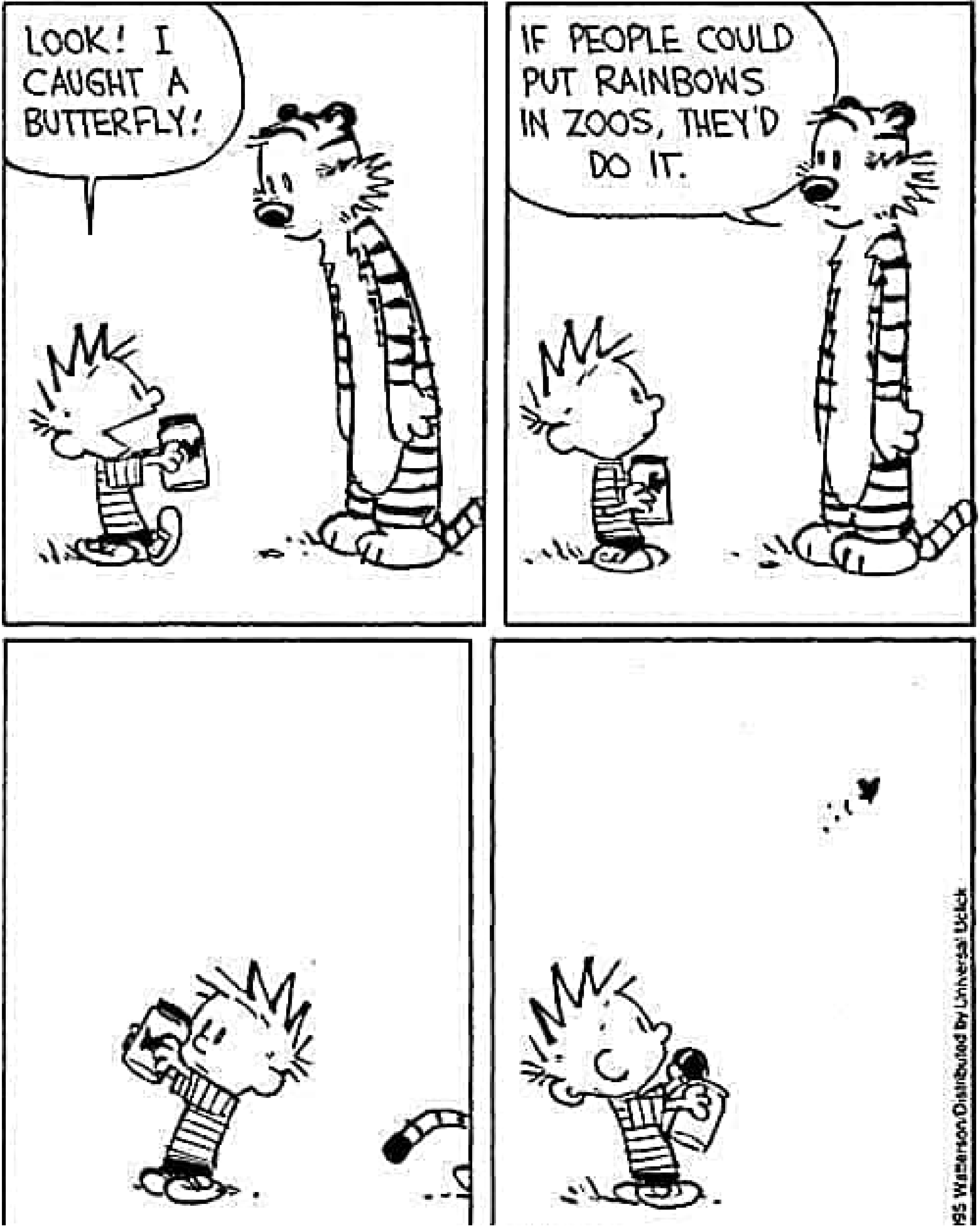

LOOK! I CAUGHT A BUTTERFLY!
IF PEOPLE COULD PUT RAINBOWS IN ZOOS, THEY'D DO IT.
'95 Watterson/Distributed by Universal Uclick

SYLLABUS (2016-17)

CONTROL SYSTEMS
(ETEE–212)

Instructions to Paper Setters:

1. *Question No. 1 should be compulsory and cover the entire syllabus. This question should have objective or short answer type questions. It should be of 25 marks.*
2. *Apart from Question No. 1, rest of the paper shall consist of four units as per the syllabus. Every unit should have two questions. However, student may be asked to attempt only 1 question from each unit. Each question should be of 12.5 marks.*

UNIT I

Control Systems—Basics & Components

Introduction to basic terms, classifications & types of Control Systems, block diagrams & signal flow graphs. Transfer function, determination of transfer function using block diagram reduction techniques and Mason's Gain formula Control System components. Electrical/ Mechanical/Electronic/AC/DC Servo Motors. Stepper Motors, Tacho Generators, Synchros, Magnetic Amplifiers, Servo Amplifiers. **[T1, T2][No. of hrs. 11]**

UNIT II

Time—Domain Analysis

Time domain performance specifications, transient response of first & second order systems, steady state errors and static error constants in unity feedback controls systems, response with P, PI and PID controllers, limitations of time domain analysis.

[T1, T2] [No. of hrs. 10]

UNIT III

Frequency Domain Analysis

Polar and inverse polar plots, frequency domain specifications and performance of LTI systems. Logarithmic plots (Bode plots), gain and phase margins, relative stability. Correlation with time domain performance closes loop frequency responses from open loop response. Limitations of frequency domain analysis, minimum/non-minimum phase systems.

[T1, T2] [No. of hrs. 10]

UNIT IV

Stability & Compensation Techniques

Concepts, absolute, asymptotic, conditional and marginal stability. Routh-Hurwitz and Nyquist stability criterion. Root locus technique and its application.

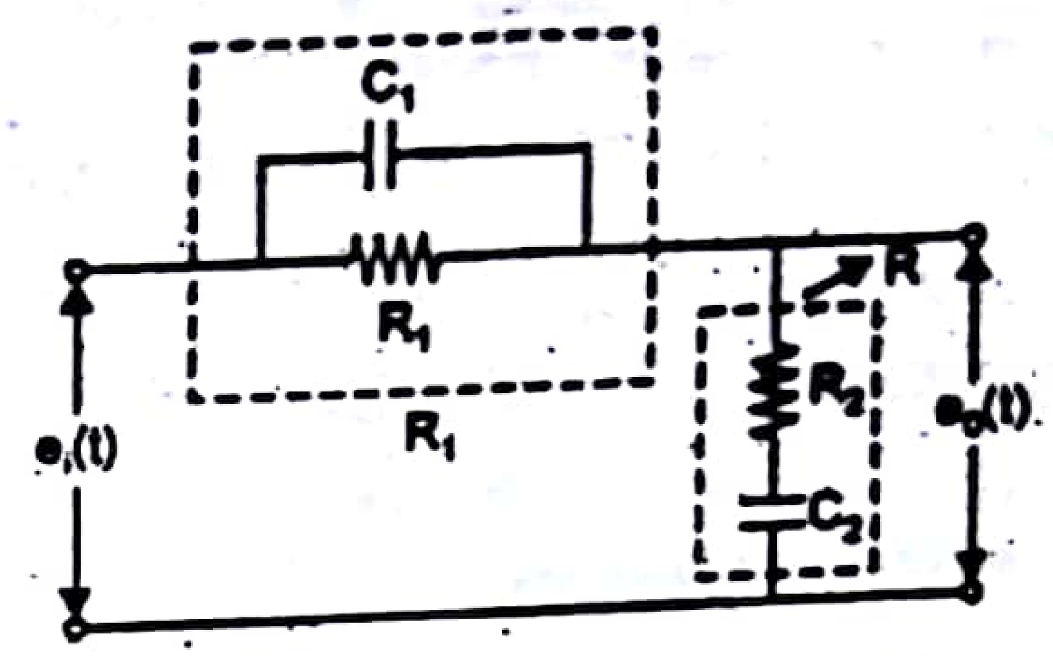
Concept of compensation, series/parallel/series-parallel/feedback compensation. Lag/Lead/ Lag-Lead networks for compensation, compensation using P, PI, PID controllers.

[T1, T2] [No. of hrs. 11]

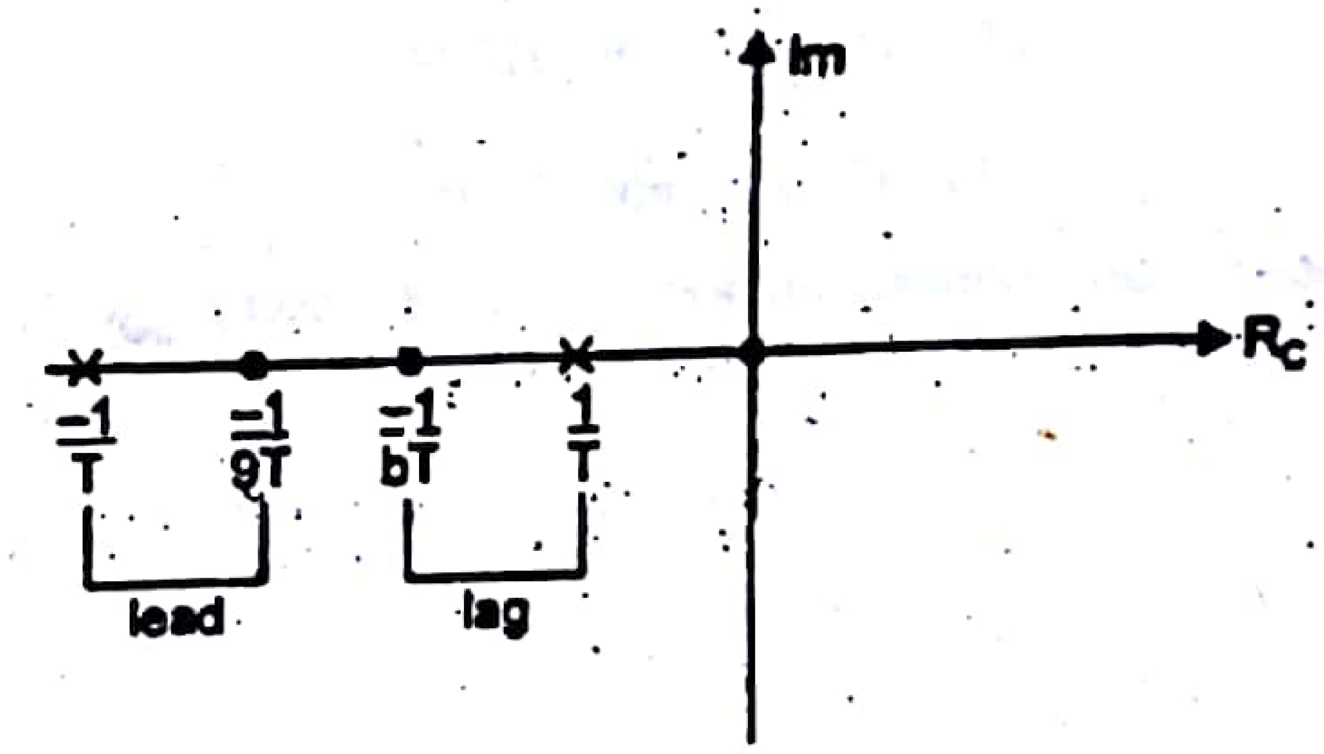
IMPORTANT QUESTION

Q.1. What do you understand by Lag-Lead Compensator.

Ans.

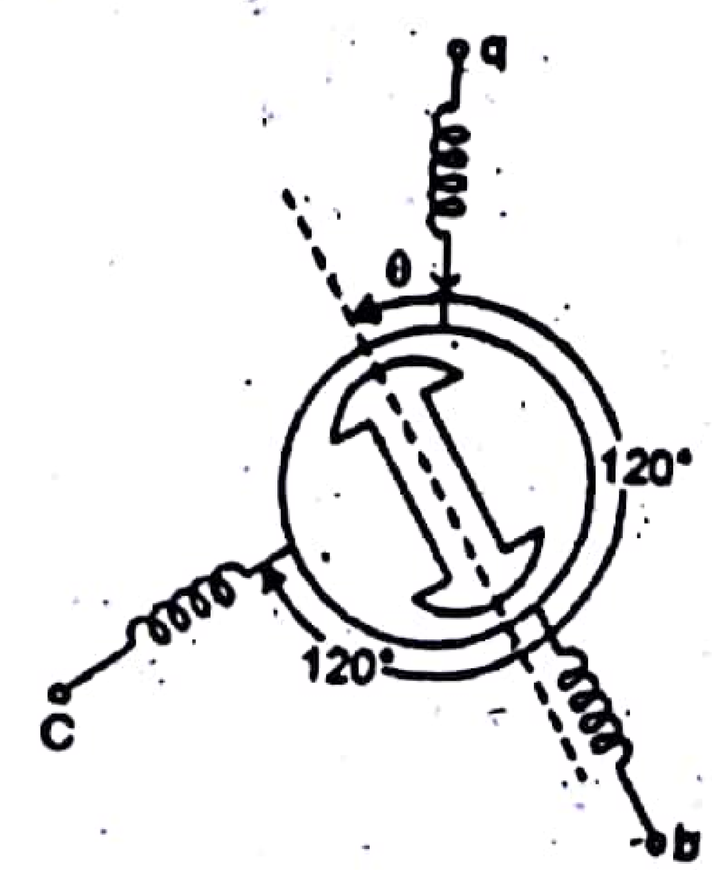


$$\frac{E_0(S)}{E_i(S)} = \frac{(1+R_1C_1S)(1+R_2C_2S)}{1+(R_1C_1+R_2C_2+R_1C_2)S+R_1R_2C_1C_2S_2}$$



Q.2. Explain the constructional detail and working of Synchros. With a ntTat sketch explain how it can be used in position control system. **(6)**

Sol. Synchro-Transmitter: The construction of synchro-transmitter is very similar to that of a three phase alternator. The stator is made of laminated Splicon steel and carries three phase star connected windings. The rotor is a rotating part, dumbell shaped magnet with a single winding.



A single phase a.c. voltage is applied to the rotor through slip rings. Let applied a.c. voltage to the rotor is

$$e_r = E_r \sin \omega t \quad ...(1)$$

due to this applied voltage a magnetizing current will flow in the rotor coil. This magnetizing current produces sinusoidally varying flux and distributed in the air gap. Because of transformer action voltages get induced in all stator coil which is proportional to cosine of angle between stator and rotor coil axes.

Now, Consider the rotor of synchro transmitter is at an angle 0, then voltages in each stator coil with respect to neutral are

$$E_{an} = KE_r \sin w_0 t \cos \theta \quad ...(2)$$

$$E_{bn} = KE_r \sin w_0 t \cos (\theta + 120°) \quad ...(3)$$

$$E_{cn} = KE_r \sin w_0 t \cos (\theta + 240°) \quad ...(4)$$

Magnitudes of stator terminal voltage are

$$E_{cb} = E_{cn} - E_{bn} = KE_r \sin w_0 t[\cos(\theta + 240°) - \cos(\theta + 120°)]$$

$$E_{cb} = KE_r \sin w_0 t[\sqrt{3} \sin \theta] = \sqrt{3} KE_r \sin w_0 t \sin \theta \quad ...(5)$$

Similarly $$E_{ac} = \sqrt{3} KE_r \sin w_0 t \sin(\theta + 120°) \quad ...(6)$$

$$E_{ba} = \sqrt{3} KE_r \sin w_0 t \sin(\theta + 240°) \quad ...(7)$$

When $\theta = 0$ the maximum induced voltage. Will be E_{ax} and E_{cb} will be zero.

Q.2. (b) Draw the Bode diagram of the transfer function $G(s) = \dfrac{200(s+2)}{s(s^3+10s+100)}$ **(5)**

Ans. Bode Plot

$$H(s) = 1$$

$$G(s) = \frac{200(s+2)}{s(s^2+105+100)}$$

Standard form of open loop transfer function

$$G(s)H(s) = \frac{k(1+sT_1)(1+sT_2)}{S^N(1+ST_a)(1+ST_b)(1+sT_c)} \quad ...(1)$$

To make actual open loop function comparable with equation (1)

$$G(s)H(s) = \frac{200 \times 2(s/2+1)}{s(5-8.6j)\left(\frac{s}{5-8.6j}+1\right)(5+8.6j)\left(\frac{8}{5+8.6j}+1\right)}$$

$$= \frac{400(1+s/2)}{98.96s\left(1+\frac{s}{5-8.6j}\right)\left(1+\frac{s}{5+8.6j}\right)}$$

$$= \frac{4.042(1+s/2)}{s\left(1+\frac{s}{5-8.6j}\right)\left(1+\frac{s}{5+8.6j}\right)}$$

Corner frequenies

$$w_1 = 2 \quad \text{(zero)}$$

$$w_2 = 5, 5 \quad \text{(double pole)}$$

It can never be complex.

System type = 1

$\therefore$ Initial slope $= -20 \times 1 = -20$ dB/decate $\quad (\because N = 1)$

This line will pass through

$$w = k^{1/N} \text{ rad/sec} = 4.042 \text{ rad/sec}$$

For Magnitude Graph

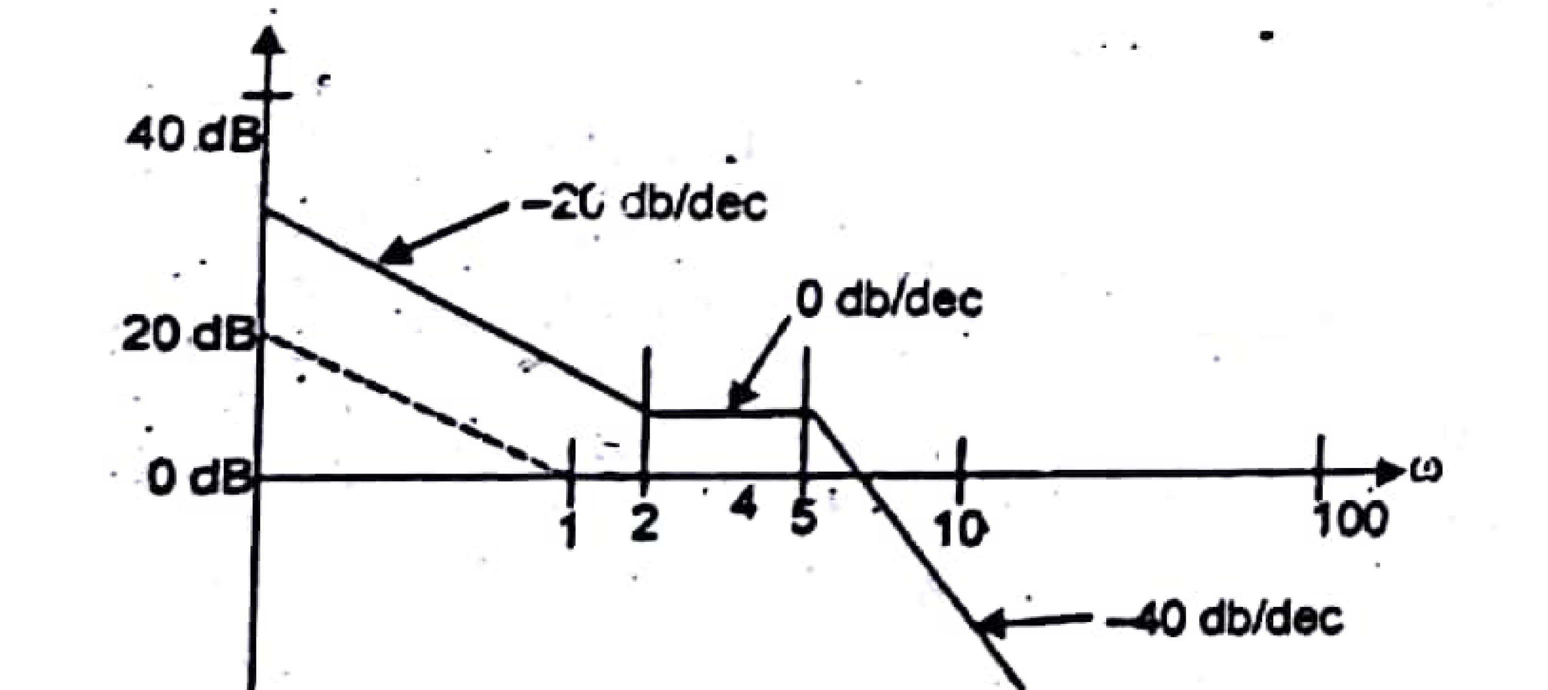
Factor	Corner freq (rad/sec)	Slope (dB/dec)	Angle (degree)
$\dfrac{4.042}{jw}$	—	–20 (initial)	–90°
$1+jw/2$	2	+ 20	$\tan^{-1}(w/2)$
$\dfrac{1}{\left(1+\dfrac{jw}{5-8.6j}\right)}$	5	– 20	$-\tan^{-1}\left(\dfrac{w}{5-8.6j}\right)$
$\dfrac{1}{\left(1+\dfrac{jw}{5+8.6j}\right)}$	5	– 20	$-\tan^{-1}\left(\dfrac{w}{5+8.6j}\right)$

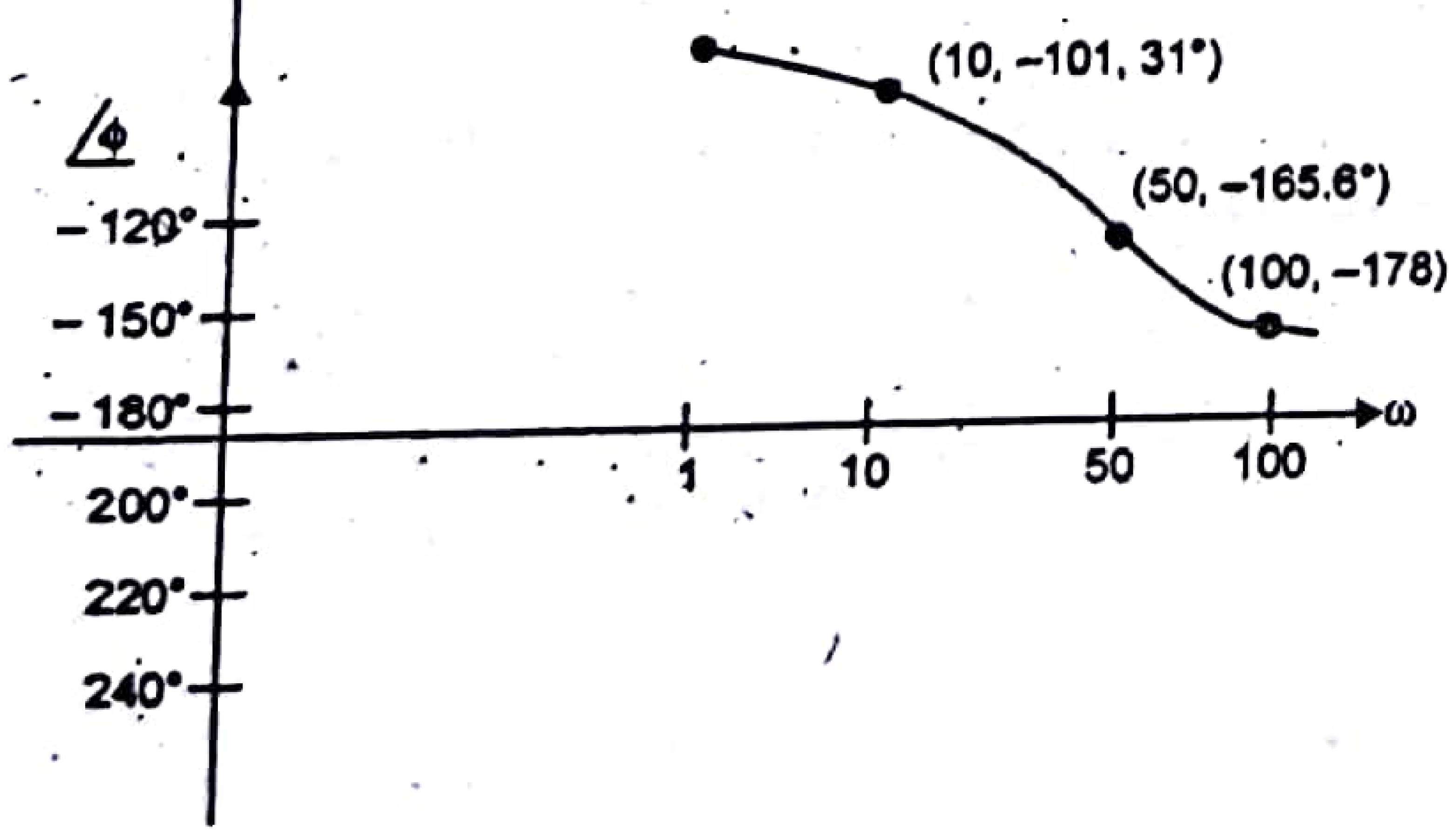
For Phase Graph

Total angle for the system.

$$\theta = -90+\tan^{-1}\left(\frac{w}{2}\right)-\tan^{-1}\left(\frac{w}{5-8.6j}\right)-\tan^{-1}\left(\frac{w}{5+8.6j}\right)$$

w	θ
10	–101.31°
50	–165.68°
100	–178.1°





FIRST TERM EXAMINATION (2015)
FOURTH & FIFTH SEMESTER (B.TECH)
CONTROL SYSTEM (ETEE-212)

Time : 1HR **M.M. : 30**

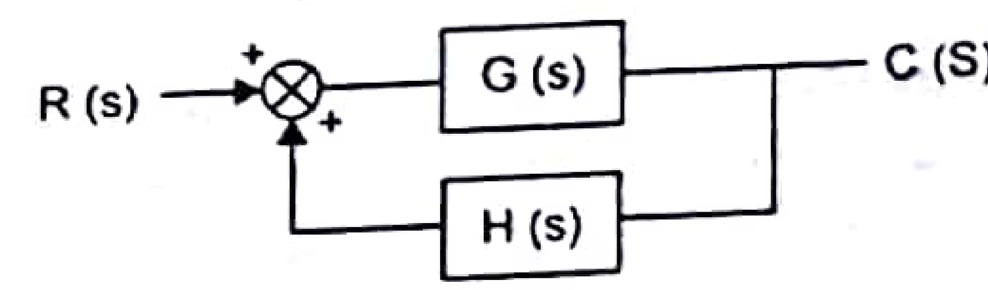
Note: *Attempt Q.No.1 And Any Two More*

Q.1. (a) How feedback control systems classified? **5x2**

Ans: Feedback control system can be classified by the type of feedback signal.

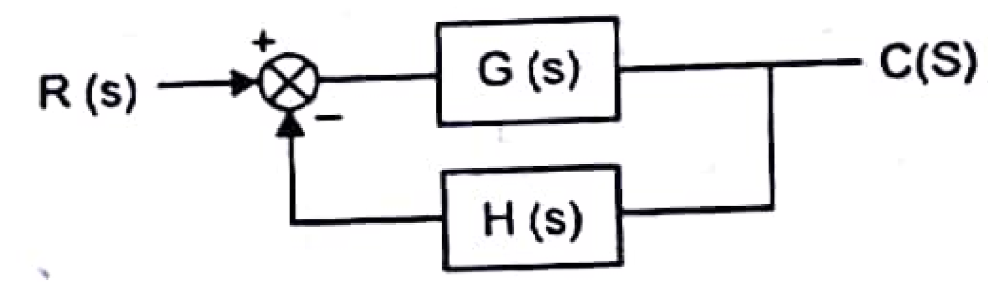
(1) Positive feedback → where H (s) is the feedback gain. Transfer function is

$$T = \frac{G(s)}{1-G(s)H(s)}$$



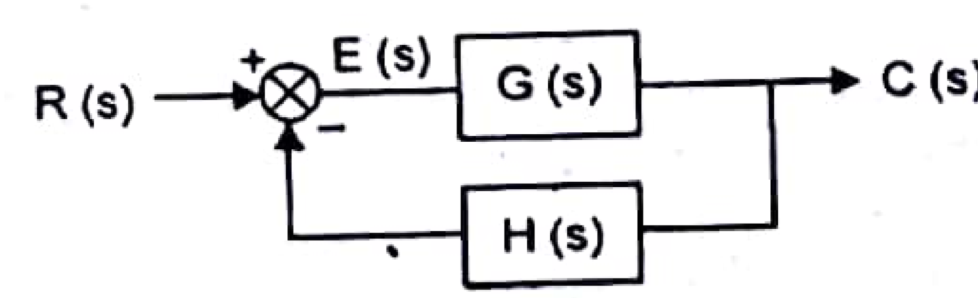
(2) Negative feedback

where
$$T = \frac{G(s)}{1+G(s)H(s)}$$



Q.1. (b) Derive the formula of steady state error for unity feedback?

Ans:



Steady-state error is the difference between the input & output of the system during steady state. For accuracy the steady state error should be minimum.

$$\frac{E(S)}{R(S)} = \frac{1}{1+G(s)H(S)}$$

$$E(S) = \frac{R(s)}{1+G(s)H(s)}$$

steady state error

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s.E(s) \quad \text{\{Final value theorem\}}$$

$$e_{ss} = \lim_{s\to 0} \cdot \frac{sR(s)}{1+G(s)H(s)}$$

for unity feedback system H(s) = 1 (2)

$$e_{ss} = \lim_{s \to 0} s.\frac{G(s)}{1+G(s)}$$

Q.1. (c) Discuss three properties of signal flow graphs.

Ans: (1) Signal flow graph is applicable to linear time-invariant systems.

(2) The signal flow is only along the directions of arrows.

(3) The value of variable at each node is equal to the algebric sum of all signals entering at that node.

(4) The gain of signal flow graph is given by Mason's formula

(5) The signal gets multiplied by the branch gain when it travels along it.

(6) The signal flow graph is not to be the unique property of the system.

Q.1. (*d*) Differentiate between signal flow graphs & block diagrams?

Ans:

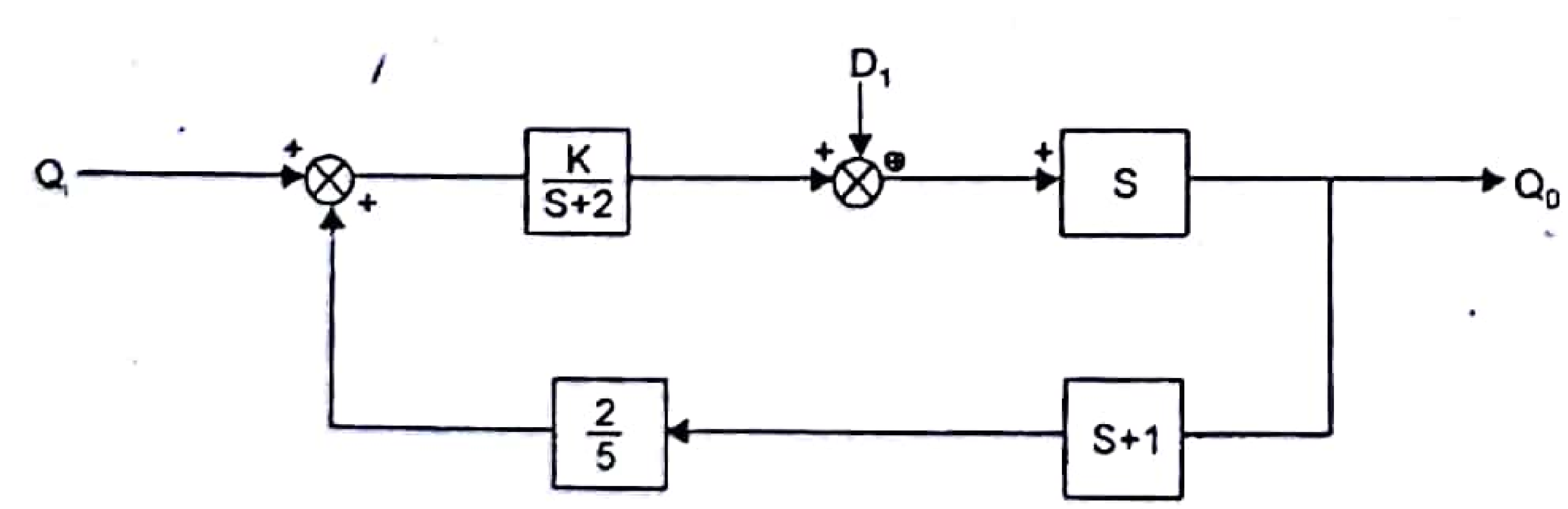
Block Diagram	Signal Flow graph
1. Applicable to linear time invariant systems only	Applicable to linear time invariant system
2. Each element is represented by block.	Each variable is represented by node.
3. Summing point & take off points are seperate	Summing & take off points are absent.
4. Self loop do not exist.	Self loop can exist.
5. It is time consming method.	Require less time by using Mason's gain formula.
6. Transfer function of the element is shown inside the corresponding block	Transfer function is shown along branches connecting the nodes
7. Feedback path is present	Feedback loops are used.

Q.1. (*e*) Discuss the use of steeper motor in closed loop mode?

Ans. In Stepper motors, the movement of the rotor is in discrete steps. Stepper motors are used in computer peripheral system such as printers, tape drives e.t.c. Stepper motor is as similer as that of servo motor. Stepper motor can be used as feedback sensor for angular displacement or angular position. with each input into the stepper motor, there is an angular displacement.

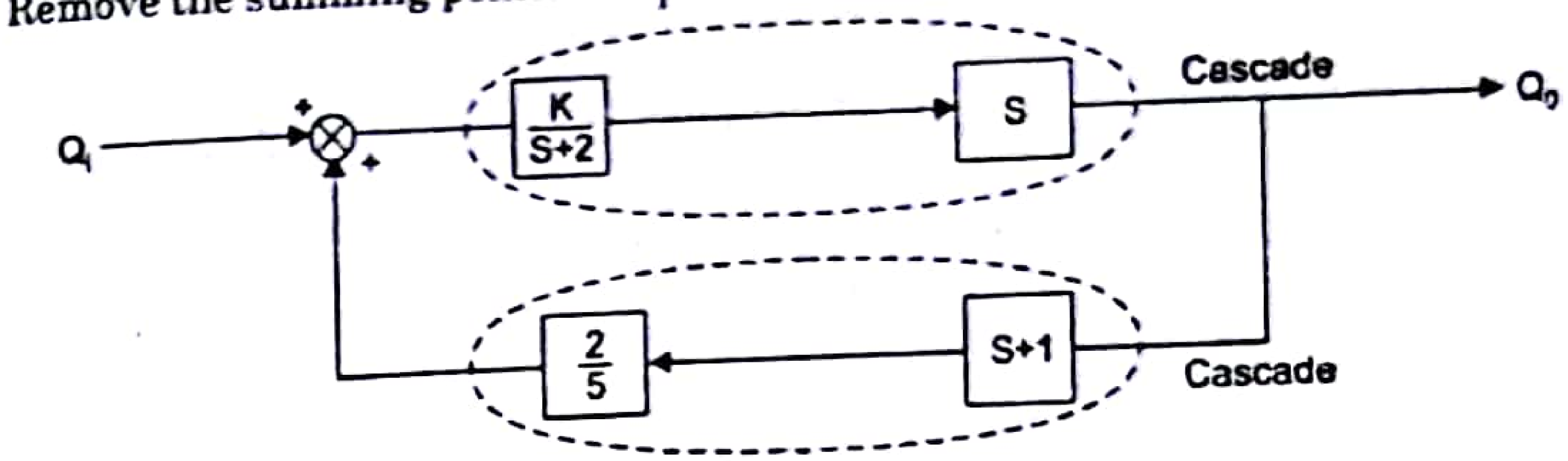
Q.2. (*a*) Find the Overall Transfer function of the system shown in following figure 6

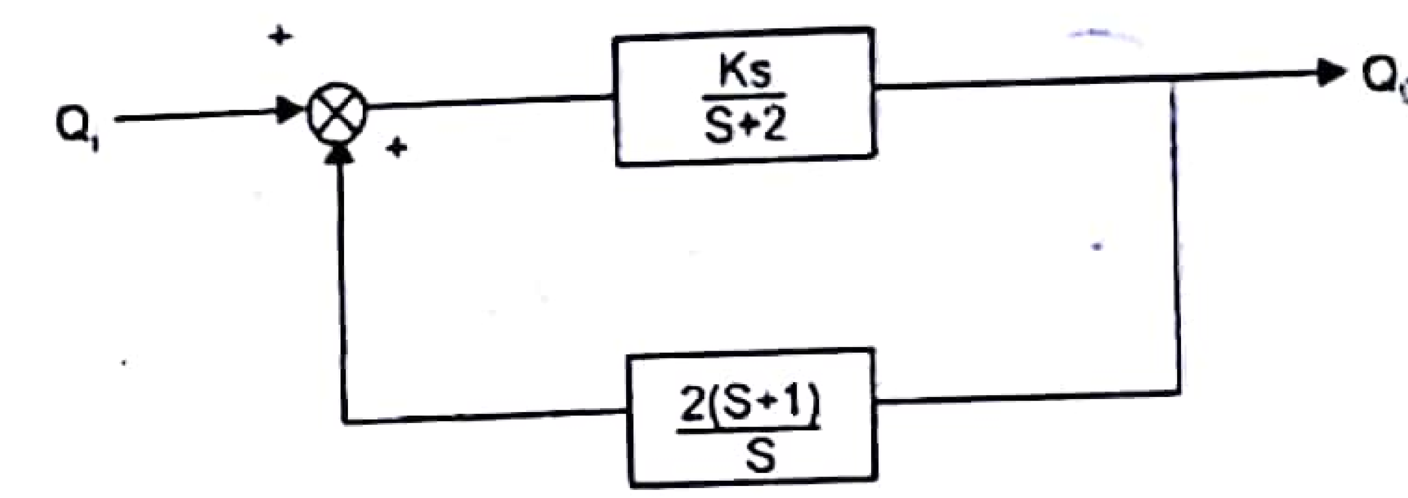
Ans:



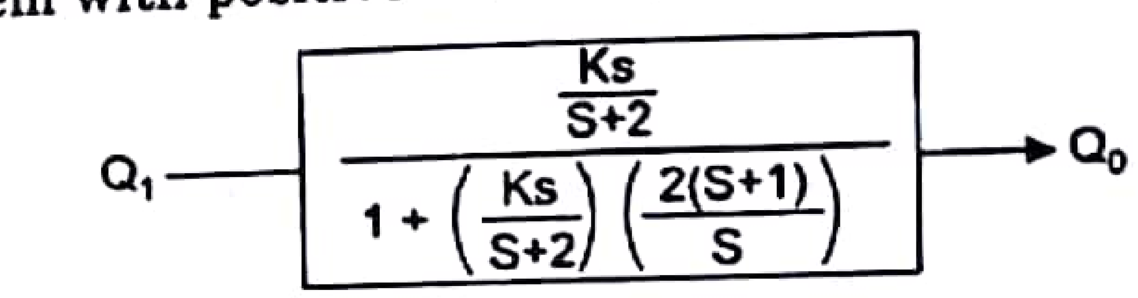
Ans. (*a*) When $D_1 = 0$,

Remove the summing points of D_1





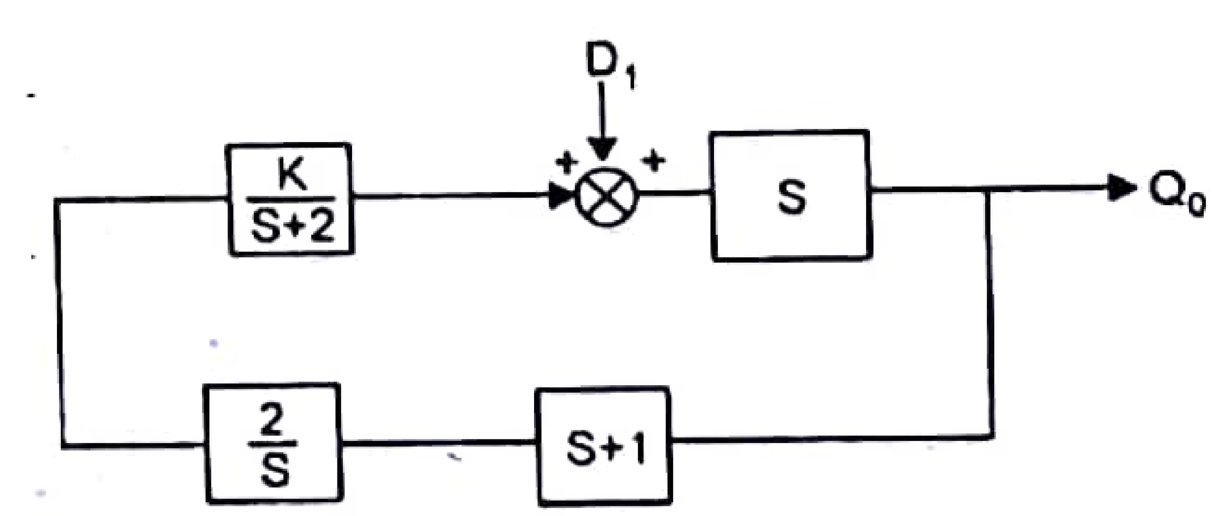
Closed loop system with positive feedback

