instruments designed for low operating torque. Such errors may be minimized by adopting a moving system of a light construction and large deflecting torque *i.e.*, large torque-weight ratio and mounting the moving system on a vertical, rather than a horizontal, spindle for preference, between jewelled bearings. The user can minimize frictional effects by tapping gently on the case with his finger tip before taking an observation. Vigorous tapping will defeat the purpose and may even injure the bearing.

3. ***Observational Error.*** Such errors are due to misreading of the scale, parallax in readings and errors of estimation. Precision of readings becomes difficult if the pointer has a broad tip. Interpolation errors may enter, particularly if the scale readings are too large or too small.

Parallax error is caused by the observer not having his line of sight on the pointer exactly at right angle to the plane of the scale. Such an error is eliminated by providing a mirror beneath the scale and a knife-edged pointer.

Lack of balance in the moving system and variations in the strength of permanent magnets (if used) are other possible sources of error which are common to several types of instruments.

9.9. MOVING IRON INSTRUMENTS

These instruments are widely used in laboratories and switchboards at commercial frequencies because these are cheaper in cost, robust in construction and can be manufactured with required accuracy.

There are two general types of such instruments *i.e.*, (*i*) the *attraction type* and (*ii*) the *repulsion type*. The attraction type instruments operate on the principle of attraction of a single piece of soft iron into a magnetic field and repulsion type instruments operate on the principle of repulsion of two adjacent iron pieces magnetised by the same magnetic field. Repulsion type instruments are more sensitive as in these instruments large operating torque is developed by having two iron elements positioned close together inside the field coil where the magnetizing effect is maximum. In both types of these instruments the current under measurement (~~or a definite fraction of it or proportional to the voltage under measurement~~), is passed through a coil of wire. This current carrying coil sets up the necessary field. Depending on the magnitude of the current to be measured, the coil may be of a few turns of very heavy conductor or of many turns of fine wire. The instrument to be used as an ammeter is provided with a coil of few turns of thick wire in order to have low resistance and carry large current and that to be used as a voltmeter is provided with a coil of large number of turns of fine wire in order to have high resistance and draw as small current as possible.

1. ***Attraction Type Moving Iron Instruments.*** The earliest and simplest form of attraction moving iron instrument uses a solenoid and moving oval shaped soft iron pivoted eccentrically as shown in Fig. 9.7. To this iron a pointer is attached so that it may deflect along with the moving iron over a graduated scale. The iron is made of sheet metal specially shaped to give a scale as nearly uniform as possible. The moving iron is drawn into the field of solenoid when current flows through it. The movement of the iron is always from weaker magnetic field outside the coil into the stronger magnetic field inside the coil regardless the direction of flow of current.

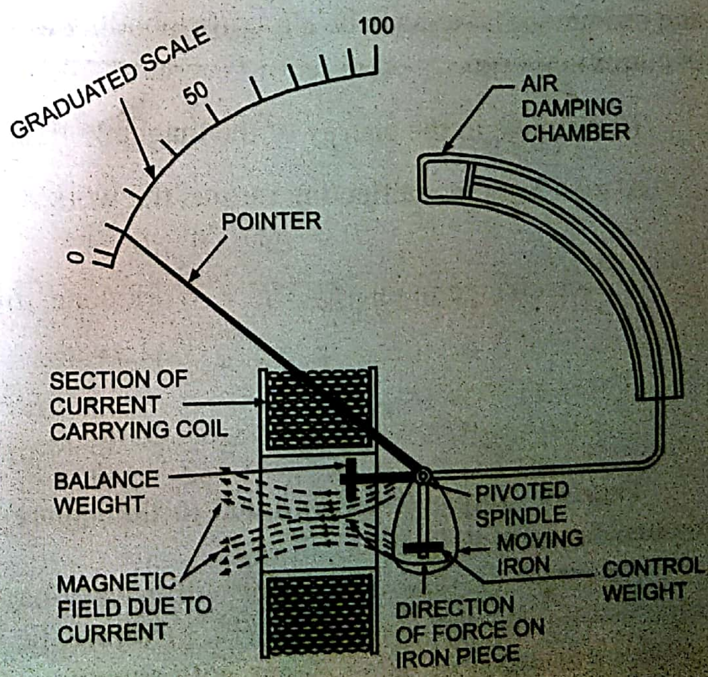


Fig. 9.7 *Moving Iron Attraction Type Instrument*

When the current to be measured (or a definite fraction of the current to be measured or proportional to the voltage to be measured) is passed through the solenoid, a magnetic field is set up inside the solenoid, which in turn magnetises the iron. Thus the iron is attracted into the coil causing the spindle and the pointer to rotate. Such instruments normally have spring control and pneumatic damping. In Fig. 9.7 gravity control attraction type instrument, which have been now superceeded by spring control instruments owing to their merits over gravity control instruments, is shown.

Such an instrument has a scale cramped at the lower end and greatly expanded at the upper end, as the operating torque is quite low when the moving iron is just entering the solenoid and increases rapidly as iron is drawn further in. However, it does permit the development of large working torque and is still used in some instruments requiring high operating torque.

2. *Repulsion Type Moving Iron Instruments.* Repulsion type moving iron instrument, in its simplest form, is shown in Fig. 9.8. In this type of instrument there are two irons. A curved 'iron' of soft iron or of mumetal is fixed to the inside of the bobbin former and another curved 'iron' is mounted on a spindle which passes axially through the solenoid. The two irons lie in the magnetic field produced by a solenoid. When there is no current in the solenoid the two irons, moving one and fixed one, are almost touching each other and the pointer rests on zero position. When the current to be measured (or a definite fraction of it or proportional to the voltage to be measured) is passed through the solenoid, which is wound with insulated copper wire on a cylindrical non-magnetic former (a few turns of thick wire in case of an ammeter and a large number of turns of fine wire in case of a voltmeter), a magnetic field is set up inside the solenoid and the two irons are magnetized in the same direction. This sets up a repulsive force so moving iron is repelled by fixed iron, thereby results in the motion of the moving iron carrying the pointer. The pointer comes to rest in a deflected position when equilibrium is attained between the repulsion force of the working elements and the controlling force. Control of the movement is either by hair springs, when the instrument is to be employed in any position, or by gravity, when the instrument is to be operated only vertically. Damping used is air friction (pneumatic) one. Eddy current damping is not possible because of presence of permanent magnet needed for such purpose would affect the deflection *i.e.*, reading of the instrument. The repulsion of irons is proportional to the square of current, and so the scale is uneven, being crowded at the lower values and wide spread at the higher values. The iron may be so shaped and arranged that the scale is spread out to some extent at the lower values of current.

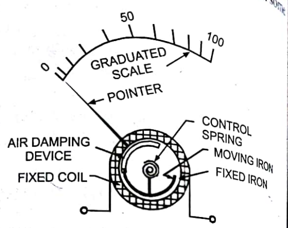


Fig. 9.8 *Repulsion Type Moving Iron Instrument*

Deflecting Torque. Let L be the self-inductance corresponding to a total angular deflection of θ radians and change in inductance be dL corresponding to small change in deflection angle $d\theta$ due to small change in current.

The change in the energy of the magnetic field, $dE = \frac{1}{2} I^2 dL$

Also if T_d is the deflecting torque, the work done during the movement of the iron,

$$dw = T_d d\theta$$

Since change in energy, dE = Work done, dw,

$$T_d d\theta = \frac{1}{2} I^2 dL$$

$$\text{or } T_d = \frac{1}{2} I^2 \frac{dL}{d\theta} \text{ N-m if I is in amperes, L in henrys and } \theta \text{ in radians.}$$

Thus the torque is proportional to the square of the instrument current and to the rate of change of inductance with deflection.

An attraction type instrument will usually have a lower inductance than the corresponding repulsion type instrument, and voltmeters will, therefore, be accurate over a wider range of frequency and there is greater possibility of using shunts with ammeters. On the other hand, repulsion type instruments are more suitable for economical production in manufacture and a nearly uniform scale is more easily obtained; they are, therefore, much more common than the attraction type.

Ranges. ***Ammeters*** — from about 20 mA to 750 A maximum without current transformer.

Voltmeters — from about 0–1 V to 750 V maximum without potential transformer.

Errors in Moving Iron Instruments. The errors usually occurring in moving iron instruments are (*i*) friction (*ii*) temperature (*iii*) stray magnetic field (*iv*) hysteresis (*v*) frequency and (*vi*) waveform. The first four errors affect the instruments on ac and dc while last two affect only on ac. Friction and temperature errors have already been dealt earlier. *Stray magnetic field errors* may be serious if not guarded against because of the weakness of the operating magnetic field in case of moving iron instruments. Such errors are minimized by shielding the instrument with the help of steel case or sheet steel lining to the case. *Hysteresis error* is serious type of error in moving iron instruments specially when used on dc circuits. This error may be reduced by making the iron parts short so that they demagnetize themselves or by choosing low value of flux density in the iron or by using mumetal or permalloy having negligible hysteresis. Change in frequency may cause errors due to change of reactance of the operating coil and also to the changes in magnitude of eddy currents set up in the metal parts of the instrument near the operating portion. The change in instrument coil reactance owing to change in frequency causes serious error in case of voltmeters (not in case of ammeters). The eddy current error affects both ammeters and voltmeters equally. The frequency error may be compensated for by connecting a suitable capacitor C in parallel with the swamping resistance R (C being equal to L/R^2 if frequency is not too high).

Moving iron instruments may be seriously affected by waveform both on account of the change in the form of flux waveform and in case of voltmeters, the effect of harmonics upon the inductance.

Merits and Demerits of Moving Iron Instruments

Merits. (*i*) These instruments can be used both in dc as well as in ac circuits. This is due to the fact that the deflecting torque depends on the square of the current.

(*ii*) These instruments are robust, owing to the simple construction of the moving parts and the fact that there are no current leads to these parts.

(*iii*) The stationary parts of the instruments are also simple.

(*iv*) The simplicity of construction, as described in (*ii*) and (*iii*) results in low cost.

(*v*) These instruments possess high operating torque.

(*vi*) These instruments can withstand overload momentarily.

(*vii*) These instruments are capable of giving an accuracy within the limits of both precision and industrial grades. Instruments which are required to have industrial grade accuracy on both ac and dc require very considerable care in design.

The moving iron instruments, owing to the above advantages, have become most widely used instruments in ac circuits, in addition, they are largely used on dc circuits in cases where the superior accuracy of moving coil instruments is not an essential requirement.

Demerits. (*i*) Scales of these instruments are not uniform. ←

(*ii*) Power consumption is higher for low voltage range.

