

(iii) *Forbidden energy gap.*

*The separation between conduction band and valence band on the energy level diagram is known as **forbidden energy gap**.*

No electron of a solid can stay in a forbidden energy gap as there is no allowed energy state in this region. The width of the forbidden energy gap is a measure of the boundage of valence electrons to the atom. The greater the energy gap, more tightly the valence electrons are bound to the nucleus. In order to push an electron from valence band to conduction band (*i.e.*, to make the valence electron free), external energy equal to forbidden energy gap must be supplied.

1.7 Classification of Solids on the Basis of Energy Bands

We know that some solids are good conductors of electricity while other are insulators. There is also an intermediate class of semiconductors. The difference in the behaviour of solids as regards their electrical conductivity can be beautifully explained in terms of energy bands. The electrons in the lower energy band are tightly bound to the nucleus and play no part in the conduction process. However, the valence and conduction bands are of particular importance in ascertaining the electrical behaviour of various solids.

(a) Insulators or Non-Conductors or Dielectrics

Insulators (*e.g.*, wood, glass etc.) are those substances which do not allow the passage of electric current through them. In terms of energy band, the valence band is completely filled while the conduction band is empty. Further, the energy gap between valence and conduction bands is very large (≥ 15 eV) as shown in Fig. 1.7.

Therefore, a very high electric field is required to push the valence electrons to the conduction band. For this reason, the electrical conductivity of such materials is extremely small and may be regarded as nil under ordinary conditions.

At room temperature, the valence electrons of the insulators do not have enough energy to cross over the conduction band. However when the temperature is raised, some of the valence electrons may acquire enough energy to cross over to the conduction band. Hence the resistance of an insulator decreases with increase in temperature *i.e.*, an insulator has negative temperature coefficient of resistance.

(b) Conductors

A second possibility is that of a solid in which the valence band is completely filled with electrons but there is no forbidden energy gap *i.e.*, the valence and conduction bands overlap

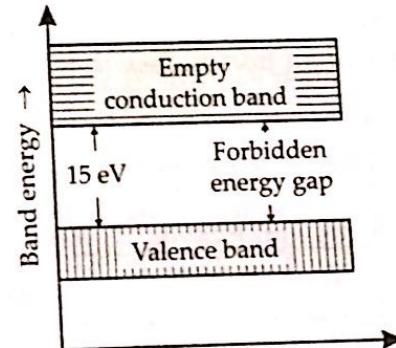


Figure 1.7 Energy bands of insulators.

is possible to change the energy of the valence electrons by accelerating them by external electric field ; the solid can carry a current. It will be a good conductor, for example : metals (copper, aluminium etc.)

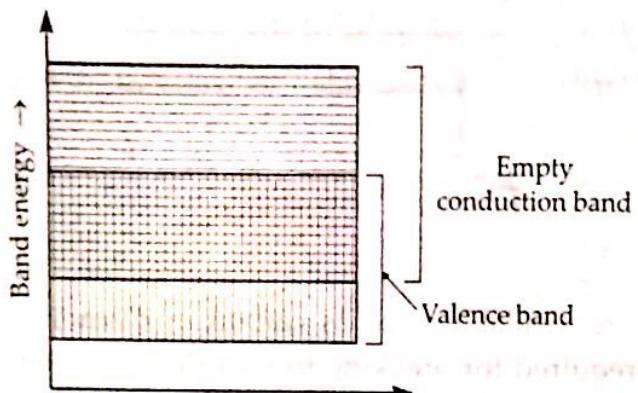


Figure 1.8 Energy bands of metals.

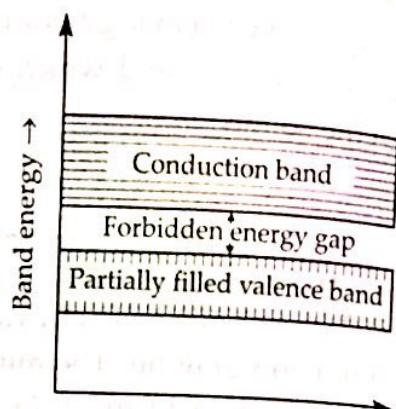


Figure 1.9 Energy bands of alkali metals.

The third possibility is that of a solid in which the valence band is only partially filled (Fig. 1.9), so that unfilled levels are available adjacent to the filled levels and external electronic field is able to accelerate the valence electrons and make them carry a current. The solid is, thus a good conductor. This is a situation of alkali metals.

(c) Semiconductors

(i) *Intrinsic Semiconductors.* If the valence band is completely filled, conduction band is empty but the forbidden energy is small, of the order of about 1 eV [Fig. 1.10], the solid will be insulator at 0 K. At higher temperatures such that the thermal energy $k_B T$ is more than the gap energy ΔE_g , some of the valence electrons are able to jump over the gap into the conduction band. This creates free electrons in the conduction band and free holes in valence band. Thus the electron-hole pairs are thermally generated and can carry a small current through the solid. This is the case of an *intrinsic semiconductor*.

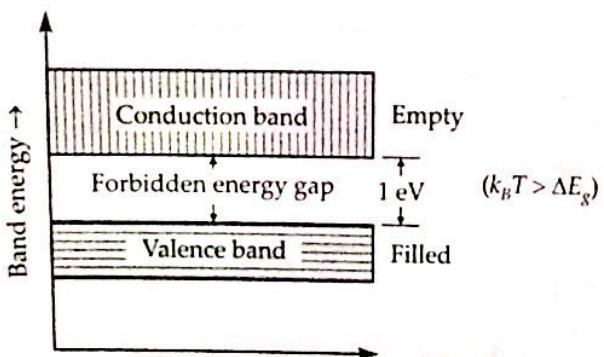


Figure 1.10 Energy bands of intrinsic semiconductor

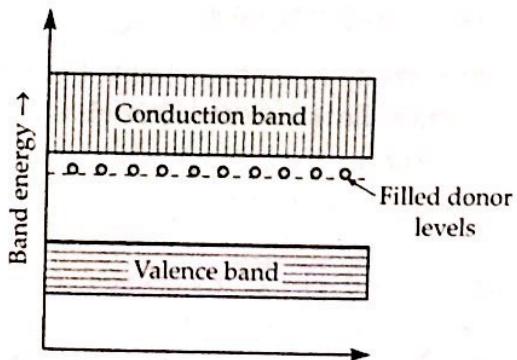


Figure 1.11 N-type semiconductor.

(ii) *Extrinsic Semiconductors.* In an *N-type extrinsic semiconductor*, the energy levels of the valence electrons of the impurity atoms lie just below the bottom of the conduction band (Fig. 1.11). These electrons are very easily excited into the conduction band and are then capable of carrying a current through the solid.

Likewise, in a *P-type semiconductor*, the impurity atoms have unfilled energy levels, which lie just above the top of the valence band (Fig. 1.12). Electrons from the valence band are easily excited into these vacant levels leaving behind vacancies or free holes in the valence band which can carry a current through the solid.

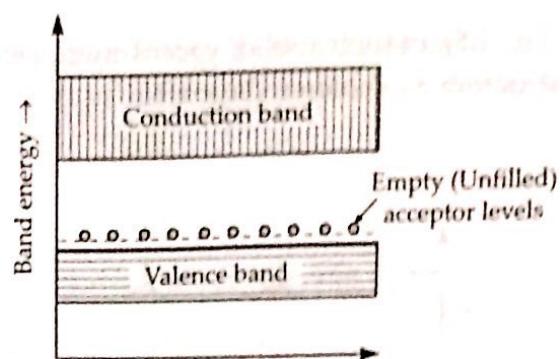


Figure 1.12 P-type semiconductor.

1.8 Commonly Used Semiconductor Materials

A large number of tetravalent materials are available such as carbon in diamond state, germanium and gray tin. The minimum energy required for breaking the covalent bond in these materials is 7 eV, 1.12 eV, 0.75 eV and 0.1 eV respectively. Carbon in diamond state having forbidden energy gap of 7 eV, behaves more or less as an insulator. Whereas gray tin having forbidden energy gap 0.1 eV behaves as a conductor. The energies of 0.72 eV and 1.1 eV required to break a covalent bond in germanium and silicon respectively are of such an appropriate value so as to make these materials to have conductivity between the insulators and conductors. Therefore, germanium and silicon are considered to be most suitable semiconductor materials.

(i) Germanium (Ge)

It was discovered in 1886. It is an earth element recovered from the ash of certain coals and from the flue dust of zinc smelters. The recovered germanium is in the form of germanium dioxide powder. It is then reduced to pure germanium. For the applications in electronics, it has become the model substance since it can be purified relatively well and crystallised easily than any other semiconductor material.

The atomic structure of germanium is shown in Fig. 1.13. Its atomic number is 32. Therefore, it has 32 protons in the nucleus and 32 electrons distributed in the four orbits around the nucleus. The number of electrons in the first, second, third and fourth orbits are 2, 8, 18 and 4 respectively. It is clear that germanium atom has four valence electrons *i.e.*, the electrons in the outmost (valence) orbit. Since germanium has 4 valence electrons, it is known as tetravalent element. The various germanium atoms are held together through covalent bonds as shown in Fig. 1.14.

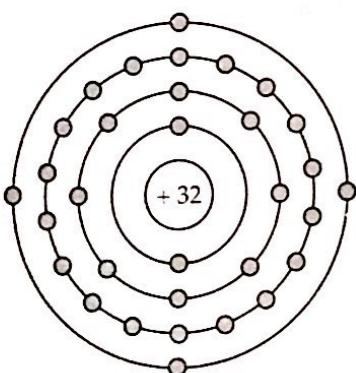


Figure 1.13 Atomic structure of Ge

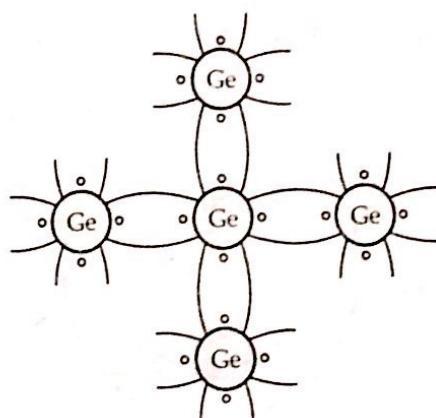


Figure 1.14 Covalent bonds in Ge

The atoms of germanium are arranged in an orderly pattern and form a crystalline structure as shown in Fig. 1.15.

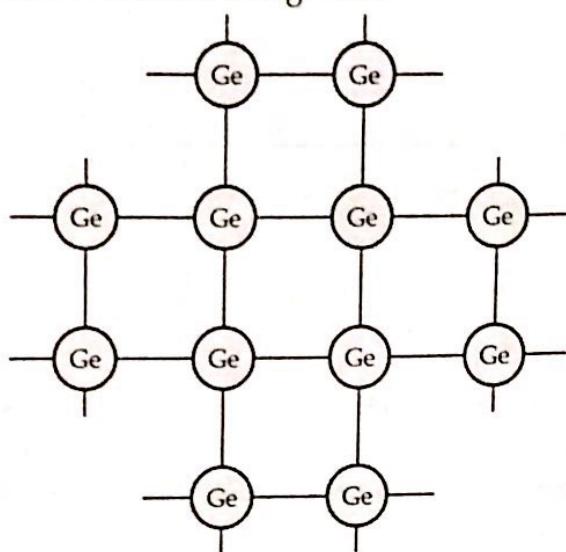


Figure 1.15 Crystalline structure of Ge.

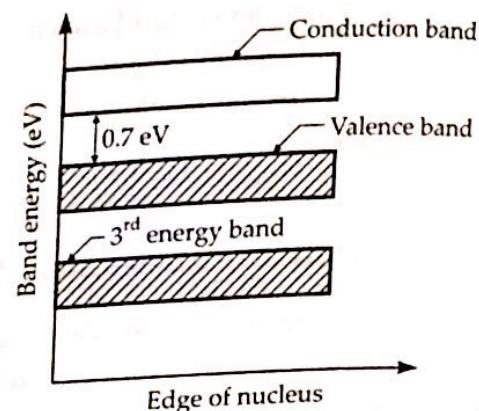


Figure 1.16 Energy band diagram of Ge.

Figure 1.16 shows the energy band diagram of germanium. The forbidden gap i.e., gap between valance band and conduction band in this material is very small i.e., 0.7 eV. Therefore, a very small energy is sufficient to lift the electrons from the valance band to conduction band. Even at room temperature, minor quantity of electrons are lifted to the conduction band. This is the reason, that even at room temperature germanium does not behave as an insulator, but at the same time it does not behave as a conductor because electrons available in the conduction band are very minor and hence called a semiconductor.

(ii) Silicon (Si)

It is an element available in most of the common rocks. Actually, sand is silicon dioxide. The silicon dioxide is treated chemically and reduced to pure silicon which can be used for the preparation of electronic devices.

Figure 1.17 shows the atomic structure of silicon. Its atomic number is 14. Therefore, it has 14 protons in the nucleus and 14 electrons are distributed in the three orbitals around the nucleus. The number of electrons in the first, second and third orbit are 2, 8 and 4 respectively.

The silicon atom also has four valence electrons and is known as tetravalent element. The various silicon atoms are held together through covalent bonds as shown in Fig. 1.18.

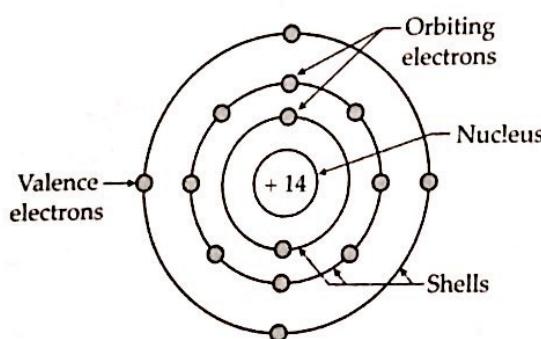


Figure 1.17 Atomic structure of Si.

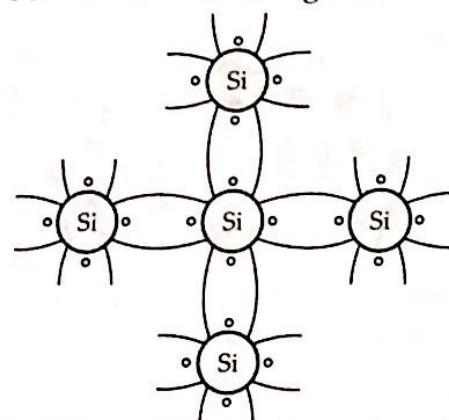


Figure 1.18 Covalent bonds of Si.

The atoms of silicon are arranged in an orderly pattern and form a crystalline structure as shown in Fig. 1.19.

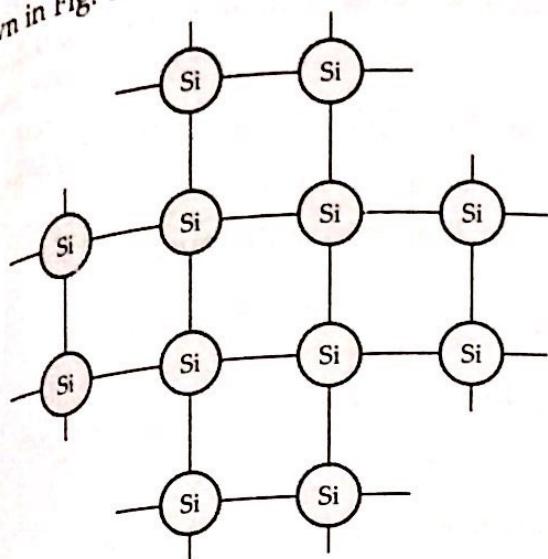


Figure 1.19 Crystalline structure of Si.

Figure 1.20 shows the energy band diagram of silicon. The forbidden energy gap in this material is quite small i.e., 1.1 eV. It also needs a small energy to lift the electrons from valence band to conduction band.

Therefore, even at room temperature, a minute quantity of valence electrons is lifted to conduction band and constitute current conduction if a high electric field is applied. However at room temperature, the number of electrons lifted to the conduction band in case of silicon are quite less than the germanium. This is reason why silicon semiconductor devices are preferred over germanium devices.

Formulae at a Glance

1. Empirical formula for several series :

$$\frac{1}{\lambda} = R \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$$

where λ = wavelength.

R = Rydberg's constant,

$m = 1, 2, 3, 4, \dots$,

$n = (m+1), (m+2), (m+3), \dots$

2. $mvr = n\hbar$

where m = mass of the particle

v = velocity of the particle

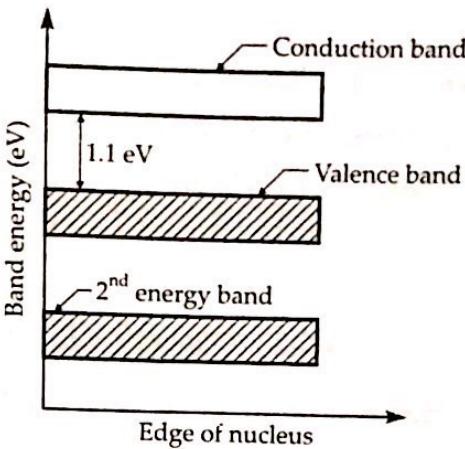


Figure 1.20 Energy band diagram of Si.

r = radius of the orbit

$$\hbar = \frac{h}{2\pi}$$

$$3. h\nu_{mn} = E_m - E_n$$

$$4. K.E., T = \frac{ke^2}{2r}, k = \frac{1}{4\pi\epsilon_0}$$

$$5. \text{Radius } r_n = \frac{n^2 \hbar^2}{kme^2}$$

$$6. P.E., V(r) = -\frac{ke^2}{r}$$

$$7. E_n = -\frac{k^2 me^4}{2\hbar^2} \cdot \frac{1}{n^2} = -\frac{13.6}{n^2} \text{ eV}$$

This is shown in Fig. 2.1.

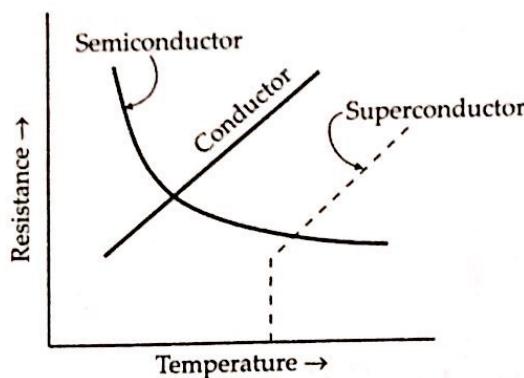


Figure 2.1 Temperature vs. resistance curves for different materials.