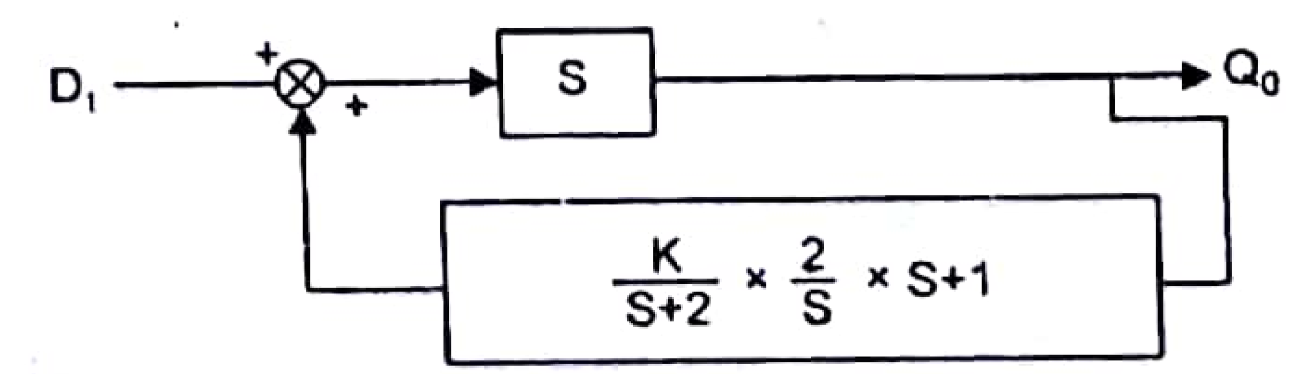


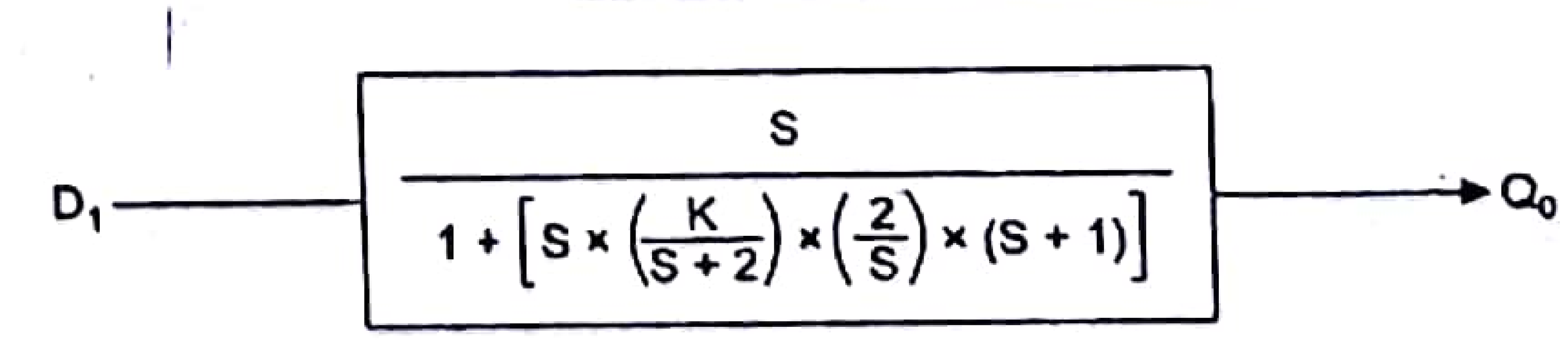
$$\frac{Q_0}{Q_i} = \frac{KS^2}{s^2 + 4s + Ks + 2}$$

(*b*) When $\theta_i = 0$,

Remove the summing point of θ_i and find $\frac{\theta_0}{D_1}$





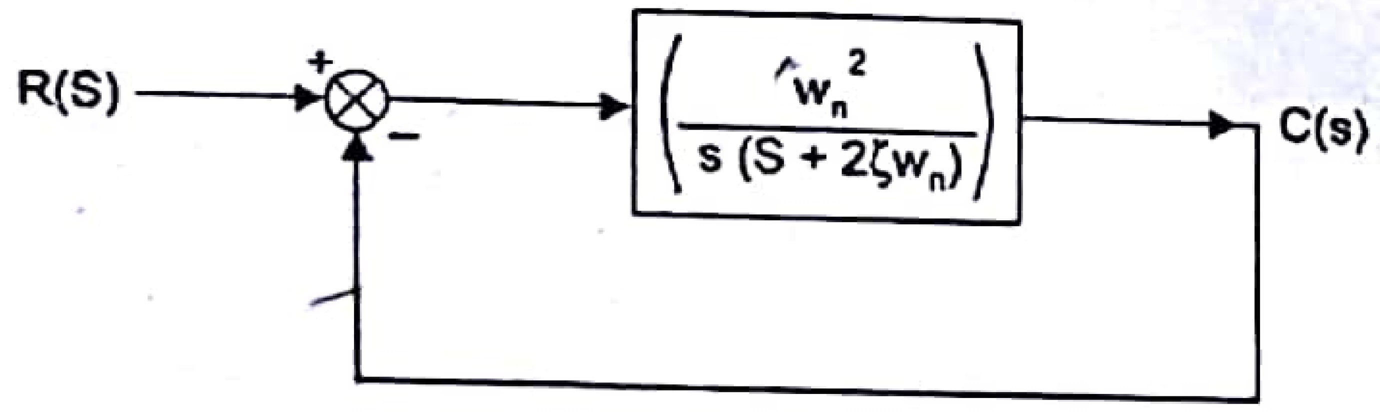


$$\boxed{\frac{Q_0}{D_1} = \frac{s+2}{s^3 + (4+K)s^2 + 6s + 4}}$$

Q.2. (b) Find the step Response of second order system for damping factor equal to 1 (ζ = 1) **4**

Ans:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$



$$H(s) = 1, \zeta = 1$$

$$G(s) = \frac{w_n^2}{s(s+2w_n)}$$

$$\frac{C(s)}{R(s)} = \frac{w_n^2/s(s+2w_n)}{1+\dfrac{w_n^2}{s(s+2w_n)}} = \frac{w_n^2}{s^2+2w_n s+w_n^2}$$

For unit step input, $R(s) = 1/s$

$$C(s) = \frac{1}{s}\cdot\frac{w_n^2}{s^2+2w_n s+w_n^2}$$

$$C(s) = \frac{1}{s}\cdot\frac{w_n^2}{(s+w_n)^2} \qquad (1)$$

Break the equation (1) by partial fraction

$$\frac{w_n^2}{s(s+w_n)^2} = \frac{A}{s}+\frac{B}{(s+w_n)}+\frac{C}{(s+w_n)^2}$$

we get.

$$C = -w_n,$$
$$A = 1,$$
$$B = -1$$

$$\therefore \quad \frac{w_n^2}{s(s+w_n)^2} = \frac{1}{s}-\frac{1}{s+w_n}-\frac{w_n}{(s+w_n)^2}$$

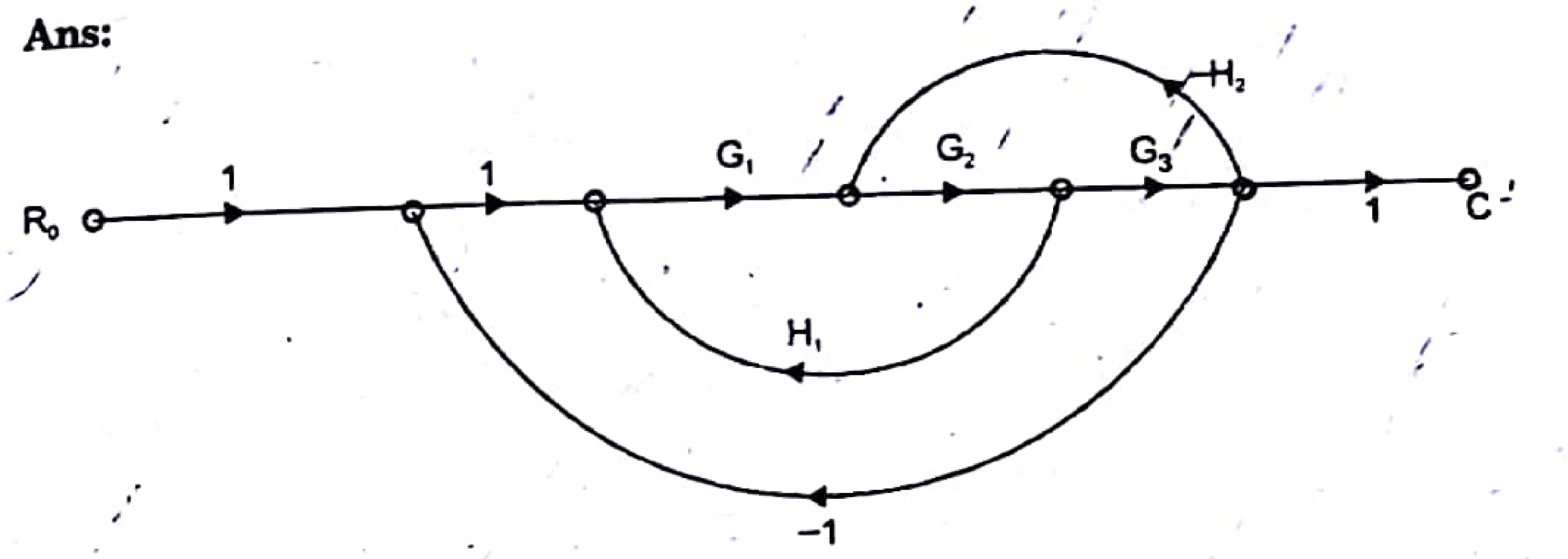
Taking inverse laplace

$$\pounds^{-1}[(C(s)] = \pounds^{-1}(1/s) - \pounds^{-1}\left(\frac{1}{s+w_n}\right) - \pounds^{-1}\left[\frac{w_n}{(s+w_n)^2}\right]$$

$$\boxed{c(t) = 1 - e^{-w_n t} - t w_n e^{-w_n t}}$$

Q.3. (a) Find the transfer function C/R for the following signal flow graph **6**

Ans:



(1) Forward path gain $\quad g_1 = G_1G_2$

(2) Loops

$$L_1 = G_1G_2H_1$$
$$L_2 = -G_2G_3H_2$$
$$L_3 = -G_1G_2G_3$$

(3) 2 Non touching loops ⇒ 0

⇒ Applying Mason's gain formula:

$$T = \frac{\Sigma g_k \Delta_k}{\Delta}$$

$$\therefore \quad T = \frac{g_1\Delta_1}{\Delta}$$

(4)

$$\Delta = 1-[L_1+L_2+L_3]+[0]$$
$$\Delta = 1-[G_1G_2H_1-G_2G_3H_2-G_1G_2G_3]$$
$$\Delta = 1-G_1G_2H_1+G_2G_3H_2+G_1G_2G_3$$

(5) Find Δ_1 of forward path

$$\Delta_1 = 1-0 = 1 \rightarrow \text{all there loops are touching } g_1$$

$$\therefore \textit{Transfer function} = \quad T = \frac{C}{R} = \frac{G_1G_2}{1-G_1G_2H_1+G_2G_3H_2+G_1G_2G_3}$$

Q.3. (b) A unity feedback control system has an open loop transfer function

$$G(S) = \frac{10}{s(s+2)}$$

Find the rise time, percentage overshoot for a step input of 10 units. (4)

Ans:

$$G(s) = \frac{10}{s(s+2)}$$
$$H(s) = 1$$

Characteristics equation $\Rightarrow 1 + G(S)H(S) = 0$

$$1+\frac{10}{s(s+2)} = 0$$

$$1+\frac{10}{s^2+2s} = 0$$

$$s^2+2s+10 = 0$$

compare with

$$s^2+2\zeta\omega_n s+\omega_n^2 = 0$$
$$\omega_n^2 = 10,$$
$$\omega_n = \sqrt{10} = 3.16 \text{ rad/sec}$$
$$2\zeta\omega_n = 2$$
$$2\zeta\times 3.16 = 2$$
$$\zeta = 0.316$$
$$M_P = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}\times 100$$
$$M_P = e^{-\frac{\pi(0.316)}{\sqrt{1-(0.316)^2}}}\times 100$$
$$= e^{-\frac{\pi(0.316)}{0.948}}\times 100$$
$$\boxed{M_P = 35.34\ \%}$$

Rise time

$$t_r = \frac{\pi-\tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}}{\omega_n\sqrt{1-\zeta^2}}$$
$$= \frac{\pi-\tan^{-1}\frac{\sqrt{1-(0.316)^2}}{0.316}}{3.16\sqrt{1-(0.316)^2}}$$
$$\boxed{t_r = 0.506 \text{ sec}}$$

Q.4. Write short note on any two 10

(a) Discuss Construction and principal of Synchros with neat diagram

Ans: A synchro is an electromagnetic transducer which converts the angular position of a shaft into an electric signal.

Synchro Transmitter:- it consists of a stator and rotor. The stator is made of laminated silicon steel and carries three phase star connected windings. The rotor is a rotating part, dumbell shaped magnet with a single winding. A single phase a.c. voltage is applied to the rotor through slip rings.

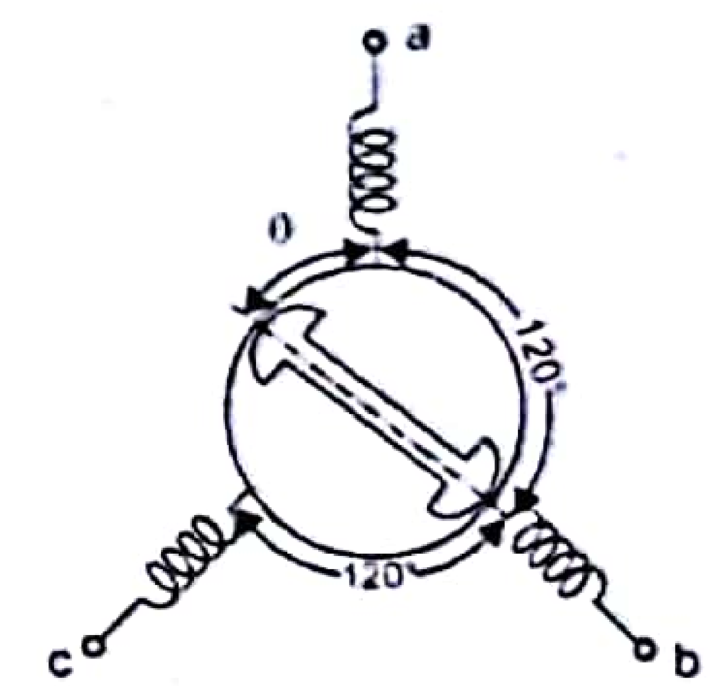
$$e_r = E_r \sin\omega_0 t$$

is the a.c. voltage to the rotor.

Due to this applied voltage a magnetizing current will flow in rotor coil. This magnetizing current produces sinsoidally varying flux & distibuted in the air gap. Because of transformer action voltages get induced in all stator coil which is proportional to cosine of angle between stator & rotor coil axes.

Consider the rotor of synchro transmitter is at an angle θ, then voltages in each stator coil with respect to neutral are:

$$E_{an} = KE_r \sin w_0 t\cos\theta$$
$$E_{bn} = KE_r \sin w_0 t\cos(\theta+120°)$$
$$E_{cn} = KE_r \sin w_0 t\cos(\theta+240°)$$



Magnitudes of stator terminal voltages are

$$E_{cb} = E_{cn} - E_{bn}$$
$$E_{cb} = KE_r \sin w_0 t\left[\sqrt{3}\sin\theta\right] \quad (6)$$
$$E_{cb} = \sqrt{3}k_r \sin w_0 t\sin\theta$$

When $\theta = 0$, the maximum induced voltage will be E_{an} & E_{cb} will be zero. This position of rotor is definced as electrical zero of the transmitter & is used as the refrence for indicating the angular position of the rotor.

Thus, the input to the synchro transmitter is the angular position of the rotor shaft & output are the three single phases voltages which are the function of the shaft position.

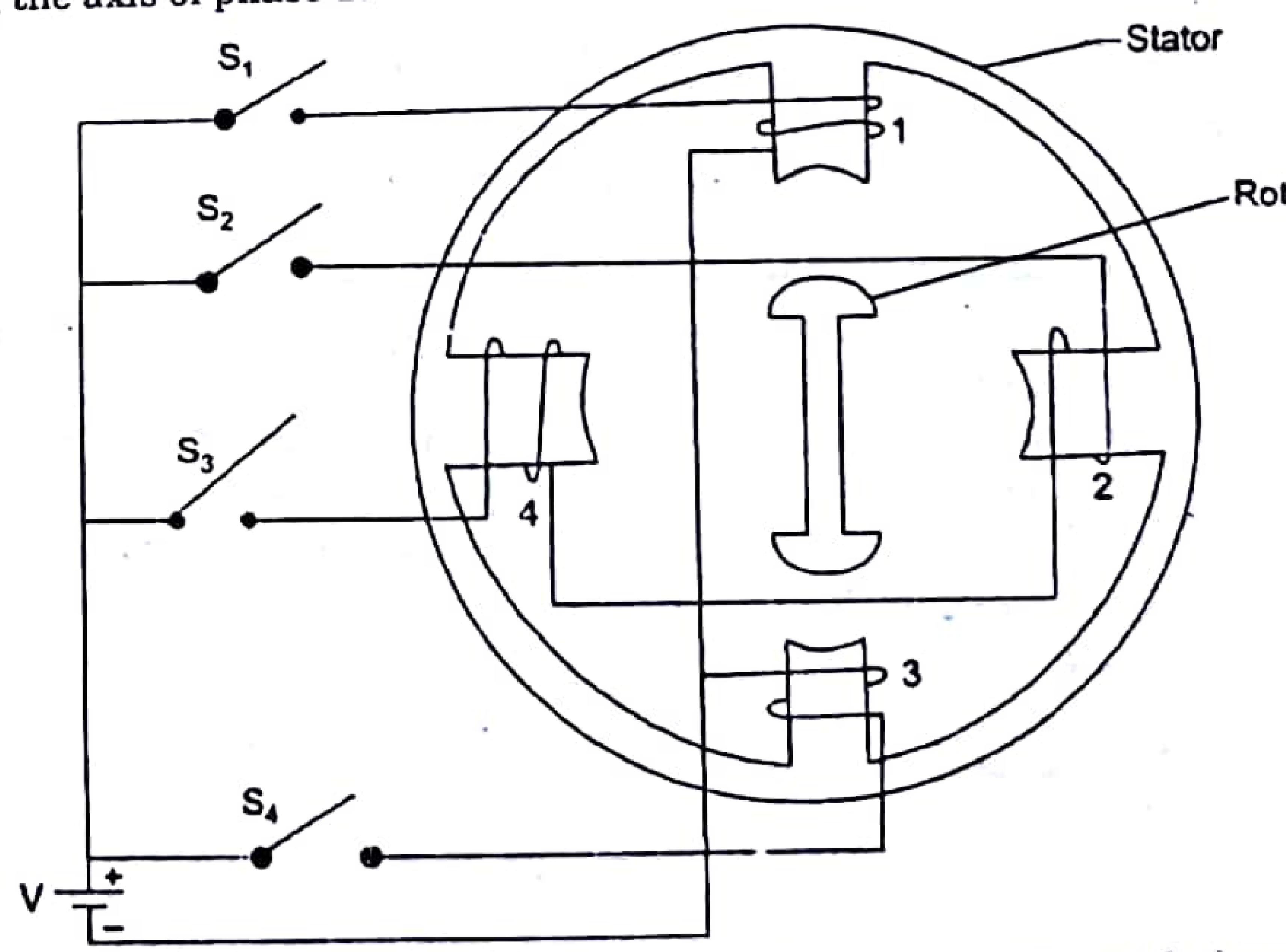
Q.4. (b) Write short note on Variable Reluctance Stepper Motor

Ans: Variable reluctance stepper motors are of two types:-

(1) Single-stack variable reluctance motor

(2) Multi stack variable reluctance motor.

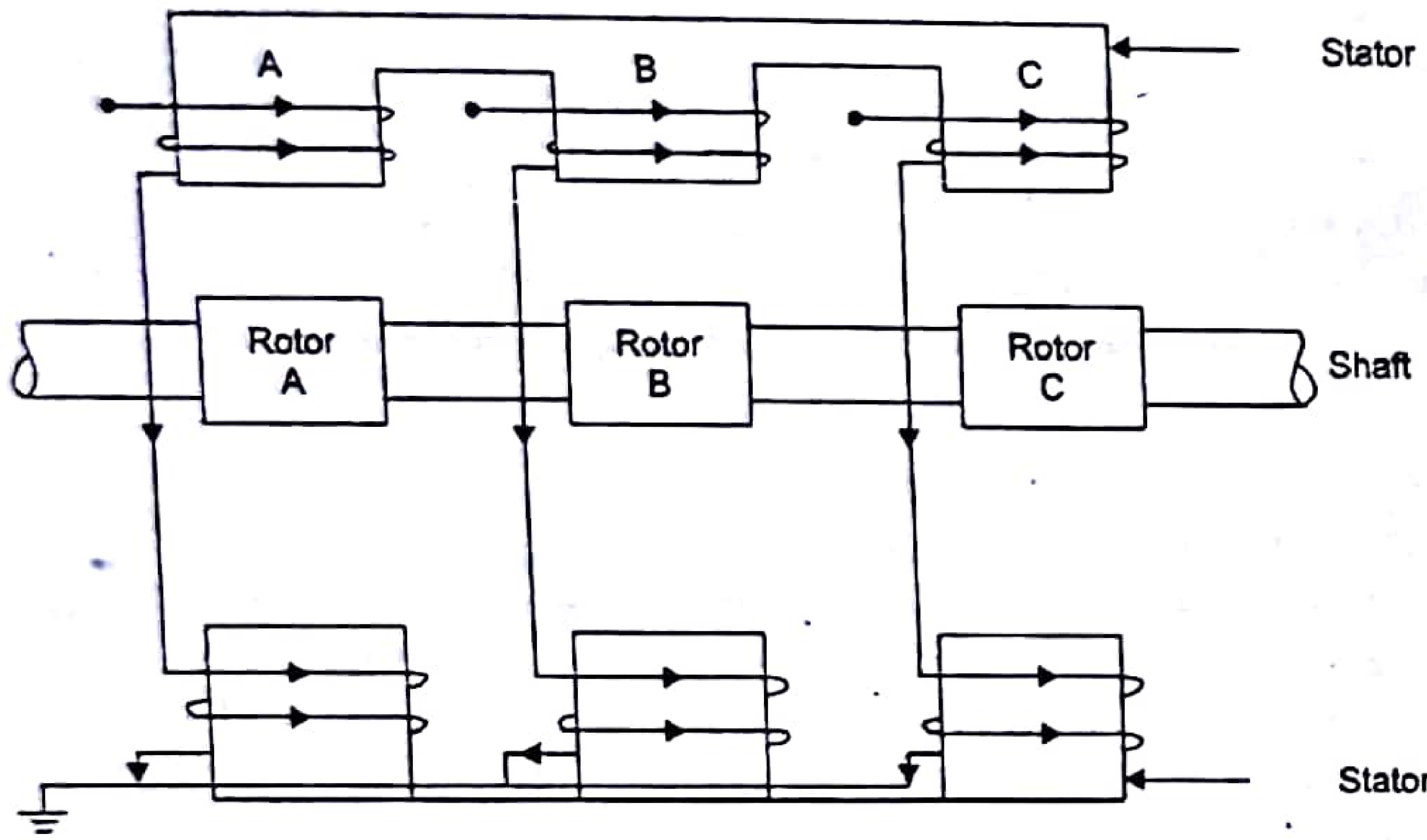
(1) Single-stack variable reluctance motor:- It has 4-phases which are connected to d.c. source through switches S_1, S_2, S_3, and S_4. When phase 1 is excited, the rotor aligns with the axis of phase 1. If now switch S_1 off & S_2 switched on, the rotor moves through 90° in clockwise direction to align with the resultant air gap field which lies along the axis of phase 2.



If phase 3 is excited and phase 2 is disconnected, the rotor aligns with the resultant airgap field which now lies along phases 3. The windings of the stator are energized in sequence 1,2,3,4,1. As the phases are energized in sequence, the rotor moves through a step of 90° in clockwise direction.

Rotor can be made to rotate in anticlockwise direction by reversing the switching sequence.

2. Multistack Variable Reluctance Stepper Motor: In this type of stepper motor, rotor have a common shaft & stator has a common frame. The stator & rotor have same number of poles with same teeth size. The stators are pulse excited while rotors are unexcited. The windings of all stator poles are excited simultaneously. Since, stators and rotors have same sumber of poles, therefore pole pitch is also same.



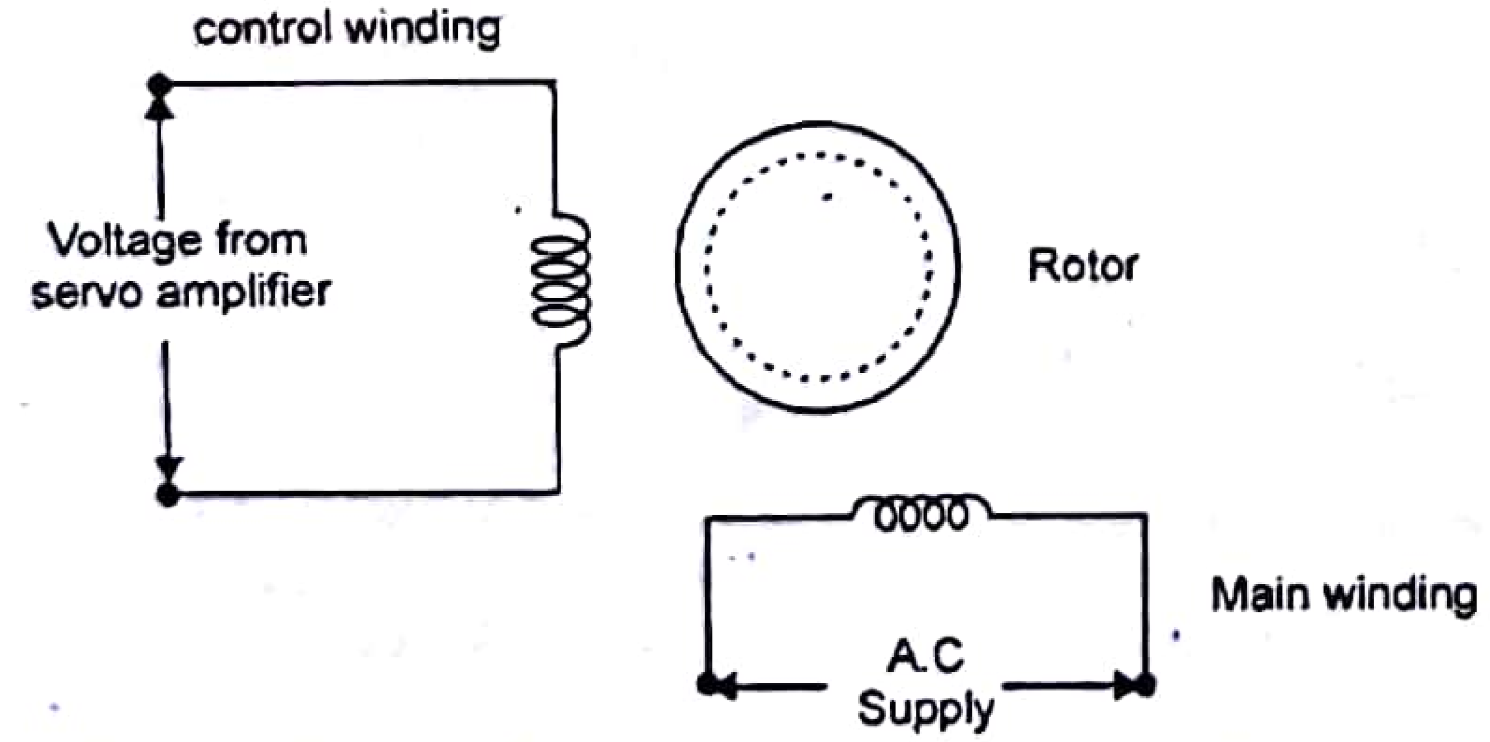
If T_r is the number of rotor teeth & n is the number of stacks or phases, then tooth pitch = $\frac{360°}{T_r}$, angular displacement or step angle = $\frac{360°}{nT_r}$.

Q.4. (c) Write short note on AC and DC Servomotors.

Ans: Servomotors are two phase induction motor. The stator has two distributed windings. These windings are displaced from each other by 90° electrical.

(1) Main winding or Reference winding is excited by constant A.C. voltage.

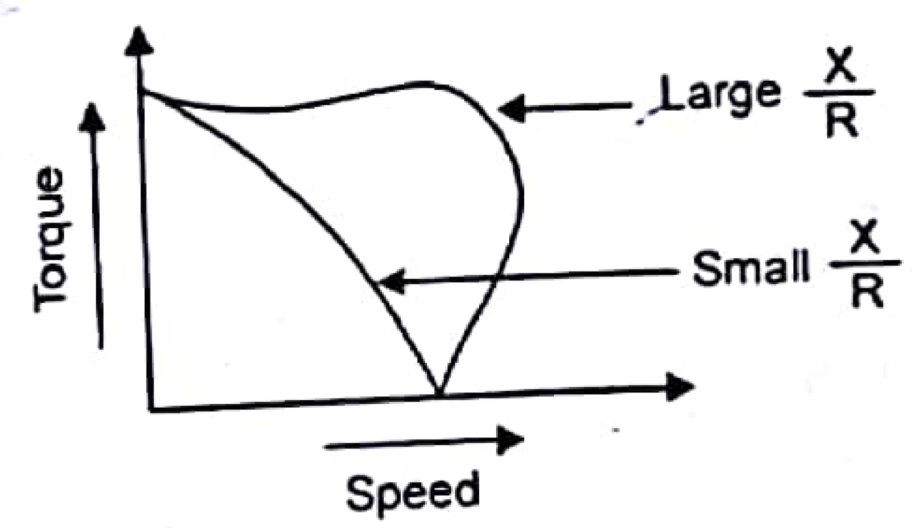
(2) Control winding is excited by variable control voltage of the same frequency as the refrence winding, but having a phase displacement of 90° electrical.

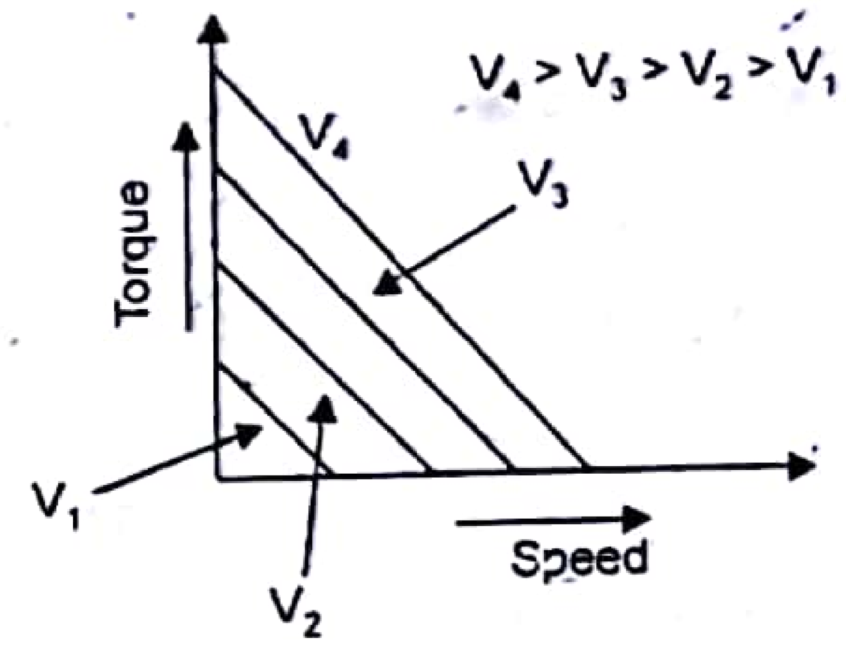


The variable control voltage for control winding is obtained from a servoamplifier. The direction of rotation of the rotor depends upon phase relationship of voltages applied to the two windings. The direction of rotation of the rotor can be reversed by reversing

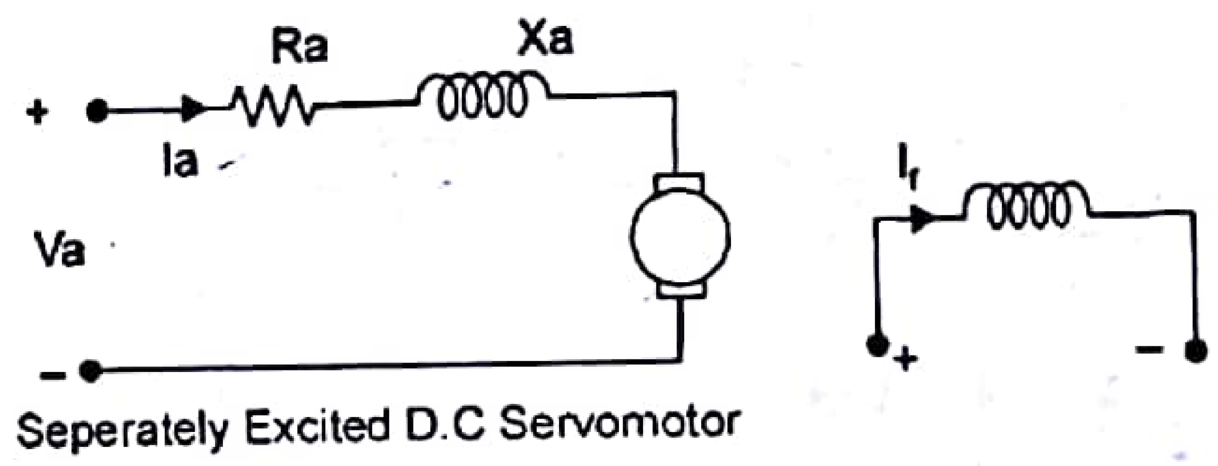
the phase difference between control voltage reference voltage.

Torque-Speed Characteristics:- The torque-speed characteristics of two phase induction motor depends upon the ratio of reactance to resistance. For high resistance & low reactance, the characteristics is linear & as its ratio is large, it becomes non linear.

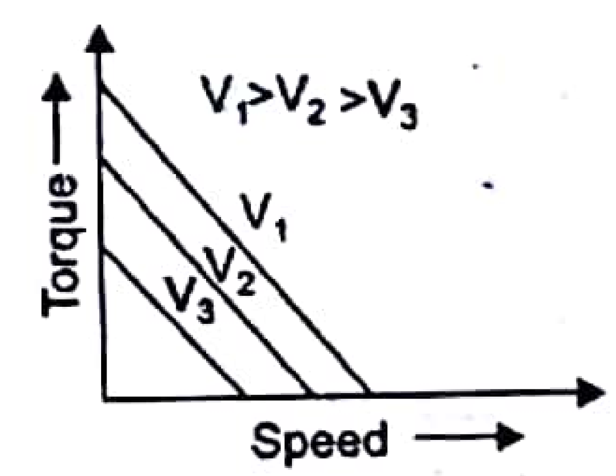




D.C. Servomotors: These motors are separately excited or permanent magnet d.c. servomotors. The armature of d.c. servomotor has a large resistance, therefore torque speed characteristics is linear.



Seperately Excited D.C Servomotor



Torque Speed Characteristics

The d.c. servometer can be controlled from the armature side or from field. In field controlled d.c. servomotor, the ratio of L/R is large, i.e. time constant for field circuit is large. Due to large time constant, the reponse is slow & therefore they are not commonly used.

SECOND TERM EXAMINATION (2015)
FOURTH & FIFTH SEMESTER (B.TECH)
CONTROL SYSTEM (ETEE-212)

Time : 1 Hr **M.M. : 30**

Note: *Attempt Q.No.1 And Any Two More.*

Q.1. (a) What is the actuating signal or output signal from PID Controller **5 × 2**

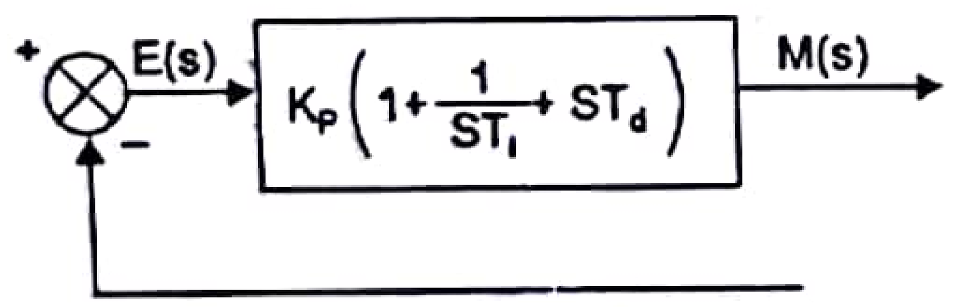
Ans: In PID control, the actuating signal consists of proportional error signal added with derivative & integral of error signal

$$m(t) = K_p e(t) + K_p \frac{1}{T_i}\int_0^t e(t)dt + K_p T_d \frac{d}{dt}e(t)$$

Laplace transform

$$M(s) = K_p E(s) + \frac{K_p}{ST_i}E(s) + KpT_d SE(s)$$

$$\frac{M(s)}{E(s)} = K_p\left(1 + \frac{1}{ST_i} + ST_d\right)$$



where

K_p = proportional gain
T_i = integral time
T_r = derivative time
M(s) = actuating signal.

Q.1. (b) Define Minimum Phase Systems and Non-Minimum phase systems.

Ans: Minimum Phase System: The transfer function having no poles and zeroes in the right half s-plane are called minimum phase transfer functions. System with minimum phase transfer functions are called minimum phase system. The magnitude and phase angle plots of minimum phase systems are uniquely related, that if the magnitude curve is specified for the frequency from zero to infinity, then the phase angle curve is uniquely related.