(*iii*) The stiffness of the spring decreases with the increase in temperature.

(*iv*) The errors are caused due to hysteresis in the iron of the operating system and due to stray magnetic field.

(*v*) Change in frequency and in waveform causes serious errors in ac measurements.

(*vi*) There is a difference between dc and ac calibrations because of inductance effect of the instruments and eddy currents on ac. The instruments should be calibrated for frequencies at which these are to be used.

Example 9.4

A spring-controlled moving iron voltmeter reads correctly on 250 V dc. Calculate the scale reading when 250 V, 50 Hz ac is applied. The instrument has a resistance of 500 Ω and an inductance 1 H and the series resistance of 2,000 Ω.

[G.G.S.I.P. Univ. Delhi Second Term Exam. April 2009]

Solution: Total resistance of the moving iron voltmeter circuit, $(R + r) = 500 + 2{,}000 = 2{,}500\ \Omega$

When the instrument is used on dc system the coil inductance does not affect the instrument readings, so if the instrument reads correctly, then when the instrument is used on ac system the voltmeter reading will be affected by the coil inductance and the instrument readings will be incorrect.

At 50 Hz frequency, impedance of the instrument circuit,

$$Z = \sqrt{(R+r)^2 + (2\pi \times f\, L)^2} = \sqrt{(2{,}500)^2 + (2\pi \times 50 \times 1)^2} = 2{,}520\ \Omega$$

Current drawn by the instrument, when connected to 250 V, 50 Hz ac supply = $\dfrac{250}{2{,}520}$ A

Since voltmeter reads correctly on dc supply on 250 V, so its current corresponding to 250 V = $\dfrac{250}{2{,}500} = 0.1$ A

So voltmeter reading, when connected to 250 V, 50 Hz ac supply = $250 \times \dfrac{250/2{,}520}{0.1} = 248$ V **Ans.**

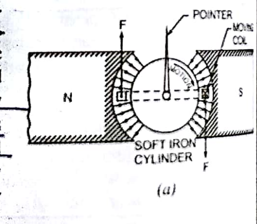
9.10. MOVING COIL INSTRUMENTS

There are two types of moving coil instruments namely the *permanent magnet type* and *dynamometer type*.

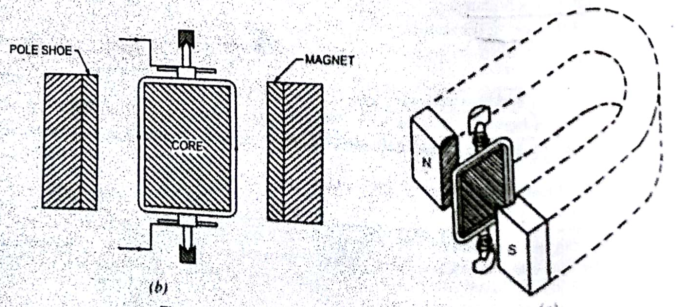
Permanent magnet moving coil (PMMC) *instrument* is the most accurate and useful for dc measurements. In fact, were it not for the need of other instruments for ac measurements, it may be doubted whether any other type of indicating instruments would have survived except where cheapness is main requirement, in which case the simple moving iron instrument holds its place. *Dynamometer type moving coil instruments* can be used both on dc as well as on ac. Their calibration is the same for both dc as well as ac and so these instruments are very useful as transfer instruments.

9.11. PERMANENT MAGNET MOVING COIL (PMMC) INSTRUMENTS

A permanent magnet type moving coil instrument is shown in Fig. 9.9. It consists of permanent powerful magnet with soft iron pole pieces. A cylindrical iron core is mounted between the two poles of the magnet giving very narrow air gap in which the sides of a pivoted light rectangular coil lies. The rectangular coil is wound of many turns from fine wire on light aluminium or copper former and acts as moving element. The purpose of using core is to make the field uniform and to reduce the reluctance of the magnetic circuit. A low reluctance helps to retain permanence of the magnet for a longer period.



(a)



(b) (c)

Fig. 9.9 *Moving Coil (Permanent Magnet Type) Instrument*

The current is led into and out of the coil by means of phosphor bronze hair springs provided at both ends. The springs also provide the controlling torque. The springs are spiralled in opposite directions to neutralise the effect of change in temperature.

Torque Equation. When the current to be measured (or a definite fraction of it or proportional to the voltage to be measured) is passed through the coil, say in the direction shown in Fig. 9.9, a deflecting torque is produced on account of reaction of the permanent magnetic field with the coil magnetic field. The direction of deflecting torque can be determined by applying Fleming's left hand rule.

If i is the current in amperes flowing through the coil of turns N and length l *metres and* B *is the flux density in teslas in the air gap*

Then deflecting force, $F = BilN$ newtons

If r is the distance in metres between the centre of the coil and the force F

Then deflecting torque, $T_d = F \times r = BilNr$ Nm

From the above expression it is obvious that if flux density B in the air gap is constant, then

Deflecting torque, $T_d \propto i$

Since such instruments are spring controlled,

Controlling torque, $T_c \propto$ deflection θ

Since in steady deflection position, $T_c = T_d$

$\therefore \theta \propto i$

Since θ is directly proportional to current, the scale markings of the basic dc PMMC instrument are usually linearly spaced. If low frequency ac is applied to the moving coil, the pointer will deflect up-scale for one half-cycle of the input waveform and down-scale for the next half cycle. At power frequencies of 50 Hz and above, the pointer will not be able to follow the rapid variations in direction and will quiver slightly around the zero mark, seeking the average value of ac *i.e.*, zero. That is why, PMMC instrument is unsuitable for ac measurements, unless the current is rectified before applying it to the moving coil.

Ranges. (*i*) *DC Ammeters* : 0 – 5 μA and up to 20 mA with internal shunts
and 0 – 5,000 A with external shunts.

(*ii*) *DC Voltmeters* : 0 – 100 mV without series resistance
and up to 20 kV or 30 kV with external series resistance.

Voltmeter Sensitivity and Loading Effect. We know that PMMC movement requires series resistors of different ranges. For example a movement requiring 5 mA for full-scale deflection can be converted into a voltmeter of range 250 V, 25 V and 2.5 V by connecting series resistances of different values so as to make total resistance of 50,000 Ω, 5,000 Ω and 500 Ω respectively. In either case if we divide the number of ohms by the scale range *i.e.*, 50,000/250 or 5,000/25 or 500/2.5 we have the same quotient in ohms per volt *i.e.*, 200 Ω/V. This is called the sensitivity of the voltmeter. The current drawn at full scale is the reciprocal of this number *i.e.*, 0.005 A.

Merits And Demerits of PMMC Instruments

Merits. (*i*) Uniform scale.

(*ii*) Low power consumption (say from 25 microwatt to 200 microwatt) because of small driving power.

(*iii*) No hysteresis loss as the former is of copper or aluminium.

(*iv*) Very effective and reliable eddy current damping.

(*v*) High torque-weight ratio resulting in high accuracy (2-5% of full-scale reading).

(*vi*) Use of single instrument for measurement of currents and voltages by employing shunts and multipliers of different resistances.

(*vii*) No effect of stray magnetic fields because of use of intensive polarised or unidirectional field.

(*viii*) The instrument using core magnet is very suitable in aircraft and aerospace applications, where a multiplicity of instruments are mounted in close proximity to each other. This is because of self-shielding property of core magnets.

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Demerits. (*i*) These instruments cannot be used for ac measurements.

(*ii*) These are costlier in comparison to moving iron instruments because of delicate construction and the necessary accurate machining and assembly of various parts.

(*iii*) Friction and temperature might introduce errors, as in case of other instruments, which can be reduced as already discussed.

(*iv*) Ageing of control springs and of the permanent magnets might cause errors which can be considerably obviated by careful choice of material and preageing during manufacture.

Example 9.5

The resistance of a moving coil voltmeter is 12,000 Ω. The moving coil has 100 turns and is 4 cm long and 3 cm wide. The flux density in the airgap is 6×10^{-2} Wb/m². Find the deflection produced by 300 V if the spring control gives a deflection of one degree for a torque of 25×10^{-7} N-m. [*Allahabad Univ. Elec. Engg. 1974*]

Solution:

Voltage applied, V = 300 V

Coil resistance, R = 12,000 Ω

Current flowing through the coil, $i = \frac{V}{R} = \frac{300}{12,000} = 0.025$ A

Deflecting torque, $T_d = NBilr = 100 \times 6 \times 10^{-2} \times 0.025 \times 0.04 \times 0.03 = 18 \times 10^{-5}$ N-m

Controlling torque, $T_c = 25 \times 10^{-7}\,\theta$ N-m where θ is the deflection in degrees produced by 300 V

Since for steady deflection state $T_c = T_d$

$\therefore\ 25 \times 10^{-7}\,\theta = 18 \times 10^{-5}$

or $\theta = \frac{18 \times 10^{-5}}{25 \times 10^{-7}} = 72°$ **Ans.**

Example 9.6

A moving coil milli-voltmeter has a resistance of 200 Ω and the full-scale deflection is reached when a pd of 100 mV is applied across the terminals. The moving coil has effective dimensions of 30 mm × 25 mm and is wound with 100 turns. The flux density in the gap is 0.2 Wb/m². Determine the control constant of the spring if the final deflection is 100°.

Solution : Full-scale deflection current, $i = \frac{\text{Full-scale potential difference}}{\text{Voltmeter resistance}} = \frac{100\text{ mV}}{200\ \Omega} = 0.5$ mA

Deflecting torque, $T_d = BilrN = 0.2 \times 0.5 \times 10^{-3} \times 30 \times 10^{-3} \times 25 \times 10^{-3} \times 100$

$= 75 \times 10^{-7}$ N-m

Full-scale deflection, $\theta_f = 100°$

Control constant of spring $= \frac{T_d}{\theta_f} = \frac{75 \times 10^{-7}}{100} = 0.75 \times 10^{-7}$ N-m/degree **Ans.**

9.12. DYNAMOMETER TYPE INSTRUMENTS

Dynamometer type instruments are very similar to PMMC instruments except that the permanent magnetic field is replaced by a coil (usually two fixed air-cored coils are used) which carry the current to be measured (or a definite fraction of it or proportional to the voltage to be measured). The coils are usually air-cored to avoid hysteresis, eddy currents and other errors when the instrument is used on ac. The fixed coil is divided into two halves, connected in series with the moving coil, and placed together and parallel to each so as to provide a fairly uniform field within the range of the movement of the moving coil. However, the space between the two halves of the fixed coil must be sufficient enough to allow the movement of the moving coil shaft.

