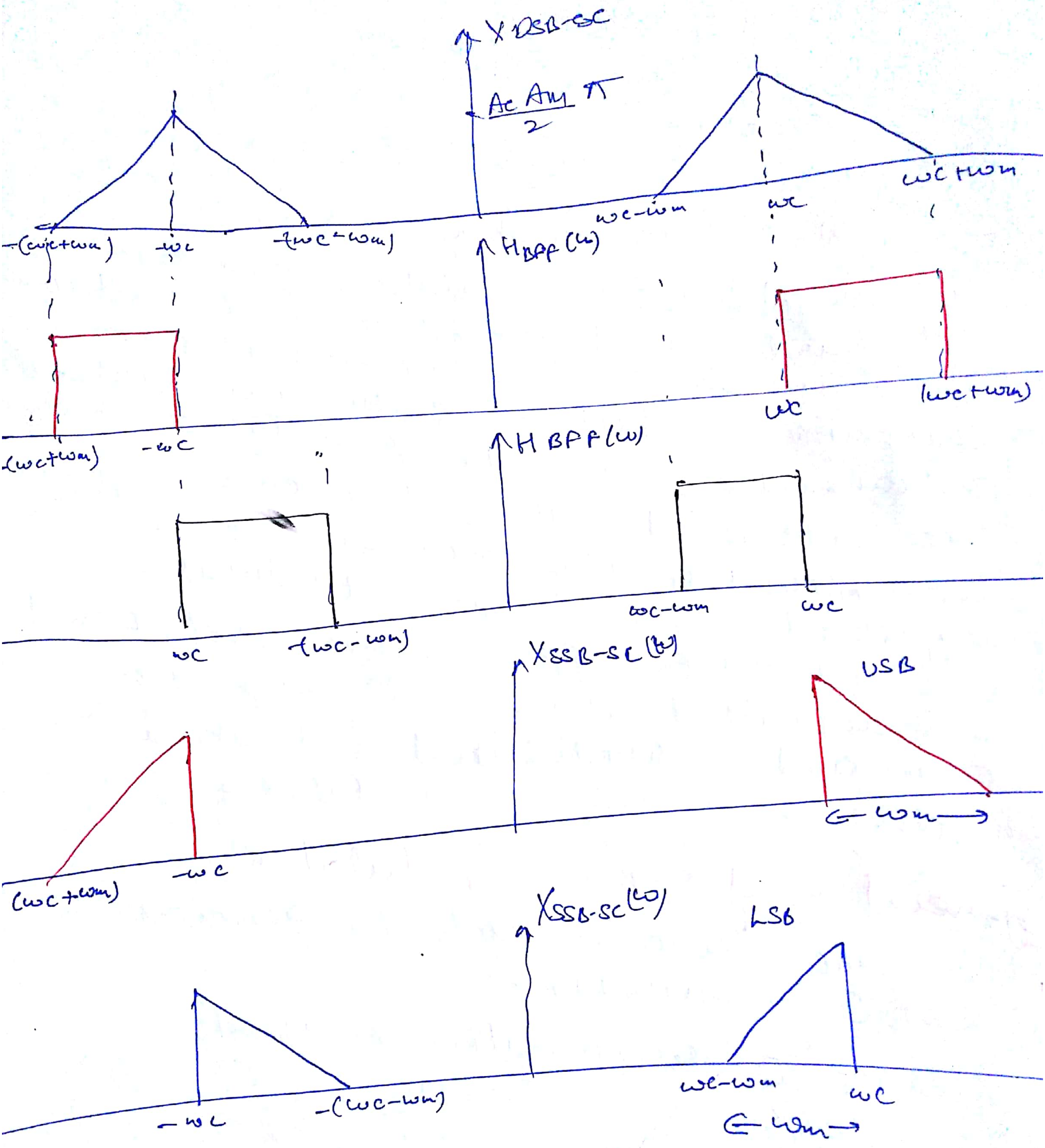
In case of DSB-SC modulation, the two side bands are transmitted which contains the same information. So the DSB-SC transmission is further redundant. Instead of transmitting both the side bands, only one side band either USB or LSB is transmitted while another one is suppressed. Such type of modulation in which either upper or lower side band is transmitted while suppressing the carrier and unwanted side band is known as SSB-SC modulation.

Generation of SSB-SC modulation

There are two methods for generation of SSB-SC modulation.

① frequency discrimination method





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Disadvantage of freq discrimination method

The freq discrimination method can be used only when there is definite gap b/w upper and lower SB.
If gap is very very small or both are melting at carrier freq. then this method can not be used

② Phase discrimination method

Hilbert transform

It is a method of seperating the sig wrt there phase contents. The easiest phase generation is π (180°).
Hilbert transformer is a special type of transfr that seperates the signal with a phase difference of $\pi/2$ b/w them. The hilbert transform of sig $x(t)$ is given by

$$\bar{x}(t) = \int_{-\infty}^{\infty} \frac{x(\tau)}{\pi(t-\tau)} d\tau$$

$$\bar{x}(t) = x(t) * \frac{1}{\pi t}$$

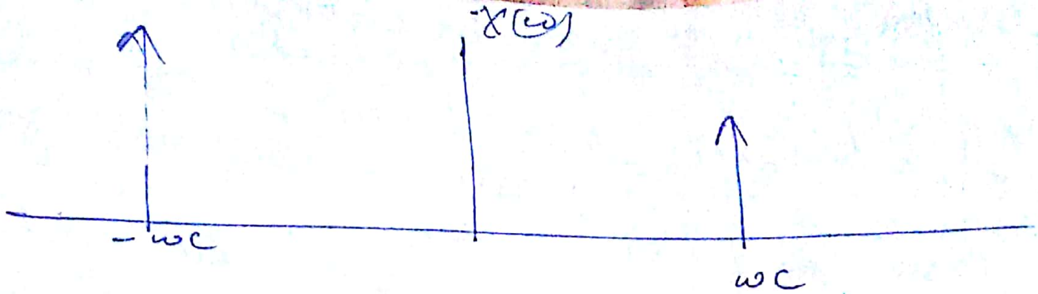
$$\bar{X}(\omega) = X(\omega)(-j\,\mathrm{sgn}(\omega))$$

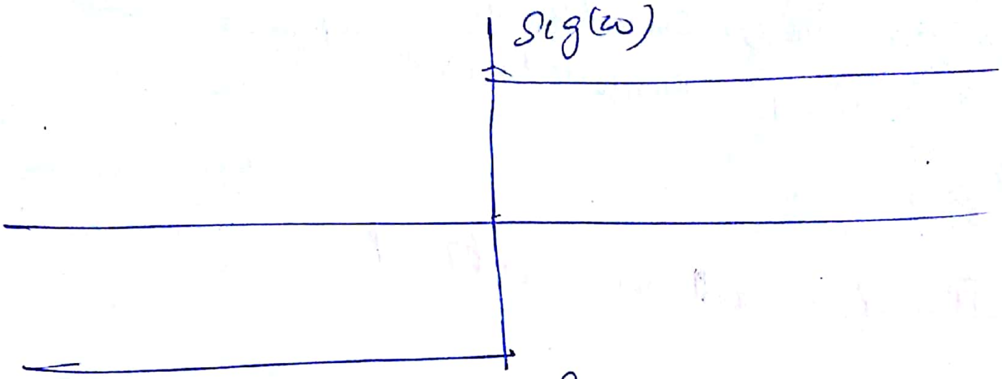
Q $x(t) = \cos\omega_c t \xleftrightarrow{HT} \sin\omega_c t$

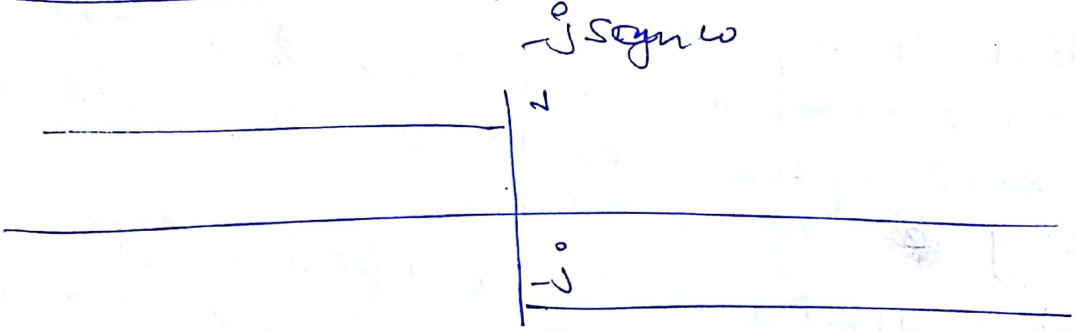
$$X(\omega) = \pi(\delta(\omega-\omega_c) + \delta(\omega+\omega_c))$$

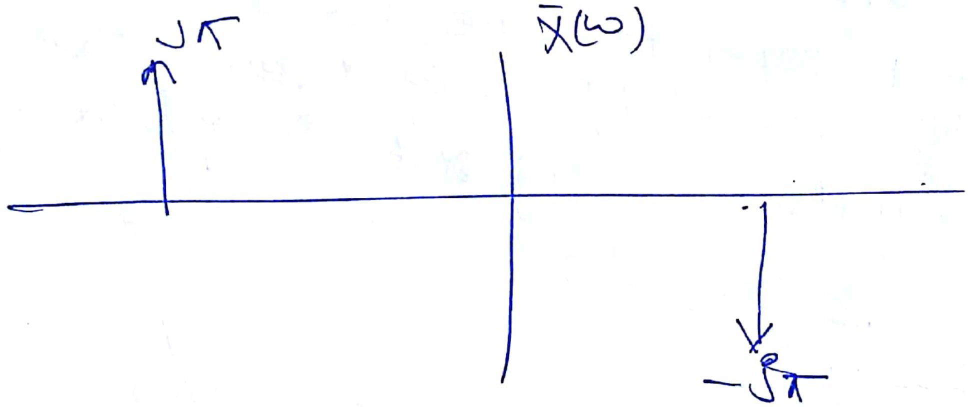
$$\bar{X}(\omega) = \pi(\delta(\omega-\omega_c) + \delta(\omega+\omega_c))(-j\,\mathrm{sgn}(\omega))$$

$$= -j\pi$$









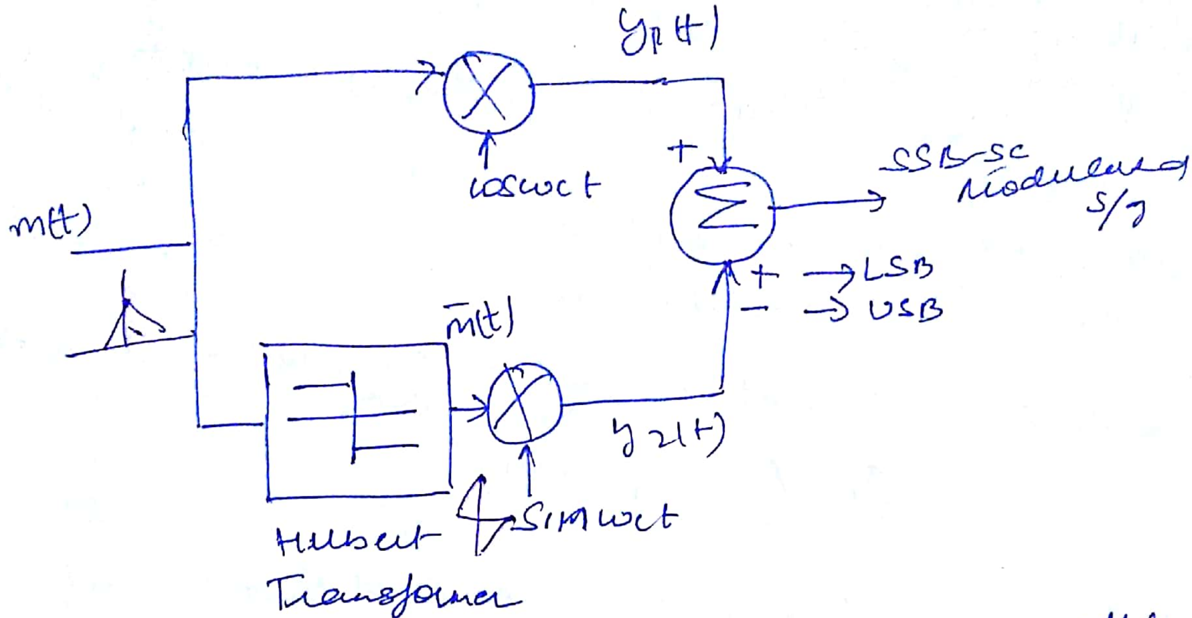
$$j\pi\left(\delta(\omega+\omega_c)-\delta(\omega-\omega_c)\right)$$

$$\hat{X}(\omega) = \frac{\pi}{j}\left(\delta(\omega-\omega_c)-\delta(\omega+\omega_c)\right)$$

$$\hat{x}(t) = \sin \omega_c t$$

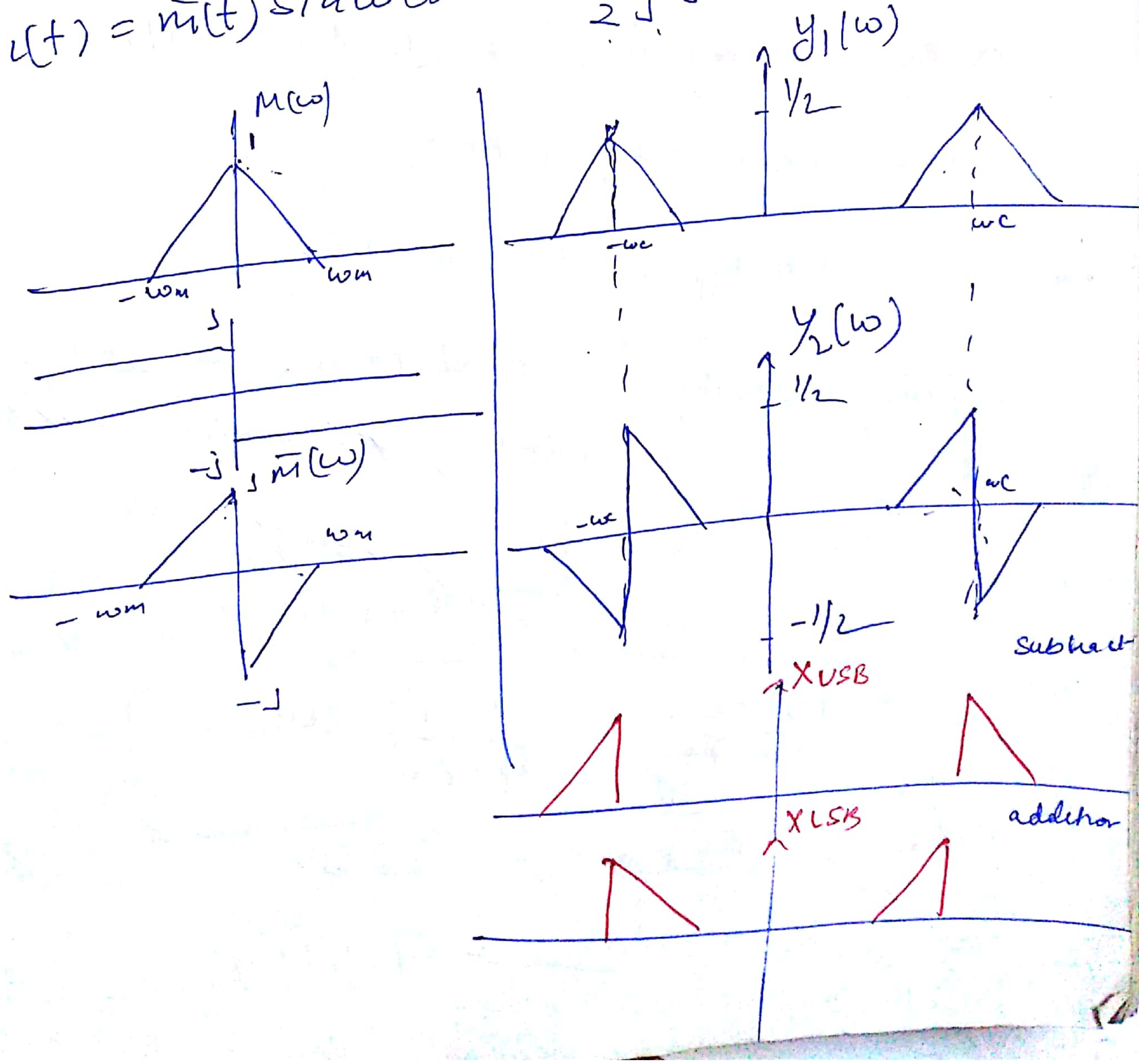
Assignment

$$\sin \omega_c t \xrightarrow{HT} -\cos \omega_c t$$



$$y_1(t) = m(t)\cos\omega_c t \xleftrightarrow{FT} \frac{1}{2}\left(M(\omega-\omega_c) + M(\omega+\omega_c)\right)$$

$$y_2(t) = \bar{m}(t)\sin\omega_c t \xleftrightarrow{FT} \frac{1}{2j}\left(\bar{M}(\omega-\omega_c) - \bar{M}(\omega+\omega_c)\right)$$



$$X_{SSB-SC} = m(t)\cos\omega_c t \pm \hat{m}(t)\sin\omega_c t$$

$$m(t) = A_m\cos\omega_m t \xrightarrow{HT} A_m\sin\omega_m t$$

$$X_{SSB-SC}(t) = A_m\cos\omega_m t\cos\omega_c t \pm A_m\sin\omega_m t\sin\omega_c t$$

$$X_{SSB-SC}(t) = \cos\omega_c t\cos\omega_m t \pm_{USB}^{LSB} \sin\omega_c t\sin\omega_m t$$

$$= \cos(\omega_c \mp \omega_m)t$$

$\cos(\omega_c - \omega_m)t \rightarrow$ lower side band

$\cos(\omega_c + \omega_m)t \rightarrow$ upper side band

Power saving in SSB-SC Modulation

$$P_t = P_c + P_c\frac{m^2}{4} + P_c\frac{m^2}{4}$$

Total power saving $= P_c\left(1 + \frac{m^2}{4}\right)$

$$\% \text{ power saving} = \frac{\text{Total power saved}}{\text{total Tx power}} \times 100\%$$

$$= \frac{P_c\left(1 + \frac{m^2}{4}\right)}{P_c\left(1 + \frac{m^2}{2}\right)}$$

$$\boxed{\% \text{ power saving} = \frac{4 + m^2}{4 + 2m^2} \times 100\%}$$

$m = 1 \Rightarrow \frac{5}{6} \times 100\% = 83.33\%$

$m = 0.5 \Rightarrow \frac{4.25}{4.5} = 94.4\%$

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Advantage of SSB-SC mod. over DSB-SC & Full-AM

① The spectrum space occupied by the SSB-SC s/g is only f_m which is only half that of AM & DSB-SC s/g So. SSB-SC modulation requires only half the transmission BW as compared to AM & DSB-SC s/g. that is why "SSB-SC modulation is Standard for FDM"

total available spectrum

100KHz

for DSB-SC & full AM

$f_m = 5KHz$

$BW = 10KHz$

total no. of channel

$= \frac{100}{10} = 10$

for SSB-SC

$f_m = 5KHz$

$BW = 5KHz$

Total no of channel $= 20$

② due to suppression of carrier and one side band, more power is saved and saved power can be used to produce a stronger s/g that will carry farther and it will be reliably received at greater distances

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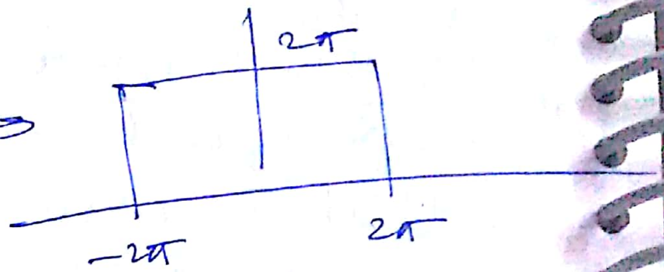
$$\left(\frac{2\sin 2\pi t}{t}\cos 200\pi t + \frac{\sin 199\pi t}{t}\right)\cos 200\pi t$$

$$\frac{2\sin 2\pi t}{t}\cos^2 200\pi t + \frac{\sin 199\pi t}{t}\cos 200\pi t$$

$$m(t) = \pi \frac{2\sin 2\pi t}{\pi t} \xleftrightarrow{FT}$$



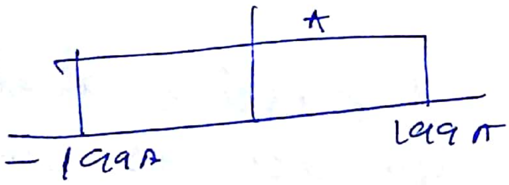
$$m(t)\left(1 + \frac{\cos 400\pi t}{2}\right)$$

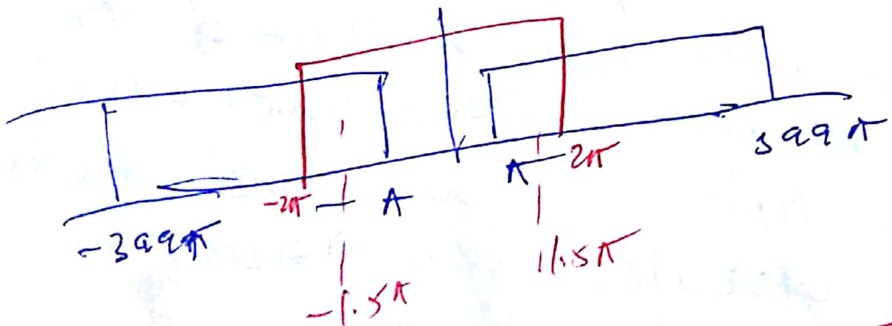
$$\frac{m t}{2} + \frac{m t \cos 400\pi t}{2}$$ not pass

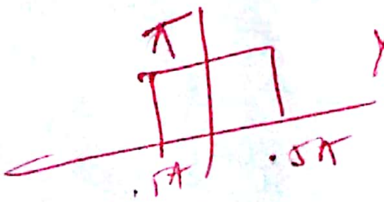
$$\frac{\sin 199\pi t}{t}\cos 200\pi t + \frac{1}{2}\left(M(\omega - 200\pi) + M(\omega + 200\pi)\right)$$

πt

$\frac{1}{2}\sin$







$\times \cos 1.5\pi$

$$\frac{\pi \sin 0.5\pi}{\pi t}$$

Detection of AM waves

→ The recovery of Baseband signal from the modulated signal is known as demodulation or detection of waves.

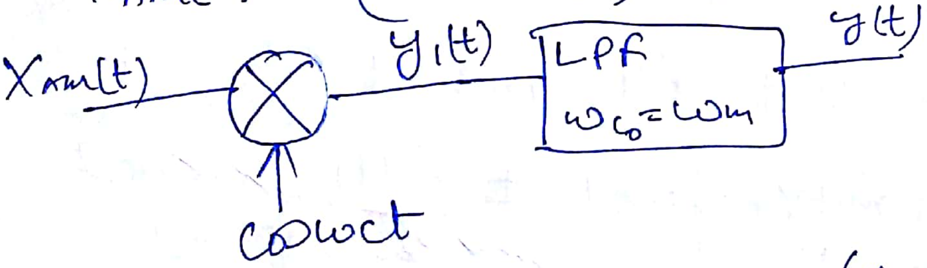
→ there are two methods for detection of AM sig.

(I) synchronous detection

in this method of detection, a local carrier is generated at the receiving end whose phase is exactly synchronized with the Txed carrier sig phase. This locally generated carrier is multiplied by the Rxed s/g and the product is passed through a LPF to recover the original baseband s/g.

