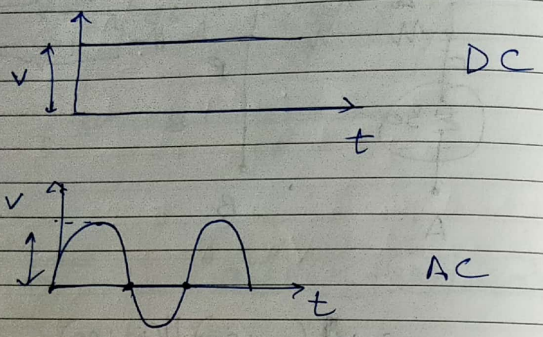
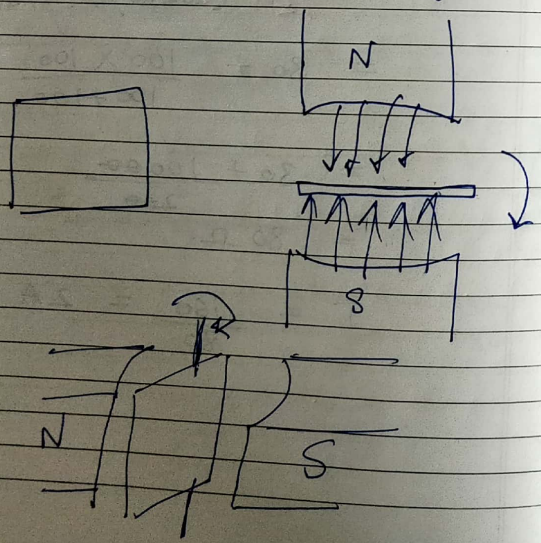
AC Analysis



1. Safety → AC
2. Transmission loss over long distance



SI – webers – Mag Field – flux

$$e \propto \frac{d\phi}{dt}$$

ϕ = flux

B = flux density = Wb/m^2

Flux density = Wb/m^2

Flux = F.d × Area

$$e = -N \frac{d\phi}{dt}$$

$$e = -N \frac{d}{dt}(\phi_m \cos \omega t)$$

$$e = -N \phi_m \frac{d}{dt}(\cos \omega t)$$

$$e = \omega N \phi_m \sin \omega t$$

N – number of turns

ϕ_M – Max. flux

ω – Angular frequency = $2\pi F$

$E = E_m \cos \omega t$ — Max amplitude

$E = E \sin \omega t$ — Amplitude

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Few definitions in AC

(1) **Amplitude (Peak Value)** : It is the max. value (+ve or -ve) of an alternating quantity.

(2) **Instantaneous Value** : It is the value of alternating qty. at any particular instant.

(3) **Cycle** : It is ~~the~~ one complete set of +ve & -ve values of an alternating quantity.

(4) **Time Period** : Time required to complete one cycle.

In a Signal of 50 Hz, $T = \frac{1}{50}$ s

$$\boxed{T = \frac{1}{f}}$$

(5) **Frequency** : No. of cycles in seconds.

(6) **Phase** : It is fraction of Time Period that has elapsed since that quantity Passed through the zero position.

(7) **Phase Angle** : It is the phase in radians or degrees. It is given by

$$\frac{2\pi t}{T}$$

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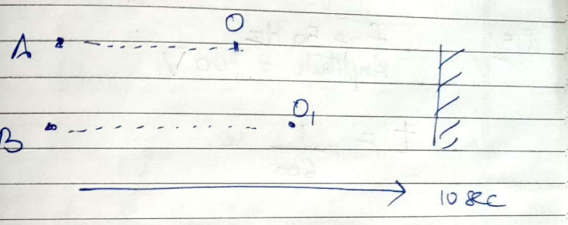
$t \rightarrow$ Instantaneous time
$T \rightarrow$ Time period.