This instrument, like other instruments already discussed, develops deflecting torque by the interaction of magnetic fields, one field due to current in a moving coil and the other due to current in the fixed coil. The important difference is that the field due to fixed coil is not constant, but varies with the magnitude of current flowing through the fixed coil. Hence the deflecting torque of this instrument is determined not only by the moving coil current but also by the fixed coil current.

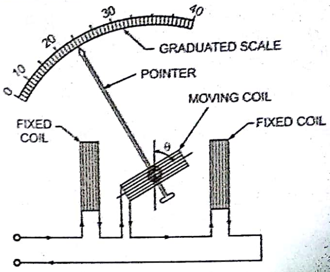


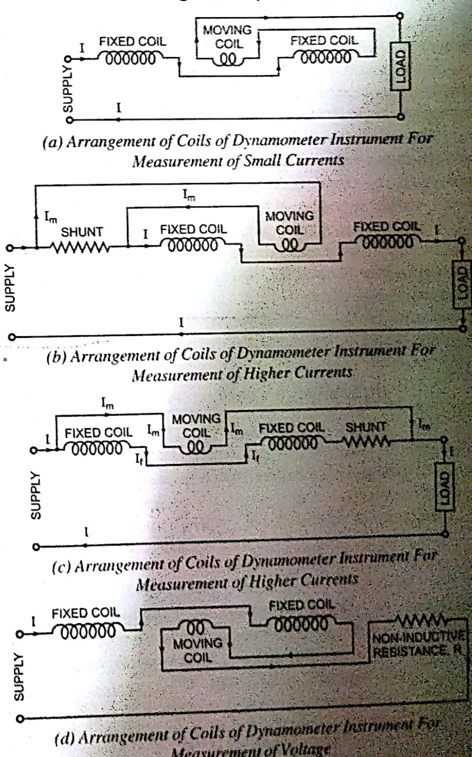
Fig. 9.10 *Dynamometer Type Instrument*

This instrument, with its fixed and moving coils connected in series, is readily adapted for voltage measurement, provided the proper series resistor is used, as shown in Fig. 9.11 (*d*).

This instrument is seldom used as an ammeter because the lead-in spirals to the moving coil can carry limited current, frequency variations influence the inductance of the coils and introduce error and the resistance of the moving and fixed coils in series may cause an undesirably high voltage drop across the shunt.

Since the deflecting torque is determined by variations in either the fixed or the moving coil currents, the dynamometer instrument is a versatile measuring device for several other applications such as for measurement of power, reactive volt-ampères and, with some modifications, for measuring power factor and frequency in ac circuits.

Though these instruments can be used both, for dc and ac measurements but most of its practical uses are in ac as these instruments have higher cost, higher power consumption, lower torque-weight ratio, non-uniform scale and other several drawbacks in comparison to PMMC instruments.



(*a*) *Arrangement of Coils of Dynamometer Instrument For Measurement of Small Currents*

(*b*) *Arrangement of Coils of Dynamometer Instrument For Measurement of Higher Currents*

(*c*) *Arrangement of Coils of Dynamometer Instrument For Measurement of Higher Currents*

(*d*) *Arrangement of Coils of Dynamometer Instrument For Measurement of Voltage*

Fig. 9.11

The dynamometer type ammeters and voltmeters are usually portable and laboratory standard instruments of the highest attainable precision, but owing to the fact that they are costly to build in consequence of the care to be taken in their manufacture and calibration and their operating power required are usually considerably higher than those of moving iron instruments they are not usually used as ammeters and voltmeters for general laboratory and shop measurements at power frequency *i.e.*, where sturdier and cheaper instruments are adequate.

then

Deflecting torque, $T_d \propto i_f i_m$

In spring-controlled instruments controlling torque,

$T_c \propto \theta$, the deflection

But in steady deflected position, $T_c = T_d$

$\therefore \theta \propto i_f i_m$

or $\theta \propto i^2$ since $i_f \propto i$ and $i_m \propto i$

The instrument, therefore, has a "square law" response and it indicates in terms of effective (or rms) current or voltage.

Ranges. (1) Ammeters : 0 – 0.01 A up to 0–0.05 A with fixed and moving coils connected in series [Fig. 9.11 (*a*)] and up to 0–20 A with moving coil shunted [Fig. 9.11 (*b*)] or parallel connections [Fig. 9.11 (*c*)].

(2) Voltmeters: Up to 0–750 V

Merits and Demerits of Dynamometer Type Instruments

Merits. (*i*) As the instrument has square law response, so can be used on both dc as well as on ac.

(*ii*) These instruments are free from hysteresis and eddy current errors because of absence of iron in the operating parts of the instrument.

(*iii*) Ammeters up to 10 A and voltmeters up to 600 V can be constructed with precision grade accuracy.

(*iv*) Dynamometer type voltmeters are very useful for accurate measurement of rms values of voltages irrespective of waveforms.

(*v*) Because of precision grade accuracy and same calibration for dc and ac measurements these instruments are used as transfer and calibration instruments.

A transfer instrument is that which may be calibrated with dc source and then used on ac without any modification.

Demerits. (*i*) The scale is not uniform as the instrument has square-law response.

(*ii*) These instruments have small torque-weight ratio, as already explained, so the friction error is considerable.

(*iii*) Owing to heavy moving system friction losses in these instruments are somewhat more than those in other instruments.

(*iv*) As a result of measures taken to reduce the frictional error; their cost is more in comparison to moving iron and PMMC instruments and they are more sensitive to overloads and mechanical impacts and are to be handled with care.

(*v*) Adequate screening of the movements against stray magnetic fields is essential.

(*vi*) The sensitivity of the instrument is typically very low due to poor deflecting torque. The sensitivity of dynamometer type voltmeter is 10 to 30 Ω/V in comparison to the sensitivity of 20 kΩ/V in case of D'Arsonval movement.

(*vii*) Power consumption of these instruments is comparatively high because of their construction.

Errors in Dynamometer Type Instruments. The errors that usually occur in dynamometer type instru-nents are (*i*) frictional error (*ii*) temperature error (*iii*) error due to stray magnetic fields and (*iv*) frequency error.

(*i*) *Frictional Error.* Since the coils are air-cored, magnetic field produced is of small strength (near about .005 T–0.006 T) thereby requiring large number of ampere-turns to create necessary deflecting torque. This esults in heavy moving system and, therefore, small torque-weight ratio. Thus the frictional losses in dyna-nometer type instruments are somewhat larger than those in other types. However, frictional error is minimised y a reasonable reduction in weight of the moving element, proper selection of materials for the bearings and pindle and adequate polishing of these parts. These measures, however, increase the cost of instrument and nake it more sensitive to overloads and mechanical impacts.

9.13. COMPARISON BETWEEN MOVING IRON AND MOVING COIL INSTRUMENTS

1. *Construction.* Moving iron instruments are simple and robust in construction and are therefore, cheaper in cost.
   Moving coil instruments are costlier in comparison to moving iron instruments because of delicate construction and the necessary accurate machining and assembly of various parts.
2. *Damping torque.* In moving iron instruments, air friction damping is provided which is not effective while in moving coil instruments very effective and reliable eddy current damping is provided.
3. *Power consumption.* Moving coil instruments consume less power as compared to moving iron instruments because of small driving power.
4. *Scale.* Moving coil instruments have uniform scale while moving iron instruments have non-uniform scale (crowded at the beginning).
5. Sensitivity. Moving coil instruments are more sensitive as compared to moving iron instruments because of high torque-weight ratio.
6. *Accuracy and reliability.* As compared to moving iron instruments, moving coil instruments are more accurate and reliable because they have no hysteresis loss and they are not affected by stray magnetic fields.
7. *Applications.* Moving iron instruments can be used both in dc as well as in ac circuits but moving coil instruments cannot be used in ac circuits.

9.14. COMPARISON BETWEEN MOVING IRON AND DYNAMOMETER TYPE INSTRUMENTS

1. *Torque-Weight Ratio.* Dynamometer type instruments have small torque-weight ratio.
2. *Friction Error.* Dynamometer type instruments have considerable friction error.
3. *Friction Loss.* Owing to heavy moving system, dynamometer type instruments have more friction losses.
4. *Cost and Sensitivity To Overloads.* As a result of measures taken to reduce the frictional error, the cost of dynamometer type instruments is more in comparison to moving iron. Dynamometer type instruments are more sensitive to overloads and mechanical impacts and are to be handled with care.
5. *Sensitivity.* The sensitivity of dynamometer instruments is typically very poor due to poor deflecting torque.
6. *Power Consumption.* Dynamometer instruments have comparatively higher power consumption.
7. *Effect of Stray Magnetic Field.* There is no effect of stray magnetic field on moving iron instruments because of use of intensive polarised or unidirectional field while dynamometer type instruments are more sensitive to the stray magnetic field and need adequate screening.
8. *Hysteresis and Eddy Current Errors.* Dynamometer type instruments are free from hysteresis and eddy current errors because of absence of iron in the operating parts while moving iron instruments have these errors.
9. *Effect of Waveform.* Dynamometer type voltmeters are very useful for accurate measurement of rms values of voltages irrespective of waveforms while change in frequency and in waveform causes serious errors in ac measurements with moving iron instruments.
10. *Calibration.* Dynamometer type instruments have same calibration for dc and ac measurements and have precision grade accuracy and are, therefore, widely used as transfer and calibration instruments while moving iron instruments have a difference between dc and ac calibrations.

Dynamometer type instruments are superior because of its inherent advantages given above in para (8), (9) and (10).

9.15. EXTENSION OF INSTRUMENT RANGE

The range of an electrical measurement is limited by the current, which can be carried by the coil of the instrument safely. For example, the moving coil and the spiral springs used as coil connections can be designed for a maximum current of only about 50 mA and a potential drop of about 50 millivolts. Hence for measurement of large currents or voltage or power, some means for increasing the range of the instrument is to be adopted.

There are four common devices employed for extending the range of instruments; these are shunts, multipliers, current transformers and potential transformers. When instruments are supplied with such external devices, the instrument is calibrated and its scale is correspondingly marked, over the range of the associated shunt, multiplier or transformer.

9.15.1. *Ammeter Shunts.* An ammeter shunt is merely a low resistance that is placed in parallel with the coil circuit of the instrument in order to measure fairly large currents. The greater part of the current in main circuit is then diverted around the coil through the shunt.

The connection diagram for a shunt and milliammeter for measuring large currents is shown in Fig. 9.13. The shunt has four terminals, two of large current carrying capacity known as *current terminals* for inserting it in series with the main circuit and the other two of smaller size, known as *potential terminals*, for connecting the ammeter across it.