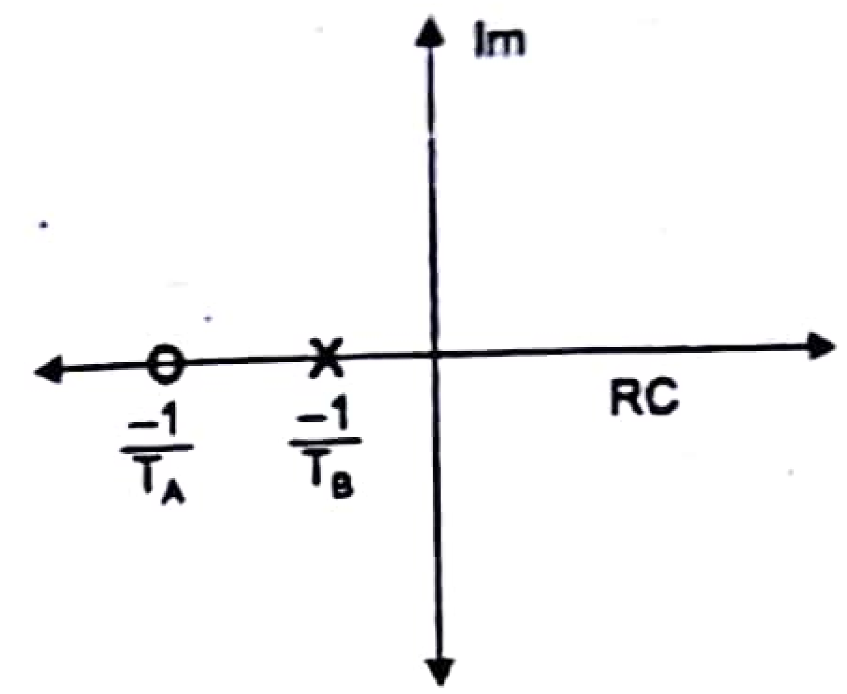
Let

$$G_1(jw) = \frac{1 + j\omega T_A}{1 + j\omega T_B},$$

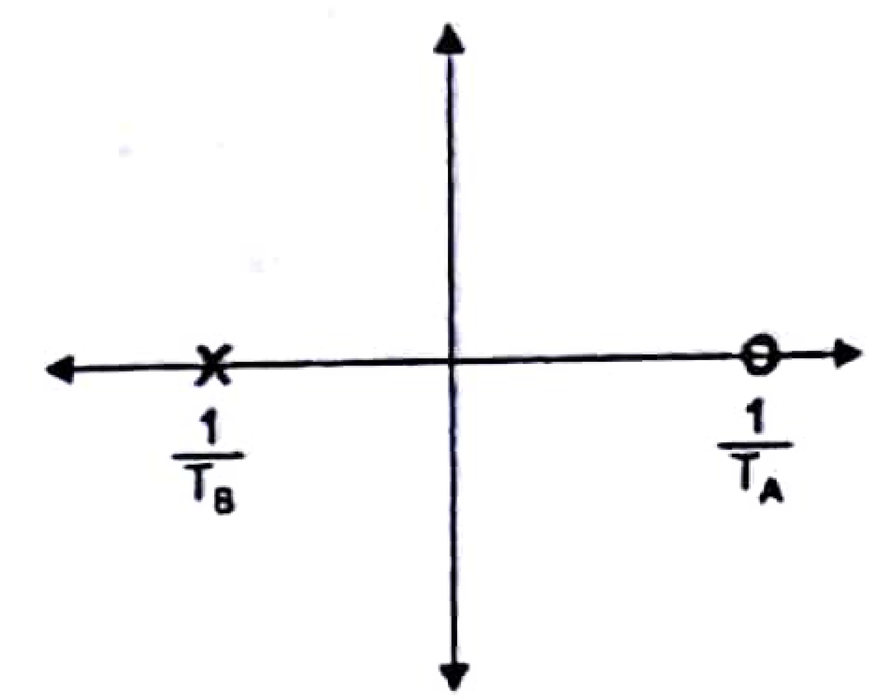
$$G_2(jw) = \frac{1 - j\omega T_A}{1 + j\omega T_B}$$

then G_1 (jw) is a minimum phase transfer function & G_2 (jw) is a non-minimum phase transfer function

The transfer functions having poles and/ or zeroes in the right half s-plane are called non-minimum phase transfer function & such system are known as non-minimum phase system.



Minimum Phase



Non-Minimum Phase

Non-minimum phase systems are slow in response.

Q.1. (c) The damping ratio and natural frequency of oscilication of a second order system is 0.5 & 8 rad/sec respectively. Determine the Resonant peak and Resonant frequency?

Ans: Resonant peak $= M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = \frac{1}{2\times 0.5\sqrt{1-0.25}}$

$$= \frac{1}{2\times 0.5\sqrt{0.75}} = \frac{1}{2\times 0.5\times 0.86} = 1.16$$

Resonant frequency= $\omega_r = \omega_n\sqrt{1-2\zeta^2} = 8\sqrt{1-2\times 0.25}$

$$= 8\sqrt{0.5} = 5.65 \text{ rad/sec}$$

Q.1. (d) Derive the values of static error coefficients and steady state error for a Type 1 system with unit step input.

Ans: Type '1' system:

$$G(s)H(s) = \frac{K(1+sT_1)(1+ST_2).....}{s(1+sT_a)(1+sT_b).....}$$

$$K_p = \lim_{s\to 0} G(s)H(s) = \lim_{s\to 0}\frac{K(1+sT_1)(1+sT_2)........}{s(1+sT_a)(1+sT_b).....}$$

$\boxed{K_p = \infty}$ static error coefficients.

$$\text{Steady state error} = e_{ss} = \frac{1}{1+K_p} = 0$$

$$\boxed{e_{ss} = 0}$$

Q.1. (e) What is necessary but not sifficient conditon for stability?

Ans: Let a system has a characteristic equation as

$$a_0 s^m + a_1 s^{m-1} + ...+ a_m = 0$$

(a) All the coefficient of the equation should have same sign.

(b) There should be no missing term.

If above two conditions are not satisfied, then the system will be unstable. But if all the coefficients have same sign and there is no missing term, there is no guarantee that the system will be stable.

Q.2. (a) A unity feedback system has the forward loop transfer function

$$G(S) = \frac{K(S+2)^2}{S(S+3)} \qquad (6)$$

Determine the range of K for stable operation using Routh Stability criterion.

Ans: Characteristic equation

$$1 + G(s)H(s) = 0$$

$$1 + \frac{1(s+2)^2}{s(s+3)} = 0$$

$$s^2 + 3s + ks^2 + 4sk + 4k = 0$$

$$s^2(1+k) + s(3+4k) + 4k = 0$$

$$\begin{array}{c|cc} S^2 & (1+k) & 4k \\ S^1 & (3+4k) & 0 \\ S^0 & \frac{4k(3+4k)}{(3+4k)} & = 4k \end{array}$$

so, $4k \geq 0$

& Hence, Range of

$K = [0,4]$ **Ans.**

Q.2. (b) Sketch the polar plot for the transfer function $G(S) = \frac{S}{(1+ST)}$ (4)

Ans: $G(s) = \frac{s}{(1+sT)}$

Put $s = jw$

$$G(jw) = \frac{jw}{1+jwT}$$

$$G(jw) = \frac{w}{\sqrt{1+w^2T^2}} \underline{|90° - \tan^{-1}\omega T}$$

taking the limit for the magnitude of $G(jw)$

$$\lim_{\omega\to 0}|G(jw)| = \lim_{w\to 0}\frac{w}{\sqrt{1+w^2T^2}} = 0$$

$$\lim_{\omega\to\infty}|G(jw)| = \lim_{w\to\infty}\frac{w}{\sqrt{1+w^2T^2}} = \frac{1}{T}$$

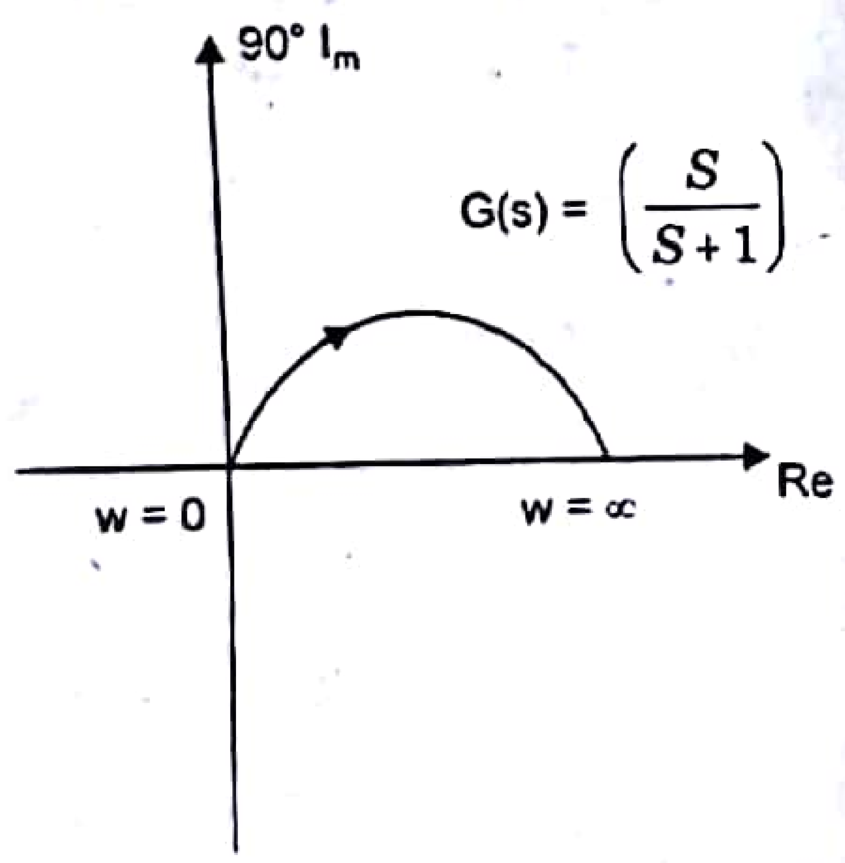
Taking the limit for the phase angle of G (jw)

$$\lim_{\omega\to\infty}\underline{|G(jw)} = \lim_{\omega\to 0}\underline{|90° - \tan^{-1}wt} = 90°$$

$$\lim_{\omega\to\infty}\sqrt{G(jw)} = \lim_{\omega\to\infty}\underline{|90° - \tan^{-1}wT} = 0°$$

Seperating the real & imaginary terms of G (jw)

$$G(jw) = \frac{jw}{1+jwT} \times \frac{(1-jwT)}{(1-jwT)}$$



$$= \frac{w^2T}{1+w^2T^2} + j\frac{w}{1+w^2T^2}$$

Equate the imaginary part to zero

$$\frac{w}{1+w^2T^2} = 0$$

$$\omega = \infty$$

$\therefore$

Polar plot intersects the real axis at $\omega = \infty$

at which $G(j\omega) = \frac{1}{T}\underline{|0°}$

Equate the real part to zero

$$\frac{w^2T}{1+w^2T^2} = 0$$

$$\omega = 0$$

$\therefore$

Polar plot intersects the imaginary axis at $\omega = 0$ at which $G(j\omega) = 0\underline{|90°}$

Q.3. (a) Sketch the Root Locus for 7

$$G(S) = \frac{k}{s(s+4)(s^2+4s+20)}$$

Ans: Step 1 → Plot poles and zeros

Poles are at

$$s = 0,$$

$$s = -4$$

$$s^2 + 4s + 20 = 0$$

$$s = -2 \pm j4$$

Step 2 → The segment between s = 0 & s = – 4 is the part of the root locus.

Step 3 → Number of root locii

N = P = 4

Step 4 → Centroid of asymptotes

$$\sigma_A = \frac{\text{sum of poles} - \text{sum of zeros}}{P - Z}$$

$$= \frac{0 - 4 - 2 + jw - 2 - j4 - 0}{4}$$

$$\boxed{\sigma_A = -2}$$

Step 5 → Angle of asymptotes

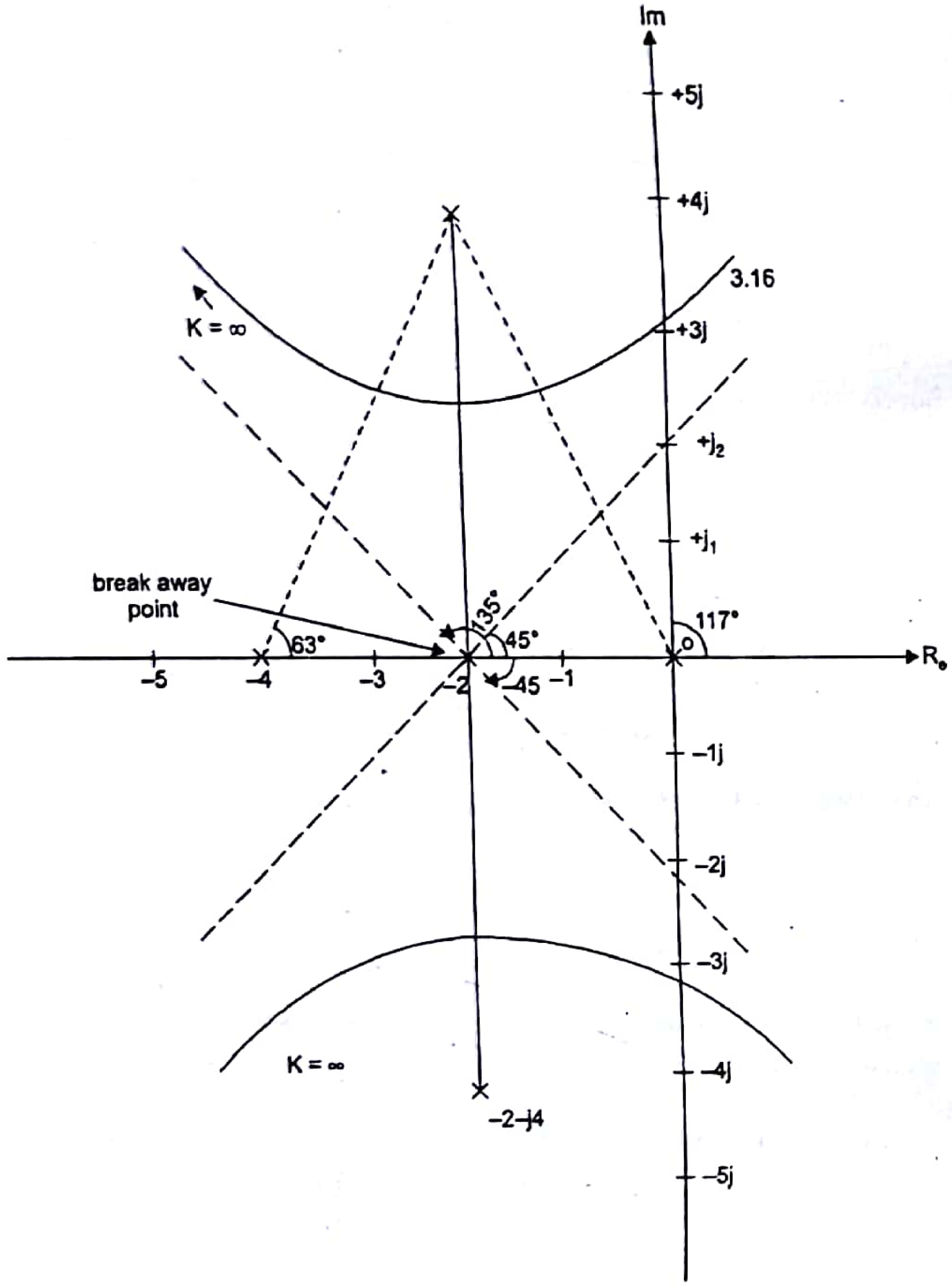
$$\phi = \frac{2k+1}{P-Z}180°$$

$k = 0 \quad \phi_1 = 45°$

$k = 1 \quad \phi_2 = 135°$

$k = 2 \quad \phi_3 = 225°$

$k = 3 \quad \phi_4 = 315°$



Step 6 Breakaway point

The characteristics equation : 1 + G (s) H (s) = 0

$$1+\frac{k}{s\,(s+4)\,(s^2+4s+20)} = 0$$

$$s^4+8s^3+36s^2+80s+k = 0$$

$$k = -\left[(s^4)+8s^3+36s^2+80s\right]$$

$$\frac{dk}{ds} = -(4s^3+24s^2+72s+80) = 0$$

Breakaway point is s = -2 & two complex breakaway points are $-2 \pm j\,2.45$

Step 7 Points of intersection of root locii on imaginary axis Routh array

s^4	1	36 k
s^3	8	80
s^3	26	k
s^1	$80-0.307k$	
s^0	k	

for stability K > 0

$$80–0.307\,k > 0 \text{ or } k < 260$$

at k = 260, the auxiliary equation

$$A(s) = 26\,s^2+k$$

$$26\,s^2+260 = 0$$

$$s = \pm j\,3.16$$

Step 8 The angle of departure at upper complex pole

$$\phi_d = 180° - (117° + 90° + 63°)$$

$$\phi_d = -90°$$

Q.3. (b) Define relative stability and absolute stability. 3

Ans: Absolute stability means whether the system is stable or not, i.e. the system is stable or unstable. If the system is stable then we determine how stable it is, i.e., we measure the degree of stability, it is known as relative stability.

In time domain, the relative stabilty is measured by maximum overshoot and damping ratio. In frequency domain relative stability is measured by resonant peak (M_r).

Q.4. (a) Construct the Bode plots for a unity feedback control system having 7

$$G(S) = \frac{200}{s(s+10)(s+50)}$$

From Bode plot determine the (a) Phases Crossover Frequency (b) Gain Margin (c) Gain Crossover Frequency (d) Phases Margin

Ans:

$$G(s) = \frac{200}{s(s+10)(s+50)}$$

$$= \frac{200}{10\times 50\times s(0.1s+1)(0.02sH)}$$

$$= \frac{0.4}{s(0.1s+1)(0.02s+1)}$$

Step 1 Put $s = jw$

$$G(jw) = 0.4/(jw)\,(0.1jw+1)\,(jw0.02+1)$$

$$= 0.4/(jw)\,(j0.1w+1)\,(j0.\,02w+1)$$

Step 2 Draw the magnitude curve

corner frequencies $w_1 = \frac{1}{0.1} = 10$ rad/sec

$$w_2 = \frac{1}{0.02} = 50 \text{ rad/sec}$$

Initial slope of curve will be -20 db/decade due to 1/s term. From the starting point to the first corner frequency the slope will be- 20 db/decade, then from corner frequency 10 the slope will db/decade, theb from corner frequency 10 the slope will be – 20+ (–20) = – 40 db /dec upto 50.

After 50, the slope will be –40 + (–20)

= – 60 db/decade

Step- 3 Draw the phase curve

$$\phi = -90° - \tan^{-1} 0.1w - \tan^{-1} 0.02\, w.$$

w	$Arg(jw)$	$\arg(1 + j0.1w)$ $-\tan^{-1}(0.1w)$	$Arg(1 + j0.02jw)$ $-\tan^{-1}(0.02w)$	Resultant
1	-90	-5.71	-1.14	-96.85
5	-90°	-26.5	-5.7	-122.2
10	-90°	-45	-11.30	-146.3
30	-90°	-71.56	-30.96	-192.52
50	-90°	-78.6	-45	-213.6
100	-90°	-84.2	-63.43	-237.63

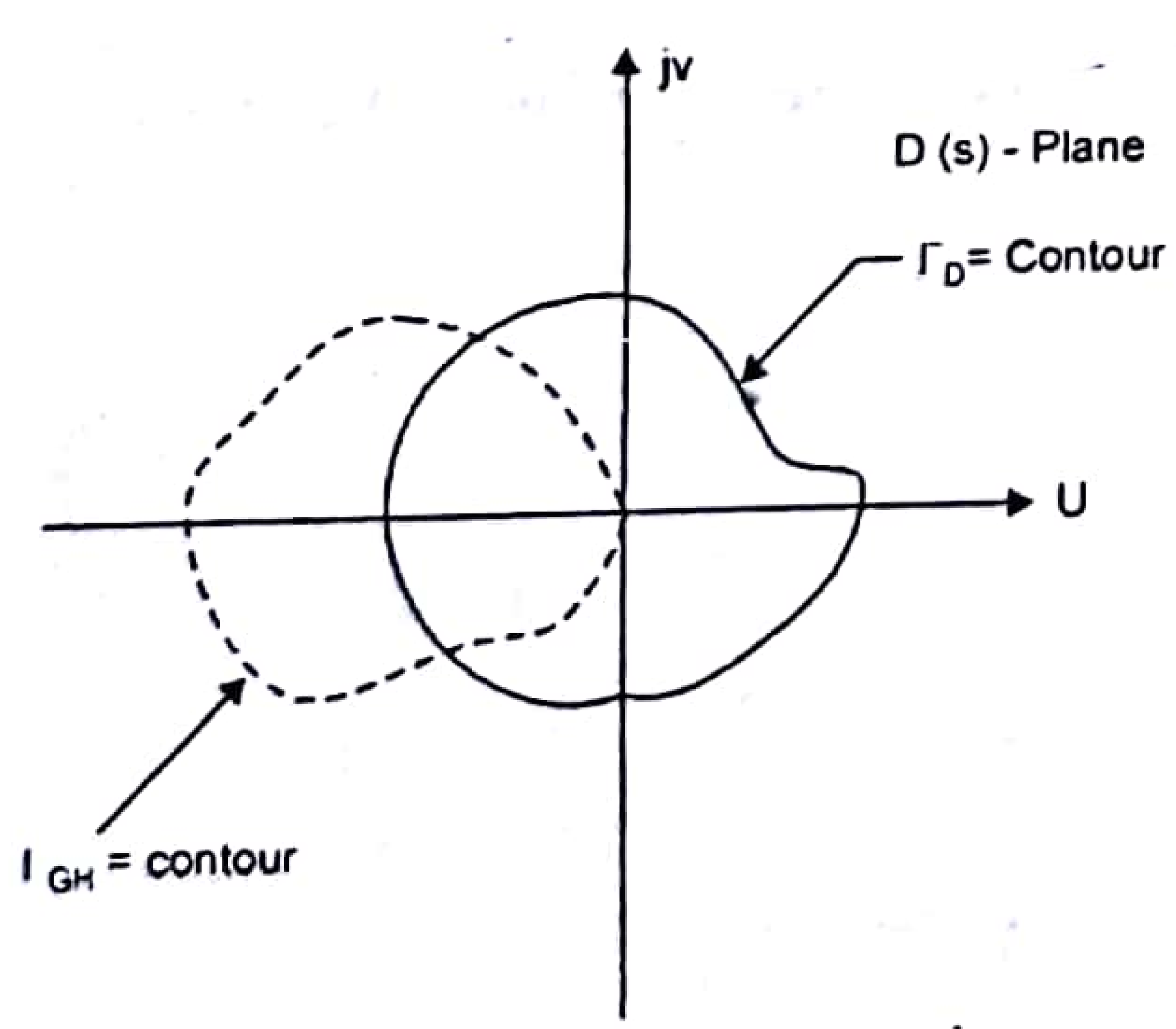
Step 4 From Bode plot

(1) Gain cross over frequency (2) Phase crossover frequency

(3) Gain Margin (4) Phase margin

Q.4. (*b*) State and explain the Nyquist Criterion for stability 3

Ans: A feedback system or closed loop system is stable if the contour Γ_{GH} of the open loop transfer function G(s) H(s) corresponding to the Nyquist contour is the s-plane encircles the point (–1+j0) in clock wise direction & number of counter clockwise encirelement above the (–1+j0) equals to the number of poles of G (s) H (s) in the right half of splane i.e with positive axis. The closed loop system is stable if the contour Γ_{GH} of G (s) H (s) does not pass through or does not encircle (–1+j0) point i.e. net encirclement is zero.



END TERM EXAMINATION (2015)
FOURTH & FIFTH SEMESTER (B.TECH)
CONTROL SYSTEM (ETEE-212)

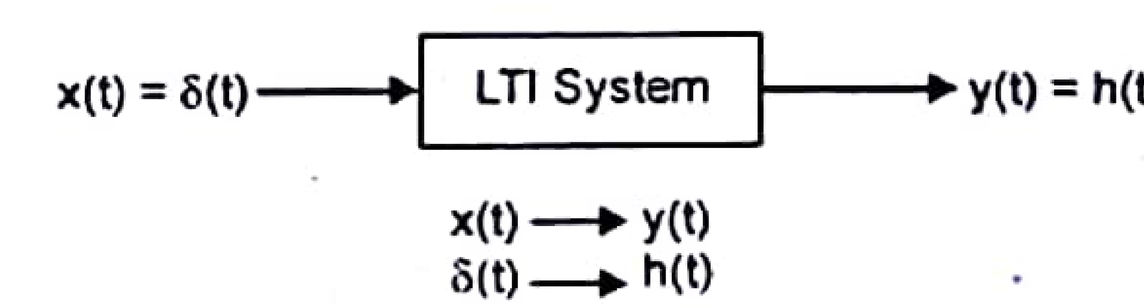
Time : 3 Hrs. **M.M. : 75**

Note: *Attempt five question inclusding Q.no.1 wich is compulsory. Select one question from each unit. Assume missing data, if any.*

Q.1. (*a*) Define transfer function of a linear time invariant system in terms of its impulse response. What are limitations of transfer function approach? **(5 × 5 = 25)**

Ans: When the parameters of a control system are stationary with respect to time during the operation of the system, the system is called a time-invariant system.

An LTI system in terms of its inpulse response, defined as the output of an LTI system due to a unit impulse signal input applied at $t = 0$. The impulse response completely characterizes the behaviour of any LTI system.



Any linear system can be represented by a mathematical model in the form of a transfer function of the system.Transfer function is the ratio of Laplace transform of the output to the Laplace transform of the input. The following are the limitations of the transfer function approach

(*i*) All initial conditions are to be assumed as zero.

(*ii*) The system should be linear and time invariant.That is, the system parameters should not vary with time.

Q.1. (*b*) Differentiate between terms "servo system" and "regulator".

Ans: A servo mechanism is a control system in which the output is a mechanical position, or the rate of change of position, or rate of change of velocity, that is, velocity or acceleration. A robot, automatic positioning of anti-aircraft gun, radar antenna are some examples of servomechanism.

Regulating system (Regulator) is a feedback control system in which for a present value of the refrence input, the output is kept constant at its desired value. A regulating system differes from a servomechanism in that the main function of a regulator is usually to maintain a constant output for a fixed input, while that of a servomechanism is mostly to cause the output of the system to follow the varying input. A temperature regulator, speed governor are examples of regulating system.

Q.1. (*c*) What is meant by minumum phases transfer function and non-minimum phase transfer function? Give an example of each.

Ans: Minimum-phase and non-minimum phase system:

The transfer functions having no poles and zeros in the right half s-plane are called minimum phase transfer function. Systems with minimum phase transfer function are called minimum phase systems.

The transfer functions having poles and/or zeros in the right half s-plane are called non-minimum phase transfer functions. Systems with non-minimum phase transfer functions are called non-minimum phase systems.

Q.1. (d) Define peak overshoot, settling time, delay time, peak time and rise time for step response of second order system.

Ans: t_d–Delay time: The time that the system output response takes for the step input to reach 50% of its final value is called delay time.

t_r–Rise time: The rise time is the time required for the response to rise from 10% to 90% or 0% to 100% of its final value.

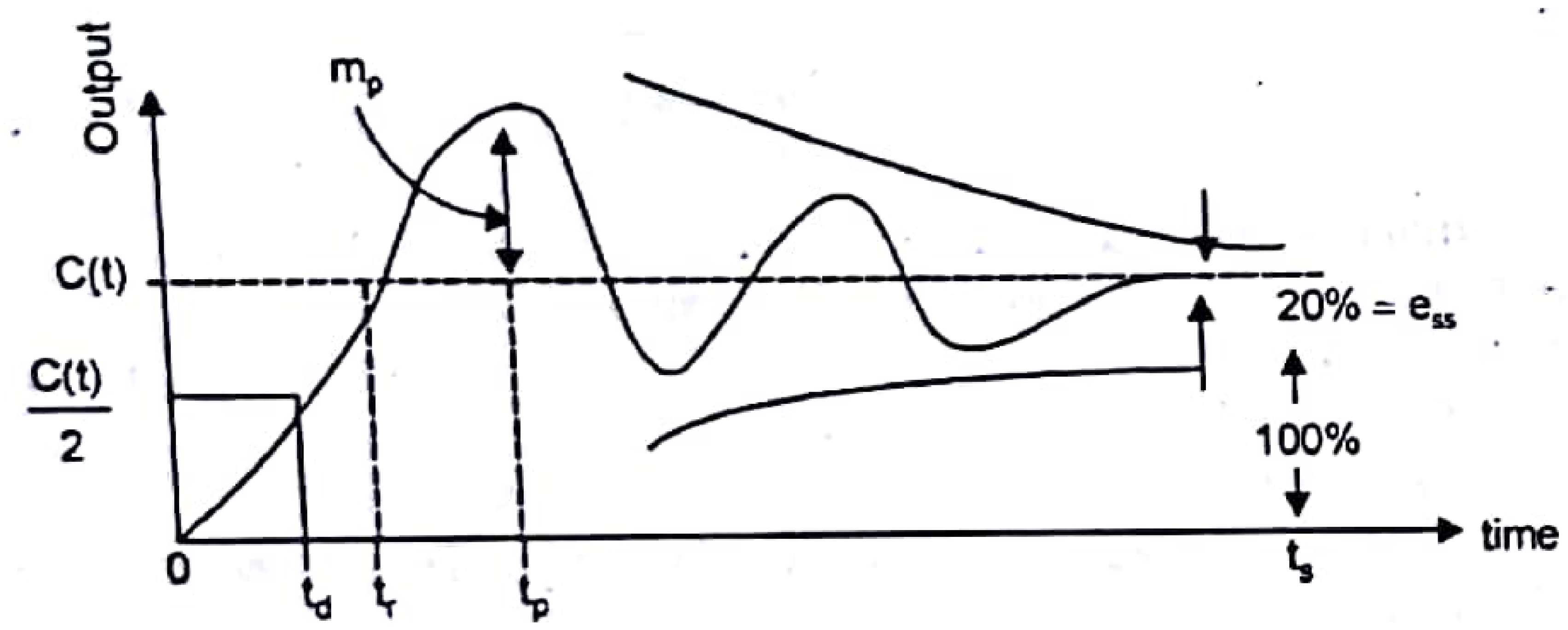
For underdamped second order systems, the 0% to 100% rise time is normally used. For overdamped systems, the 10% to 90% rise time is commonly used.

t_p–Peak Time: The time needed to reach the maximum overshoot is called peak time.

t_s–Settling time: The time needed to settle down aforesaid oscillations within 2% of desired value of the output is known as setting time.

m_p–Maximum overshoot: The maximum positive deviation of the output with respect to its desired value is known as maximum overshoot.

e_{ss}–Steady State error: It is the difference between the input and the output for a prescribed test input as $t \to \infty$. It is a measure of system accuracy.



Q.1. (e) What is the input should be given to the system whose frequency response is desired? What are advantages and limitations of frequency domain analysis?

Ans: Advantages of Frequency-Response Analysis:

The advantages of frequency-response analysis are:

1. Sinusoid test signals for various ranges of frequencies and amplitude are easily available. Thus, the experimental determination of the frequency response is easily accomplished and is the most reliable.

2. The transfer function describing the sinusoidal stedy-state behaviour of a system is obtained by replacing s with $j\omega$ in the system transfer function. The transfer function then becomes the function of the complex variable $j\omega$ and it has a magnitude and phase angle. The magnitude and phases angle can be graphically plotted. These graphical plots provide significant insight into the analysis and design of control system.

3. The design and parameter adjustment of the open-loop transfer function of a system for a specified closed-loop performance can be carried out easily in frequency domain as compared to time domain.

4. In frequency domain it is very easy to visualize the effects of noise disturbance and parameter variations.

5. The transient response of a system can be obtained from its frequency response.

6. It can be extended to certain linear system.

7. The absolute and relative stability of a closed-loop control system can be estimated from the knowledge of the open-loop frequency response.

Limitations Of Frequency-response Analysis:

The limitations of frequency-response analysis are:

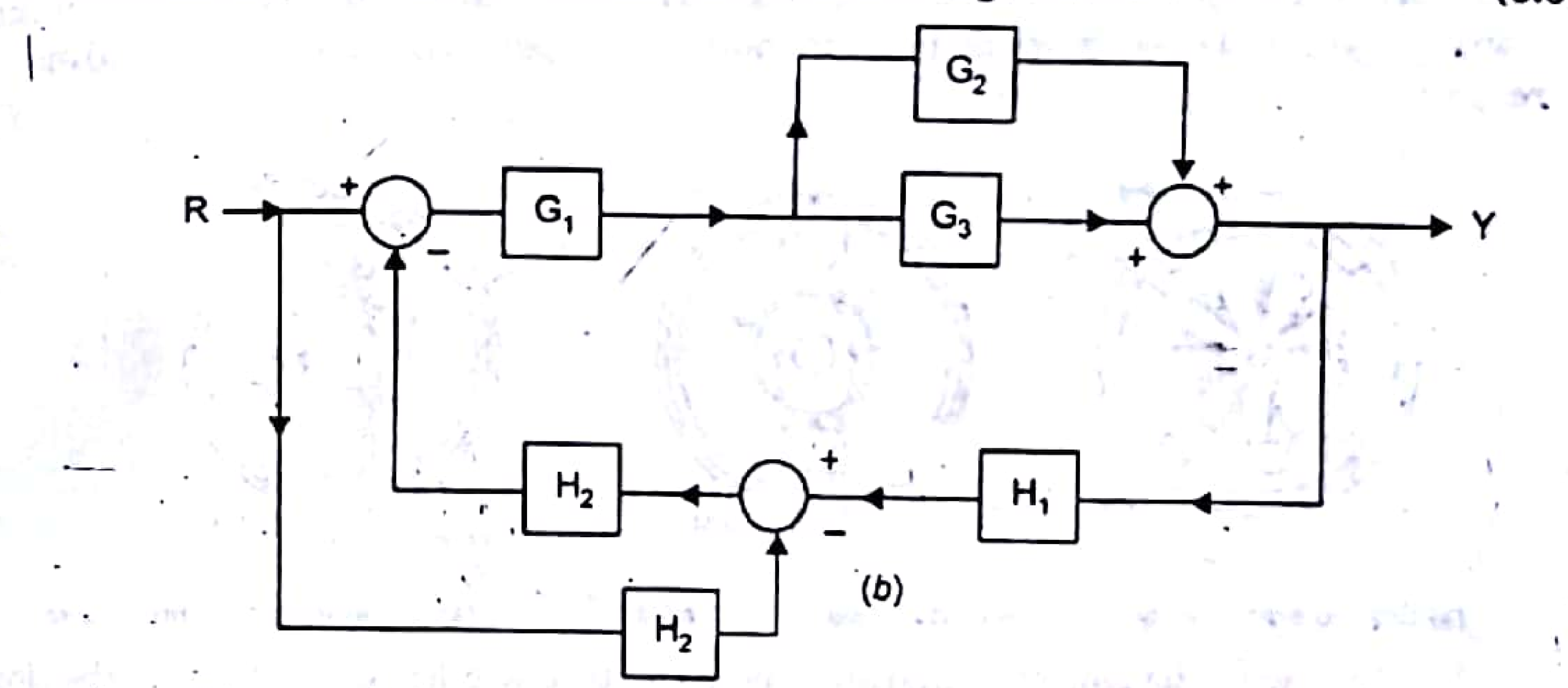
1. These methods are basically applicable to linear systems.

2. For systems with large time constants, the frequency reponse test is cumbersome to perform.

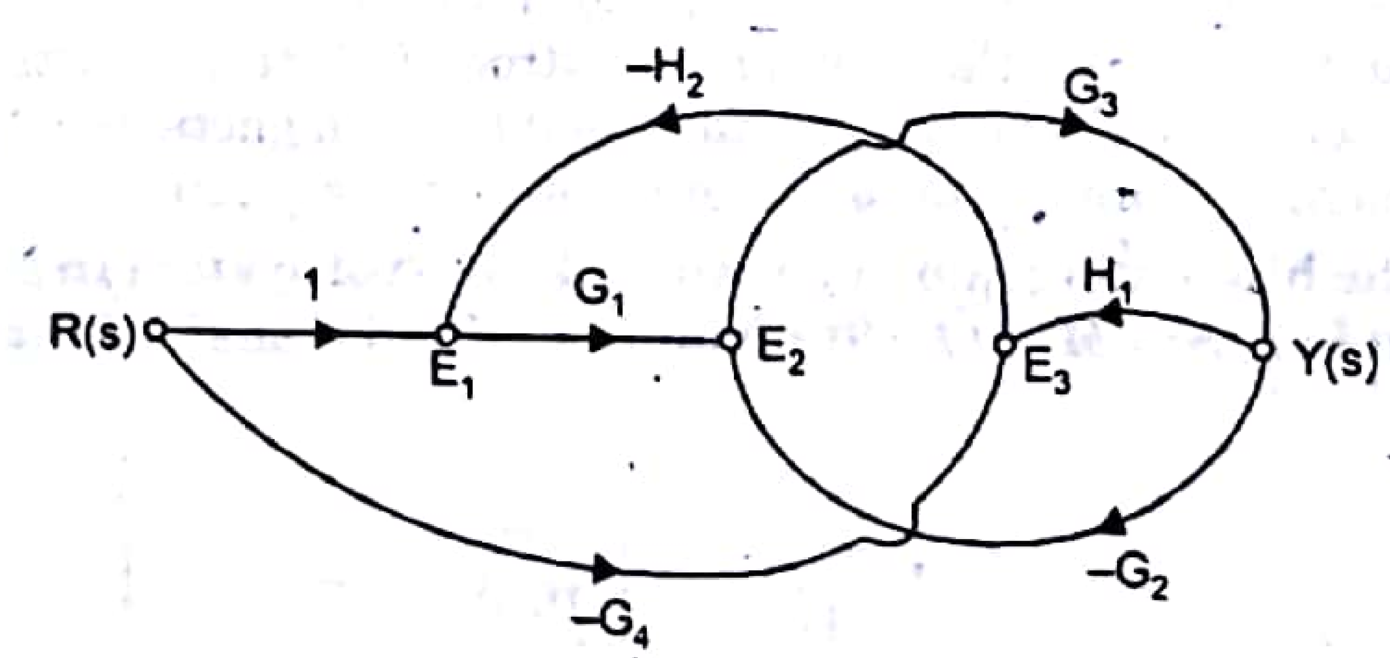
3. Frequency-response methods are time-consuming.

