Syllabus

Digital Communication
(ETEC-303)

UNIT I

Introduction to Digital Communication: Line coding: NRZ, RZ, Manchester encoding, differential Manchester encoding, AMI coding, high density bipolar code, binary with n-zero substitution codes, Review of Sampling theorem, uniform and non-uniform quantization, companding, i-Law and A-Law compressors, Concept and Analysis of PCM, DPCM, DM and ADM modulators and demodulators, M-ary waveforms, S/N ratio for all modulation, probability of error for PCM in AWGN Channel and other modulation techniques, Duo Binary pulse.

[T1, R2][No. of Hours: 11]

UNIT- II

Random Signal Theory: Probability, Concept of Random variable (Stationary, Non stationary, WSS, SSS), Random process, CDF, PDF, Joint CDF, Joint PDF, marginal PDF, Mean, Moments, Central Moment Auto-correlation & Cross-correlation, covariance functions, ergodicity, power spectral density, Gaussian distribution, Uniform distribution, Rayleigh distribution, Binomial distribution, Poission distribution, Weiner distribution, Wiener-Khinchin theorem, Central limit theorem.

[T1, T2, R2] [No. of Hours: 11]

UNIT- III

Designing of Receiver: Analysis of digital receiver, Prediction Filter, Design and Property of Matched filter, Correlator Receiver, Orthogonal Signal, Gram-Schmidt Orthogonalization Procedure, Maximum likelihood receiver, Coherent receiver design, Inter Symbol Interference, Eye Pattern.

[T1, T2, R1, R2] [No. of Hours: 11]

UNIT- IV

Digital Modulation Schemes: Coherent Binary Schemes: ASK, FSK, PSK, QPSK, MSK, G-MSK. Coherent M-ary Schemes, Incoherent Schemes (DPSK and DEPSK), Calculation of average probability of error for different modulation schemes, Power spectra of digitally modulated signals, Performance comparison of different digital modulation schemes. Review of 2 Latest Research Paper.

[T1, T2, R2][No. of Hours: 11]

FIRST TERM EXAMINATION

FIFTH SEMESTER [B.Tech.], September, 2016

Digital Communication (ETEC-303)

Time : 1.5 hours *Maximum Marks : 30*

Note: ***Question One is Compulsory. Attempt any two from the rest of the Questions.***

Question 1. ***(2×5=10)***

(a) ***Explain the terms***
  (i) ***Probability***
  (ii) ***Random Process***

(b) ***Discuss the variance and standard deviation of random variables.***

(c) ***What is probability of error.***

(d) ***Discuss the Gaussian Process.***

(e) ***What is scrambling.***

Solution:

(a) Refer to Q. 1(*a*) First Term 2015.

(b) Expectation can be used to describe properties of the *PDF* or *PMF* of random variables. If we let $g(X) = X^n$ for $n = 1, 2, 3, \ldots$, the equantity

$$\mathcal{E}\{X^n\} = \int_{-\infty}^{\infty} x^n fx(x)dx$$

for a continuous random variable or

$$\mathcal{E}\{X^n\} = \sum_{\forall k} k^n f_K[k]$$

for a discrete random variable is called the n^{th} moment of the distribution. The first moment ($n = 1$) is called the mean, and represents the average value of the random variable for the given *PDF*.

Second and higher order moments further characterize the *PDF*.

X^2 represents the "power" associated with the signal; therefore, the second moment $\mathcal{E}\{X^2\}$ represents average signal power.

When the mean of a random variable is not 0, then the moments centered about the mean are generally more useful than the moments defined above. These central moments are defined by

$$\mathcal{E}\{(X - m_X)^n\} = \int_{-\infty}^{\infty} (x - m_X)^n fx(x)dx \quad (a) \text{ CONTINUOUS RV}$$

$$\mathcal{E}\{(K - m_K)^n\} = \sum_{\forall k} (k - m_K)^n f_K[k] \quad (b) \text{ DISCRETE RV}$$

The second central moment is callled the variance and is given by

$$\boxed{\sigma_X^2 = \text{Var}[X] = \mathcal{E}\{(X - m_X)^2\}}$$

The square root of the variance (σ_X) is called the standard deviation and has the same units as the random variable X. The quantity σ_X is important because it is a measure of the width or spread of the distribution.

(c) The probability of error in a transmitted bit is a statistical property. The performance of any communication system in the presence of channel noise is evaluated by computing the average probability of error. The measurement or prediction of errors can be expressed in various statistical ways but four main parameters have been traditionally used:

- **Bit error rate (or ratio) (*BER*):** ratio of errored bits to the total transmitted bits in some measurement interval.
- **Error-free seconds (*EFS*) or error seconds (*ES*):** percentage or probability of one-second measurement intervals that are error free (*EFS*) or in error (*ES*)
- **Percentage of time that the *BER* does not exceed a given threshold value:** percentage of specified measurement intervals (say, 1 min) that do not exceed a given BER threshold.
- **Error-free blocks (*EFB*):** percentage or probability of data blocks that are error free.

(d) Refer to Q. 3(*b*) End Term 2013.

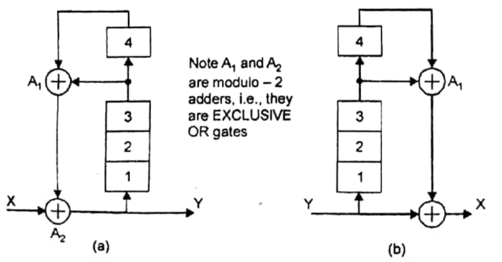
(e) Scrambling is basically a process of randomizing the binary message bit stream at the transmitting end. It has the following beneficial effects.

(*i*) It eliminates long strings of zeros or ones which might affect the receiver synchronization.

(*ii*) It makes it difficult to have unauthorized access to the data being transmitted.

It is of course necessary to undo, at the receiving end, the scrambling done at the transmitter to restore the original bit sequence of the bit stream as it existed before the scrambling.

As shown in Fig. the scrambler consists of a feedback shift register while the unscrambler consists of a feed-forward shift register.



(a) Scrambler (b) Unscrambler

Question 2.

(a) Draw the different Line Code Waveforms for the data '00110110' ***(10)***

(i) NRZ bipolar code

(ii) RZ unipolar

(iii) AMI

(iv) Manchester coding

(v) Differential Manchester coding

OR

(b) Explain the PDF and CDF and derive the relationship between CDF and PDF.

Solution:

Refer to Q. 2 First Term 2015.

Question 3.

(5+5=10)

(a) Discuss in detail modulation and demodulation in adaptive data modulation.

(b) A signal having bandwidth equal to 4 kHz is sampled, quantized and coded by a PCM system. The coded signal is then transmitted over a trasnmission channel of supporting a transmission rate of 100 K bits/sec. Determine the maximum signal to noise ratio that can be obtained by this system. The input signal has peak to peak value of 6 volts and rms value of 0.4 V.

Solution:

Refer to Q. 3 First Term 2015

Question 4.

(5+5=10)

(a) Discuss the Binomial Distribution and derive expression for PDF and CDF of Binomial Distribution.

(b) Discuss the physical significance of PSD and its different properties. Also explain Central Limit Theorem.

Solution:

Refer to Q. 4 First Term 2015.

SECOND TERM EXAMINATION

FIFTH SEMESTER [B.Tech.], Novermber, 2016
Digital Communication (ETEC-303)

Time : 1.5 hours *Maximum Marks : 30*

Note: *Question One is Compulsory. Attempt any two from the rest of the Questions.*

Question 1. *(2×5=10)*

(a) Which parameter is called figure of merit of a digital communication system.