The conductivities of solids can be explained as follows : A solid is made up of a large number of atoms arranged in a regular manner. The atoms are composed of a positively charged nucleus around which electrons revolve in certain well defined orbits as discussed in chapter 1. An electron in a orbit represents a bound electrons ; it is constrained to move in that particular orbit under the electrostatic attraction of the nucleus. Such an electron cannot carry current through the solid because it is not free to move along the direction of an external electric field. Therefore, a solid containing only the electrons that are bound in atoms will be an *insulator*. In order that a current might flow through the solid, it must contain some electrons which are free to move under the action of an external electric field without much hinderance. As we know that, a metal has a large number of such free electrons. Their movement in the absence of any electric field is random ; as such it does not result in a non-zero average current in any direction. However, when electric field is applied, they acquire an non-zero average velocity in the direction of field and result in non-zero current through the solid. Thus a perfect insulator does not have any free electron, good conductor has a very large number of them and we expect that, semiconductor will have some of them.

2.2 Classification of Semiconductors

The classification of semiconductors on the basis of crystal structure and position of their constituent elements in the Periodic Table is shown in Fig. 2.2.

The classification of semiconductors produced much interest of scientists and engineers due to crystal structure and nature of bonding between the constituent atoms those in turn related to band gap.

Primarily, a semiconducting material is either crystalline or amorphous. The crystalline materials are used in the form of a single crystal or in the polycrystal form in electronic devices. The crystalline semiconductors are further classified into single elemental material on the basis of their position in Periodic Table and compound semiconductors.

We know two most well known semiconductors of group IV i.e., silicon (Si) and germanium (Ge). There are said to be *elemental semiconductors*. Silicon and germanium both have four valence electrons and they crystallize in the diamond structure. If a semiconducting material has elements

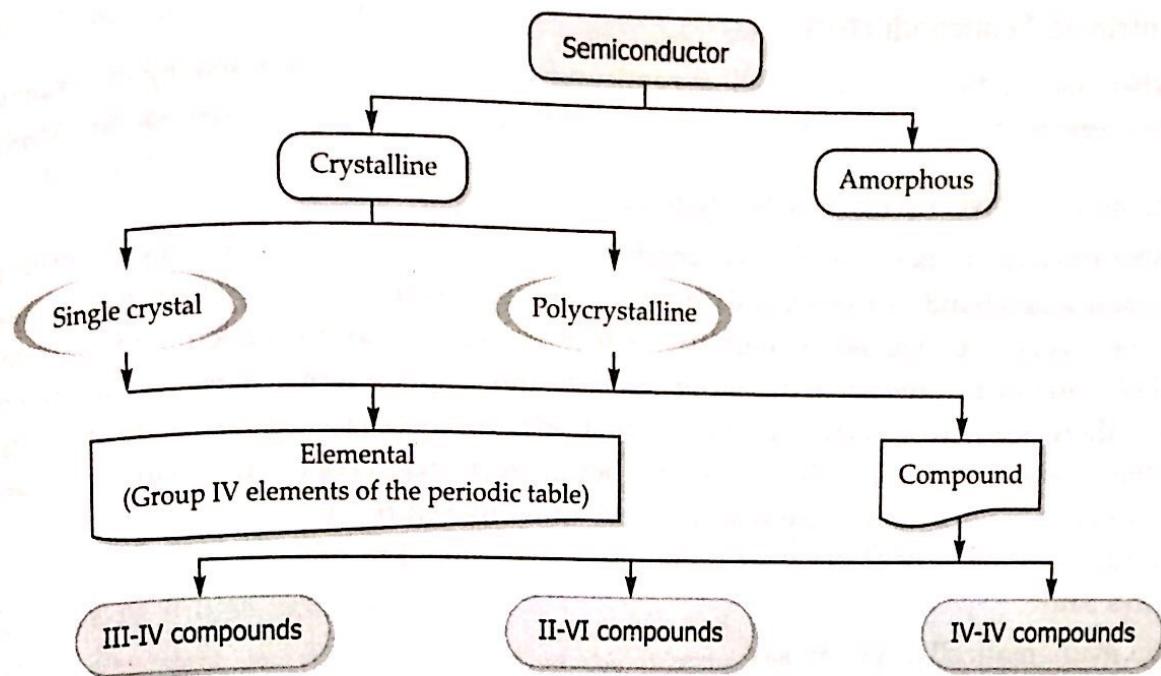


Figure 2.2 Classification chart of semiconductors.

from two different groups or from the same group of the Periodic Table, it is known as a *compound semiconductor*. This compound semiconductor has important group i.e., III-IV compounds in which one element from the third group and another element from the fifth group like gallium arsenide (GaAs), indium antimonide (InSb). On average they have four valence electrons. *II-VI compounds* are formed by combination of group II and group VI elements such as CdS and ZnO. The band gap of II-VI compounds is larger than III-V semiconductors. Another important group of semiconductors is IV-IV semiconducting compounds. This group of semiconductors is known as *narrow gap semiconductors*. Examples of IV-IV compounds are SiC, SiGe etc. These semiconductors are crystallized in simple rocksalt type structure.

Semiconductors are also further divided in terms of purity of their crystal structures as shown in Fig. 2.3.

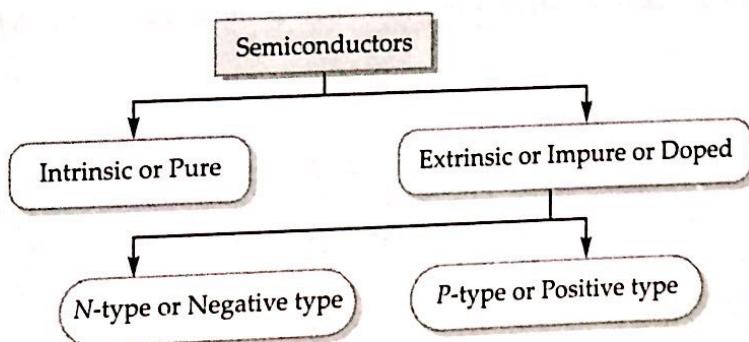


Figure 2.3 Classification of semiconductors on purity basis.

The semiconductors, which do not contain any impurity atom or defects are called *intrinsic (or pure) semiconductors*, while those which contain substitutional impurity at lattice sites are called *extrinsic (or impure or doped) semiconductors*. Extrinsic semiconductors are further divided into *N-type* or *P-type* semiconductors, depending upon the type of impurity added.

2.2.1 Intrinsic Semiconductors

When the conductivity in crystalline semiconductor is only due to breaking of covalent bonds of the electrons in the valence band, then the substance is said to be an intrinsic semiconductor.

Electrons and Positive Holes in a Semiconductor

When a covalent bond of Si or Ge crystal breaks, then a free electron n moves from valence band to conduction band of a semiconductor and it leaves behind an unfilled electronic state in the valence band (Fig. 2.4). The absence of an electron in valence band is called positive hole p . The word "hole" in a semiconductor therefore refers to the empty energy levels in valence band. So whenever there is an extra electron in the conduction band there is a simultaneous production of a hole in the valence band [Fig. 2.4(a)]. The number of electrons in conduction band and number of holes in valence band will increase with increase of temperature simultaneously. But for intrinsic semiconductor, number of electrons in conduction band n and number of holes in valence band p are always same.

So mathematically, we can say

$$n = p \quad \dots(2.1)$$

$$n = p = n_i \quad \dots(2.2)$$

where n_i is the number of charge carriers in intrinsic semiconductor. The positive hole, regarded as the active particle in the valence band, in the same way that the electron is considered to be active particle in conduction band. This leads to the concept of conductivity in semiconductors is caused not only by the motion of electrons in the conduction band which give rise to conduction current but also by the motion of the positive hole which gives rise to hole current in valence band. The movement of hole in the valence band is understood from Figs. 2.4–2.5. Hole positions can be changed from one place to other by interchanging their positions with electrons (Figs. 2.4 and 2.5). These positive holes are also physical entities whose movement constitute a flow of current, which is called *hole current*. Since movement of hole is opposite to that of electron, so the direction of hole current is same as direction of current flow. So in semiconductors, there are two charge carriers, electron and hole. But in metals electrons are the only charge carriers, because there is no question of breaking bonds and thereby forming the holes.

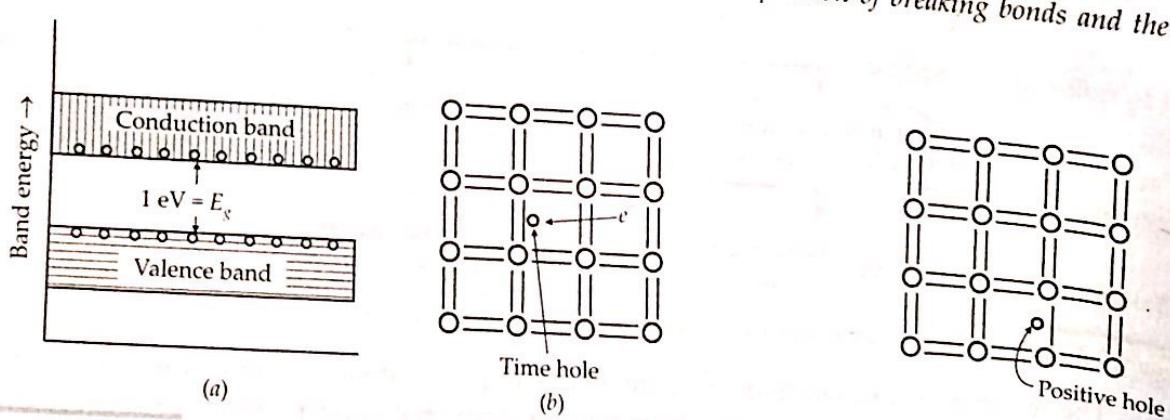


Figure 2.4 Formation of holes in V.B. and movement of holes in V.B. which give rise to hole-current.

Figure 2.5

2.2.2 Extrinsic Semiconductors

The electrical conductivity of intrinsic semiconductor is very small. To increase the conductivity of intrinsic semiconductor a small percentage of trivalent or pentavalent atom (impurities) is added to the pure semiconductor in the process of crystallization, which is called *doping* and results the impure semiconductor being called *extrinsic semiconductor*. The conductivity of extrinsic semiconductor is much higher, say for example, 12 times than intrinsic semiconductor when an impurity is added 1 part in 10^8 . The impurity atom has a size which is almost of the same order of the host lattice. Since percentage of impurity atoms is very small so every impurity atom is surrounded by a normal lattice site. So basic structure of crystal will not get altered after doping.

There are two types of extrinsic semiconductors :

➤ N-type semiconductor

➤ P-type semiconductor

Negative Type or N-type Semiconductor

If a pentavalent atom is introduced in the intrinsic semiconductor say germanium the situation is shown in Fig. 2.6. The impurity pentavalent atoms, some of the germanium atoms in the original crystal lattice and four of five valence electrons of the impurity atom will occupy four covalent bonds and fifth one will be almost free, as it does not find a place in the covalent bond. Every impurity atom thus contributes almost free electrons without creating a hole, and those free electrons act as carrier of current. A very small amount of energy is required to free the fifth electron from the atom, of the order of 0.01 eV for Ge and 0.05 eV for Si. Since those pentavalent impurities donate excess electron carrier, so they are called N-type impurity, and the crystal doped with donor impurity is called N-type semiconductor. Each ionized donor atom has a net charge of (+e) and is bound or immobile (Fig. 2.7).

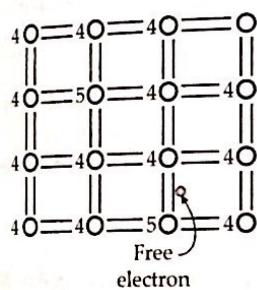


Figure 2.6 Formation of N-type semiconductor.

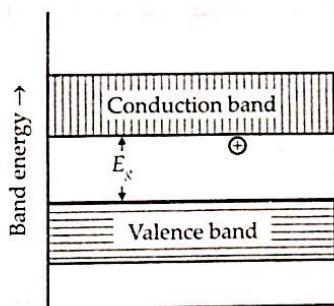


Figure 2.7 Energy band diagram of N-type semiconductor.

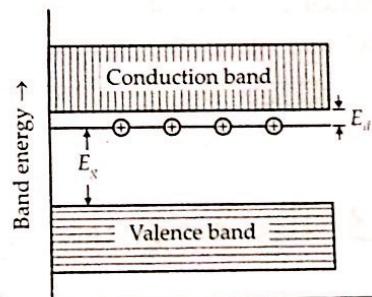


Figure 2.8 Energy band diagram of N-type semiconductor with additional discrete levels.

When a donor impurities are doped to an intrinsic semiconductor Ge or Si, then additional discrete levels are introduced just below the conduction band in the forbidden gap (Fig. 2.8). These new additional levels will be created and is known as *donor level* E_d (Fig. 2.8) because of additional impurity atoms are situated far apart in the crystal structure and hence their interaction is small. Since very small amount of energy (0.01 eV for Ge and 0.05 eV for Si) is required to free the electron from donor level to conduction band, therefore almost all the fifth electron of donor materials are raised to the conduction band at room temperature.

Positive Type or P-Type Semiconductor

When a trivalent impurity (boron $Z=65$, gallium $Z=31$ or indium $Z=49$) is added to the intrinsic semiconductor, only three of the covalent bond can be filled up and the vacancy that exists in the fourth bond constitute a hole (Fig. 2.9) without making an electron free. These holes can accept electrons and thus make available positive carriers in the valence band. So these trivalent impurities are called *acceptor* because every trivalent atom creates a *hole*, which accept electron and the intrinsic semiconductor with acceptor impurity is called *P-type* semiconductor. Each acceptor atom has a net charge ($-e$) and is bound or immobile (Fig. 2.10).

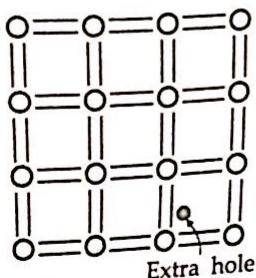


Figure 2.9 Formation of P-type semiconductor.

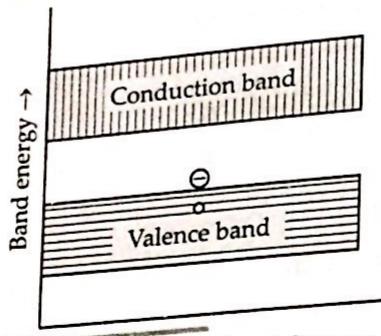


Figure 2.10 Energy band diagram of P-type semiconductor.

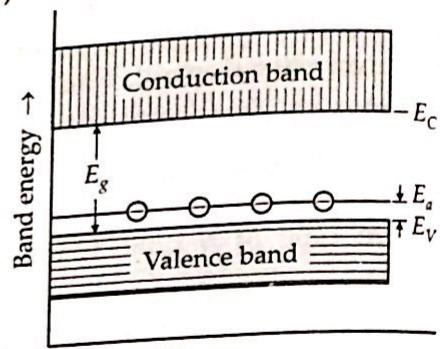


Figure 2.11 Energy band diagram of P-type semiconductor with additional discrete levels.

When acceptor impurity is doped to an intrinsic semiconductor, Ge or Si, then the additional discrete energy levels are introduced. Just above the valence band (Fig. 2.11) in the forbidden gap, which is called *acceptor level* (E_a). Since a very small amount of energy is necessary for an electron to leave the valence band and occupy the acceptor level, it thus creates a large number of holes in the valence band in the extrinsic semiconductor.

Thus by doping the impurity atoms in the intrinsic semiconductor not only conductivity is increased but it also serves to produce a conductor in which the electric carriers are mainly electrons or mainly holes.

NOTE Electrons are majority carriers and holes are minority carriers in N-type semiconductor whereas in P-type semiconductor the majority carriers are holes and minority carriers are electrons.

2.3 Mass-Action Law : Densities of Charge Carriers in a Semiconductor

In any semiconductor, the processes of thermal generation of new electron-hole pairs and recombination of the existing electrons and holes take place continuously. The number of electron-hole pairs created per unit volume per second is known as the *rate of generation* (g). It is a function of the energy gap (ΔE_g) and the temperature T , varying directly as T and inversely as ΔE_g .

$$g = f(\Delta E_g, T) \quad \dots(2.3)$$

The recombination rate r depends on the rate at which electrons and holes collide with each other. This, in turn, is directly proportional to the number densities n and p of the electrons and holes respectively in the semiconductor. Thus

$$r \propto n p \quad \dots(2.4)$$

Equilibrium is reached when the rate of generation just equal the rate of recombination resulting in constant carrier densities.

In an intrinsic semiconductor, the electrons and holes are present in equal numbers, so that $n_i = p_i$, the subscript i indicating the intrinsic material. The product $n_i p_i = n_i^2$ depends on the energy gap ΔE_g , hence it is a characteristic of the material and increase exponentially with temperature for a given material. Typical values of n_i at 300 K are 2.5×10^{19} per m³ for germanium and 1.5×10^{16} per m³ for silicon.

The relation $n_i p_i = n_i^2$ is said to be **Mass Action Law**.

In an *N*-type semiconductor, in addition to the thermally generated electron-hole pairs, extra electrons are injected into the material from the impurities, on increasing the electron density. This enhances the probability of hole colliding with an electron and recombining with it. Thus recombination rate increases so that the hole density goes below the intrinsic levels. Thus the electron density goes up while the hole density goes down such that their product remains unchanged at the intrinsic level :

$$n_n p_n = n_i^2 \quad \dots(2.5)$$

where the subscript n indicates *N*-type material. Likewise in a *P*-type semiconductor, because of the extra holes injected from the acceptor impurities, the hole density goes up while the electron density goes down in such a way that

$$n_p p_p = n_i^2 \quad \dots(2.6)$$

In order to calculate the carrier densities, we need to consider the balance of positive and negative charges in the semiconductor material which, as a whole, is electrically neutral. The positive charge is provided by the free holes and the donor atoms that have donated one electron and have become ionized. The negative charge is provided by the free electrons and the acceptor atoms that have already taken up an electron from the host crystal. For electrical neutrality, we must have

$$p + N_D = n + N_A \quad \dots(2.7)$$

where N_D and N_A are the number of donor and the acceptor atoms per unit volume respectively.