UNIT-I

Q.2. (a) Convert block diagram of the following figure 1 to signal flow graph and obtain input-output function using Masons gain rule. (6.5)



Ans: The signal flow graph of the given block diagram is shown below



Forward path Gains

$$P_1 = 1 \times G_1 \times G_3 = G_1 G_3$$

$$P_2 = -G_4 \times (-H_2) \times G_1 \times G_3 = G_1 G_3 G_4 H_2$$

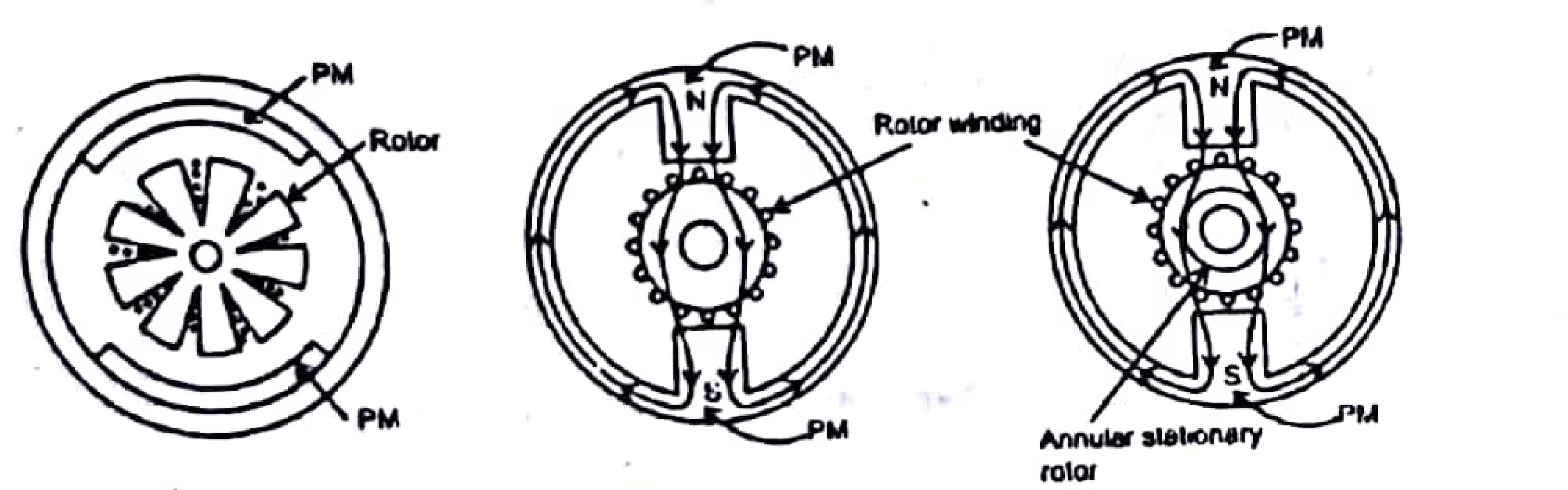
we know that

$$\frac{Y(s)}{R(s)} = \frac{\Sigma p_k \Delta_k}{\Delta}$$

$$\frac{Y(s)}{R(s)} = \frac{-G_1G_2 + G_1G_3 - G_1G_2G_4H_2 + G_1G_3G_4H_2}{1 + G_1G_3H_1H_2 - G_1G_2H_1H_2}$$

Q.2. (b) Armature control of dc motor is used in closed loop speed control system. Draw a schematic layout of the system. Describe operation of main components used, explaining cleary how error signal is formed. (6)

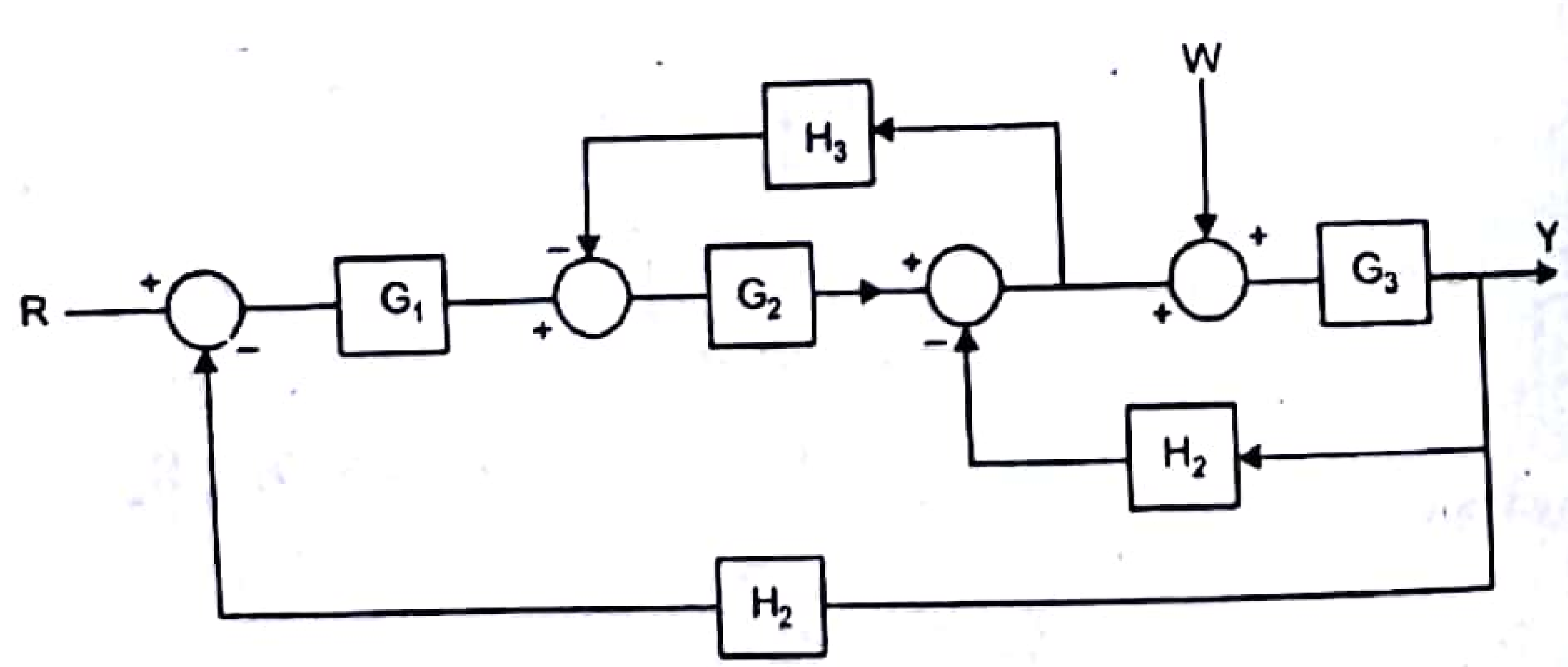
Ans: DC Servomotors : From the control point of view, an armature voltage controlled dc servomotor is similar to a conventional dc motor with fixed excitation. Due to high residual flux density and coercivity, permanent magnets (PM) are used in the construction of dc servomotors. This property gives a higher torque-inertia ration and high efficiency. The speed of a PM dc motor is directly proportional to the armature voltage at a given load torque. A PM dc motor has much more flatter speed torque characteristics than a field wound motor which has evere armature reaction effects. There are three types of dc servomotors. They arre slotted armature type, surface wound iron core type and surface would non-magnetic core type as shown in Fig. (a), (b) and (c) respectively.



(a) Slotted armature type (b) Surface wound Iron core (c) Surface wound nonmagnetic core

Fig. (a) shows the slotted armature type. Armature windings are placed in the slots similar to a conventional dc motor. Slotted armature type has large inertia. To reduce this, surface wound iron core is used as shown in Fig. (b). The main draw back in the surface would iron core is that it requires a strong PM for its construction. The inertia may be further reduced by having surface would non-magnetic core as shown in Fig. (c). This non-magnetic core rotates on an annular stationary rotor.

Q.3. The block diagram of a feedback control system in shown in figure 2. The output is $Y(S) = M(S)\,R(S) + N(s)\,W(s)$. Find transfer functions M (s) and N (s). (12.5)



(b)

Soluction: The given system is a 2-input system. We shall find the output by considering one input at a time.

(A) Excitation R_1 acting alone:

Step 1 : First we shall assume that the input R_2 is made zero. The summing point at R_2 is removed and the resulting block diagram is shown in Fig.1. Let the output be C_1.

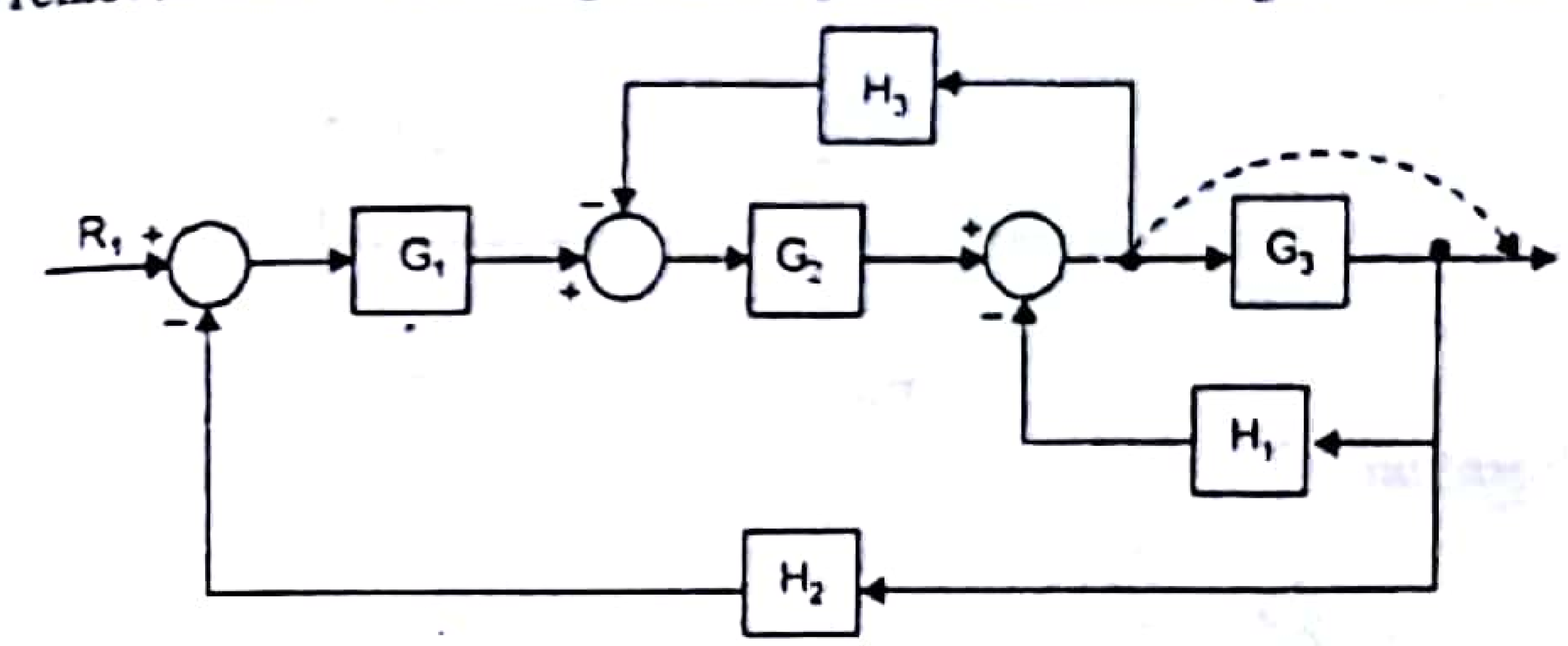


Fig. 1.

Step 2 : More the take-off point in Fig. (1) after block G_3 to obtain the block diagram as shown in fig. 2.

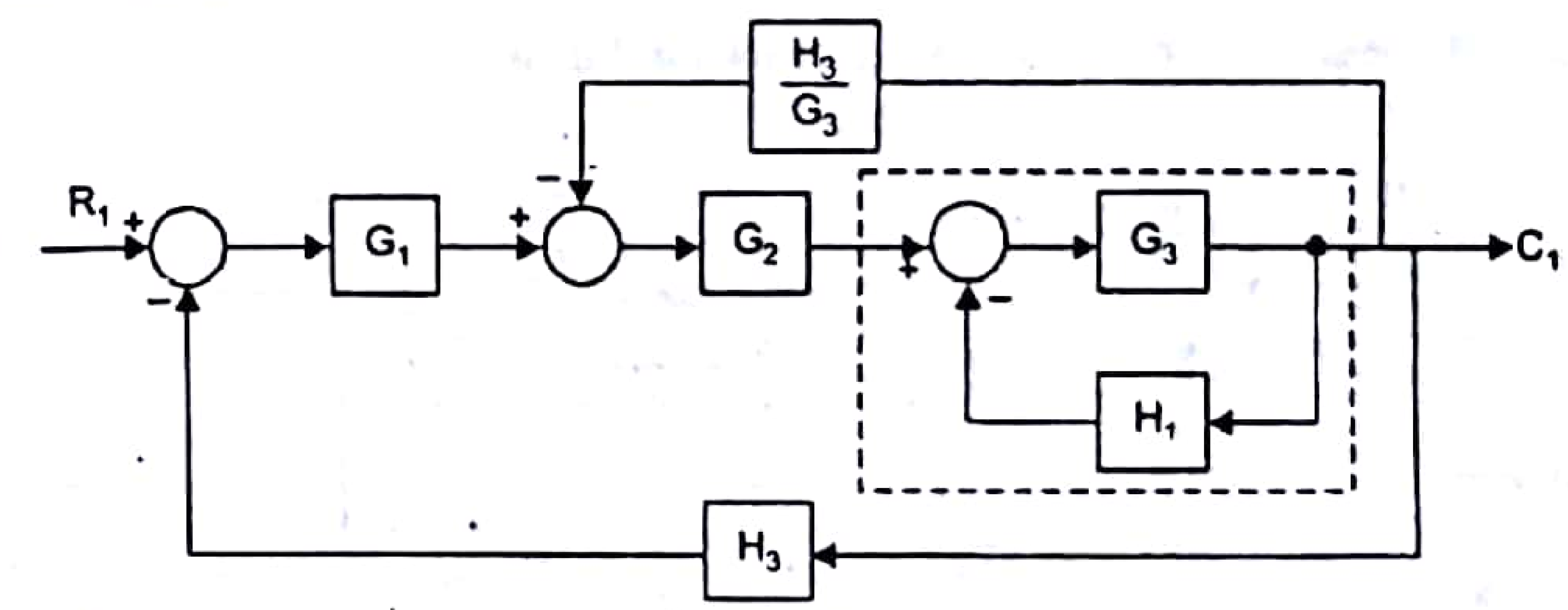


Fig. 2.

Step 3 : Eliminate the inner loop in Fig.2 and combine the result with G_2 and rearrange. This gives the block diagram as shown in Fig. 3.

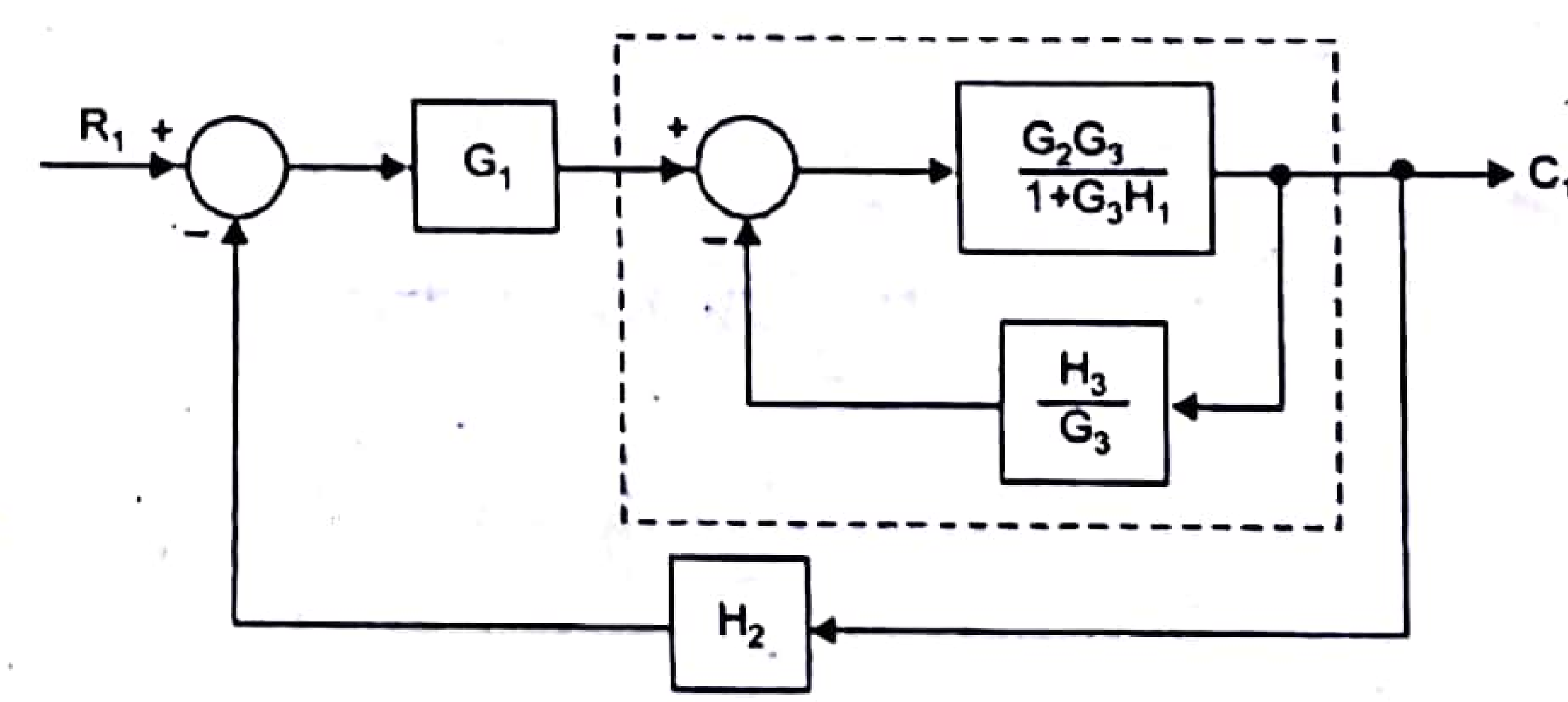


Fig. 3.

Step 4 : Eliminate the inner loop in Fig. 3. The block diagram is shown in Fig. 4.

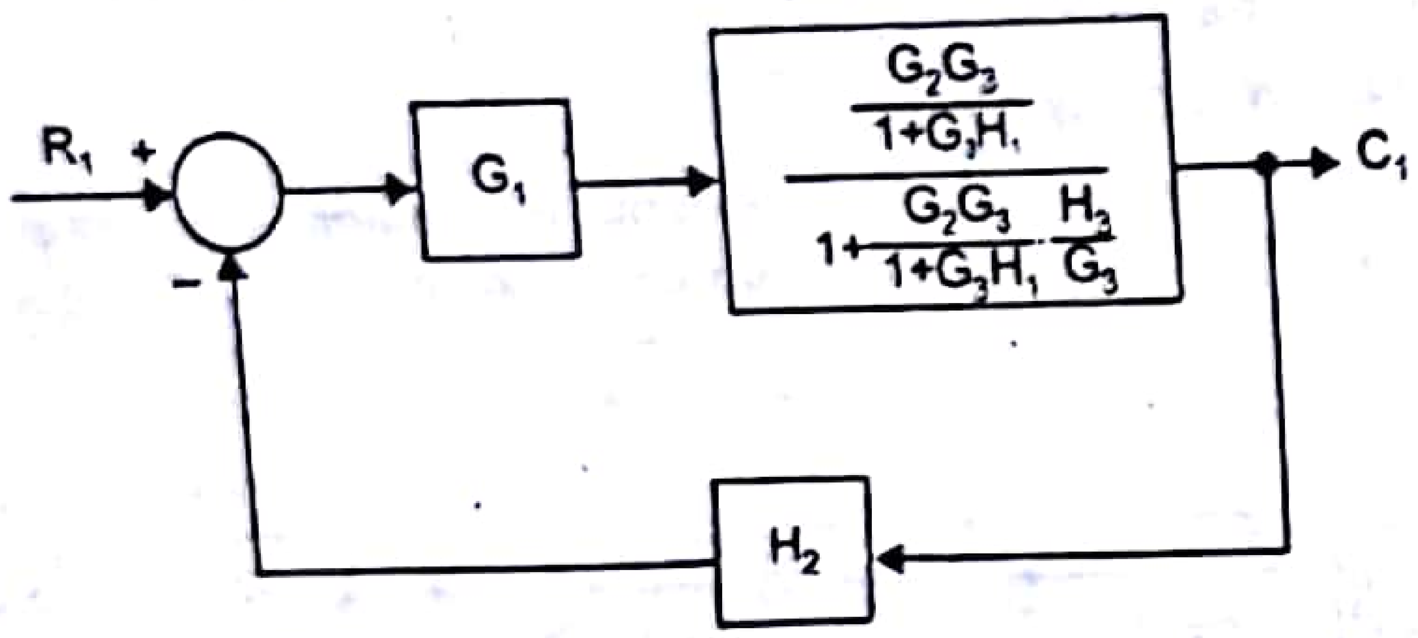


Fig. 4.

On simplification,

$$\frac{\dfrac{G_2G_3}{1+G_3H_1}}{1+\dfrac{G_2G_3}{1+G_3H_1}\cdot\dfrac{H_3}{G_3}} = \frac{G_2G_3}{1+G_3H_1+G_2H_3}$$

The block diagram in Fig. 4, will be as shown in Fig. 5.

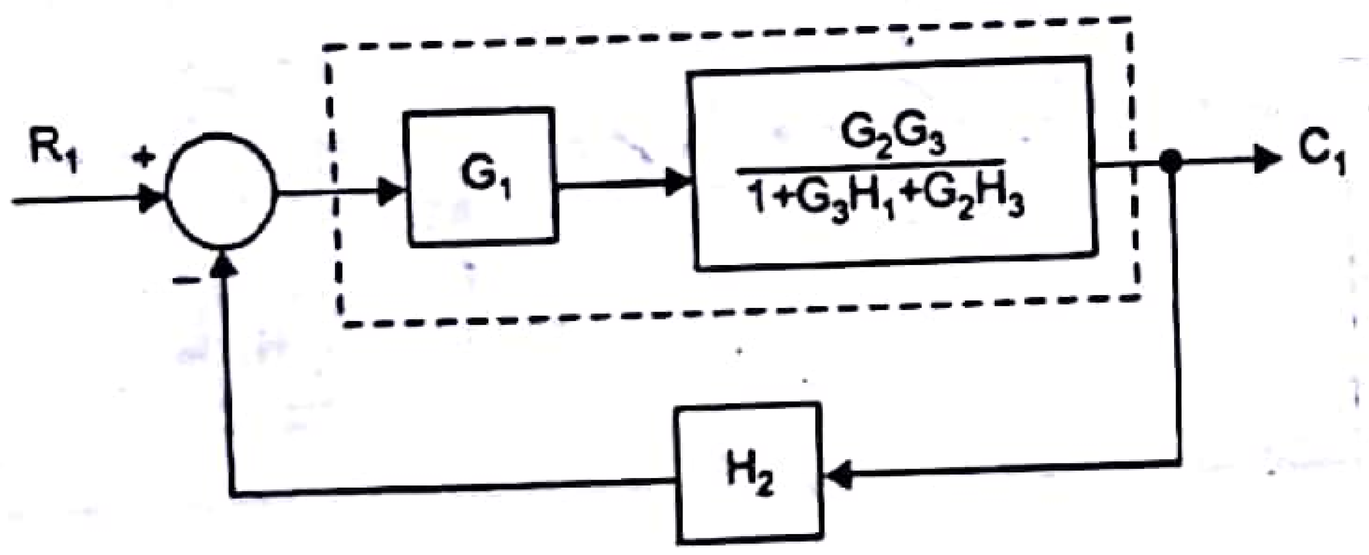


Fig.5

Step 5 : Combining the blocks in cascade in Fig. (5), the block diagram will be as shown in Fig. 6.

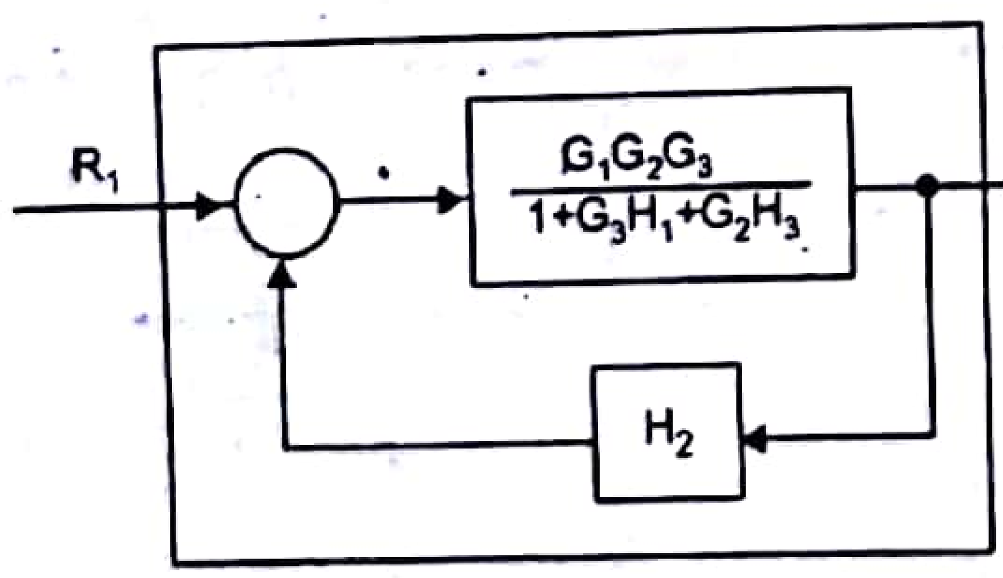


Fig. 6

Step 6 : Eliminating the only loop in Fig.6, the block diagram will be as shown in Fig. 7.

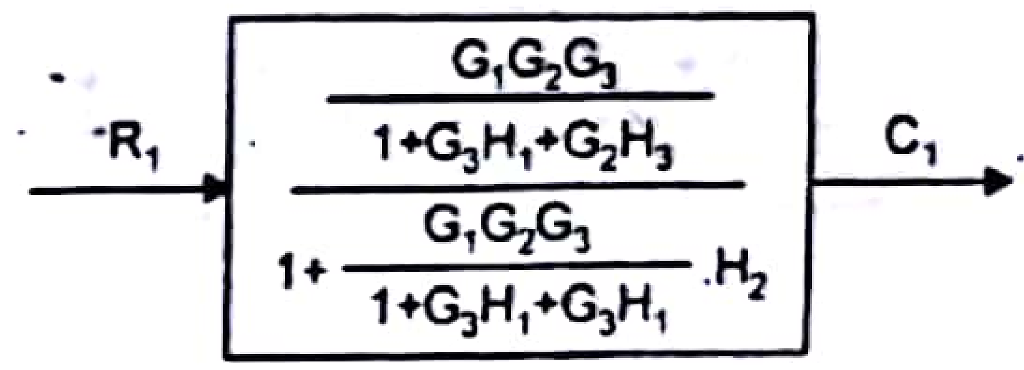


Fig. 7

The closed-loop transfer function is given by

$$\frac{C_1}{R_1} = \frac{\dfrac{G_1G_2G_3}{1+G_3H_1+G_2H_3}}{1+\dfrac{G_1G_2G_3H_2}{1+G_3H_1+G_2H_3}}$$

$$= \frac{G_1G_2G_3}{1+G_3H_1+G_2H_3+G_1G_2G_3G_4H_2}\cdot R_1$$

Therefore, the output component produced by excitation R_1 alone is

$$C_1 = \frac{G_1G_2G_3}{1+G_3H_1+G_2H_3+G_1G_2G_3G_4H_2}\cdot R_1$$

***(B)* R_2 acting alone:** The excitation R_1 is made zero and the output is produced by the excitation R_2 alone. Let the output be C_2. By making $R_1 = 0$, the summing point at R_1 is removed and a negative sign is attached to the feedback path gain H_2. The resulitng block diagram is shown in fig. 8.

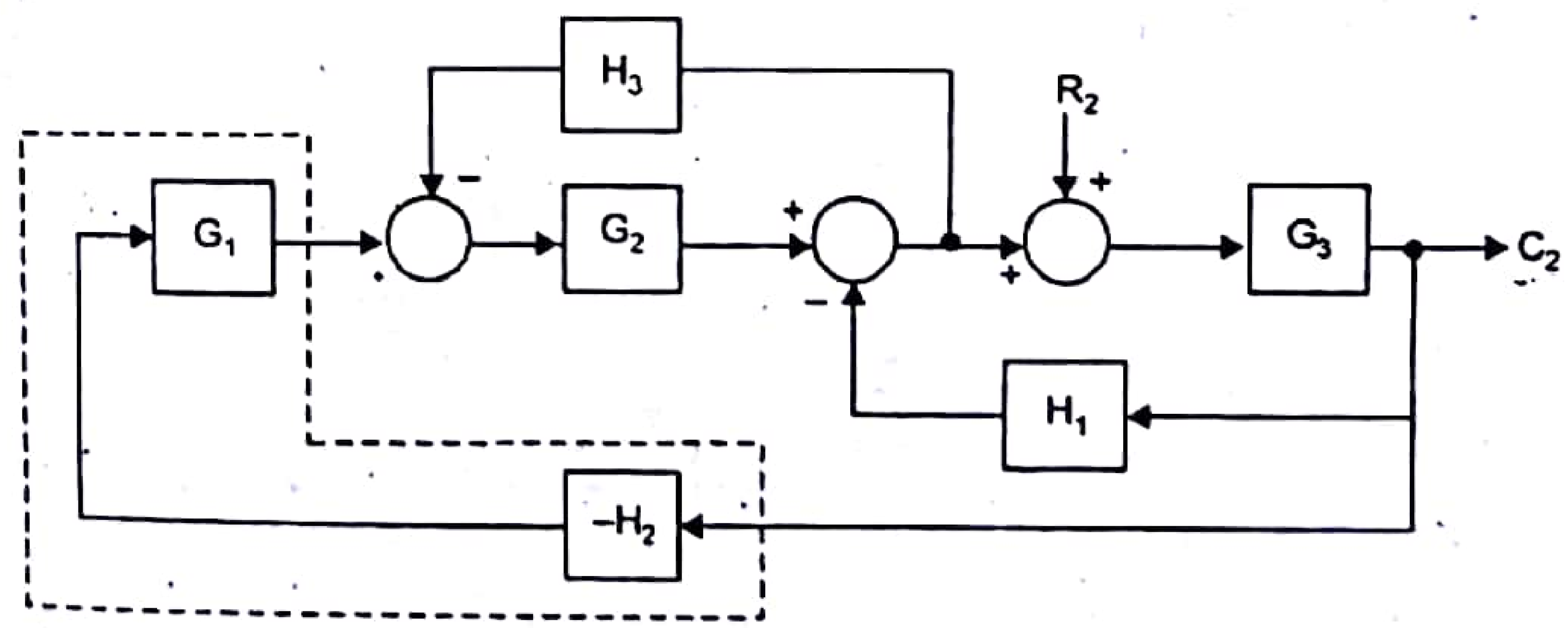


Fig. 8.

Step 1: Combining the G_1 and $-H_2$ in cascade into a single block and rearranging the diagram in Fig. 8, the resluting block diagram is as shown in Fig.9.

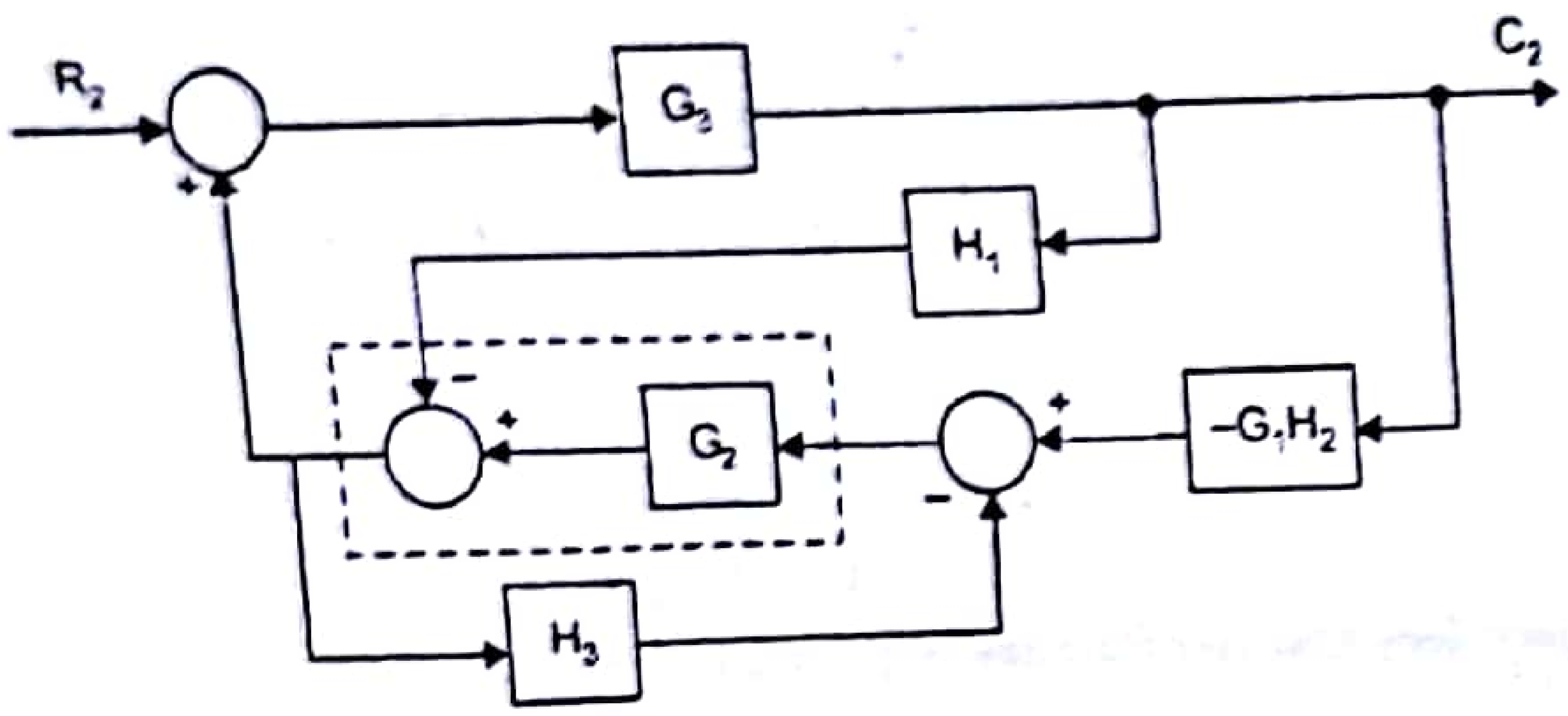


Fig. 9.

Step 2 : Moving the summing point before G_2 in Fig. (9), the block diagram will be as shown in Fig. 10.

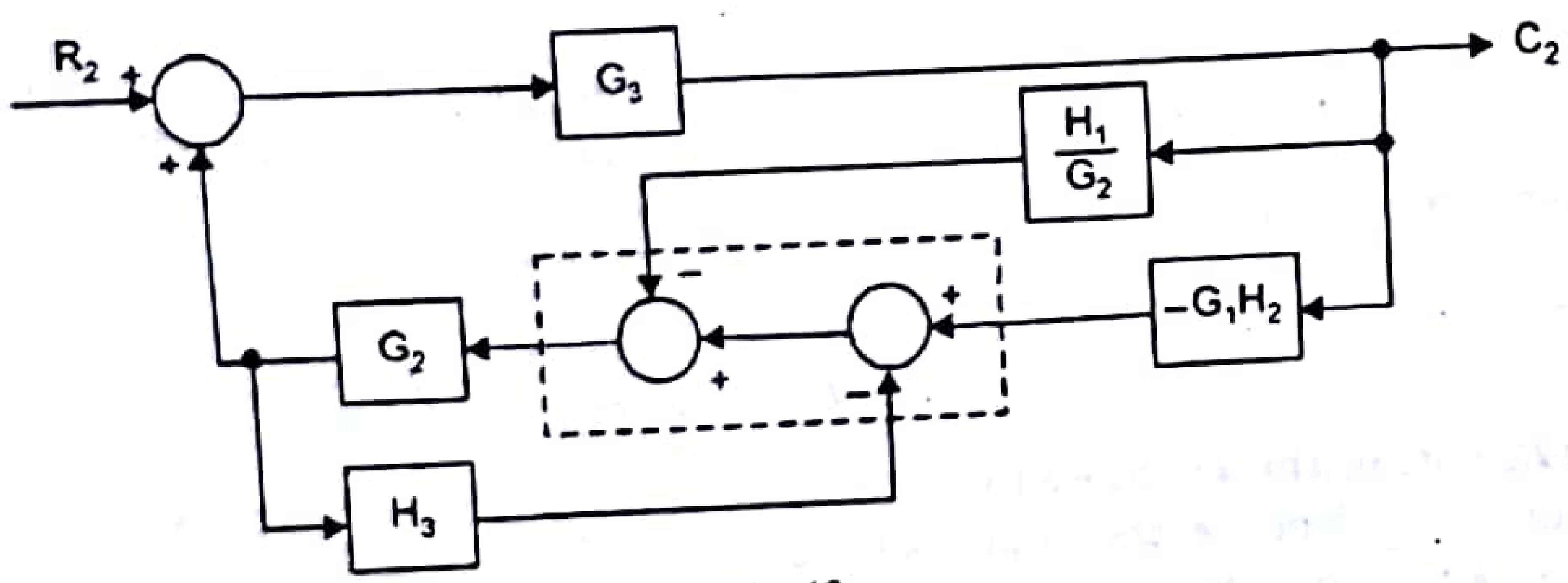


Fig. 10.

Step 3 : Interchanging the summing points in Fig. 10, the block diagram will be as shown in Fig. 11.

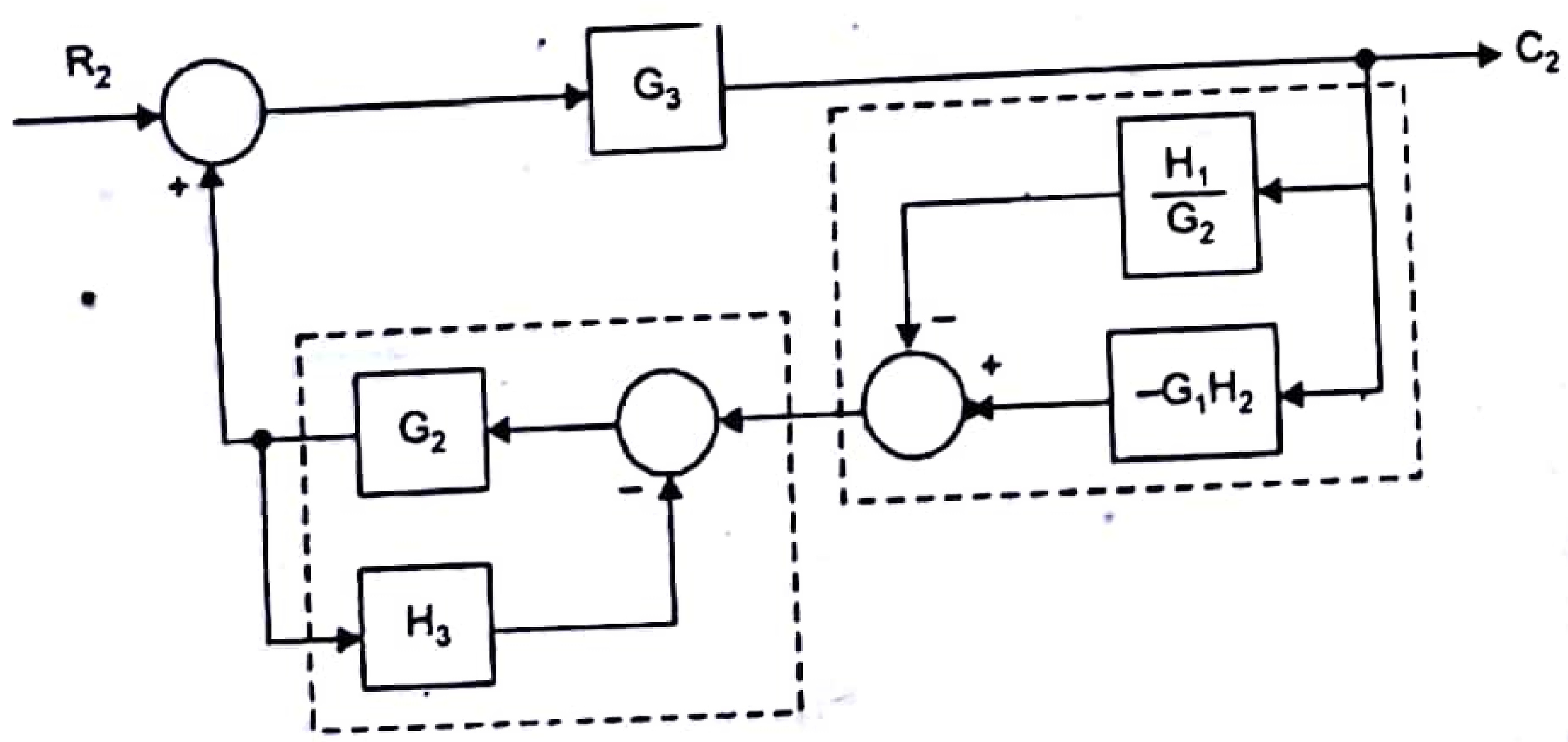


Fig. 11.