(8) **Phase Difference** : P.D b/w two alternating quantities is the fractional part of Period by which one has advanced over or lags behind the other.



at point O

Phase = 5 sec
Phase Angle = $2\pi \frac{5}{} = \pi$
Phase difference = $O_1 - O^0$ (sec)

(9) **Rms value** : $I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707\, I_{max}$

(10) **Average Value** : $\frac{2}{\pi} I_{max} = 0.637\, I_{max}$

(11) **Amplitude / Peak factor** : It is the

$= \frac{I_{max}}{I_{rms}} = \frac{\frac{I_m}{\sqrt{2}}}{\frac{I_{max}}{\sqrt{2}}} = \sqrt{2} = 1.41$

(12) **Form Factor (K_f)**

$$\frac{I_{rms}}{I_{av}} = \frac{\frac{I_{max}}{\sqrt{2}}}{\frac{2}{\pi} I_{max}} = \frac{I_{max}}{\sqrt{2}} \times \frac{\pi}{2 I_{max}}$$

$$= \frac{\pi}{2\sqrt{2}} = 1.11$$

Q5.

$f = 50$ Hz
Amplitude $= 100$ V

$$t = \frac{1}{600} \text{ sec}$$

$$2\pi f t = 2\pi \times \frac{50}{600} = \frac{\pi}{6}$$

$e = E_m \sin \omega t$

$$E = 100 \sin \frac{\pi}{6} (100\pi . t)$$

$$E = 100 \sin \frac{\pi}{6}$$

$$E_{600} = \frac{100}{2} = 50 \text{ V}$$

Quantity	Units	Symbol
Current	Ampere	I
Voltage	Volts	V
emf	Volts	E_M
flux	Wb	ϕ
flux density	Wb/m^2	B_M
Angular freq.	rad/sec	ω
freq.	Hz $(\frac{1}{T})$	f
phase	s	T

$$\omega = 2\pi f$$

Inductive Reactance	Ω	X_L
Capacitive Reactance	Ω	X_C
Capacitance	Farad	C
Resistance	Ω	R
Inductance	Henry	L
Impedance	Ω	Z

5.2

$I_{rms} = 50\ A$

$f = 60\ Hz$

$t = ?$ for $I = 50\ A$

$I_{max} = \sqrt{2}\, I_{rms}$

$= 50\sqrt{2}\ A$

$I = I_m \sin \omega t$

$50 = 50\sqrt{2} \sin \omega t$

$\omega t = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\omega t = 45$

$t = \frac{45}{2\pi \times 60}$

$= 0.119$

$I = \sqrt{2} \sin (2\pi \times 60 \times t)$

$\frac{1}{\sqrt{2}} = \sin (120\pi t)$

$\sin^{-1} \frac{1}{\sqrt{2}} = \sin^{-1} (120\pi t)$

$\frac{\pi}{4} = 120\pi t$

$t = 2.083 \times 10^{-3}$ sec

Q.

Angular freq $(\omega) = 314$ rad/sec

$V_{av} = 127.4\ V$

emf at $t = \frac{1}{300}$ s

and $t = \frac{1}{75}$ s

$\omega = 2\pi f$

$f = \frac{1}{T}$

$2\pi \frac{t}{T}$

$V_{av} = \frac{2}{\pi} V_{max}$

$V_{max} = \frac{127.4\ \pi}{2}$

$V_{max} = 200.11\ V$

$E_{1/300} = E_m \cos \omega t$

$= 200.11 \cos \left(314 \times \frac{1}{300}\right)$

$= 200.11 \cos \left(\frac{180}{3}\pi \times \frac{1}{300}\right)$

$= 200.11 \cos 60$

$= 100.055\ V$

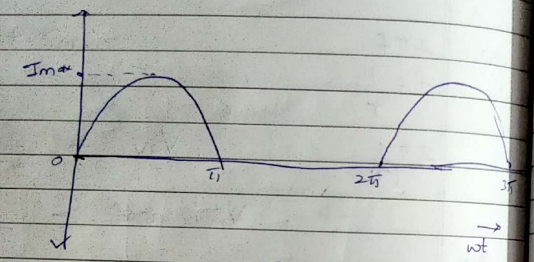
$E_{1/75} = 200.11 \cos \left(100\pi \times \frac{1}{75}\right)$