For an *N*-type semiconductor, $N_A = 0$ and the hole density is small as compared to the donor density, hence the electron density is approximately equal to the donor density :

$$n_n \approx N_D \quad \dots(2.8)$$

The hole density can then be obtained from Eq. (2.5) as $p_n = \frac{n_i^2}{N_D}$

The hole and electron densities for a *P*-type semiconductor can be obtained in a similar manner. These are

$$p_p = N_A \quad \dots(2.10)$$

The electron density can be obtained using Eq. (2.5) as $n_p = n_i^2 / N_A$.

Example 2.1 Consider an *N*-type silicon for which dopant concentration $N_D = 10^{17} / \text{cm}^3$. Find the electrons and hole concentration at $T = 300 \text{ K}$. [IGGSIPU, Dec. 2013 (2.5 marks)]

Solution. Given $N_D = 10^{17} / \text{cm}^3$, $n_i = 1.5 \times 10^{10} / \text{cm}^3$, then in *N*-type silicon electron concentration $n_n \approx N_D = 10^{17} / \text{cm}^3$.

$$\text{The hole density } p_n = \frac{n_i^2}{N_D} = \frac{(1.5 \times 10^{10})^2}{10^{17}} / \text{cm}^3 = 2.25 \times 10^3 / \text{cm}^3.$$

2.4 Electrical Conductivity of Semiconductors

If a semiconductor is placed in an external field, its charge carriers experiences a force and move in the direction of the field. This is known as the **drift** and it results in a drift current through the semiconductor.

It is often more convenient to talk in terms of the current density J rather than the total current I through a conductor. It is defined as the *current through a unit cross-sectional area normal to the direction of flow of charges*. Let ρ be the density of free charges in a conductor, consider a cylindrical region with its axis in the direction of v , its length equal to v and its cross-section unity as shown in Fig. 2.12. The charge contained in this cylinder is ρv . All this charge but no more will pass the cross-section in one second. Hence, the current through the unit cross-section i.e., the current density J will be :

$$J = \rho v \quad \dots(2.11)$$

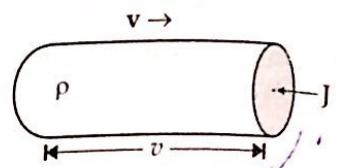


Figure 2.12

Now, let us consider a semiconductor placed in an external electric field E , so that the electrons and holes acquire average drift velocities equal to v_e and v_h respectively. Then from Eq. (2.11), the current density J_e due to the drift of electron is

$$J_e = nev_e \quad \dots(2.12)$$

and that due to the hole drift is $J_h = pve_h$...(2.13)

where n and p are respectively, the electron and hole densities in the semiconductor. In Eq. (2.12), the negative sign of the electronic charge has been left out because the electron velocity v_e will also be in opposite direction.

Thus the net current density J_{drift} through the semiconductor becomes

$$J_{drift} = e(nv_n + pve_p) \quad \dots(2.14)$$

The drift velocities are directly proportional to the field E , hence it is convenient to introduce a new parameter, the mobility μ , defined as the *drift velocity per unit electric field E*.

Thus $\mu_n = \frac{v_n}{E}$ and $\mu_p = \frac{v_p}{E}$...(2.15)

Combining Eqs. (2.14) and (2.15), the current density J_{drift} can be written in terms of the field E and the electron and hole mobilities as

$$J_{drift} = eE(n\mu_n + p\mu_p) \quad \dots(2.16)$$

Therefore, the conductivity (σ) of the semiconductor, which is equal to the current density per unit electric field, is given by

$$\sigma = e(n\mu_n + p\mu_p) \quad \dots(2.17)$$

The resistivity ρ is the inverse of the conductivity and is given by

$$\rho = \frac{1}{e(n\mu_n + p\mu_p)} \quad \dots(2.18)$$

Example 2.2 Show that the ratio of maximum resistivity to intrinsic resistivity is expressed as

$$\frac{\rho_{\max}}{\rho_i} = \frac{\mu_n + \mu_p}{2\sqrt{\mu_n \mu_p}}.$$

Solution. Intrinsic conductivity $\sigma_i = n_i e(\mu_n + \mu_p)$

The conductivity at any other doping level is expressed as

$$\sigma = e(n_n \mu_n + n_p \mu_p) \quad [\because n_n n_p = n_i^2]$$

Hence

$$\sigma = e \left[n_n \mu_n + \frac{n_i^2 \mu_p}{n_n} \right]$$

For resistivity to be maximum, conductivity has to be minimum, for that

$$\frac{d\sigma}{dn_n} = 0 = e \left(\mu_n - \frac{n_i^2 \mu_p}{n_n^2} \right)$$

$$n^2 = \frac{n_i^2 \mu_p}{\mu_n} \quad \text{or} \quad n = n_i \sqrt{\frac{\mu_p}{\mu_n}}$$

or

Similarly,

$$p = n \sqrt{\frac{\mu_n}{\mu_p}}$$

Hence

$$\sigma_{\max} = e(\sqrt{\mu_n - \mu_p} + \sqrt{\mu_n + \mu_p}) n_i = 2e n_i \sqrt{\mu_n \mu_p}$$

Therefore

$$\frac{\sigma_i}{\sigma_{\max}} = \frac{\rho_{\max}}{\rho_i} = \frac{e n_i (\mu_n + \mu_p)}{2e n_i \sqrt{\mu_n \mu_p}} = \frac{(\mu_n + \mu_p)}{2\sqrt{\mu_n \mu_p}}.$$

Example 2.3 An intrinsic silicon specimen at approximately 300 K has a conductivity of $4.3 \times 10^{-4} \Omega^{-1} m^{-1}$. What is the intrinsic carrier concentration? If a current is passed through a specimen, what proportion of it is carried by the electrons? The same specimen is now doped to make N-type. The donor concentration is $10^{21} / m^3$. Find the hole density of the doped specimen and also the proportion of current that would now be carried by electrons. Assume that the mobilities are substantially unchanged by the doping process. Given μ_n at 300 K = $0.135 \text{ m}^2 / \text{Vs}$; μ_p at 300 K = $0.048 \text{ m}^2 / \text{Vs}$.

Solution. For intrinsic semiconductor, $\sigma = e(n\mu_n + p\mu_p)$ with $n = p = n_i$

$$\text{Hence } n_i = \frac{\sigma}{e(\mu_n + \mu_p)} = \frac{4.3 \times 10^{-4}}{1.6 \times 10^{-19} \times (0.135 + 0.048)} = 1.47 \times 10^{16} / \text{m}^3$$

The proportion of current carried by electrons is given by

$$\frac{n\mu_n}{n(\mu_n + \mu_p)} = \frac{\mu_n}{(\mu_n + \mu_p)} = \frac{0.135}{0.183} = 0.738$$

For extrinsic N-type material, we have

$$p = \frac{n_i^2}{N_D} \text{ with } n_i = 1.47 \times 10^{16} / \text{m}^3 \text{ and } N_D = 10^{21} / \text{m}^3.$$

Hence the concentration of holes in the N-type material is

$$p = \frac{n_i^2}{N_D} = \frac{(1.47 \times 10^{16})}{10^{21}} = 2.16 \times 10^{11} / \text{m}^3 = 2.16 \times 10^{11} / \text{m}^3.$$

Hence the proportion of current in this case is

$$\frac{n\mu_n}{n\mu_n + p\mu_p} = \frac{1}{1 + \frac{p\mu_p}{n\mu_p}} = \left(1 + \frac{p\mu_p}{n\mu_n}\right)^{-1} = \left(1 - \frac{p\mu_n}{n\mu_p}\right) = \left[1 - \frac{2.16 \times 10^{11} \times 0.048}{10^{21} \times 0.135}\right]$$

i.e., $= [1 - 10^{-10}] \approx 1$ and so the current is mainly due to electrons.

Example 2.4 Find the resistivity of (a) intrinsic silicon (b) P-type silicon with $N_A = 10^6 / \text{cm}^3$, use $n_i = 1.5 \times 10^{10} / \text{cm}^3$ and assume that for intrinsic silicon $\mu_n = 1350 \text{ cm}^2/\text{Vs}$ and $\mu_p = 480 \text{ cm}^2/\text{Vs}$ for the doped silicon $\mu_n = 1110 \text{ cm}^2/\text{Vs}$ and $\mu_p = 400 \text{ cm}^2/\text{Vs}$. [GGSIPU, Dec. 2013 (12.5 marks)]

Solution. Given $N_A = 10^6 / \text{cm}^3$, $n_i = 1.5 \times 10^{10} / \text{cm}^3$

For intrinsic silicon $\mu_n = 1350 \text{ cm}^2/\text{Vs}$ and $\mu_p = 480 \text{ cm}^2/\text{Vs}$.

For P-type silicon $\mu_n = 1110 \text{ cm}^2/\text{Vs}$ and $\mu_p = 400 \text{ cm}^2/\text{Vs}$.

(a) Resistivity of intrinsic silicon

$$\begin{aligned} \text{Intrinsic conductivity } \sigma_i &= e(n\mu_e + p\mu_p) = en_i(\mu_e + \mu_p) \\ &= (1.6 \times 10^{-19}) \times 1.5 \times 10^{10} \frac{1}{\text{cm}^3} \times (1350 + 480) \frac{\text{cm}^2}{\text{Vs}} \\ &= 1.6 \times 1.5 \times 10^{-9} \times 1830 \frac{1}{\Omega \text{cm}} = 4.392 \times 10^{-6} \times 10^2 (\Omega \text{m})^{-1} = 4.392 \times 10^{-4} (\Omega \text{m})^{-1} \end{aligned}$$

Then the resistivity $\rho_i = \frac{1}{\sigma_i} = 2.27 \times 10^3 \Omega \text{m}$

(b) Resistivity of P-type silicon

Conductivity $\sigma_p = e_p \mu_p$

$$p = N_A = 10^6 / \text{cm}^3$$

$$\sigma_p = 1.6 \times 10^{-19} \times 10^6 \times 480 \times 10^2 \frac{1}{\Omega \text{m}} = 7.68 \times 10^{-11} \frac{1}{\Omega \text{m}}$$

Then the resistivity $\rho_p = \frac{1}{\sigma_p} = 1.30 \times 10^{10} \Omega \text{m}$

Example 2.5 Find the concentration of holes and electrons in a P-type germanium at 300 K if the conductivity is $100 (\Omega \text{cm})^{-1}$. Also find these values for N-type silicon if the conductivity is $0.1 (\Omega \text{cm})^{-1}$.

Given that for Ge, $n_i = 2.5 \times 10^{13} / \text{cm}^3$, $\mu_n = 3800 \text{ cm}^2 / \text{Vs}$ and $\mu_p = 1800 \text{ cm}^2 / \text{Vs}$ and for Si, $n_i = 1.5 \times 10^{10} / \text{cm}^3$, $\mu_n = 1300 \text{ cm}^2 / \text{Vs}$ and $\mu_p = 500 \text{ cm}^2 / \text{Vs}$.

Solution. Concentration in P-type germanium : fed

Conductivity of a P-type germanium, $\sigma = e p \mu_p$

$$100 = 1.6 \times 10^{-19} \times p \times 1800$$

$$\therefore N_A = p = \frac{100}{2.88 \times 10^{-16}} = 3.47 \times 10^{17} / \text{cm}^3$$

Concentration of electrons,

$$N_D = n = \frac{n_i^2}{p} = \frac{(2.5 \times 10^{13})^2}{3.47 \times 10^{17}} = 1.8 \times 10^9 / \text{cm}^3$$

Concentration in N-type silicon

Conductivity of an N-type silicon, $\sigma = e n \mu_n$

$$N_D = n = \frac{\sigma}{e \mu_n} = \frac{0.1}{1.6 \times 10^{-19} \times 1300} = 4.8 \times 10^{14} / \text{cm}^3$$

$$\text{Number of holes, } p = N_A = \frac{n_i^2}{n} = \frac{(1.5 \times 10^{10})^2}{4.8 \times 10^{14}} = 4.7 \times 10^5 / \text{cm}^3$$

Example 2.6 A pure semiconductor (Ge) is simultaneously doped with donor and acceptor impurities to the extent of $1:10^7$ donor atoms and $1:10^{11}$ acceptor. Find its conductivity. Given total number of atoms $= 4.421 \times 10^{22} / \text{cm}^3$, $n_i = 2.5 \times 10^{13} \text{ atoms/cm}^3$, $\mu_n = 3800 \text{ cm}^2 / \text{Vs}$ and $\mu_p = 1800 \text{ cm}^2 / \text{Vs}$.

[GGSIPU, Oct. 2013 (2.5 marks)]

Solution. (a) Number of germanium atoms $= 4.421 \times 10^{22} / \text{cm}^3$

Hence with the given impurity of 1 donor atom per 10^7 germanium atom

$$N_D = 4.421 \times 10^{15} \text{ atom/cm}^3$$

Further $n \approx N_D$

$$\text{Hence } p = \frac{n_i^2}{N_D} = \frac{(2.5 \times 10^{13})^2}{4.421 \times 10^{15}} = 1.413 \times 10^{11} \text{ holes/cm}^3$$

Since $n \gg p$, we may neglect p in calculating the conductivity,

then

$$\sigma = n q \mu_n = 4.421 \times 10^{15} \times 1.6 \times 10^{-19} \times 3800 = 2.69 \text{ S/cm}$$

(b) With the given acceptor impurity of 1 acceptor atom per 10^{11} germanium atom

$$N_A = 4.421 \times 10^{11} \text{ atom/cm}^3$$

Further $p = N_A$

$$\text{Hence } n = \frac{n_i^2}{N_A} = \frac{(2.5 \times 10^{13})^2}{4.421 \times 10^{11}} = 1.413 \times 10^{15} \text{ electrons/cm}^3$$

Since $p \gg n$, we may neglect n in calculating conductivity,

$$\sigma = p q \mu_p = 4.421 \times 10^{11} \times 1.6 \times 10^{-19} \times 1800 = 1.273 \times 10^{-4} \text{ S/cm}$$

2.5 Fermi-Dirac Function, Fermi Level and Effect of Temperature in Intrinsic Semiconductor

As we know that at absolute zero all the electronic states of the valence band are full and those of the conduction band are empty, so a semiconductor becomes an insulator at 0 K. But as the temperature increases some electrons from the valence band get sufficient energy and become free. They move to the conduction band and take part in conduction and give rise to conductivity in the semiconductor.

Now we want to discuss the phenomenon with quantum mechanical point of view. Classically all electrons have zero energy at 0 K, but quantum mechanically electrons cannot have zero energy at 0 K. The maximum energy that electrons may possess at 0 K is the Fermi energy (E_F). So quantum mechanically the electrons actually have energies extending from 0 to E_F at absolute zero temperature (Fig. 2.13).

Now in order to know how many of the electronic energy states in the valence band and conduction band will be occupied at different temperatures, we introduce a Fermi-factor $f(E)$, which is the number that expresses probability that a state of a given energy (E) is occupied by an electron under condition of thermal equilibrium. This number has a value between zero and unity and is a function of energy and temperature (or distribution function for Fermi-Dirac statistics) as

$$f(E) = \frac{1}{e^{(E-E_F)/(k_B T)} + 1} \quad \dots(2.19)$$

where k_B is Boltzmann constant, T is temperature in K and E_F is Fermi level of energy in eV.

The Fermi factor is independent of the energy density of states, it is the probability that the states occupied at that level irrespective of the number of states actually present. That is, it is the fractional occupancy of possible states.

The variation of Fermi factor $f(E)$ with energy at different temperatures for intrinsic semiconductor is as shown in Fig. 2.14, where full line curve shows $f(E)$ at 0 K and dotted line curve at T K.

When $T = 0$ K; $f(E)$ can take two values

(i) $E > E_F$

$$f(E) = \frac{1}{1 + \exp(\infty)} = 0 \quad (\text{because } e^\infty = \infty) \quad \dots(2.20)$$

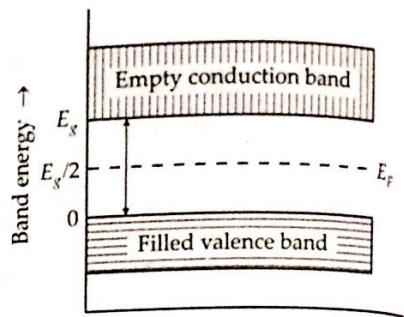


Figure 2.13 Intrinsic semiconductor with Fermi level $E_F = (E_g/2)$

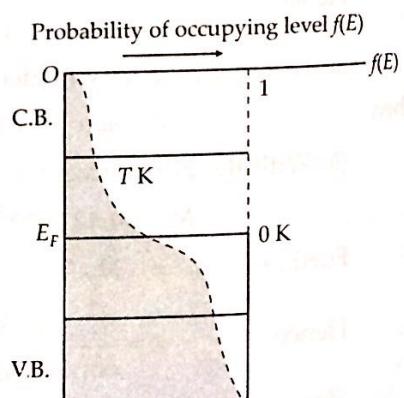


Figure 2.14 Variation of Fermi factor $f(E)$ with temperature.

(ii) $E < E_F$

$$f(E) = \frac{1}{1 + \exp(-\infty)} = 1 \quad (\text{because } e^{-\infty} = 0) \quad \dots(2.21)$$

Thus at $T=0\text{ K}$, $f(E)=1$, when $E < E_F$, which means that all the level below E_F i.e., valence band are filled up by electron and $f(E)=0$, when $E > E_F$ i.e., all level above E_F i.e., conduction bands are empty. Because at 0 K , no heat energy is present, so no covalent bonds are being broken and the semiconductor behaves as an insulator.

But when $t=T\text{ K}$, then at $E=E_F$, $f(E)$ becomes

$$f(E) = \frac{1}{e^0 + 1} = \frac{1}{2} \quad [:: e^0 = 1] \quad \dots(2.22)$$

which means that when the temperature is not 0 K but some higher value say $T=1000\text{ K}$, then some covalent bonds will break and some electrons will be available in conduction band whereas some electron vacancy i.e., hole will be available at valence band. This is shown by the dotted line in Fig. 2.14.

Example 2.7 Fermi energy of an intrinsic semiconductor is 0.6 eV . The low lying energy level in the conduction band is 0.2 eV above the Fermi level. Calculate the probability of occupation of this level by an electron at room temperature.