(b) How can BER of an system be improved.

(c) Draw the PSK waveform for 101010.

(d) What is Gram Schmidt ortogonalization procedure (GSOP)?

(e) What do you inter from the constellation representation of a signal?

Solution:

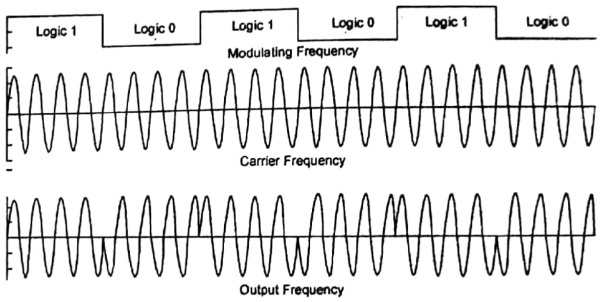
(*a*) In communication systems, message signal travels from the transmitter to the receiver via a channel (radio, or line). The channel introduces additive noise in the message and, hence the message reaching the receiver becomes corrupted. As the receiver detects both noise and message signals, it reproduces a noisy message at the output. The noise characteristics of a modulation system is evaluated by a parameter known as figure of merit denoted by γ. It is defined as the ratio of output signal to noise ratio to input signal to noise ratio of a receiver.

$$\gamma = \frac{S_o / N_o}{S_i / N_i}$$

(*b*) *BER* can be affected by a number of factors. By manipulating the variables that can be controlled, it is possible to optimize a system to provide the performance levels that are required.

- **Increase transmitter power:** It is also possible to increase the power level of the system so that the power per bit is increased.
- **Lower order modulation:** Lower order modulation schemes can be used, but this is at the expense of data throughput.
- **Reduce bandwidth:** Another approach that can be adopted to reduce the bit error rate is to reduce the bandwidth.
- **Interference:** The interference levels present in a system are generally set by external factors and cannot be changed by the system design.

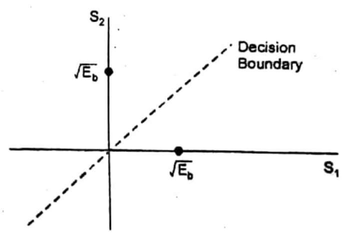
(*c*)



(*d*) Refer to Q. 2 (*a*) Second Term 2015.

(*e*) A constellation diagram is a representation of a signal modulated by a digital modulation scheme such as quadrature amplitude modulation or phaseshift keying. It displays the signal as a two-dimensional scatter diagram in the complex plane at symbol sampling instants. In a more abstract sense, it represents the possible symbols that may be selected by a given modulation scheme as points in the complex plane. Measured constellation diagrams can be used to recognize the type of interference and distortion in a signal.

Constellation diagram of *BFSK* is shown below:



Question 2. *(5+5=10)*

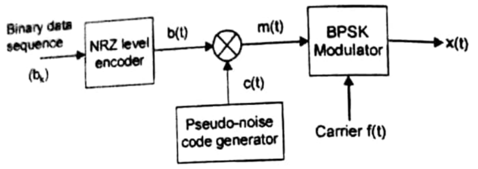
(a) Draw the block diagram of a direct sequence spread BPSK system. Explain their significance.

(b) Explain the basis of operation of a matched filter receiver with suitable diagrams. Derive the condition for which the output signal-to-noise ratio is maximized.

Solution:

(*a*) Fig. shows transmitter of Direct Sequence Spread Spectrum with *BPSK*.

As shown in the above figure, the binary data sequence is given to *NRZ* level encoder. This encoder converts b_k into bipolar *NRZ* waveform.



The pseudo-noise sequence generator generates and encodes this sequence in bipolar *NRZ* signal. The multiplier multiplies the two signals $b(t)$ and $c(t)$. The output of multiplier is direct sequence spread signal $m(t)$. This signal is given as modulating signal to *BPSK* transmitter. The direct sequence *BPSK* (or *DS/BPSK*) signal is generated at the output (i.e. $x(t)$). Let's say that the carries is represented as,

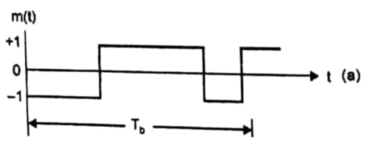
$$\phi(t) = \sqrt{2P_s}\,\sin(2\pi f_c t)$$

Then the transmitted signal is

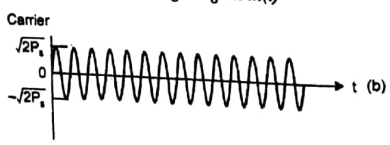
$$x(t) = \sqrt{2P_s}\,m(t)\sin(2\pi f_c t)$$

Thus when $m(t)$ is positive, there is phase shift of '0' and if it is negative, there is phase shift of 180°.

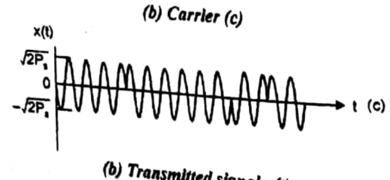
Fig. shows the waveform of message signal $m(t)$, carrier signal $\phi(t)$ and modulated signal $x(t)$.



(a) Messages signal m(t)



(b) Carrier (c)

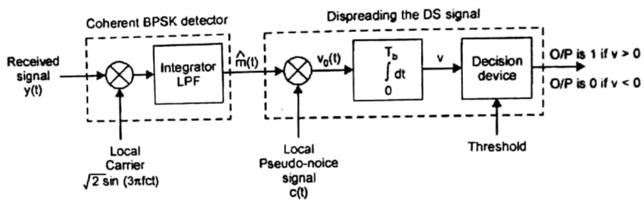


(b) Transmitted signal x(t)

Waveforms of DB/BPSK system

***DS-SS BPSK* Receiver**

Fig. shows the block diagram of *DS/BPSK* receiver. There are two stages of demodulation. The received signal $y(t)$ is applied to the multiplier which is also supplied with locally generated coherent carrier. The output of the multiplier is then applied the low pass filter (integrator). The bandwidth of this low pass filter is equal to that of $m(t)$.



Block diagram of DS/BPSK receiver or decoder

(*b*) Refer to Q. 4(*b*) End Term 2014.

Refer to Q. 4 End Term 2013.

uestion 3.

(4+6=10)

(a) Derive the expression for bit error probability due to a matched filter. Explain the basis of operation of a matched filter receiver with suitable diagrams.

(b) Highlight the difference between Binary orthogonal FSK and MSK modulation schemes. Draw their corresponding signal constellations and obtain an expression for the probability of bit error for both cases.

olution:

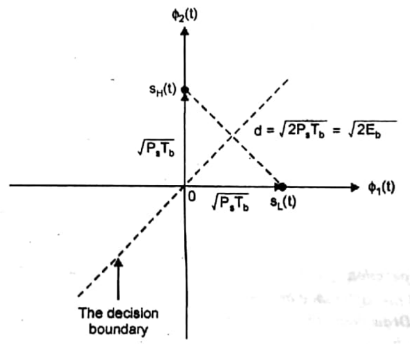
(*a*) Refer to Q. 3(*a*) Second Term 2015.