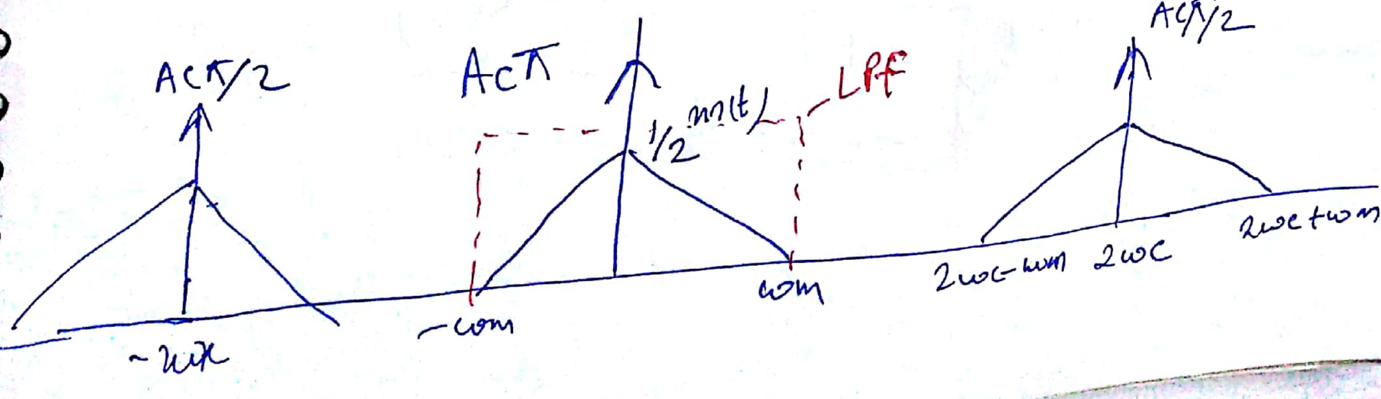
1. detection of full AM wave

$$X_{AM}(t) = (A_c + m(t))\cos\omega_c t$$



$$y_1(t) = (A_c + m(t))\cos^2\omega_c t = (A_c + m(t))\frac{1+\cos 2\omega_c t}{2}$$

$$= \frac{A_c}{2} + \frac{m(t)}{2} + \frac{A_c\cos 2\omega_c t}{2} + \frac{m(t)}{2}\cos 2\omega_c t$$



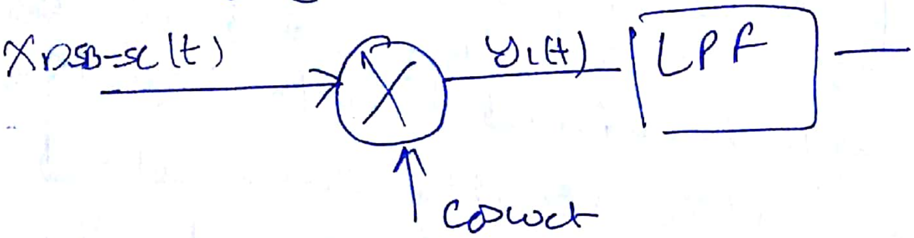
$y(t) = \frac{A_c}{2} + \frac{1}{2}m(t)$

$\frac{A_c}{2}$ can be removed by passing this sig. to capacitor

$$\boxed{y(t) = \frac{1}{2}m(t)}$$

2. Detection of DSB-SC wave

$X_{DSB-SC}(t) = A_c m(t)\cos\omega_c t$



$y_1(t) = A_c m(t)\cos^2\omega_c t = A_c m t \frac{(1+\cos 2\omega_c t)}{2}$

$= \frac{A_c m(t)}{2} + \frac{A_c m(t)\cos 2\omega_c t}{2}$ — blocked —

$y(t) = \frac{A_c}{2} m(t)$

3. Detection of SSB-SC wave

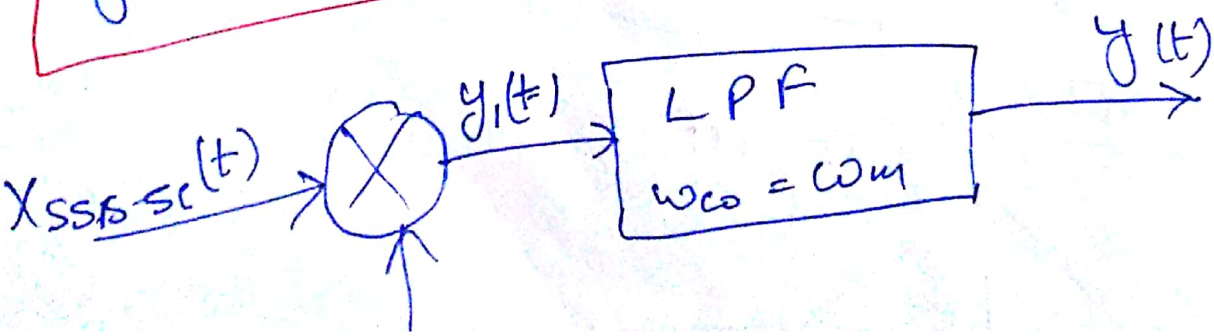
$X_{SSB-SC}(t) = m(t)\cos(\omega_c t) \pm \hat{m}(t)\sin\omega_c t$

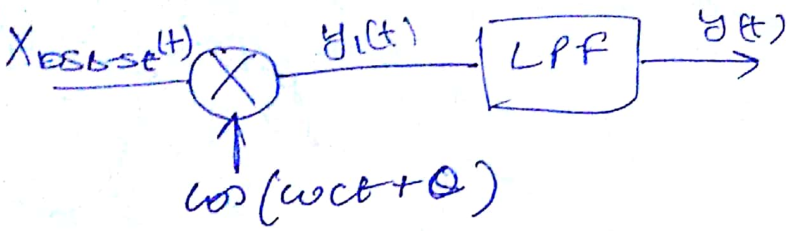
$y_1(t) = m(t)\cos^2\omega_c t \pm \hat{m}(t)\sin\omega_c t\cos\omega_c t$

$y_1(t) = \frac{1}{2}m(t) + \frac{1}{2}m(t)\cos 2\omega_c t \pm \frac{\hat{m}(t)}{2}\sin 2\omega_c x$

(— blocked —) (— blocked —)

$$\boxed{y(t) = \frac{1}{2}m(t)}$$





$y_1(t) = A_c m(t) \cos\omega_c t \cos(\omega_c t+\theta)$

$$y_1(t) = A_c m(t) \cos\omega_c t \cos(\omega_c t+\theta)$$

$$= \frac{A_c m(t)}{2}\left[\cos(2\omega_c t+\theta) + \cos\theta\right]$$

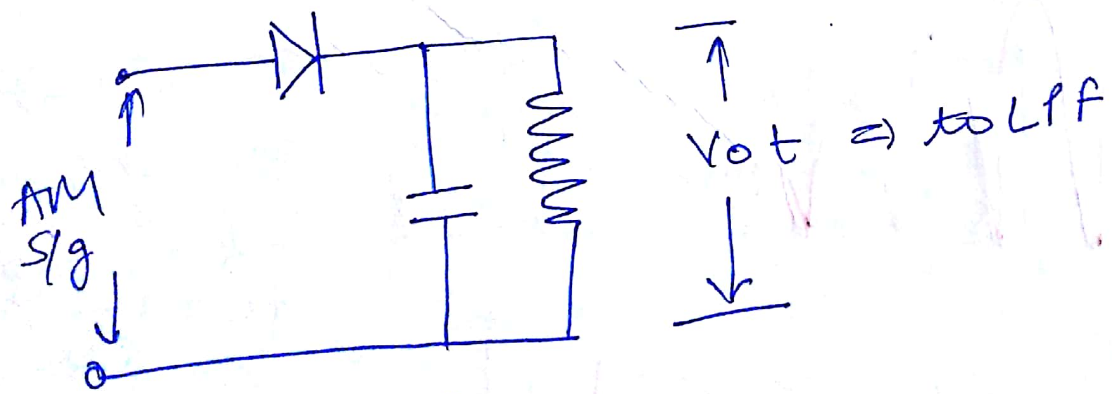
$$\boxed{y_2(t) = \frac{A_c}{2} m(t) \cos\theta}$$

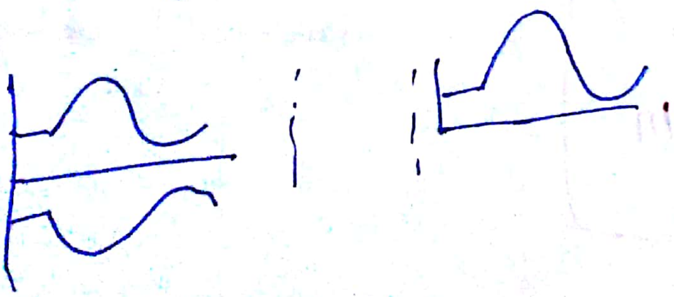
at $\theta = 90° \Rightarrow y(t) = 0 \Rightarrow$ Quadrature Null effect

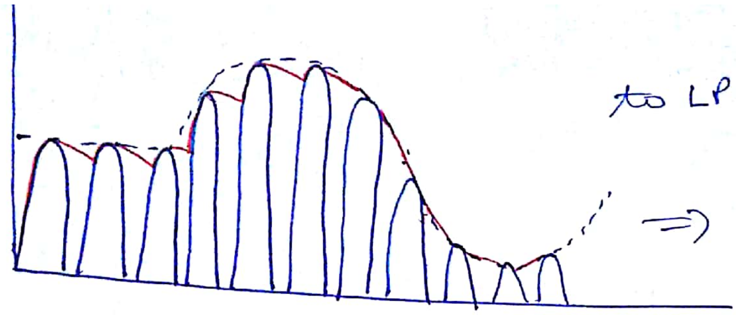
→ When the txed ~~carrier~~ phase and locally generated carrier phase are not synchronised, we always obtain a distorted sig at the o/p and when $\theta = 90°$ $y(t) = 0$, This effect is known as Quadrature Null effect

II Asynchronous Detection Method

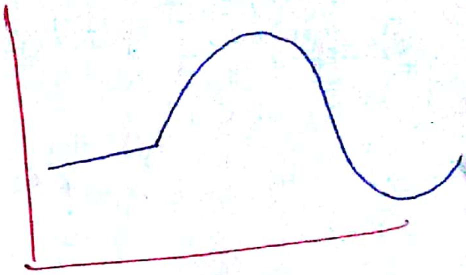
This method is also known as Envelope detector or diode detector.





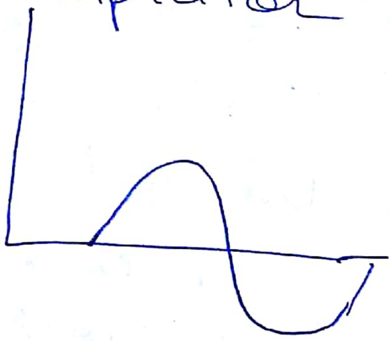


to LPF ⇒



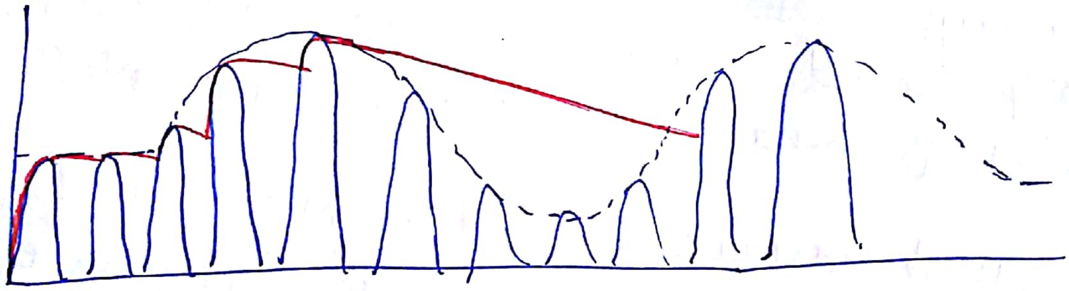
to capacitor

⇒

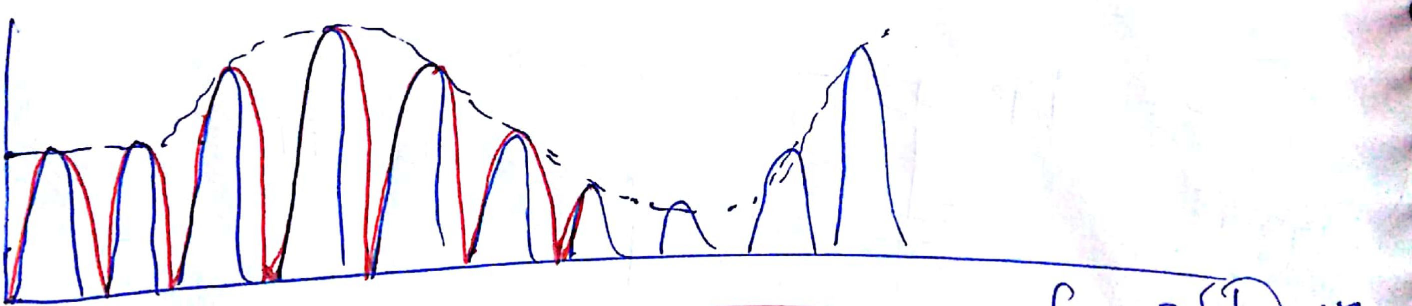


ROC → low as possible

<u>Case 1</u> <u>RC is very high</u>



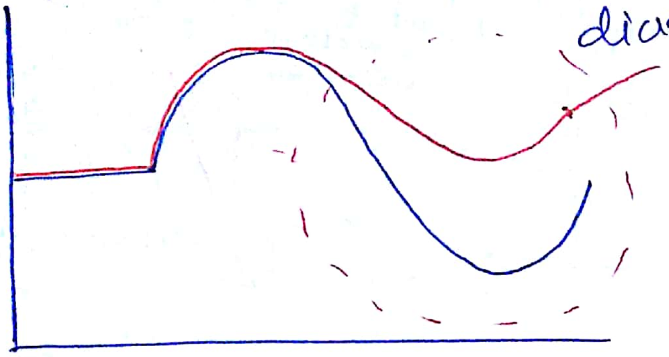
<u>Case 2</u> <u>RC is very low</u>



$f_c \rightarrow \frac{1}{f_c}$ — very low

$f_m \rightarrow \frac{1}{f_m} \rightarrow$ high

$$\boxed{\frac{1}{f_c} << RC << \frac{1}{f_m}}$$



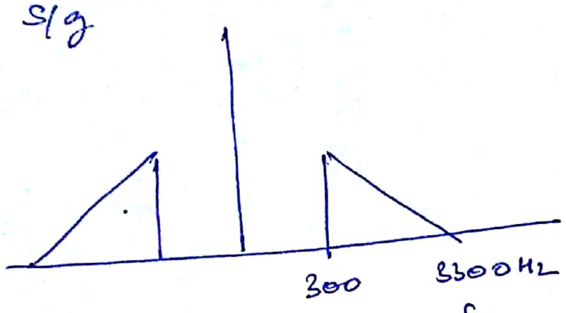
Condition to avoid diagonal clipping

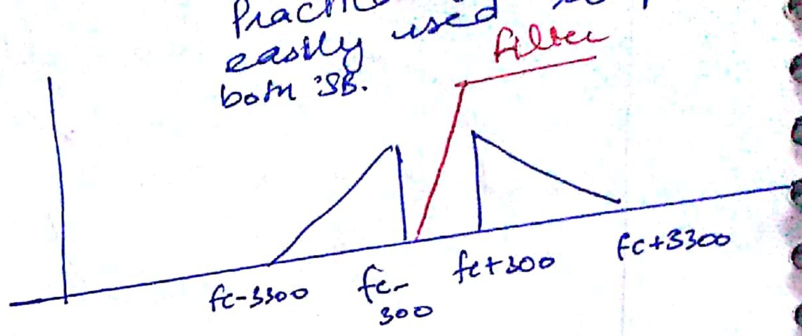
when $\underline{RC}$ is $\underline{\text{high}}$

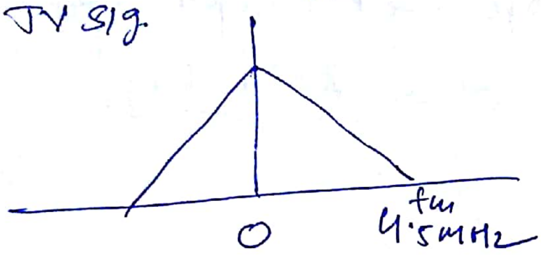
$$\boxed{RC \leq \frac{1}{\omega_m} \frac{\sqrt{1-m^2}}{m}}$$

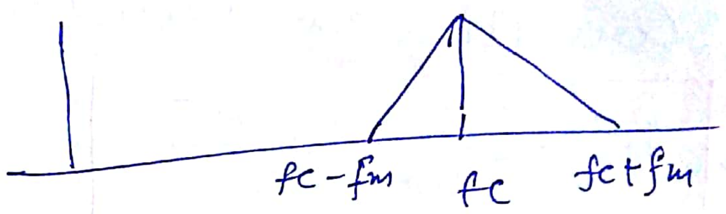
Vestigial Side Band Modulation

SSB modulation is well suited for transmission of voice because of freq. gap that exist in the spectrum of voice sigs b/w 0 and a few hundred HZs. when a baseband sig contains significant component at extremely low freq's as in case of TV picture sig's, the lower & upper SB meet at carrier freq. this means that use of SSB modulation is inappropriate for trsmsn of such baseband sig due to difficulty of isolating one side band

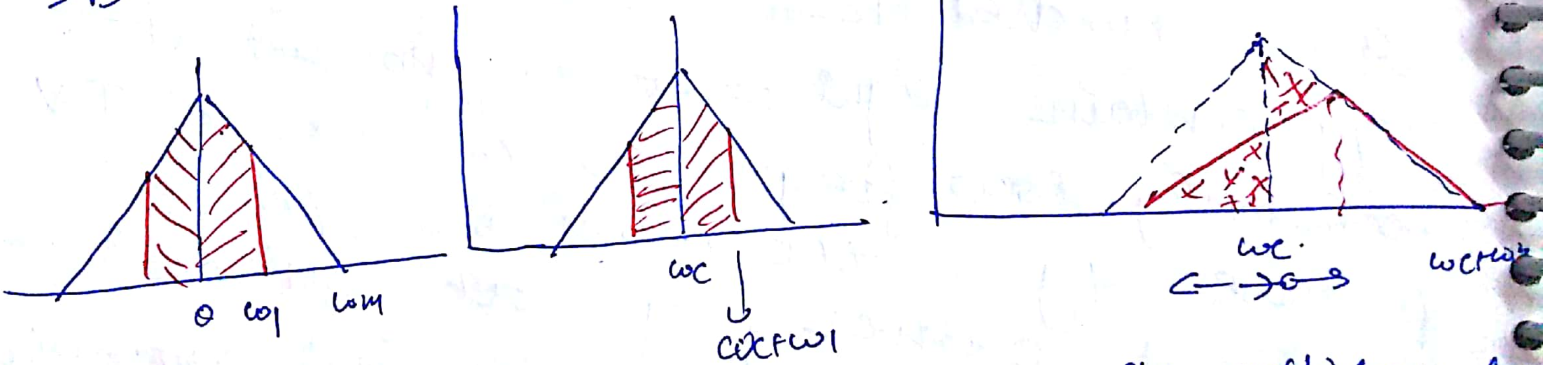


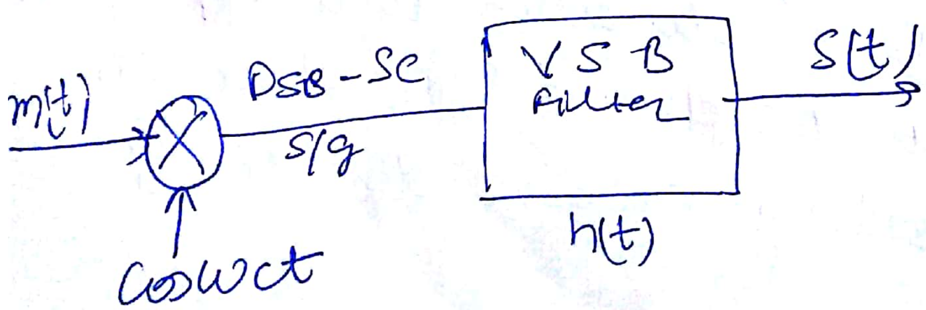






In VSB Modulation, one side band is passed almost completely where as just a trace or vestige of unwanted SB is retained. Specifically the vestige part of unwanted SB compensates for the part removed from the desired SB.

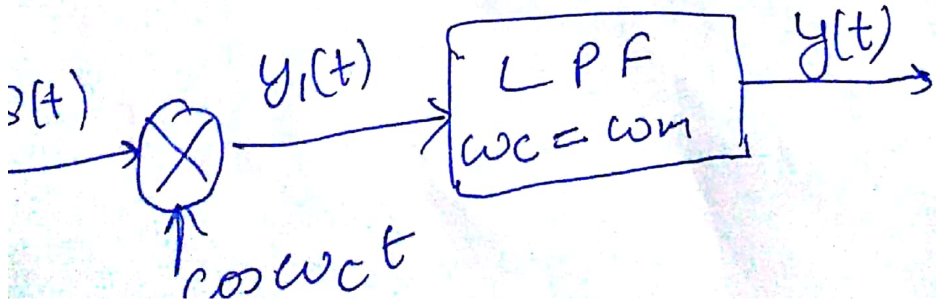




$x_{DSB\text{-}SC}(t) = m(t)\cos\omega_c t$

$s(t) = m(t)\cos\omega_c t * h(t)$

$S(\omega) = \frac{1}{2}\left(M(\omega-\omega_c)+M(\omega+\omega_c)\right) \times H(\omega)$



$y_1(t) = s(t)\cdot\cos\omega_c t$

$Y_1(\omega) = \frac{1}{2}\left[S(\omega-\omega_c) + S(\omega+\omega_c)\right]$

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$$S(\omega) = \frac{1}{2}\left(M(\omega-\omega_c) + M(\omega+\omega_c)\right] \cdot H(\omega)$$

$$Y_1(\omega) = \frac{1}{2}\left[S(\omega-\omega_c) + S(\omega+\omega_c)\right]$$

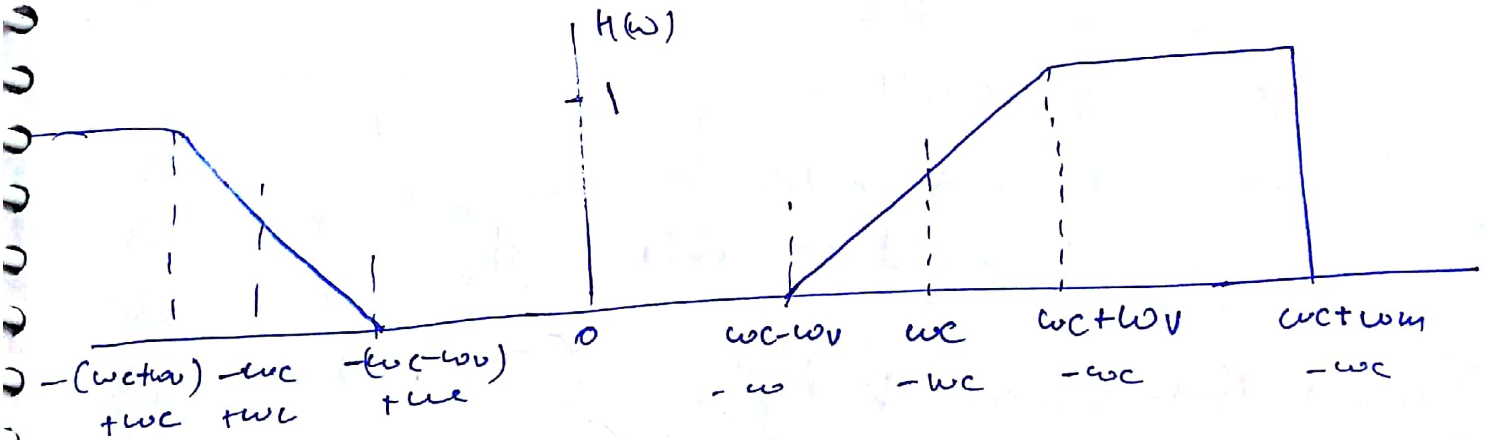
$$Y_1(\omega) = \frac{1}{4}\Big(\underbrace{M(\omega-\omega_c-\omega_c)}_{\text{blocked}} + M(\omega+\cancel{\omega_c}-\cancel{\omega_c})\Big] \cdot H(\omega-\omega_c)$$

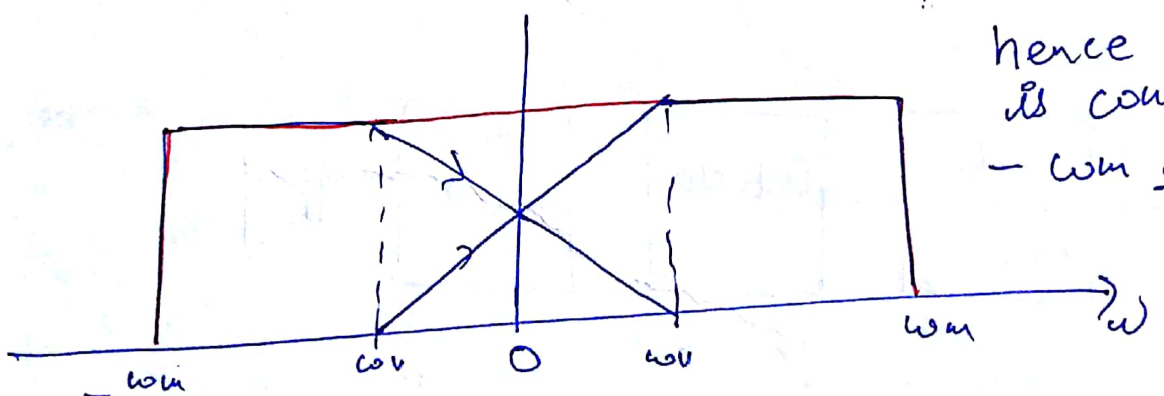
$$+\frac{1}{4}\Big(\underbrace{M(\omega+\omega_c+\omega_c)}_{\text{blocked}} + M(\omega-\cancel{\omega_c}+\cancel{\omega_c})\Big] \cdot H(\omega+\omega_c)$$

$$Y(\omega) = \frac{1}{4} M(\omega)\left[H(\omega-\omega_c) + H(\omega+\omega_c)\right]$$

for $-\omega_m \le \omega \le \omega_m$

$$\boxed{\left[H(\omega-\omega_c) + H(\omega+\omega_c)\right] = \text{constant}\ ;\ -\omega_m \le \omega \le \omega_m}$$





hence its spectrum is constant from $-\omega_m \le \omega \le \omega_m$

in TV transmission, the picture sig is transmitted using VSB modulation. & sound sig is transmitted using FM. and total transmission BW is 6 MHZ

$$0.098m^2 = \frac{m^2}{1-m^2}$$

$$1.098m^2 = 1.$$

$$m = 0.95$$

ANGLE Modulation

In angle Modulation the phase angle of high freq. carrier s/g is varied in accordance with instantaneous value of modulating s/g keeping amplitude constant.

$$C(t) = A_c \cos \underbrace{\omega_c t}_{\text{angle}}$$

$$\theta_c(t) = \omega_c t$$

$$\omega = \frac{d\theta}{dt}$$

$$\theta = \int_0^t \omega \, dt$$

Frequency Modulation (FM)

in FM, the freq of high freq carrier s/g is varied in accordance with instantaneous value of Modulating s/g keeping Amplitude constant, phase is varied indirectly.