Step 4 : Eliminating the loop and combining the parallel block in the feedback path in Fig. 11, the block diagram is shown in Fig. 12.

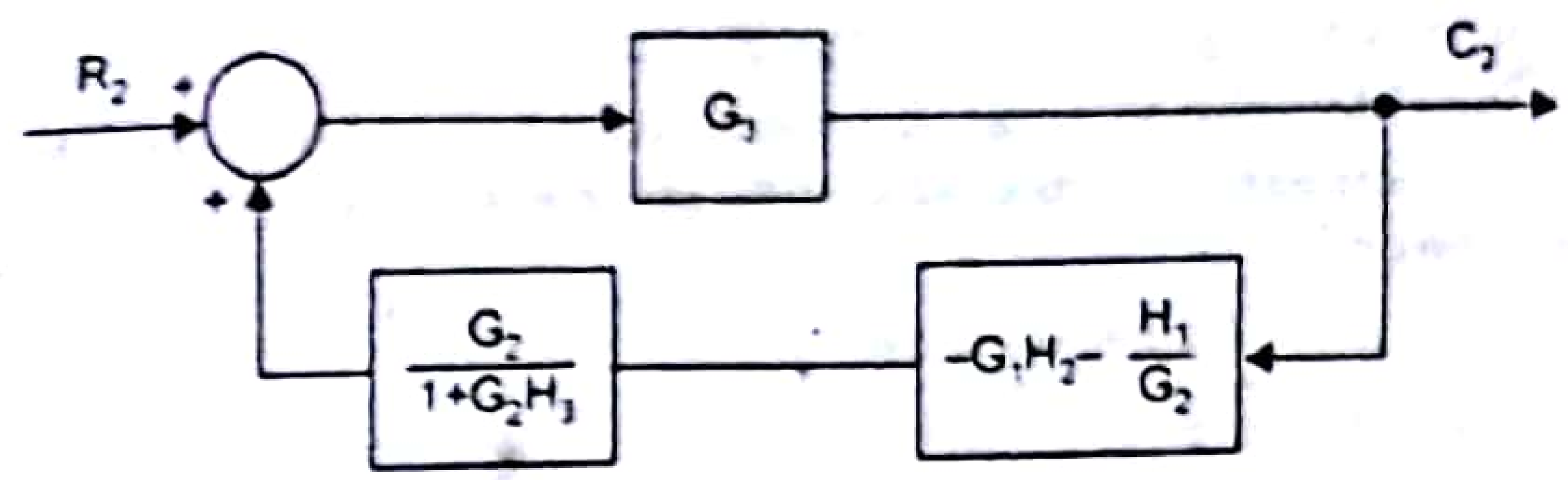


Fig. 12

Step 5 : Combining the blocks in casade in Fig. 12, the block diagram is shown in Fig. 13.

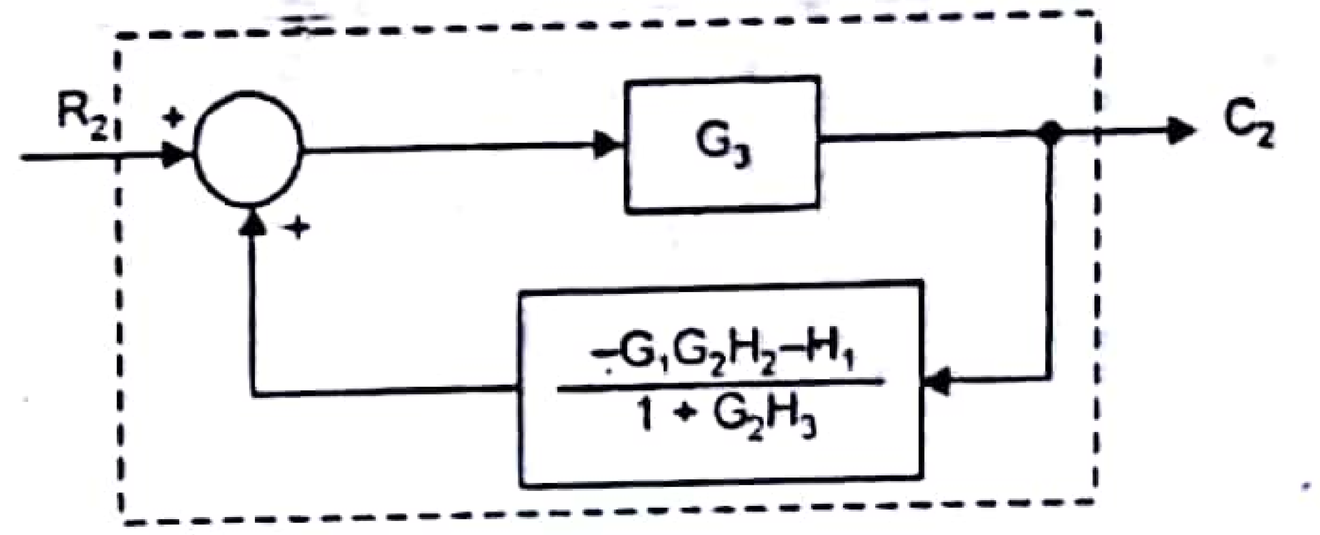


Fig. 13.

Step 6 : Eliminating the only loop in Fig. 13, the transfer function will be given by

$$\frac{C_2}{R_2} = \frac{G_3}{1 - \dfrac{G_3(-G_1G_2H_2 - H_1)}{1 + G_2H_3}} = \frac{G_3(1 + G_2H_3)}{1 + G_2H_3 + G_3H_1 + G_1G_2G_3H_2}$$

Therefore, the output component produced by excitation R_2 alone is

$$C_2 = \frac{G_3(1 + G_2H_3)}{1 + G_2H \mapsto + G_3H_1 + G_1G_2G_3H_2} \cdot R_2$$

By the principle of superpostion, the total output due to both excitations R_1 and R_2 is

$$C = C_1 + C_2 = \frac{G_1G_2G_3}{1 + G_3H_1 + G_2H_3 + G_1G_2G_3G_4H_2} \cdot R_1$$

$$+ \frac{G(1 + G_2H_3)}{1 + G_2H_3 + G_3H_1 + G_1G_2G_3H_2} \cdot R_2$$

Unit-II

Q.4. (a) List some limitations of time domain analysis. **(2.5)**

Ans. (*a*) Limitations of time domain analysis are.

(*i*) They do not give any information on the steady-state error when the inputs are other than the three standard test inputs-step, ramp and parabolic.

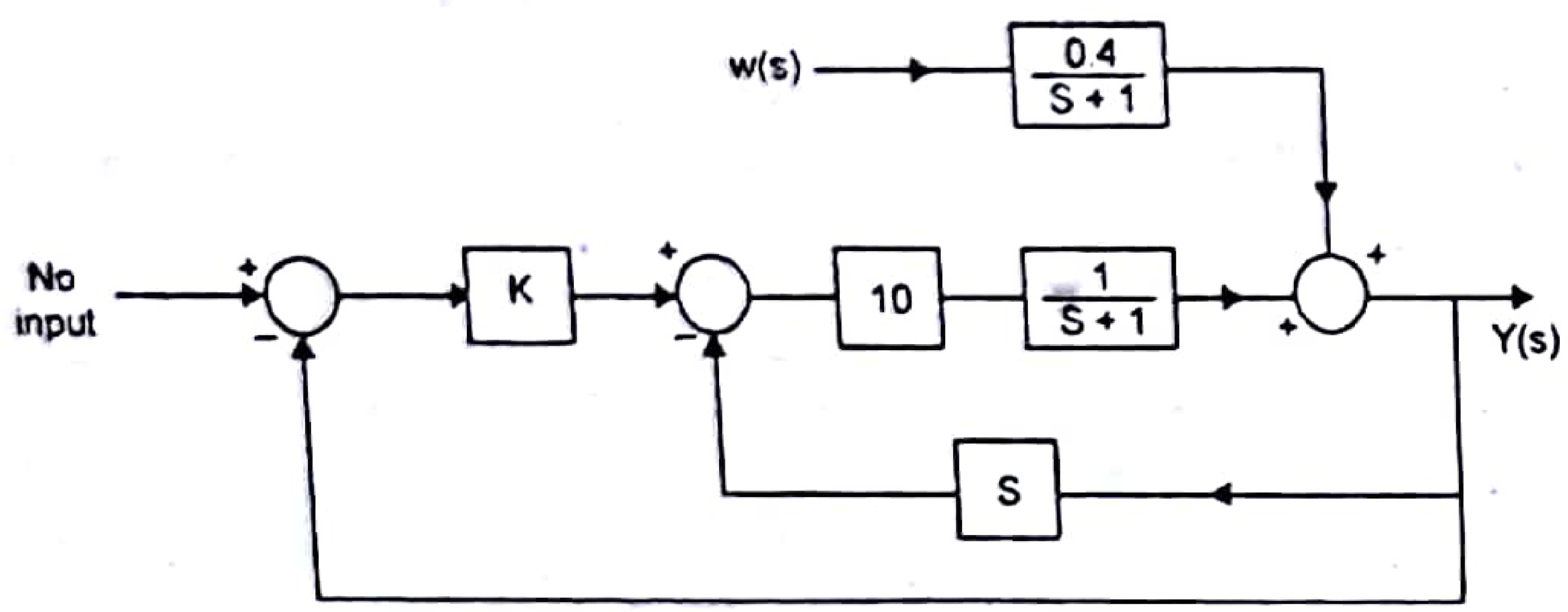
(*ii*) This does not provide precise value of error.

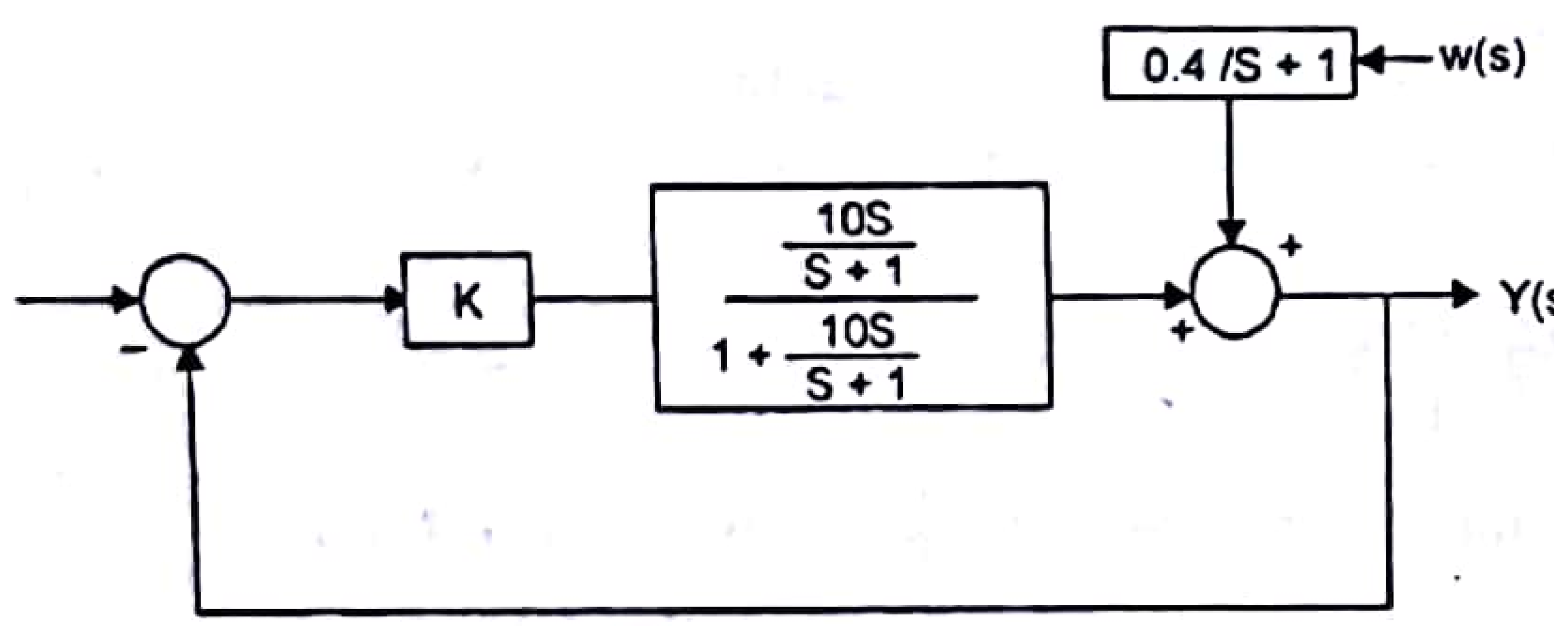
(*iii*) It does not provide variation of error with respect to time which may be needed for design purpose.

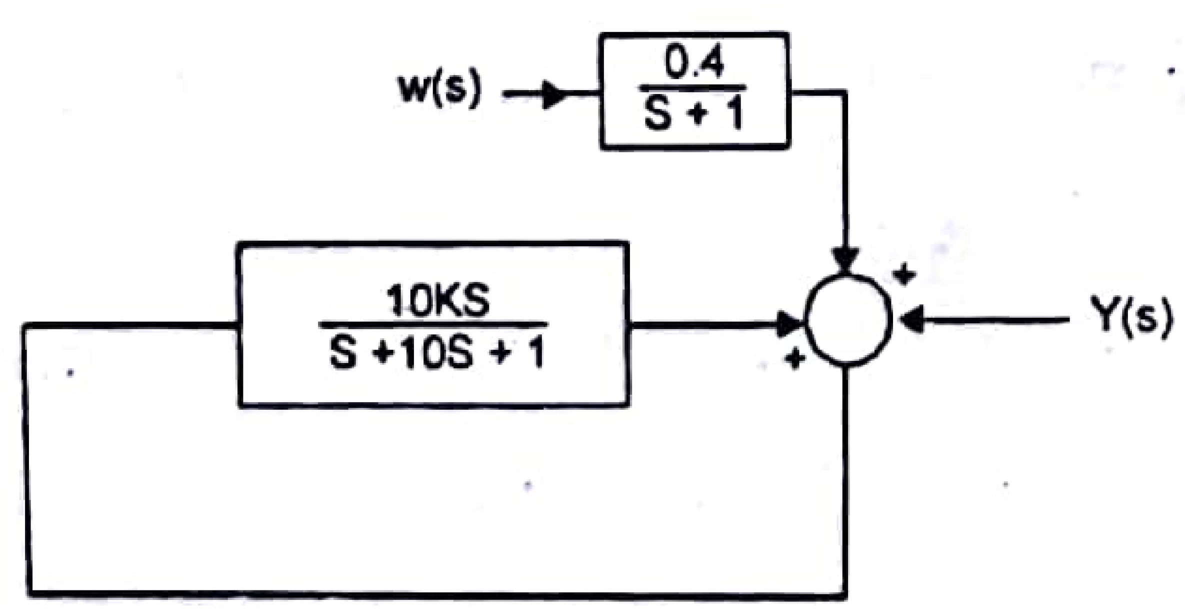
(iv) The method is applicable to stable system.

Q.4.(b) Consider the system shown is figure 3, where w(t) is unit step disturbance. Calculate K such that steady state error in y(t) due to w(t) is less than one percent of w(t). (10)

Ans: From the given figure, we have







$$\frac{y(s)}{w(s)} = \frac{\dfrac{0.4}{S+1}}{\dfrac{10KS}{5+10S+1}} = \frac{0.4(S+10S+1)}{(S+1)(10KS)}$$

We have, for type-0 system

$$e_{ss} = \frac{1}{1+K}$$

Given that $w(t)$ is less than 1% of $w(t)$ which means

$$\frac{1}{100} = \frac{1}{1+K} \Rightarrow 1+K = 100$$

$$\Rightarrow \boxed{K = 99}$$

Q.5. (a) Show with help of examples that introduction of dervatives mode of control in feedback system with proportional control makes the system less oscillatory. What is its effect on steady state error? (6.5)

Ans: Proportional Plus Derivative (Pd) Control: In proportional plus Derivative (PD) controllers, the actuating signal $e_a(t)$ is proportional to the error signal $e(t)$ and also proportional derivative of the error signal. Thus, the actuating signal for proportional plus derivative control is given by

$$e_a(t) = K_p e(t) + K_D \frac{d}{dt} e(t) \quad ...(1)$$

Taking the Laplace transform of both the sides of Eq. (1), we get

$$E_a(s) = K_p E(s) + K_D sE(s)$$

or

$$E_a(s) = (K_p + sK_D) E(s) \quad ...(2)$$

Fig.1. shows the block diagram of a PD control for a second-order system.

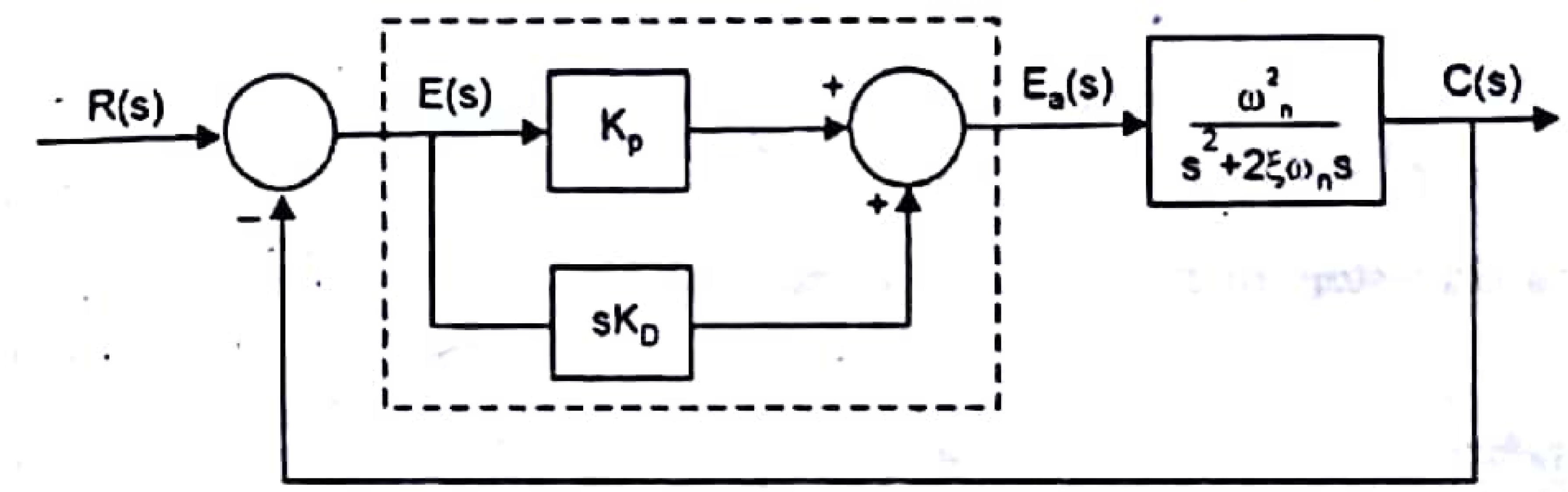


Fig. 1.

From Fig. open-loop transfer function is

$$G(s) = \frac{C(s)}{E(s)}$$

$$= (K_p + sK_D) . \frac{\omega_n^2}{s^2 + 2\xi\omega_n s} \quad ...(3)$$

and

$$H(s) = 1 \quad ...(4)$$

Therefore, closed-loop transfer function of the system is given by

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$= \frac{(K_p + sK_D)\omega_n^2}{s^2 + (2\xi\omega_n + K_D\omega_n^2)s + \omega_n^2 K_p} \quad ...(5)$$

The characteristic equation of the system given by the denominator of Eq. (5) is

$$s^2 + (2\xi\omega_n + K_D\,\omega_n^2)s + K_p\,\omega_n^2 = 0 \quad ...(6)$$

The standard equation of a second-order system is

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \quad ...(7)$$

We shall compare (Eq.6) with standard Eq. (7).

Now,
$$2\xi\omega_n + K_D\omega_n^2 = 2\left(\xi + \frac{1}{2}K_D\omega_n\right)\omega_n = 2\xi'\omega_n$$

where
$$\xi' = \left(\xi + \frac{1}{2}K_D\omega_n\right) \quad ...(8)$$

Equation (8) shows that effective damping has increased uisng PD control. This makes the system response slower with less overshoots increasing delay time. Proportional derivative control will not affect the steady-state error of the system.

Q.5. (b) Consider a second order model

$$\frac{y(s)}{R(s)} = G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n S + \omega_n^2}, 0 < \xi < 1.$$

Find the reponse y(t) to the input (*i*) $r(t) = \mu(t)$, a unit step function. (ii) $r(t) = t\mu(t)$, a unit ramp function. **(6)**

Ans: (*i*) The output of the system is given by

$$C(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n + \omega_n^2}R(s)$$

For a Unit-step input $r(t) = 1$ and $R(s) = \frac{1}{s}$

Therefore,
$$C(s) = \frac{\omega_n^2}{(s^2 + 2\xi\omega_n s + \omega_n^2)s} \quad ...(1)$$

$$= \frac{1}{s} - \frac{s + 2\xi\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

or
$$C(s) = \frac{1}{s} - \frac{s + \xi\omega_n}{(s + \xi\omega_n)^2 + \omega_d^2} - \frac{\xi\omega_n}{(s + \xi\omega_n)^2 + \omega_d^2} \quad ...(2)$$

We know that,
$$= L^{-1}\left[\frac{s + \xi\omega_n}{(s + \xi\omega_n)^2 + \omega_d^2}\right] = e^{-\xi\omega_n t}\cos\omega_d t \quad ...(3)$$

$$L^{-1}\left[\frac{\omega_d}{(s + \xi\omega_n)^2 + \omega_d^2}\right] = e^{-\xi\omega_n t}\sin\omega_d t \quad ...(4)$$

Taking the inverse Laplace transform of Eq. (2), we get

$$c(t) = L^{-1}C(s) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi\omega_n}{\omega_d}e^{-\xi\omega_n t}\cos\omega_d t \quad ...(5)$$

Since
$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

Eq. (5) may be written as

$$c(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi}{\sqrt{1-\xi^2}}e^{-\xi\omega_n t}\sin\omega_d t$$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\left(\sqrt{1-\xi^2}\right)\cos\omega_d t + \xi\sin\omega_d t\right] \quad ...(6)$$

Put
$$\sqrt{1-\xi^2} = \sin\phi$$

Therefore,
$$\cos\phi = \xi \text{ and } \tan\phi = \frac{\sqrt{1-\xi^2}}{\xi}$$

Therefore Eq. (6) can be written as

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}[\sin\phi\cos\omega_d t + \cos\phi\sin\omega_d t] \quad ...(7)$$

$$c(t) = 1 - \frac{e - \xi\omega_n t}{\sqrt{1-\xi^2}}\sin(\omega_d t + \phi) \quad ...(8)$$

$$\tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{\sqrt{1-\xi^2}}{\xi}$$

$$\phi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} \text{ radians} \quad ...(9)$$

Equation (8) can be written as

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\left[\omega_d t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right] \text{ for } t \geq 0 \quad ...(10)$$

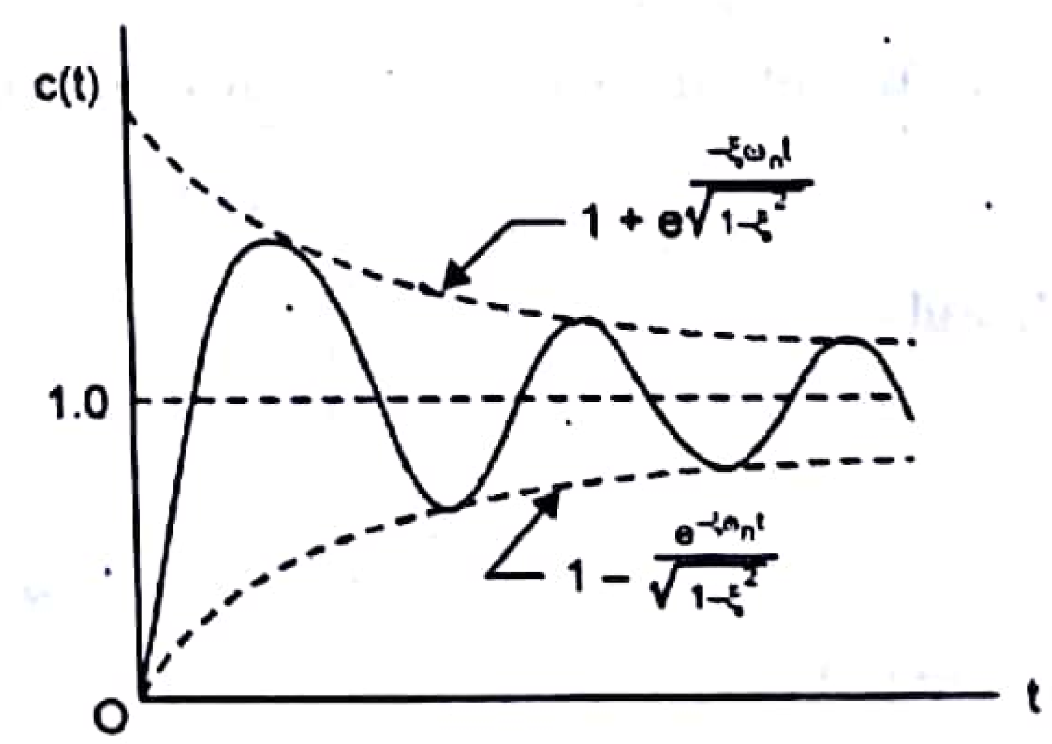


Fig.(1) Unit-step response of a second-order system

Figure 11 shows the response c(t) of Eq. (10) for $0<\xi<1$. It is a sinusoid decaying on an exponetial envelope.

For a step of A units, the expression for c(t) is given by

$$c(t) = A\left[1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t+\phi)\right] \quad ...(11)$$

(ii) Time Response of Second-Order System with Unit Ramp Functions

We have, $$C(s) = \frac{\omega_n^2}{s^2+2\zeta w_n s+\omega_n^2}R(s) \quad ...(1)$$

If the input is a unit ramp function

$$r(t) = t, R(s) = \frac{1}{s^2}$$

Therefore, $$C(s) = \frac{\omega_n^2}{s^2\left(s^2+2\zeta\omega_n s+\omega_n^2\right)} \quad ...(2)$$

Resolving Eq. (2) into partial fractions

$$\frac{\omega_n^2}{s^2\left(s^2+2\zeta\omega_n s+\omega_n^2\right)} = \frac{A_1}{s}+\frac{A_2}{s^2}+\frac{A_3 s+A_4}{s^2+2\zeta\omega_n s+\omega_n^2} \quad ...(3)$$

Therefore $\omega_n^2 = A_1 s\,(s^2+2\zeta\omega_n s+\omega_n^2)+A_2\,(s^2+2\xi\omega_n s+\omega_n^2)+(A_3 s+A_4)s^2$ ...(4)

Equating the coefficients of s^3 on both sides, we get

$$A_1+A_3 = 0 \quad ...(5)$$

Equating the coeffcients of s^2 on both sides gives

$$2\zeta\omega_n A_1+A_2+A_4 = 0 \quad ...(6)$$

Equating the coefficients of s on both sides

$$A_1\omega_n^2+2\xi\omega_n A_2 = 0 \quad ...(7)$$

Equating the coefficients of the terms without s on both sides,

$$A_2\omega_n^2 = \omega_n^2 \quad ...(8)$$

From Eq. (8), $$A_2 = 1 \quad ...(9)$$

From Eqs. (7) and (9),

$$A_1\omega_n^2+2\zeta\omega_n.1 = 0$$

Therefore, $$A_1 = -\frac{2\xi}{\omega_n} \quad ...(10)$$

From Eqs. (5) and (10),

$$A_3 = -A_1 = \frac{2\zeta}{\omega_n} \quad ...(11)$$

Subsitituting the values of A_1 and A_2 in Eq. (6), we get

$$2\zeta\omega_n\left(-\frac{2\zeta}{\omega_n}\right)+1+A_4 = 0$$

Therefore,

$$A_4 = -1+4\zeta^2 \quad ...(12)$$

Substitution of the values of $A_1, A_2\ A_3$ and A_4 in Eq. (3) gives

$$C(s) = \frac{-2\xi}{\omega_n s}+\frac{1}{s^2}+\frac{\frac{2\xi}{\omega_n}s-(1-4\xi^2)}{s^2+2\xi\omega_n s+\omega_n^2} \quad ...(13)$$

$$s^2+2\zeta\omega_n s+\omega_n^2 = (s+\zeta\omega_n)^2+\omega_n^2-\zeta^2\omega_n^2$$
$$= (s+\zeta\omega_n)^2+\omega_n^2(1-\zeta^2)$$

Let $$\beta = \sqrt{1-\zeta^2} = \sin\phi \quad ...(14)$$

Therefore,

$$s^2+2\zeta\omega_n s+\omega_n^2 = (s+\zeta\omega_n)^2+(\beta\omega_n)^2 \quad ...(15)$$

Therefore, Eq. (13), becomes

$$C(s) = \frac{-2\zeta}{\omega_n s}+\frac{1}{s^2}+(s+\zeta\omega_n)^2+(\beta\omega_n)^2\frac{2\zeta}{\omega_n}s-(1-4\zeta^2)$$

$$= \frac{-2\zeta}{\omega_n s}+\frac{1}{s^2}+\frac{\frac{2\zeta}{\omega_n}(s+\zeta\omega_n)-2\zeta^2-1+4\zeta^2}{(s+\zeta\omega_n)^2+(\beta\omega_n)^2}$$

$$= \frac{-2\zeta}{\omega_n s}.\frac{1}{s^2}+\frac{1}{s^2}+\frac{2\zeta}{\omega_n}.\frac{(s+\zeta\omega_n)}{(s+\zeta\omega_n)^2+(\beta\omega_n)^2}$$

$$-\frac{(1-2\zeta^2).(\beta\omega_n)}{\beta\omega_n\left[(s+\zeta\omega_n)^2+(\beta\omega_n)^2\right]} \quad ...(16)$$

Taking the inverse Laplace of Eq. (16), we obtain

$$c(t) = t-\frac{2\zeta}{\omega_n}+e^{-\zeta\omega_n t}\left[\frac{2\zeta}{\omega_n}\cos\beta\omega_n t-\frac{1-2\zeta^2}{\beta\omega_n}\sin\beta\omega_n t\right]$$

Unit-III

Q.6. Draw bode magnitude and phase plot of the following transfer function. Find its gain cross over frequency, gain magin and phases margin

$$G(S) = \frac{200(s+2)}{s(s^2+10s+100)} \quad (12.5)$$

Ans: Bode Plot

$$H(s) = 1$$

$$G(s) = \frac{200(s+2)}{s(s^2+105+100)}$$

Standard form of open loop transfer function

$$G(s)H(s) = \frac{k(1+sT_1)(1+sT_2)}{S^N(1+ST_a)(1+ST_b)(1+sT_c)} \quad ...(1)$$

To make actual open loop function comparable with equation (1)

$$G(s)H(s) = \frac{200\times 2(s/2+1)}{s(5-8.6j)\left(\frac{s}{5-8.6j}+1\right)(5+8.6j)\left(\frac{8}{5+8.6j}+1\right)}$$

$$= \frac{400(1+s/2)}{98.96s\left(1+\frac{s}{5-8.6j}\right)\left(1+\frac{s}{5+8.6j}\right)}$$

$$= \frac{4.042(1+s/2)}{s\left(1+\frac{s}{5-8.6j}\right)\left(1+\frac{s}{5+8.6j}\right)}$$

Corner frequenies

$w_1 = 2$ (zero)

$w_2 = 5, 5$ (double pole)

It can never be complex.

System type = 1

$\therefore$ Initial slope $= -20 \times 1 = -20$ dB/decate ($\because$ N = 1)

This line will pass through

$w = k^{1/N}$ rad/sec = 4.042 rad/sec

For Magnitude Graph

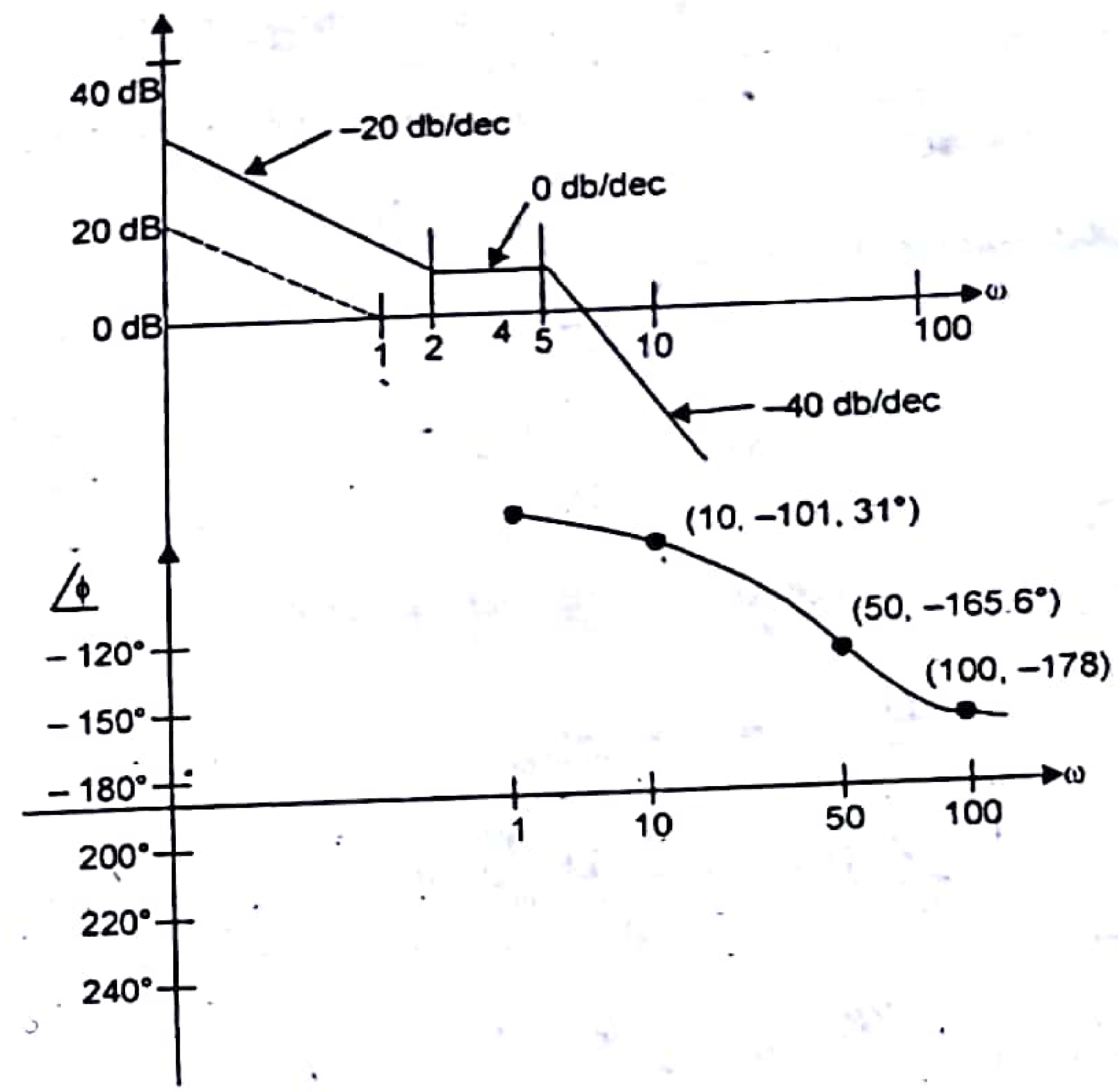
Factor	Corner freq (rad/sec)	Slope (dB/dec)	Angle (degree)
$\frac{4.042}{jw}$	—	–20 (initial)	–90°
$1 + jw/2$	2	+ 20	$\tan^{-1}(w/2)$
$\frac{1}{\left(1+\frac{jw}{5-8.6j}\right)}$	5	– 20	$-\tan^{-1}\left(\frac{w}{5-8.6j}\right)$
$\frac{1}{\left(1+\frac{jw}{5+8.6j}\right)}$	5	– 20	$-\tan^{-1}\left(\frac{w}{5+8.6j}\right)$

For Phase Graph

Total angle for the system.

$$\theta = -90 + \tan^{-1}\left(\frac{w}{2}\right) - \tan^{-1}\left(\frac{w}{5-8.6j}\right) - \tan^{-1}\left(\frac{w}{5+8.6j}\right)$$

w	θ
10	–101.31°
50	–165.68°
100	–178.1°



Q.7. (a) Draw polar plot of the given system $G(s)H(s) = \frac{20}{s(s+2)(s+4)}$.(6.5)

Ans: Given that the system

$$G(s)H(s) = \frac{20}{s(s+2)(s+4)}$$

Put $s = j\omega$, then

$$G(j\omega)H(j\omega) = \frac{20}{jw(jw+2)(jw+4)}$$

$$M = |G(jw)H(jw)| = \frac{20}{w\sqrt{w^2+4}\sqrt{w^2+16}}$$

$$\phi = \angle G(jw)H(jw) = -90° - \tan^{-1}\frac{w}{2} - \tan^{-1}\frac{w}{4}$$

ω is varied from 0 to ∞ and the values of M and ϕ are calculated as below

ω	0	0.05	0.1	0.4	0.8	1	5	10	100	∞
M	∞	50	25	6.098	2.845	2.5	0.116	0.0182	0.0000199	0
ϕ	−90°	−92.148°	−94.29°	−107°	−123°	−130°	−209°	−236°	−266.550	−270°

Q.7. (*b*)Exaplain in detail effect of adding a pole and zero on polar plot.(6)

Ans: Addition of a non-zero pole to the transfer function results in further rotation of the end points of polar plot through an angle of 90°.

When a pole at origin is added to the transfer function, it rotates the entire polar plot by a further angle of 90°.