(*b*)

Parameter	BFSK	MSK
Information is transmiited by change in	Frequency	Frequency
Equation of the transmitted	$s(t) = \sqrt{2P_s}$ cos signal $s(t)$	$s(t) = b_0(t)\sin 2\pi$ $[(2\pi f_C + d(t)\Omega)f]$ $\left[f_C + b_e(t)b_0(t)\frac{f_b}{4}\right]t$ $b_e(t), b_0(t)$ = odd/even sequence
Bits per symbol	one	Two
Detection method	non coherent	coherent

Minimum Euclidean distance signal points	$\sqrt{2E_b}$	$2\sqrt{E_b}$
Minimum Bandwidth (BW)	$4f_b$	$1.5f_b$
Probability of error $P(e)$	$\frac{1}{2}\,erfc\sqrt{\frac{E_b}{2No}}$	$erfc\sqrt{\frac{E_b}{N_o}}$
Symbol duration (T_s)	T_b	$2T_b$

Binary OFSK



Probability of Error

Refer to Q. 6(b) End Term 2013.

MSK

Refer to Q. 7(b) End Term 2011.

Question 4.

(4+6=10)

(a) Explain Maximum likely hood receiver.

(b) Explain the QPSK modulation scheme with suitable transmitter and receiver block diagrams. Also derive the average probability of error in the presence of AWGN using the signal space approach.

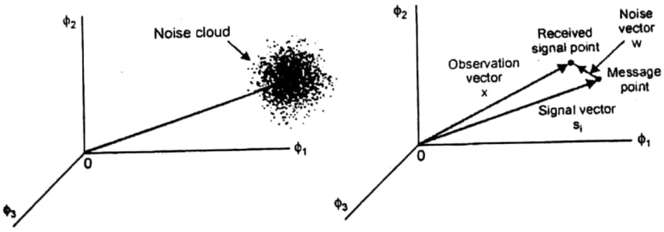
Solution:

(a) The Maximum Likelihood Receiver

The maximum likelihood receiver is that receiver that selects the most likely signal sent, given a waveform $r(t)$ that it has observed. Roughly speaking, the receiver tries to find the transmitted signal $s_i(t)$ that maximizes the conditional probability $f[s_i(t)/r(t)]$ (f denotes a probability density function). Heuristically, we can use Bayes' rule to express this probability as

$$\frac{f_R[r(t)/s_i(t)]\ \Pr[s_i(t)]}{f_R[r(t)]} \qquad (1)$$

Here we have imagined that the received function of time $r(t)$ has a density Suppose that in each time slot of duration T seconds, one of the M possible signals $s_1(t), s_2(t), \dots, s_M(t)$ is transmitted with equal probability, $1/M$. For geometric signal representation, the signal $s_i(t)$, $i = 1, 2, \dots, M$, is applied to a bank of correlators, with a common input and supplied with an appropriate set of N orthonormal basis functions. The resulting correlator outputs defien the signal $s_i(t)$ itself, and vice versa, we may represent $s_i(t)$ by a point in a Euclidean space of dimension $N \le M$. We refer to this point as the transmitted signal point or message point.



However, the representation of the received signal $x(t)$ is complicated by the presence of additive noise $w(t)$. We note that when the received signal $x(t)$ is applied to the bank of N correlators, the correlator outputs define the observation vector x.

the vector x differs from the signal vector s_i by the noise vector w whose orientation is completely random.

This presents a signal detection problem:

Given the observation vector x, perform a mapping from x to an estimate $\hat{m} = m_i$. The probability of error in this decision, which we denote by $P_e(m_i|x)$, is supply

$$P_e(m_i|x) = P(m_i \text{ not sent} | x)$$
$$= 1 - P(m_i \text{ sent} | x)$$

The decision-making criterion is to minimize the probability of error in mapping each giver. observation vector x into a decision. On the basis of Equation, we may therefore state the optimum decision rule:

Set $\hat{m} = m_i$ if

$$P(m_i \text{ sent} | x) \ge P(m_k \text{ sent} | x) \qquad \text{for all } k \ne i$$

where $k = 1, 2, \dots, M$. This decision rule is referred to as the maximum a posteriori probability (MAP) rule.

The condition of above Equation may be expressed in terms of the a priori probabilities of the transmitted signals and in terms of the likelihood functions. Using Bayes' rule we may restate the MAP rule as follows:

Set $\hat{m} = m_i$ if

$$\frac{p_k f_x(x|m_k)}{f_x(x)} \text{ is maximum for } k = i$$

where p_k is the a priori probability of transmitting symbol m_k. $f_x(x|m_k)$ is the conditional probability density function of the random observation vector X given the transmission of symbol m_k, and $f_x(x)$ is the unconditional probability density function of X. In Equation we may note the following:

- The denominator term $f_x(x)$ is independent of the transmitted symbol.
- The a priori probability $p_k = p_i$ when all the source symbols are transmitted with equal probability.
- The conditional probability density function $f_x(x \mid m_k)$ bears a one-to-one relationship to the log-likelihood function $l(m_k)$.

Accordingly, we may restate the decision rule of above Equation in terms of $l(m_k)$ simply as follows:

Set $\hat{m} = m_i$ if

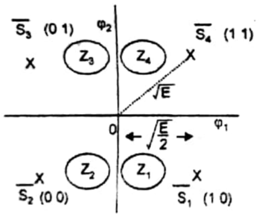
$l(m_k)$ is maximum for $k = i$

This decision rule is referred to as the maximum likelihood rule, and the device for its implementation is correspondingly referred to as the maximum likelihood receiver.

(b) Refer to Q. 1(*a*) Second Term 2011.

Error Performance of coherent *QPSK* in *AWGN*

Following fig. shows the *QPSK* signal constellation along with the four decision zones. As we have noted earlier, all the four signal points are equidistant from the origin. The dotted lines divide the signal space in four segments.



A time-limited *QPSK* modulated signal is expressed as,

$$S_i(t) = \sqrt{\frac{2E}{T}} \cdot \cos[(2i-1)\pi/4]$$

$$\cos w_c t - \sqrt{\frac{2E}{T}} \cdot \sin[(2i-1)\frac{\pi}{4}] \sin w_c t,\ 1 \le i \le 4$$

The corresponding signal at the input of a *QPSK* receiver is $r(t) - s_i(t) + w(t)$, $0 < t < T$, where '$w(t)$' is the noise sample function and 'T' is the duration of one symbol. Following our discussion on correlation receiver, we observe that the receiver vector $\bar{r}$, at the output of a bank of I-path and Q-path correlators, has two components:

$$r_1 = \int_0^T r(t)\,\varphi_1(t)\,dt = \sqrt{E}\,.\,\cos\left[(2i-1)\frac{\pi}{4}\right] + w_1$$

and
$$r_2 = \int_0^T r(t)\,\varphi_2(t)\,dt = \sqrt{E}\,.\,\cos\left[(2i-1)\frac{\pi}{4}\right] + w_2$$

Note that if $r_1 > 0$, it implies that the receiver vector is either in decision zone Z_1 or in decision zone Z_4. Similarly, if $r_2 > 0$, it implies that the receiver vector is either in decision zone Z_3 or in decision zone Z_4.

Further, r_1 and r_2 are sample values of independent Gaussian random variables with means $\sqrt{E}\cos\left[(2i-1)\frac{\pi}{4}\right]$ and $-\sqrt{E}\sin\left[(2i-1)\frac{\pi}{4}\right]$ respectively and with variance $\frac{N_0}{2}$.

Let us now assume that $S_4(t)$ is transmitted and that we have received $\bar{r}$. For a change, we will first compute the probability of correct decision when a symbol is transmitted.

Let, $P_{c_{s_4(t)}}$ = Probability of correct decision when $s_4(t)$ is transmitted.