Phase Modulation (PM)

in PM, the phase angle of high freq carrier s/g is varied directly in accordance with instantaneous value of modulati s/g keeping Amplitude constant.
freq is varied indirectly

$m(t)$ = Arbitrary modulating sig
max frq = ω_m

$$c(t) = A_c \cos \omega_c t$$

$$c(t) = A_c \cos \theta_c(t)$$

$$\theta_c(t) = \omega t$$

FM

$$X_{FM}(t) = A_c \cos \theta_i(t)$$

$$\omega_i = \omega_c + K_f\, m(t)$$

$$\theta_i(t) = \int_0^t \omega_i\, dt$$

$$\theta_i(t) = \int_0^t \left[\omega_c + K_f m(t)\right] dt$$

$$\theta_i(t) = \omega_c t + K_f \int_0^t m(t)\, dt$$

$$X_{FM}(t) = A_c \cos\left[\omega_c t + K_f \int m t\, dt\right]$$

PM

$$X_{PM}(t) = A_c \cos \theta_i(t)$$

$$\theta_i(t) = \theta_c(t) + K_p m(t)$$

$$X_{PM}(t) = A_c \cos\left(\theta_c(t) + K_p m(t)\right)$$

$$= A_c \cos\left[\omega_c t + K_p m(t)\right]$$

$$X_{FM}(t) = A_c \cos\left[\omega_c t + K_f \int_0^t m(t)\, dt\right]$$

$$X_{PM}(t) = A_c \cos\left[\omega_c t + K_p\, m(t)\right]$$

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Relation b/w FM & PM signal

$$X_{FM}(t) = A_c \cos\left[\omega_c t + K_f \int_0^t m(t)\,dt\right]$$

$$X_{PM}(t) = A_c \cos\left[\omega_c t + K_p m(t)\right]$$

$m(t) \rightarrow \boxed{\frac{d}{dt}} \rightarrow \boxed{FM} \rightarrow PM$

$$X_{PM}(t) = A_c \cos\left[\omega_c t + K_p \int_0^t \frac{dm}{dt}\,dt\right]$$

$$= A_c \cos(\omega_c t + K_p m t)$$

$m(t) \rightarrow \boxed{\int_0^t dt} \rightarrow \boxed{PM} \rightarrow FM$

$$= A_c \cos\left(\omega_c t + K_f \int_0^t m(t)\,dt\right)$$

this method of generation of FM using PM is used, worldwide.

Sinusoidal FM

$m(t) = A_m \cos \omega_m t$

$c(t) = A_c \cos \omega_c t$

$\theta_c(t) = \omega_c t$

$X_{FM}(t) = A_c \cos \theta_i(t)$

$\omega_i = \omega_c + K_f A_m \cos \omega_m t$

$\boxed{\text{peak freq deviation } \Delta\omega = K_f A_m}$

$\omega_i = \omega_c + \Delta\omega \cos \omega_m t$

$\theta_i(t) = \int^t \omega_i\,dt$

$\theta_i(t) = \int^t (\omega_c + \Delta\omega \cos \omega_m t)\,dt$

PM

$m(t) = A_m \cos \omega_m t$

$\theta_c(t) = \omega_c t$

$X_{PM}(t) = A_c \cos \theta_i t$

$\theta_i(t) = \theta_c(t) + K_p A_m \cos \omega_m t$

$\boxed{\text{Peak ~~freq~~ deviation } (\beta_{PM}) = K_p A_m}$

$\theta_i(t) = \omega_c t + \beta_{PM} \cos \omega_m t$

$X_{PM}(t) = A_c \cos(\omega_c t + \beta_{PM} \cos \omega_m t)$

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$$\boxed{\text{Modulation index } (\beta_{FM}) = \frac{\Delta\omega}{\omega_m} = \frac{\Delta f}{f_m}}$$

$$\theta_i(t) = \omega_c t + \beta_{FM} \sin \omega_m t$$

$$\boxed{X_{FM}(t) = A_c \cos\left(\omega_c t + \beta_{FM} \sin \omega_m t\right)}$$

$$\boxed{X_{PM}(t) = A_c \cos\left(\omega_c t + \beta_{PM} \cos \omega_m t\right)}$$

Difference b/w FM & PM S/g

$$\theta_i(t)\Big|_{FM} = \omega_c t + \frac{\Delta\omega}{\omega_m} \sin \omega_m t$$

$$\omega_i = \frac{d\,\theta_i(t)}{dt} = \omega_c + \Delta\omega \cos\omega_m t$$

$$\boxed{\Delta\omega\Big|_{FM} = K_f A_m}$$

$$\theta_i(t)\Big|_{PM} = \omega_c t + K_p A_m \cos \omega_m t$$

$$\omega_i\Big|_{PM} = \omega_c - K_p A_m \omega_m \sin \omega_m t$$

$$\boxed{\Delta\omega\Big|_{PM} = K_p A_m \omega_m}$$

$$\Delta\omega = K_p A_m \omega_m$$

$$\boxed{\frac{\Delta\omega}{\omega_m} = \beta_{PM} = K_p A_m}$$

(10) $s(t) = \cos[2\pi(2\times10^6 t + 30\sin 150t + 40\cos 150t)]$

$\theta_i = 2\pi(2\times10^6 t + 30\sin 150t + 40\cos 150t)$

$= 2\pi(2\times10^6 t + \sqrt{30^2+40^2}\cos(150t+\theta))$

$= 2\pi(2\times10^6 t + 50\cos(150t+\theta))$

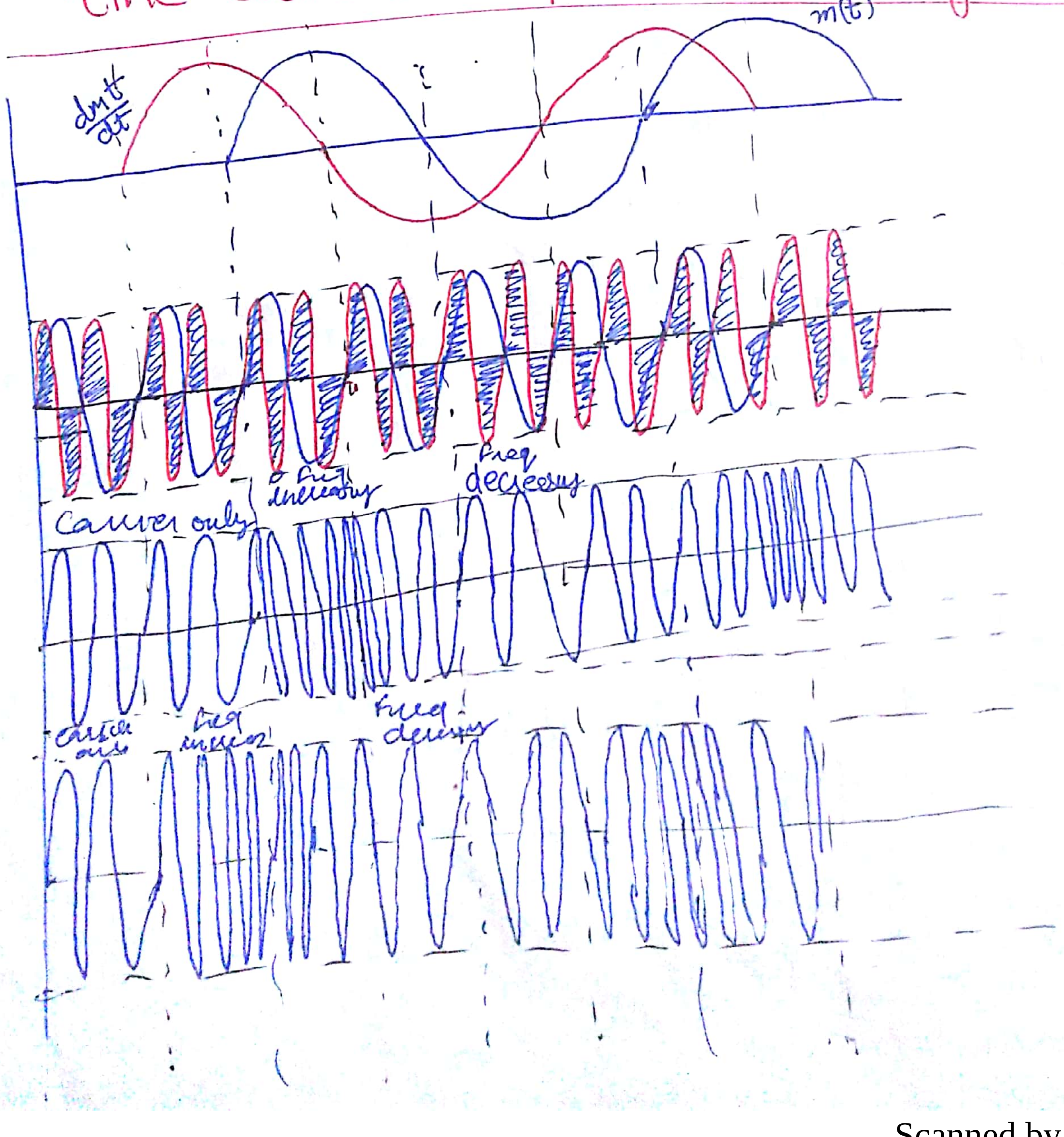
Peak phase deviation = 100π

$\omega_i = 4\pi\times10^6 - 2\pi\times50\times150\sin(150t+\theta))$

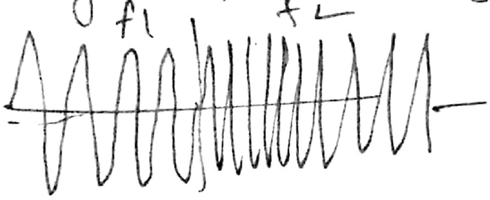
$\Delta f = \frac{\Delta\omega}{2\pi} = \frac{2\pi\times50\times150}{2\pi} = 7500$

$= 7.5\ KHz$

time domain Representation of FM & PM

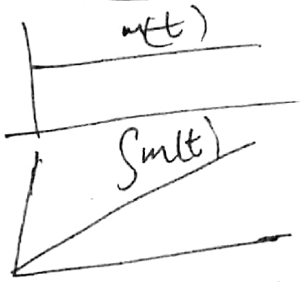


if $m(t)$ is step sig

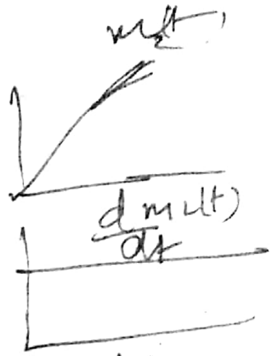


$\omega_i = \omega_c + k_f m(t)$

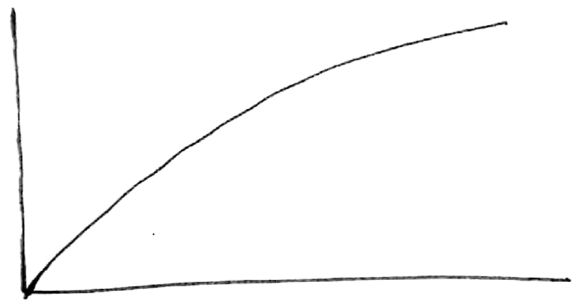
$\omega_i = \omega_c + k_f$

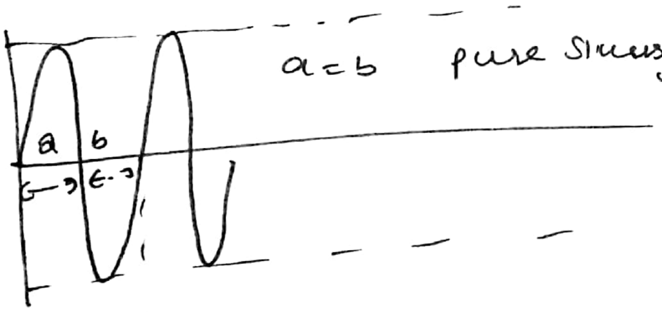


Freq modulation



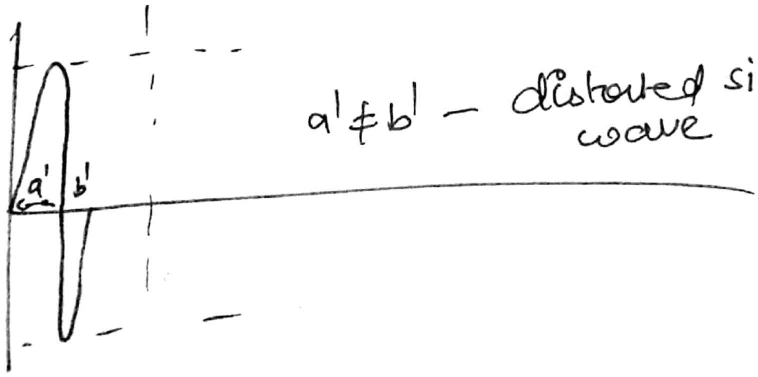
phase modul





$a = b$ pure sinusoidal wave

$\underline{a > a'}$



$a' \neq b'$ – distorted sinusoidal wave

Types of Freq. Modulation

→ depending upon the value of β, there are two types of freq. modulation

$$X_{fm}(t) = A_c \cos(\omega_c t + \beta \sin \omega_m t)$$

$$X_{fm}(t) = A_c \left[\cos \omega_c t \cos(\beta \sin \omega_m t) - \sin \omega_c t \sin(\beta \sin \omega_m t) \right]$$

(1) Narrow band FM

→ when β is very very small

$\cos \theta = 1$, $\theta \to 0$ $\qquad$ $\sin \theta = \theta$, $\theta \to 0$

$\cos(\beta \sin \omega_m t) \simeq 1$, $\beta \to 0$

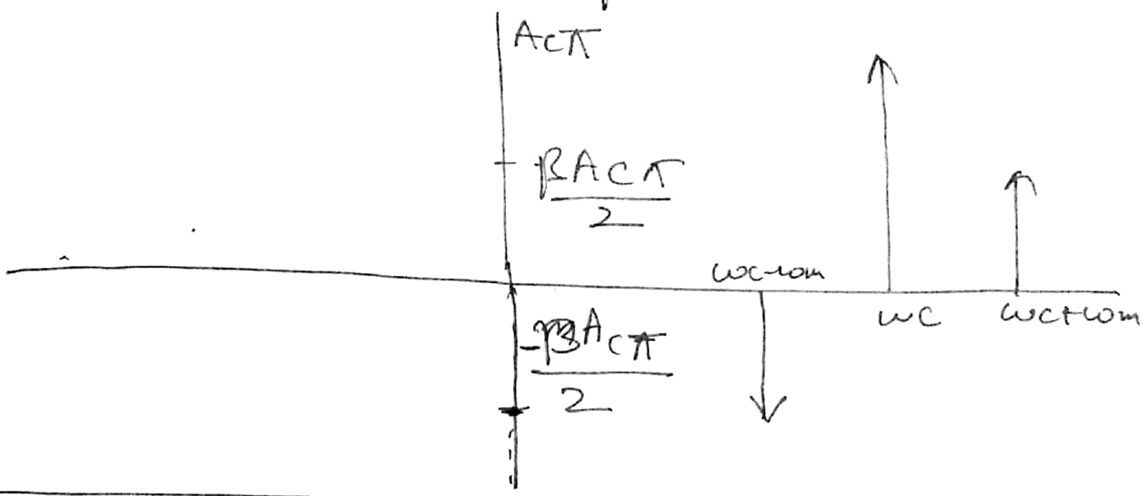
$\sin(\beta \sin \omega_m t) \simeq \beta \sin \omega_m t$, $\beta \to 0$

$$X_{NBFM}(t) = A_c \cos \omega_c t + A_c \beta \sin \omega_m t \sin \omega_c t$$

$$\simeq A_c \cos \omega_c t - \frac{A_c \beta}{2} \cos(\omega_c - \omega_m)t + \frac{A_c \beta}{2} \cos(\omega_c + \omega_m)t$$

$$X_{AM}(t) = A_c \cos \omega_c t + \frac{m A_c}{2} \cos(\omega_c - \omega_m)t + \frac{m A_c}{2} \cos(\omega_c + \omega_m)t$$

⇒ The NBFM is similar to AM except that in AM, all the three components i.e., carrier, USB & LSB are in phase but in NBFM, the LSB is 180° out of phase w.r.t carrier & USB freq. domain representation



$\boxed{B = 2\omega_m}$

~~Phase~~ in NBFM, transmission BW does not effected

② Wide band FM

Since change in freq causes the time required to complete one half of a cycle to differ from the time required to complete the next half cycle, the actual wave is a distorted sinusoidal oscillations the higher mathematical analysis using Bessels function shows that the distorted oscillation corresponding to a wave with sinusoidal freq modulation is made up of infinite no. of freq components spaced by modulating freq.

here $\cos(\beta \sin\omega_m t) \Leftarrow \cos\omega_m t$ &
$\sin(\beta \sin\omega_m t) \Leftarrow \sin\omega_m t$ are periodic s/g's with fundamental freq ω_m.

Thus the forier transform of each of these sig is an impulse train with impulses at integer multiple of ωm and amplitude $\propto$ to Bessels function of Ist kind

WBFM

$$\cos(\beta \sin \omega mt) = J_0(\beta) + 2J_2 \cos 2\omega mt + 2J_4 \cos 4\omega mt + \cdots$$

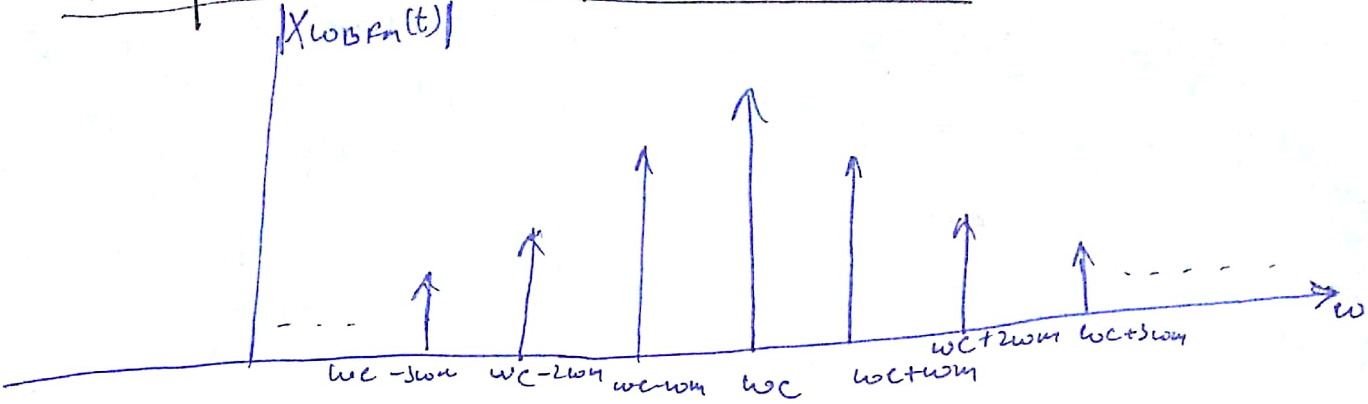
$$\sin(\beta \sin \omega mt) = 2J_1(\beta) \sin \omega mt + 2J_3(\beta) \sin 3\omega mt + 2J_5(\beta) \sin 5\omega mt + \cdots$$

$$\cos \omega ct \cos(\beta \sin \omega mt) = J_0(\beta) \cos \omega ct + 2J_2(\beta) \cos \omega ct \cdot \cos 2\omega mt + 2J_4(\beta) \cos \omega ct \cdot \cos 4\omega mt + \cdots$$

$$\sin \omega ct \sin(\beta \sin \omega mt) = 2J_1 \beta \sin \omega ct \sin \omega mt + 2J_3(\beta) \sin \omega ct \sin 3\omega mt + 2J_5(\beta) \sin \omega ct \sin 5\omega mt + \cdots$$

$$X_{WBFM}(t) = \Big[J_0(\beta) \cos \omega ct + J_1(\beta) \big[\cos(\omega c + \omega m)t - \cos(\omega c - \omega m)t\big] + J_2 \beta \big[\cos(\omega c + 2\omega m)t + \cos(\omega c - 2\omega m)t\big] + J_3(\beta) \big[\cos(\omega c + 3\omega m)t - \cos(\omega c - 3\omega m)t\big] + \cdots$$

Freq. domain Representation



① → the spectrum of WBFM s/g contains infinite no. of side band component centre around carrier freq. ωc [fc] and the spacing b/w each spectral component is maximum modulating s/g freq ωm [fm]

② → the bessel's function $A_c J_n(\beta)$ ← modulation index; n^{th} pair of S.B.