Theory. Let I be the current of the circuit to be measured, R_s be the resistance of the shunt and R_m be the resistance of the ammeter plus its leads to the potential terminals.

The main circuit current will be split in two parts, I_m, the current flowing through the ammeter and I_s, the current through the shunt.

$$\text{Thus } I = I_m + I_s \qquad ...(i)$$

Since the voltage drop across the shunt and the instrument is same,

$$I_m R_m = I_s R_s$$

$$\text{or } R_s = \frac{I_m R_m}{I_s} = \frac{I_m R_m}{I - I_m}$$

$$= \frac{R_m}{I/I_m - 1} \qquad ...(ii)$$

Since from Eq. (*i*) $I_s = I - I_m$

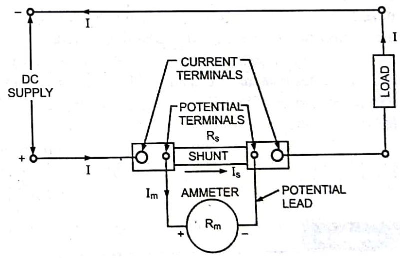


Fig. 9.13 *Diagram Showing The Method of Connecting a Shunt and Milliammeter For Measuring High Currents*

The ratio of the total current to the instrument current (*i.e.*, I/I_m) is known as the 'multiplying power' of the shunt. Denoting the ratio by N, we have shunt resistance,

$$R_s = \frac{R_m}{N-1} \qquad ...(iii)$$

Construction of Shunts. The shunts used for extension of range of instruments must have temperature coefficient of resistance very low and as far as possible equal to that of the instrument coil so that multiplying power of the shunt is independent of temperature. The resistance of the shunt should also not vary with the time of usage. Other requirements are that the thermoelectric emf of the shunt should be very small and it must be capable of carrying required current without appreciable heating.

The material employed for shunts is manganin, which has a low temperature coefficient, in the form of thin strips, the ends of which are soldered to two large copper blocks. Each copper block carry two terminals — one current terminal and other potential terminal. The strips, which form the shunt are spaced from each other to promote a good circulation of air and thus efficient cooling.

Precautions. The resistance of the leads, as already mentioned, used with the instruments forms the part of the resistance of the instrument circuit and, therefore, must be considered while determining the required shunt resistance. A special pair of leads is usually supplied with the instrument by the manufacturer intended to be used. These leads should always be used in order to obtain the stated accuracy as the instrument is calibrated with these leads in use.

Advantages. (*i*) The same instrument can be used for measurement of currents of different ranges by employing different shunts.

(*ii*) The shunt enables the ammeter to be mounted remote from the main circuit, which results in reduction of the possible effect of stray field and have the advantage of remote indication.

9.15.2. Voltmeter Multipliers. Voltmeter multiplier is a high non-inductive resistance connected in series with the voltmeter coil and is used for increasing the range of the voltmeter. A voltmeter connected in series with the high non-inductive resistance (voltmeter multiplier) for measurement of high voltage is shown in Fig. 9.14.

Theory. Let R_v be the resistance of voltmeter, I_m be the full-scale deflection current, V be the voltage of the circuit to be measured and *R* be the external series resistance.

Now since voltage across supply leads = Voltage drop across the voltmeter + voltage drop across external resistance

$$\therefore V = I_m R_v + I_m R$$

$$\text{or } R = \frac{V}{I_m} - R_v$$

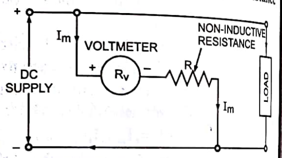


Fig. 9.14

The main requirements of voltage multipliers to be used for dc measurements are that (*i*) resistance should not change with time of usage (*ii*) their temperature coefficient of resistance must be very low so that change in temperature may not affect their resistance values and (*iii*) should have ample provision for cooling to dissipate the heat developed, as an appreciable amount of power is developed in such resistors.

The series resistance is usually of manganin, either in the form of a bobbin or in the form of strip wound on a narrow sheet of bakelite or other insulating material. It may be inside the instrument case or particularly for higher voltages, in a separate ventilated case.

The question of temperature compensation does not normally arise in the case of a voltmeter, as any slight change in the coil resistance is negligible in comparison with the total resistance, the major portion of which consists of manganin wire. The series resistance, in fact acts as a completely effective swamp resistance.

Example 9.7

The coil of a moving coil instrument is wound with 80 turns. The coil is 5 cm long and 3 cm wide. The flux density in the air gap is 0.5 Wb/m². The control spring exerts a torque of 2.0×10^{-4} N-m, at the full-scale deflection of 100 divisions. Calculate the resistance which must be connected in series with the coil to measure 2 V/division (Assume the coil resistance be zero).

[*G.G.S.I.P. Univ. Delhi Second Term Exam. April 2008*]

Solution: Let the full-scale deflection current be I_m amperes

Full-scale deflecting torque, $T_d = NBIlr = 80 \times 0.5 \times 0.05 \times 0.03\ I_m = 0.06\ I_m$ N-m

Full-scale controlling torque, $T_c = 2 \times 10^{-4}$ N-m

Since for steady deflection state, $T_c = T_d$

So $2 \times 10^{-4} = 0.06\ I_m$

$$\text{or } I_m = \frac{2\times10^{-4}}{0.06} = 0.00333 \text{ A or } 3.333 \text{ mA}$$

Full-scale deflection voltage, $V = 2 \times 100 = 200$ V

Resistance required for full-scale deflection with 200 V

$$R = \frac{V}{I_m} - R_m = \frac{200}{0.00333} - 0 = 60{,}000\ \Omega \text{ or } 60\ k\Omega \textbf{ Ans.}$$

Example 9.8

A PMMC instrument gives a reading of 25 mA when the potential difference across its terminals is 75 mV. Calculate shunt resistance for full-scale deflection corresponding to 50 A. [*G.G.S.I.P. Univ. Delhi Electrical Science June 2006*]

Solution:

Meter resistance, $R_m = \frac{75 \text{ mV}}{25 \text{ mA}} = 3\ \Omega$

Main circuit current, I = 50 A

Meter current for full-scale deflection, I_m = 25 mA = 0.025 A

Multiplying factor, $N = \frac{I}{I_m} = \frac{50}{0.025} = 2{,}000$

Shunt resistance, $R_s = \frac{R_m}{N-1} = \frac{3}{2{,}000-1} = \frac{3}{1{,}999}\ \Omega$ **Ans.**

Example 9.9

A moving iron instrument having an internal resistance of 50 Ω indicates full-scale deflection with a current of 10 mA. How can it be made to work as (*i*) voltmeter to read 100 V on full scale (*ii*) an ammeter of 1 A on full scale.

[*G.G.S.I.P. Univ. Delhi Electrical Science May 2005*]

Solution: Full scale deflection current, I_m = 10 mA = 0.01 A

Instrument resistance, $R_m = 50\ \Omega$

(*i*) Resistance required for full-scale deflection with 100 V

$$R = \frac{V}{I_m} - R_m = \frac{100}{0.01} - 50 = 9{,}950\ \Omega \textbf{ Ans.}$$

(*ii*) Shunt resistance required for full-scale deflection at 1A

$$R_s = \frac{R_m}{\frac{I}{I_m}-1} = \frac{50}{\frac{1}{0.01}-1} = \frac{50}{99}\ \Omega \textbf{ Ans.}$$

Note: Inductance of the instrument has been neglected as it is not given.

Example 9.10

A moving coil instrument has a resistance of 10 Ω and gives full-scale deflection, when carrying a current of 50 mA. Show how it can be adopted to measure (*i*) Voltage up to 750 V (*ii*) Currents up to 100 A.

[*G.G.S.I.P. Univ. Delhi Electrical Science May 2005*]

Solution: Full-scale deflection current, I_m = 50 mA = 0.05 A

Instrument resistance, $R_m = 10\ \Omega$

(*i*) Series resistance required for full-scale deflection at 750 V,

$$R = \frac{V}{I_m} - R_m = \frac{750}{0.05} - 10 = 15{,}000 - 10 = 14{,}990\ \Omega \textbf{ Ans.}$$

(*ii*) Shunt resistance required for full-scale deflection at 100 A

$$R_s = \frac{R_m}{\frac{I}{I_m}-1} = \frac{10}{\frac{100}{0.05}-1} = \frac{10}{2{,}000-1} = \frac{10}{1{,}999}\ \Omega \textbf{ Ans.}$$

Example 9.11

A moving coil instrument has a resistance of 2 Ω and it reads up to 250 V when a resistance of 5,000 Ω is connected in series with it. Find the current range of the instrument when it is used as an ammeter with the coil connected across a shunt resistance of 2 milli ohms. [*U.P. Technical Univ. Electrical Engineering February 2002; January 2005*]

Solution: Instrument coil resistance, $R_m = 2\ \Omega$

Current flowing through the instrument for full-scale deflection,

$$I_m = \frac{\text{Full-scale reading in volts}}{R_m + \text{Series resistance}} = \frac{250}{2 + 5{,}000} = 0.04998 \text{ A or } 49.98 \text{ mA}$$

Shunt resistance, $R_s = 2 \times 10^{-3}\ \Omega$

Current through shunt resistance, $I_s = \frac{I_m R_m}{R_s} = \frac{49.98 \times 10^{-3} \times 2}{2 \times 10^{-3}} = 49.98$ A

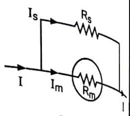


Fig. 9.15

Current range of instrument = Full-scale deflection current

$= I_m + I_s = 0.04998 + 49.98 = 50$ A **Ans.**

Example 9.12

How will you use a PMMC instrument which gives full-scale deflection at 50 mV pd and 10 mA current (*i*) ammeter 0–10 A range (*ii*) voltmeter 0–250 V range. [*G.G.S.I.P. Univ. Delhi Electrical Science May-June 2009*]

Solution: Meter resistance $= \frac{50 \text{ mV}}{10 \text{ mA}} = 5\ \Omega$

Full-scale deflection current, $I_m = 10$ mA $= 0.01$ A

(*i*) Shunt resistance required for full-scale deflection at 10 A

$$R_s = \frac{R_m}{\frac{I}{I_m} - 1} = \frac{5}{\frac{10}{0.01} - 1} = \frac{5}{999}\ \Omega \textbf{ Ans.}$$

(*ii*) Series resistance required for full-scale deflection at 250 V

$$R = \frac{V}{I_m} - R_m = \frac{250}{0.01} - 5 = 24{,}995\ \Omega \textbf{ Ans.}$$

Example 9.13

A moving coil ammeter has a fixed shunt of 0.02 Ω with a coil circuit resistance of R = 1 kΩ and needs potential difference of 0.5 V across it for full-scale deflection.