$P_{c_{s_4(t)}}$ = Joint probability of the event that, $r_1 > 0$ and $r_2 > 0$

As $s_4(t)$ transmitted,

Mean of $\quad r_1 = \sqrt{E}\cos\left[7\frac{\pi}{4}\right] = \sqrt{\frac{E}{2}}$ and

Mean of $\quad r_2 = -\sqrt{E}\sin[7\pi/4] = \sqrt{\frac{E}{2}}$

$$\therefore \quad P_{c_{s_4(t)}} = \int_0^\infty \frac{1}{\pi N_0}.\exp\left[-\frac{\left(r_1-\sqrt{\frac{E}{2}}\right)^2}{N_0}\right]dr_1 .$$

$$\int_0^\infty \frac{1}{\pi N_0}.\exp\left[-\frac{\left(r_2-\sqrt{\frac{E}{2}}\right)^2}{N_0}\right]dr_2$$

As r_1 and r_2 are statistically independent, putting $\dfrac{r_j - \sqrt{\frac{E}{2}}}{\sqrt{N_0}} = Z, j = 1, 2,$ we get,

$$P_{c_{s_4(t)}} = \left[\frac{1}{\sqrt{\pi}}\cdot\int_{-\sqrt{\frac{E}{2N_0}}}^{\infty} \exp(-Z^2)\,dz\right]^2$$

Now, note that, $\frac{1}{\sqrt{\pi}}\int_{-a}^{\infty} e^{-x^2}dx = 1-\frac{1}{2}erfc(a)$.

$$\therefore \quad P_{c_{S_4(t)}} = \left[1-\frac{1}{2}erfc\left(\sqrt{\frac{E}{2N_0}}\right)\right]^2$$

$$= 1 - erfc\left(\sqrt{\frac{E}{2N_0}}\right) + \frac{1}{4}erfc^2\left(\sqrt{\frac{E}{2N_0}}\right)$$

So, the probability of decision error in this case, say, $P_{c_{S_4(t)}}$ is

$$P_{c_{S_4(t)}} = 1 - P_{c_{S_4(t)}} = erfc\left(\sqrt{\frac{E}{2N_0}}\right) - \frac{1}{4}erfc^2\left(\sqrt{\frac{E}{2N_0}}\right)$$

Following similar argument as above, it can be shown that $P_{e_{s1(t)}} = P_{e_{s2(t)}} = P_{e_{s3(t)}} = P_{e_{s4(t)}}$.

Now, assuming all symbols as equally likely, the average probabilty of symbol error

$$= Pe = 4 \times \left[erfc\left(\sqrt{\frac{E}{2N_0}}\right) - \frac{1}{4}erfc^2\left(\sqrt{\frac{E}{2N_0}}\right)\right]$$

The value of $erfc(x)$ decreases fast with increase in its argument. This implies that, for moderate or large value of E_b/N_0, the second term on the *R.H.S.* of Eq. may be neglected to obtain a shorter expression for the average probability of symbol error, *Pe*:

$$P_e \cong erfc\left(\sqrt{\frac{E}{2N_0}}\right) = erfc\left(\sqrt{\frac{2.E_b}{2.N_0}}\right) = erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$$

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END TERM EXAMINATION

FIFTH SEMESTER [B.Tech.], December, 2016

Digital Communication (ETEC-303)

Time : 3 hours | Maximum Marks : 75

Note: *Attempt any five questions including Q.No. 1 which is compulsory. Select one question from each unit.*

Question 1.

Attempt all:

(a) Discuss the properties and statistical characteristics of AWGN Channel. **(5)**

(b) Represent the data 010011110 in Bipolar Return to Zero line coding format and AMI Line coding format. **(5)**

(c) Differentiate between Strict Sense Stationary (SSS) and Wide Sence Stationary (WSS) Random Processes with proper example. **(5)**

(d) Explain the carrier recovery technique using Costas Receiver with help of block diagram. **(5)**

(e) Obtain the relation for bandwidth requirement of FSK Modulated signal. **(5)**

Solution:

(a) Additive White Gaussian Noise

AWGN is the term given to basic and generally accepted model for thermal noise in communication channels which is based on a set of assumptions that:

- the noise is additive, i.e., the received signal equals the transmit signal plus some noise, where the noise is statisticaly independent of the signal.
- the noise is white, i.e, the power spectral density is flat, so the autocorrelation of the noise in time domain is zero for any non-zero time offset.
- the noise samples have a Gaussian distribution.

An *AWGN* channel has the following properties:

(*i*) The channel provides distortion-free transmission over the channel bandwidth *B* and any transmission loss incurred in the channel is compensated by amplification.

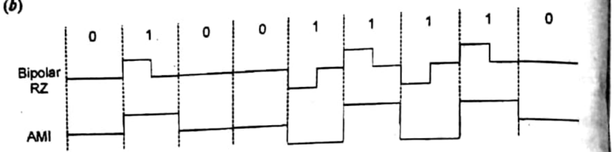
(*ii*) The channel constraints the input from the source to be a band limited signal $x(t)$ with fixed average power $S = \overline{x^2}$.

(*iii*) The signal received at the destination is contaminated by the addition of band limited white Gaussian noise $n(t)$ with zero mean and average noise power $N = \overline{x^2} = \eta B$, where η is the one sided noise *PSD*.

(*iv*) The signal and noise are independent so that the power of the received signal $y(t)$ is obtained by simple addition of the transmitted signal power and the noise power.

$$\overline{y^2} = \overline{x^2} + \overline{y^2} = S+N$$

(*b*)



(*c*) **Strict-Sense Stationary (*SSS*) :**

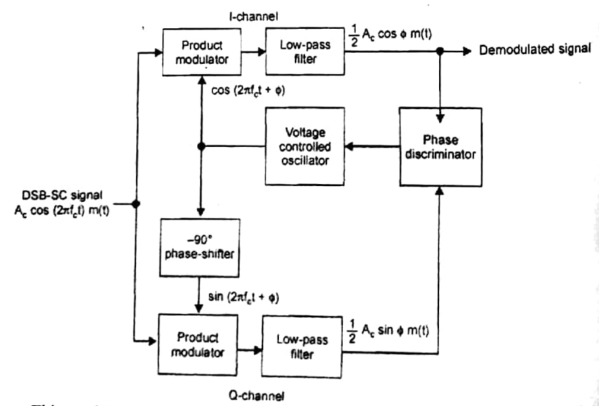
- A random process $X(t)$ (or X_n) is said to be *SSS* if all its finite order distributions are time invariant, i.e., the joint *cdf* (*pdf*, or *pmf*) of $X(t_1), X(t_2), \ldots, X(t_k)$ is the same as for $X(t_1+\alpha), X(t_2+\alpha), \ldots, X(t_k+\alpha)$, for all k, all $t_1, t_2, \ldots, t_k$, and all time shifts α.
- So for a *SSS* process, the first order distribution is independent of t, and the second order distribution, i.e., the distribution of any two samples $X(t_1)$ and $X(t_2)$, depends only on $\tau = t_2 - t_1$ for any t, the joint distribution of $X(t_1)$ and $X(t_2)$ is the same as the joint distribution of $X(t_1+(t-t_1))$ and $X(t_2+(t-t_1)) = X(t+(t_2-t_1))$.

Wide-Sence Stationary (*WSS*) :

- A random process $X(t)$ is said to be *WSS* if its mean and autocorrelation functions are time invariant, i.e., $E(X(t)) = \mu$, independent of t and $R_X(t_1, t_2)$ is only a function of (t_2, t_1).
- Since $R_X(t_1, t_2) = R_X(t_2, t_1)$, if $X(t)$ is *WSS*, $R_X(t_1, t_2)$ is only a function of $|t_2 - t_1|$.