③ → since the spectrum of WBFM s/g contains infinite SBs so the actual transmission BW required for the transmission of WBFM is infinite. But as we move away from the carrier freq, the significant amplitude of sidebands go on decreasing & decreasing so. if higher SB's components are neglected, it does not effect the quality of transmission. So the approximate BW for transmission in WBFM and WBPM s/g's are given by Carson's rule

$$BW = 2(\Delta\omega + \omega m) \text{ rad/sec}$$
$$= 2(\Delta f + fm) \text{ Hz}$$
$$= 2(\beta + 1) fm$$

$(BW)_{FM} = (BW)_{PM} = 2(75+5) = 160 KHz$

If $f_m = 15 KHz$

$\Delta f = 75 KHz$

$(BW)_{FM} = 2(75+15)$
$= 180 KHz$

PM

$\Delta f = 75 \times 3 = 225$

$(BW)_{PM} = 2(225+15)$
$= 480 KHz$

$\Delta f = K_f A_m f_m$ — PM

$\frac{\Delta f}{f_m} = K_f A_m = 15$

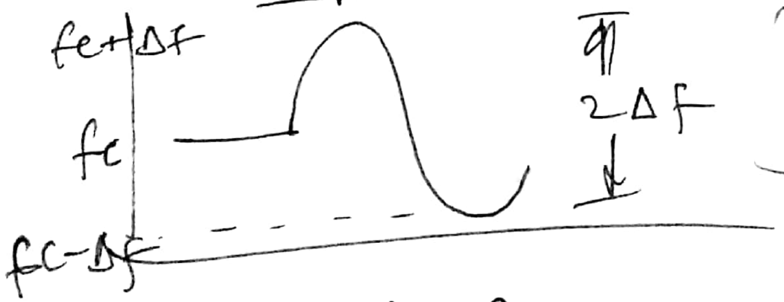
$\Delta f' = K_f A_m f_m' = 15 \times 15 = \underline{225}$

Approximated BW

$BW = 2(\Delta f + f_m)$

$\Delta f >> f_m$

$\boxed{BW \simeq 2\Delta f} \Rightarrow$ Carrier Swing



% Modulation

$$\boxed{\% \text{ Modulation} = \frac{\Delta f_{actual}}{\Delta f_{max}} \times 100\%}$$

$\Delta f_{actual} = 25 KHz$

$\% \text{ mod} = \frac{25}{75} \times 100\% = 33.33\%$

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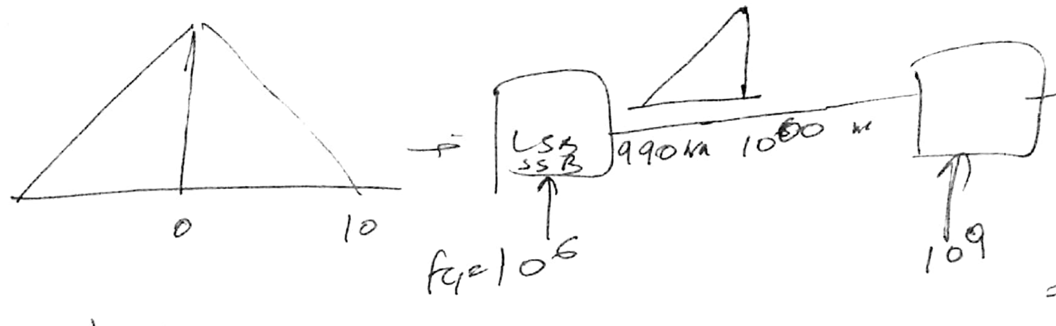
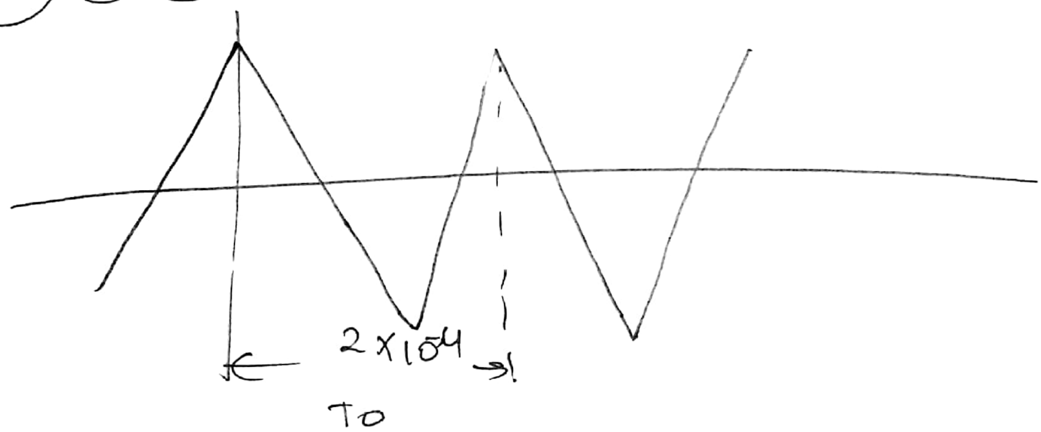


$= 2 \times 1000$
$= 2000$ KHz
$= 2 \times 10^6$ KHz

$$\frac{\Delta f'}{\Delta f''} = \frac{K_f A_m}{K_f A_m} = \frac{1}{1}$$

22 23 24



$$f_0 = \frac{1}{T_0} = \frac{10^4}{2} = 5\text{KHz}$$

$$3 f_0 = \underline{\underline{15\text{KHz}}}$$

$$X_{FM}(t) = A_c \cos\left[\omega_c t + K_f \int_0^t m(t)\,dt\right]$$

$$\theta_i(t) = \omega_c t + K_f \int_0^t m(t)\,dt$$

$$\omega_i(t) = \omega_c + K_f\, m(t)$$

$$\Delta\omega = K_f\, m(t)\Big|_{max}$$

$$\Delta\omega = 2\pi \times 10^5$$

$$\Delta f = 100\text{KHz}$$

$$BW = 2(100 + 15)$$
$$= 230\text{KHz}$$

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$x_{PM}(t) = A\cos(\omega_c t + k_p m t)$

$\theta_i(t) = \omega_c t + k_p m t$

$\cancel{\Delta\omega = k_p A_m \omega_m}$

$= 5\pi \times 2\pi \times 15$

$= 2\pi^2 \cdot 75$

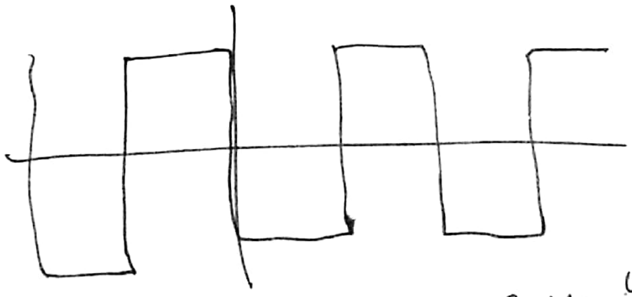
$\Delta f = \frac{150\pi^2}{2\pi} = 75\pi$

$2(\Delta f + f_m) = 2(75\pi + 15)$

$\omega_i(t) = \frac{d\theta_i}{dt} = \omega_c + k_p \frac{dm(t)}{dt}$

$\Delta\omega = k_p \frac{dm(t)}{dt}\Big|_{max}$

$\frac{dm(t)}{dt}$



$\text{slope of } m(t) = \frac{-2}{10^{-4}} = -2\times10^4$

$\text{slope} = 2\times10^4$

$= 5\pi \times 2\times10^4$

$\Delta\omega = 10\pi\times10^4$

$\Delta f = 50\ KHz$

$BW = 2(50+15) = 130\ KHz$

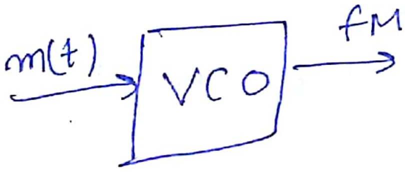
Generation of FM waves

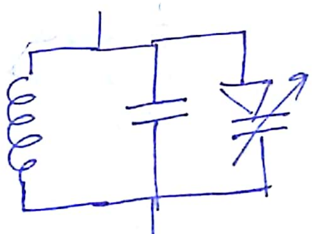
there are two methods for generation of FM waves

① Direct Method

the basic concept of FM is to vary the carrier freq in accordance with the modulating sig the carrier is generated by an LC oscillator. in an LC oscillator the carrier freq is fixed by components of tank ckt. the carrier freq can be varied by varing either L or C of tank ckt. The oscilator whose freq is varied in accordance with voltage (modulating sig in case of FM) is known as VCO (voltage controlled oscillator)

$$f_c = \frac{1}{2\pi\sqrt{LC}}$$





Disadvantage of Direct method

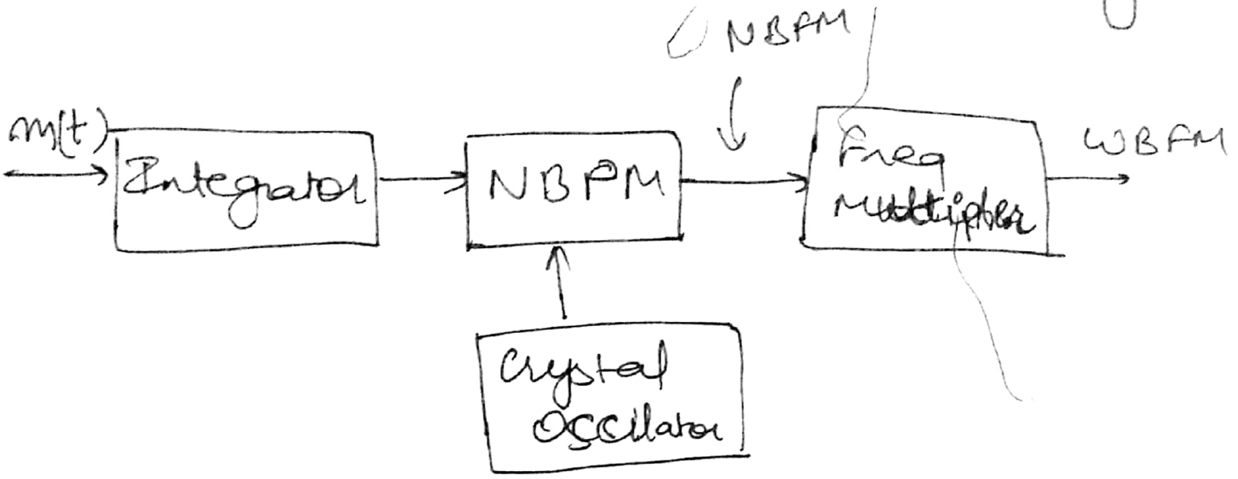
① Most LC oscillators are not stable enough to provide a carrier sig. The carrier freq usually vary due to temperature variation, humidity, aging of component etc.

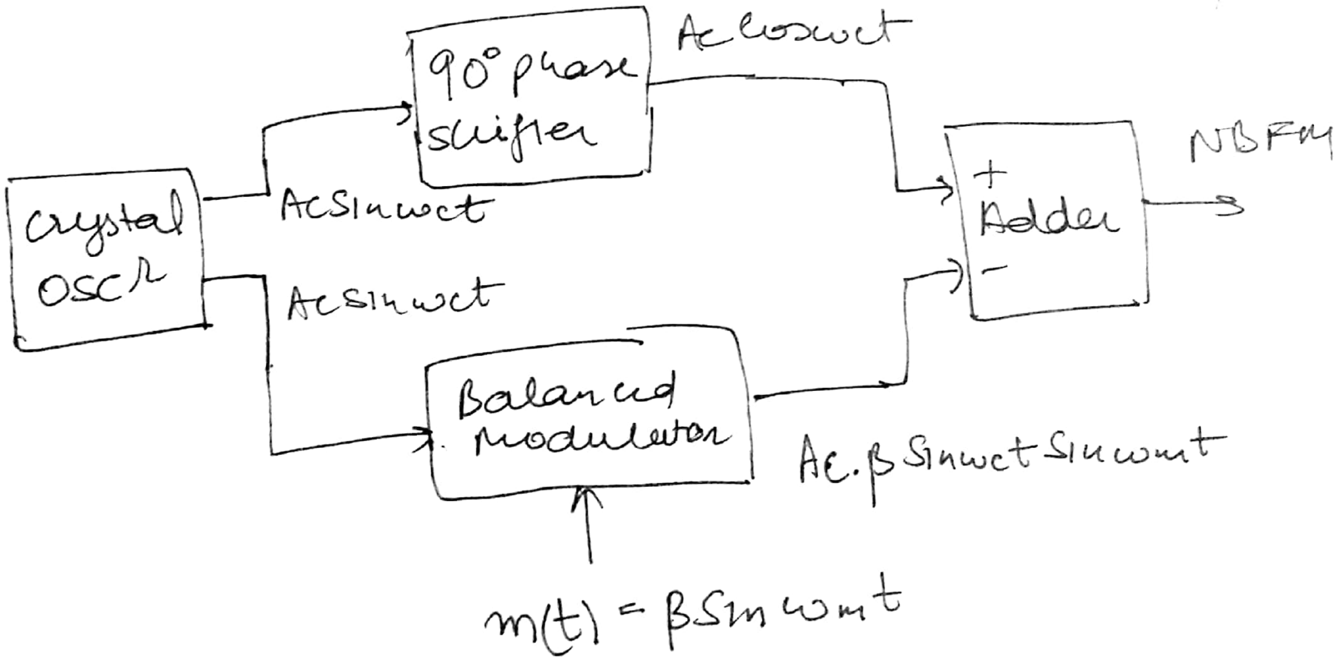
→ So instead of using LC oscilators, a crystal oscilator must be used.

→ But since they provide highly stable carrier freq so. only a very small freq deviation is possible.

what is why, indirect method of FM generation is used.

② Indirect method (Armstrong Method)





$$X_{NBFM}(t) = A_c \cos\omega_c t - A_c\beta \sin\omega_c t \sin\omega_m t$$

Frequency multiplier

$$y(t) = x^n(t)$$

Let $y(t) = x^2(t)$

$$X_{FM}(t) = A\cos(\omega_c t + \beta \sin\omega_m t)$$

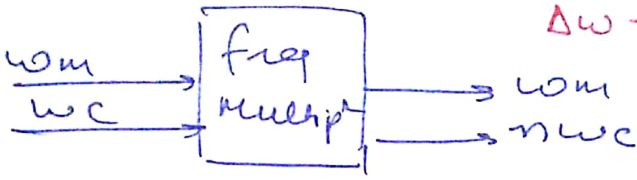
$$y(t) = \frac{A_c^2\left(1 + \cos(2\omega_c t + 2\beta \sin\omega_m t)\right)}{2}$$

$\omega_c \rightarrow 2\omega_c$
$\omega_m \rightarrow \omega_m$
$\beta \rightarrow 2\beta$
$\Delta\omega \rightarrow 2\Delta\omega$

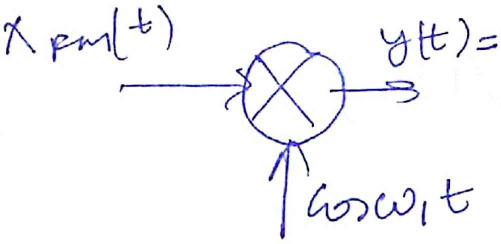
generalised.

$y(t) = x^n(t)$

$\omega_c \rightarrow n\omega_c$
$\omega_m \rightarrow \omega_m$
$\beta \rightarrow n\beta$
$\Delta\omega \rightarrow n\Delta\omega$



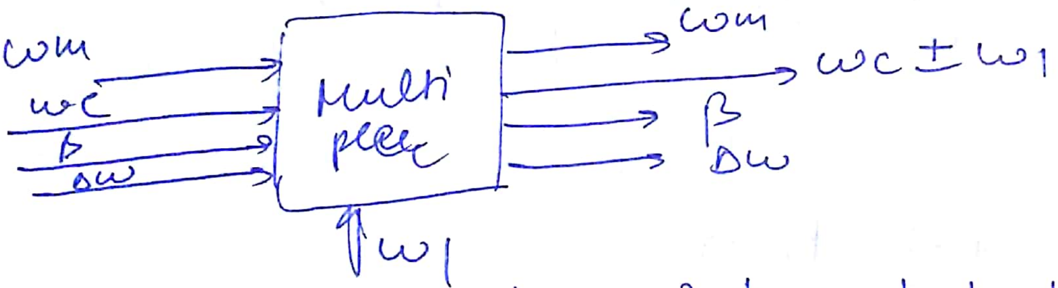
in this, there is no change in ω_m



$y(t) = A_c \cos(\omega_c t + \beta \sin\omega_m t)\cos\omega_1 t$

$\frac{A_c}{2}\Big[\cos\big((\omega_c+\omega_1)t + \beta\sin\omega_m t\big)$

$+ \cos\big((\omega_c-\omega_1)t + \beta\sin\omega_m t\big)\Big]$



only carrier freq is translated by ω_1

---

(120) Q 21 $\phi(t) = 10\cos(\omega_c t + 5\sin 3000t + 10\sin 2000\pi t$

$\theta_i(t) = \omega_c t + 5\sin 3000t + 10\sin 2000\pi t$

$\omega_i(t) = \omega_c + 15000\cos 3000t + 20000\pi\cos 2000\pi t$

$\Delta\omega_{max} = 15000 + 20000\pi$

$\beta = \frac{\Delta\omega}{\omega_m}$

Deviation ratio $(D) = \frac{\Delta\omega}{\omega_{m(max)}}$

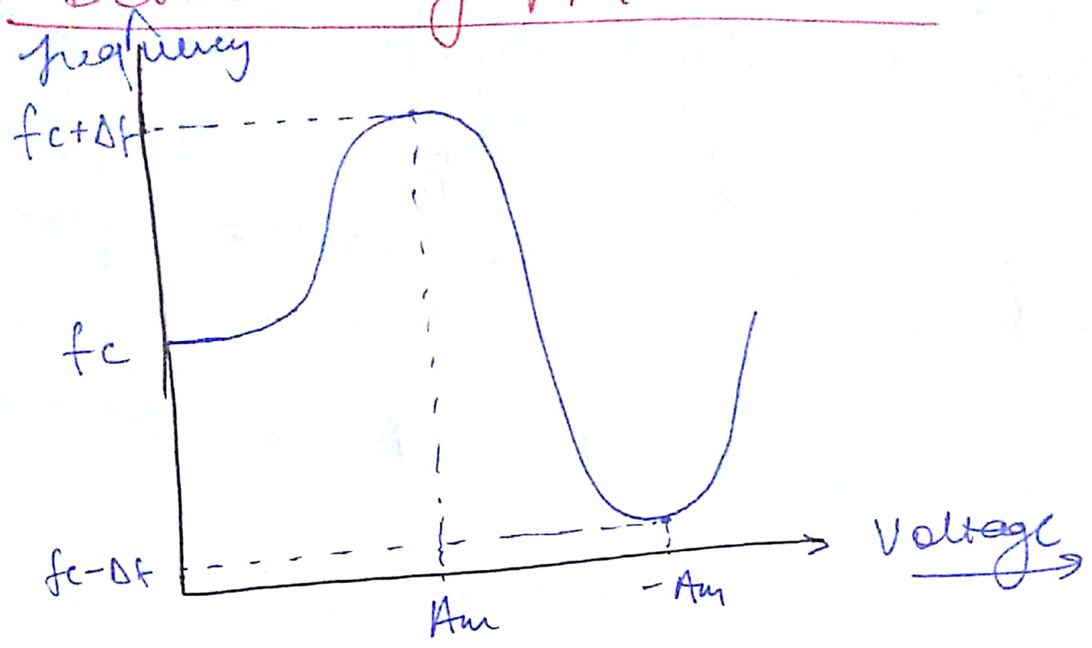
when there are more than one modulating signal then we calculate D