Ex. Sketch polar plot of $G(s) = \frac{1}{s^2}$

Put $$s = j\omega$$

$$G(j\omega) = \frac{1}{(j\omega)(j\omega)}$$

$$|G(j\omega)| = \frac{1}{\omega^2} \text{ and } \phi = -180°$$

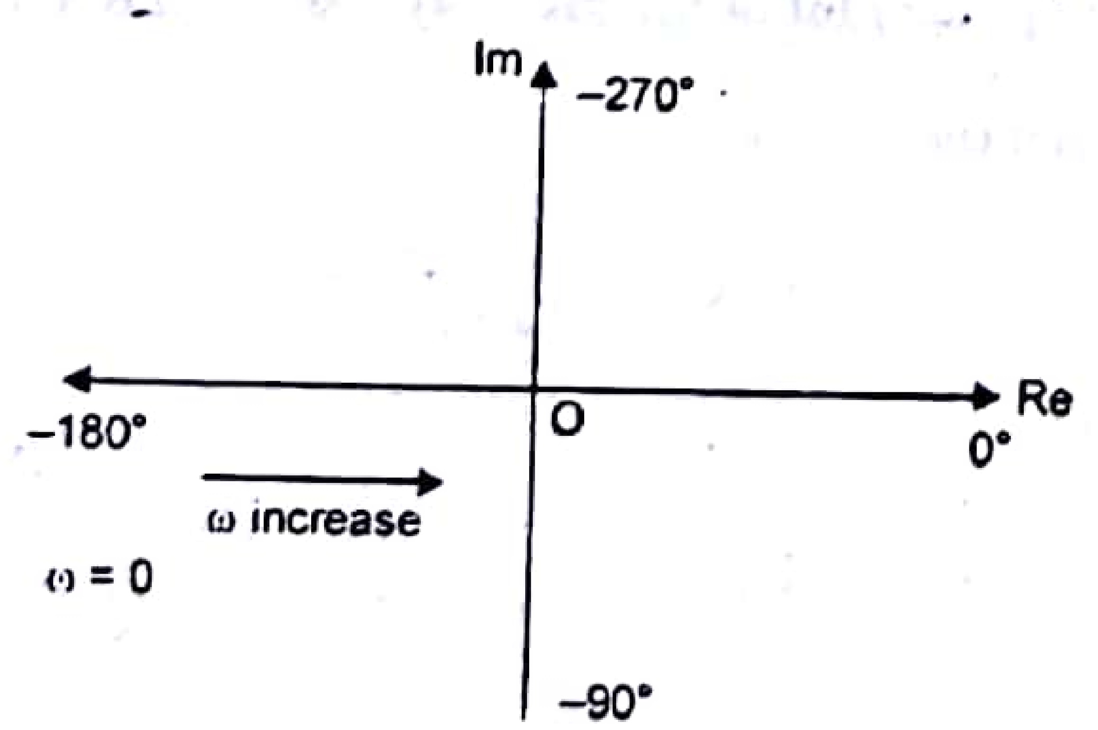
$$\underset{\omega\to 0}{\text{Lt}} |G(j\omega)| = \underset{\omega\to 0}{\text{Lt}} \frac{1}{\omega^2} = \infty$$

$$\underset{\omega\to \infty}{\text{Lt}} |G(j\omega)| = \underset{\omega\to \infty}{\text{Lt}} \frac{1}{\omega^2} = 0$$

Therefore,

$$G(j\omega)\big|_{\omega=0} = \infty < -180°$$

$$G(j\omega)\big|_{\omega\to\infty} = 0 < -180°$$



Unit-IV

Q.8. Sketch root locus of the given characteristic equation $F(s) = s\,(s + 4)(s^2 + 4s + 20) + k = 0$. (12.5)

Ans: Open loop poles are roots of denominator polynomial.

(i.e) $$s\,(s+4)\,(s^2 + 4\,s + 20) = 0$$

$$p_1 = 0, p_2 = -4, p_{3,4} = \frac{-4 \pm \sqrt{16-80}}{2} \text{ or}$$

$$p_{3,4} = -2 \pm j4$$

Mark the poles on a graph sheet with × symbol.

Step 1 : There are four open loop poles, hence the number of branches in the root locus is four.

Step 2 : The four root branches start at $p_1 = 0, p_2 = -4, p_3 = -2 + j4$ and $p_4 = -2 - j4$ when k = 0 and terminate at infinity when k = ∞.

Step 3 : All the points between −4 and 0 on the real axis lie on the root locus since odd number of poles and zeros on the real axis to the right of these points.

Step 4: The four brancehs that terminate at infintiy do so along the asymptotes with angles.

$$\phi_A = \frac{(2q+1)180°}{n-m}; q = 0, 1, 2, 3 \ldots (n-m-1)$$

$$\phi_A = \frac{(2q+1)180°}{4}; q = 0, 1, 2, 3$$

For q = 0; $\phi_A = 45°$

For q = 1; $\phi_A = 135°$

For q = 2; $\phi_A = 225°$

For q = 3 ; $\phi_A = 315°$

Step 5 : The asymptotes meet at a point known as centroid.

$$\sigma_A = \frac{\text{sun of real part of poles-sum of real partz of zeros}}{\text{Number of poles-Number of zeros}}$$

$$\sigma_A = \frac{0-4-2-2}{4} = -2$$

Mark the centroid on the real axis and draw the asympotes with angles calculated in step 4 using protractor.

Step 6 : The break away points of the root locus are the solution of $\frac{dk}{ds} = 0$.

Given $$G(s) = \frac{k}{s(s+4)(s^2+4s+20)} = 0; H(s) = 0$$

The characteristic equation is

$$1 + \frac{k}{s(s+4)(s^2+4s+20)} = 0$$

$$k = -s\,(s+4)\,(s^2 + 4s + 20)$$
$$= -(s^2 + 4\,s\,)\,(s^2 + 4\,s + 20)$$
$$= s^4 + 8s^3 + 36\,s^2 + 80\,s$$

$$\frac{dk}{ds} = 4s^3 + 24\,s^2 + 72s + 80 = 0$$

$$4s^3 + 24s^2 + 72s + 80 = 0 \Rightarrow s^3 + 6s^3 + 18s + 20 = 0$$

$$\begin{array}{c|cccc} & 1 & 6 & 18 & 20 \\ -2 & 0 & -2 & -8 & -20 \\ \hline & 1 & 4 & 10 & \lfloor 0 \end{array}$$

When $s = 0$ $\quad \frac{dk}{ds} = 80$ (+ve)

$$s^2 + 4s + 10 = 0$$

When $s = -4$ $\quad \frac{dk}{ds} = 80$ (–ve)

When $s = -2$ $\quad \frac{dk}{ds} = 0$

$\therefore$ The break away point is -2

The roots of $s^2 + 4s + 10 = 0$ are

$$\frac{-4 \pm \sqrt{16-40}}{2};\ \frac{-4 \pm j2\sqrt{6}}{2};\ -2 \pm j\sqrt{6}$$

The other breakway points are $-2 \pm j2.45$.

Step 7 : The angle of departure $\phi_{p3} = 180° + \phi$

Where $\phi = -\theta_1 - \theta_2 - \theta_4$

$$= -\left(180° - \tan^{-1}\tfrac{4}{2}\right) - \tan^{-1}\frac{4}{2} - 90°$$

$$\therefore \quad \phi_{p3} = 180° - (180° - 63.43°)$$
$$- 63.43° - 90°$$
$$= 180° - 270° = -90°$$

Similarly,

$$\phi_{p4} = +90°$$

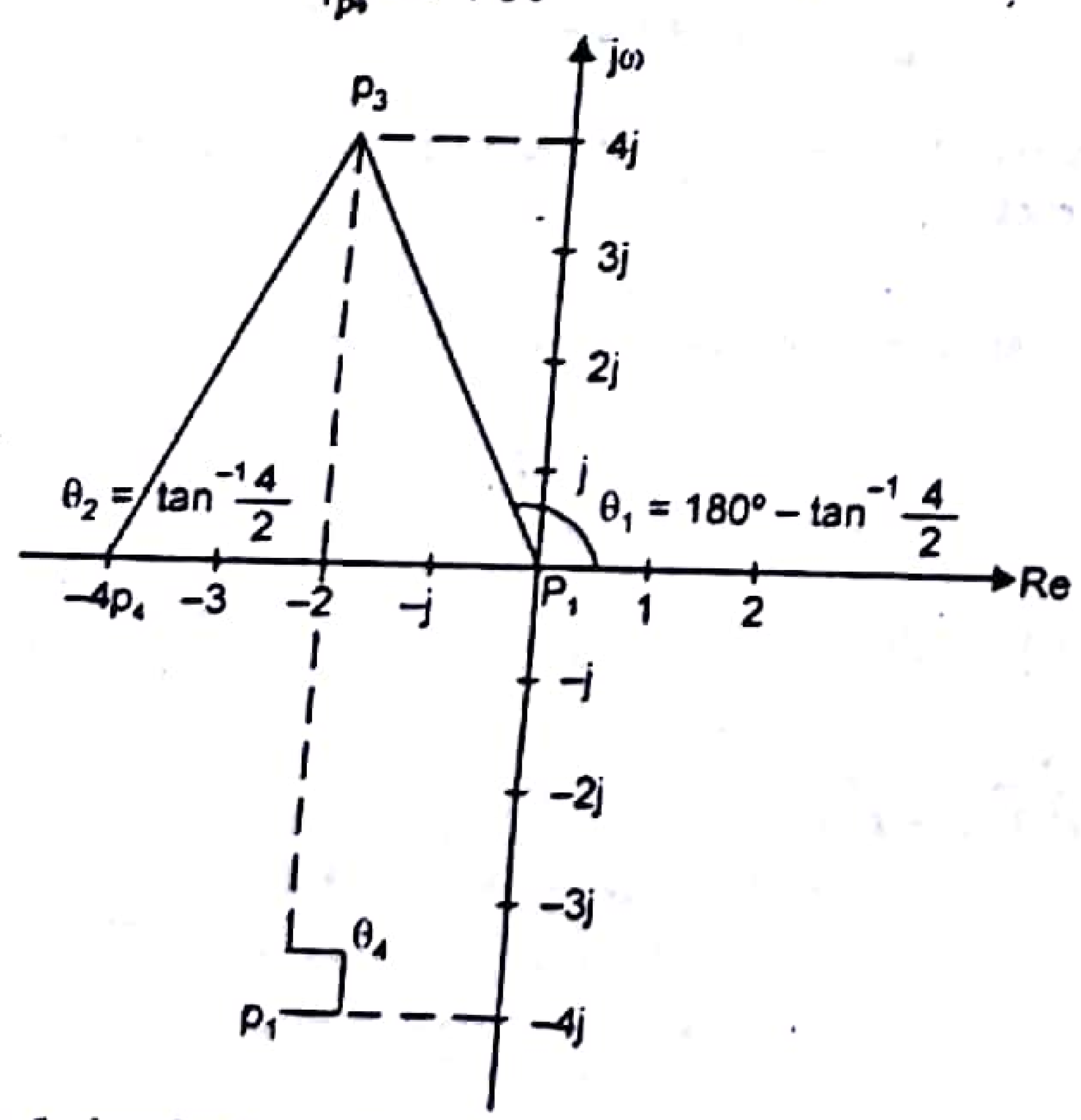


Fig. 1. Angle Contribution of poles p_1, p_2, p_4, to p_3

Step 8: The crossing point on the imaginary axis can be find using the routh criterion. The characteristic equation is given by

$$1 + G(s)H(s) = 0$$

$$1 + \frac{k}{s(s+4)(s^2+4s+20)} = 0$$

$$s(s+4)(s^2+4s+20) + k = 0$$

$$s^4 + 8s^3 + 36s^2 + 80s + k = 0$$

$$\begin{array}{c|ccc} s^4 & 1 & 36 & k \\ s^3 & 8 & 80 & 0 \\ s^2 & 26 & k & \\ s^1 & \frac{2080-8k}{26} & 0 & \\ s^0 & k & & \end{array}$$

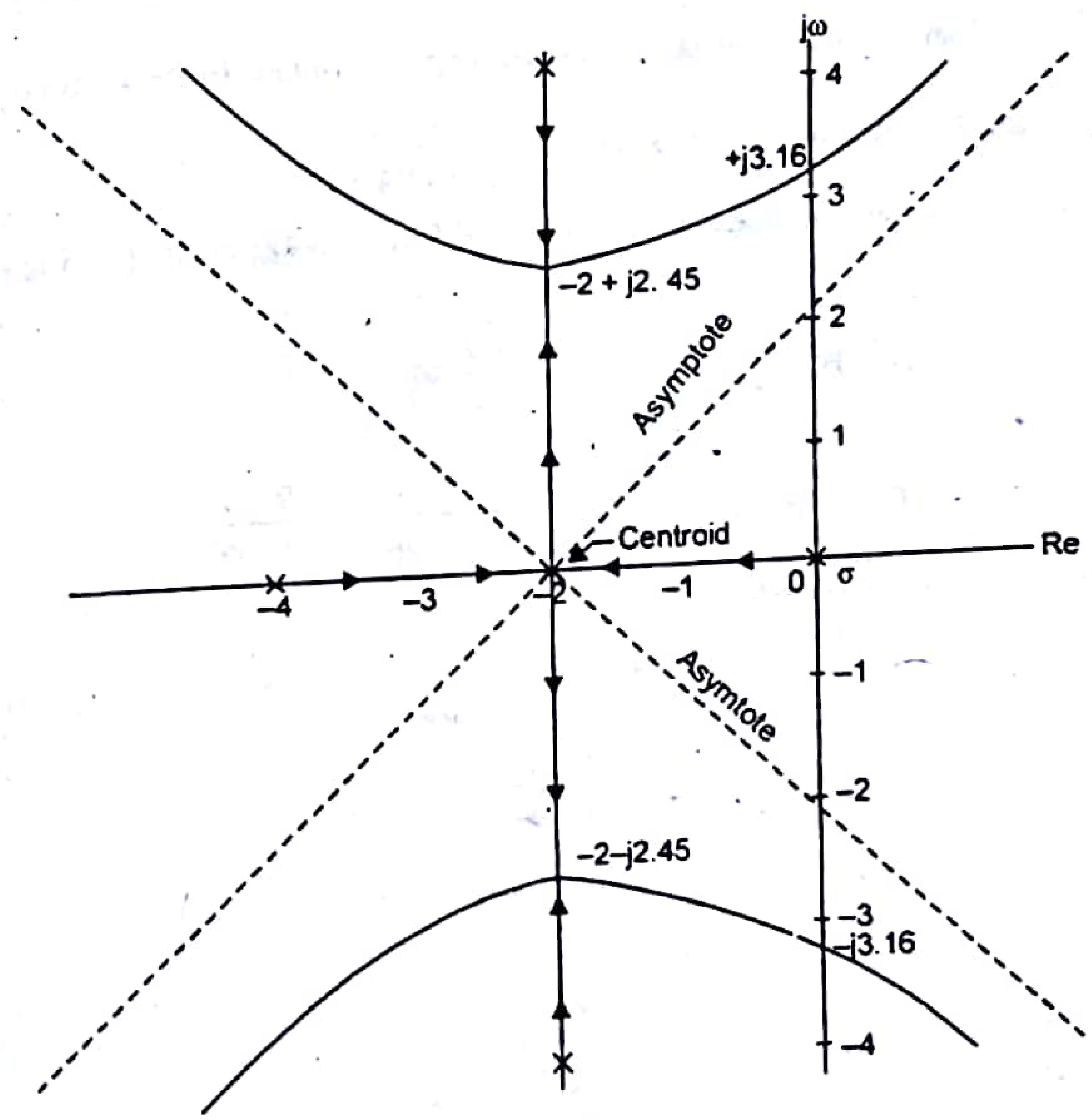


Fig.2. Root locus of the septem

$$\frac{2080-8k}{26} = 0$$

$$\Rightarrow \quad k = 260$$

$$\therefore \qquad 0 < k < 260$$

when k = 260 the root locus corsses imaginary axis, For k = 260m the auxlia equation is

$$26s^2 + 260 = 0$$

$$s^2 = -\frac{260}{26} = -10$$

$$\therefore \qquad s = \pm j\sqrt{10} = \pm j3.16.$$

The complete root locus plot is shown in Fig.2. (2)

Q.9. (a) State and explain Nyquist Stability Criteria. (2.5)

Ans: Nyquist criterion is used to identify the presence of roots of a characteristic equation of a control systemin a specific region of s-plane. From the stability view point the specified region being the entire right hand side beyond the imaginary axis of complex s-plane.

Q.9. (b) Find the stability of the system using nyquist stability criteria whose open loop transfer function is $G(s)H(s) = \frac{(s+2)}{(s+1)(s-1)}$. (10)

Ans: The given transfer function in sinusodal from is obtained by putting $s = j\omega$ in G (s) H (s).

$$G(s)H(s)\big|_{s=j\omega} = \frac{j\omega + 2}{(j\omega+1)(j\omega-1)}$$

$$G(j\omega)H(j\omega) = -\frac{2+j\omega}{(1+j\omega)(1-j\omega)} = -\frac{2+j\omega}{1+\omega^2}$$

The Nyquist path is shown in Fig.1.

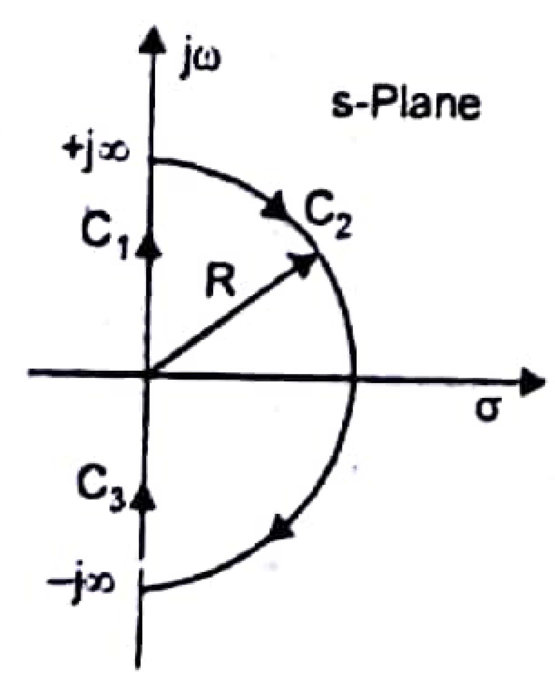


Fig. 1

It is divided into three section C_1, C_2 and C_3. We map each section from s plane to GH plane.

***(i)* Mapping of section C_1**

In this section of the Nyquist contour on the jω axis, s various from 0^+ to $+\infty$

$$G(j\omega)H(j\omega) = \frac{-2}{1+\omega^2} - j\frac{\omega}{1+\omega^2}$$

At $\omega = 0^+$
$$G(j\omega)H(j\omega) = \frac{-2}{1+(0)^2} - j\frac{(0)}{1+(0)^2} = 2 - j0$$

At $\omega = \infty$
$$G(j\omega)H(j\omega) = \frac{-2}{1+(\infty)^2} - j\frac{\infty}{1+(\infty)^2} = -0 - j0$$

As the exact shape of the $G(s)H(s)$ is not required, we draw the part of GH contour by noting that the magnitude of GH changed form −2 to 0 as ω changing from 0 to ∞.

The mapping of section C_1 from C contour to C_5 contour is shown in Fig. (2).

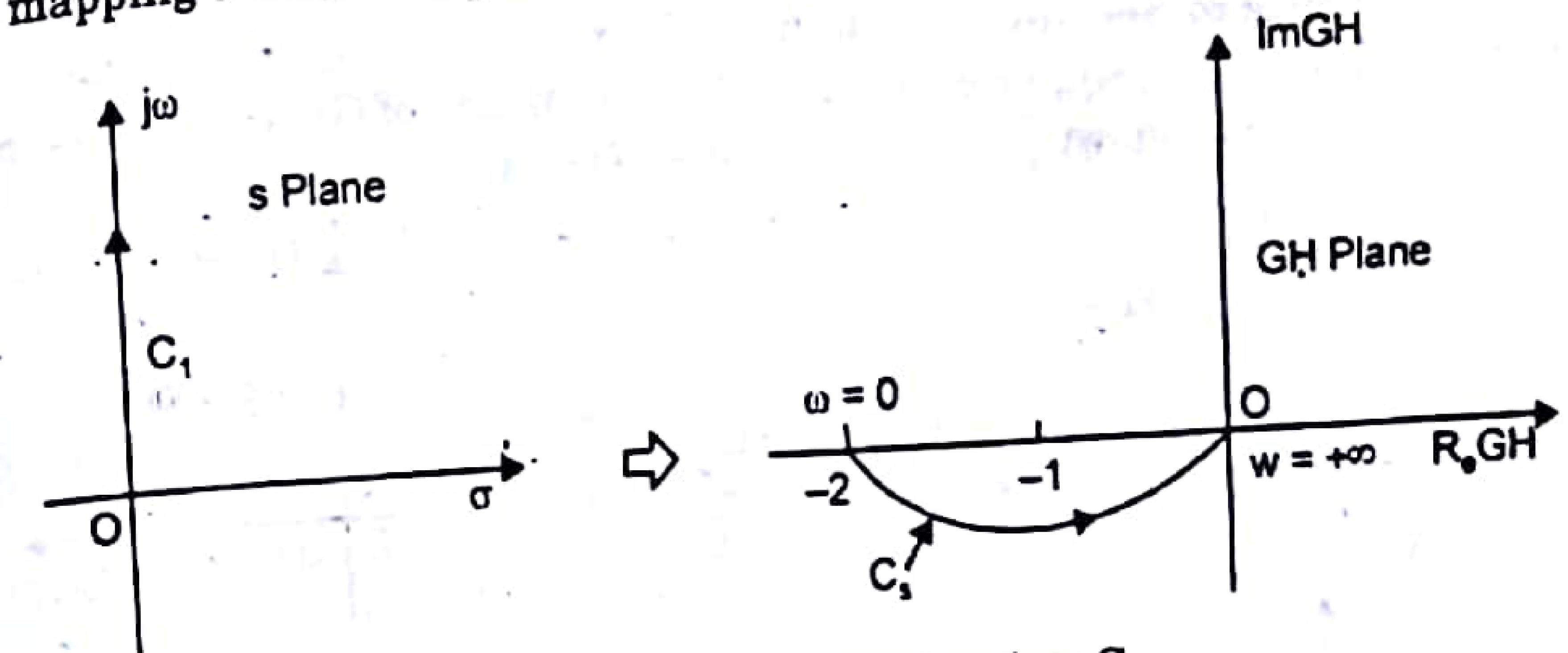


Fig. 2. Mapping of Section C_1

***(ii)* Mapping of section C_2 :** Section C_2 in is plane is a semicircle of infinite radius.

This section can be mapped in G(s) H (s) plane by substituting $s = \lim_{R\to\infty} Re^{j\theta}$ in G (s) H (s) and varying θ form + 90° to 90° through 0°.

$$\lim_{R\to\infty} G(s)H(s)\Big|_{s=Re^{j\theta}} = \lim_{R\to\infty}\frac{1}{Re^{j\theta}}$$

As R →∞,

$(Re^{j\theta}+2)$ is approximated to $Re^{j\theta}$, $(Re^{j\theta}+1)$ is approximated to $Re^{j\theta}$ and $(Re^{j\theta}-1)$ is approximated to $Re^{j\theta}$.

Therefore,

$$\lim_{R\to\infty} G(s)H(s)\Big|_{s=Re^{j\theta}} = \lim_{R\to\infty}\frac{Re^{j\theta}}{(Re^{j\theta})(Re^{j\theta})}$$

$$= \lim_{R\to\infty}\frac{1}{Re^{j\theta}} = 0e^{-j\theta}$$

That is, for θ varying form + 90° to −90° through 0°, the section C_2 in the s plane is mapped into a point at the origin of G (s) H (s) plane.

The points A, B, C of the C counter correspond to the mapped C_s plot at points A', B' and C' respectively.

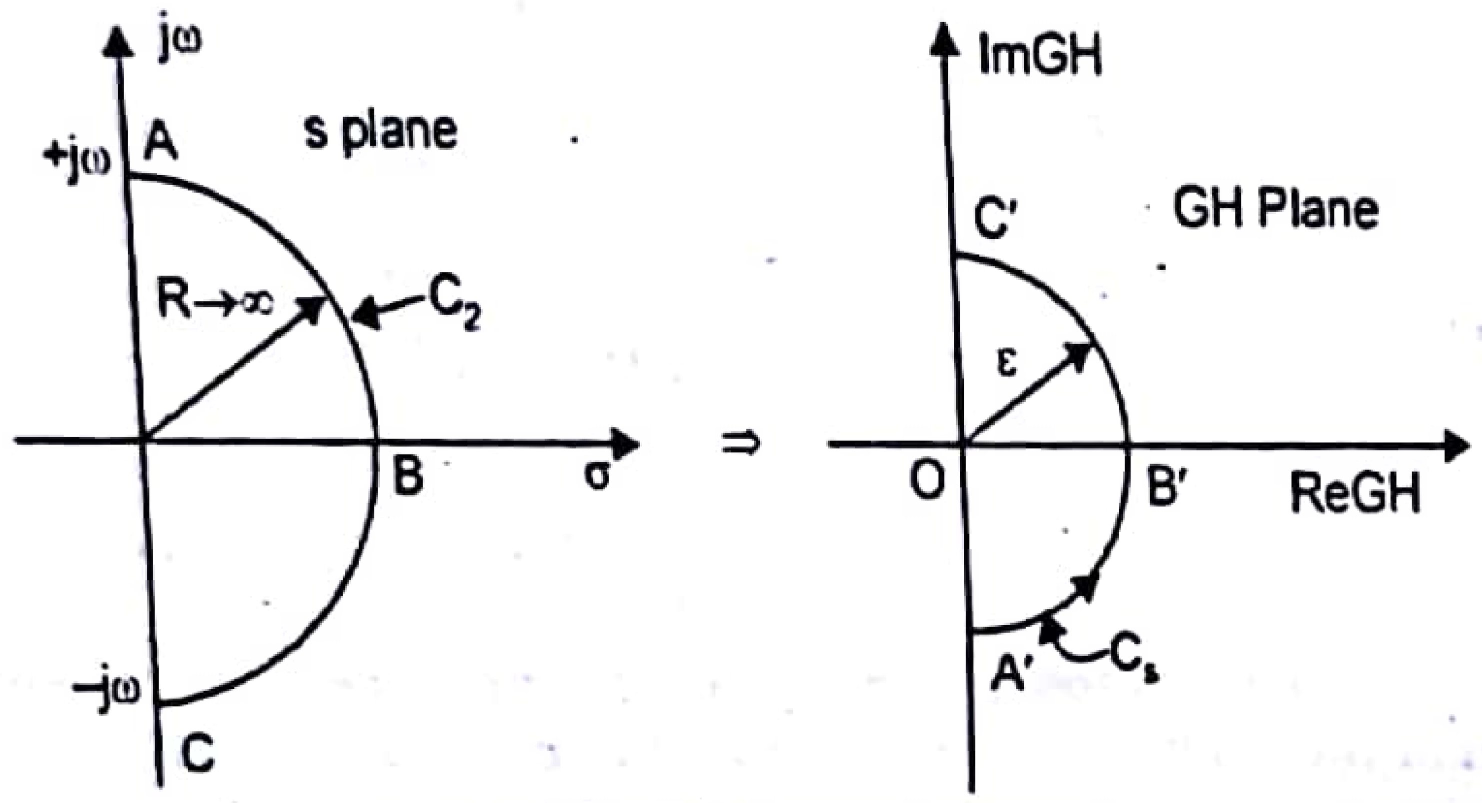


Fig. 3. Mapping of section C_2

***(iii)* Mapping of Section C_3:** In this section the frequency varies form $-\infty$ to 0. The plot of G (jω) H(jω) will be symmetric to the real axis of GH plane. It is the mirr image of Fig. 2 and is shown by the dotted line in fig. 4.

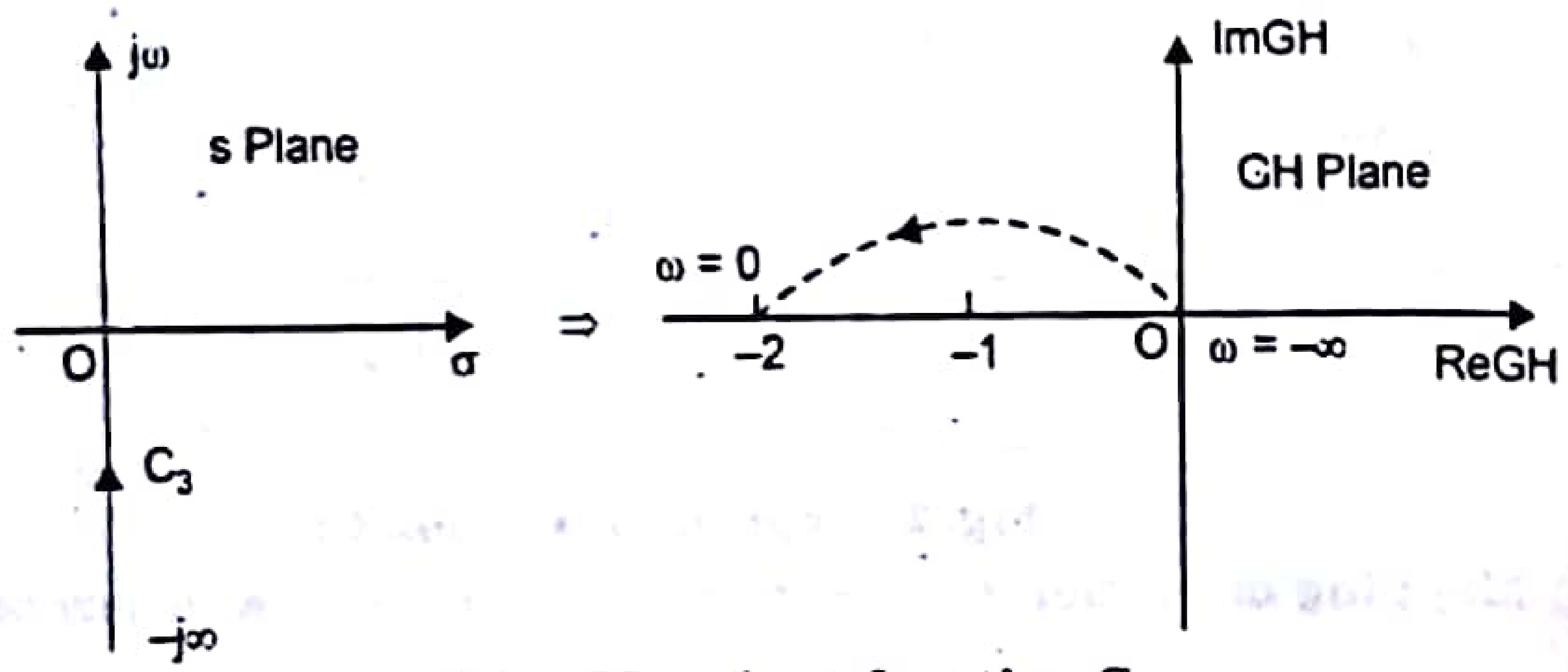


Fig. 4. Mapping of section C_3

In order to get the complete Nyquist plot C_s in G(s) H(s) H(s) corresponding to C contour in s plane, we add part-by-part of C_s contour. The complete Nyquist plot is shown in fig. (5).

From the Nyquist plot it is observed that the encirclement N of the (–1 j0) point by C_s contour is once in the anticlockwise direction. Therefore, N = – 1. From the loop transfer function G (s)H(s), the number of poles lying in the right half of the s plane is P = 1

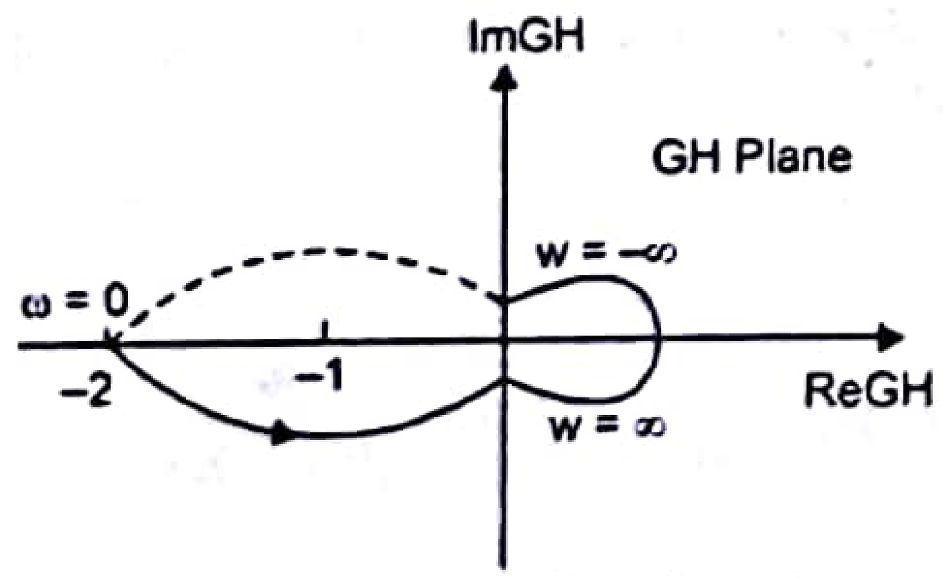


Fig. 5. Complete Nyquist plot

Therefore,

$$Z = N + P = -1 + 1 = 0$$

Thus, there is no zero of 1 + G (s) H (s). that is, no pole of the closed-llop system that lies in the right-hand side of the s plane.

Hence, according to Nyquist critertion, the closed-loop system is stable.

FIRST TERM EXAMINATION [FEB.–2016]
FOURTH SEMESTER [B.TECH]
CONTROL SYSTEMS [ETEE–212]

Time : 1: 30 hrs. **M.M. : 30**

Note: *1. Q no. 1 compulsory.*

2. Attempt any two question from the remaining.

Q.1. Answer the following questions in brief: **(2.5 × 4)**

***(i)* Differentiate between open loop and closed loop systems.**

Ans. An open loop system is one in which the output is independent on input, but controlling action or input is totally independent of the output or changes in output of the system.

A system in which the controlling action or input is somehow dependent on the output or changes in output is called closed-loop system.

***(ii)* Define PI and PID controller.**

Ans. In proportional plus integral controllers, the actuating signal consists of proportional error signal added to the integral of the error signal. The actuating signal in time domain is given by

$$e_a(t) = K_p e(t) + K_I \int_0^t e(\tau)d\tau$$

A PID controller produces an output signal consisting of three terms–one proportional to error signal, another one proportional to integral of error signal and third one proportional to derivative of error signal. The actuating signal or output signal from a PID controller in time domain is given by

$$e_a(t) = K_p e(t) + K_I \int_0^t e(\tau)d\tau + K_D \frac{de}{dt}$$

***(iii)* Differentiate between type and order of a system.**

Type of a system represents the power of that pole which lies on the origin, and order represents the maximum power of a 's' for example

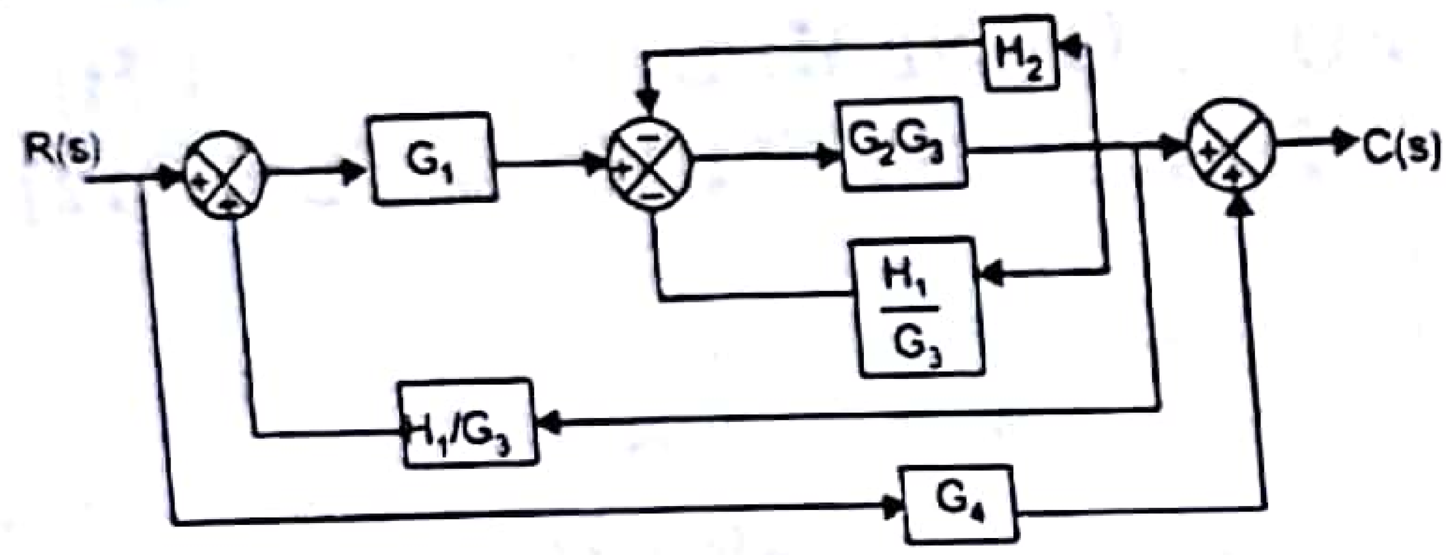
$$G(s) = \frac{1}{s^2(1+0.1s)(1+0.2s)}$$

$$\text{Type} = 2$$
$$\text{Order} = 4$$

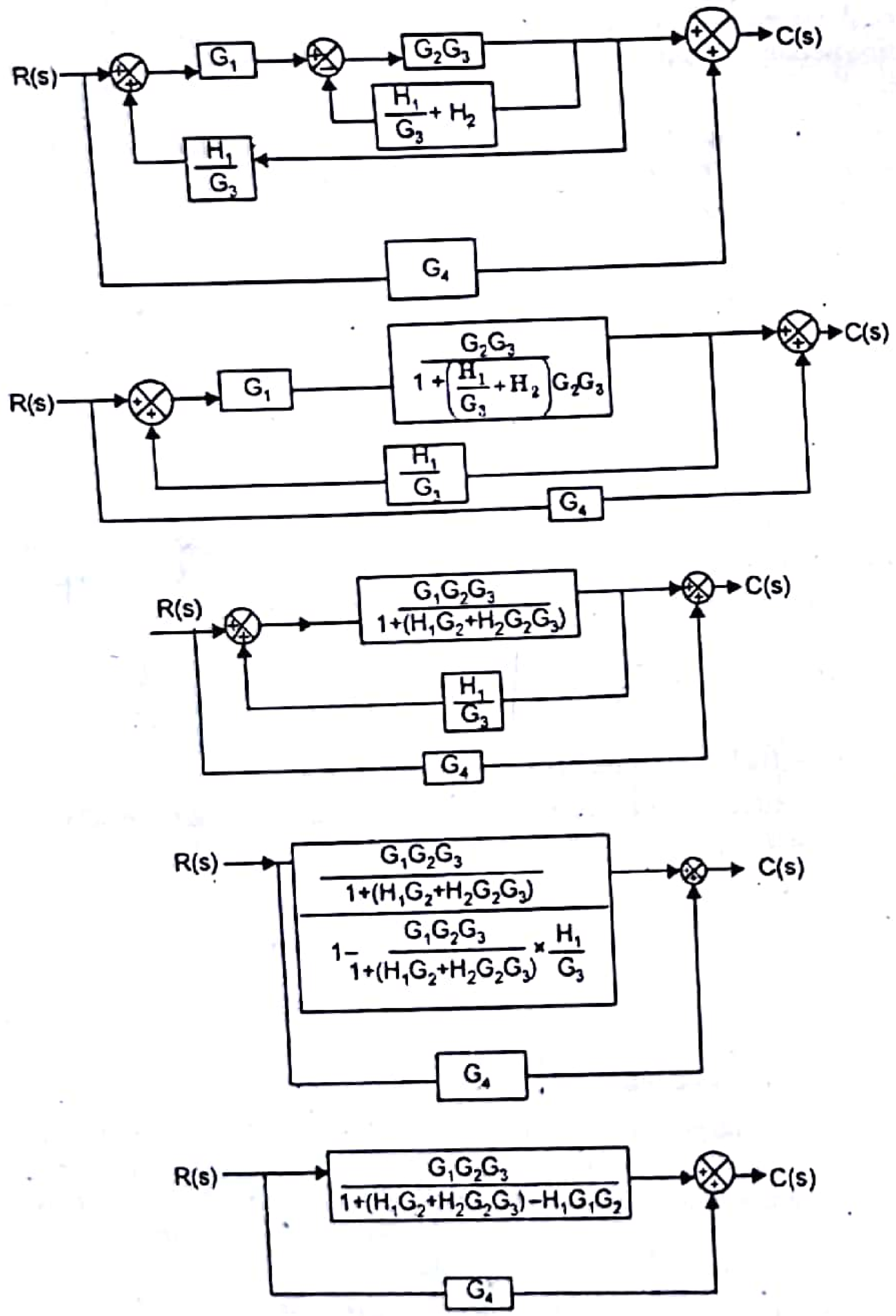
***(iv)* Briefly discuss synchros.**

Ans. It is a transformer whose primary-to-secondary coupling may be varied by physically changing the relative orientation of the two windings. Synchros are often used for measuring the angle of a rotating machine such as an antenna platform. The primary winding of the transformer, fixed to the rotor, is excited by an alternating current, which by electromagnetic induction, causes currents to flow