(1) To what total current does this correspond ?

(2) Calculate the value of shunt to give full-scale deflection when the total current is 10 A and 75 A.

[*Nagpur Univ. Measurement and Instrumentation*]

Solution: Current through shunt, $I_s = \frac{\text{PD across shunt}}{\text{Shunt resistance}} = \frac{0.5}{0.02} = 25$ A

Current through instrument, $I_m = \frac{V}{R_m} = \frac{0.5}{1{,}000} = 0.0005$ A

$\therefore$ Total line current, $I = I_s + I_m = 25.0 + 0.0005 = 25.0005$ A **Ans.**

Resistance required for the shunt to give full-scale deflection for total current of 10 A

$$R_s' = \frac{R_m}{\frac{I'}{I_m} - 1} = \frac{1{,}000}{\frac{10}{0.0005} - 1} = 0.05\ \Omega \textbf{ Ans.}$$

Resistance required for the shunt to give full-scale deflection for total current of 75 A,

$$R_s'' = \frac{R_m}{\frac{I''}{I_m} - 1} = \frac{1{,}000}{\frac{75}{0.0005} - 1} = 0.00667\ \Omega \textbf{ Ans.}$$

Example 9.14

A moving coil ammeter has a fixed shunt of 0.02 Ω with a coil resistance of R = 100 Ω and a pd of 500 mV across it, full-scale deflection is obtained.

(*a*) To what shunted current does this respond ?

(*b*) Calculate the value of R to give full-scale deflection when shunted current is (*i*) 10 A (*ii*) 75 A and (*iii*) with what value of R is 40% deflection obtained with I = 100 A.

[*G.G.S.I.P. Univ. Delhi Electrical Science May 2011*]

Solution: (*a*) Current through shunt, $I_s = \frac{\text{PD across shunt}}{\text{Shunt resistance}} = \frac{500 \times 10^{-3}}{0.02} = 25$ A **Ans.**

Current through instrument, $I_m = \frac{V}{R_m} = \frac{500 \times 10^{-3}}{100} = 0.005$ A

Total current, $I = I_s + I_m = 25 + 0.005 = 25.005$ A

(*b*) For full-scale deflection

(*i*) When shunted current, $I_s = 10$ A

PD applied across the instrument, $V = I_s R_s = 10 \times 0.02 = 0.2$ V

Full-scale deflection instrument current $I_m = 0.005$ A as determined above

Required instrument resistance, $R_m = \frac{V}{I_m} = \frac{0.2}{0.005} = 40\ \Omega$ **Ans.**

(*ii*) When shunted current, $I_s = 75$ A

PD applied across the instrument $= 75 \times 0.02 = 1.5$ V

Required instrument resistance, $R_m = \frac{1.5}{0.005} = 300\ \Omega$ **Ans.**

(*iii*) Instrument current for 40% deflection, $I_m = \frac{40}{100} \times 0.005 = 0.002$ A   assuming $\theta \propto I$

Shunted current, $I_s = I - I_m = 100 - 0.002 = 99.998$ A

PD applied across the instrument, $V = I_s R_s = 99.998 \times 0.02 = 1.99996$ V

Required instrument resistance, $R = \frac{1.99996}{0.002} = 999.98\ \Omega$ **Ans.**

9.16. DIGITAL VOLTMETERS (DVMs)

The digital voltmeter (DVM) displays measurements of ac or dc voltages as discrete numerals instead of a pointer deflection on a continuous scale as in analog instruments. It is a versatile and accurate instrument that is employed in many laboratory measurement applications. Because of development and perfection of IC modules, the size, power requirements and cost of the digital voltmeter has been drastically reduced and, therefore, DVMs can actively compete with conventional analog instruments, both in price and portability.

The block diagram of a simple digital voltmeter is shown in Fig. 9.16. The unknown voltage signal is fed to the pulse generator which generates a pulse whose width is directly proportional to the input unknown voltage. The output of the pulse generator is applied to one leg of an AND gate. The input signal to the other leg of the AND gate is a train of pulses. The output of the AND gate is, thus, a positive trigger train of duration t seconds and the

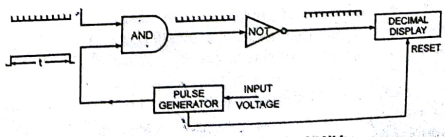


Fig. 9.16 *Block Diagram of DVM*

inverter converts it into a negative trigger train. The counter, counts the number of triggers in *t* seconds which is proportional to the voltage under measurement. Thus, the counter can be calibrated to indicate voltage in volts directly.

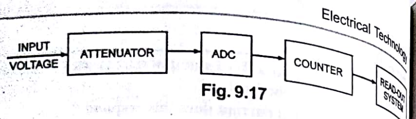


Fig. 9.17

Thus, we see that the DVM described above is an ADC which converts an analog signal into a train of pulses, the number of which is proportional to the input voltage. So a digital voltmeter can be made by using any one of the A/D conversion methods and can be represented by a block diagram shown in Fig. 9.17. Thus the DVMs can be classified on the basis of ADCs used.

The input range of the DVM may vary form ± 1.00000 V to ±1,000.00 V and its limiting accuracy is as high as ± 0.005 per cent of the reading. Its resolution may be 1 part in 10^6, giving 1 μV reading of the 1 V input range. It has a high input resistance of the order of 10 MΩ and input capacitance of the order of 40 pF.

9.17. DIGITAL MULTIMETERS

Digital multimeter (DMM) is basically a digital voltmeter and may be used for the measurement of voltage, current (dc or ac) and resistance. All quantities other than dc voltage are first converted into an equivalent dc voltage by some device.

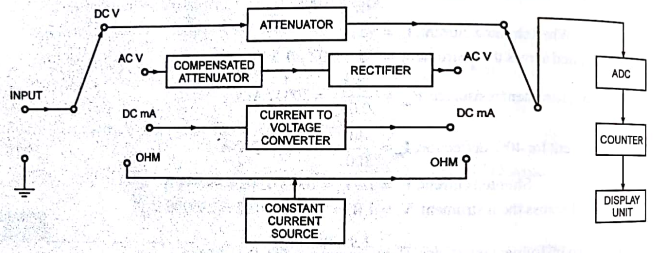


Fig. 9.18 *Block Diagram of Digital Multimeter*

The block diagram of a basic digital multimeter is given in Fig. 9.18.

For measurement of ac voltage the input voltage is converted into a dc voltage by means of a rectifier. A compensated attenuator is employed. Many manufacturers provide the same attenuator for both dc and ac measurements.

For measurement of resistance a constant current, depending on the range, supplied from a battery or constant current source is passed through the resistance under measurement and the voltage developed across it is measured. The resistance value is displayed in ohms.

For measurement of current, the unknown current is passed through a precision resistor in many commercial digital multimeters and the voltage developed across the precision resistor is measured. The current value is displayed in mA.

For measurement of current, a current-to-voltage converter may also be used, as illustrated by block diagram in Fig. 9.19.

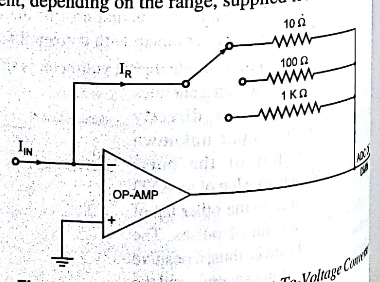


Fig. 9.19 *Block Diagram of Current-To-Voltage Converter*

The current under measurement is applied to the summing junction at the input of the op-amp. The current in the feedback resistor I_R is equal to the input current I_{IN} because of very high input impedance of the op-amp. The current I_R causes a voltage drop across one of the resistors, which is proportional to the input current I_{IN}. Different resistors are employed for different ranges.

The performance of a digital multimeter largely depends on the A/D converter. Low-cost hand held multimeters use a single chip $3\frac{1}{2}$-digit ADC whereas high precision $5\frac{1}{2}$ and $6\frac{1}{2}$ digit multimeters make use of microprocessor controlled discrete ADCs. Digital multimeters give very high input impedance, better accuracy and resolution. Such multimeters have autoranging, autopolarity and auto zero facilities.

The hand-held (or pocket type), $3\frac{1}{2}$-digit digital multimeter DMM with auto-range and data hold function is shown in Fig. 9.20. The main features and technical specifications for the DMM shown in Fig. 9.20 are as follows:

$3\frac{1}{2}$-digits LCD maximum reading 1999 and backlight
12 functions for DCV, ACV, DCA, ACA and OHM
Diode check
Auto power off
Display: 1999 counts and digit is 16 mm high
DC voltage: 200 m/2/20/200/600 V
AC voltage: 200 m/2/20/200/600 V
DC current: 200 μ/2000 μ/20 m/200 mA/2 A/10 A
AC current: 200 μ/2000 μ/20 m/200 mA/2 A/10 A
DC current with clamp: 200/2,000A (0.1 mV/0.1 A)
AC current with clamp: 200/2,000 A (0.1 mV/0.1 A)
Resistance: 200/2 k/20 k/200 k/2 M/20 MΩ
Temperature: 20 to 1,000 degrees C/0 to 1,800 degrees F
Battery test: 1.5, 3 and 9 V (test current approximately 60 mA)
Continuity: Yes
Diode check: Yes


Fig. 9.20 *$3\frac{1}{2}$-Digit DMM With Auto-range and Data-hold Function*

The central knob (function switch) has many positions and can be switched to any position which is required for the measurement to be carried out. For example, if the meter is switched to 200 V DC, then 200 V is the maximum voltage that can be measured. This is sometimes known as 200 V FSD (full-scale deflection).

A minus sign also appears in front of the numerical display, when required, to indicate a negative measured quantity.