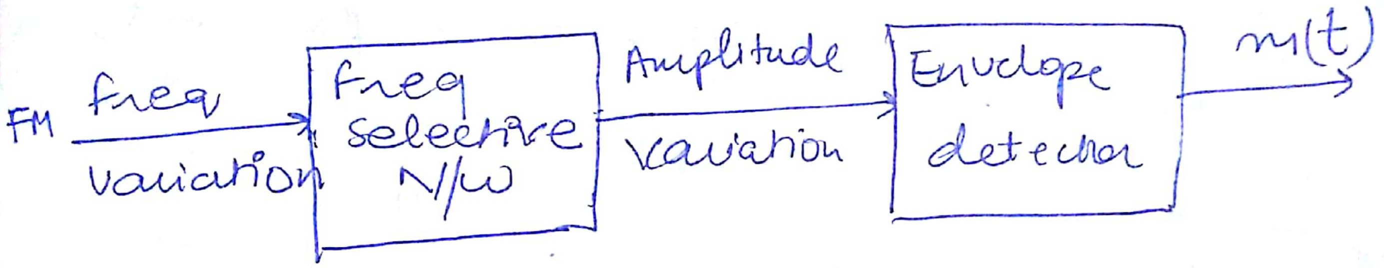
$= \frac{15000 + 20000\pi}{2000\pi}$

$10 + \frac{15000}{2000\pi}$ $10 + \frac{10}{\pi}$

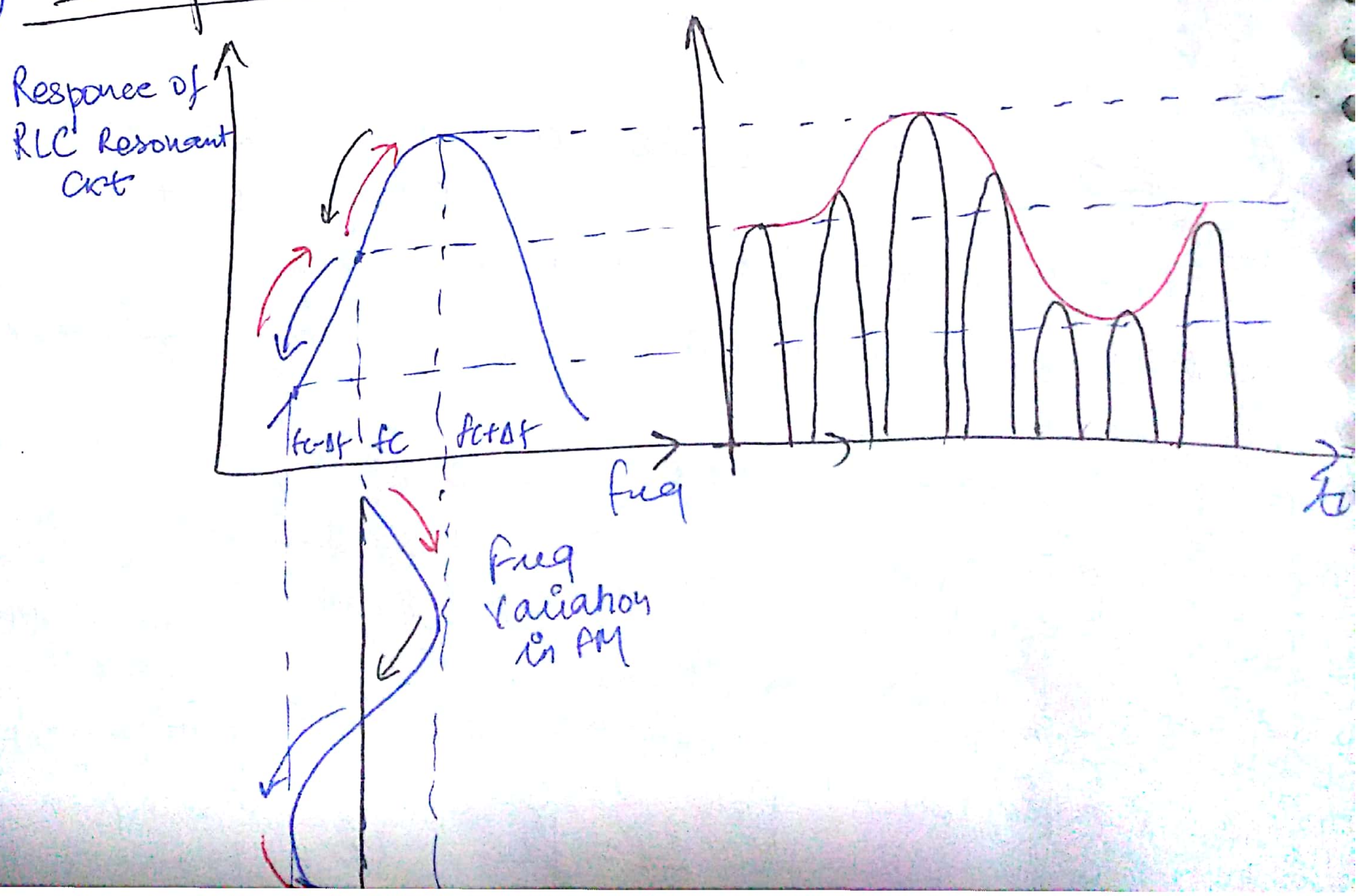
~ 12

Detection of FM wave





① Slope detection



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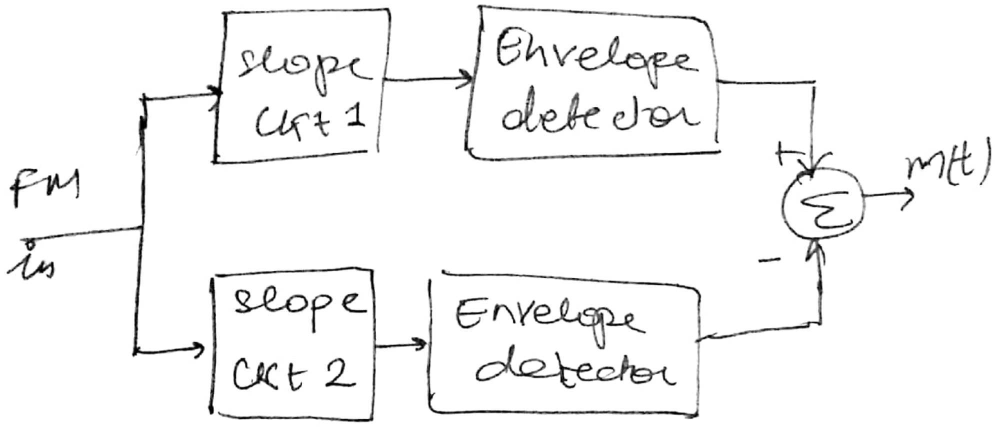
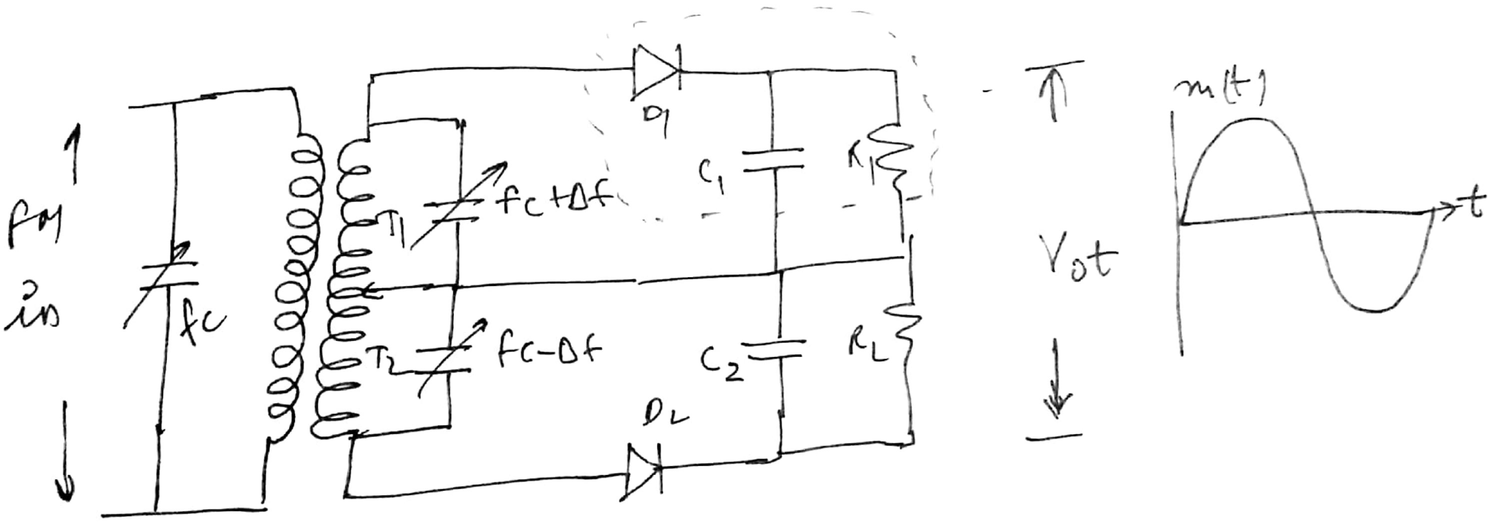
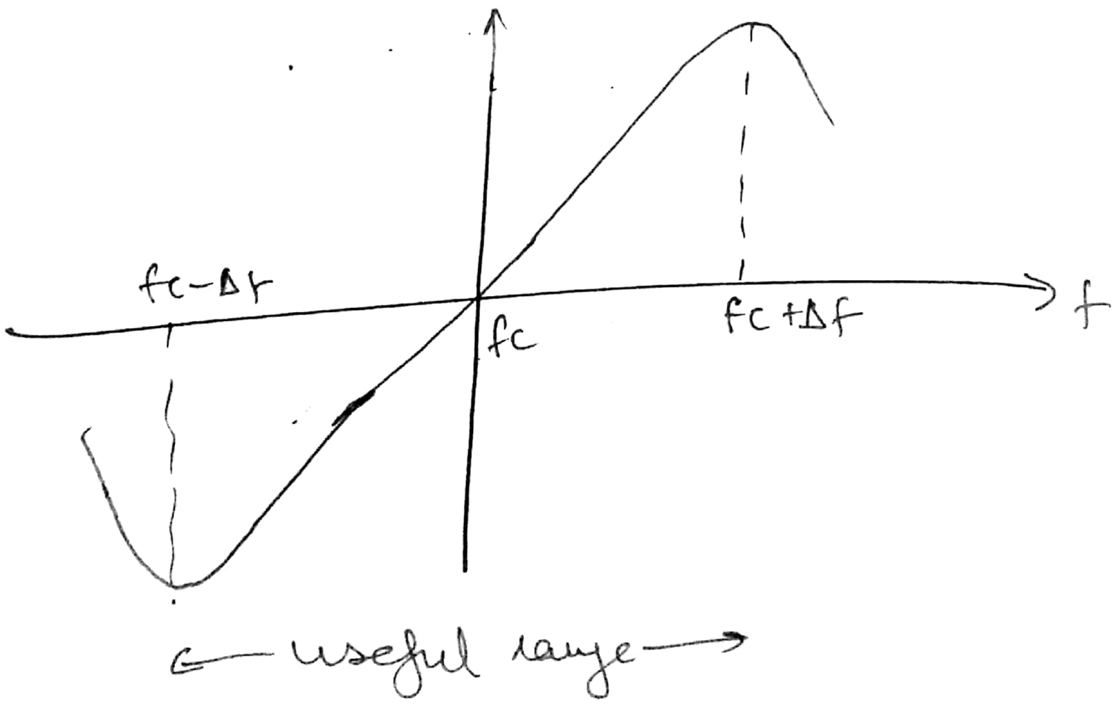
② Balanced slope detector







→ Balanced slope detector is also known as triple tuned ckt as 3 tuned ckt are used. The i/p is tuned as carrier freq fc, t_1 at fc + Δf and t_2 at fc − Δf

→ when fin = fc, the gain provided by t_1 & t_2 will be same so the o/p of both tuned ckt will be same but in phase opposition. So the net o/p is 0.

→ when fin is closer to fc + Δf, the o/p of t_1 will be positive large as compared to −ve voltage of t_2 so the net o/p voltage will be in +ve direction.

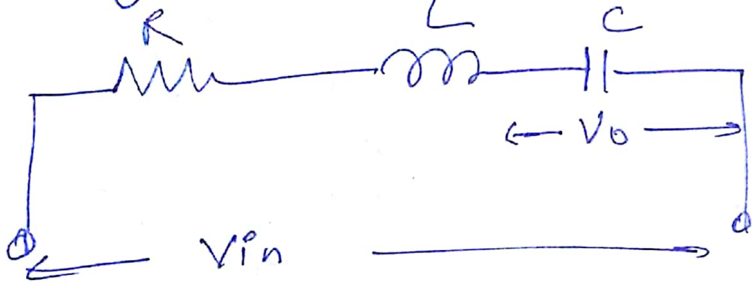
→ when fin is closer to fc − Δf, the o/p of t_2 will be more positive as compared to t_1 and net o/p will be in the −ve direction

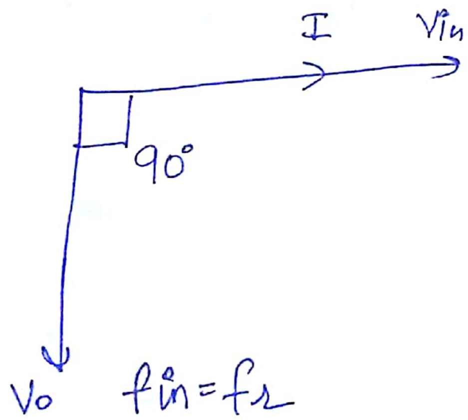
t_2 will provide more gain

FOSTER Seeley discriminator

(Phase shift discriminator)

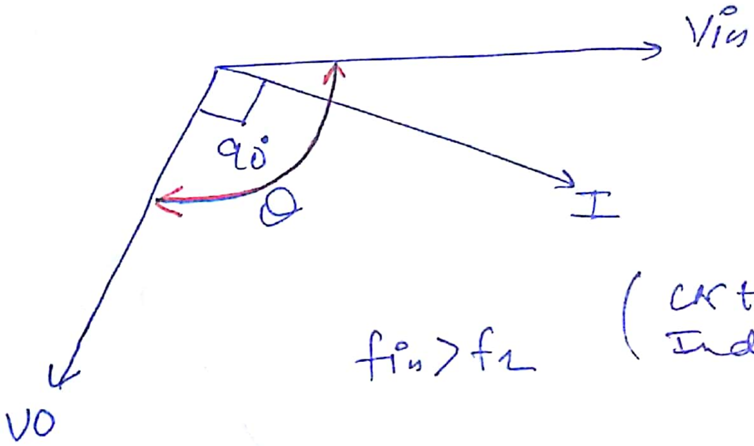
→ This method make use of freq. sensitive nature of series RLC resonant ckt and provides a freq. dependent phase shifting of the modulated s/g. when the phase shifted s/g is added to the portion of original s/g, the resultant s/g will have a magnitude varying with freq.





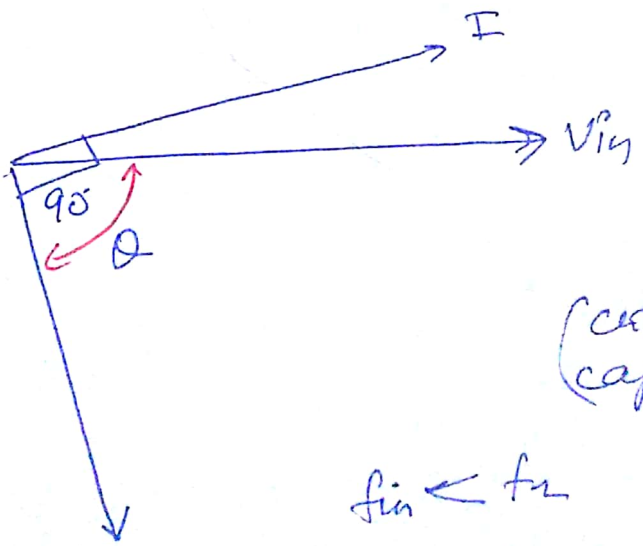
$f_{in} = f_r$

(ckt is Resistive)



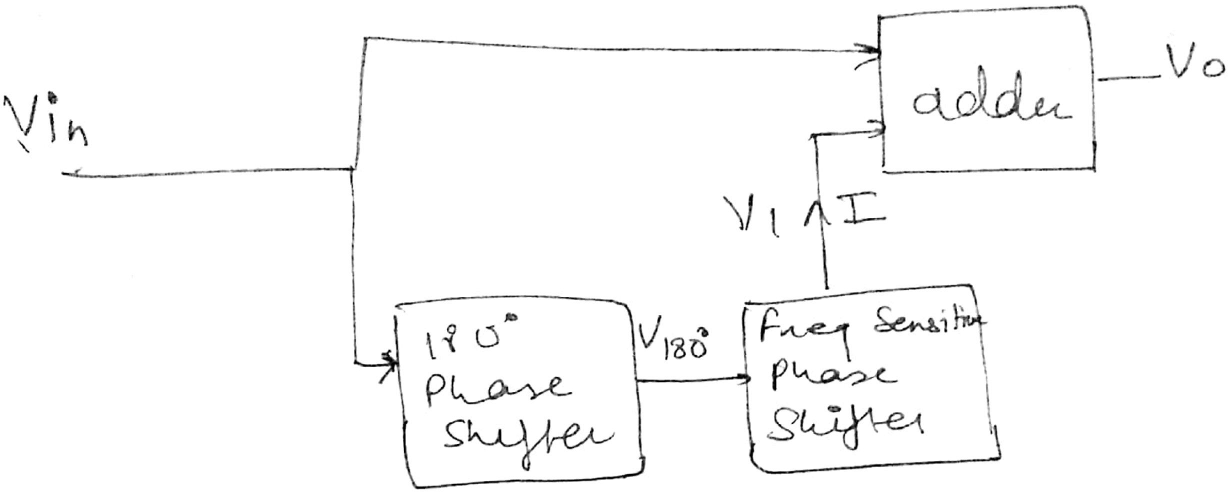
$f_{in} > f_r$

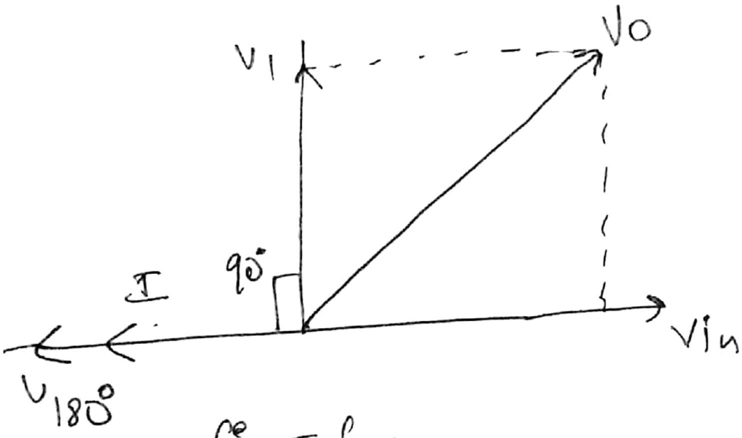
(ckt is Inductive)



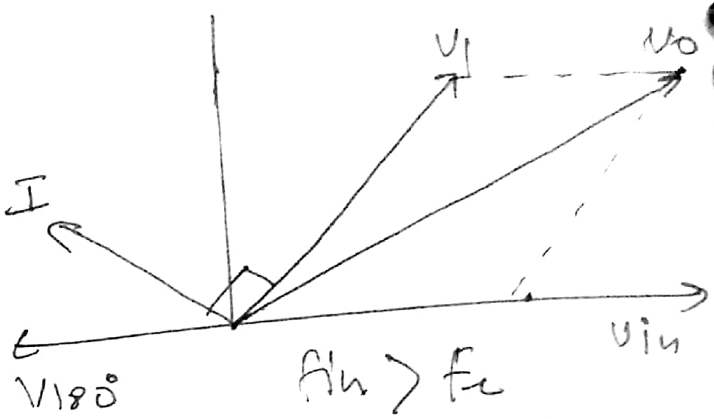
(ckt is capacitive)

$f_{in} < f_r$

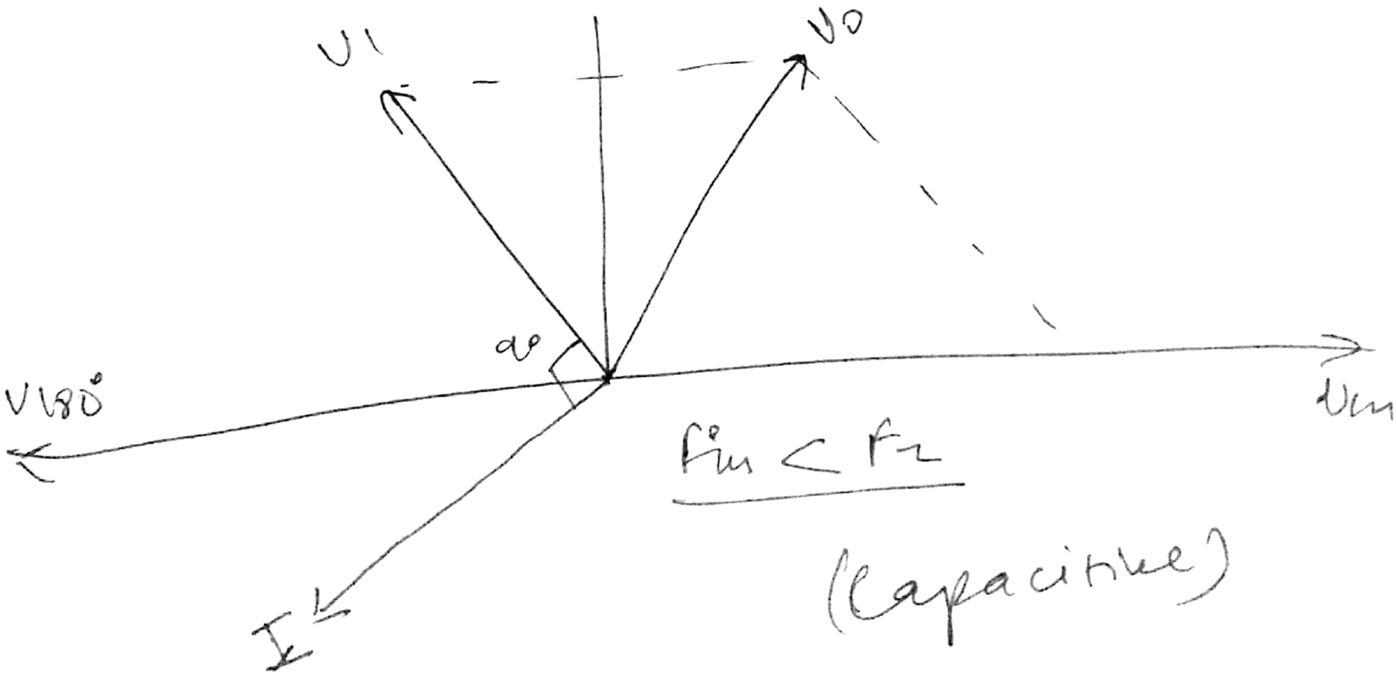




fin = fr
(ckt resistive)



fin > fr
(ckt is inductive)



fin < fr

(capacitive)

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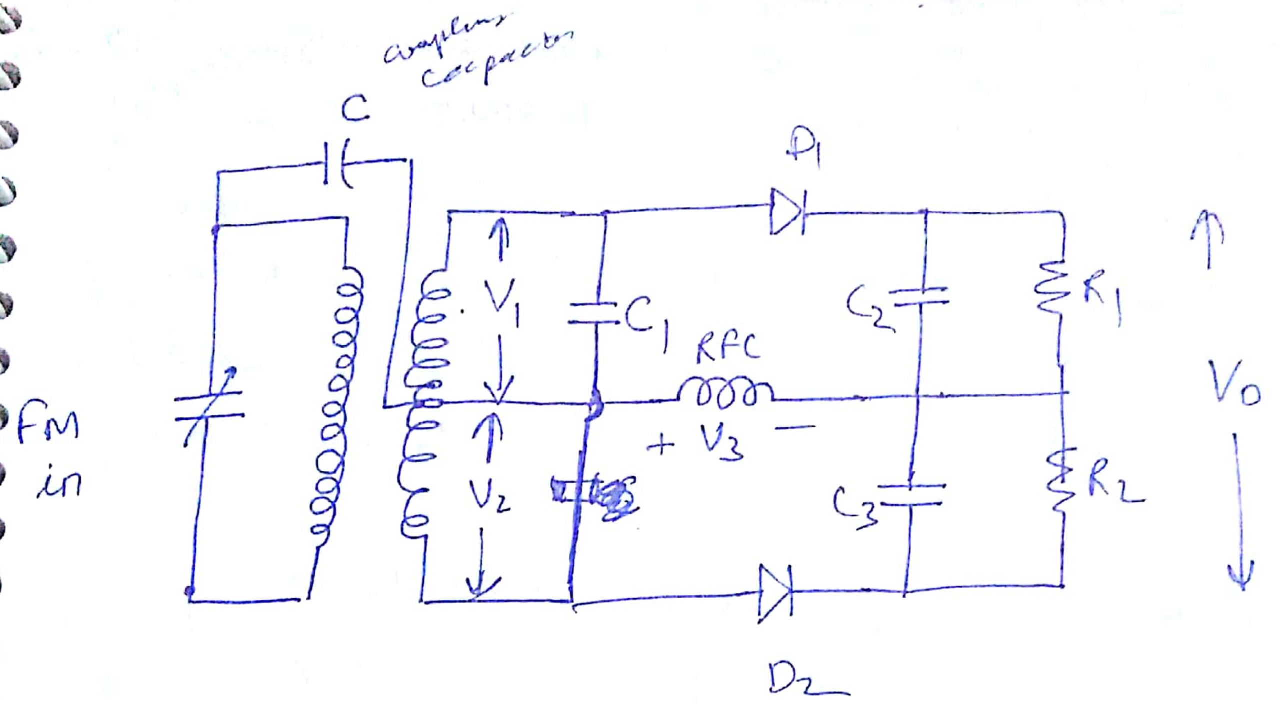
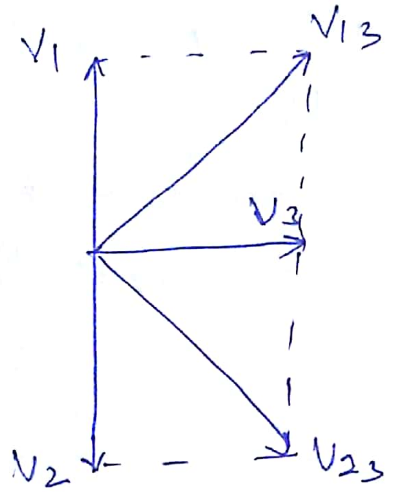
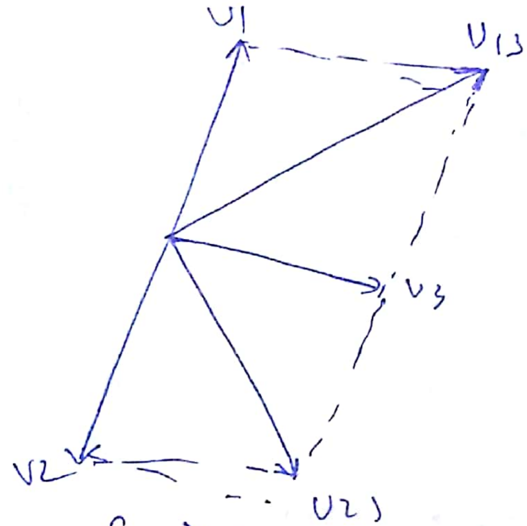




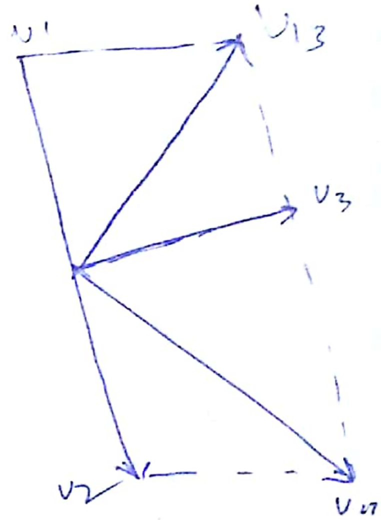
fin = fc

(Resistive)



fin > fc

(inductive)



fin < fc

(capacitive)

Working

① → the coupling capacitor C is used to couple the input voltage to the secondary of centre tap transformer

② → capacitor C_1 is so chosen that voltage V_3 is exactly 90° out of phase wrt to voltage V_1 & V_2 at $f_{in} = f_c$

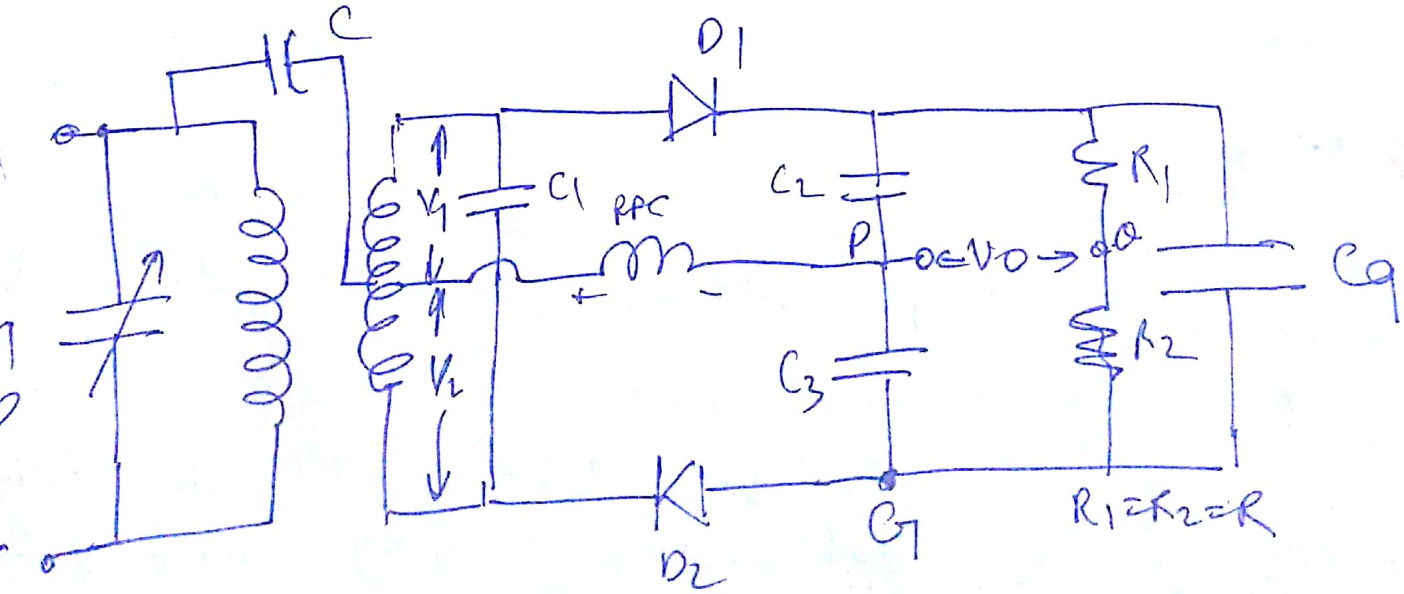
③ → with the help of centre tap arrangement voltage V_1 & V_2 are always 180° out of phase

(4) → the Resonant freq of phase shifting n/w is adjusted to carrier frequency of FM sig. when the freq of i/p sig increases and becomes more than carrier freq fc, the resultant of V1 & V3 becomes greater than resultant of V2 & V3. and Net o/p is in +ve direction

(5) → when fin < fc, the Resultant of V2 & V3 becomes greater than resultant of V1 & V3 and net o/p voltage is in -ve direction

6 → when fin = fc, both the resultants become same but in phase opposition the net o/p voltage is Zero

Ratio detector



The Ratio detector differ from the Foster Seeley discriminator in 3 ways

1. direction of D_2 has been Reversed
2. Method of taking o/p voltage Vout
3. A large value capacitance C_9 has been included

$\underline{C_9}$ the capacitor C_9 provides Amplitude limiting, when Vin is const. & C_9 is charged upto max. value then there is no current to charge or discharge C_9. if any short duration noise effects the i/p s/g then it will not cause any harm to the detected s/g as the capacitor C_9 will take large time to charge upto this voltage. Generally RC time constant is kept at a value $\underline{0.2 \text{ sec}}$.