Clearly *SSS* $\Rightarrow$ *WSS*, the convrese, however, is not necessarily true.

(*d*) **Costas Receiver:** One method of obtaining a practical synchronous receiver system, suitable for demodulating *DSB-SC* waves, is to use the Costas receiver shown in Fig.



This receiver, consists of two coherent detectors supplied with the same input signal, namely, the incoming *DSB-SC* wave $A_c \cos(2\pi f_c t)\, m(t)$, but with individual local oscillator signals that are in phase quadrature with respect to each other. The

frequency of the local oscillator is adjusted to be the same as the carrier frequency f_c, which is assumed known a priori. The detector in the upper path is referred to as the in-phase coherent detector or *I*-channel, and that in the lower path is referred to as the quadrature-phase coherent detector or *Q*-channel. These two detectors are coupled together to form a negative feedback system designed in such a way as to maintain the local oscillator synchronous with the carrier wave.

(*e*) we can write *BFSK* signal $s(t)$ as,

$$s(t) = \sqrt{2P_s}\, P_H(t)\cos(2\pi f_H t) + \sqrt{2P_s}\, P_L(t)\cos(2\pi f_L t)$$

in *BFSK* the coefficient $P_H(t)$ or $P_L(t)$ are unipolar.

Therefore let us convert those coefficients in bipolar form as follows

$$P_H(t) = \frac{1}{2} + \frac{1}{2}P_H'(t)$$

and

$$P_L(t) = \frac{1}{2} + \frac{1}{2}P_L'(t)$$

Here $P'_H(t)$ and $P'_L(t)$ will be bipolar (i.e. +1 or −1). Putting those values in equation

$$s(t) = \sqrt{2P_s}\left[\frac{1}{2} + \frac{1}{2}P_H'(t)\right]\cos(2\pi f_H t) + \sqrt{2P_s}\left[\frac{1}{2} + \frac{1}{2}P_L'(t)\right]\cos(2\pi f_L t)$$

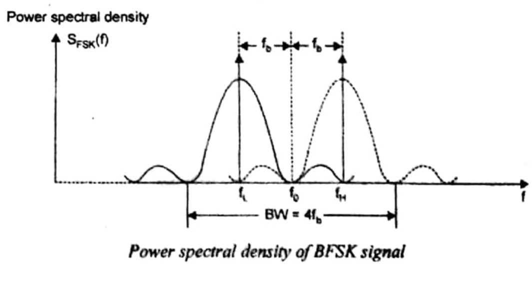
$$= \sqrt{\frac{P_s}{2}}\cos(2\pi f_H t) + \sqrt{\frac{P_s}{2}}\cos(2\pi f_L t) + \sqrt{\frac{P_s}{2}}P'_H(t)\cos(2\pi f_H t) + \sqrt{\frac{P_s}{2}}P'_L(t)\cos(2\pi f_L t)$$

Now, we can write the power spectral density of *BFSK* as,

$$S(f) = \sqrt{\frac{P_s}{2}}\left\{\delta(f - f_L) + \delta(f + f_L) + \frac{P_s T_b}{2}\right.$$

$$\left\{\frac{\sin(\pi f_H T_b)}{\pi f_H T_b}\right\}^2 + \frac{P_s T_b}{2}\left\{\frac{\sin(\pi f_L T_b)}{\pi f_L T_b}\right\}^2$$

Fig. shows the plot of power spectral density of *BFSK* signal given by above equation.



Power spectral density of BFSK signal

f_H and f_L are selected. Such that,

$$f_H - f_L = 2f_b$$

Bandwidth of *BFSK* Signal : From Fig. it is clear that the width of one lobe is $2f_b$. The two lobes due to f_H and f_L are place such that the total width due to both main lobes is $4f_b$. i.e.,

$$\text{Bandwidth of } BFSK = 2f_b + 2f_b$$

or $$BW = 4f_b$$

UNIT–I

Question 2.

Explain the significance of Companding in digital system. Discuss the following compression techniques. **(12.5)**

(i) μ-Law Compression

(ii) A-Law Compression

Solution:

Refer to Q. 1(*c*) First Term 2010

Refer to Q. 1(*c*) End Term 2011

Question 3.

With the help of Block diagrams explain the functionality of ADM Transmitter and receiver. Interpret the signal representation after each block. **(12.5)**

Solution:

Adaptive delta modulation (*ADM*) is a delta modulation where the step size δ of the staircase waveform is varied depending upon the slope or amplitude characteristics of the analog input signal. When the *DM* output is a string of consecutive 1*s* or 0*s*, this indicates the staircase waveform is not tracking the analog waveform and the possibility of slope overload occurring is high. When an alternating sequence of 1*s* and 0*s* is occurring, this indicates that the possibility of granular noise occurring is high. A typical schematic block diagram of *ADM* transmitter is shown in Figure.

The effects of slope overload distortion and granular noise can be overcome by making the staircase step size $\delta(nT_s)$, a function of the slope of the input analog signal. When the slope of the input analog signal is large, $\delta(nT_s)$ in increased and when the slope is fairly constant, $\delta(nT_s)$ is decreased. In practice, the staircase step size $\delta(nT_s)$ is bounded by lower and upper limits δ_{min} and δ_{max}, respectively. That is,

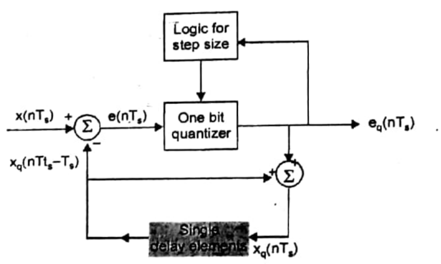
$$\delta_{min} \le \delta(nT_s) \le \delta_{max}$$

The lower limit δ_{min} controls the amount of granular noise and the upper limit δ_{max} controls the amount of slope overload distortion. Inside these limits, the adaptation alogrithm is

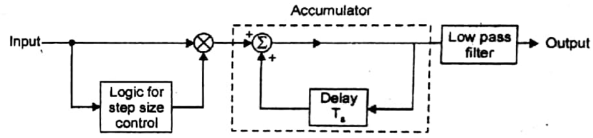
$$\delta(nT_s) = g(nT_s)\,\delta(nT_s - T_s)$$

The time varying multiplier $g(nT_s)$ depends on the previous and the present outputs of the delta modulator. The algorithm is begun with $\delta(nT_s - T_s) = \delta_{min}$. It may be noted that

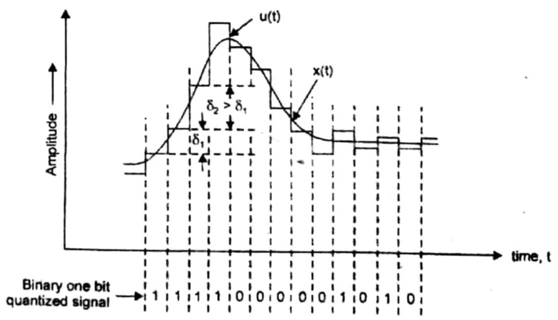
$$g(nT_s) = \begin{cases} K, & \text{if } e_q(nT_s) = e_q(nT_s - T_s) \\ K^{-1} & \text{if } e_q(nT_s) \ne e_q(nT_s - T_s) \end{cases}$$



In the receiver of adaptive delta modulator shown in figure, there are two portions.