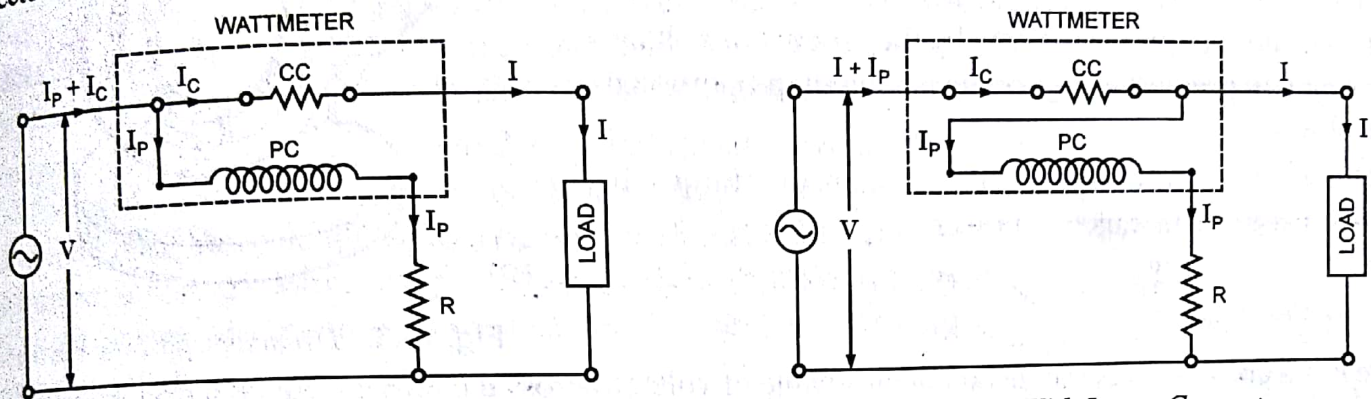
For multifunction instruments, it is very important to use correct terminals and to set the function switch correctly for the meter desired application. Incorrect connection and function selection may cause damage to the circuit under test and to the instrument.

HIGHLIGHTS

1. Ammeters and voltmeters, except electrostatic voltmeters, operate on the same principle. The ammeter carries the current to be measured or a definite fraction of it and this current or its definite fraction produces the deflecting torque whereas voltmeter carries the current proportional to the voltage to be measured which produces the deflecting torque.
2. Instruments may be dc, ac or dc/ac type instruments.

10.2. WATTMETERS

A wattmeter is essentially an inherent combination of an ammeter and a voltmeter and, therefore, consists of two coils known as *current coil* and *pressure coil*. The operating torque is produced due to interaction of fluxes on account of currents in current and pressure coils. The current coil is inserted in series with the line carrying current to be measured and the pressure coil in series with a high non-inductive resistance R is connected across the load or supply terminals, as shown in Fig. 10.2.



(*a*) *Circuit With Small Currents* (*b*) *Circuit With Large Currents*

Fig. 10.2 *Wattmeter Connections*

The wattmeter gives reading which is proportional to current flowing through its current coil, pd across potential coil and cosine of the phase angle between voltage and current. The wattmeter indicates the power lost either in CC or in PC in addition to load power. Normally the power lost in CC or in PC is very small as compared with that measured and, therefore, can be neglected.

There are two methods of connecting wattmeters in the circuit for measurement of power, as shown in Fig. 10.2. Circuit shown in Fig. 10.2 (*a*) is used for measurement of power in circuits carrying small current while the circuit shown in Fig. 10.2 (*b*) is used for measurement of power in circuits carrying large currents.

There are four types of wattmeters namely (*i*) dynamometer type wattmeters (*ii*) induction type wattmeters (*iii*) electrostatic type wattmeters and (*iv*) thermal type wattmeters. Of these first two *i.e.*, the dynamometer type and induction type are most commonly employed. The third one *i.e.*, electrostatic type is a standard wattmeter and it is very useful for the measurement of power of small magnitude, particularly when the voltage is high and power factor is low.

The switchboard and portable wattmeters are of the direct indicating type with pivoted moving elements, whereas the standard wattmeters are of indirect reading type having a torsion head suspension.

10.3. DYNAMOMETER TYPE WATTMETER

This instrument is similar in design and principle to the dynamometer type ammeter and voltmeter already described in Chapter 9. When the instrument of this type is used as a wattmeter, the fixed coil, which is divided into two equal portions in order to provide uniform field, is employed as current coil and the moving coil is used as a pressure coil *i.e.*, the fixed coil carries the current flowing through the circuit and the moving coil carries the current proportional to the voltage across the circuit. A high non-inductive resistance is connected in series with the moving coil in order to limit the current in it. The magnetic fields of the fixed and moving coils react on one another causing the moving coil to turn about its axis. The movement is controlled by hair springs which also lead the current into and out of the moving element. Damping is provided by light aluminium vanes moving in an air dash pot. Electromagnetic or eddy current damping cannot be used as introduction of a permanent magnet, required for damping purpose, will greatly distort the weak working magnetic field. The knife edge pointer is fixed to the moving coil spindle and moves over a suitably calibrated mirror type scale.

Theory. Let v by the supply voltage, i the load current and R the total resistance of the moving coil circuit.

Current through fixed coil, $i_f = i$

Current through moving coils, $i_m = \frac{v}{R}$

Deflecting torque, $T_d \propto i_f i_m \propto \frac{iv}{R}$

In a dc circuit, power is given by the product of voltage and current, hence deflecting torque is directly proportional to the power.

However, in ac circuits, the instantaneous torque is proportional to instantaneous power

i.e., $T_{instantaneous} \propto vi$

or $T_{instantaneous} = kvi$

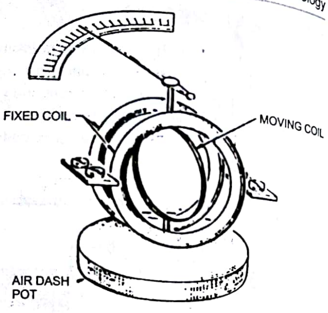


Fig. 10.3 *Dynamometer Type Wattmeter*

where k is a constant, v is the instantaneous value of voltage across the moving coil and i is the instantaneous value of current flowing through fixed coils.

Because of large time constant of the moving system, it cannot follow the rapid variation of the torque having frequency double of the voltage and the instrument takes up a position at which the average deflecting torque is balanced by the controlling torque.

Average deflecting torque is given by

$T_{av} \propto$ Average value of vi

$$\frac{1}{2\pi}\int_o^{2\pi} V_{max} \sin\theta \times I_{max} \sin(\theta - \phi)\, d\theta$$

Taking $v = V_{max} \sin\theta$ and $i = I_{max} \sin(\theta - \phi)$

$$\propto \frac{V_{max} I_{max}}{2\pi}\int_o^{2\pi} \sin\theta \sin(\theta - \phi)\, d\theta$$

$$\propto \frac{V_{max} I_{max}}{2\pi}\int_o^{2\pi} \frac{\cos\phi - \cos(2\theta - \phi)}{2} d\theta$$

$$\propto \frac{V_{max} I_{max}}{4\pi}\left[\theta\cos\phi - \frac{\sin(2\theta - \phi)}{2}\right]_0^{2\pi}$$

$$\propto \frac{V_{max} I_{max}}{4\pi} \times 2\pi\cos\phi \propto \frac{V_{max} I_{max}}{2}\cos\phi \propto \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}}\cos\phi \propto VI\cos\phi$$

where V and I are the rms values.

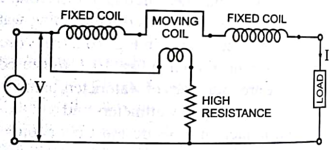


Fig. 10.4 *Connection Diagram of Wattmeter For Measurement of Power in AC Circuits*

Thus $T_d \propto VI\cos\phi \propto$ True power.

Thus an electrodynamic instrument, when connected as shown in Fig. 10.4, indicates the average power, irrespective of the fact it is connected in an ac or a dc circuit.

The scale of the dynamometer type wattmeter is more or less uniform, because its deflection is proportional to the average power and for spring control torque is proportional to deflection, therefore, $\theta \propto$ power.

The dynamometer wattmeter has four external terminals to which connections must be made in order to measure power. Two of them are designated as the *voltage terminals* and the other two as *the current terminals*. The current terminals provide connections to the fixed coils, while the voltage terminals provide connections to the moving coil branch. One terminal of each type is marked ±. It is necessary to connect the ± current terminal and the ± voltage terminal to the same wire of the incoming supply line. In that way the fixed coils and the moving coil will be at about the same potential (because most of the voltage across the voltage branch is dropped by series resistor, being of resistance much larger than that of the voltage or potential coil). Then no electric field will exist between the fixed and the moving coils. An electric field would arise between the potential and the current coils if they were at different potentials. The force of attraction due to the field could slightly restrict the movement of the moving coil and give an erroneous reading.

The meter will always read up-scale when the instrument is correctly connected in the circuit in which the power is to be measured. If for any reason (as in the two wattmeter method of measuring 3-phase power), the meter should read backward, the current coil connections (not the potential coil connections) should be reversed.

The overall errors in commercially manufactured dynamometer instruments lie between ± 0.1 and ± 0.5 per cent when operated between their specified frequencies. The high accuracy instruments are used as laboratory standards of power.

The wattmeter is rated in terms of its maximum current, voltage and power. Each of these ratings must be observed to prevent damage to the instrument. Excess current could harm the current coil (CC) and their insulation. Excess voltage could cause the potential coil (PC) branch to suffer in a similar fashion. In low-power factor circuits, either of these limits could be exceeded without exceeding the wattage rating.

Ranges. (*i*) Current circuit 0.25 to 200 A without employing CTs.

(*ii*) Potential circuit 5 to 750 V without employing PTs.

Advantages. (*i*) Since deflecting torque is proportional to true power in both the cases *i.e.*, dc and ac and the instrument is spring controlled, the scale of the instrument is uniform.

(*ii*) high degree of accuracy can be obtained by careful design, hence these are used as standard for calibration purposes.

Disadvantages: (*i*) As there is no iron core in the coils, the field is very weak. Hence a fairly large number of ampere-turns are required for the fixed as well as the moving coil. This makes the moving system heavy, as well as the power loss in the instrument large.

(*ii*) A heavy moving element makes the torque-weight ratio small resulting in large frictional error.

(*iii*) At low power factor the error due to the inductance of pressure coil is very serious unless special features are incorporated to reduce its effect.

(*iv*) The reading of the instrument may be affected by stray field acting on the moving coil. To reduce it magnetic shielding is provided by enclosing the instrument in an iron case.