Working of ckt

let $V_{C9} = 10V$. Since $R_1 = R_2 = R$ so V_{Cg} is always maintained at 5V.

① → when $f_{in} = f_c$, the resultant of V_1 & V_2 is same as resultant of V_2 & V_3 so two diodes conducts equally well and $V_{C_2} = V_{C_3} = 5V$ and Net o/p voltage is 0.

② → when $f_{in} > f_c$, Resultant of V_1 & V_3

becomes greater than resultant of V_2 & V_3 so diode D_1 conducts more heavily as compared to D_2 and $\underline{V_{C2} > V_{C3}}$

so net o/p is in +ve direction

③ when $f_{in} < f_c$, Diode D_2 conducts more heavily as compared to D_1 and $\underline{V_{C3} > V_{C2}}$ and Net o/p voltage is in −ve direction. Depending upon the i/p freq, the Ratio of capacitor voltage C_2 & C_3 changes. This Ratio change is detected by the ckt hence the name ratio detector.

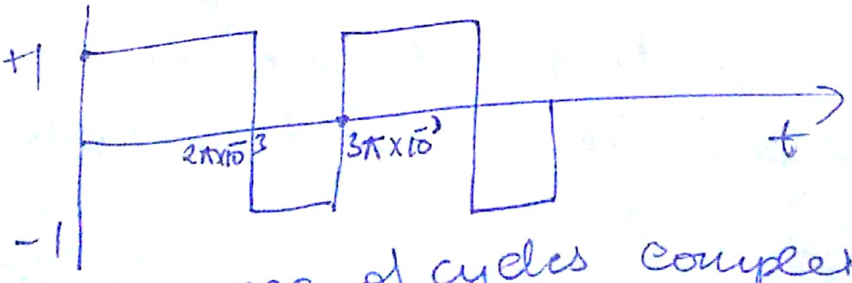
Q Consider a sig $x(t)$ given by $\cos(50t + \sin 5t)$ calculate its instantaneous freq. in rad/sec at $t=0$

$\theta_i = 50t + \sin 5t$

$\omega_i = 50 + 5\cos 5t$

$\omega_i|_{t=0} = 50 + 5 = 55$

Q a modulating sig $m(t)$ is shown below



assume the constant $K_f = 1000$ rad/sec/v

$\omega_c = 3000$ rad/sec

ⓐ the no. of cycles completed by resultant fm wave (when m(t) is +1) are equal to

$f_m = \frac{1}{3\pi \times 10^{-3}} = \frac{10^3}{3\pi}$

$\omega_m = \frac{2}{3} \times 10^3$

~~cos(ωct + kf ∫m(t)dt)~~

$\cos(\omega_c t + k_f m t)$

~~3000t + 1000mt~~

~~$\omega_i = 3000 + 1000 \frac{dmt}{dt}$~~

$\omega_i = \omega_c + k_f m t$

$= 3000 + 1000(1)$

$= 4000$

$T = \frac{2\pi}{4000}$ sec ——— 1 cycle

1 sec ———→ $\frac{4000}{2\pi}$

$2\pi \times 10^{-3}$ ———→ $\frac{4000 \times 2\pi \times 10^{-3}}{2\pi} = 4$ cycles

$\omega_i \Big|_{m(t)=-1} = 3000 - 1000$
$= 2000$

$T_i = \frac{2\pi}{2000}$

$\pi \times 10^{-3} = \frac{\cancel{\pi} \times 10^{-3} \times 2000}{2\cancel{\pi}} = 1$ cycles

Q

The following waveform is received
$x(t) = 5\cos[2\pi \times 10^6 t + 200 \sin(2\pi \times 500)t]$
if sig represents an FM wave
then modulating sig $m(t)$ is given by
assume $k_f = 10^5$ Hz/V

$$\beta = \frac{\Delta f}{f_m} = \frac{k_f A_m}{f_m} = 200$$

$$\frac{A_m \times 10^5}{500} = 200$$

$$A_m = \frac{200 \times 500}{10^5} = 1$$

$m(t) = \cos(2\pi \times 500 t)$

$(\omega_c t + k_f \int_0^t m(t)\,dt)$

$$k_f \int_0^t m(t)\,dt = 200 \sin(2\pi \times 500)t$$

Q a ~~[illegible]~~ sig is given by $\cos(\omega_c t + \phi_1(t))$
The FM sig is generated using the armstrong method. the modulating sig $m(t) = 3\cos(2\pi \times 3000)t$ if the final carrier freq is 100 MHz and max phase deviation is 10 rad. calculate the integer value n by which carrier freq f_c is multiplied assume $k_f = 20\pi$ rad/sec/volt

$$\omega_i = \omega_c + K_f m t$$

$$\omega_i = \omega_c + K_f \times 3 \cos(2\pi \times 3000) t$$

$$\theta_i = \omega_c t + \phi_1(t)$$

$$\omega_i = \omega_c + \frac{d\,\phi_1(t)}{dt}$$

$$K_f \times 3 \cos(2\pi\, 3000 t) = \frac{d\,\phi_1 t}{dt}$$

$$\frac{3 K_f \sin(2\pi\, 3000 t)}{2\pi\, 3000} = \phi_1 t$$

$$\frac{3 \times 20\pi\, n}{2\pi \times 3000} \qquad \frac{1}{100} = 10$$

$$\frac{1}{100} \sin(2\pi\, 3000 t) = \phi_1 t$$

$$n = 1000$$

$$\phi_1(t)_{max} = \frac{1}{100}$$

$$N = \frac{10}{\frac{1}{100}} = \underline{\underline{1000}}$$

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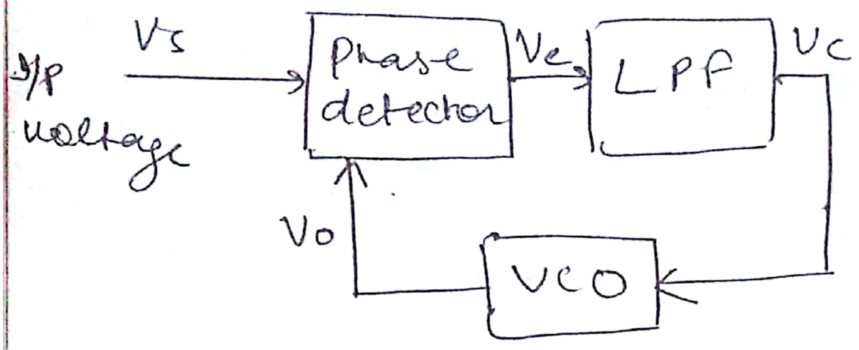
PLL (Phase locked loop)



Phase locked loop is essentially a feed-back system that consists of three essential components

① phase detector
② LPF
③ VCO (Voltage control oscillator)

If an i/p sig V_s of freq f_s is applied to the PLL, the phase detector compares the phase and freq of incoming sig to that of o/p V_o of Vco. If the two sig differ in freq &/or phase, an error voltage is generated, the phase detector is basically a multiplier and produces the sum & difference components at its o/p. The high freq component is removed by the LPF and the difference freq component is amplified and then applied as control voltage V_c to VCO.

$V_s(t) = A\cos\omega_s t$

$V_o(t) = B\cos(\omega_o t + \phi)$

$V_E(t) = V_s(t)V_o(t) = AB\cos\omega_s t\cos(\omega_o t + \phi)$

$= \frac{AB}{2}\left[\cos((\omega_s+\omega_o)t+\phi) - \cos((\omega_s-\omega_o)t-\phi)\right]$

then LPF block $(\omega_s+\omega_o)t$

$V_c(t) = \cos[(\omega_s-\omega_o)t-\phi]$

at $\omega_s = \omega_o$ or $\omega_s - \omega_o = 0 \Rightarrow$ at synchronization

$V_c(t) = \cos(-\phi) = \cos\phi$

at $\phi = 90°$

$\boxed{V_c(t) = 0}$ at exact synchronization

at this condition exact locking occurs

→ The s/g V_c shifts the VCO freq. in a direction to reduce the freq difference b/w f_s & f_o. Once this action starts, we say that the s/g is in capture mode → The VCO continues to change freq till its o/p freq is exactly the same as the i/p s/g freq, the ckt is then said to be locked.

→ Exact locking is achieved when the two s/g are having same freq but a phase difference of 90°

→ The LPF controls the capture range. If VCO freq is far away the difference freq will be too high to pass through the filter and PLL will not respond.

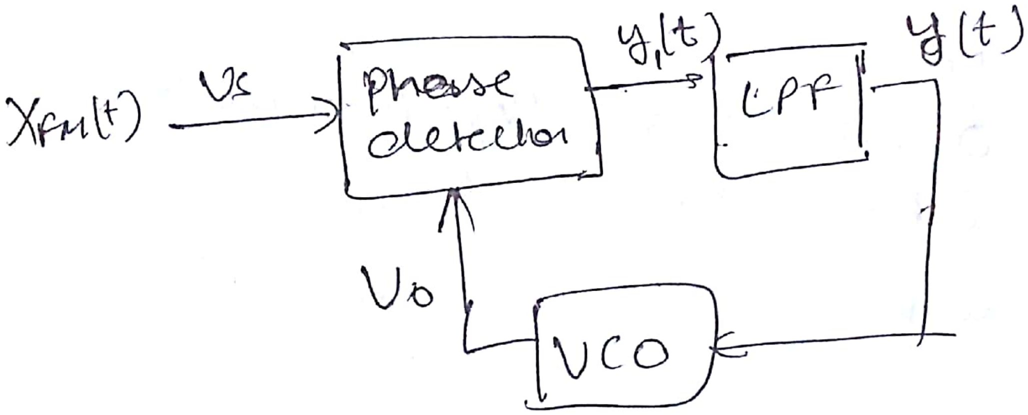
We say that sig is out of capture band

However once locked, the filter no longer restricts the PLL. the VCO can track the sig well beyond the capture band

"thus the capture Range is always less than or equal to the locking Range"

Detection of FM sig using PLL

$$X_{FM}(t) = A\cos\left(\omega_c t + K_f \int_0^t m(t)\,dt\right)$$



$$X_{FM}(t) = A\cos\omega_c t$$

$$V_0(t) = B\cos(\omega_c t + \phi)$$

$$y_1(t) = AB\cos(\omega_c t + \phi)\cos\omega_c t$$

$$y(t) = 0.5\,AB\cos\phi$$

$$\phi = 90° \; ; \; y(t) = 0$$

$$V_0(t) = B\cos(\omega_c t + 90°) = -B\sin\omega_c t$$

$$X_{FM}(t) = A\cos(\omega_c t + \phi_i t);$$

$$\phi_i(t) = K_f \int_0^t m(t)\,dt$$

$$\phi_i(S) = K_f \frac{(m(S))}{S}$$

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$$V_o(t) = -B\sin(\omega_c t + \phi_o t)$$

$$\phi_o(t) = K_1 \int_0^t y(t)\,dt$$

$$\frac{d\phi_o(t)}{dt} = K_1 y(t)$$

$$s\,\phi_o(s) = K_1 Y(s)$$

$$y_1(t) = -AB\cos(\omega_c t + \phi_i(t))\sin(\omega_c t + \phi_o(t))$$

$$= -\frac{AB}{2}\left[\sin(2\omega_c t \cancel{+} \phi_i(t) + \phi_o(t)) + \sin(\phi_o(t) - \phi_i(t))\right]$$

$$y(t) = -\frac{AB}{2}\sin(\phi_o(t) - \phi_i t)$$

$(\phi_i(t) - \phi_o(t))$ is very low

$\sin\theta = \theta$

$\theta \to 0$

$$y(t) = \frac{AB}{2}[\sin(\phi_i(t) - \phi_o t)$$

$$\frac{1}{K_1}\frac{d\phi_o(t)}{dt} = \frac{AB}{2}[\phi_i(t) - \phi_o(t)]$$

$$s\,\phi_o(s) = K[\phi_i(s) - \phi_o(s)]$$

$$\phi_o(s) = \frac{K}{s+K}\phi_i(s)$$

$$\frac{K_1}{s}Y(s) = \frac{K}{s+K}\frac{K_f}{s}M(s)$$

↓ as K is large

so using dominant pole concept

$$\frac{1}{s+K} \Rightarrow \frac{1}{K}$$

$$\boxed{Y(s) = \frac{K_f}{K_1}M(s)}$$

Comparison b/w FM detection method

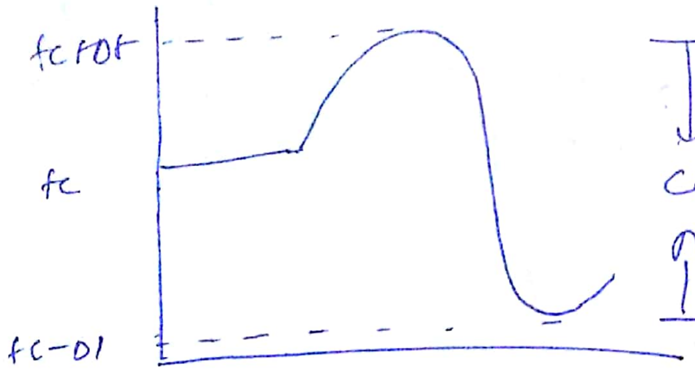
Parameter	Slope detector	Balanced slope detector	Foster Selley Discriminator	Ratio detector	PLL
No of tuned circuits	1	3	2	2	1
Linearity	very poor	poor	very good	good	excellent
Sensitivity to Noise & amplitude variation	sensitive	sensitive	sensitive	Insensitive	sensitive
Application	Never used in Practice	Never used in Practice	Used in applications where linearity is critical ex Reception of TV sound carrier	Used where linearity is not critical as compared to sensitivity to noise variation ex Satellite Station RX	Now a days in every application PLL is used for Reception of FM S/G

(127) CQ

① $f_m = 500\,Hz$ $\Delta f_1 = 5\,kHz$

$f_c = 10\,MHz$

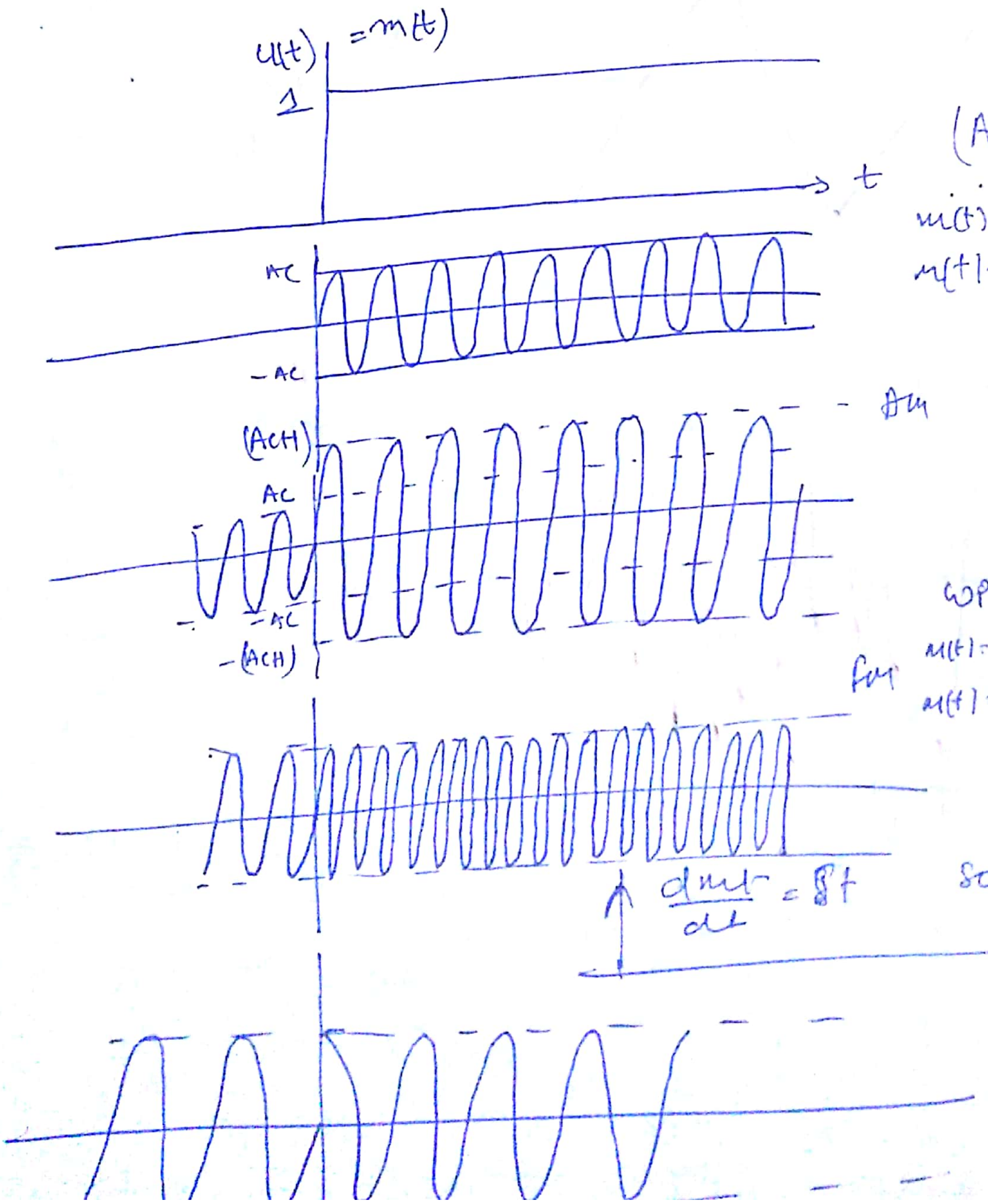
$$\beta = \frac{\Delta f}{f_m} = \frac{5\times10^3}{500} = 10$$



Capture Range = Carrier Swing $= 2\Delta f$ $= 10\,kHz$

(iii) Prove $y(t) = m(t)$

$$\Delta f = K_f\, m(t)\Big|_{max}$$



$(A_c + m(t)) \cos \omega_c t$

$m(t) = 0$; $A_c \cos \omega_c t$

$m(t) = 1$; $(A_c + 1) \cos \omega_c t$

$\omega_i = \omega_c + K_f\, m(t)$

$m(t) = 0$ $\omega_i = \omega_c$

$m(t) = 1$ $\omega_i = \omega_c + K_f$

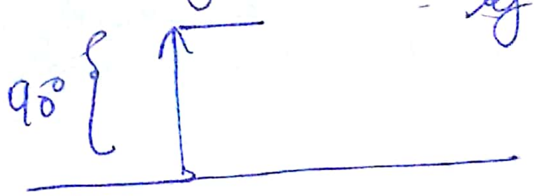
So only phase change occur by 90°

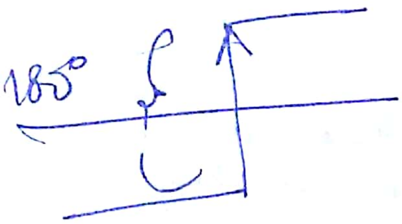
freq remains same.