Solution. Given $E_F = 0.6\text{ eV}$, then $E = (0.6 + 0.2)\text{ eV} = 0.8\text{ eV}$

The probability of occupation of an energy level by an electron is given by the Fermi-Dirac distribution law

$$\begin{aligned} f(E) &= \frac{1}{1 + \exp\left(\frac{E - E_F}{k_B T}\right)} \\ &= \frac{1}{1 + \exp\left[\frac{(0.8 - 0.6)\text{ eV} \times 1.6 \times 10^{-19} \text{ J}}{1.38 \times 10^{-23} \text{ JK}^{-1} \times 300 \text{ K}}\right]} = \frac{1}{1 + \exp(7.7)} = 0.0004 = 0.04\% \end{aligned}$$

2.6 Fermi Level in Extrinsic Semiconductors

In an intrinsic semiconductor the number of electrons is equal to number of holes ($n_i = p_i$). But in N -type extrinsic semiconductor number of electrons are increased due to doping of pentavalent atom ($n_e > n_i$) and number of holes are decreased ($p_p < p_i$) than which would be available for intrinsic semiconductor. The number of holes are decreased because of the large number of electrons present which increase the rate of recombination of electrons and holes. With similar argument it can be shown that in P -type extrinsic semiconductor the number of holes are increased due to doping of trivalent impurity atom ($p_p > p_i$) and number of electrons decreased ($n_e < n_i$) than which would be available for intrinsic semiconductor. Now since the Fermi level (E_F) is a measure of the probability of occupancy of the allowed energy states by the electron, so for an intrinsic semiconductor when $n_i = p_i$; Fermi level is at the centre of the forbidden gap as shown in Fig. 2.15(a). But for N -type semiconductor, since $n_e > p_e$, it is clear that E_F must move closer to

conduction band as shown in Fig. 2.15(b), to indicate that many of the energy states in the band are filled by the donor electrons, and of few holes exist in the valence band. Similarly, for P-type semiconductor, since $p_e > n_e$ so E_F must move from the centre of the forbidden gap closer to the valence band as shown in Fig. 2.15(c).

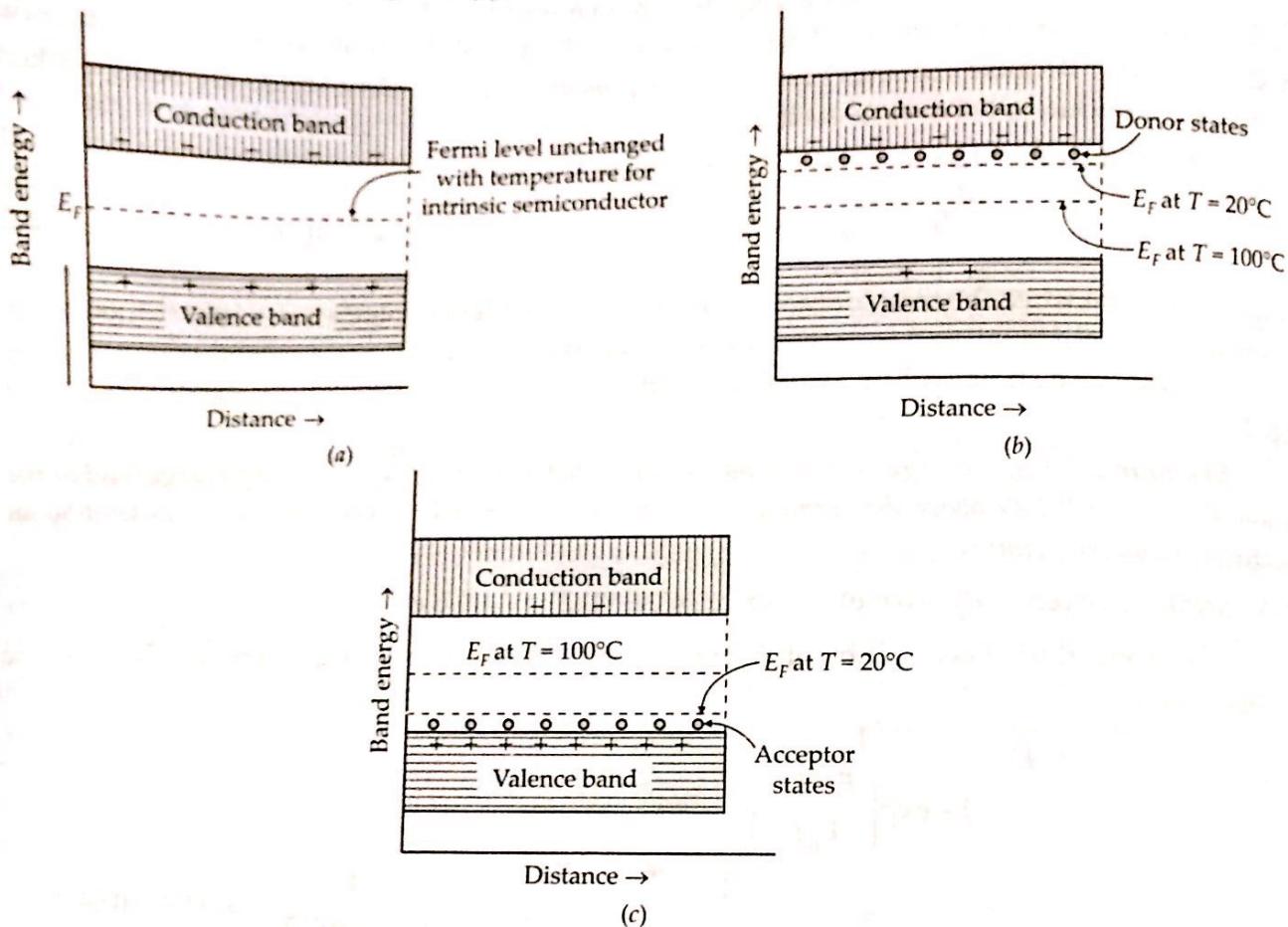


Figure 2.15 Position of Fermi level (E_F) for (a) intrinsic semiconductor ; (b) N-type semiconductor ; (c) P-type semiconductor.

2.7 Variation of Fermi Level with Temperature in Extrinsic Semiconductors

For an intrinsic semiconductor $n_i = p_i$ and as temperature increases both n_i and p_i will increase. Thus the Fermi level (E_F) will remain approximately at the centre of the forbidden gap. Thus for intrinsic semiconductor Fermi level is independent of temperature as shown in Fig. 2.15(a).

But in an extrinsic semiconductor it is different. Say for N-type material the electrons are coming from two different sources. Some electrons come from donor state, which are easily separated from their parent atom and they do not vary much as the temperature is increased. The other electrons in the conduction band are present because of the breaking of a covalent bond. Such intrinsic effect will increase in number as temperature raised. So proportion of the total number of conduction electrons will increase from valence band. Thus *as the temperature rises the material becomes more and more intrinsic and Fermi level moves closer to the intrinsic position, i.e., at the centre of the forbidden gap as shown in Fig. 2.15(b)*.

increasing intrinsic semiconductors more and more like intrinsic material at high temperature. This places a limit on the operating temperature of an extrinsic semiconductor device.

2.8 Carrier Concentration and Fermi Level in Intrinsic Semiconductors

We have assumed that the width of the allowed energy band is comparable to the forbidden gap of width E_g . Conduction band electrons have energy lying between E_C and ∞ , and, valence band electrons have energy lying between $-\infty$ to E_V . This is shown in Fig. 2.16.

2.8.1 Electron Concentration in Conduction Band

We know the total numbers of electrons per unit volume in conduction band is given by in terms of Fermi-Dirac distribution function $f(E)$ and density of state function $g(E)dE$ as

$$n_e = \int_{E_C}^{\text{top}} f(E)g(E)dE \quad \dots(2.23)$$

Here $f(E)$ decreases as we rise through the conduction band because $f(E)=0$ for $E \gg E_g$.

So we have set the upper limit for energy as infinity for conduction band.

Hence, Eq. (2.23) becomes

$$n_e = \int_{E_C}^{\infty} f(E)g(E)dE$$

$$\text{Here } f(E) = \frac{1}{(e^{(E-E_F)/k_B T} + 1)} \quad \text{and } g(E)dE = \frac{4\pi}{h^3} (2m_e^*)^{3/2} \cdot (E - E_C)^{1/2} dE$$

Hence putting the values of $f(E)$ and $g(E)dE$, we have

$$n_e = \int_{E_C}^{\infty} \left[\frac{1}{e^{(E-E_F)/k_B T} + 1} \right] \times \left[\frac{4\pi}{h^3} (2m_e^*)^{3/2} \cdot (E - E_C)^{1/2} dE \right]$$

As $(E - E_F) \gg k_B T$, we can ignore the unit term in the $f(E)$ and so

$$n_e = \frac{4\pi}{h^3} (2m_e^*)^{3/2} \int_{E_C}^{\infty} [(E - E_C)^{1/2} \times e^{-(E_F - E)/k_B T}] dE$$

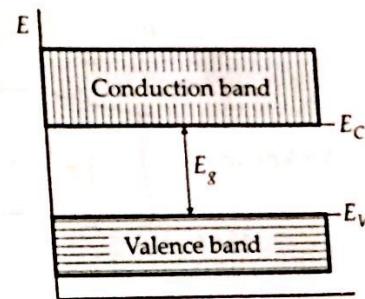


Figure 2.16 Energy band diagram of an intrinsic semiconductor.

$$\Rightarrow n_e = \frac{4\pi}{h^3} (2m_e^*)^{3/2} \int_{E_C}^{\infty} [(E - E_C)^{1/2} \times e^{[(E_F - E_C)/k_B T]} \times e^{[(E - E)/k_B T]} dE] \quad \dots(2.24)$$

Let

$$y = (E - E_C) / k_B T$$

$$\Rightarrow dE = k_B T dy \text{ and } [E - E_C]^{1/2} = [y]^{1/2} [k_B T]^{1/2}$$

Putting these into Eq. (2.24), we have

$$n_e = \frac{4\pi}{h^3} (2m_e^*)^{3/2} [k_B T]^{1/2} \times e^{[(E_F - E_C)/k_B T]} \int_{E_C}^{\infty} [(y)^{1/2} \times e^{-y} \times (k_B T) dy] \quad \dots(2.25)$$

We know

$$\boxed{\int_0^{\infty} y^{1/2} \times e^{-y} dy = \left(\frac{\pi}{4}\right)^{1/2}}$$

Hence, from Eq. (2.25)

$$\begin{aligned} n_e &= \frac{4\pi}{h^3} (2m_e^*)^{3/2} [k_B T]^{1/2} \times e^{[(E_F - E_C)/k_B T]} \times \left(\frac{\pi}{4}\right)^{1/2} \\ &= 2 \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{3/2} \times e^{[(E_F - E_C)/k_B T]} \end{aligned} \quad \dots(2.26)$$

2.8.2 Hole Concentration in Valence Band (VB)

Here the probability for a state of energy E to be unoccupied is given by $[1 - f(E)]$.

Thus the total numbers of holes per unit volume is given by

$$n_h = \int_{\text{bottom}}^{E_V} [1 - f(E)] g(E) dE \quad \dots(2.27)$$

As earlier, we can change the lower limit as $-\infty$

$$\text{Now } 1 - f(E) = 1 - \frac{1}{e^{[(E - E_F)/k_B T]} + 1} = \frac{e^{[(E - E_F)/k_B T]}}{e^{[(E - E_F)/k_B T]} + 1}$$

However, for valence band, $E < E_F$, we have

$$e^{[(E - E_F)/k_B T]} \ll 1$$

$$\text{Hence } 1 - f(E) \approx e^{[(E - E_F)/k_B T]} \quad \dots(2.28)$$

Thus, in valence band, $[1 - f(E)]$ decreases exponentially as we move downwards, from top of the valence band. Now using Eq. (2.28) into Eq. (2.27), we have

$$n_h = \int_{-\infty}^{E_V} e^{[(E - E_F)/k_B T]} \times \left[\frac{4\pi}{h^3} (2m_e^*)^{3/2} (E_V - E)^{3/2} dE \right]$$

On solving as in above Eq. (2.25), we have

$$n_h = 2 \left(\frac{2 \pi m_h^* k_B T}{h^2} \right)^{3/2} \times e^{[(E_V - E_F)/k_B T]} \quad \dots(2.29)$$

2.8.3 Value of Fermi Energy

In intrinsic semiconductor $n_e = n_h$

So equating Eqs. (2.26) and (2.29), we have

$$\begin{aligned} & 2 \left(\frac{2 \pi m_e^* k_B T}{h^2} \right)^{3/2} \times e^{[(E_F - E_C)/k_B T]} = 2 \left(\frac{2 \pi m_h^* k_B T}{h^2} \right)^{3/2} \times e^{[(E_V - E_F)/k_B T]} \\ \Rightarrow & (m_e^*)^{3/2} \times e^{[(E_F - E_C)/k_B T]} = (m_h^*)^{3/2} \times e^{[(E_F - E_C)/k_B T]} \\ \Rightarrow & e^{[(2E_F - E_C - E_V)/k_B T]} = \left(\frac{m_h^*}{m_e^*} \right)^{3/2} \\ \Rightarrow & \left\{ \frac{2E_F - E_C - E_V}{k_B T} \right\} = \frac{3}{2} \log \left(\frac{m_h^*}{m_e^*} \right) \\ \Rightarrow & E_F = \frac{E_C + E_V}{2} + \frac{3}{4} \log \left(\frac{m_h^*}{m_e^*} \right) \quad \dots(2.31) \\ & \qquad \qquad \qquad \left[\text{for } m_e^* = m_h^*, \log \frac{m_h^*}{m_e^*} = \log 1 = 0 \right] \end{aligned}$$

$$\Rightarrow E_F = \frac{E_C + E_V}{2} \quad \dots(2.32)$$

So Fermi level lies exactly in the middle of the top of valence band and the bottom of the conduction band.

NOTE Show that for an intrinsic semiconductor, Fermi level lies at the middle of the band gap.

Now putting the values of E_F into Eqs. (2.26) and (2.29) separately, we have

$$\begin{aligned} n_e &= 2 \left(\frac{2 \pi m_e^* k_B T}{h^2} \right)^{3/2} \times e^{[(E_F - E_C)/k_B T]} \\ &= 2 \left(\frac{2 \pi k_B T}{h^2} \right)^{3/2} \times [(m_e^*)^2]^{3/4} e^{[(E_C + E_F)/2 - E_C/k_B T]} \end{aligned}$$

$$\text{From Eq. (2.30), } n_e = 2 \left(\frac{2 \pi k_B T}{h^2} \right)^{3/2} [m_e^* m_h^*]^{3/4} e^{[(E_V - E_C)/2k_B T]} \quad \dots(2.33)$$

$$= 2 \left(\frac{2 \pi k_B T}{h^2} \right)^{3/2} [m_e^* m_h^*]^{3/4} e^{[-E_g/2k_B T]} \quad \dots(2.34)$$

and

$$n_h = 2 \left(\frac{2 \pi k_B T}{h^2} \right)^{3/2} [m_e^* m_h^*]^{3/4} e^{[-E_g/2k_B T]} \quad \dots(2.35)$$

Hence $n_h = n_e = n_i$, the intrinsic carrier concentration of the semiconductor.

Example 2.8 In a certain semiconductor, the mobility of electrons is $0.3 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$. The mobility of holes is $0.2 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$ and forbidden energy band is 0.7 eV . Calculate the intrinsic carrier concentration in semiconductor at 300 K , if effective mass of electrons and holes are respectively 0.55 and 0.37 times the rest mass of the electron.

Solution. Given : $T = 300 \text{ K}$, $E_g = 0.7 \times 1.6 \times 10^{-19} \text{ J}$, $m_e^* = 0.55$, $m_e = 0.55 \times 9.1 \times 10^{-31} \text{ kg}$, $m_h^* = 0.37$, $m_h = 0.37 \times 9.1 \times 10^{-31} \text{ kg}$, $k_B = 1.38 \times 10^{-23} \text{ JK}^{-1}$ and $h = 6.626 \times 10^{-34} \text{ Js}$

$$\text{The intrinsic concentration } n_i = 2 \left\{ \frac{2 \pi k_B T}{h^2} \right\}^{3/2} (m_e^* m_h^*)^{3/4} \exp \left[\frac{-E_g}{2k_B T} \right]$$

$$= 2 \left\{ \frac{2 \times \pi \times 1.38 \times 10^{-23}}{(6.626 \times 10^{-34})^2} \right\}^{3/2} [(0.55 \times 9.1 \times 10^{-31} \times 0.37 \times 9.1 \times 10^{-31})]^{3/4} \exp \left[-\frac{0.7 \times 1.6 \times 10^{-19}}{2 \times 1.38 \times 10^{-23} \times 300} \right]$$

$$= 1.35 \times 10^{13} \text{ m}^{-3}$$

2.9 Carrier Concentration and Fermi Level in Extrinsic Semiconductors

2.9.1 N-Type Semiconductor

Fermi level corresponds to the centre of gravity of the electrons and holes. In the case of intrinsic semiconductor, the number of electrons are equal to number of holes. The Fermi level lies midway between the valence band and conduction band. In N-type semiconductor the Fermi level is lifted towards the conduction band as the conduction electrons are the majority carriers.

For N-type semiconductor a donor level E_d is formed below the Fermi level. At a temperature T , the density of conduction electron from Eq. (2.26) is given by

$$n_e = 2 \left(\frac{2 \pi m_e^* k_B T}{h^2} \right) e^{[(E_F - E_C)/k_B T]} \quad \dots(2.35)$$

The density of vacancies in the donor levels of energy E_d is given by

$$N_d^+ = n_d [1 - f(E)] = n_d \left[1 - \frac{1}{e^{(E_d - E_F)/k_B T} + 1} \right] = \left[\frac{e^{(E_d - E_F)/k_B T}}{e^{(E_d - E_F)/k_B T} + 1} \right]$$

Here N_d^+ is concentration for ionised donor atoms S. Assuming the position a little above the donor level by a few $k_B T$ the above expression can be approximated

$$N_d^+ = n_d [e^{(E_d - E_F)/k_B T}] \quad \dots(2.36)$$

However, the electron concentration in the conduction band and the vacancy concentration in donor level should be equal.

Hence equating Eqs. (2.26) and (2.36), we have

$$n_e = 2 \left(\frac{2 \pi m_e^* k_B T}{h^2} \right) e^{[(E_F - E_C)/k_B T]} = n_d [e^{(E_d - E_F)/k_B T}]$$

$$\text{or } \log \left[2 \left(\frac{2 \pi m_e^* k_B T}{h^2} \right)^{3/2} \right] + \frac{E_F - E_C}{k_B T} = \log n_d + \frac{E_d - E_F}{k_B T}$$

$$\Rightarrow \frac{2 E_F - E_C - E_d}{k_B T} = \log \frac{n_d}{2 \left(\frac{2 \pi m_e^* k_B T}{h^2} \right)^{3/2}}$$

$$\Rightarrow E_F = \frac{E_C - E_d}{2} + \frac{k_B T}{2} \log \frac{n_d}{2 \left(\frac{2 \pi m_e^* k_B T}{h^2} \right)^{3/2}} \quad \dots(2.37)$$

At absolute zero temperature, $E_F = \frac{E_C + E_d}{2}$. This means that the Fermi level lies at half way between donor level and the bottom of the conduction band.