The first portion produces the step size from each incoming bit. Exactly the same process is followed as that in transmitter. The previous input and present input decides the stepsize. It is then applied to an accumulator which builds up staircase waveform. The lowpass filter then smoothens out the staircase waveform to reconstruct the original signal.



Waveforms for adaptive delta modulation

Advantages of Adaptive Delta Modulation :

Adaptive delta modulation has certain advantages ove delta modulation as under:

(*i*) The signal to noise ratio becomes better than ordinary delta modulation because of the reduction in slope overload distortion and idle noise.

(*ii*) Because of the variable step size, the dynamic range of *ADM* is wider than simple *DM*.

(*iii*) Utilization of bandwidth is better than delta modulation.

UNIT–II

Question 4.

Discuss about the following entities used for statistical analysis of Random signals.

(i) Power Spectral Density

(ii) Joint PDF

(iii) Marginal PDF

Give the relevant mathematical treatment. **(12.5)**

Solution:

Refer to Q. 2(*a*) (*i*) End Term 2013

Refer to Q. 2(*b*) End Term 2013

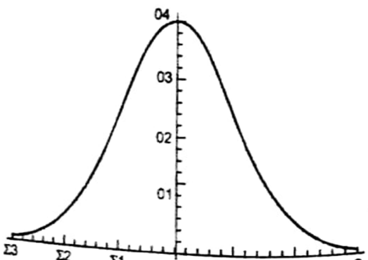
Refer to Q. 2(*b*) Second Term 2011

Question 5.

Obtain the relation of Probability Density Function (PDF) of Gaussian distribution. Discuss the role of central limit Theorem in the analysis of Gaussian distribution. **(12.5)**

Solution:

Data are said to be normally (Gaussian) distributed if the rate at which the frequencies fall off is proportional to the distance of the score from the mean, and to the frequencies themselves.



In practice, the value of the bell shaped curve is that we can find the proportion of the scores which lie over a certain interval. In a probability distribution, this is the area under the curve over the interval: a typical calculus problem. However, in order to use calculus to find these areas, we need a formula for the curve. We can find such a formula because our definition gives us the following differential equation.

$$\frac{df}{dx} = -k(x-\mu)f(x)$$

Where k is a positive constant. Note that to the right of the mean, the curve will be decreasing and to the left, it will be increaing. We can separate the varaibles

$$\frac{df}{f} = -k(x-\mu)\, d(x)$$

and we can integrate both sides.

$$\int\frac{df}{f} = -k\int(x-\mu)\, dx$$

$$\text{In} f = \frac{-k(x-\mu)^2}{2} + \text{In}C$$

Take exponentials of both sides.

$$f(x) = Ce^{-\frac{k}{2}(x-\mu)^2}$$

The question now becomes, what is C, not to mention, what is k? We can use the fact that the normal distribution is a probability distribution, and the total area under the curve is 1. If $f(x)$ is a probability measure, then

$$C\int_{-\infty}^{\infty} e^{-\frac{k}{2}(x-\mu)^2} dx = 1$$

This is actually somewhat humorous. It is a function which does not have an elementary function for its integral. Howe ever, there is a trick for getting the total area under the curve. First, let

$$u = \sqrt{\frac{k}{2}}(x-\mu)$$

Then,

$$\frac{k}{2}(x-\mu)^2 = u^2$$

and

$$du = \sqrt{\frac{k}{2}}dx$$

so

$$dx = \sqrt{\frac{2}{k}}du$$

So our integral becomes

$$C\int_{-\infty}^{\infty} e^{-\frac{k}{2}(x-\mu)^2} dx = C\sqrt{\frac{2}{k}}\int_{-\infty}^{\infty} e^{-u^2} du = 1$$

Square the integral. When we write down two factors of the integral, we can use different variables for the two integrals.

$$= \frac{2C^2}{k}\left(\int_{-\infty}^{\infty} e^{-x^2} dx\right)\left(\int_{-\infty}^{\infty} e^{-y^2} dy\right) = 1$$

$$= \frac{2C^2}{k} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-|x^2+y^2|}\, dx\, dy = 1$$

Transfer to polar coordinates.

$$= \frac{2C^2}{k} \int_{0}^{2\pi}\int_{0}^{\infty} re^{-r^2}\, dr\, d\theta = 1$$

We now have something we can integrate. Let

$$v = -r^2$$

$$dv = -2rdr$$

$$rdr = -\frac{1}{2}du$$

So our integral becomes

$$= -\frac{C^2}{k} \int_{0}^{2\pi}\int_{0}^{\infty} e^{-u} du\, d\theta = 1$$

$$= -\frac{C^2}{k} \int_{0}^{2\pi} d\theta = 1$$

$$= \frac{2\pi C^2}{k} = 1$$

$$C^2 = \frac{k}{2\pi}$$

or

$$C = \sqrt{\frac{k}{2\pi}}$$

so

$$f(x) = \sqrt{\frac{k}{2\pi}}\, e^{-\frac{k}{2}(x-\mu)^2}$$

Central Limit Theorem

Refer to Q. 3(c) End Term 2011

UNIT–III

Question 6.

Discuss analysis of following digital Receivers. ***(12.5)***

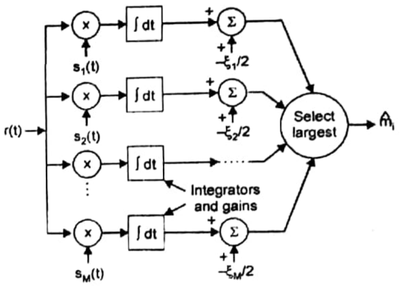
(a) Correlator Receiver

(b) Maximum Likelihood Receiver.

Solution:

The Correlator Receiver

A correlation receiver is an optimal receiver which cross-correlates the received signal with the transmitted pulse and feeds the resulting correlation to a slicer.



The receiver analysis begins with Parseval's identity,

$$\|r - s_i\| = \int_T [r(t) - s_i(t)^2]dt$$

of this identity. But we can just as well minimize the right-hand side; after expanding the integral, this operation becomes

$$\|r - s_i\|^2 = \int_T [r(t) - s_i(t)^2]dt$$

the *ML* receiver works by minimizing the left-hand sideing the integral, this operation becomes

$$\min_i \left\{ \int r(t)^2 dt + \int s_i(t)^2 dt - 2\int r(t)s_i(t)dt \right\}$$

The first term here is a constant during the minimization and can therefore be dropped. The second term is the energy of the *i*th signal. Let ξ_i denote this energy; conceivably, all *M* of the ξ_i could be precomputed before the receiver begins to operates. If all the signals have the same energy, a common case, then the second term is constant over *i* and can also be dropped.

Find *i* that achieves $\max_i \int r(t)s_i(t)\, dt$

If the signals are not equal-energy,

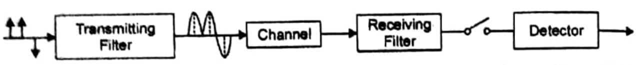
Find *i* that achieves $\max_i \int r(t)s_i(t)\, dt - \frac{1}{2}\xi_i$

The heart of these expressions is a correlation integral. It evaluates the similarity between the received signal *r*(*t*) and each possible transmitted signal, an entirely reasonable idea for a receiver, which we have just known is the optimal design, the one that minimizes the error probability in the *AWGN* channel case. This kind of detector is called a correlator receiver.

Question 7.

How would avoid Inter Symbol Interference (ISI) in Base band Digital Communication systems. Discuss in detail about any one of the methods to minimize ISI. How eye pattern is useful in determining ISI? ***(12.5)***

Solution:



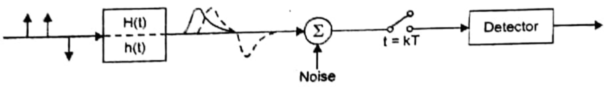
Throughout the system there are many filtering (and reactive circuit elements) that distort the original signal.