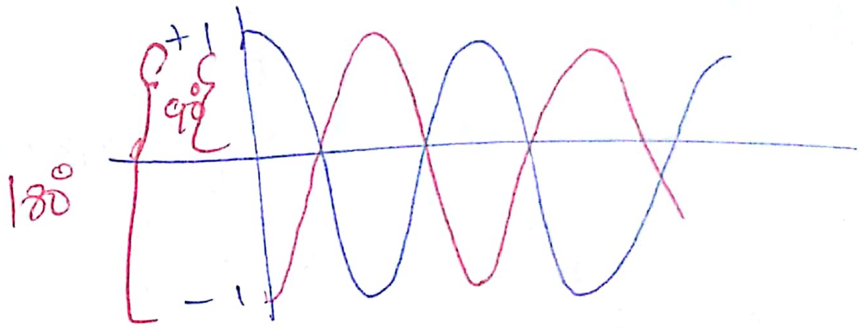
in case of abrupt discontinuity

if

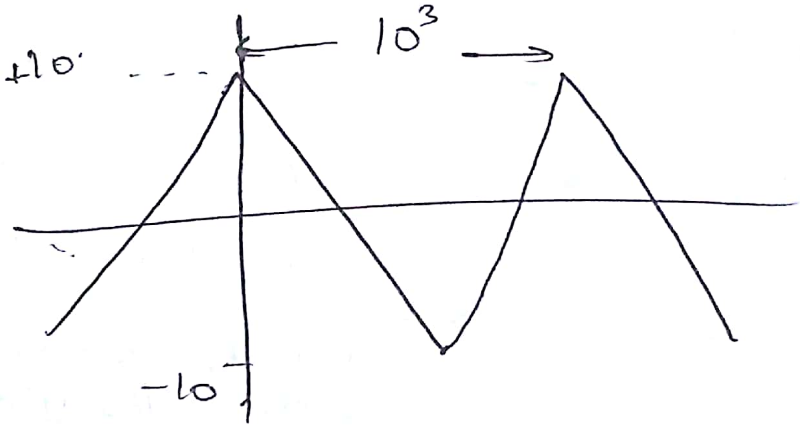
- if abrupt discontinuity is fro 0 to 1 then phase change by 90°







C.Q Pg 121



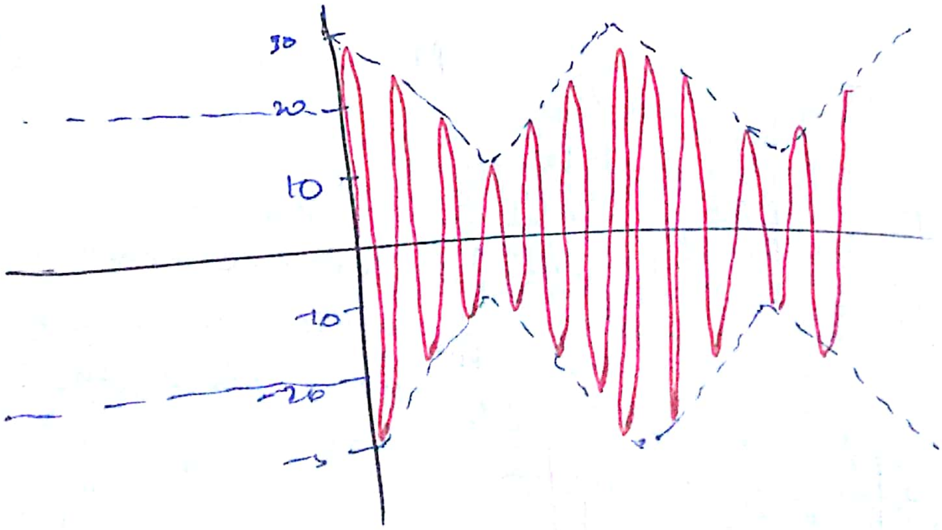
$m = 0.5$

$A_m = 10$

$A_c = 20$

$A_c + A_m = 30$

$A_c - A_m = 10$



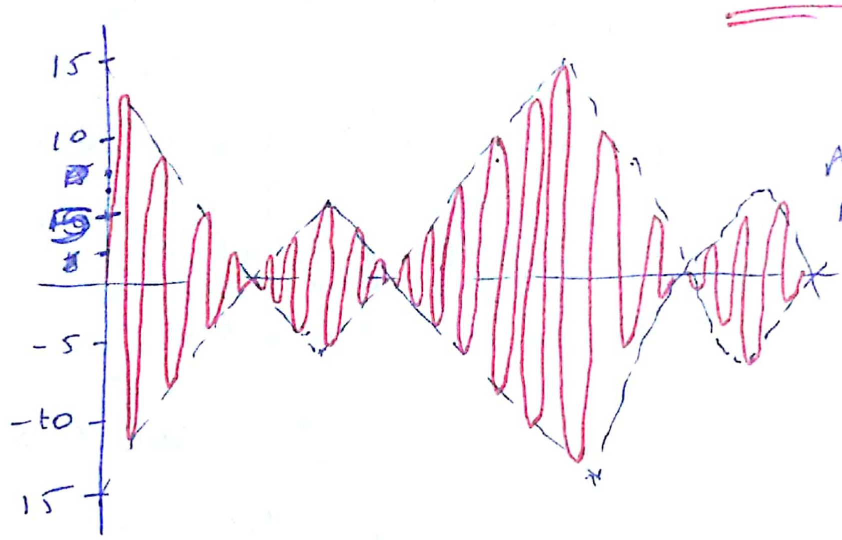
$m = 2$

$A_m = 10$

$A_c = 5V$

$A_m + A_c = 15V$

$A_c - A_m = -5V$



$M = \infty = \frac{A_c}{A_c}$

$A_c = \frac{A_m}{\infty} = 0$

$A_m + A_c = 10$

$A_c - A_m = -10$

$M = \infty$

no modulatio

⇒ when $m = \infty$, no carrier is present only the baseband s/g is transmitted which represents no modulation

(2)

(a) $A_c = \frac{10}{0.8} = \frac{100}{8} = 12.5$

$P_c = \frac{(12.5)^2}{2} = \frac{156.25}{2} = 78.125$

(b) $m(t)\cos\omega_c t = \frac{1}{2} \cdot P_m = \frac{1}{2} \times \frac{10^2}{3} = \frac{100}{6} = \frac{50}{3} = 16.67W$

$P_m = 4\left(\frac{1}{T}\int_0^{T/4} \frac{16A^2}{T^2} \cdot t^2 + A^2 - \frac{8A^2}{T}t\right] dt$

$4\left(\frac{1}{T}\left[\frac{16A^2}{T^2}\frac{T^3}{3 \cdot 64} + \frac{A^2T}{4} - \frac{8A^2}{T} \times \frac{T^2}{2 \cdot 16 \cdot 4}\right]\right.$

$\frac{A^2}{3} + A^2 - 4A^2$

$= \frac{A^2}{3}$

$$\eta = \frac{16.67}{78.125 + 16.67} \times 100\% = 17.5\%$$

Random Variable & Noise

Experiment – An experiment is a process conducted to get some results

Outcome – An outcome is the Result obtained after performing the experiment

Event – An event is combination of outcome

Sample space – The set of all possible outcomes of an experiment is called as sample space. It is also known as Ensemble space (not a function of time)

Random variable (X)

→ It is a numerical description of the outcomes

→ A Random variable is a numerically valued function defined over sample space

→ There are two types of Random Variable

① Discrete Random Variable (DRV)

If X takes only a countable no. of distinct values then X is called a DRV

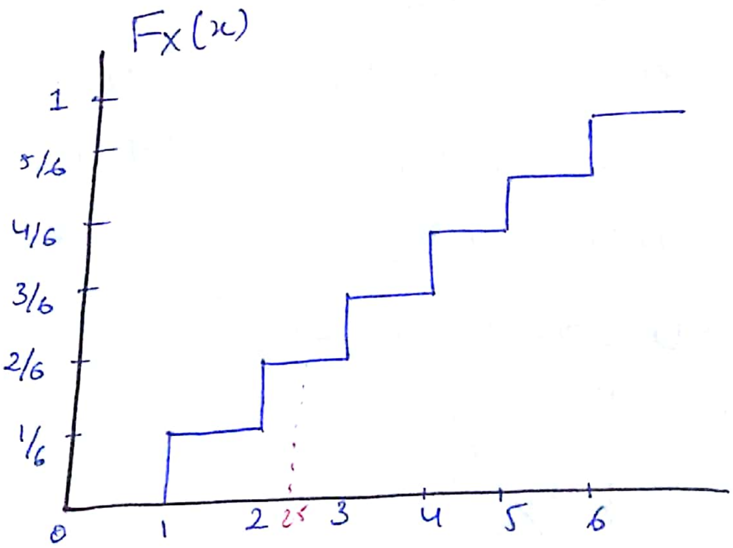
(2) Continuous Random Variable (CRV)

→ If X can assume any value then it is called a continuous Random Variable. For ex.— Noise Voltage generated in an electronic ampr has a continuous amplitude.

Cumulative distribution function (CDF)

CDF is the probability of all possible values in the total available Range it contains all probabilistic information of Random variable and is given by

$$F_X(x) = P\{X \leq x\} \quad ; -\infty \leq x \leq \infty$$



$F_X(2.5) = P\{(X \leq 2.5)\} = \frac{2}{6}$

$F_X(3.8) = P\{X \leq 3.8\} = \frac{3}{6}$

$F_X(5) = P\{X \leq 5\} = \frac{5}{6}$

$F_X(6.2) = P\{X \leq 6.2\} = 1 \Rightarrow$ all possible event

$F_X(0.8) = P\{X \leq 0.8\} = 0 \Rightarrow$ no. possible event

Properties of CDF

① $0 \leq F_X(x) \leq 1$

② $F_X(-\infty) = P\{X \leq -\infty\} = 0$

③ $F_X(\infty) = 1$

④ $F_X(x_1) \leq F_X(x_2)$; if $x_2 > x_1$

it is always a Non decreasing function of x

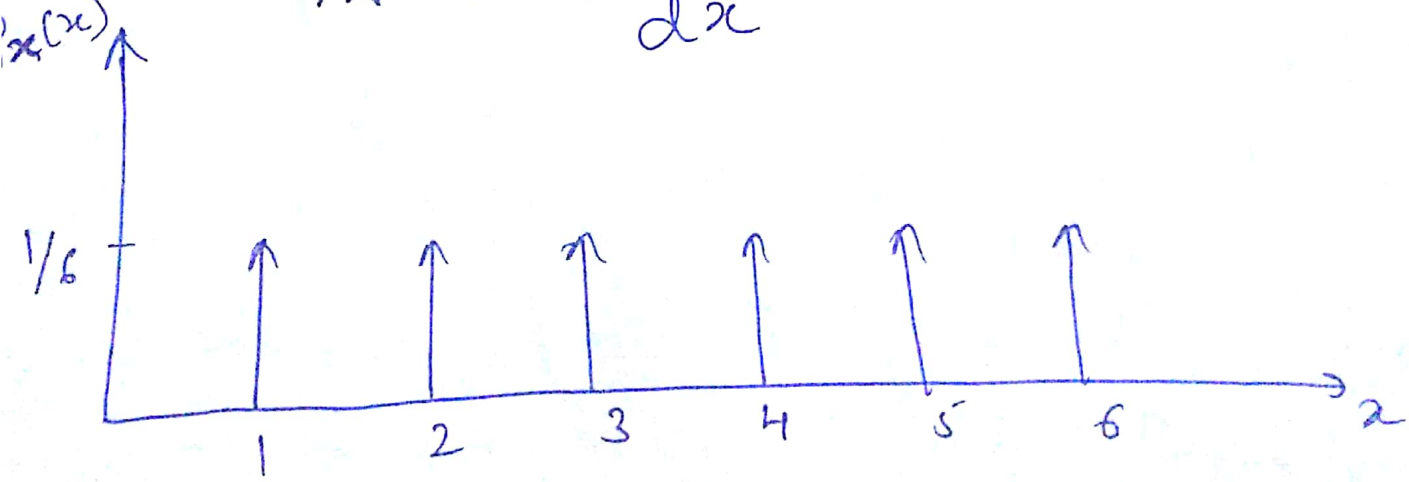
⇒ CDF is non decreasing fn of x

PDF

Probability density or distribution function

→ a CRV can't be described by any mathematical eq. but we can definitly describe the probability of CRV lying within an Range around any possible value by a Mathematical function. This analytical function is known as PDF and is given by

$$P_X(x) = \frac{dF_X(x)}{dx}$$



PDF of outcomes of Dice

Property of PDF

(1) $f_X(x) \geq 0$

Since CDF is a non decreasing fn. of x so its differenciation will be always ≥ 0

(2) Area under the PDF curve is always equal to 1

$$\int_{-\infty}^{\infty} f_X(x)\,dx = 1$$

$$\int_{-\infty}^{\infty} \frac{dF_X(x)}{dx}\,dx = F_X(\infty) - F_X(-\infty)$$
$$= 1 - 0 = 1$$

(3) $F_X(x) = \int_{-\infty}^{x} f_X(x)\,dx$

(4) $\boxed{P\{x_1 \leq X \leq x_2\} = \int_{x_1}^{x_2} f_X(x)\,dx}$

Q Consider a Random variable X having CDF given by $F_X(x) = 1 - e^{-0.01x}$; $x \geq 0$
Compute probability that random variable has $P\{X \geq 100\}$

$$f_X(x) = 0.01\, e^{-0.01x}$$

$$P(X \geq 100) = \int_{100}^{\infty} 0.01\, e^{-0.01x}\,dx$$

$$= -\frac{0.01}{0.01}\left(e^{-0.01x} \Big|_{100}^{\infty} \right)$$

$$= e^{-1}$$

Method ②

$$P\{X \geq 100\} = \int_{100}^{\infty} f_X(x)\, dx$$

$$= \int_{100}^{\infty} \frac{d F_X(x)}{dx}\, dx$$

$$= F_X(x)\Big|_{100}^{\infty} = F(\infty) - F_X(100)$$

$$= 1 - (1 - e^{-1})$$

$$= e^{-1} = \frac{1}{e}$$

Q Consider a Random variable X whose PDF is given by $f_X(X) = a e^{-b|x|}$ calculate

(a) Relation b/w a & b

(b) Value of CDF for $x < 0$

(c) value of CDF for $x > 0$

$$f_X(x) = \begin{cases} a e^{bx} & ; x < 0 \\ a e^{-bx} & ; x > 0 \end{cases}$$

area under curve

$$\int_{-\infty}^{0} a e^{bx}\, dx + \int_{0}^{\infty} a e^{-bx}\, dx = 1$$

$$\frac{a}{b}(1) + \left(+\frac{a}{b}\right)(1) = 1$$

$$\frac{2a}{b} = 1$$

$$2a = b$$

(b) $\cancel{\int_{-\infty}^{0} \frac{a}{b} e^{bx}\, dx = \frac{a}{b^2}[1-0] = \frac{a}{b^2}}$

(c) $\cancel{\int_{0}^{\infty} -\frac{a}{b} e^{-bx}\, dx = \frac{a}{b^2}[}$

(b) $F_X(x) = \int_{-\infty}^{x} a e^{bx} dx \quad ; x < 0$

$$= \frac{a}{b} e^{bx}$$

(c) $F_X(x) = \int_{-\infty}^{0} a e^{bx} dx + \int_{0}^{x} a e^{-bx} \cdot dx \quad ; x > 0$

$$= \frac{a}{b} + \frac{a}{-b}\left(e^{-bx} - 1\right)$$

$$= \frac{a}{b} + \frac{a}{b}\left(1 - e^{-bx}\right)$$

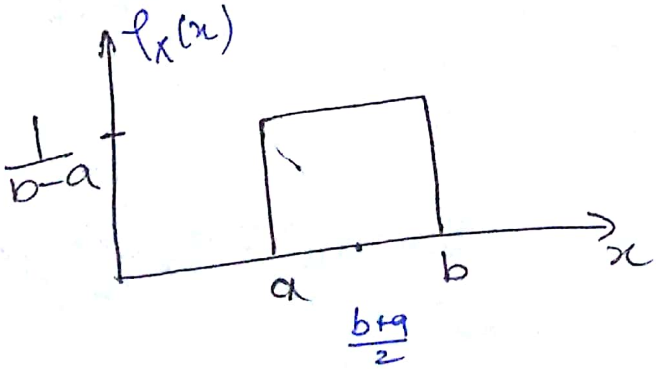
$$= \frac{a}{b}\left(2 - e^{-bx}\right)$$

Statistical Averages

(1) Mean or First Moment

mean is a measure of where distribution curve is centered and is given by expectation of X

$$\boxed{E[X] = m_x = \int_{-\infty}^{\infty} x \cdot f_X(x)\, dx}$$



$$m_x = \frac{1}{b-a} \int_{a}^{b} x\, dx =$$

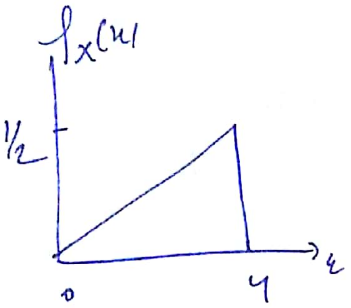
$$= \frac{1}{b-a} \left| \frac{x^2}{2} \right|_a^b$$

$$= \frac{b+a}{2}$$

② Mean square value or second moment

the mean square value or second moment is given by expectation of X^2

$$\boxed{E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx}$$

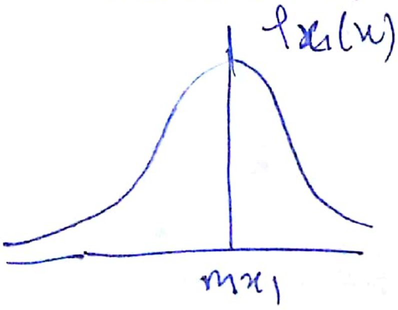


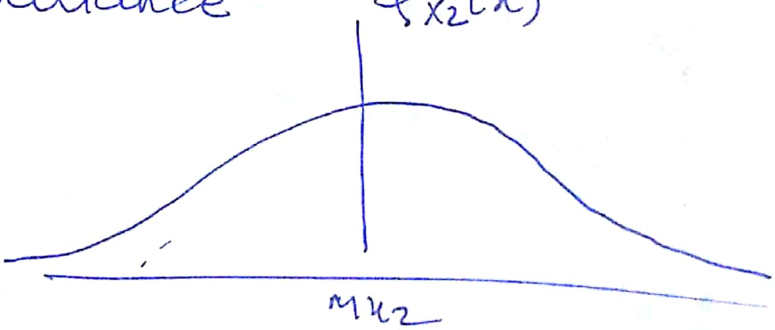
$f_X(x) = \frac{1}{8}x$

$$E(X^2) = \frac{1}{8}\int_0^4 x^3\, dx = \frac{1}{8}\left|\frac{x^4}{4}\right|_0^4 = 8$$

③ Variance or Second Central Moment

→ it measures the spread of distribution about its mean. The less the spreading, smaller is the variance





$$\boxed{\sigma_x^2 = E[X - m_x]^2 = \int_{-\infty}^{\infty} (x - m_x)^2 \cdot f_X(x)\, dx}$$

properties of Expectation operator E[ ]

① Expectation of a constant is constant itself' eg → $E[C] = C$

$$E[C] = \int_{-\infty}^{\infty} C f_X(x)\,dx = C \underbrace{\int_{-\infty}^{\infty} f_X(x)\,dx}_{\text{area under the curve} = 1}$$

$$= C$$

② Expectation of a constant time Random variable is equal to the constant times the expectation of Random variable

$$\boxed{E[CX] = C E[X]}$$

— homogeneity property

$$E[CX] = \int_{-\infty}^{\infty} C x f_X(x)\,dx$$

$$= C E(X)$$

③ Expectation of sum of Random variable is equal to sum of expectation

$$E\left[\sum_i X_i\right] = \sum_i E[X_i]$$

$$= \int_{-\infty}^{\infty} \sum_i X_i f_x(x)\,dx$$

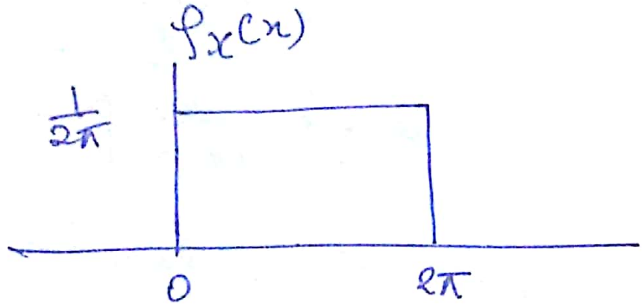
$$= \sum_i \int_{-\infty}^{\infty} X_i f_X(x)\,dx$$

$$= \sum_i E[X_i]$$

_additivity

"expectation operator is a Linear operator"

Q Consider a Random variable X uniformly distributed b/w 0 & 2π calculate
(a) mean value (b) mean sq. value
(c) variance (d) $E[\cos x]$



(a) mean = π

(b) mean sq. val = $\frac{1}{2\pi}\int_0^{2\pi} x^2\,dx = \left|\frac{x^3}{3}\right|_0^{2\pi}\frac{1}{2\pi}$

$= \left(\frac{8\pi^3}{3}\right)\frac{1}{2\pi}$

$= \frac{4\pi^2}{3}$

(c) variance = $\int_0^{2\pi}\frac{(x-\pi)^2}{2\pi}\,dx = \frac{1}{2\pi}\left[\frac{(x-\pi)^3}{3}\right]_0^{2\pi}$

$= \frac{1}{2\pi \times 3}\left[\left(\frac{2\pi^3}{3}-\frac{3\pi}{3}\right)^3\right]$

$\frac{1}{2\pi^3}\left(\frac{\pi}{3}\right)^3$

$\cancel{\frac{\pi^2}{3}}$

$= \frac{1}{2\pi}\left[\frac{\pi^3}{3}\right]$

$= \frac{\pi^2}{3}$

(d) $E[\cos X] = \frac{1}{2\pi}\int_0^{2\pi}\cos x\,dx = 0$

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Random Process [X(t)]

→ The time domain Representation of Random variable is known as Random process

→ The time description of Random variable is called as Random process

— There are two types of Random Process

① **Stationary Random process**

a Random process is said to be stationary if its statistical properties are not function of time

→ it is further of two types

(i) Strictly stationary Random process

→ if the R.P. has the same statistical properties that is mean, pdf, autocorrelation function (ACF) etc. at any instant of time then x(t) is stationary in strict sense.

(ii) Wide sense stationary R.P. (WSS)

→ if only mean & auto correlation fn. are stationary then X(t) is said to be WSS

"ALL strictly stationary R.P. are WSS but converse is not true"

② Non stationary Random process

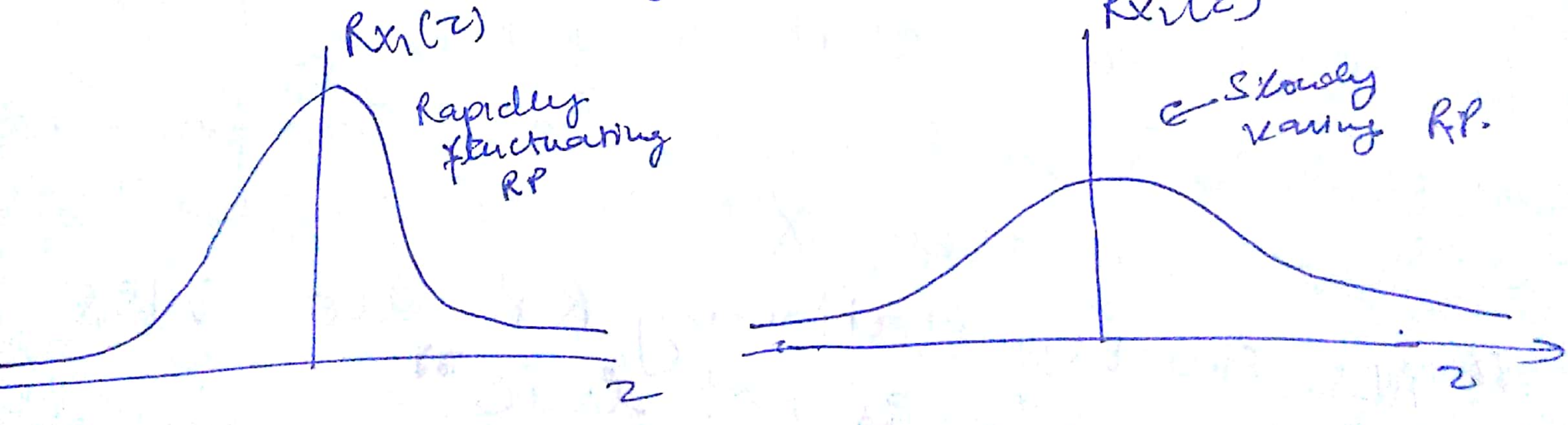
if statistical properties are function of time then the process is non-stationary

Auto correlation function (ACF)

→ It is also known as Ensemble ACF. the ACF of a Random process $X(t)$ is given by $R_X(\tau) = E[X(t) \cdot X(t+\tau)]$

physical significance

the physical significance of ACF is that it provides the mean of describing the interdependence of RP at time τ sec. apart it is therefore clear that more rapidly the Random process changes with time, more Rapidly will the ACF decrease from its max. value



Q Consider a Random process X(t) given by $A\cos(2\pi f_c t + \theta)$ where θ is a uniformly distributed Random variable b/w 0 and 2π. Calculate its autocorrelation fn.

$$X(t) = A\cos(2\pi f_c t + \theta)$$

$$R_X(\tau) = E\left[A\cos(2\pi f_c t + \theta) \cdot A\cos(2\pi f_c t + 2\pi f_c \tau + \theta)\right]$$

$$= E\left[\frac{A^2}{2}\left(\cos(4\pi f_c t + 2\pi f_c \tau + 2\theta) + \cos(2\pi f_c \tau)\right)\right]$$

$$R_X(\tau) = \underbrace{E\left[\frac{A^2}{2}\cos(4\pi f_c t + 2\pi f_c \tau + 2\theta)\right]}_{=0} + E\Big[\underbrace{\frac{A^2}{2}\cos(2\pi f_c \tau)}_{\text{const}}\Big]$$

$$R_X(\tau) = \frac{A^2}{2}\cos(2\pi f_c \tau)$$

Properties of ACF

→ Correlation has its max value at $\tau = 0$

→ $\boxed{R_X(0) \geq R_X(\tau)}$

→ the autocorrelation func. is always a even function of time τ

$$\boxed{R_X(-\tau) = R_X(\tau)}$$

→ the mean square value of Random process may be obtained from $R_X(\tau)$ by simply putting $\tau = 0$

$$R_X(0) = E[X^2(t)] = \text{mean square value of R.P.}$$

Q

if $R_x(\tau)$ is autocorrelation func. of a zero mean WSS R.P. $X(t)$ then which one of the following is not true

(a) $R_x(\tau) = R_x(-\tau)$

(b) $R_x(\tau) = -R_x(-\tau)$

(c) $\boxed{\sigma_x^2 = R_x(0)}$ for mean = 0

d) $|R_x(\tau)| \leq R_x(0)$

Ergodic Random process

A R.P. is said to be ergodic. when

1) → time average of a Random process becomes equals to ensemble Mean.

$$\frac{1}{2T}\int_{-T}^{T} x(t)\,dt = m_x$$

→ when time averaged autocorrelation function becomes equal to ensemble ACF

$$\frac{1}{2T}\int_{-T}^{T} x(t)\,x(t+\tau)\,dt = R_x(\tau) = E[x(t)\,x(t+\tau)]$$

Properties of Ergodic RP

① The ensemble mean m_x gives the dc component of a Random Process

② $[m_x]^2$ is the dc power of ergodic random process

③ mean square value of Random process is the total average power of ergodic R.P. at $R_x(\tau)$ $\underline{\underline{\tau=0}}$

$$R_X(0) = E[X^2(t)] = \frac{1}{2T}\int_{-T}^{T} X^2(t)\,dt$$

$$\sigma_x^2 = E[(X - m_x)^2] = \int_{-\infty}^{\infty} (x - m_x)^2 f_x(x)\,dx$$

$$= \int_{-\infty}^{\infty} x^2 f_x(x)\,dx + \int_{-\infty}^{\infty} m_x^2 \cdot f_x(x)\,dx - 2\int_{-\infty}^{\infty} m_x x \cdot f_x(x)\,dx$$

$$= \int_{-\infty}^{\infty} x^2 f_x(x)\,dx + m_x^2 \int_{-\infty}^{\infty} f_x(x)\,dx - 2m_x\int_{-\infty}^{\infty} x\, f_x(x)\,dx$$

$$= E[X^2] + m_x^2 - 2m_x m_x$$

$$\sigma_x^2 = E[X^2] - m_x^2$$

σ_x^2 ↓ ac power; $E[X^2]$ ↓ total average power; m_x^2 ↓ dc power

at $m_x = 0$

$$\sigma_x^2 = E[X^2]$$

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Q a stationary RP $X(t)$ has a au... given by $R_x(\tau) = 20 + 5\cos 2\tau + 10e^{-4|\tau|}$

calculate (a) Total average power (b) dc power (c) ac power (d) mean (e) variance (f) rms value of random process.