With an increase in T , E_F decrease and at room temperature it comes below the donor level.

Substituting the value of E_F from Eq. (2.37) into Eq. (2.24), we have the free electron concentration in the conduction band as

$$n_e = 2 \left(\frac{2 \pi m_e^* k_B T}{h^2} \right)^{3/2} \exp \frac{E_F - E_C}{k_B T}$$

$$= 2 \left(\frac{2 \pi m_e^* k_B T}{h^2} \right)^{3/2} \exp \left[\frac{\frac{E_C + E_d}{2} + \frac{k_B T}{2} \log \frac{n_d}{\left[2 \left(\frac{2 \pi m_e^* k_B T}{h^2} \right) \right]^{1/2}} - E_C}{k_B T} \right]$$

$$\begin{aligned}
 &= 2 \left(\frac{2 \pi m_e^* k_B T}{h^2} \right)^{3/2} \exp \left[\frac{E_d - E_C}{k_B T} + \frac{1}{2} \log \left[\frac{n_d}{2 \left(\frac{2 \pi m_e^* k_B T}{h^2} \right)^{3/2}} \right] \right] \\
 &= 2 \times \left(\frac{2 \pi m_e^* k_B T}{h^2} \right)^{3/2} e^{(E_d - E_C)/2k_B T} \times \frac{n_d}{2 \left(\frac{2 \pi m_e^* k_B T}{h^2} \right)^{2/3}} \\
 &= (2 n_d)^{1/2} \left(\frac{2 \pi m_e^* k_B T}{h^2} \right)^{3/4} e^{(E_d - E_C)/2k_B T}
 \end{aligned}$$

The ionization energy is given by $\Delta E = E_C - E_d$ of the donors, then, we have

$$n_e = (2 n_d)^{1/2} \left(\frac{2 \pi m_e^* k_B T}{h^2} \right)^{3/4} e^{(-\Delta E)/2k_B T} \quad \dots(2.38)$$

From Eq. (2.38), we say that electron concentration is proportional to the square root of the donor concentration of the semiconductor. With more increase E_F reaches up to the middle of the conduction band and valence band to make the material intrinsic.

Example 2.9 In an N-type semiconductor, the Fermi level lies 0.3 eV below the conduction band at 300 K. If the temperature is increased to 330 K, find the new position of the Fermi level.

Solution. Given $(E_C - E_F) = 0.3 \text{ eV}$ or $(E_F - E_C) = -0.3 \text{ eV}$ at temperature 300 K

$$(E_C - E_F) = ? \text{ at temperature } 330 \text{ K}$$

We know that $n_e = 2 \left(\frac{2 \pi m_e^* k_B T}{h^2} \right)^{3/2} \exp \left(\frac{E_F - E_C}{k_B T} \right)$

Assuming that the density of electrons in the conduction band of intrinsic semiconductor (n_e) remains unchanged by changing temperature 300 K to 330 K

$$(n_e)^{300 \text{ K}} = (n_e)^{330 \text{ K}}$$

$$\exp \left[\frac{E_F - E_C}{k_B \times 300} \right] = \exp \left[\frac{E_F - E_C}{k_B \times 330} \right]$$

$$\Rightarrow \frac{0.3}{300 \text{ K}} = \frac{(E_C - E_F)_{330}}{330 \text{ K}}$$

$$(E_C - E_F) \text{ at } 330 \text{ K} = \frac{0.3 \times 330}{300} = 0.33 \text{ eV}$$

Thus at 330 K, the Fermi energy level lies 0.33 eV below the conduction band.

2.9.2 P-type Semiconductors

For P-type semiconductors an acceptor level E_a is formed above the Fermi level and the valence band. At a temperature T , the density of holes is similar to Eq. (2.39) and is given by

$$n_h = 2 \left(\frac{2 \pi m_h^* k_B T}{h^2} \right)^{3/2} e^{(E_V - E_F)/k_B T} \quad \dots(2.39)$$

The density of electrons in the acceptor levels of energy E_a is given by

$$N_a^- = n_a [f(E)] = n_a \left[\frac{1}{e^{(E_a - E_F)/k_B T} + 1} \right] \quad \dots(2.40)$$

Here N_a^- is ionized acceptor atoms concentration. However, the hole concentration in the valence band and the electron concentration in the acceptor level should be equal.

Also, assuming $E_a - E_F \gg a$ few $k_B T$, equating Eqs. (2.39) and (2.40), we have

$$2 \left(\frac{2 \pi m_h^* k_B T}{h^2} \right)^{3/2} e^{(E_V - E_F)/k_B T} = n_a \left[\frac{1}{e^{(E_a - E_F)/k_B T}} \right]$$

On simplification

$$E_F = \left\{ \frac{E_a + E_V}{2} \right\} - \frac{k_B T}{2} \log \frac{n_a}{\left[2 \left(\frac{2 \pi m_h^* k_B T}{h^2} \right) \right]^{3/2}} \quad \dots(2.41)$$

At absolute zero temperature

$$E_F = \left\{ \frac{E_a + E_V}{2} \right\}$$

⇒ The Fermi level lies at half way between acceptor level and the top of the valence band.

With an increase in T , E_F increase and at room temperature it comes above the acceptor level slightly. Hence we have

$$\Rightarrow n_h = (2 n_d)^{1/2} \left(\frac{2 \pi m_h^* k_B T}{h^2} \right)^{3/4} e^{-\Delta E/k_B T} \quad \dots(2.42)$$

where $\Delta E = E_a - E_V$.

Example 2.10 In a P-type semiconductor, the Fermi level lies 0.4 eV above the valence band. If the concentration of the acceptor atom is tripled, find new position of the Fermi level.

Solution. Given P-type semiconductor, $E_F - E_V = 0.4$ eV, $n'_a = 3 n_a$,

also $k_B T = 0.03$ eV, $(E'_F - E_V) = ?$

If σ_∞ be the (extrapolated) conductivity for $T = \infty$, then

$$\sigma_\infty = K$$

$$\sigma = \sigma_\infty e^{-E_g/2k_B T}$$

and we write

$$\log_e \sigma = \log_e \sigma_\infty - \frac{E_g}{2k_B T}$$

...(2.45)

The plot between $\log_e \sigma$ and $\frac{1}{T}$ gives a straight line, whose slope is $-\frac{E_g}{2k_B}$ as shown in Fig. 2.17.

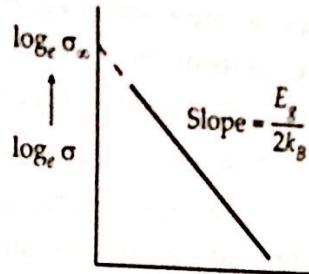


Figure 2.17 $\log_e \sigma$ vs. $\frac{1}{T}$.

2.12 Drift and Diffusion Currents

Drift Current

Although this is already discussed in Section 2.4 at page 30. Under the action of an electric field, the carriers in the semiconductor materials drift and produce drift current. Two types of charge carriers (electrons and holes) produce two drift current components.

The electron drift in the conduction band produces a component J_A given by

$$J_A (\text{drift}) = n e \mu_n E \quad \dots(2.46)$$

The hole drift in the valence band causes a component J_p

$$J_p (\text{drift}) = p e \mu_p E \quad \dots(2.47)$$

Although electrons and holes flow in opposite directions, the direction of conventional current flow due to both the carriers in the same direction. Hence their contributions add to each other. Thus, the total drift current density is given by

$$\begin{aligned} J_{\text{drift}} &= J_n (\text{drift}) + J_p (\text{drift}) \\ &= e(n\mu_n + p\mu_p) E \end{aligned} \quad \dots(2.48)$$

Equation (2.48) is applicable to both, intrinsic as well as extrinsic semiconductors.

Hence drift current depends upon *two* variables :

- the carrier concentration
- the electric field

Diffusion Current

When the concentration of atoms of one element is higher at one point than another point, the atom will diffuse from the region of higher concentration to that of lower concentration. This process of movement of atoms and molecules is known as diffusion. In case of semiconductors, the moving species are charge carriers. Hence the directional movement of charge carriers due to their concentration gradient produces a component of current known as diffusion current. This transport of charge carriers occurs without the assistance of an electric field.

If a junction is formed between a *P*-type and an *N*-type semiconductors, the electron density charges by several orders of magnitude as we go from *N*-side having electrons as majority charge carriers to the *P*-side, where the electrons are in minority. The statement is true for hole density also as we go from the *P*-side to the *N*-side. Thus there exist steep concentration gradients dp/dx and dn/dx in the junction region as shown in Fig. 2.18.

This sets in a diffusion process from the regions of higher concentration to the region of lower concentration. Thus electrons diffuse from *N*-side to *P*-side and holes diffuse from *P*-side to *N*-side resulting in diffusion currents through the junction. The rate of diffusion is proportional to the magnitude of the concentration gradient. This rate multiplied by the charge of carrier gives the diffusion current density.

Thus

$$J_p = -eD_p \frac{dp}{dx}$$

and

$$J_n = eD_n \frac{dn}{dx} \quad \dots(2.49)$$

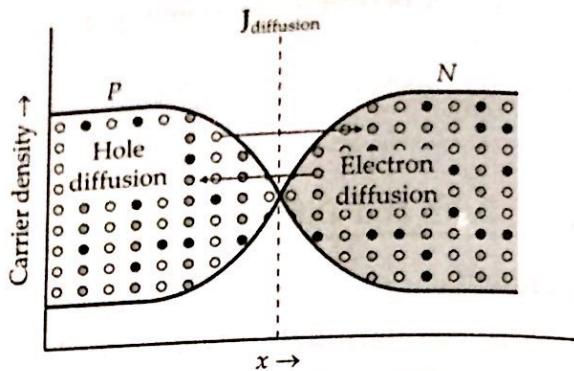


Figure 2.18 Concentration gradients in a *P*-*N* junction.

where D_p and D_n are known as the diffusion constants for the hole and electron, respectively. Since the electron and hole movements are in opposite directions, their currents add up to give the net diffusion current through the junction as

$$J_{\text{diff}} = e \left(D_n \frac{dn}{dx} + D_p \frac{dp}{dx} \right) \quad \dots(2.50)$$

It may be observed from Eq. (2.50) that diffusion transport of carriers depends only on one variable, namely *concentration gradient*. Diffusion transport of carriers is not encountered in metals.

Drift and diffusion current exists in semiconductors. The total current density due to drift and diffusion of electrons may be written as

$$J_n = J_n(\text{drift}) + J_n(\text{diffusion})$$

or

$$J_n = e \left[n\mu_n E + D_n \frac{dn}{dx} \right] \quad \dots(2.51)$$

Similarly, for holes, we can write

$$J_p = e \left[p\mu_p E - D_p \frac{dp}{dx} \right] \quad \dots(2.52)$$

Example 2.11 Calculate the diffusion current density in a piece of germanium having concentration gradient of 1.5×10^{22} electrons/m⁴, the diffusion coefficient $D_n = 0.0012 \text{ m}^2/\text{s}$.

Solution. Given $e = 1.6 \times 10^{-19}$, $D_n = 0.0012 \text{ m}^2/\text{s}$, $\frac{dn}{dx} = 1.5 \times 10^{22} \text{ A/m}^4$

Then the diffusion current density

$$J_n = e D_n \frac{dn}{dx}$$

$$= 1.6 \times 10^{-19} \times 0.0012 \times 1.5 \times 10^{22} = 2.88 \text{ A/m}^2$$

Einstein's Relationship

The diffusivity D and the mobility μ are related to each other through the following relationship. This relationship is known as Einstein's equation.

$$\frac{D_p}{\mu_p} = \frac{D_n}{\mu_n} = \frac{k_B T}{e} \quad \dots(2.53)$$

where k_B is the Boltzmann constant, T the absolute temperature and e the electronic charge. The quantity $\frac{k_B T}{e}$ is referred as thermal voltage.

At room temperature ($T = 300 \text{ K}$), putting $k_B = 1.38 \times 10^{-23} \text{ J/K}$ and $e = 1.6 \times 10^{-19} \text{ Coulomb}$, we obtain from Eq. (2.53)

$$\frac{D_p}{\mu_p} = \frac{D_n}{\mu_n} = \frac{k_B T}{e}$$

$$= \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} = 0.026 \text{ V}$$

For electrons in Ge at 300 K, the electron mobility is $\mu_n = 0.38 \text{ m}^2/\text{Vs}$.

Therefore $D_n = 0.026 \times 0.38 = 0.0099 \text{ m}^2/\text{s}$.

2.13 Carrier Life Time

When the concentration of minority carriers in an impure semiconductor increases over the thermal equilibrium value the number of these excess carriers decays with time through recombination.

Suppose that n_0 and p_0 are the thermal equilibrium concentrations of free electrons and holes respectively. Let Δn and Δp be the small changes of the electron and hole concentrations from their thermal equilibrium values. The recombination rate R' is proportional to the product of the electron and hole concentrations and is therefore given by

$$R' = K(n_0 + \Delta n)(p_0 + \Delta p) \quad \dots(2.54)$$

where K is the proportionality constant. In thermal equilibrium, $\Delta n = \Delta p = 0$ and the thermal generation or recombination rate equals $K n_0 p_0$. The recombination rate R for the excess carriers is given by the difference of R' and $K n_0 p_0$. Hence

$$R = K[(n_0 + \Delta n)(p_0 + \Delta p) - n_0 p_0] = K(n_0 \Delta p + p_0 \Delta n) \quad \dots(2.55)$$

where the product $\Delta n \Delta p$ of the two small quantities is neglected.

If the semiconductor is *N*-type, we have $n_0 \Delta p \gg p_0 \Delta n$, since $n_0 \gg p_0$ and $\Delta n = \Delta p$. Therefore Eq. (2.55) gives

$$R = Kn_0 \Delta p \quad (\text{for } N\text{-type}) \quad \dots(2.56)$$

Similarly, for a *P*-type material, $p_0 \Delta n \gg n_0 \Delta p$ and we obtain from Eq. (2.55)

$$R = Kp_0 \Delta n \quad (\text{for } P\text{-type}) \quad \dots(2.57)$$

Equations (2.56) and (2.57) show that the recombination rate is proportional to the concentration of the excess minority carriers, which are holes in an *N*-type semiconductor and electrons in *P*-type semiconductor.

For an *N*-type semiconductor, we can write from Eq. (2.56)

$$R = -\frac{d\Delta p}{dt} = \frac{\Delta p}{\tau_p} \quad \dots(2.58)$$

where

$$\tau_p = \frac{1}{Kn_0} \quad \dots(2.59)$$

The quantity τ_p has the dimension of time and is called the *lifetime of an excess minority carrier* i.e., hole. The solution of the differential Eq. (2.58) gives

$$\Delta p(t) = \Delta p(0) \exp \left(-\frac{t}{\tau_p} \right) \quad \dots(2.60)$$

where $\Delta p(0)$ is the initial change in the hole concentration at time $t=0$. Clearly, τ_p is the time constant with which Δp decays exponentially with time.

In the similar fashion, we can show that the excess electron concentration in a *P*-type material decreases with time obeying the relationship

$$\Delta n(t) = \Delta n(0) \exp \left(-\frac{t}{\tau_n} \right) \quad \dots(2.61)$$

where

$$\tau_n = \frac{1}{Kp_0} \quad \dots(2.62)$$

τ_n is known as lifetime of an excess electron in the sample.

Recombination Centres

In the direct recombination process an electron jumps from the conduction band to the valence band to merge with a hole, so that mobile electron-hole pair disappears. Since the momentum of pair vanishes after recombination, the momentum conservation law requires that the momenta of the electron and hole prior to recombination must be equal and opposite. For this reason, the probability of the direct recombination is rather small.

The recombination rate is greatly enhanced through the introduction of traps and combination centres which give available electronic states deep in the forbidden gap. These states originate from the impurities such as copper or imperfections in the crystal. To understand

combination through trapping centres, suppose that an available energy state is located in the forbidden gap. This state is neutral but can be negatively charged by trapping an electron from the conduction band. The negatively charged centre attracts a hole and when the latter comes close enough, the trapped electron is transferred to it. Thus electron hole recombination occurs. Since the trapping centres are strongly coupled to the crystal, the energies and the momenta of the interacting electrons and holes are absorbed by the lattice vibrations. Thus recombination involving traps emits little or no electromagnetic radiation.

2.14 The Continuity Equation

The carrier concentration in semiconductor is a function of time and distance both. The differential equation governing this functional relationship is called *continuity equation*.

Let us consider an infinitesimal element of volume of semiconductor material of unit area and length dx as shown in Fig. 2.19. Let mobile charge carriers be produced in it at a rate g by some process at some time. Simultaneously, let them disappear at a rate r through the process of recombination when free electrons fall into empty covalent bonds. The particle may also flow away from a point (Fig. 2.19) at a rate which depends upon the differential current $\frac{dJ}{dx}$. Adding all these particle-current components, we get the time rate of change of particle density ρ .

$$\frac{d\rho}{dt} = g - r - \frac{dJ}{dx} \quad \dots(2.63)$$

But diffusion current J is given by

$$J = D \frac{dp}{dx} \quad \dots(2.64)$$

where D is called diffusion current or diffusivity. Therefore, combining Eqs. (2.63) and (2.64), we get the diffusion equation

$$\frac{d\rho}{dt} = g - r + D \frac{d^2\rho}{dx^2} \quad \dots(2.65)$$

If in addition to diffusion, drift also takes place simultaneously then J will represent the total electric current and is given by

$$J = -D \frac{dp}{dx} + \mu E p \quad \dots(2.66)$$

where μ is the mobility (average drift velocity per unit electric field) of a carrier. Then Eq. (2.63) is modified to

$$\frac{d\rho}{dt} = g - r + D \frac{d^2\rho}{dx^2} - \mu \frac{d(\rho E)}{dx} \quad \dots(2.67)$$

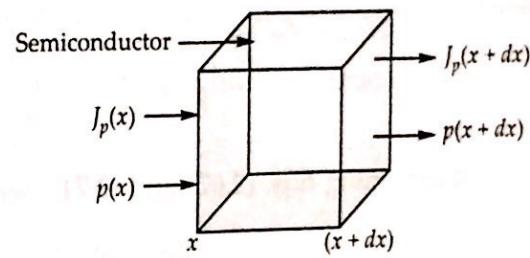


Figure 2.19 Illustration for continuity equation.