10.4. MEASUREMENT OF POWER IN THREE PHASE CIRCUITS

10.4.1. Three Wattmeter Method. This method is employed for measurement of power in 3-phase, 4-wire circuits, the connections being as shown in Fig. 10.5 (*a*). As the neutral wire is common to the three phases each wattmeter reads power in its own phase, and the total power of the load circuit is given by the sum of the readings of the three wattmeters

i.e., Total power of load circuit, $P = W_1 + W_2 + W_3$

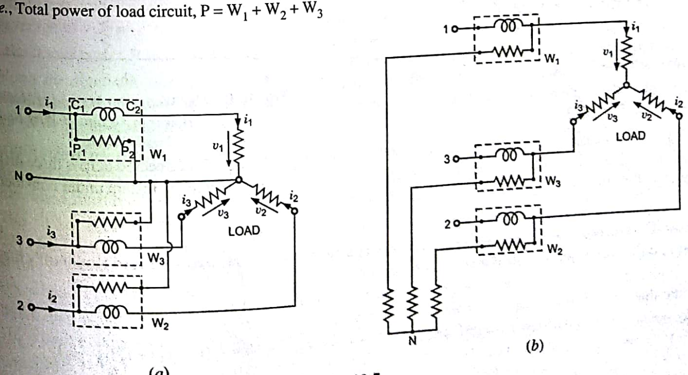
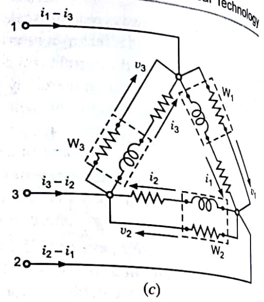


Fig. 10.5

The above method is only useful for measuring power in 3-phase, 4-wire load circuits. In case of 3-phase, 3-wire star-connected circuits difficulty is experienced in getting neutral. In special cases when it is necessary to employ this method for measurement of power in 3-phase, 3-wire circuits an "artificial star" can be formed by connecting three equal high resistances in star to the three line conductors, as shown in Fig. 10.5 (*b*). (In case of low-voltage circuits three potential coils may be connected to form the common star).

In case of delta-connected circuits the difficulty in adopting the above mentioned method for measurement of power is due to the fact that the phase coils are required to be broken for inserting current coils of wattmeters, as shown in Fig. 10.5 (*c*).



(*c*)

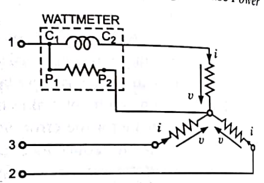
Fig. 10.5 *Three Wattmeter Method of Measuring 3-Phase Power*

10.4.2. One Wattmeter Method. In a balanced 3-wire, 3-phase load circuit the power in each phase is equal and, therefore, total power of the circuit can be determined by multiplying the power measured in any one phase by three.

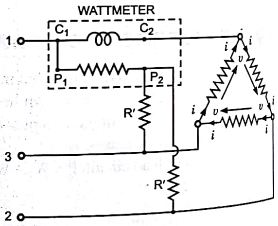
Figure 10.6 (*a*) shows connections for measurement of power in one phase of the 3-phase, 3-wire star-connected balanced load circuit. Total power of the load circuit will be 3 times the reading of wattmeter.

Figure 10.6 (*b*) shows connections for measurement of power in 3-phase, 3-wire delta-connected balanced load circuits by one wattmeter method. In this method of measurement of power, resistance coils of resistance equal to that of pressure coil is connected in each of remaining two phases, as shown in Fig. 10.6 (*b*). The pressure coil and resistance coils form a balanced star connection.

It should be noted that the one wattmeter method has the disadvantage that even a slight degree of unbalance in the loading produce a large error in the measurement.



(*a*)



(*b*)

Fig. 10.6 *One Wattmeter Method of Measuring Power in 3-Phase 3-Wire Balanced Load Circuits*

10.4.3. Two Wattmeter Method. This is the generally used method for measurement of power in 3-phase, 3-wire load circuits. The current coils of two wattmeters are inserted in any two lines and pressure coil is connected from its own current coil to the line without a current coil. The connections are shown in Fig. 10.7.

Let v_1, v_2, v_3 and i_1, i_2, i_3 be the voltages and currents of the three loads connected across three different phases at any particular instant. These being instantaneous values, the power at the instant under consideration is equal to the sum of their products regardless of power factor

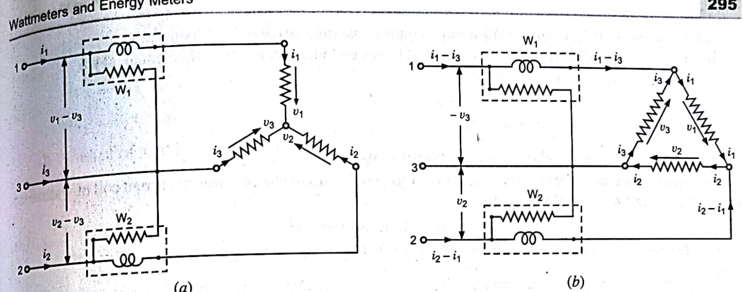
i.e., instantaneous power, $p = v_1 i_1 + v_2 i_2 + v_3 i_3$ watts ...(10.1)

(*a*) **Star-Connected System.** Since all the three phases meet at a star point so according to Kirchhoff's first law the algebraic sum of three instantaneous currents is zero.

i.e., $i_1 + i_2 + i_3 = 0$ or $i_3 = -(i_1 + i_2)$

Substituting $i_3 = -(i_1 + i_2)$ in Eq. (10.1), we get

Instantaneous power, $p = v_1 i_1 + v_2 i_2 - v_3 (i_1 + i_2) = i_1 (v_1 - v_3) + i_2 (v_2 - v_3)$



(*a*) (*b*)

Fig. 10.7 *Two Wattmeter Method of Measuring 3-Phase 3-Wire Power*

Since i_1 is the instantaneous current flowing through the current coil and $(v_1 - v_3)$ is the instantaneous potential difference across pressure coil of wattmeter W_1, $(v_1 - v_3) i_1 = w_1$, the instantaneous power measured by wattmeter W_1.

Similarly i_2 is the instantaneous current flowing through the current coil and $(v_2 - v_3)$ is the instantaneous potential difference across pressure coil of wattmeter W_2, therefore, $(v_2 - v_3) i_2 = w_2$ the instantaneous power measured by wattmeter W_2.

Hence total instantaneous power, $p = w_1 + w_2$

or Total average power, $P = W_1 + W_2$ ...(10.2)

Hence the algebraic sum of two wattmeter readings gives the total power in the 3-phase, 3-wire star-connected load circuits whether the load is balanced or unbalanced.

(*b*) **Delta-Connected System.** In delta-connected system the three phases form a closed loop and according to Kirchhoff's second law.

$$v_1 + v_2 + v_3 = 0 \text{ or } v_1 = -(v_2 + v_3)$$

Instantaneous power, $p = v_1 i_1 + v_2 i_2 + v_3 i_3 = -(v_2 + v_3) i_1 + v_2 i_2 + v_3 i_3 = -v_3 (i_1 - i_3) + v_2 (i_2 - i_1)$ ...(10.3)

Since $-v_3$ is the instantaneous pd across pressure coil and $(i_1 - i_3)$ is the instantaneous current flowing through current coil of wattmeter W_1, wattmeter W_1 reads average of $-v_3 (i_1 - i_3)$ and similarly wattmeter W_2 reads average of $v_2 (i_2 - i_1)$.

Hence total power, $P = W_1 + W_2$ ...(10.4)

Hence the algebraic sum of two wattmeter readings gives the total power of the circuit irrespective of the fact that the circuit is balanced or unbalanced and star-connected or delta-connected.

Determination of Power Factor From Wattmeter Readings. If load is balanced then pf of the load can also be determined from the wattmeter readings.

The phasor diagram for a balanced star-connected inductive load is shown in Fig. 10.8. Let V_1, V_2, and V_3 be the rms values of phase voltages and I_1, I_2 and I_3 be the rms values of phase currents.

Since load is balanced,

(*i*) Phase voltages V_1, V_2 and V_3 will be equal (say, equal to V_P)

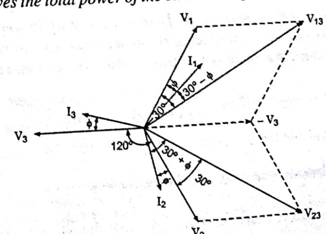


Fig. 10.8 *Phasor Diagram For Balanced Star-connected Inductive Load*

(*ii*) Phase currents I_1, I_2 and I_3 will be equal (say, equal to I_P or I_L)

(*iii*) Phase angles between respective phase voltages and phase currents will be equal, say ϕ

The current in current coil of wattmeter W_1

$$= I_1 = I_L \text{ lagging behind } V_1 \text{ by } \phi.$$

The pd across pressure coil of wattmeter W_1

$$= V_{13} = \sqrt{3}\ V_P = V_L \text{ lagging behind } V_1 \text{ by } 30°.$$

Therefore, phase angle between voltage across potential coil and current through current coil of wattmeter W_1 is $(30° - \phi)$ and reading of wattmeter

$$W_1 = V_L I_L \cos(30° - \phi) \quad ...(10.5)$$

The current in current coil of wattmeter $W_2 = I_2 = I_L$ lagging behind V_2 by ϕ.

The pd across pressure coil of wattmeter $W_2 = V_{23} = \sqrt{3}\ V_P = V_L$ leading V_2 by 30°.

Therefore, phase angle between pd across potential coil and current through current coil of wattmeter W_2 is $(30° + \phi)$.

Hence reading of wattmeter $W_2 = V_L I_L \cos(30° + \phi)$ ...(10.6)

The sum of two wattmeter readings

$$W_1 + W_2 = V_L I_L \cos(30° - \phi) + V_L I_L \cos(30° + \phi)$$
$$= V_L I_L \times 2\cos 30° \cos\phi$$
$$= \sqrt{3}\ V_L I_L \cos\phi = \text{True power of load} \quad ...(10.7)$$

and
$$W_1 - W_2 = V_L I_L \cos(30° - \phi) - V_L I_L \cos(30° + \phi)$$
$$= V_L I_L \times 2\sin 30° \sin\phi = V_L I_L \sin\phi \quad ...(10.8)$$

Dividing Eq. (10.8) by Eq. (10.7), we get

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{\operatorname{Tan}\phi}{\sqrt{3}}$$

$$\text{or } \phi = \operatorname{Tan}^{-1}\left(\frac{W_1 - W_2}{W_1 + W_2}\right) \times \sqrt{3}$$

and pf of load circuit,
$$\cos\phi = \operatorname{Cos}\tan^{-1}\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2} \quad ...(10.9)$$

Hence phase angle ϕ and pf, $\cos\phi$ can be determined from readings of two wattmeters.

Alternative Method of Determination of Power Factor From Wattmeter Readings. This is convenient method for determining the power factor from wattmeter readings for a balanced load. In this method of determination of power factor from the wattmeter readings, for various values of phase angles the ratio of two wattmeter readings, $\frac{\operatorname{Cos} 30° + \phi}{\operatorname{Cos} 30° - \phi}\left(i.e., \frac{\text{smaller reading}}{\text{larger reading}}\right)$ and the corresponding power factor values are determined and plotted as shown in Fig. 10.9. Now by using the watt-ratio curve (Fig. 10.9) the power factor is read directly, the ratio, $\frac{W_2}{W_1}$ being known.