(a) $\int_{-\infty}^{\infty}$ $R_X(0) = 35$

(b) 20

(c) 15

(d) $\sqrt{20}$

(e) 15

(f) $\sqrt{35}$

Q. $x(n) \xrightarrow{\text{variance}} \sigma_x^2$

$d(n) \xrightarrow{\text{"}} \frac{\sigma_x^2}{10}$

$E[d^2(n)] = E[(x(n) - x(n-1))^2]$

$= E[x^2(n)] + E[x^2(n-1)] - 2E[x(n)\cdot x(n-1)]$

$\frac{\sigma_x^2}{10} = \sigma_x^2 + \sigma_x^2 - 2(R_x(1))$

$\frac{\sigma_x^2}{10} - \frac{20\sigma_x^2}{10} = -2R_x(1)$

$\frac{-19\sigma_x^2}{10} = -2R_x(1)$

$\frac{R_x}{\sigma_x^2} = \frac{19}{20}$

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Wiener - Khinchine theorem

It states that autocorrelation function & Power spectral density (PSD) forms the Fourier transform pair

$$R_x(\tau) \xleftrightarrow{FT} PSD$$

$$S_x(\omega) = \int_{-\infty}^{\infty} R_x(\tau) e^{-j\omega\tau} d\tau$$

$$= \int_{-\infty}^{\infty} R_x(\tau) e^{-j2\pi f\tau} d\tau$$

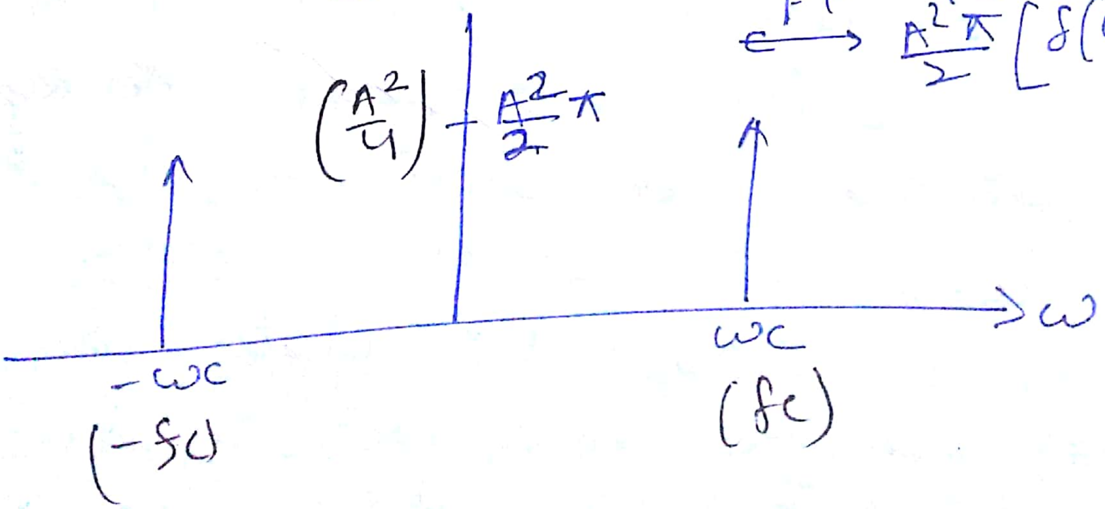
$$R_x(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_x(\omega) e^{j\omega\tau} d\omega$$

$$= \int_{-\infty}^{\infty} S_x(f) e^{j2\pi f\tau} df$$

$$X(t) = A\cos(2\pi f_c t + \theta)$$

$$R_x(\tau) = \frac{A^2}{2} \cos 2\pi f_c \tau \xleftrightarrow{FT} \frac{A^2}{4}\left[\delta(f-f_c) + \delta(f+f_c)\right]$$

$$\xleftrightarrow{FT} \frac{A^2\pi}{2}\left[\delta(\omega-\omega_c) + \delta(\omega+\omega_c)\right]$$



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Power spectral density PSD

1. Power spectral density is a measure of how total average power is distributed w.r.t freq. & is defined as "power / unit BW" in watt/HZ or watt/Rad/sec

→ It is also called as power spectrum

Let us consider a Random process x(t) whose power spectral density curve is shown above as it can be clearly seen that the max. noise power is concentrated with in low freq and as we go higher on freq's, the effect of noise go on reducing & Reducing & after a freq. f1, the noise completely disappear so system can be optimised accordingly. If it has to be used at lower freq's, the sig power should be high but if the system can be used at higher freq, only a small amount of sig power can be used to have high SNR.

Properties of PSD

① the zero freq value of PSD of a WSS R.P. equals the total area under the graph of autocorrelation function.
(central ordinate theorem)

$$S_X(0) = \int_{-\infty}^{\infty} R_X(\tau)\, d\tau$$

total area under the curve.

② The mean square value of WSS R.P. equals the total area under the graph of power spectral density

$$E[X^2(t)] = R_X(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_X(\omega)\, d\omega$$
$$= \int_{-\infty}^{\infty} S_X(f)\, df$$

③ the PSD is always non-negative

$$S_X(\omega) \geq 0$$
$$S_X(f) \geq 0$$

$$PSD = \frac{Power}{BW}$$

④ PSD is always an even function of freq.

i.e. $$S_X(-\omega) = S_X(\omega)$$
$$S_X(-f) = S_X(f)$$

since ACF is an even fn^t of time τ so its FT is also a even fn^t

Q

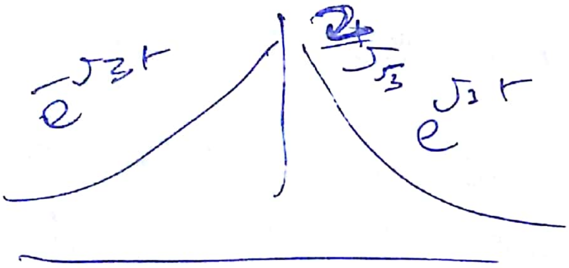
a WSS R.P. $x(t)$ has power spectrum given by

$$S_X(\omega) = \frac{4}{\omega^2+3}$$

Calculate (a) ~~auto~~ - ACR ACF $R_x(\tau)$ $\qquad e^{-a|t|} = \frac{2a}{\omega^2+a^2}$

(b) total avg pw

(a) - $\frac{2}{\sqrt{3}} e^{-\sqrt{3}|t|}$



(b) - $R_X(0) = \frac{2}{\sqrt{3}}$

$\delta\ t \to a$

$2\pi \to 2a$

Q given the following power spectrum of a R.P. $X(t)$

$$S_X(\omega) = 8\pi \cdot \delta(\omega) + 1.2\pi\,\delta(\omega-2.3\pi) + 1.2\pi\,\delta(\omega+2.3\pi) + 4\pi\,\delta(\omega-5) + 4\pi\,\delta(\omega+5)$$

Calculate

(a) $R_X(\tau) = 1.2\cos 2.3\tau + 4\cos 5\tau + 4$

(b) Total average power, dc, ac power

9.2 $\qquad$ 4 $\qquad$ 5.2

Q Consider the two spectrums given by

$S_1(\omega) = e^{-(\omega-2)^2}$ ⇒ no. It is not even func of freq.

$S_2(\omega) = \frac{\omega^2}{1+2\omega^2+j\omega^4}$ ⇒ No, it is not power spectrum function
it is a imaginary function

since $S_2(\omega)$ has an imaginary part so it ~~can't~~ be PSD because PSD is always a Real function

Q $S_1(\omega) = \frac{\cos(2\omega)}{1+2\omega^2+\omega^4} = \frac{\cos(2\omega)}{(\omega^2+1)^2}$ since $\cos 2\omega$ can be negative so it can't be PSD

$S_2(\omega) = \frac{\omega^2+5}{(\omega^2+1)(\omega^2+3)}$

it is real, non negative and even also
so it is PSD

Assignment

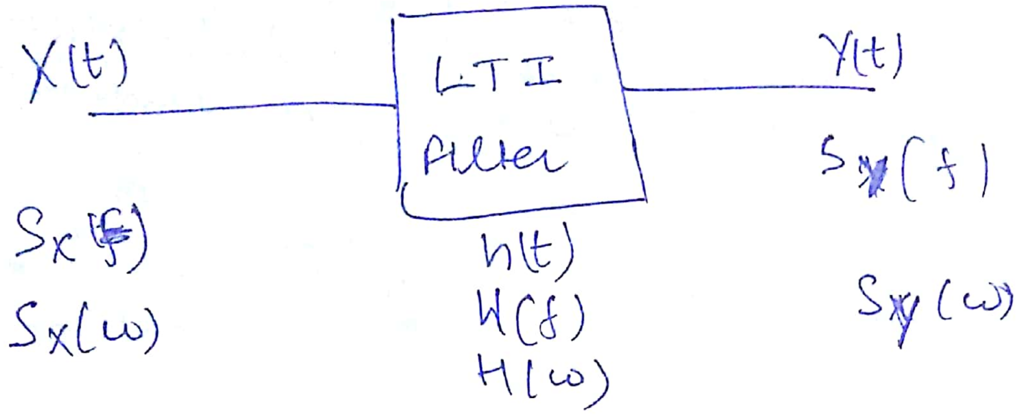
a R.P. $x(t)$ has a power spectrum given by

$S_x(\omega) = \frac{\omega^2+4}{(\omega^4+8\omega^2+7)}$

Calculate ⓐ ACF ⓑ Total average power

Inverse Fourier of $S_x(\omega)$

Transmission of a Random process through linear filter



it states that mean square value of the o/p of stable LTI filter in Responce to wss process is equal to the integral over all freq.es of the PSD of input Random process multiplied by square magnitude of freq. Responce of filter

$$\boxed{E[Y^2(t)] = \int_{-\infty}^{\infty} S_X(f) \cdot |H(f)|^2 \, df}$$

$$E[Y^2(t)] = \int_{-\infty}^{\infty} S_Y(f) \, df$$

$$\boxed{S_Y(f) = S_X(f) |H(f)|^2}$$

$$\boxed{E(Y^2(t)) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_X(\omega) \, |H(\omega)|^2 \, d\omega}$$

$$E(Y^2(t)) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_Y(\omega) \, d\omega$$

$$\boxed{S_Y(\omega) = S_X(\omega) |H(\omega)|^2}$$

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Q a R.P. $x(t)$ with PSD :

$$S_X(f) = \frac{1}{2}\left[\delta\left(f-\frac{1}{2\pi}\right)+\delta\left(f+\frac{1}{2\pi}\right)\right]$$

is the input to a linear system having freq. Responce $H(f) = \frac{1}{1+j2\pi f}$

Calculate the average power of o/p random process.

$$= \int_{-\infty}^{\infty} \frac{1}{2}\left[\delta\left(f-\frac{1}{2\pi}\right)+\delta\left(f+\frac{1}{2\pi}\right)\right]\left|\frac{1}{1+j2\pi f}\right|^2 df$$

$$= \int_{-\infty}^{\infty} \frac{1}{2}\left[\delta\left(f-\frac{1}{2\pi}\right)+\delta\left(f+\frac{1}{2\pi}\right)\right]\frac{1}{1+4\pi^2 f^2}\, df$$

$$= \frac{1}{2}\left[\frac{1}{1+4\pi^2\left(\frac{1}{2\pi}\right)^2}+\frac{1}{1+4\pi^2\left(\frac{1}{2\pi}\right)^2}\right]$$

$$= \frac{1}{1+1} = \frac{1}{2}$$

Q consider a Random process $x(t)$ with ACF $R_X(\tau) = e^{-a|\tau|}$ with $a \ll \omega$ the processed is passed through an ideal differentiator the approximate PSD at o/p $Y(t)$

$$\boxed{\frac{d}{dt}}$$

$j\omega$

$$S_X(f) = \frac{2a}{\omega^2+a^2} = \frac{2a}{\omega^2} = \cancel{\frac{2a}{\omega^3}} = \cancel{\frac{4a}{\omega^3}}$$

$$\int_{-\infty}^{\infty} \frac{2a}{\omega^2+a^2}|j\omega|^2\, d\omega = \int_{-\infty}^{\infty} \frac{2a\,\omega^2}{\omega^2+a^2} = 2a$$

$$\boxed{H(\omega) = j\omega} \qquad |H(\omega)|^2 = \omega^2$$

$$S_Y(\omega) = S_X(\omega)\,|H(\omega)|^2$$

$$= \frac{2a}{\omega^2+a^2}\,\omega^2 = 2a$$

Assignment

Q. a system with TF $H(s) = \dfrac{s-1}{s^2+3s+2}$ is excited by a Random process having uniform power spectral density N_o, the ACF of o/p process $Y(t)$ is given by ....

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Gaussian PDF

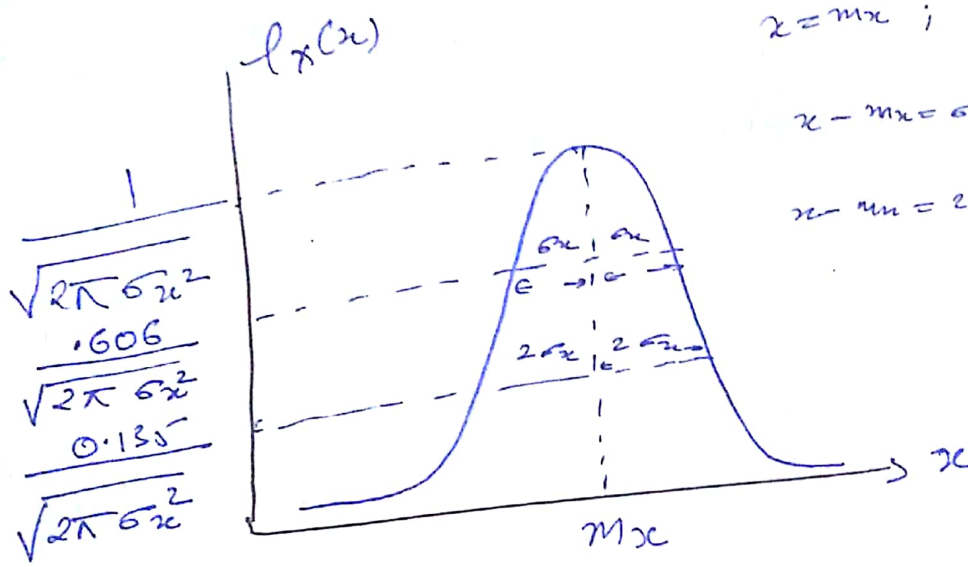
Gaussian PDF provides a good mathematical model for various physically observed random phenomena and is given by

$$\boxed{f_X(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}} \exp\left[\frac{-(x-m_x)^2}{2\sigma_x^2}\right]}$$

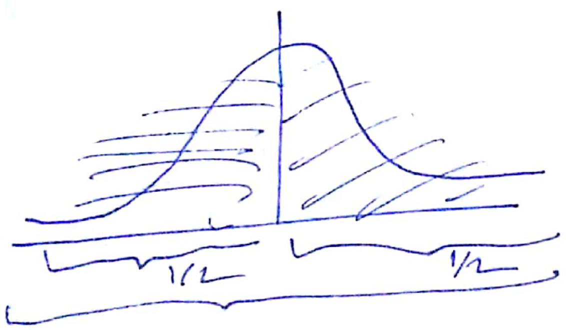
$x = m_x$; $f_X(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}}$

$x - m_x = \sigma_x$; $f_X(x) = \frac{.606}{\sqrt{2\pi\sigma_x^2}}$

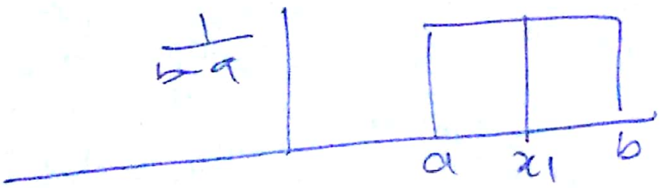
$x - m_x = 2\sigma_x$; $f_X(x) = \frac{e^{-2}}{\sqrt{2\pi\sigma_x^2}}$



Q For a zero mean gaussian random variable, the probability $P\{X \geq 0\}$ will be. $\frac{1}{2}$



Q ②



$$P\{X = x_1\} = \frac{1}{b-a}\int_{x_1}^{x_1} dx$$

$$= \frac{1}{b-a}(x_1 - x_1)$$

$$= 0$$

"For a continuous R.V., the probability at a single pt is always equal to Zero"

Not for Discrete

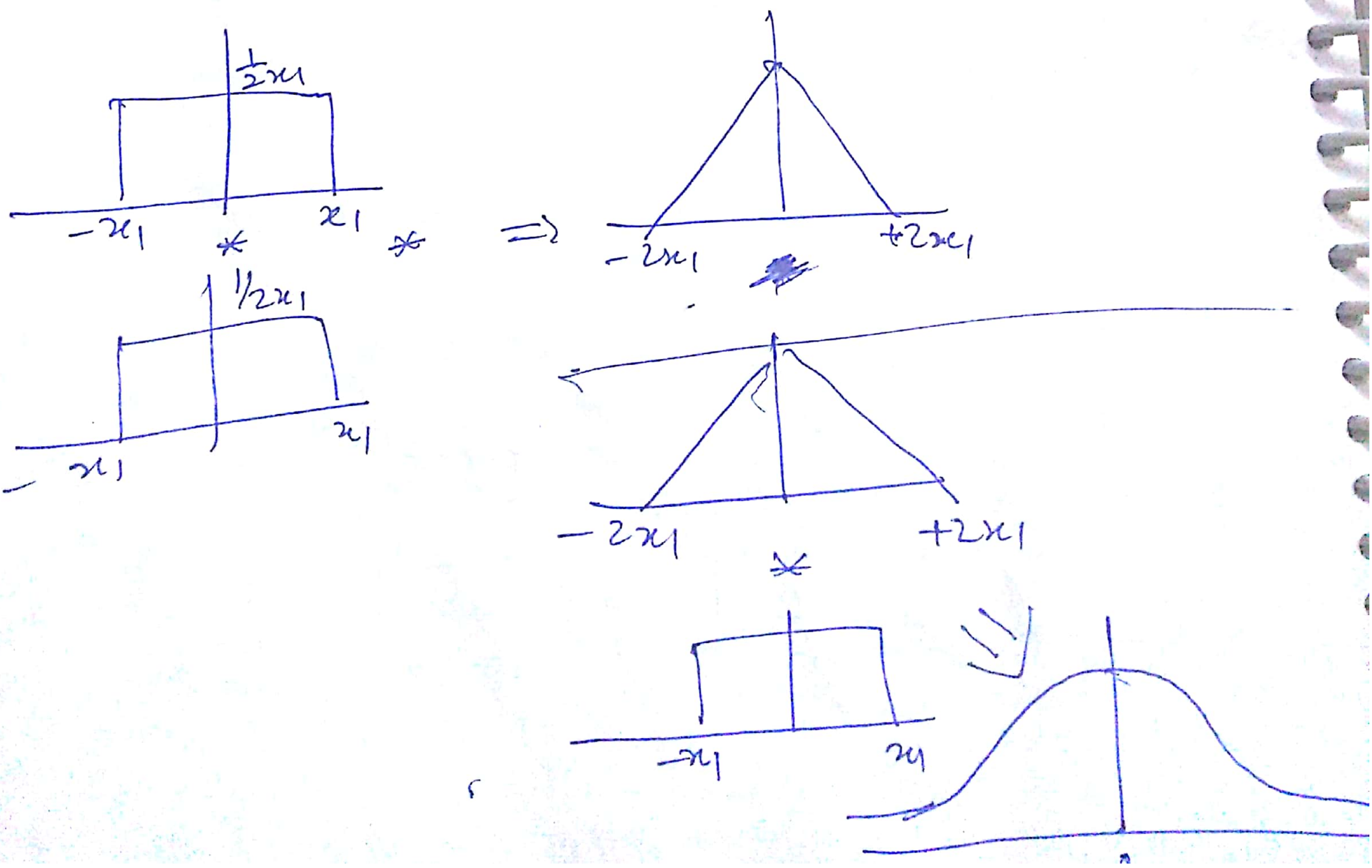
!!!!

Central Limit Theorem

It states that the P.D. function of a sum of "N" independent Random variables tend to approach a gaussian density as the no. N increases. This theorem applies even ~~when~~ individual R.V. are not gaussian

$$\sum_{i=1}^{N} X_i = X_1 + X_2 + X_3 + \dots + X_N = \text{Gaussian } \underline{R.V.}$$

$$f_{X_1}(x) * f_{X_2}(x) * f_{X_3}(x) \dots * f_{X_N}(x) = \underline{\text{Gaussian PDF}}$$



Q a voltage $V(t)$ which is a gaussian ergodic R.P. with a mean zero and a variance of $4V^2$, is measured by (a) a dc meter (b) a true RMS meter (c) a meter which squares $V(t)$ and reads its dc component. Calculate the o/p of each meter

$$E|V(t)^2| = \sigma_v^2 = 4V^2$$

<u>mean of square</u>

$\overline{V^2 t}$ = total average power

(a) 0

(b) 2 V

(c) $4V^2$

$|\overline{V(t)}|^2$ = dc power

<u>square of mean</u>