$= 200.11 \cos \left(\frac{4\pi}{3}\right)$

$= -100.055\ V$

Q. Form factor = 2
Amplitude factor = 2.5
Find I_{av}.
$I_{max} = 500\ A$
Find rms & av. value.

Sol



$$I_{av} = \frac{1}{T}\int_0^T f(x)\,dx$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T f(x)^2\,dx}$$

$$I_{av} = \frac{1}{2\pi}\int_0^{2\pi} I_m \sin(\omega t)\,d(\omega t)$$

$$= \frac{I_m}{2\pi}\left[-\cos\omega t\right]_0^{\pi}$$

$$\int \sin x\,dx$$

$$\left[-\cos x\right] + C$$

$$= \frac{I_m}{2\pi}\left[-\cos\pi + \cos 0\right]$$

$$= \frac{I_m}{2\pi}\left[1 + 1\right]$$

$$= \frac{I_m}{\pi}$$

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} I_m^2 \sin^2(\omega t)\,d\omega t}$$

$$= \sqrt{\left(\frac{I_m}{2\pi}\right)\left[\frac{1-\cos 2\omega t}{2}\right]_0^{2\pi}}$$

$$= \sqrt{\frac{I_m}{2\pi}\left[-\frac{\cos^2\omega t}{2}\right]_0^{2\pi}}$$

$$= \frac{I_m}{2\pi}\int_0^{\pi} \sin^2\omega t\,d\omega t$$

$$= \frac{I_m}{2\pi}\int_0^{\pi} \frac{1-\cos 2\omega t}{2}\,d\omega t$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$= \frac{I_m}{2\pi}\sqrt{\frac{I_m^2}{4\pi}\int_0^{2\pi}(1-\cos 2\omega t)\,d\omega t}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{\pi}}$$

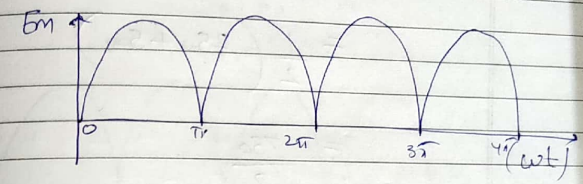
$$= \sqrt{\frac{I_m^2}{4\pi}\left[\pi - \frac{\sin 2\pi}{2} + \frac{\sin 0}{2}\right]}$$

$$\sqrt{\frac{I_m}{4}}$$

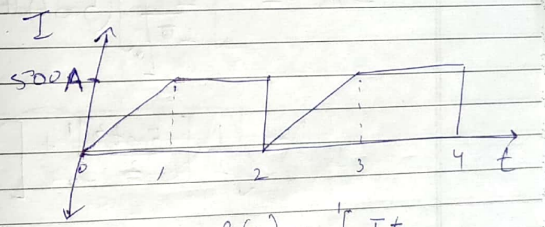
$$= \sqrt{\frac{I_m^2}{4\pi}\left(\pi - \frac{0+0}{2}\right)}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\cdot\pi} = \sqrt{\frac{I_m^2}{4}}$$

$$= \frac{I_m}{2}$$



E_{av} E_{rms} ?



I_{av} I_{rms} ? $f(x) = \int_0^t I\,t$

$I_{t=0} = 0$

$I_{t=1} = 500$

$$I_{av} = \frac{1}{2}\int_0^2 500t\,dt + \int_1^2 500\,dt$$

$$= \frac{1}{2}\left[\left[\frac{500t^2}{2}\right]_0^1 + \left[500t\right]_1^2\right]$$

$$= \frac{1}{2}\left[\left[\frac{500}{2}\right] + \left[1000 - 500\right]\right]$$

$$= \frac{1}{2}\left(250 + 500\right)$$

$$= \frac{750}{2}$$

$$= 375 \text{ A}$$

$$I_{rms} = \sqrt{\frac{1}{2}\left[\int_0^1 (500t)^2 dt + \int_1^2 (500)^2 dt\right]}$$

$$= \sqrt{\frac{1}{2}\left[\frac{500t^2}{2}\right]_0^1 + \left[500t\right]_1^2}$$

$$= \sqrt{\frac{1}{2}\left[(500)^2 \frac{t^3}{3}\right]_0^1 + \left[(500)^2 t\right]_1^2}$$