Even when we assume absence of noise, the filtering characteristics still are capable enough to cause significant distortion.

Assuming transmitter is modelled using a transfer function $H_t(f)$, the channel is modelled using a transfer function $H_C(f)$ and the receiver is modelled using the transfer function $H_r(f)$, then the overall equivalent system transfer function is

$$H(f) = H_t(f)\,H_C(f)\,H_r(f)$$

and a simplified respresentation can be



As observed in the diagram above, the pulse which are received can overlap each other the range of transmisison as a result of broadening due to filtering process. The trail of such a pulse can 'smear' into the adjacent symbol pulse intervals thereby causing interference in the direction process and degrading the system performance. Such interference is called Intersymbol of Interference (*ISI*). The process of controlling the *ISI* thus remains with the criteria of controlling $H(f)$. Intersymbol interference can be avoided by following the Nyquist criterion for distortionless baseband transmission. It provides a method for contructing band-limited functions to overcome the effects of intersymbol interference.

Ideal Solution

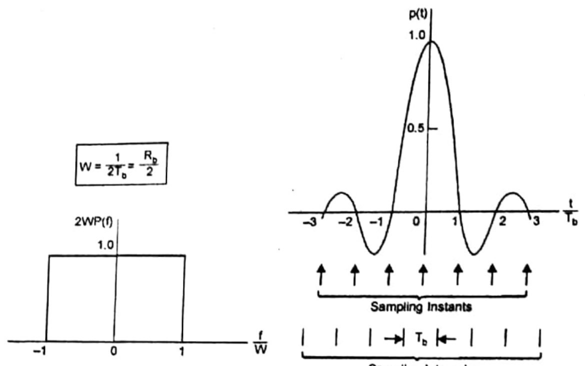
One signal waveform that produces zero intersymbol interference is defined by the sinc function:

$$p(t) = \frac{\sin(2\pi Wt)}{2\pi Wt}$$
$$= \text{sinc}\,(2Wt)$$

The special value of the bit rate R_b, $= 2W$ is called the Nyquist rate, and W is itself called the Nyquist bandwidth.

Correspondingly, the ideal baseband pulse transmission system is described by following equation in the frequency domain:

$$P(f) = \begin{cases} \frac{1}{2W}, & -W < f < W \\ 0 & |f| W \end{cases}$$
$$= \frac{1}{2W}\,rect\left(\frac{f}{2W}\right)$$



There are two practical difficulties in this design:

1. It requires that the magnitude characteristics of $P(f)$ be flat from $-W$ to W, and zero elsewhere. This is physically unrealizable because of the abrupt transitions at the band edges $\pm W$.
2. The function $p(t)$ decreases as $1/|t|$ for large $|t|$, resulting in a slow rate of decay. This is also caused by the discountinuity of $P(f)$ at $\pm W$. Accordingly, there is practically no margin of error in sampling times in the receiver.

Practical Solution: Raised Cosine Spectrum

We may overcome the practical difficulties encountered with the ideal Nyquist channel by extending the bandwidth from the minimum value $W = R_b/2$ to an adjustable value between W and $2W$. We now specify the overall frequency response to satisfy a condition more elaborate than that for the ideal Nyquist channel.

A particular form that embodies many desirable features is provided by a raised cosine spectrum. This frequency response consists of a flat portion and a roll off portion that has a sinusoidal form, as follows:

$$P(f) = \begin{cases} \frac{1}{2W}, & 0 \le |f| < f_1 \\ \frac{1}{4W}\left\{1 - \sin\left[\frac{\pi(|f| - W)}{2W - 2f_1}\right]\right\}, & f_1 |f| < 2W - f_1 \\ 0 & |f| \ge 2W - f_1 \end{cases}$$

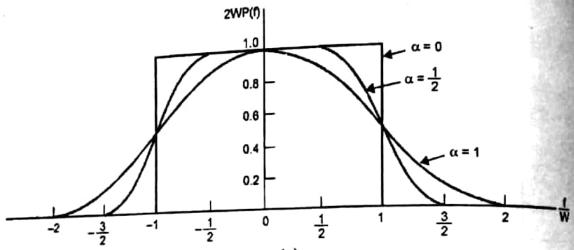
The frequency parameter f_1 and bandwidth W are related by

$$\alpha = 1 - \frac{f_1}{W}$$

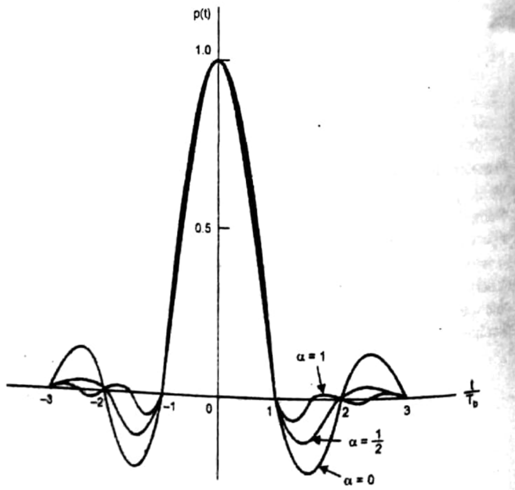
The parameter α is called the rolloff factor; it indicates the excess bandwidth over the ideal solution, W. Specifically, the transmission bandwidth B_T is defined by

$$B_T = 2W - f_1$$
$$= W(1 + \alpha)$$

$$p(t) = (\text{since } (2Wt))\left(\frac{\cos(2\pi\alpha Wt)}{1 - 16\alpha^2 W^2 t^2}\right)$$

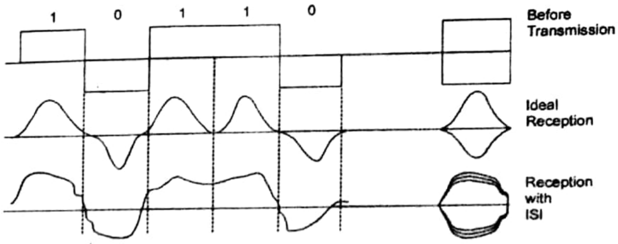


(a)

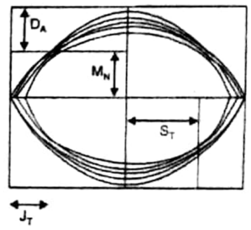


(b)

Eye diagram is a technique or representing the baseband signals in a specific way. On the vertical plates of an oscilloscope we connect the receiver's response to any random sequence. On the horizontal plates we connect a sawtooth wave at the signalling frequency. Hence repeated plots over the symbol time are observed on the CRO which appear superimposed onto each other.



This pattern when observed looks like an eye. This it is known as eye diagram. Thus bigger the eye opening lesser the error due to *ISI* and vice versa.



D_A = Distortion caused by *ISI*

M_N = Noise margin

J_T = Timing Jitter

S_T = Sensitivity to timing error

Information Obtained from Eye Pattern :

(*i*) The width of the eye opening defines the time interval over which the received wave can be sampled, without an error due to *ISI*. The best time for sampling is when the eye is open widest.

(*ii*) The sensitivity of the system to the timing error is determined by the rate of closure of the eye as the sampling rate is varied.

(*iii*) The height of eye opening at a specified sampling time defines the margin over noise.

(*iv*) When the effect of *ISI* is severe, the eye is completely closed and it is impossible to avoid errors due to the combined presence of *ISI* and noise in the system.