Q.2. *(a)* Obtain the transfer function of the given system by block diagram reduction. **(8)**



Ans.



$$\frac{C(s)}{R(s)} = G_4 + \frac{G_1G_2G_3}{1+(H_1G_2+H_2G_2G_3) - H_1G_1G_2}$$

Q.2. (b) Find out the closed loop response of a first order system with $G(s) = \frac{1}{Ts}$ and $H(s) = 1$ when a unit inpulse imput is applied to it. **(2)**

Ans. Given that $G(s) = \frac{1}{ST}$ and $H(s) = 1$

Input is a unit impulse hence $R(s) = 1$

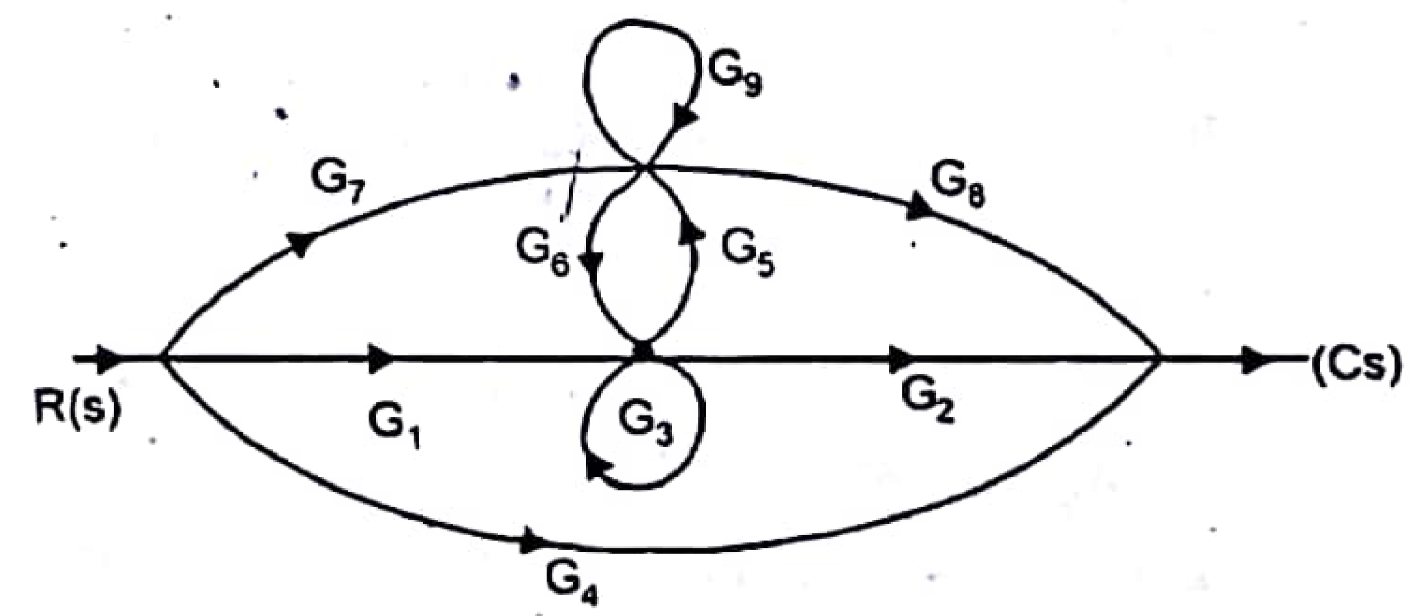
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$C(s) = 1 \times \frac{\frac{1}{ST}}{1+\frac{1}{ST}\times 1} = \frac{1}{1+ST} = \frac{1}{T\left(\frac{1}{T}+S\right)}$$

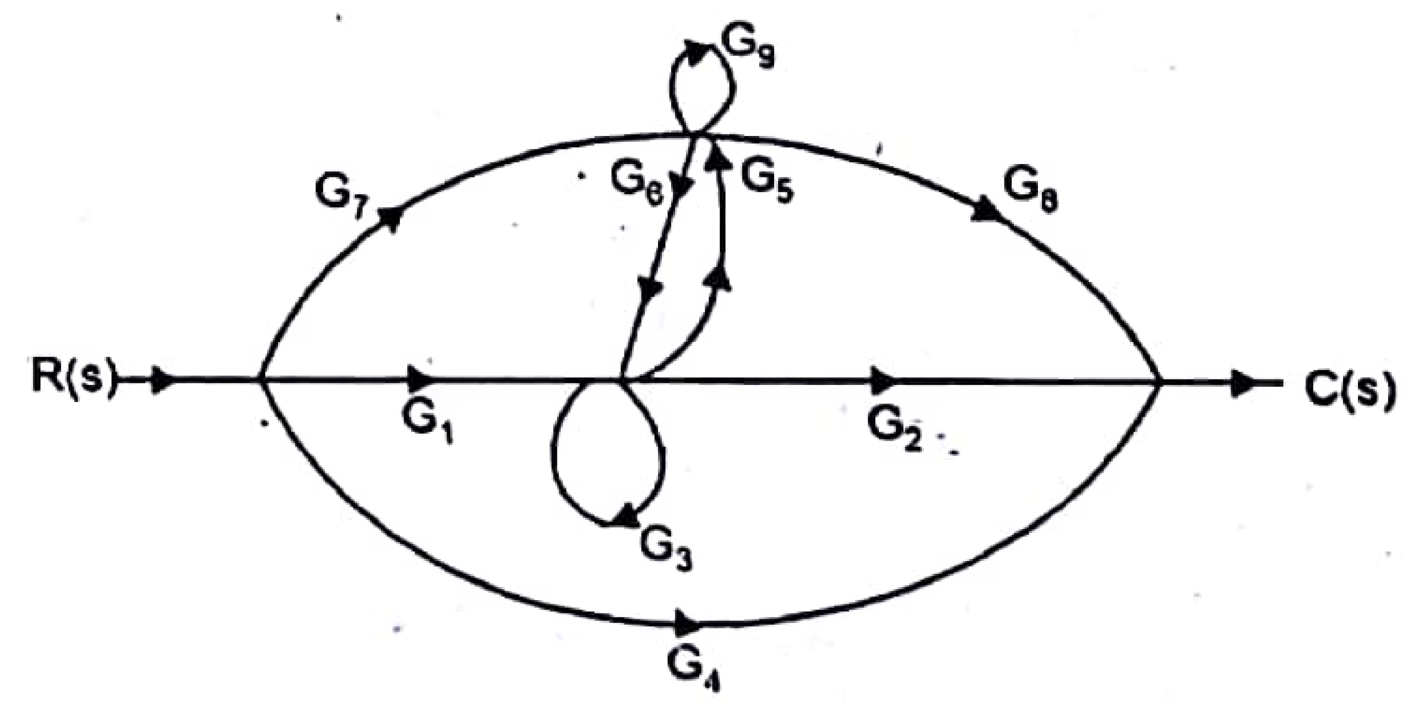
Taking Inverse Laplace transform.

$$C(t) = \frac{1}{T}e^{-t/T}$$

Q.3. (a) Using Masons Gain formula find out the closed loop transfer function. **(8)**



Ans.



$$\begin{array}{lll}
g_1 = G_1G_2 & L_1 = G_3 & \Delta_1 = 1 - G_9\alpha \\
g_2 = G_7G_8 & L_2 = G_9 & \Delta_2 = 1 - G_3 \\
g_3 = G_4 & L_3 = G_5G_6 & \Delta_3 = 1-(G_3+G_9+G_5G_6)+(G_3G_9) \\
g_4 = G_2G_6G_7 & & \Delta_4 = 1 \\
g_5 = G_1G_5G_8 & & \Delta_5 = 1
\end{array}$$

$$\frac{C(s)}{R(s)} = \frac{g_1\Delta_1 + g_2\Delta_2 + g_3\Delta_3 + g_4\Delta_4 + g_5\Delta_5}{\Delta}$$

$$= \frac{G_1G_2(1-G_9)+G_7G_8(1-G_3)+G_4[1-(G_3+G_9+G_5G_6)+(G_3G_9)]+G_2G_6G_7+G_1G_5G_8}{1-[G_3+G_9+G_5G_6]+[G_3G_9]}$$

Q.3. (b) Discuss different types of stepper motors. **(2)**

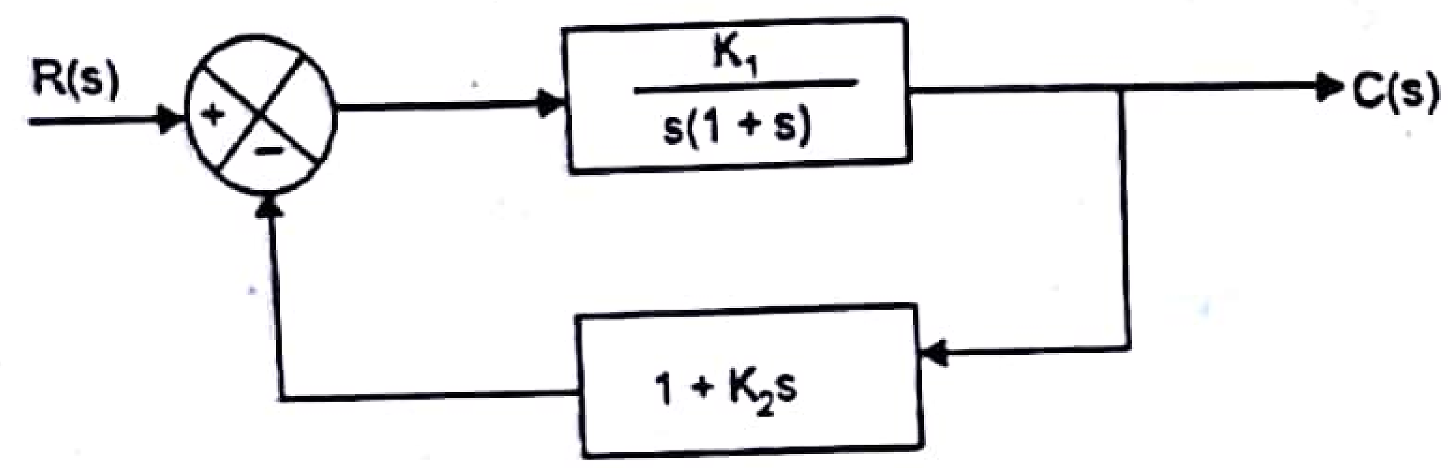
Ans. There are three types of stepper motors

(*i*) Variable reluctance motors

(*ii*) Permanent magnet moters

(*iii*) Hybrid type.

Q.4. (a) A feedback control system is shown below. Find the value of K_1 and K_2 so that max overshoot is 1.2% and peak time is 1 sec for unit step input. **(5)**



Ans.

$$\frac{C(s)}{R(s)} = \frac{\frac{K_1}{s(1+s)}}{1+\frac{K_1}{s(1+s)}(1+K_2s)} = \frac{K_1}{s(1+s)+K_1(1+K_2s)}$$

$$\frac{C(s)}{R(s)} = \frac{K_1}{s+s^2+K_1+K_1K_2s} = \frac{K_1}{s^2+s(1+K_1K_2)+K_1}$$

Compare $s^2+s(1+K_1K_2)+K_1$ with $s^2+2\xi\omega_n s+w_n{}^2$

$$2\xi w_n = 1+K_1K_2 \text{ and } \omega_n = \sqrt{K_1}$$

$$\therefore \quad \xi = \frac{1+K_1K_2}{2\sqrt{K_1}}$$

Given that

$$M_p = 1.2\%$$

Then $$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100$$

$$1.2 = e^{-3.14\xi/\sqrt{1-\xi^2}} \quad ...(1)$$

Also given that $$T_p = 1 \text{ sec.}$$

$$\therefore \quad 1 = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} \quad ...(2)$$

From (1) $$27.72 = \frac{\xi}{\sqrt{1-\xi^2}}$$

$$768.65 = \frac{\xi^2}{1-\xi^2} \text{ or, } 768.65\text{-}768.65\,\xi^2 = \xi^2$$

$$768.65 = \xi^2 769.65 \Rightarrow \xi^2 = 0.99$$

$$\therefore \quad \xi = 0.99 \cong 1$$

Q.4. (b) Derive the expression for the response of a second order underdamped system when a unit step input is given to it. **(5)**

Ans. The output of the system is given by

$$C(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}R(s)$$

For a unit-step input $$r(t) = 1 \text{ and } R(s) = \frac{1}{s}$$

Therefore, $$C(s) = \frac{\omega_n^2}{(s_2+2\zeta\omega_n s+\omega_n^2)s} \quad ...(1)$$

$$= \frac{1}{s} - \frac{s+2\zeta\omega_n}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$= \frac{1}{s} - \frac{s+\zeta\omega_n}{s(s+\zeta\omega_n)^2+\omega_d^2} - \frac{\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2} \quad ...(2)$$

We know that, $$L^{-1}\left[\frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}\right] = e^{-\zeta\omega_n t}\cos\omega_d t \quad ...(3)$$

$$L^{-1}\left[\frac{\omega_d}{(s+\zeta\omega_n)^2+\omega_d^2}\right] = e^{-\zeta\omega_n t}\sin\omega_d t \quad ...(4)$$

Taking the inverse Laplace transform of Eq. (2) we get

$$c(t) = L^{-1}C(s) = 1-e^{-\zeta\omega_n t}\cos\omega_d t - \frac{\zeta\omega_n}{\omega_d}e^{-\zeta\sin\omega_d t} \quad ...(5)$$

Since $$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

Eq. (5) may be written as

$$c(t) = 1-e^{-\zeta\omega_n t}\cos\omega_d t - \frac{\zeta}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_d t$$

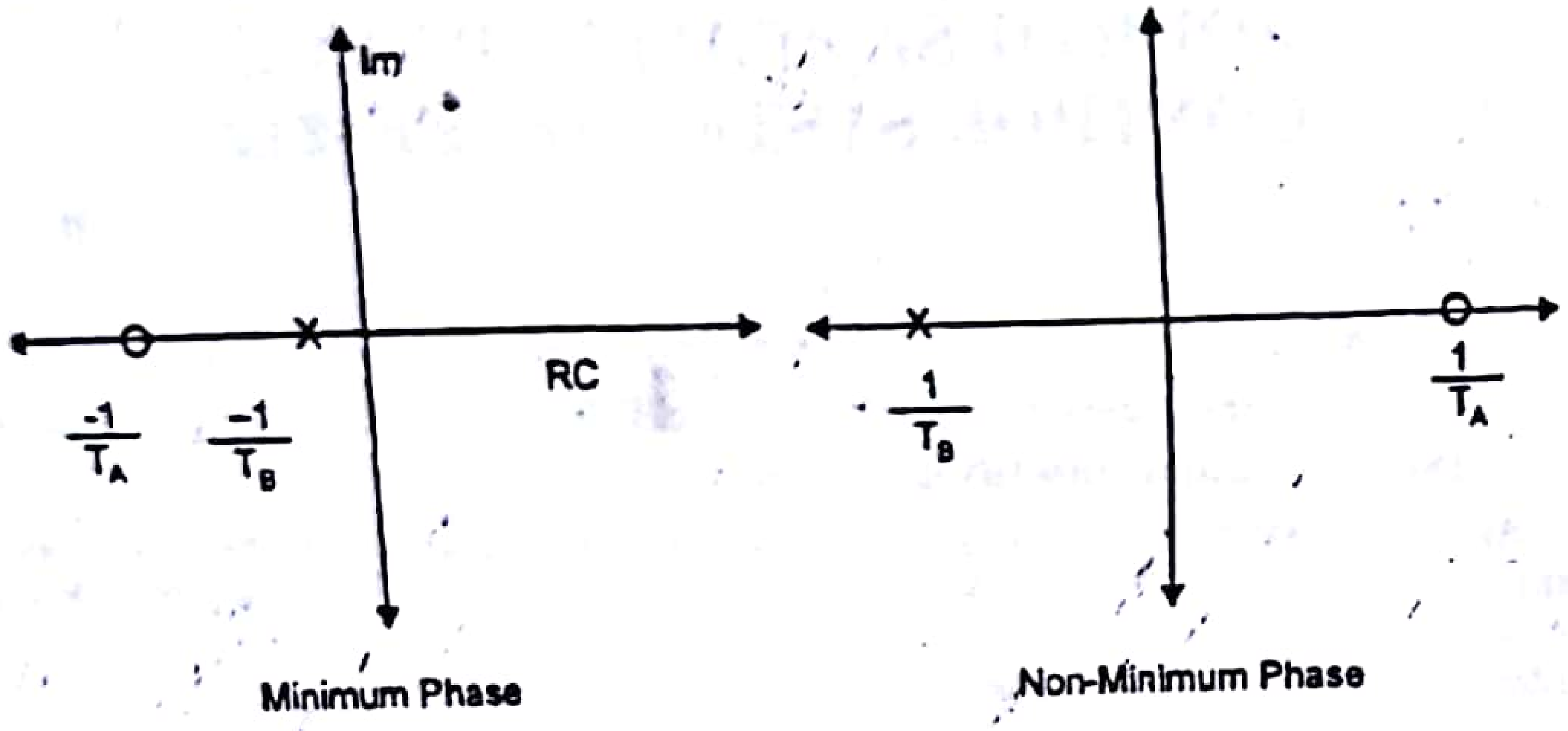
$$= 1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left[\left(\sqrt{1-\zeta^2}\right)\cos\omega_d t+\zeta\sin\omega_d t\right] \quad ...(6)$$

Put $\sqrt{1-\zeta^2} = \sin\phi$

Therefore, $$\cos\phi = \zeta \text{ and } \tan\phi = \frac{\sqrt{1-\zeta^2}}{\zeta}$$

Therefore Eq. (6) can be written as

$$c(t) = 1-\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left[\sin\phi\cos\omega_d t+\cos\phi\sin\omega_d t\right] \quad ...(7)$$



Non-minimum phase systems are slow in response.

(iii) Define Gain Margin and Phase Margin.

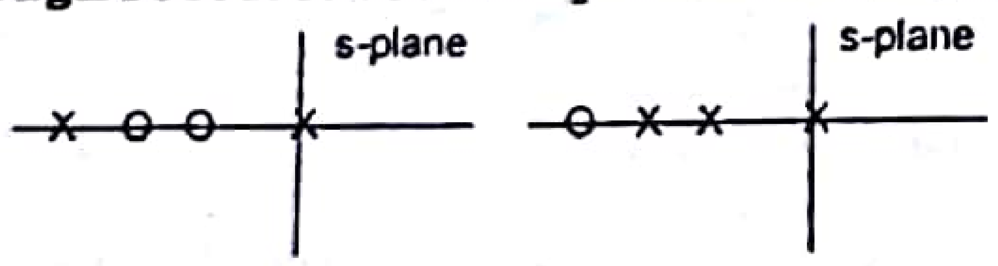
Ans. Phase margin: The phase margin is that amount of additional phase lag and the gain crossover frequency required to bring the system to the verge of instability.

The P.M. is equal to 180° plus the angle of $G(j\omega)$ at the gain crossover point *i.e.* $\phi_m = 180° + \phi$.

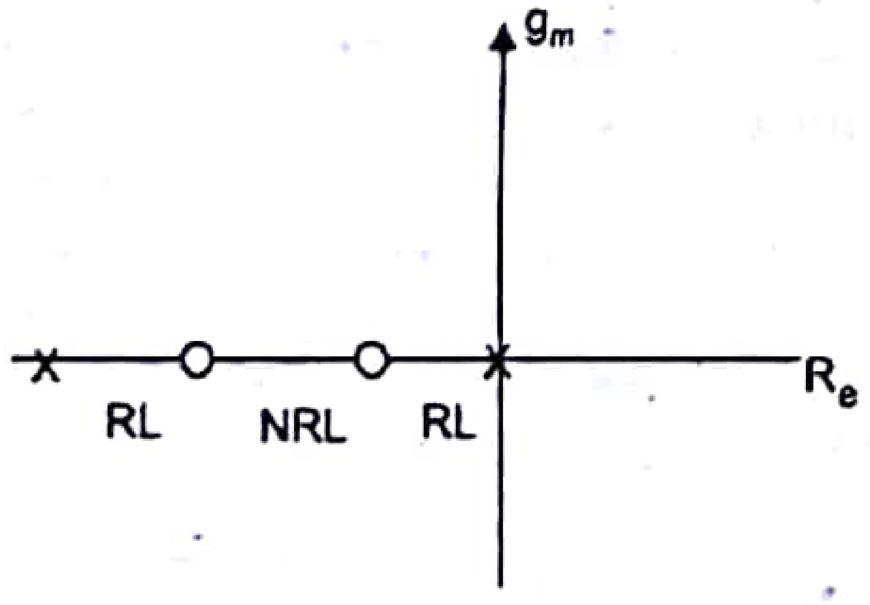
Gain Marging: The G.M. is the reciprocal of the magnitude $|G(j\omega)|$ at the frequency at which the phase angle is-180°

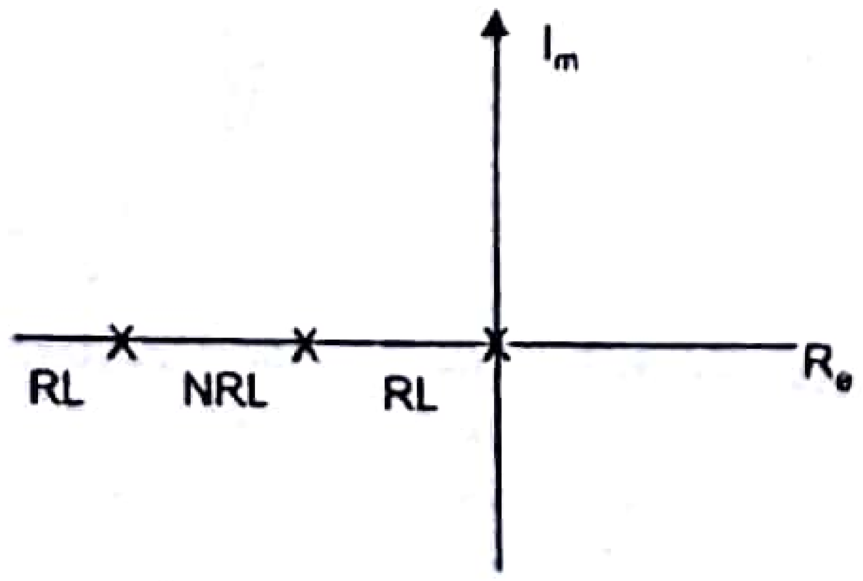
$$\text{G.M(kg)} = \frac{1}{|G(j\omega_c)|}$$

(iv) Draw the rough root locus for the pole-zero location given below:



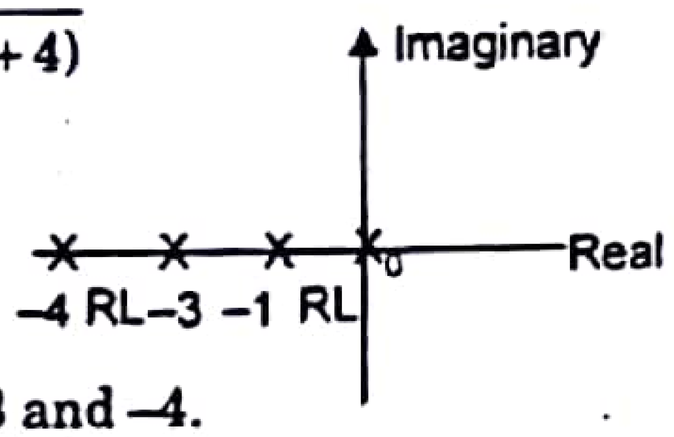
Ans.





Q.2. (a) Plot the root locus for the given open-loop transfer function $G(s)H(s) = \frac{K}{s(s+1)(s+3)(s+4)}$. Also find the point of intersection with imaginary axis. **(8)**

Ans. Given that $$G(s)H(s) = \frac{k}{s(s+1)(s+3)(s+4)}$$



Step 1: Plot the poles and zeros

poles are at $s = 0, -1, -3, -4$

No. of poles $p = 4$

No. of zero $z = 0$

Step 2: The root locus exists between 0 and – 1 and –3 and –4.

Step 3: Centroid of asymptotes

$$\sigma_A = \frac{\sum p - \sum z}{p - z} = \frac{(-1-3-4)-0}{4} = \frac{-8}{4} = -2$$

Step 4: Angle of asymptotes

$$\phi = \left(\frac{2k+1}{p-z}\right)180°$$

$$k = 0,\ \phi_1 = 45°$$

$$k = 1,\ \phi_2 = 135°$$

$$k = 2\ \phi_3 = 225°$$

$$k = 3,\ \phi_4 = 315°$$

Step 5: Calculation of breakways point. The characteristic equation $1 + G(s)H(s) = 0$

$$1 + \frac{k}{s(s+1)(s+3)(s+4)} = 0 \Rightarrow (s^2+s)(s^2+7s+12)+k = 0$$

$$\Rightarrow s^4 + s^3 + 7s^3 + 7s^2 + 12s^2 + 12s + k = 0$$

$$\Rightarrow s^4 + 8s^3 + 19s^2 + 12s + k = 0$$

$$\Rightarrow k = -s^4 - 8s^3 - 19s^2 - 12s$$

$$\frac{dk}{ds} = -4s^3 - 24s^2 - 38s - 12 = 0$$

$$\Rightarrow 4s^3 + 24s^2 + 38s + 12$$

$$s_1 = -0.41,\ s_2 = -3.58,\ s_3 = -2$$

Step 6: Determination of point of interesetion of branches of root locus with imaginary axis, by Routh Hurwitz the characteristic equation is

$$s^4 + 8s^3 + 19s^2 + 12s + k = 0$$

s^4	1	19	k
s^3	8	12	0
s^2	17.5	k	0
s^1	$\frac{210-8k}{17.5}$		
s^0	k		

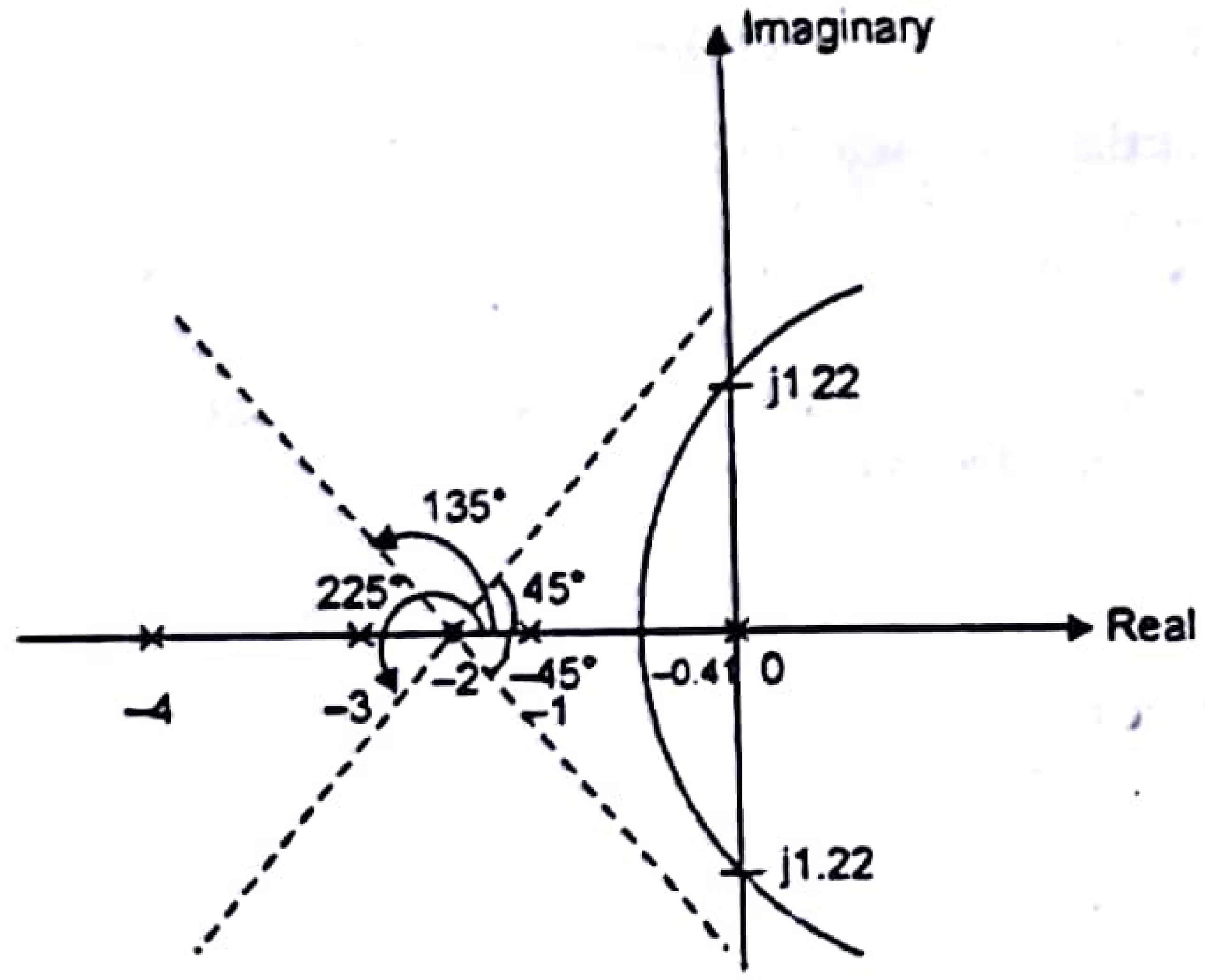
For $k = 26.25$, the auxiliary equation

$$A(s) = 17.5s^2 + k$$

or $17.5s^2 + k = 0 \Rightarrow 17.5s^2 + 26.25 = 0$

or $s = \pm j1.22$

The complete root locus is shown below



Q.2. (b) Draw the Polar Plot for $G(s)H(s) = \dfrac{1}{s(1+sT)}$ **(2)**

Ans. The sinusoidal transfer function is obtained by putting $s = j\omega$

$$G(j\omega) = \frac{1}{j\omega(1 + j\omega T)} \quad ...(1)$$

$$G(j\omega) = \frac{1+j0}{(0+j\omega)(1+j\omega T)} \quad ...(2)$$

The transfer function can be written as

$$G(j\omega) = |G(j\omega)| \angle G(j\omega) = M\angle\phi$$

$$M = |G(j\omega)| = \frac{1}{\omega\sqrt{1+\omega^2T^2}} \quad ...(3)$$

$$\phi = \angle G(j\omega) = \frac{\tan^{-1}\left(\frac{0}{T}\right)}{\left(\tan^{-1}\frac{\omega}{0}\right)\tan^{-1}\left(\frac{\omega T}{1}\right)}$$

$$\phi = \frac{0°}{(90°)\tan^{-1}\omega T}$$

$$\phi = 90° - \tan^{-1}\omega T \quad ...(4)$$

From Eq. (3), when $\omega = 0$

$$M = \lim_{\omega\to 0}|G(j\omega)| = \lim_{\omega\to 0}\frac{1}{0} = \infty$$

From Eq. (4), When $\omega = 0$

$$\phi = \lim_{\omega\to 0}|G(j\omega)| = 90° - \tan^{-1}0 = -90°$$

Therefore,

$$G(j\omega)|_{\omega=0} = \infty\angle -90°$$

From Eqs.(3) and (4), whene $\omega\to\infty$

$$M = \lim_{\omega\to\infty}|G(jw)| = \frac{1}{\infty} = 0$$

$$\phi = \lim_{\omega\to 0}|G(j\omega)| = \frac{1}{\infty} = 0$$

$$\phi = -90° - \tan^{-1}\infty = -90° - 90° = -180°$$

Therefore, $\quad G(j\omega)_{\omega\to\infty} = 0 < -180° \quad ...(6)$

It is seen from Eq. (5) that the magnitude of the function $G(j\omega)$ approaches infinity as the value of ω approaches zero.

From equation (1), we have

$$G(j\omega) = \frac{1}{j\omega(1+j\omega T)}$$

Rationalizing, we get

$$G(j\omega) = \frac{-j\omega(1-j\omega T)}{(j\omega)(1+j\omega T)(-j\omega)(1-j\omega T)}$$

$$= \frac{-j\omega - \omega^2 T}{\omega^2(1+\omega^2T^2)}$$

or $\quad G(j\omega) = \dfrac{-T}{1+\omega^2T^2} - j\dfrac{1}{\omega(1+\omega^2T^2)}$

Therefore, $V_x = \lim_{\omega\to 0}\mathrm{Re}[G(j\omega)]$

or $\quad V_x = \lim_{\omega\to 0}\left[\dfrac{-T}{1+\omega^2T^2}\right] = -T$

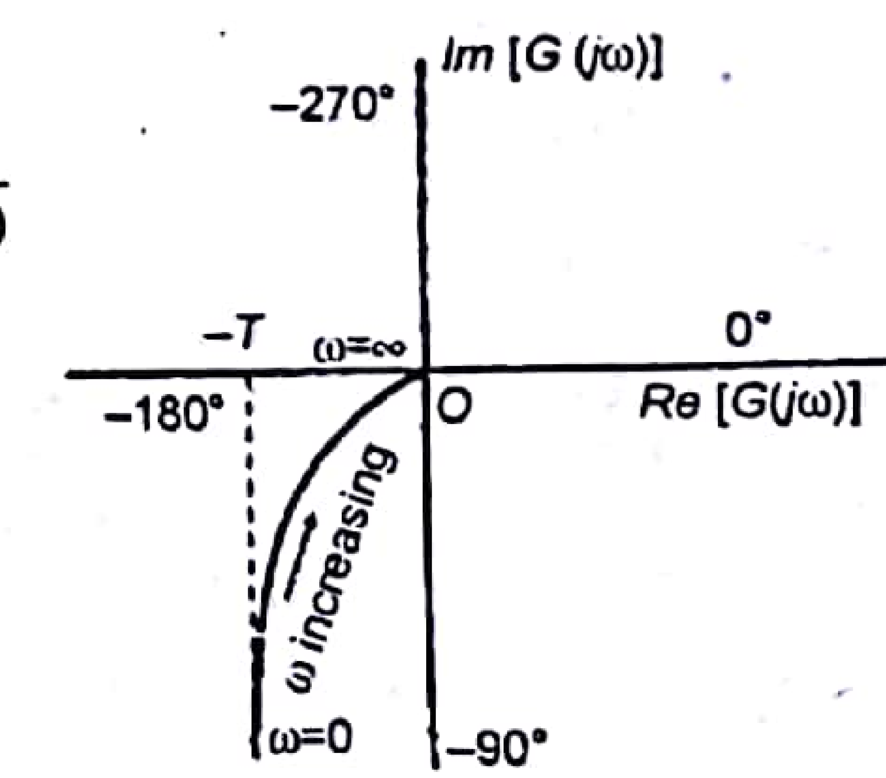


Fig. Polar plot of $G = \dfrac{1}{j\omega(1+j\omega T)}$

That is, the plot is asymptotic to the vertical line passing through the point $(-T, 0)$ The polar plot starts at $M = \infty$, with an angle $-90°$. As ω increases the magnitude M decreases and approaches zero at $\omega = \infty$. The polar plot is shown in Fig.

Q.3. (a) Draw the Bode plot for the given transfer function.

$$G(s)H(s) = \frac{20}{s(s+2)(s+10)} \quad \textbf{(8)}$$

From Bode Plot find out gain margin, phase margin, gain crossover frequency. and phase crossover frequency. Comment on the stability of the system.

Ans. Given that $G(s)H(s) = \dfrac{20}{s(s+2)(s+10)}$

$$G(s) = \frac{1}{s\left(1+\frac{s}{2}\right)\left(1+\frac{s}{10}\right)} = \frac{1}{s(1+0.5s)(0.1s)}$$

$$G(j\omega) = G(s)|_{s=j\omega} = \frac{1}{j\omega(1+j0.5\omega)(1+j0.1\omega)}$$

Constant gain K = 1

Pole at the origin $= \frac{1}{j\omega}, s = 0$

Pole at S = – 2

Pole at S = – 10

Conrner frequencies

(I) For the factor K = 1, no corner frequency

(II) For the factor $\frac{1}{j\omega}$, no corner frequency

(III) For the factor $\frac{1}{1+j0-5\omega}$ the corner frequency is 2 rad/s

(IV) For the factor $\frac{1}{1+j0-1\omega}$ no conrer frequency 10 rad/s

Magnitude Plot:

Factor	Corner freq.	Asymptotic log-magnitude characteristics
K = 1	None	20 log K = 201 log 1 = 0
$\frac{1}{j\omega}$	None	Straight line of constant slope -20dB/decode through ω = 1 rad/sec.
$\frac{1}{1+j0.5\omega}$	$\omega_1 = 2$	Straight line of constant slope -20dB/dec and originating from ω_1 = 1 rad/sec.
$\frac{1}{1+j0.1\omega}$	$\omega_2 = 0$	Straight line of constant slope -20dB/dec and originating from $\omega>_2$ = 10 rad/sec.

Phase Angle Plot

$$G_2(j\omega) = \frac{1}{j\omega(1+j0.5\omega)(1+j0.1\omega)}$$

$$\angle G(j\omega) = 0^2 - 90^2 - \tan^{-1}(0.5\omega) - \tan^{-1}(0.1\omega)$$

ω	$\angle\frac{1}{j\omega}$	$-\tan^{-1}(0.5\omega)$	$-\tan^{-1}(0.1a)$	φ
0.1	– 90°	– 2.86°	– 0.57°	– 93.43°
1	– 90°	– 45°	– 5.71°	– 140.71°
3	– 90°	– 56.30°	– 16.69°	– 162.99°
5	–90°	– 68.19°	– 26.56°	– 184.75°
10	– 90°	– 78.69°	– 75°	– 213.69°
50	– 90°	– 87.70°	– 78.69°	– 256.39°
∞	– 90°	– 90°	– 90°	–270°

Q.3. (b) Discuss Lag-lead Compensator. (2)

Ans. Refer Q.1 of Important Questions Page 37-2014.

Q.4. (a) Consider the open-loop unstable system $G(s)H(s) = \frac{s+2}{(s+1)(s-1)}$ (5)

Find out closed loop stability by Nyquist plot.

Ans. The given transfer function in sinusoidal form is obtained by putting $s = j\omega$ in $G(s)H(s)$.

$$G(s)H(s)|_{s=j\omega} = \frac{j\omega+2}{(j\omega+1)(j\omega-1)}$$

$$G(j\omega)H(j\omega) = -\frac{2+j\omega}{(1+j\omega)(1-j\omega)} = -\frac{2+j\omega}{1+\omega^2}$$

The Nyquist path is shown in Fig. (1)

It is divided into three sections C_1, C_2 and C_3. We map each section from s plane to GH plane.

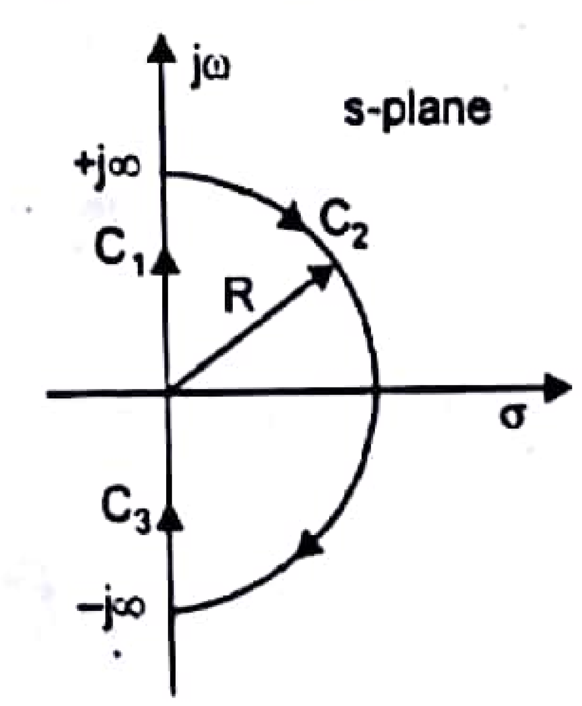


Fig. 1.