Formulae at a Glance

2.1 The Mass action law

$$\begin{array}{ll} (i) n_i p_i = n_i^2 & (ii) n_n p_n = n_i^2 \\ (iii) n_p p_p = n_i^2 & (iv) p + N_D = n + N_A \\ (v) p = \frac{n_i^2}{N_D} & (vi) n = \frac{n_i^2}{N_A} \end{array}$$

where i = stand for intrinsic material

n_n = electron density in N-type semiconductor

p_n = hole density in N-type semiconductor

n_p = electron density in P-type

p_p = hole density in P-type

N_D = Donor atoms per unit volume

N_A = Acceptor atoms per unit volume

2.2 Electrical conductivity of semiconductors

$$(i) \text{Current density } J = \rho v$$

where ρ = density of free charges

v = length per unit time

$$(ii) J_e = n_e v_e \text{ and } J_p = p_e v_p$$

where v_e and v_p = drift velocities of electrons and holes respectively

$$(iii) J_{\text{drift}} = e(nv_n + pv_p) = eE(n\mu_n + p\mu_p)$$

$$\text{where } \mu_n = \frac{v_n}{E} \text{ and } \mu_p = \frac{v_p}{E}$$

are mobilities for electrons and holes respectively.

2.3 Distribution function for Fermi-Dirac statistics

$$f(E) = \frac{1}{e^{(E-E_F)/k_B T} + 1}$$

2.4 Fermi Energy (intrinsic semiconductor)

$$E_F = \frac{E_C + E_V}{2}$$

E_C = energy level in conduction band

E_V = energy level in valence band.

2.5 Carrier concentration and Fermi level (Intrinsic semiconductor)

(i) Electron concentration in conduction band

$$n_e = 2 \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{3/2} \exp(E_F - E_C)/k_B T$$

(ii) Hole concentration in valence band

$$n_h = 2 \left(\frac{2\pi m_h^* k_B T}{h^2} \right)^{3/2} \exp(E_V - E_F)/k_B T$$

$$(iii) \text{Fermi energy } E_F = \frac{E_C + E_V}{2}$$

(iv) The intrinsic carrier concentration

$$n_e = n_h = n_i = 2 \left(\frac{2\pi k_B T}{h^2} \right)^{3/2} (m_e^* m_h^*)^{3/4} \exp[-E_g/2k_B T]$$

2.6 Electron concentration in N-type semiconductor

$$n_e = (2n_a)^{1/2} \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{3/4} \exp[-\Delta E/k_B T]$$

[Here $\Delta E = E_C - E_d$]

2.7 Hole concentration in P-type semiconductor

$$n_h = (2n_a)^{1/2} \left(\frac{2\pi m_h^* k_B T}{h^2} \right)^{3/4} \exp[-\Delta E/k_B T]$$

[Here $\Delta E = E_a - E_V$]

2.8 Diffusion current density

$$J_{\text{diffusion}} = e \left(D_e \frac{dn}{dx} + D_h \frac{dp}{dx} \right)$$

2.9 Total current density due to drift and diffusion of electron is

$$J_n = J_n(\text{drift}) + J_n(\text{diffusion})$$

$$= e \left[n\mu_n E + D_n \frac{dn}{dx} \right]$$

Similarly, for holes is

$$J_p = e \left[p\mu_p E + D_p \frac{dp}{dx} \right]$$

2.10 Einstein's relationship

$$\frac{D_p}{\mu_p} = \frac{D_n}{\mu_n} = \frac{k_B T}{e}$$

2.11 Carrier life time

$$\tau_p = \frac{1}{Kn_0} \text{ and } \tau_n = \frac{1}{Kp_0}$$

2.12 Continuity equation

$$(i) \frac{dp}{dt} = -\frac{p - p_0}{T_p} + D \frac{d^2 p}{dx^2} - \mu \frac{d(pE)}{dx} \quad (\text{General equation})$$

$$(ii) \frac{dn}{dt} = -\frac{n - n_0}{T_n} + D \frac{d^2 n}{dx^2} - n \frac{d(nE)}{dx} \quad (\text{for N-type})$$

$$(iii) \frac{dp}{dt} = -\frac{p - p_0}{T_p} + D \frac{d^2 p}{dx^2} - \mu \frac{d(pE)}{dx} \quad (\text{for N-type})$$

2.13 The Hall effect

(i) Hall coefficient

$$R_H = -\frac{1}{CNe} \quad [\text{in e.m.u.}]$$

$$R_H = -\frac{1}{Ne} \quad [\text{in e.s.u.}]$$

(ii) Mobility

$$\mu = R_H \cdot \sigma \cdot C \quad (\text{in e.m.u.}) \quad \left[\because \frac{J_x}{E_x} = \sigma \right]$$

$$(iii) \tan \phi = \mu B_z \\ \text{or } \phi = \tan^{-1}(\mu B_z)$$

Miscellaneous Solved Numerical Problems

Problem 2.1 Pure germanium at 300 K has a density of charge carriers $2.5 \times 10^{19}/m^3$. A specimen of pure germanium is doped with donor impurity atoms at the rate of one impurity atom for every 10^6 atoms of germanium. Assuming that all the impurity atoms are ionised, find the resistivity of doped germanium if the electron and hole mobilities are $0.36 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$ and $0.18 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$ respectively and the number of germanium atoms per unit volume is $4.2 \times 10^{28} \text{ atoms/m}^3$.

Solution. Given $n_i = 2.5 \times 10^{19}/m^3$, $\mu_n = 0.36 \text{ m}^2/\text{Vs}$, $\mu_p = 0.18 \text{ m}^2/\text{Vs}$

$$\text{Ratio of impurity atoms to germanium atoms} = \frac{1}{10^6} = 10^{-6}$$

$$\text{Number of germanium atoms/unit volume} \quad N = 4.2 \times 10^{28}/m^3$$

$$\begin{aligned} \text{No. of impurity atoms (in per m}^3) &= \text{The ratio of impurity atoms to germanium atoms (in per m}^3) \\ &= 10^{-6} \times 4.2 \times 10^{28} \text{ atoms/m}^3 = 4.2 \times 10^{22} \text{ atoms/m}^3 \end{aligned}$$

But all atoms are ionized

\therefore Number of free electrons due to impurity atoms/ m^3 , $n = 4.2 \times 10^{22}$ but from Mass Action Law, we have

$$np = n_i^2$$

or

$$p = \frac{n_i^2}{n} = \frac{(2.5 \times 10^{19})^2}{4.2 \times 10^{22}} = 1.488 \times 10^{16}/m^3$$

The electrical conductivity (σ) is given by

$$\begin{aligned}\sigma &= ne\mu_n + pe\mu_p \\ &= (4.2 \times 10^{22}) \times (1.6 \times 10^{-19}) \times (0.36) + (1.488 \times 10^{16}) \times (1.6 \times 10^{-19}) \times (0.18) \\ &= 2419 + 4.19 \times 10^{-4} = 2419 \text{ s/m}\end{aligned}$$

83

3.2)

But the resistivity of doped material $\rho = \frac{1}{\sigma}$

$$\rho = \frac{1}{2419} = 4.133 \times 10^{-4} \Omega \text{m}$$

3)

∴ The resistivity of doped germanium is $4.133 \times 10^{-4} \Omega \text{m}$.

Problem 2.2 Pure silicon has an intrinsic concentration of $1.5 \times 10^{16}/\text{m}^3$ at 300 K. A specimen of pure silicon is doped with donor type impurity atoms. If the doping is done at the rate of one impurity atom for every 10^7 atoms of silicon, calculate conductivity of the doped silicon material at 300 K. Assume that all the impurity atoms are ionised. Given : for silicon, atomic weight = 28.9, density = $2.33 \times 10^3 \text{ kg/m}^3$, electron mobility = $0.14 \text{ m}^2/\text{Vs}$ and hole mobility = $0.05 \text{ m}^2/\text{Vs}$.

Solution. Given $n_i = 1.5 \times 10^{16}/\text{m}^3$, $\rho = 2.33 \times 10^3 \text{ kg/m}^3$, $\mu_n = 0.14 \text{ m}^2/\text{Vs}$, $\mu_h = 0.05 \text{ m}^2/\text{Vs}$, $A = 28.09$ and ratio of impurity atoms to silicon atoms = $\frac{1}{10^7} = 10^{-7}$, $\sigma = ?$

The number of silicon atoms per unit volume N , is given by

$$N = \frac{\text{Avogadro's Number} \times \rho}{A} = \frac{6.025 \times 10^{26} \times 2.33 \times 10^3}{28.09} = 5 \times 10^{28} \text{ atoms/m}^3$$

Since there is one impurity atom for every 10^7 silicon atom, the number of free electrons per unit volume i.e., electron concentration

$$n = \frac{N}{10^7} = \frac{5 \times 10^{28}}{10^7} = 5 \times 10^{21} \text{ atoms/m}^3$$

From the Mass Action Law, we have $n_p = n_i^2$

$$\text{or } p = \frac{n_i^2}{n} = \frac{(1.5 \times 10^{16})^2}{5 \times 10^{21}} = 4.5 \times 10^{10}$$

∴ Electrical conductivity $\sigma = e(n\mu_n + p\mu_p)$

$$= 1.6 \times 10^{19} [(5 \times 10^{21}) \times 0.14 + (4.5 \times 10^{10}) \times 0.05] = 112 \text{ S/m.}$$

Problem 2.3 The electron concentration in an N-type semiconductor is $5 \times 10^{11} \text{ m}^{-3}$. Calculate the conductivity of the material if the drift velocity of electron is 350 m/s in an electric field of 1000 V/m.

Solution. Given $n = 5 \times 10^{17} \text{ m}^{-3}$, $v = 350 \text{ m/s}$, $E = 1000 \text{ V/m}$, $\sigma = ?$

The conductivity of the material

$$\sigma = ne\mu = ne \frac{v}{E} = 5 \times 10^{17} \times 1.6 \times 10^{-19} \times \frac{350}{1000} = 0.028 \Omega \text{m.}$$

Problem 2.4 The electron mobility and hole mobility of silicon are $0.17 \text{ m}^2/\text{Vs}$ and $0.035 \text{ m}^2/\text{Vs}$ respectively at room temperature. If the carrier density is known to be $1.1 \times 10^{16} \text{ per m}^3$, calculate the resistivity of silicon semiconductor.

Solution. Given : $\mu_n = 0.17 \text{ m}^2/\text{Vs}$, $\mu_p = 0.035 \text{ m}^2/\text{Vs}$, $n_i = 1.1 \times 10^{16} \text{ m}^{-3}$, $\rho = ?$

We have the relation for conductivity

$$\sigma = n_i e(\mu_n + \mu_p)$$

But resistivity $\rho = \frac{1}{\sigma}$

$$\therefore \rho = \frac{1}{n_i e(\mu_n + \mu_p)} = \frac{1}{(1.1 \times 10^{16}) \times (1.6 \times 10^{-19}) (0.17 + 0.035)} = 2768 \Omega \text{m}$$

Problem 2.5 The resistivity of intrinsic germanium at 27°C is equal to $0.47 \Omega \text{m}$. Assuming electron and hole mobilities as 0.38 and $0.18 \text{ m}^2/\text{Vs}$ respectively, calculate intrinsic carrier density.

Solution. Given $\rho_i = 0.47 \Omega \text{m}$, $\mu_n = 0.38 \text{ m}^2/\text{Vs}$, $\mu_p = 0.18 \text{ m}^2/\text{Vs}$, $n_i = ?$

The carrier density n_i is given by

$$n_i = \frac{1}{\rho_i e(\mu_n + \mu_p)} = \frac{1}{0.47 \times (1.6 \times 10^{-19}) (0.38 + 0.18)} = 23716 \times 10^{19} \text{ m}^{-3}$$

Problem 2.6 Calculate the concentration at which donor atoms need to be added to a silicon semiconductor so that it results in an N-type semiconductor with conductivity of $2.2 \times 10^{-4} \text{ mho/m}$, and mobility of electrons being $1.25 \times 10^{-3} \text{ m}^2/\text{Vs}$.

Solution. $\sigma_n = 2.2 \times 10^{-4} \text{ mho/m}$, $\mu_n = 1.25 \times 10^{-3} \text{ m}^2/\text{Vs}$

\therefore The electron conductivity σ_n is given by

$$\sigma_n = n e \mu_n \quad \text{or} \quad n = \frac{\sigma_n}{e \mu_n} = \frac{2.2 \times 10^{-4}}{(1.6 \times 10^{-19} \times 125 \times 10^{-3})} = 1.1 \times 10^{16} \text{ m}^{-3}$$

Problem 2.7 An intrinsic sample of Ge crystal has a hole density of 10^{13} per m^3 at room temperature. When doped with antimony, the hole density decreased to 10^{11} per cm^3 at the same temperature. Calculate the majority carrier density.

Solution. Given $n_i = 10^{13}/\text{cm}^3$, $p = 10^{11}/\text{cm}^3$

For N-type semiconductor, $np = n_i^2$

$$\therefore \text{Majority charge carriers } n = \frac{n_i^2}{p} = \frac{(10^{13})^2}{10^{11}} = 10^{15}/\text{cm}^3$$

Problem 2.8 Calculate the conductivity of pure germanium if the carrier concentration is $2.1 \times 10^{19}/\text{m}^3$. The mobilities of holes and electrons are $0.4 \text{ m}^2/\text{Vs}$ and $0.2 \text{ m}^2/\text{Vs}$ respectively.

Solution. Given, $n_i = 2.1 \times 10^{19}/\text{m}^3$, $\mu_p = 0.4 \text{ m}^2/\text{Vs}$ and $\mu_p = 0.2 \text{ m}^2/\text{Vs}$

$$\begin{aligned} \text{We know the conductivity } \sigma &= n_i e (\mu_n + \mu_p) = (1.6 \times 10^{-19} \times 2.1 \times 10^{19}) [0.4 + 0.2] \\ &= 1.6 \times 2.1 \times 0.6 = 2.01 \text{ Sm}^{-1} \end{aligned}$$

Problem 2.9 Find the resistivity of intrinsic germanium at 300 K. Given that the intrinsic density of carriers is $2.5 \times 10^{19} / m^3$, $\mu_n = 0.39 \text{ m}^2 / Vs$ and $\mu_p = 0.19 \text{ m}^2 / Vs$

Solution. Given $n_i = 2.5 \times 10^{19} / m^3$, $\mu_n = 0.39 \text{ m}^2 / Vs$ and $\mu_p = 0.19 \text{ m}^2 / Vs$

The electrical conductivity, $\sigma = e n_i (\mu_n + \mu_p)$

$$= (1.6 \times 10^{-19}) \times (2.5 \times 10^{19}) [0.39 + 0.19] = 4 \times 0.58 = 2.32 \text{ S/m}$$

and the electrical resistivity,

$$\rho = \frac{1}{\sigma} = \frac{1}{2.32} = 0.43 \Omega \text{m}$$

Problem 2.10 A sample of intrinsic germanium at room temperature has a carrier concentration of $2.4 \times 10^{19} / m^3$. It is doped with antimony atoms per million atoms of germanium. If the concentration of germanium atoms is $4 \times 10^{28} / m^3$, determine the hole concentration.

Solution. Given $n_i = 2.14 \times 10^{19} / m^3$, $N = 4 \times 10^{28} \text{ atoms/m}^3$, $N_D : N = 1 : 10^6$

$$N_D = \frac{4 \times 10^{28}}{10^{26}} = 4 \times 10^{22} \text{ donors/m}^3$$

$$n = N_D = 4 \times 10^{22} \text{ electrons/m}^3$$

$$\text{and } p = \frac{n_i^2}{n} = \frac{(2.4 \times 10^{19})^2}{4 \times 10^{22}} = 14.4 \times 10^{15} \text{ holes/m}^3.$$

Problem 2.11 Find the concentration of holes and electrons in a P-type germanium at 300 K if the conductivity is $100 / \Omega \text{ cm}$. Also find these values for N-type silicon if the conductivity is $0.1 / \Omega \text{ cm}$. Given that for Ge : $n_i = 2.5 \times 10^{13} / cm^3$, $\mu_n = 3800 \text{ m}^2 / Vs$; $\mu_p = 1800 \text{ cm}^2 / Vs$ and for Si : $n_i = 1.5 \times 10^{11} / cm^3$, $\mu_n = 1300 \text{ cm}^2 / Vs$ and $\mu_p = 500 \text{ cm}^2 / Vs$.

Solution. Concentration in P-type germanium

Conductivity $\sigma = ep\mu_p$

$$100 = 1.6 \times 10^{-19} \times p \times 1800$$

$$\text{No. of holes } p = \frac{100}{2.88 \times 10^{-16}} = 3.47 \times 10^{17} \text{ per cm}^3$$

$$\text{No. of electrons } n = \frac{n_i^2}{p} = \frac{(2.5 \times 10^{13})^2}{3.47 \times 10^{17}} = 1.8 \times 10^9 / cm^3$$

Concentration in N-type silicon

Conductivity $\sigma = ep\mu_n$

$$\text{No. of electrons } n = \frac{\sigma}{e\mu_n} = \frac{0.1}{1.6 \times 10^{-19} \times 1300} = 4.8 \times 10^{14} / cm^3$$

$$\text{No. of holes } p = \frac{n_i^2}{n} = \frac{(1.5 \times 10^{10})^2}{4.8 \times 10^{14}} = 4.7 \times 10^5 / cm^3.$$

Problem 2.12 Calculate the current produced in a small Ge plate of area 1 cm^2 and of thickness 0.3 mm when potential difference of 2 V is applied across the faces. Given concentration of free electrons in Ge is $2 \times 10^{19}/\text{m}^3$ and mobilities of electrons and holes are $0.36\text{ m}^2/\text{Vs}$ and $0.17\text{ m}^2/\text{Vs}$ respectively.