It is seen that, when $\frac{W_2}{W_1} = 1.0$, the power factor is unity; when $\frac{W_2}{W_1} = 0$, the power factor is 0.5; when $\frac{W_2}{W_1}$ is negative *i.e.*, when it becomes necessary to reverse W_2, the power factor is less than 0.5.

Variation in Wattmeter Readings. Wattmeter readings for inductive loads

$$W_1 = V_L I_L \cos(30° - \phi)$$
$$W_2 = V_L I_L \cos(30° + \phi)$$

(*i*) When pf of load is unity *i.e.* $\phi = 0$ then

$$W_1 = V_L I_L \cos 30° = \frac{\sqrt{3}}{2} V_L I_L = \text{Half of total power}$$
$$W_2 = V_L I_L \cos 30° = \frac{\sqrt{3}}{2} V_L I_L = \text{Half of total power}$$

Therefore, readings of both wattmeters are same, positive and equal to half the total power of the circuit.

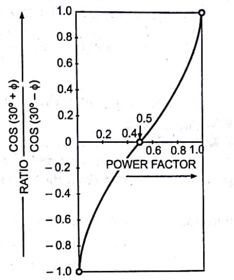


Fig. 10.9 *The Watt-ratio Curve*

(*ii*) When pf of load is 0.5 *i.e.*, $\phi = 60°$ then

$$W_1 = V_L I_L \cos(30° - 60°) = \frac{\sqrt{3}}{2} V_L I_L = \text{Total power}$$
$$W_2 = V_L I_L \cos(30° + 60°) = 0$$

Hence reading of wattmeter W_1 will give total power of the load.

When the pf of load is zero *i.e.*, $\phi = 90°$, then

$$W_1 = V_L I_L \cos(30° - 90°) = \frac{1}{2} V_L I_L$$
$$W_2 = V_L I_L \cos(30° + 90°) = -\frac{1}{2} V_L I_L$$

Hence the readings of two wattmeters are equal but of opposite sign.

So wattmeter W_2 gives –ve reading when phase angle ϕ varies from 60° to 90°. For obtaining the reading of wattmeter W_2 either the connection of current coil or pressure coil should be changed and readings obtained after the reversal of connections should be subtracted from the other wattmeter reading in order to get the total power.

Wattmeter Readings For Capacitive Loads. In the above discussion inductive load has been considered. If the circuit is capacitive the expression for readings of two meters are obtained by substituting in ϕ by $(-\phi)$ in the above expressions for wattmeter readings.

$$W_1 = V_L I_L \cos[30° - (-\phi)] = V_L I_L \cos(30° + \phi)$$
$$W_2 = V_L I_L \cos[30° + (-\phi)] = V_L I_L \cos(30° - \phi)$$

i.e., readings of wattmeters are interchanged.

Example 10.1

In the two wattmeter method of power measurement in a 3-phase circuit, the readings of the wattmeters are 1,200 W and 300 W. What is the pf of the load : Prove the formula used. [*M.D. Univ. Rohtak Electrical Technology May 2003*]

Solution:

$$W_1 = 1{,}200 \text{ watts and } W_2 = 300 \text{ watts}$$

Power factor of load,
$$\cos\phi = \operatorname{Cos}\tan^{-1}\frac{W_1 - W_2}{W_1 + W_2}\sqrt{3} = \operatorname{Cos}\tan^{-1}\frac{1{,}200 - 300}{1{,}200 + 300}\sqrt{3} = 0.6934 \text{ (lag) } \textbf{Ans.}$$

Example 10.2

Two wattmeters connected to measure the total power in a 3-phase balanced circuit. One measures 4,800 W, while the other reads backwards. On reversing the latter it is found to read 400 W. What is the total power and power factor ? Draw the connection diagram and phasor diagram of the circuit.

[*R.G.P.V. Bhopal Basic Electrical Engineering 2001; G.G.S.I.P. Univ. Delhi Second Term Exam. April 2011*]

Solution: Wattmeter reading, $W_1 = 4{,}800$ W

Wattmeter reading $W_2 = -400$ W

As with the two wattmeter method previously discussed, it will be seen that, if the current of the instrument has to be reversed to obtain an upscale reading for either position, that wattmeter reading is subtracted from the other.

It should be noted that, as with other one wattmeter method, the errors in the measurement will be large if the load is at all unbalanced. The normal two wattmeter method is very much preferable.

10.5. ENERGY METERS

The essential difference between an energy meter and a wattmeter is that the former is fitted with some type of registration mechanism whereby all the instantaneous readings of power are summed over a definite period of time whereas the latter indicates the value at particular instant when it is read.

Energy meters are generally of the following three types *viz.* (*i*) electrolytic meters (*ii*) motor meters and (*iii*) clock meters.

Out of above three types of energy meters, the motor meters are the most commonly used meters, where as the clockmeters, because of their complexity, have practically disappeared from use, although they have some application as standards because of their inherent high accuracy. Electrolytic meters are still used to some extent.

10.5.1. Motor Meters. These meters can be used on dc as well as on ac circuits. In principle the motor meter is a small motor of dc or ac type whose instantaneous speed of rotation is proportional to the circuit current in case of an ampere-hour meter and to the power of the circuit in case of a watt-hour meter. The number of revolutions made by the revolving portion (rotor) in any given time is proportional, in case of A- H meter, to the quantity of electricity supplied during that time; and in case of energy meter to the energy supplied. A train of wheels connected to the spindle (rotor) count the number of revolutions made and thus register the AH or kWh directly.

These meters essentially consist of the following parts:

1. *An operating system,* which develops a torque, called the *driving torque*, that causes rotation of the moving system continuously. The driving torque may be compared with the deflecting torque in an indicating instrument.

2. *A braking device* is used to produce a braking torque. It is due to this torque that the moving system of the instrument (armature) rotates at a particular speed for a particular rate of energy consumption at any time. The braking torque is usually produced by a permanent magnet, called the *brake magnet*, that induces current in some part of the moving system which in turn produces the braking torque. Therefore, the braking torque is proportional to the induced currents whereas induced currents are proportional to the speed of the moving system and hence the braking torque is proportional to the speed of the moving system (disc). The moving system attains a steady speed when the braking torque is equal to the driving torque.

3. A device for registering the number of revolutions made by rotating element. This is obtained by having a worm cut on the spindle of the instrument. The worm engages with a pinion and thus drives the train of wheels and registers A-H or W-H directly.

Mechanical Details of Construction. The construction of the meter must be such that the functioning of the meter is not readily interfered with by tilting or any external means. For safety the meters are enclosed in an insulating or metal casing provided with glass windows for the purpose of reading the meter readings. House service meters are so designed that both the terminal block and the remainder of the meter can be sealed off separately in order to prevent access to both the leads to the meter and to the internal working parts. Thus complete protection is provided aganist electric shock by accidental contact with the meter.

Errors Common To All Motor Meters. There are two principal errors common to all motor meters namely friction errors and braking errors.

1. ***Friction Errors.*** The friction error, which is due to friction at the pivots and bearings, plays a considerably more important part than the corresponding error in most indicating instruments, since it continuously operates and affects the speed of the rotating element (disc) for any given value of current or power. The friction torque, which exists when the disc is just starting to rotate, may prevent it from starting at all, if load

is small and will cause its registration to be low on small loads. This part of the friction torque may be assumed to remain constant when the moving system of the meter is in rotation and may be compensated for by providing a small constant driving torque on the moving system independent of load. When the moving system of the meter is rotating normally, a frictional torque proportional to the speed of rotating element also exists, but this torque is not of considerable importance since it merely adds a retarding torque and helps the braking magnet action.

In some meters, such as those of the mercury motor type, a frictional torque which is proportional to the square of the speed also exists and has to be compensated for. In order to reduce the frictional torque to the minimum, the weight of the rotating system should be made as small as possible, the jewels and pivots should be kept well in order and spring loaded bearing should be employed as their use reduces the friction and increases the life of the jewels.

2. ***Braking Errors.*** Variation in braking action affects the speed of the rotating element of the meter, for a given driving torque, and so affects the number of revolutions made in a given time.

The rotating element of the meter attains steady speed when the braking torque, which is proportional to speed, is equal to the driving torque. The braking torque is also proportional to the strength of the brake magnet.

$$\text{The braking torque, } T_b \propto \Phi i R$$

where Φ is the flux of the brake magnet, i is the eddy current induced in the rotating element (disc) due to rotation in the field of the brake magnet and R is the effective radius of the disc from its axis.

Since the induced emf, $e \propto \Phi n$ where n is the speed of rotation.

$$\therefore \text{ Induced current, } i = \frac{e}{r} \text{ where } r \text{ is the resistance of the path of induced current } i.$$

$$\text{or } i \propto \frac{\Phi n}{r} \text{ and braking torque } T_b \propto \Phi^2 \frac{NR}{r}$$

Since at steady speed (say of N) the braking torque is equal to the driving torque T_d,

$$T_d \propto \Phi^2 \frac{NR}{r}$$

$$\text{or } N \propto T_d \frac{r}{\Phi^2 R}$$

From the above equation of steady speed it is obvious that the steady speed attained for a constant driving torque T_d is directly proportional to the resistance of eddy current path and inversely proportional to the square of the flux of the brake magnet. Hence it is necessary that the strength of the brake magnet should remain constant during the use of the meter. Careful design and treatment of brake magnet during its manufacture are necessary in order to ensure this constancy.

With the increase in temperature, the resistance of eddy current path r will increase and, therefore, the braking torque will decrease and cause an error in the instrument registration. Though it is somewhat difficult to compensate completely the reduction in braking torque due to increase in temperature but in some meter driving torque also decreases with the increase in temperature and thus automatically compensates partially.

It is necessary that the resistance of the path of induced current, r should be low, the field produced by the permanent magnet should be strong and the effective radius of the disc large so that the steady speed of the meter be low. The radius of the disc cannot be made unduly large, otherwise the size and weight of the disc will increase and so result in increase in friction torque. An aluminium disc is usually preferred to a copper disc in order to have resistance per unit weight smaller. Strong magnets having large pole shoes and smallest possible air gap from mechanical consideration are used.

The braking torque can be adjusted by adjusting the effective radius R.

There are three types of motor meters, which are common in use, *viz.* mercury motor meter, commutator motor meter and induction motor meter. The first two types are used for dc circuits.