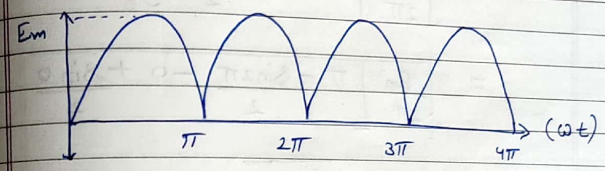
$$= \sqrt{\frac{1}{2}\left[500^2 \left(\frac{1}{3} + 1\right)\right]}$$

$$= \sqrt{\frac{500^2}{2}\left(\frac{4}{3}\right)}$$

$$= \sqrt{166666.667} \text{ A}$$

$$= 408.667 \text{ A}$$

Q.



$$E_{av} = \frac{1}{T}\int_0^T f(x)\, dx$$

$$E_{av} = \frac{1}{\pi}\int_0^{\pi} E_m \sin(\omega t)\, d\omega t$$

$$= \frac{E_m}{\pi}\left[-\cos \omega t\right]_0^{\pi}$$

$$= \frac{E_m}{\pi}\left[-\cos(\pi) + \cos 0\right]$$

$$= \frac{E_m}{\pi}\left[1 + 1\right] = \frac{2E_m}{\pi}$$

$$I_{rms} = \sqrt{\frac{E_m^2}{\pi}\int_0^{\pi} \sin^2 \omega t\, d\omega t}$$

$$= \sqrt{\frac{E_m^2}{\pi}\int_0^{2\pi} \frac{1 - \cos 2\omega t}{2}\, d\omega t}$$

$$= \sqrt{\frac{E_m^2}{2\pi}\int_0^{\pi}(1-\cos 2\omega t)\,d\omega t}$$

$$= \sqrt{\frac{E_m^2}{2\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{\pi}}$$

$$= \sqrt{\frac{E_m^2}{2\pi}\left[\pi - \frac{\sin 2\pi}{2} - 0 + \frac{\sin 0}{2}\right]}$$

$$= \sqrt{\frac{E_m^2}{2\pi}\cdot\pi}$$

$$= \frac{E_m}{\sqrt{2}}$$

Q. Form factor = 2
Amplitude factor = 2.5
$I_{max} = 500A$
$I_{rms} = ?$ $I_{av} = ?$

$$\text{Amplitude factor} = \frac{I_{max}}{I_{rms}}$$

$$I_{rms} = \frac{500}{2.5}$$

$$I_{rms} = 200\,A$$

$$\text{Form factor} = \frac{I_{rms}}{I_{av}}$$

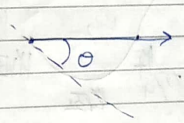
$$I_{av} = \frac{I_{rms}}{F.F}$$

$$I_{av} = \frac{200}{2}$$

$$\boxed{I_{av} = 100\,A}$$

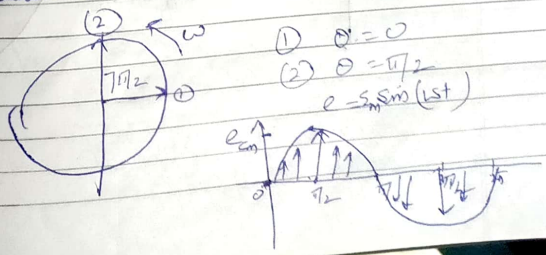
3/9/17

Phasor Representation



AC I / AC V Phasors

A phasor is a vector rotating at a constant angular velocity. Projection of this vector on the vertical axis gives the instantaneous value of the physical quantity.



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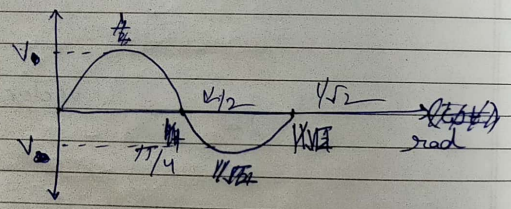
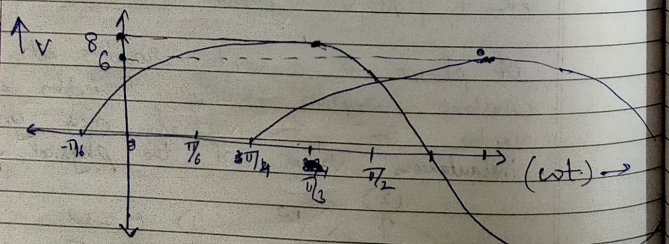
Add / Subtract 2 AC quantities

↳ Graphical
↳ Analytical.