Solution. Given $A = 1\text{ cm}^2$, $t = 0.3\text{ mm}$, $V = 2\text{ V}$, $\mu_n = 0.36\text{ m}^2/\text{Vs}$, $\mu_p = 0.17\text{ m}^2/\text{Vs}$ and $n_i = 2 \times 10^{19}/\text{m}^3$

$$\text{The current } I = \frac{n_i(\mu_n + \mu_p)AV}{t}$$

$$= \frac{(2 \times 10^{19}) \times (0.36 + 0.17) \times 1 \times 10^{-4} \times 2}{0.3 \times 10^{-3}} = 1.13\text{ A.}$$

Problem 2.13 Consider a bar of silicon in which a hole concentration profile described by $p(x) = p_0 e^{-x/L_p}$ is established. Find the hole-current density at $x = 0$. Let $p_0 = 10^{16}/\text{cm}^3$ and $L_p = 1\text{ }\mu\text{m}$. If the cross sectional area of bar is $100\text{ }\mu\text{m}^2$, find current I_p . [GGSIPU, Dec. 2013 (2-5 marks)]

Solution. Given $p_0 = 10^{16}/\text{cm}^3$, $L_p = 1\text{ }\mu\text{m}$, $A = 100\text{ }\mu\text{m}^2$, $x = 0$ and D_p = Diffusion constant for holes = 13.

$$\text{We know that } I_p = -qA \frac{dp(x)}{dx} \cdot D_p = \frac{qA p_0 e^{-x/L_p}}{L_p} \cdot D_p$$

$$= \frac{1.6 \times 10^{-19} \text{ C} \times 100 \times 10^{-6} \text{ m}^2 \times e^{-0/L_p} \times 13 \times 10^{16} \times 10^{-6}}{1 \times 10^{-6} \text{ m}}$$

$$= \frac{1.6 \times 10^{-19} \text{ C} \times 100 \text{ m}^2 \times 13 \times 10^{-6} \times 10^{10}}{1 \times 10^{-6}}$$

$$= 20.80 \times 10^{-7} \text{ A} = 208 \times 10^{-6} \text{ A} = 208 \mu\text{A}$$

Now hole-current density at $x = 0$

$$J_p = \frac{I_p}{A} = \frac{208 \times 10^{-6} \text{ A}}{100 \times 10^{-6} \text{ m}^2} = \frac{208 \mu\text{A}}{100 \mu\text{m}^2} = 2.08 \text{ A/m}^2.$$

Problem 2.14 An intrinsic semiconductor has an energy gap of 0.4 eV . Calculate the probability of occupation of the lowest level in conduction band at 0°C , 50°C and 100°C .

Solution. $E_g = 0.4\text{ eV} = 0.41 \times 1.6 \times 10^{-19}\text{ J} = 6.4 \times 10^{-20}\text{ J}$

In intrinsic semiconductor, Fermi level lies in the middle of the energy gap. Hence $E_F = \frac{1}{2}E_g$. The occupation level in question is the lowest level in the conduction band. But the energy at the bottom of the conduction band is E_g i.e., $E = E_g$ and $f(E) = ?$

We have the relation for $f(E)$ as

$$f(E) = \frac{1}{e^{(E-E_F)/k_B T} + 1}$$

Since $E = E_g$ and $E_F = \frac{1}{2}E_g$

$$f(E) = \frac{1}{e^{E_g/2k_B T} + 1} \quad \dots(i)$$

(i) For 0°C or $T = 300\text{ K}$

∴ By substituting for T in Eq. (i), the probability of occupation at 0°C is

$$f(E) = \frac{1}{e^{\frac{6.408 \times 10^{-20}}{2 \times 1.38 \times 10^{-23} \times 273} + 1}} = \frac{1}{e^{\frac{2321.74}{373} + 1}} = \frac{1}{4937 + 1} = 2.025 \times 10^{-4}$$

Similarly, the probability of occupation at

$$(ii) 50^\circ\text{C} \text{ or } T = 323\text{ K}; \quad f(E) = \frac{1}{e^{\frac{2321.74}{323} + 1}} = 7.55 \times 10^{-4}$$

$$(iii) 100^\circ\text{C} \text{ or } T = 373\text{ K}; \quad f(E) = \frac{1}{e^{\frac{2321.74}{373} + 1}} = 19.76 \times 10^{-4}$$

Problem 2.15 In an N-type semiconductor, the Fermi level lies 0.5 V below the conduction band. If the concentration of donor atoms is tripled, find the new position of the Fermi level, taking the value of $k_B T = 0.03\text{ eV}$.

Solution. The donor atom density N_D is given by $N_D = N_C e^{-(E_C - E_F)/k_B T}$

where E_C is the energy corresponding to the bottom of conduction band and E_F is the Fermi level energy.

Let initially, $N_D = N_{D_1} = N_C e^{-(E_C - E_{F_1})/k_B T}$ and $E_F = E_{F_1}$

$$\text{Then } N_{D_1} = N_C e^{-(E_C - E_{F_1}) \cdot k_B T} \quad \dots(i)$$

After tripling the donor concentration

$$3N_{D_1} = N_C e^{(-E_C - E_{F_2})/k_B T} \quad \dots(ii)$$

Now divide Eq. (ii) by Eq. (i)

$$3 = e^{-(E_C - E_{F_2})/k_B T - (E_C - E_{F_1})/k_B T}$$

$$\log_e 3 = \frac{(E_C - E_{F_2})}{0.03} - \frac{0.5}{0.03} = \frac{(E_C - E_{F_2}) - 0.5}{0.03}$$

$$\text{or } E_C - E_{F_2} = 0.5 - 0.03 \log_e 3$$

$$\text{or } E_C - E_{F_2} = 0.5 - 0.03 \times 2.3026 \times \log_{10} 3 = 0.5 - 0.032 = 0.468\text{ eV}$$

Thus after tripling the impurity, Fermi level will lie 0.468 eV below the conduction band.

Problem 2.16 In an intrinsic semiconductor, the energy gap is 1.2 eV . What is the ratio between its conductivity at 600 K and that of 300 K ? (Given $1\text{ eV} = 1.6 \times 10^{-19}\text{ J}$).

Solution. Given $E_g = 1.2\text{ eV}$, $T_1 = 300\text{ K}$, $T_2 = 600\text{ K}$, $1\text{ eV} = 1.6 \times 10^{-19}\text{ J}$

The conductivity is given by

$$\sigma = n_i e(\mu_n + \mu_p) \quad \dots(i)$$

The conductivity at 300 K is given by

$$\sigma_{300} = n_{i300} e(\mu_n + \mu_p) \quad \dots(ii)$$

The conductivity at 600 K given by

$$\sigma_{600} = n_{i600} e(\mu_n + \mu_p) \quad \dots(iii)$$

Dividing Eq. (iii) by Eq. (ii), we get $\frac{\sigma_{600}}{\sigma_{300}} = \frac{n_{i600}}{n_{i300}}$

The intrinsic concentration of carriers is given by

$$n_i = 2 \left[\frac{2\pi k_B T_m}{h^2} \right]^{3/2} \times \left[\frac{m_n^* m_p^*}{m^2} \right]^{3/4} \exp \left[-\frac{E_g}{2k_B T} \right]$$

$$\frac{n_{i600}}{n_{i300}} = \left(\frac{600}{300} \right)^{3/2} \times \exp \left[\frac{-1.2 \times 1.6 \times 10^{-19}}{2 \times 1.38 \times 10^{-23} \times 600} \right] \exp \left[\frac{-1.2 \times 1.6 \times 10^{-19}}{2 \times 1.38 \times 10^{-23} \times 300} \right]$$

$$= \left(\frac{600}{300} \right)^{3/2} \times \exp \left[\frac{1.2 \times 1.6 \times 10^{-19}}{2 \times 1.38 \times 10^{-23}} \times \left[\frac{1}{300} - \frac{1}{600} \right] \right] = 306.79.$$

Problem 2.17 The energy gaps of two intrinsic semiconductors A and B are 0.36 eV and 0.72 V respectively. Compare the intrinsic carrier density of A and B at 300 K. [Given $m_p^* = m_n^* = 9.1 \times 10^{-31}$ kg and $2k_B T = 0.052$ eV].

Solution. Given

$$E_{g_1} = 0.36 \text{ eV}, E_{g_2} = 0.72 \text{ eV}, T = 300 \text{ K}, m_p^* = m_n^* = 9.1 \times 10^{-31} \text{ kg and } 2k_B T = 0.052 \text{ eV}$$

The intrinsic concentration of carriers is given by

$$n_i = 2 \left[\frac{2\pi k_B T_m}{h^2} \right]^{3/2} \times \left[\frac{m_n^* m_p^*}{m^2} \right]^{3/4} \exp \left[-\frac{E_g}{2k_B T} \right]$$

The intrinsic carrier concentration for the materials A and B can be written as

$$n_i(A) = 2 \left[\frac{2\pi k_B T_m}{h^2} \right]^{3/2} \times \left[\frac{m_n^* m_p^*}{m^2} \right]^{3/4} \exp \left[-\frac{0.36 \text{ eV}}{2k_B T} \right] \quad \dots(i)$$

$$n_i(B) = 2 \left[\frac{2\pi k_B T_m}{h^2} \right]^{3/2} \times \left[\frac{m_n^* m_p^*}{m^2} \right]^{3/4} \exp \left[-\frac{0.72 \text{ eV}}{2k_B T} \right] \quad \dots(ii)$$

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{n_i(A)}{n_i(B)} = \exp \left[\frac{-0.36 \text{ eV}}{2k_B T} \right] \exp \left[\frac{0.72 \text{ eV}}{2k_B T} \right] = \exp \left[\frac{0.36 \text{ eV}}{0.052 \text{ eV}} \right] = 1.0154 \times 10^3$$

Problem 2.18 For copper at 1000 K, find the energy at which the probability $f(E)$ that a conduction electron state will be occupied is 0.90.

Solution. The probability $f(E)$ of a state corresponding to energy E being occupied by an electron at temperature T is given by

$$f(E) = \frac{1}{1 + \exp \left(\frac{E - E_F}{k_B T} \right)} = \frac{1}{(1 + e^{\Delta E/k_B T})}$$

Therefore $0.90 = \frac{1}{1 + e^{\Delta E/k_B T}}$ or $\frac{\Delta E}{k_B T} = -2.20$

Thus $\Delta E = -2.20 k_B T = -2.20 \times (8.63 \times 10^{-5} \text{ eV/K}) \times (1000 \text{ K}) = -0.19 \text{ eV}$

For copper, $E_F = 7.00 \text{ eV}$ so that $E = E_F + \Delta E = 7.00 \text{ eV} - 0.19 \text{ eV} = 6.81 \text{ eV}$

Problem 2.19 A bar of N-type Si with a cross-sectional area of $1 \text{ mm} \times 1 \text{ mm}$ and a length of 1 cm is connected to a 2 V battery supply and is carrying a current of 2 mA at 300 K .

(a) Calculate the thermal equilibrium electron-hole densities in the bar.

(b) Calculate dopant concentration presuming that the only donors with an energy level of 0.148 eV below the conduction band.

Solution. (a) The resistance R of the bar is given by

$$R = \rho \left(\frac{l}{A} \right) = \rho \left[\frac{10^{-2}}{10^{-3} \times 10^{-3}} \right] = 10^4 \rho \quad \dots(i)$$

Also $R = \left(\frac{V}{I} \right) = \frac{2}{(2 \times 10^{-3})} = 10^3 \Omega \quad \dots(ii)$

Comparing (i) and (ii), we find

$$\frac{1}{\rho} = e(\mu_e n + \mu_h p) = 10 \quad \dots(iii)$$

∴ The sample is N-type, $n >> p$.

Therefore $e\mu_e n = 10 \quad \dots(iv)$

Taking $\mu_e = 1350 \text{ cm}^2/\text{Vs} = 0.135 \text{ m}^2/\text{Vs}$, we get from (iv)

$$n = \frac{10}{(1.5 \times 10^{-19} \times 0.135)} = 4.63 \times 10^{20} / \text{m}^3$$

but $p = \left(\frac{n_i^2}{n} \right) = \frac{(1.5 \times 10^{16})^2}{4.63 \times 10^{20}} = 4.86 \times 10^{11} / \text{m}^3$

Hence $n = 4.63 \times 10^{14} / \text{cm}^3$ and $p = 4.86 \times 10^5 / \text{cm}^3$

(b) The electron density is given by

$$n = N_C \exp \left[\frac{-(E_C - E_F)}{k_B T} \right] \quad \dots(v)$$

where for Si, $N_C = 2[2\pi m_n^* k_B T / h^2]^{3/2} = 2.8 \times 10^{19} / \text{cm}^3$

Therefore, from Eq. (v)

$$(E_C - E_F) = k_B T \ln \left(\frac{N_C}{n} \right) = 0.026 \ln [2.8 \times 10^{19} / 4.63 \times 10^{14}] = 0.286 \text{ eV}$$

But it is given that $(E_C - E_d) = 0.148 \text{ eV}$

We may now write

$$(E_d - E_F) = (E_C - E_F) - (E_C - E_d) = 0.286 - 0.148 = 0.138 \text{ eV.}$$

Therefore, the number of electrons in the donor state is

$$n_d = \frac{N_d}{1 + \exp\left(\frac{E_d - E_F}{k_B T}\right)} = \frac{N_d}{1 + \exp\left(\frac{0.138}{0.026}\right)} = \frac{N_d}{1 + \exp(5.318)}$$

$$\text{But } N_d = n + n_d \quad \text{or} \quad n = n_d / \{1 + \exp(-5.318)\} = 4.63 \times 10^{20}$$

Solving for N_d , one finds $N_d = 4.65 \times 10^{20} / \text{m}^3$.

(c) The semiconductor specimen becomes intrinsic at a temperature T , when $n_i(T) = N_d$, that is

$$(N_C N_V)^{1/2} \exp\left[-\left(\frac{E_g}{2k_B T}\right)\right] = N_d$$

$$N_C \exp[-(E_g / 2k_B T)] \cong N_d$$

or

Taking $E_g = 1.12 \text{ eV}$ and $k_B = 1.38 \times 10^{-23} \text{ J/K}$ and solving for T , one obtains, $T = 606.8 \text{ K}$ or $t = 333.8^\circ \text{C}$. Therefore the bar becomes intrinsic at a temperature 333.8°C .

Problem 2.20 In an intrinsic semiconductor ($E_g = 0.676 \text{ eV}$), $m_e = 0.09 \text{ m}$ and $m_h = 0.36 \text{ m}$. Calculate the concentration of intrinsic charge carriers at 300 K .

Solution. $E_g = 0.676 \text{ eV}$, $m_e = 0.09 \text{ m}$ and $m_h = 0.36 \text{ m}$

Then carrier concentration

$$n_i = 2 \left(\frac{k_B T}{2\pi\hbar^2} \right)^{3/2} \times (m_e m_h)^{3/4} \exp\left(-\frac{E_g}{2k_B T}\right)$$

$$= CT^{3/2} (0.3 \times 0.6)^{3/2} \exp\left(-\frac{0.676}{0.052}\right)$$

$$C = 2 \left(\frac{k_B}{2\pi\hbar^2} \right)^{3/2} \times (9.1 \times 10^{-31})^{3/2} = 4.83 \times 10^{21} / \text{m}^3$$

$$T^{3/2} = (300)^{3/2} = 5.19 \times 10^3 \text{ K}^{3/2}$$

$$\therefore n_i = 4.83 \times 10^{21} \times (5.19 \times 10^3) \times (0.0763) \times (2.266 \times 10^{-6}) = 4.33 \times 10^{18} / \text{m}^3$$

Problem 2.21 Hall voltage of 20 mV is found to be developed with a sample carrying a current of 15 mA is placed in a transverse magnetic field of 4 kilo Gauss . Calculate the concentration of charge carriers in the sample. The thickness of the sample along the magnetic field direction is 0.4 mm .

Solution. Given Hall voltage $V_H = 20 \text{ mV}$, current $I = 15 \text{ mA}$,

$$B = 4 \text{ kilo Gauss} = 0.4 \text{ Tesla}, \text{ thickness } d = 0.4 \text{ mm} = 0.4 \times 10^{-3} \text{ m}$$

$$\mu_e = \frac{R_H}{\rho} = \sigma R_H = 14.6 \times 8.9 \times 10^{-3} = 0.13 \text{ m}^3 / \text{Vs}$$

Finally

problem 2.26 An N-type germanium sample has donor density of $10^{21}/\text{m}^3$. It has arranged in a Hall specimen having magnetic field of 0.5 T and the current density is 500 A/m^2 . Find the Hall voltage if the sample is 3 mm wide.

Solution. Given $t = 3 \times 10^{-3} \text{ m}$, $J_x = 500 \text{ A/m}^2$, $B_z = 0.5 \text{ T}$ and $N = 10^{21}/\text{m}^3$, $e = 1.6 \times 10^{-19} \text{ C}$.

We know, Hall constant

$$R_H = \frac{1}{Ne} = \frac{E_y}{J_x B_z} = \frac{V_H}{t J_x B_z}$$

$$\Rightarrow V_H = \frac{t J_x B_z}{Ne} = \frac{(3 \times 10^{-3}) \times 500 \times 0.5}{10^{21} \times (1.6 \times 10^{-19})} \text{ V} = 4.69 \times 10^{-3} \text{ V} = 4.69 \text{ mV}$$

problem 2.27 When 90 mA current is passed through a sodium specimen under the magnetic field 2.0 Wb/m^2 , the Hall voltage is 0.09 mV. The width of the specimen is 0.04 mm. Calculate carrier concentration.

Solution. $V_H = 0.09 \text{ mV} = 0.09 \times 10^{-3} \text{ V}$, $I_x = 90 \text{ mA} = 90 \times 10^{-3} \text{ A}$;

$$t = 0.04 \text{ mm} = 0.04 \times 10^{-3} \text{ m}, \text{ and } B_z = 2 \text{ Wb/m}^2$$

$$R_H = \frac{1}{Ne} = \frac{V_H}{t J_x B_z} = \frac{V_H t}{I_x B_z}$$

$$\Rightarrow N = \frac{t I_x B_z}{V_H t e} = \frac{90 \times 10^{-3} \times 2.0}{0.09 \times 10^{-3} \times 0.04 \times 10^{-3} \times 1.6 \times 10^{-19}} = 3.1 \times 10^{26}/\text{m}^3$$

Conceptual Questions

2.1 In doped semiconductor how does the free carrier concentration change with increase in temperature ?

Ans. In doped semiconductors there are three regions as far as the carrier concentration versus temperature variation is concerned.

- ❖ At low temperature the electron (hole) from the donor (acceptor) is bound to the dopant. This is the freezeout region.

- With increase of temperature the fraction of ionized donors or acceptors increases and ultimately all the donors and acceptors are ionized and the free carrier concentration becomes equal to the dopant concentration. This is called saturation region. Semiconductor devices are normally preferred to operate in the temperature range at which saturation occurs.
- As the temperature is further increased beyond saturation region, the carrier concentration again increases due to increases the intrinsic generation across the band gap and becomes larger than the doping concentration. This is called intrinsic region.