UNIT–IV

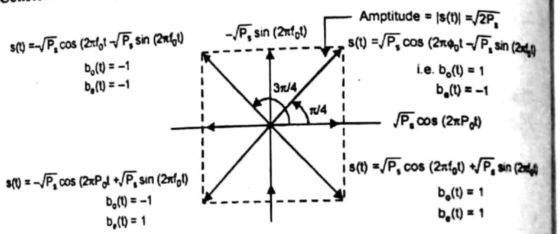
Question 8.

Discuss the functioning of QPSK modulator and Demodulator with the help of Block Diagram. Draw the constellation diagram and obtain relation for Bandwidth of QPSK signal.(12.5)

Solution:

Refer to Q. 1 (*a*) Second Term 2011

Constellation Diagram



Bandwidth

The power spectral density of input binary sequence $b(t)$ is

$$S(f) = P_s T_b \left[\frac{\sin(\pi f\ T_b)}{\pi f\ T_b}\right]^2$$

The above equation gives power spectral density of signal $b(t)$. This signal is divided into $b_e(t)$ and $b_o(t)$ each of bit period $2T_b$. If we consider that symbols 1 to 0 are equally likely, then we can write power spectral densities of $b_c(t)$ and $b_0(t)$ as,

$$S_e(f) = P_s T_s \left[\frac{\sin(\pi f\ T_s)}{\pi f\ T_s}\right]^2$$

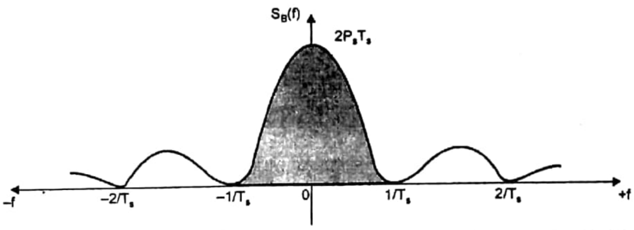
and

$$S_o(f) = P_s T_s \left[\frac{\sin(\pi f\ T_s)}{\pi f\ T_s}\right]^2$$

In the above two equations we have just replaced T_b by T_s and T_s is the period of bit in $b_e(t)$ and $b_o(t)$. Since inphase and quadrature components $[b_e(t)$ and $b_o(t)]$ are statistically independent, the baseband power spectral density of *QPSK* signal equals the sum of the individual power spectral densities $b_e(t)$ and $b_o(t)$ i.e.,

$$S_B(f) = S_e(f) + s_o(f) = 2P_s T_s \left[\frac{\sin(\pi f\ T_s)}{\pi f\ T_s}\right]^2$$

By plotting this equation, we get:



$$BW = \text{Highest frequency} - \text{Lowest frequency in main lobe}$$

$$= \frac{1}{T_s} - \left(-\frac{1}{T_s}\right) \quad \text{since carrier frequency } f_0 \text{ cancels out}$$

$$= \frac{2}{T_s}$$

We know that $T_s = 2T_b$

$$\therefore \quad BW = \frac{2}{2T_b} = \frac{1}{T_b} = f_b$$

Question 9.

Write short notes on the following:

(a) M-ary Schemes along with merits and Demerits **(6)**

(b) G-MSK Modulation Scheme **(6.5)**

Solution:

Refer to Q.1(*d*) First Term 2010

Instead of just varying the phase, frequency and amplitude of the *RF* signal, modern modulation techniques allow both envelope (amplitude) and phase (or frequency) of the *RF* carrier to vary. Because the envelope and phase provide two degrees of freedom, such modulation techniques map baseband data into four or more possible *RF* carrier signals. Such modulation techniques are known as *M*-ary modulation. Normally, the number of possible signals is, $M = 2^N$, where N is an integer.

In *M*-ary modulation scheme, two or more bits are grouped together to form symbols and one of possible signals $S_1(t), S_2(t), \ldots, S_m(t)$, is transmitted during each symbol period T_S.

Depending on whether the amplitude, phase or frequency is varied, the modulation is referred to as *M*-ary *ASK*, *M*-ary *PSK* or *M*-ary *FSK*, respectively.

Merit

M-ary modulation technique attractive for use in bandlimited channels, because these techniques achieve better bandwidth efficiency at the expense of power efficiency.

For example: an 8-*PSK* technique requires a bandwidth that is $\log_2 8 = 3$ times smaller than 2-*PSK* (also known as *BPSK*) system.

Demerit

However, *M*-ary signalling results in poorer error performance because of smaller distances between signals in the constellation diagram.

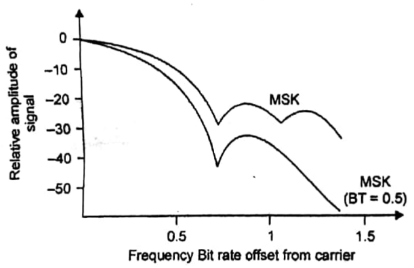
This modulation scheme also results in an increase in transmitted power.

(*b*) *GMSK* : Gaussian Minimum Shift Keying is a form of modulation used in a variety of digital radio communications systems. It has advantages of being able to carry digital modulation while still using the spectrum efficiently. One of the problems with other forms of Phase Shift Keying is that the side bands extend outwards from the main carrier and these can cause interference to other radio communications systems using nearby channels.

In view of the efficient use of the spectrum in this way, *GMSK* modulation has been used in a number of radio communications applications. Possibly the most widely used is the *GSM* cellular technology which is used worldwide and has well over 3 billion subscribers.

GMSK basics: *GMSK* modulation is based on *MSK*, which is itself a form of phase shift keying. One of the problems with standard forms of *PSK* is that side bands extend out from the carrier. To overcome this, *MSK* and its derivative *GMSK* can be used.

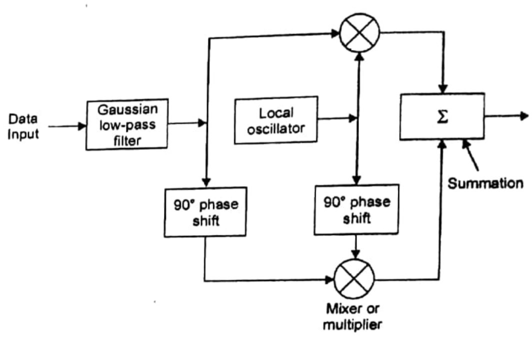
- *MSK* and also *GMSK* modulation are what is known as a continuous phase scheme.
- Here there are no phase discontinuities because the frequency changes occur at the carrier zero crossing points.
- A plot of the spectrum of an *MSK* signal shows side bands extending well beyond a bandwidth equal to the data rate.
- This can be reduced by passing the modulating signal through a low pass filter prior to applying it to the carrier. The ideal filter is known as a Gaussian filter which has a Gaussian shaped response to an impulse and no ringing.



Spectral density of MSK and GMSK signals

Generating *GMSK* Modulation :

Here, a quadrature modulator is used. The term quadrature means that the phase of a s quadrature or 90 degrees to another one. The modulator uses one signal that is said to be ii another that is in quadrature to this. In vie l phase and quadrature elements this type o*l* is often said to be an *I-Q* modulator.



Block diagram of I-Q modulator used to create GMSK

Advantages of *GMSK* Modulation:

There are several advantages to the use of *OMSK* modulation for a radio communications system.

- It provides improved spectral efficiency when compared to other phase shift keyed modes.
- It can be amplified by a non-linear amplifier and remain undistorted.
- It is immune to amplitude variations and therefore more resilient to noise.

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