Graphical :

Eg $V_1 = 8 \sin(\omega t - 30)$ (OA)
$V_2 = 6 \sin(\omega t + 45)$ (OB)





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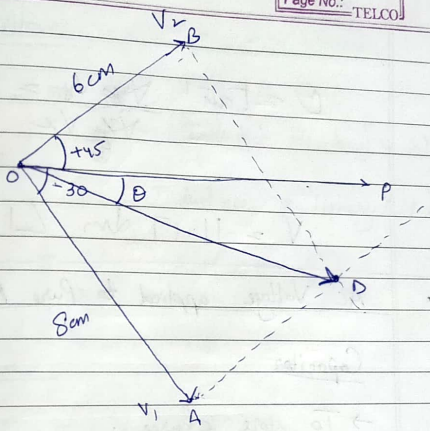
Step 1

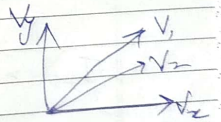


Step 2

$V_{net} = OD_{amp} \sin(\theta)$
$= 11.16 \sin(1.2^\circ)$

sin
D D P
H H B

Analytical



$V_y = 8 \sin(-30) + 6 \sin(45) = 0.242$

$V_x = 8 \cos(-30) + 6 \cos(45) = 11.170$

$V_R = \sqrt{V_x^2 + V_y^2} = \sqrt{0.24^2 + 11.17^2}$
$= 11.172 V$

$\theta = \tan^{-1} \frac{V_y}{V_x} = \tan^{-1} \frac{0.24}{11.17} = 1.23°$

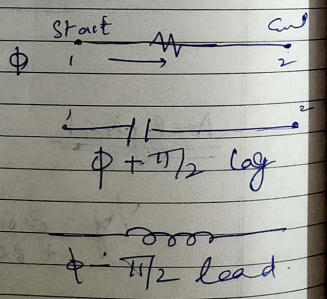
$V = 11.17 \sin \angle 1.23$

AC Voltage applied to Pure R, L and C

Capacitor

→ To store charge.
→ Used as a switch.
→ Delay circuit

R alone



$I_m = \frac{V_{max}}{R}$

$I_{rms} = \frac{V_{Rms}}{R}$

I Alone

$I = \frac{V}{X_L}$ → [Inductive Reactance] $= \omega L$

L = Inductance → Henery (H)

$i = \frac{V}{X_L} \cdot \sin\left(\omega t - \frac{\pi}{2}\right)$

C alone

$I = \frac{V}{X_c}$ → Capacitive Reactance $= \frac{1}{\omega C}$ → Farads

C → Capacitance

$\omega = 2\pi f$

$i = \frac{V}{X_c} \cdot \sin\left(\omega t + \frac{\pi}{2}\right)$

Series RL circuit

$|I| = \frac{|V|}{|Z|}$

Impedance (freq. dependant resistance)

$Z = \sqrt{R^2 + X_L^2}$

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Series R-C Circuit

$$|I| = \frac{|V|}{|Z|}$$

$$\downarrow$$

$$\sqrt{R^2 + X_C^2}$$

Series R-L-C Circuit

$$I = \frac{|V|}{|Z|}$$

$$\downarrow$$

$$\sqrt{R^2 + (X_L - X_C)^2}$$

Q5.9

$L = 50\ mH$

$X_L = ?$

$f = 100\ Hz$

$$X_L = \omega L$$
$$= 2\pi f . L$$
$$= 2\pi \times 50 \times 10^{-3} \times 100$$
$$= 10\pi\ \Omega$$

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$f = 5\ kHz$

$C = 10\ \mu F$

$X_C = ?$

$$\omega = 2\pi f = 2\pi \times 5000$$

$$X_C = \frac{1}{\omega C}$$
$$= \frac{1}{2\pi \times 5000 \times 10 \times 10^{-6}}$$
$$= \frac{1}{2\pi \times 0.05}$$
$$= 3.18\ \Omega$$