2.2 What are compensated semiconductor ?

Ans. A compensated semiconductor is one that contains both donor and acceptor impurity atoms. If acceptor impurities are diffused in N-type semiconductor or donor impurities are diffused in P-type semiconductor then compensated semiconductor are created. A completely compensated semiconductor is one for which donor impurity concentration equals acceptor impurity concentration i.e., $N_D = N_A$.

2.3 What are degenerate semiconductors ?

Ans. With increasing doping concentration of donors (acceptors) a stage will be reached when the electron (hole) concentration will exceed the effective density of states of electrons (holes) and Fermi level goes into the conduction band (N-type) or in valence band (P-type) then the semiconductor is called degenerate semiconductor.

This can be explained as follows :

As the doping concentration increases the distance between impurity atoms decreases and concentration increases the distance between impurity atoms decreases and a stage will be reached when the donor electrons will start to interact among themselves. This will lead to splitting of single discrete donor energy level into an energy band which will widen with the increase of impurity concentration and ultimately overlap the edge of the conduction band.

2.4 What do you mean by thermal equilibrium in a semiconductor ?

Ans. Equilibrium or thermal equilibrium in a semiconductor means that no external force such as electric field, magnetic field, radiation or temperature gradient act on the semiconductor. All properties of the semiconductor will be independent of time in thermal equilibrium.

2.5 Where is Fermi energy level located in an intrinsic semiconductor ?

Ans. The Fermi level is located very close to the middle of the band gap slightly upwards towards the edge of the conduction band due to the difference of the effective mass of the electrons and holes.

2.6 Why do metals have a positive temperature coefficient of resistance and semiconductor have negative temperature coefficient of resistance ?

Ans. A large number of free electrons participate in conduction in metals. With increase of temperature these electrons are scattered more frequently by increasing number of phonons and the relaxation time for collision decreases. As a result mobility decreases and resistivity increases with the increase mobility decreases and resistivity increases with the increase of temperature in metals. In case of semiconductors, as temperature increases large number of electron-hole pairs are generated thermally. Therefore, conductivity increases and resistivity decreases.

2.7 How does a doped semiconductor behave with the increase of temperature ?

Ans. A doped or extrinsic semiconductor tends to become an intrinsic one as temperature increases.

2.8 How does the Fermi level of doped semiconductor shift its position with increase of temperature ?

Ans. As temperature increases the Fermi level of a doped semiconductor shifts towards the intrinsic level i.e., towards the midgap.

2.9 Distinguish between intrinsic and extrinsic semiconductors.

Ans.

[GGSIPU, May 2010 (2.5 marks)]

Intrinsic Semiconductor	Extrinsic Semiconductor
<ol style="list-style-type: none"> It is a semiconductor in its pure form. These semiconductors have low electrical conductivity. The operating temperature is low for this type of semiconductors. Charge carriers are produced due to thermal excitation. <p>Examples : Ge, Si</p>	<p>It is a semiconductor doped with impurities.</p> <p>These semiconductors have high electrical conductivity.</p> <p>The operating temperature is high for this type of semiconductors.</p> <p>Additional charge carriers are also produced due to added impurities.</p> <p>Examples : Ge and Si doped with P, As, Bi, Sb etc.</p>

2.10 What is difference between N-type and P-type semiconductors ?

[GGSIPU, April 2010 [Reappear] (2 marks)]

Ans.

N-type Semiconductor	P-type Semiconductor
<ol style="list-style-type: none"> These are materials, which have pentavalent impurity atoms (donors) added and conduct by "electron" movement and are called N-type semiconductors. In these types of materials : <ul style="list-style-type: none"> The donors are positively charged. These are large number of free electrons. A small number of holes in relation to the number of free electrons. Doping gives : <ul style="list-style-type: none"> positively charged donors negatively charged free electrons Supply of energy gives : <ul style="list-style-type: none"> negatively charged free electrons positively charged holes. 	<p>These are materials, which have trivalent impurity atoms (acceptors) added and conduct by "hole" movement are called P-type semiconductors.</p> <p>In these types of materials :</p> <ul style="list-style-type: none"> The acceptors are negatively charged. These are large number of holes. A small number of free electrons in relation to the number of holes. Doping gives : <ul style="list-style-type: none"> negatively charged acceptors positively charged holes. Supply of energy gives : <ul style="list-style-type: none"> positively charged holes negatively charged free electrons.

Both P-and N-types as whole are electrically neutral.

2.11 Where is Fermi energy level located in an intrinsic semiconductor ?

Ans. The Fermi level is located very close to the middle of the band gap slightly upwards towards the edge of conduction band due to difference of effective mass of electrons and holes.

2.12 How does a doped semiconductor behave with the increase of temperature ?

Ans. A doped extrinsic semiconductor tends to become an intrinsic one as temperature increases.

2.13 How does the Fermi level of doped semiconductor shifts its position with increase of temperature ?

Ans. As temperature increases the Fermi level of a doped semiconductor shifts towards the intrinsic level i.e., towards the midgap.

2.14 What do you mean by Hall effect and Hall voltage ?

Ans. If a current carrying conductor is placed in a magnetic field, a potential difference is developed across the conductor in a direction perpendicular to both current and magnetic field. This phenomenon is called the Hall effect and the induced potential difference is called the Hall voltage.

2.15 What is the importance of Hall effect ?

Ans. The importance of Hall-effect is given below :

- (i) Unknown magnetic field can be estimated by the knowledge of Hall voltage.
- (ii) The nature of the current carriers can be identified.
- (iii) The mobility of the carriers can be measured directly.
- (iv) It gives the concept of negative mass.

2.16 What do you mean by mass action law ?

Ans. The law suggests that the addition of impurities to an intrinsic semiconductor increases the concentration of one type of carriers, which consequently becomes majority carrier and simultaneously decreases the concentration of other carrier, which as a result becomes the minority carrier.

2.17 What is diffusion of charge carriers ?

Ans. Due to non-uniform carrier concentration in a semiconductor, the charge carriers move from a region of higher concentration to a region of lower concentration. This process is known as diffusion of charge carriers.

2.18 What is Einstein's relation ?

Ans. The relation between mobility μ and diffusion coefficient D of charge carrier in a semiconductor is known as Einstein's relation.

2.19 What do you mean by equation of continuity ?

Ans. The equation governing the behaviour of charge carriers in a semiconductor is known as equation of continuity. This equation gives a condition of dynamic equilibrium for the density of charge carriers in any elementary volume of a semiconductor. This is based on the fact that charge can neither be created nor destroyed.

[IGCSEPU, Oct. 2013 (2.5 marks)]
[Ans. See Example 26 at page 33]

- 2.21 Define the Fermi Function. Explain the Fermi level in a semiconductor having impurities.

[IGCSEPU, Oct. 2013 (2.5 marks)]
[Ans. 99% of the donor atoms are ionized]

- 2.22 How do you realise N-type and P-type semiconductors? Mention their conducting carriers and explain them.

2.23 Explain clearly, how P-type and N-type semiconductors? Draw the respective energy band diagrams

- 2.24 What are intrinsic and extrinsic semiconductors. Explain with examples of donors indicating all the energy levels.

- 2.25 Distinguish between intrinsic and extrinsic semiconductors. Explain with examples of donors and acceptors.

- 2.26 Distinguish between drift current and diffusion current in a semiconductor diode.

- 2.27 Distinguish between P and N-type semiconductors through energy band diagram clearly showing the energy gap and Fermi level.

- 2.28 Distinguish between drift and diffusion current in a P-N device.

- 2.29 Write a brief note on drift velocity mobility and conductivity of intrinsic semiconductors.

- 2.30 Derive an expression for concentration of electrons in an intrinsic semiconductor.

- 2.31 Derive an expression for total conductivity of charge carriers in an intrinsic semiconductors.

- 2.32 Derive an expression for concentration of holes in an intrinsic semiconductor.

- 2.33 Show that the Fermi level is located midway between conduction band and valence band

- 2.34 Show that the product of electron and hole concentration is independent of the Fermi energy level in an intrinsic semiconductors.

- 2.35 Explain the position of Fermi level in different types of semiconductors and its significance with respect to conductor.

- 2.36 Write a brief note on charge densities N and P-type semiconductors.

- 2.37 Write a brief note on Fermi level in P-type and N-type semiconductors.

- 2.38 Derive a relation for conductivity in an extrinsic semiconductor.

- 2.39 Derive a relation for total drift current in an extrinsic semiconductor.

- 2.40 What is drift current? Derive a relation for total drift current in an extrinsic semiconductor.

- 2.41 What is meant by diffusion current? Obtain expressions for diffusion hole-current density and diffusion electron-current density in P and N-type semiconductors respectively.

- 2.42 Write a note on carrier life time in N-type semiconductor. Define life time of charge carrier.

- 2.43 Explain how Hall effect is used to determine the conductivity of a semiconductor.

- 2.44 State and explain drift and diffusion current. Derive the continuity equation for electrons in N-type semiconductors.

Numerical Problems

- 2.1 In an N-type semiconductor the Fermi level lies 0.3 eV below the conduction band at 300 K if the temperature is increased to 330 K, find the new position of the Fermi level.

[Ans. 0.33 eV below the conduction band]

[Ans. 99% of the donor atoms are ionized]
2.3 In a P-type semiconductor, the Fermi level lies 0.4 eV above the valence band. If the concentration of the acceptor atom is tripled find new position of the Fermi level.

[Ans. $(E_f - E_v) = 0.367 \text{ eV}$]

- 2.4 The energy gap of two intrinsic semiconductors A and B are 0.36 eV and 0.72 eV respectively. Compare the intrinsic carrier density of A to B at 300 K. (Given $m_h^* = m_e^* = 9.1 \times 10^{-31} \text{ kg}$ and $2k_B T = 0.052 \text{ eV}$).

[Ans. The ratio of the intrinsic carrier densities of the materials A and B is 1.0154×10^3]
2.5 Fermi energy of an intrinsic semiconductor is 0.6 V. The low lying energy levels in conduction band is 0.2 eV above the Fermi level. Calculate the probability of occupation of this level by an electron at room temperature.

[Ans. 99%]
2.6 In N-type semiconductor, the Fermi level lies 0.3 eV below the conduction band at 300 K. If the temperature is increased to 330 K. Find the new position of Fermi level.

[Ans. The Fermi level lies 0.33 eV below the conduction band]
2.7 Calculate the value of Hall voltage is 30 mV for a current of 70 mA flowing through a specimen of 3 mm thick placed in a magnetic field of strength $6 \times 10^3 \text{ Gauss}$. ($1 \text{ Gauss} = 10^{-4} \text{ Tesla}$)
2.8 Assuming there are $5 \times 10^{-28} \text{ atoms/m}^3$ in copper, find Hall coefficient.

[Ans. $-0.125 \times 10^{-9} \text{ m}^3/\text{C}$]
2.9 Hall voltage of 1.0 mV to be developed when a sample carrying a current of 100 mA is placed in a transverse magnetic field of 0.3 T. Calculate the charge carrier concentration of the sample, given thickness of sample along the direction of magnetic field is 0.3 mm.

[Ans. $6.25 \times 10^{27} \text{ m}^{-3}$]
2.10 Calculate the density of conduction electrons in Na, if its lattice parameter $a = 4.3 \text{\AA}$. Also calculate the Hall coefficient.

[Ans. $2.52 \times 10^{28} / \text{m}^3$; $-2.48 \times 10^{-10} \text{ m}^3/\text{C}$]
2.11 Consider a slab of Cu, 0.1 mm thick, 1.0 mm wide and 10.0 mm long:
(a) If we drive a current of 1 A down the length of slab, what is the current density?
(b) If we then put the slab in the magnetic field of 1 T with the field perpendicular to the 1 mm \times 10 mm face, what is Hall field will be produced, if the Hall coefficient is $1 \text{ mm} \times 10^{-10} \text{ m}^3 / \text{C}$.
(c) What Hall voltage will we observe across the slab?

[Ans. (a) 10^7 A/m^2 ; (b) $5.5 \times 10^{-1} \text{ V/m}$; (c) $0.55 \mu\text{V}$]

- 2.13 Calculate the Hall voltage across the width of a semiconducting specimen from the following data :
width of the specimen = 0.1 m ;
thickness of the specimen = 0.01 m ;
field applied perpendicular to both width and length = 0.6 T ;
current flowing length wise = 10 mA ;
hall coefficient = $3.84 \times 10^{-4} \text{ m}^3/\text{C}$. [Ans. $2.3 \times 10^{-4} \text{ V}$]
- 2.14 The Hall coefficient of a certain semiconductor was found - $7.35 \times 10^{-5} \text{ m}^3/\text{C}$. Is the semiconductor N-type or P-type ? The electrical conductivity of specimen is $200 \Omega/\text{m}$. Calculate density and mobility of charge carriers. [Ans. $8.5 \times 10^{22} \text{ per m}^3$, $1.47 \times 10^{-2} \text{ m/Vs}$]
- 2.15 Calculate the position of Fermi level E_F at 300 K for germanium crystal containing 5×10^{22} arsenic atoms/ m^3 . [Ans. $E_C - E_F = 0.16 \text{ eV}$]
- 2.16 Compute the carrier concentration and conductivity of an intrinsic semiconductor whose $E_g = 0.68 \text{ eV}$. Given at $T = 300 \text{ K}$; $m_n^* = m_p^* = 9.1 \times 10^{-31} \text{ kg}$, $\mu_n = 0.38 \text{ m}^2/\text{Vs}$ and $\mu_p = 0.18 \text{ m}^2/\text{Vs}$. [Ans. $n = p = 3 \times 10^{15}/\text{m}^3$, $\sigma = 2.8/\Omega\text{m}$]
- 2.17 Compare the densities of charge carriers in a pure silicon crystal at two temperatures 300 K and 330 K. The energy gap for silicon is 1.1 eV. [Ans. 0.12]
- 2.18 In an N-type semiconductor, the Fermi level lies 0.4 eV below the conduction band at 300 K. If the temperature is increased to 350 K, find the new position of Fermi level.
- 2.19 In a P-type semiconductor, the Fermi level lies 0.04 eV above the valence band. If the $k_B T = 0.03 \text{ eV}$. [Ans. 0.379 eV]
- 2.20 Find the current produced at room temperature in a pure germanium plate of area $2 \times 10^{-4} \text{ m}^2$ and of thickness $1.2 \times 10^{-3} \text{ m}$, when a potential of 5 V is applied across the faces. Concentration of carriers in germanium at room temperature is $1.6 \times 10^6/\text{m}^3$. The mobilities of electrons and holes are $0.4 \text{ m}^2/\text{Vs}$ and $0.2 \text{ m}^2/\text{Vs}$ respectively. How much heat is generated in the plate in 100 s ? [Ans. $1.28 \times 10^{-13} \text{ Amp}, 6.4 \times 10^{-11} \text{ J}$]
- 2.21 Determine the conductivity and resistivity of intrinsic silicon at 200 K, 300 K and 500 K. The electron and hole mobilities of silicon are given by $\mu_p = 4 \times 10^5 T^{-2.6} \text{ m}^2/\text{Vs}$ and $\mu_n = 2.5 \times 10^4 T^{-2.3} \text{ m}^2/\text{Vs}$. Assume that the carrier densities of pure silicon are $n_i = 3.8 \times 10^{10}/\text{cm}^3$ at 200 K and $n_i = 2 \times 10^{10}/\text{cm}^3$ at 300 K. [Ans. $3 \times 10^7 \Omega\text{m}, 1.6 \times 10^3 \Omega\text{m}$]
- 2.22 Find the density of impurity atoms that must be added to an intrinsic silicon crystal to convert it to :
(a) $10^{-1} \Omega\text{m}$ P-type silicon, and
(b) $10^{-1} \Omega\text{m}$ N-type silicon
For Si, $\mu_n = 0.135 \text{ m}^2/\text{Vs}$ and $\mu_p = 0.048 \text{ m}^2/\text{Vs}$. [Ans. $1.3 \times 10^{21}/\text{m}^3$, $4.63 \times 10^{20}/\text{m}^3$]
- 2.23 The conductivity of a semiconductor at 20°C is $250 \Omega\text{m}$ and at 100°C is $1100 \Omega\text{m}$. What is its band gap, E_g ? [Ans. $E_g = 0.2638 \text{ eV}$]

If no field is applied to the semiconductor and it is in thermal equilibrium with its surroundings, then under steady state, $J=0$, $\frac{dp}{dt}=0$ and space charge density p reaches the thermal equilibrium value p_0 , then Eq. (2.66) gives

$$g-r=0 \quad \text{or} \quad g=r$$

But rate of recombination r is proportional to charge density i.e.,

$$r=k p_0$$

Then in thermal equilibrium, the rate of generation of mobile carriers is given by

$$g=r=k p_0 = \frac{p_0}{\tau_p}$$

where τ_p is the mean life time of the carriers.

Let now due to some reason, say by illuminating the specimen by light, the space charge density increases to p from its thermal equilibrium value p_0 , then

$$g-r=\frac{p_0}{\tau_p}-\frac{p_0+\delta p}{\tau_p}=-\frac{\delta p}{\tau_p}=-\frac{p-p_0}{\tau_p}$$

Combining Eqs. (2.67) and (2.71), we get

$$\frac{dp}{dt}=-\frac{p-p_0}{\tau_p}+D \frac{d^2 p}{dx^2}-\mu \frac{d(pE)}{dx} \quad \dots(2.72)$$

Equation (2.72) is the *continuity equation* and states the condition of dynamic equilibrium for the density of mobile carriers in the elementary volume of semiconductor. It has been derived making use of the fact that *charge can neither be created nor destroyed*.

The continuity equation for electrons in *N*-type semiconductor can be written by replacing p by $-en$ and is given by

$$\frac{dn}{dt}=-\frac{n-n_0}{\tau_n}+D \frac{d^2 n}{dx^2}-\mu \frac{d(nE)}{dx} \quad \dots(2.73)$$

Similarly, continuity equation for holes is obtained by replacing p by en and is given by

$$\frac{dp}{dt}=-\frac{p-p_0}{\tau_p}+D \frac{d^2 p}{dx^2}-\mu \frac{d(pE)}{dx} \quad \dots(2.74)$$

2.15 The Hall Effect

The Hall effect is the production of a potential difference (the Hall voltage) across an electrical conductor in which an electric current flows in the presence of a magnetic field. Edwin Hall discovered this effect in 1879. The conductivity measurements are not sufficient for the determination of the number of conducting charges N and their mobility μ . Moreover, the measurements do not give any information about the sign of the prominent charge carrier. The Hall effect supplies the information of the sign of charge carrier.