(i) Mapping of Section C_1

In this section of the Nyquist contour on the $j\omega$ axis, s varies from 0 + to + ∞.

$$G(j\omega)H(j\omega) = \frac{-2}{1+(\omega)^2} - j\frac{(\omega)}{1+(\omega)^2}$$

$$\text{At } \omega = 0^+;\ G(j\omega)H(j\omega) = \frac{-2}{1+(0)^2} - j\frac{(0)}{1+(0)^2} = -2 - j0$$

At $\omega = \infty$ $$G(j\omega)H(j\omega) = \frac{-2}{1+(\infty)^2} - j\frac{\infty}{1+(\infty)^2} = -0 - j0$$

We draw the part of GH contour by noting that the magnitude of GH changed from -2 to 0 as ω changes from 0 to ∞.

The mapping of section C_1 from C control to C_s contour is shown in fig. (2) a.

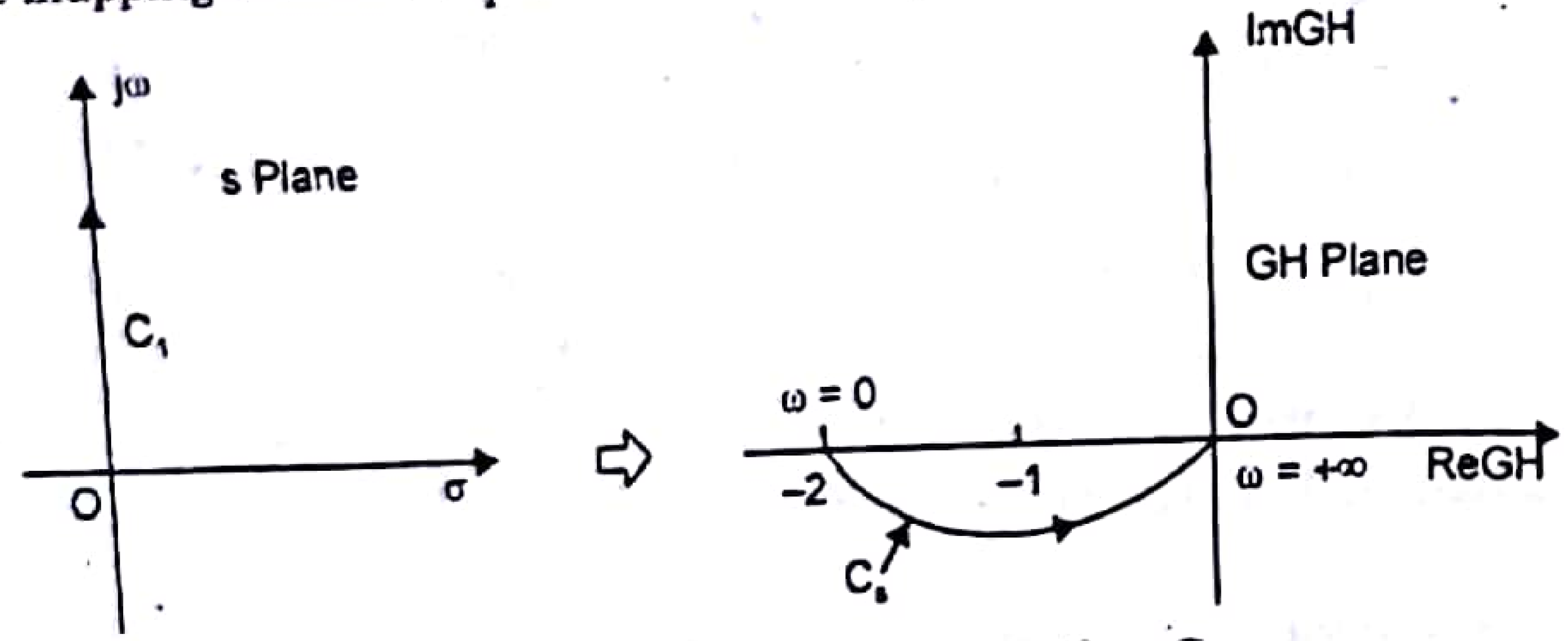


Fig. (2) (a) mapping of section C_1

(ii) Mapping of section C_2

Section C_2 in s plane is a semicircle of infinite radius.

This section can be mapped in $G(s)H(s)$ plane by substituting $s = \lim_{R\to\infty} Re^{j\theta}$ in $G(s)H(s)$ and varying θ from $+90°$ to $-90°$ through $0°$

$$\lim_{R\to\infty} G(s)H(s)\Big|_{s=Re^{j\theta}} = \lim_{R\to\infty} \frac{Re^{j\theta}+2}{(Re^{j\theta}+1)(Re^{j\theta}-1)}$$

As $R \to \infty$,

$(Re^{j\theta}+2)$ is approximated to $Re^{j\theta}$, $(Re^{j\theta}+1)$ is approximated to $Re^{j\theta}$ and $(Re^{j\theta}-1)$ is approximated to $Re^{j\theta}$.

Therefore,

$$\lim_{R\to\infty} G(s)H(s)\Big|_{s=Re^{j\theta}} = \lim_{R\to\infty} \frac{Re^{j\theta}}{(Re^{j\theta})(Re^{j\theta})}.$$

$$= \lim_{R\to\infty} \frac{1}{Re^{j\theta}} = 0e^{-j\theta}$$

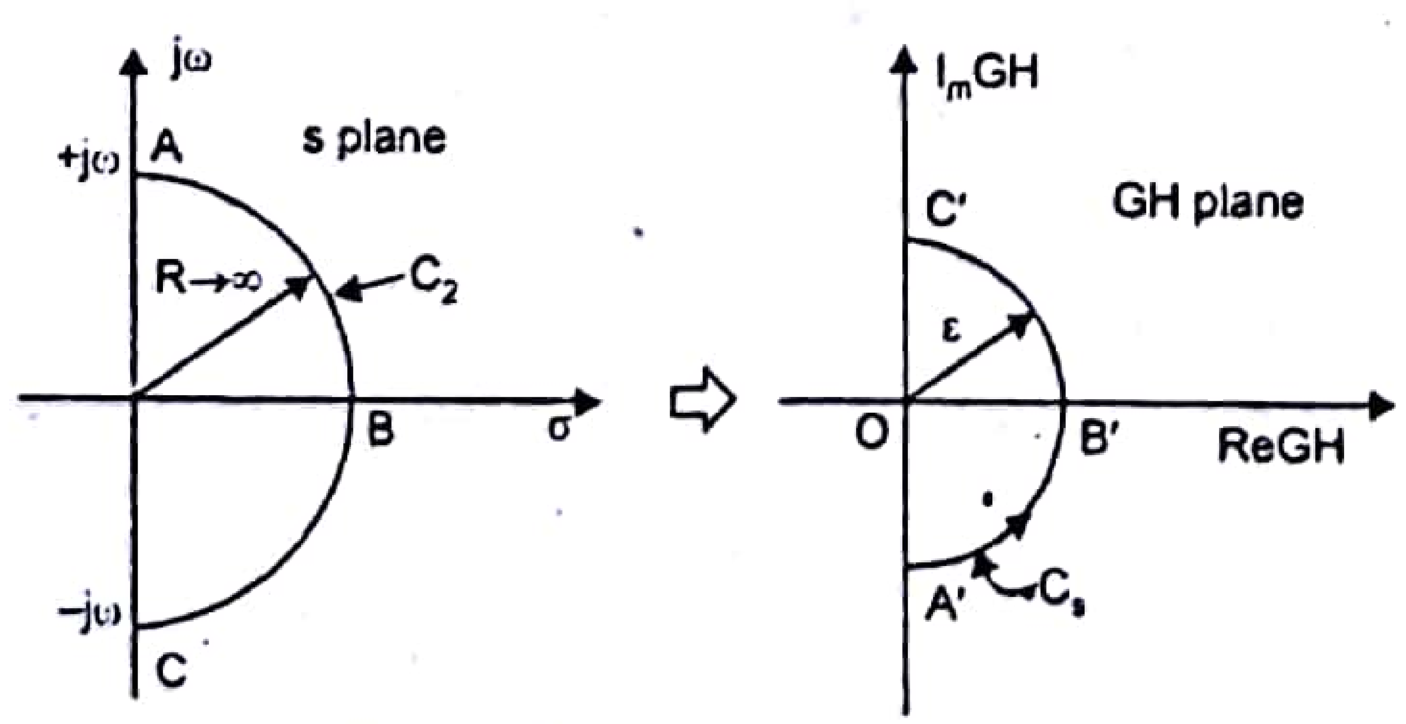


Fig. (2).(b)Mapping of section C_2

That is, for θ varying from $+90°$ to $-90°$ through $0°$, the section C_2 in the s plane is mapped into a point at the origin of $G(s)H(s)$ plane.

The points A, B, C of the C contour correspond to the mapped C_S plot at point A', B' and C' respectively.

(iii) Mapping of section C_3

In this section the frequency varies from $-\infty$ to 0^-. The plot of $G(j\omega)H(j\omega)$ will be symmetric to the real axis of GH plane. It is the mirror image of Fig. 2.(a) and is shown by the dotted line in fig. (2) c.

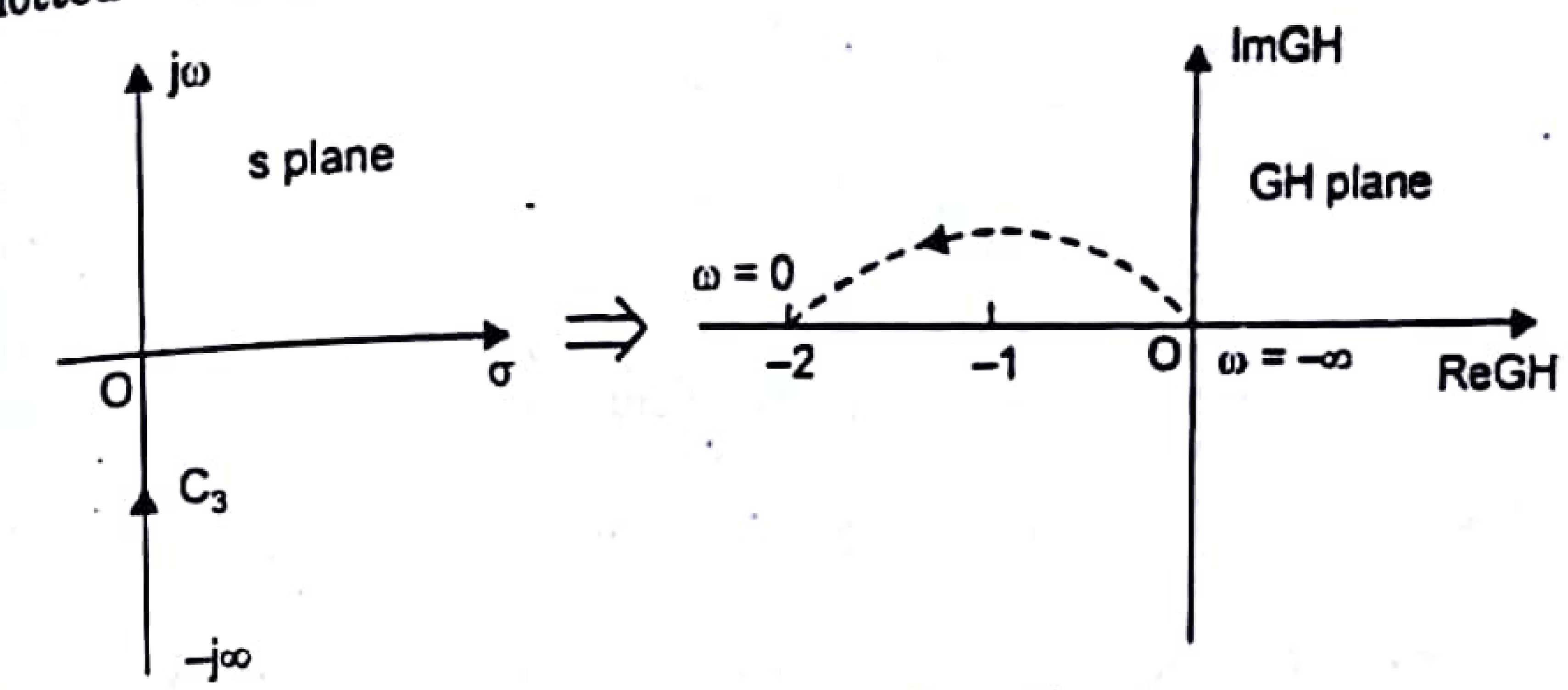


Fig.2. (c) Mapping of section C_3.

In order to get the complete Nyquist plot C_s in $G(s)H(s)$ plane corresponding to C contour in s plane we add part-by-part of C_s contour. The complete Nyquist plot is shown in Fig. 2(d).

From the Nyquist plot it is observed that the encirclement N of the $(-1, j0)$ point by C_s contour is once in the anticlockwise direction. Therefore, $N = -1$. From the loop transfer function $G(s)H(s)$, the number of poles lying in the right half of the s plane is $P = 1$.

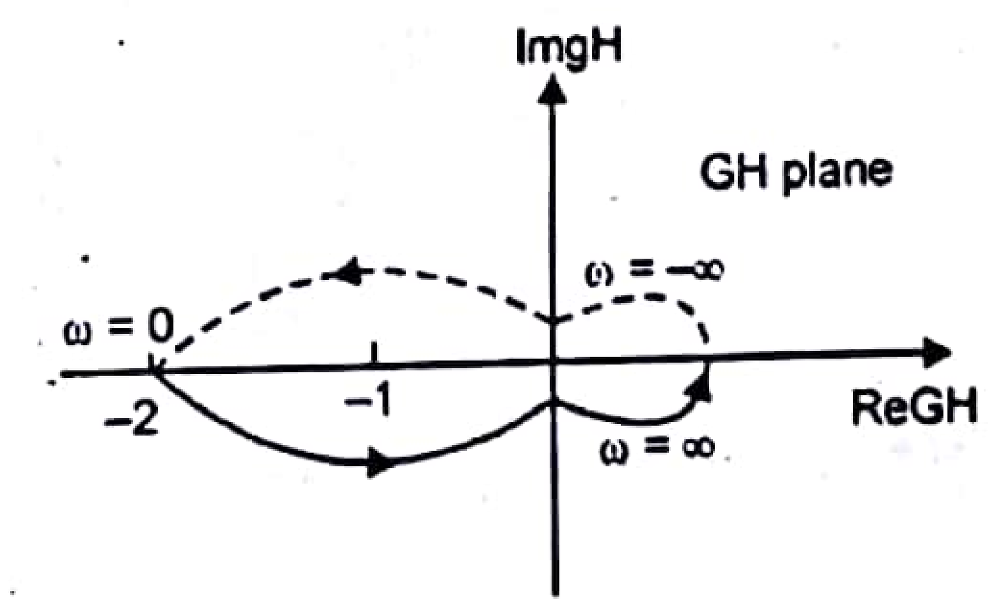


Fig.2. (d) Complete Nyquist plot

Therefore,

$$Z = N + P = -1 + 1 = 0$$

Thus, there is no zero of $1 + G(s)H(s)$ that is, no pole of the closed-loop system that lies in the right-hand side of the s plane.

Hence, according to Nyquist criterion, the closed-loop system is stable.

Q.4. (b) Show that Root Locus of the system with $G(s) = \dfrac{s+3}{s(s+2)}$ and $H(s) = 1$ is circular by angle criterion. **(5)**

Ans. For all points on the root locus, the angle condition should be satisfied.

We have

$$G(s)H(s) = \frac{k(s+3)}{s(s+2)}$$

Put $s = \sigma + j\omega$

Therefore,

$$G(s)H(s)|_{s=\sigma+j\omega} = \frac{K(\sigma + j\omega + 3)}{(\sigma + j\omega)(\sigma + j\omega + 2)}$$

$$\angle G(s)H(s)|_{s+j\omega} = 180°$$

$$\angle K + \angle(\sigma + j\omega + 3) - \angle(\sigma + j\omega) - \angle(\sigma + j\omega + 2) = 180°$$

$$\angle\left[\frac{K(\sigma + j\omega + 3)}{(\sigma + j\omega)(\sigma + j\omega + 2)}\right] = 180°$$

$$0° + \tan^{-1}\frac{\omega}{\sigma+3} - \tan^{-1}\frac{\omega}{\sigma}\tan^{-1}\frac{\omega}{\sigma+2} = 180°$$

$$\tan^{-1}\left(\frac{\omega}{\sigma+3}\right) - \tan^{-1}\left(\frac{\omega}{\sigma}\right) = 180° + \tan^{-1}\left(\frac{\omega}{\sigma+2}\right)$$

Taking the tangent on both sides of this equation.

$$\tan\left[\tan^{-1}\frac{\omega}{\sigma+3} - \tan^{-1}\frac{\omega}{\sigma}\right] = \tan\left[180° + \tan^{-1}\frac{\omega}{\sigma+2}\right] = \frac{\omega}{\sigma+2}$$

Using the relation

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\frac{\frac{\omega}{\sigma+3} - \frac{\omega}{\sigma}}{1 + \frac{\omega}{\sigma+3}\cdot\frac{\omega}{\sigma}} = \frac{\omega}{\sigma+2}$$

or, $$\sigma^2 + 6\sigma + \omega^2 = 0$$

or $$(\sigma+3)^2 + \omega^2 = (\sqrt{3})^2$$

This is the equation of a circle with centre at (–3, 0) and redius $\sqrt{3}$.

END TERM EXAMINATION [MAY-JUNE–2016]
FOURTH SEMESTER [B.TECH]
CONTROL SYSTEMS [ETEE–212]

Time : 3 hrs. **M.M. : 70**

Note: *Attempt any five questions including Q. no. 1 which is compulsory. Select one question from each unit.*

Q.1. (a) What is the difference between open loop and closed loop control systems? Discuss. **(3)**

Ans. Refer Q. no. 1. (i) of First Term Examination 2016.

Q.1. (b) Discuss the important characteristics of potentiometer. **(4)**

Ans. Characteristics of potentiometer:

(*i*) It is essentially a voltage divider used for measuring electric potential.

(*ii*) It is commonly used to control electrical devices such as volume controls on audio equipment.

(*iii*) It is a three terminal resistor with a sliding or rotating contact that forms an adjustable voltage divider.

(*iv*) It is used to adjust the level of analog signals.

(*v*) It is used to control the switching of a TRIAC and so indirectly to control the brightness of lamps.

(*vi*) Used as volume controls and as position sensors.

(*vii*) Low-power potentiometers, both linear and rotary are used to control audio equipment, changing loudness, frequency attenuation and other characteristics of audio signals.

Q.1.(c) Describe the steady state error for type 0, 1 and 2 system for step, ramp and parabolic input. **(5)**

Ans. Steady-state error for different types of Inputs

Type	Step I/p	Ramp I/P	Parabolic D/s
	e_{ss}	e_{ss}	e_{ss}
0	$\frac{1}{1+k}$	∞	∞
1	0	$\frac{1}{k}$	∞
2	0	0	$\frac{1}{k}$

Q.1. (d) What is Nyquist stability criterion? Explain. **(5)**

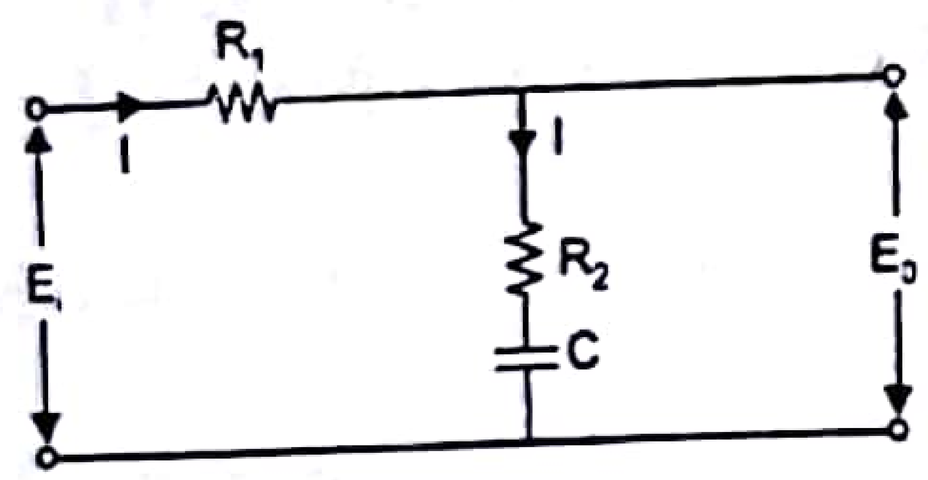
Ans. Refer Q.1. (i) Second term examination 2016.

Q.1.(e) What are the uses of polar polts? Explain. **(3)**

Ans. The uses of polar plot representation of frequency response is that both amplitude and phase angle are displayed on a single plot over a wide frequency range. It has the property to show clearly from the open loop frequency response plot whether the closed loop system is stable or unstable. The Nyquist stability criterion has been developed using polar plot only.

Q.1. (f) Discuss the Lag compensation using example. (5)

Ans. Lag Compensation:



The complex impedances are

$$Z_1(s) = R_1 \text{ and } Z_2(s) = R_2 + \frac{1}{sC} = \frac{1+sR_2C}{sC}$$

Apply KVL at input

$$E_i(s) = I(s)\,(Z_1(s)+Z_2(s))$$

Apply KVL at output

$$E_0(s) = I(s)\,Z_2(s)$$

The transfer function is given by

$$\frac{E_0(s)}{E_i(s)} = \frac{Z_2(s)}{Z_1(s)+Z_2(s)} = \frac{1+sR_2C}{1+s(R_1+R_2)C}$$

Let $R_2C = T$, and $\dfrac{R_1+R_2}{R_2} = \beta > 1$

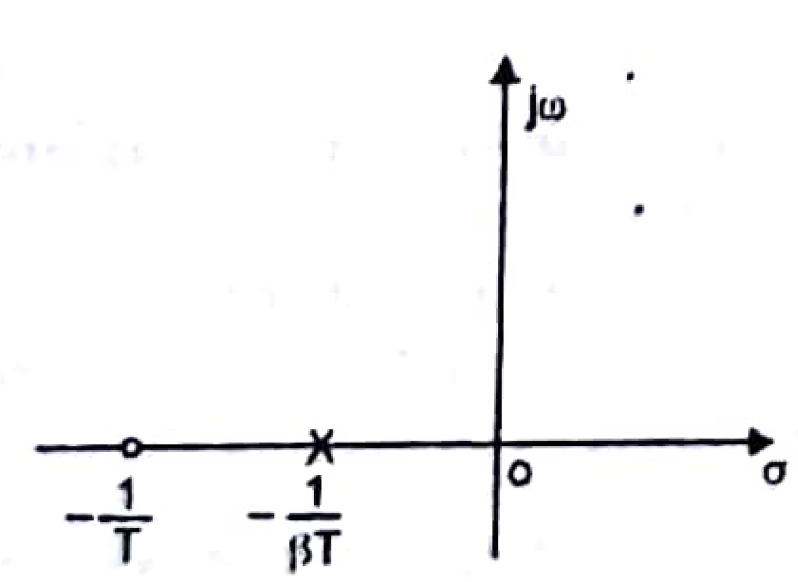
Therefore, $\beta T = (R_1+R_2)C$

and $$\frac{E_0(s)}{E_i(s)} = \frac{1+sT}{1+s\beta T};\beta>1$$

$$= \frac{1}{\beta}\left[\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}\right],\beta>1$$

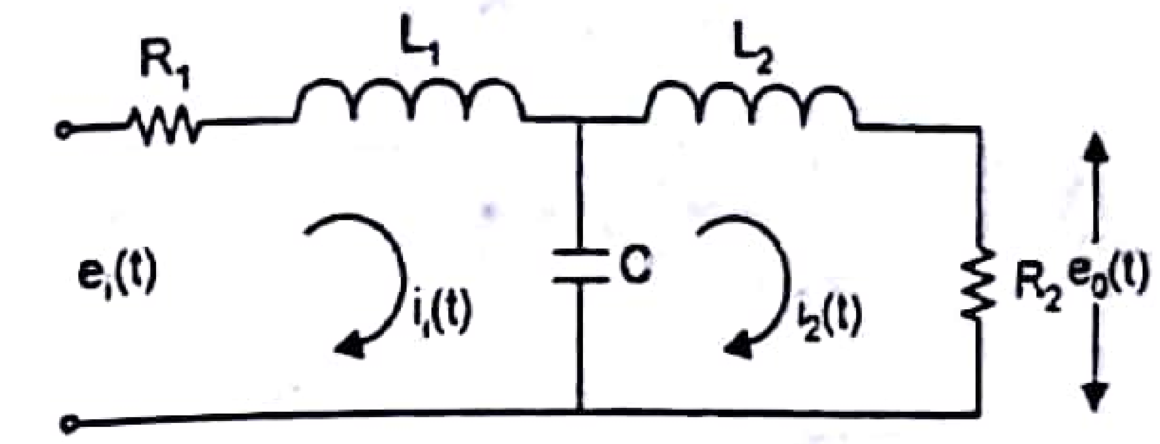
Lag compensator has a zero at $s = \dfrac{-1}{T}$ and a pole at $s = \dfrac{-1}{\beta T}$

Since $\beta > 1$

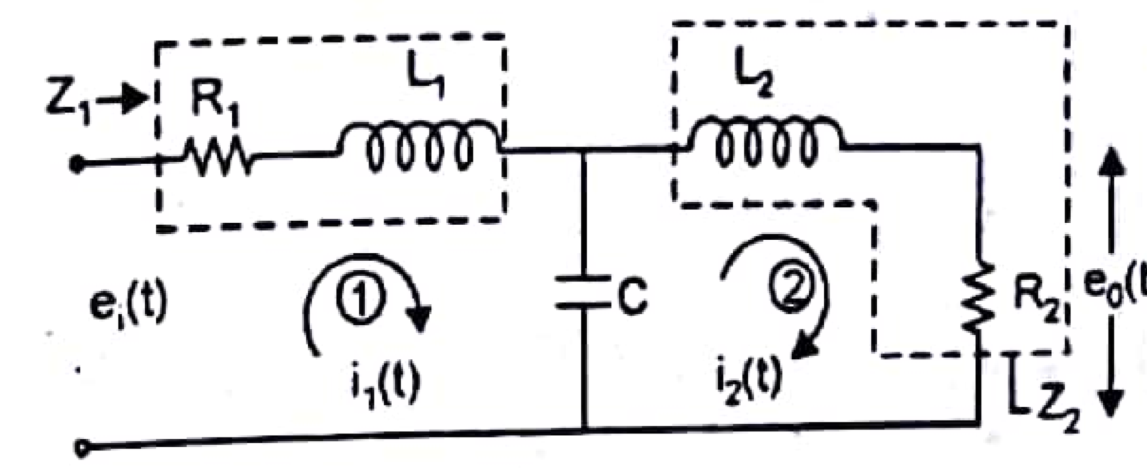


UNIT-I

Q. 2. (a) Find the transfer function of (6)



Ans.



$$Z_1 = R_1 + sL_1 \text{ and } Z_2 = R_2 + sL_2$$

Apply KVL in loop 1

$$E_i(s) = I_1(s)[R_1+sL_1] + [I_1(s)-I_2(s)]\left[\frac{1}{sC}\right]$$

$$E_i(s) = I_1(s)\left[R_1+sL_1+\frac{1}{sC}\right] - I_2(s)\frac{1}{sC} \quad \text{...(1)}$$

Apply KVL in loop 2

$$0 = I_2(s)[R_2+sL_2] + [(I_2(s)-I_1(s)]\left[\frac{1}{sC}\right]$$

$$0 = I_2(s)\left[R_2+sL_2+\frac{1}{sC}\right] - I_1(s)\frac{1}{sC}$$

$$\Rightarrow I_1(s)\frac{1}{sC} = I_2(s)\left(R_2+sL_2+\frac{1}{sC}\right) \quad \text{...(2)}$$

Also $$E_0(s) = I_2(s)R \quad \text{...(3)}$$

From eqn (1) and (2)

$$E_i(s) = I_2(s)\,\left[sCR_2+s^2L_2C+1\right]\left[R_1+sL_1+\frac{1}{sC}\right] - I_2(s)\frac{1}{sC}$$

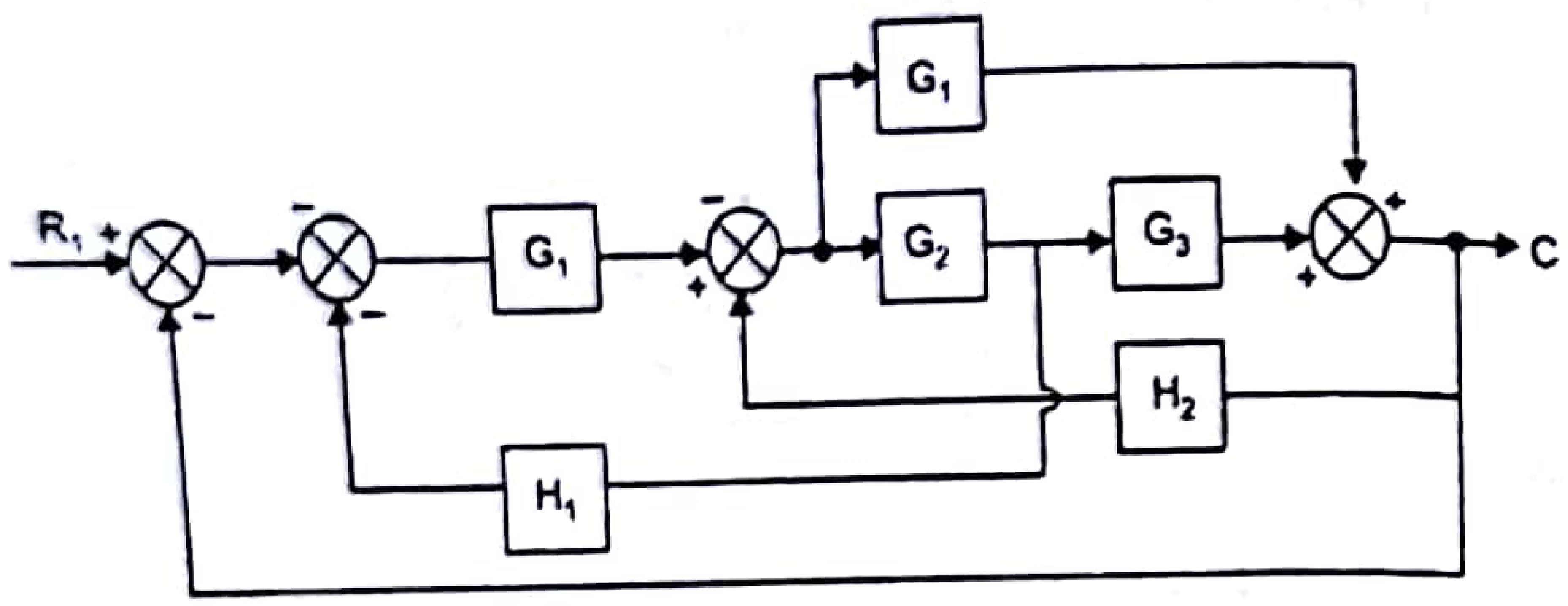
$$E_i(s) = I_2(s)\left[sR_1R_2C+s^2R_1L_2C+R_1+s^2L_1CR_2+s^3L_1L_2C+sL_1+R_2+sL_2+\frac{1}{sC}-\frac{1}{sC}\right]$$

$$E_i(s) = I_2(s)\left[s^3L_1L_2C+s^2C(R_1L_2+L_1R_2)+s(L_1+L_2+R_1R_2C)+R_1+R_2\right] \quad \text{...(4)}$$

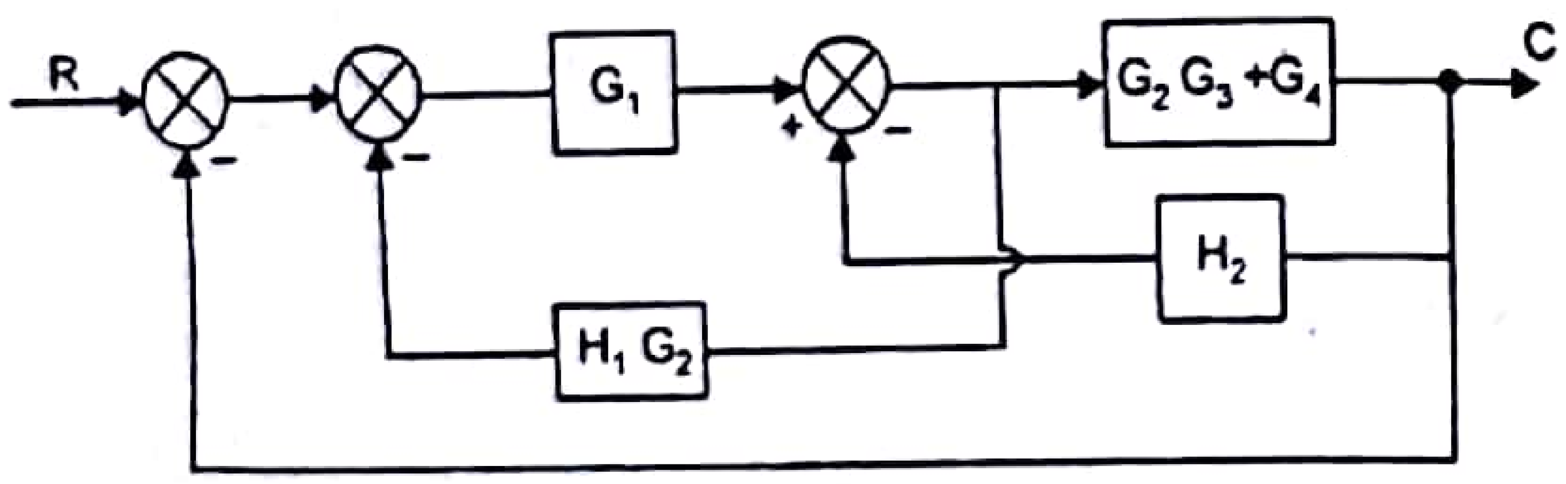
Dividing eqn (3) by eqn (4)

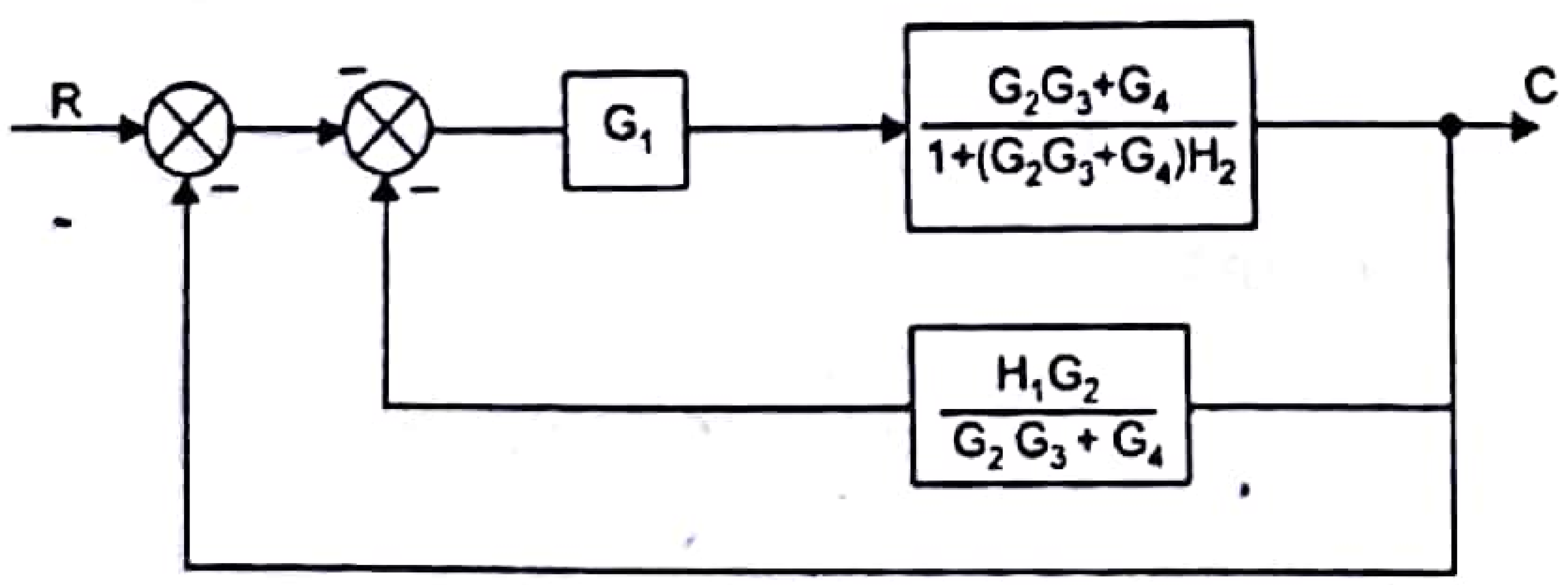
$$\frac{E_0(s)}{E_i(s)} = \frac{R}{s^3L_1L_2C+s^2C(R_1L_2+L_1R_2)+s(L_1+L_2+R_1R_2C)+R_1+R_2}$$

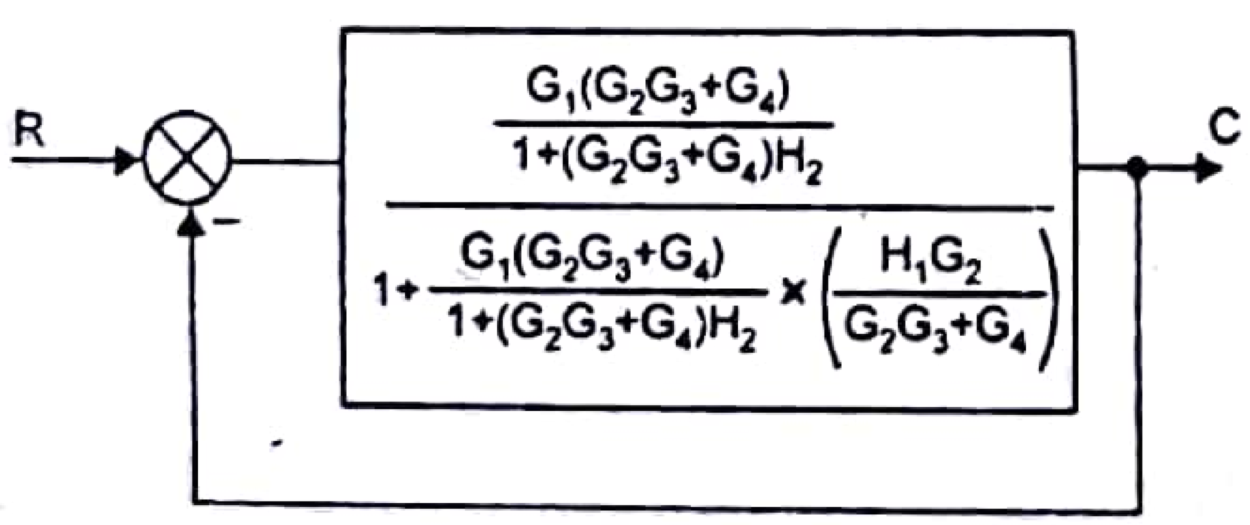
Q.2. (b) Find the closed loop transfer function of the system shown: (6.5)



Ans.



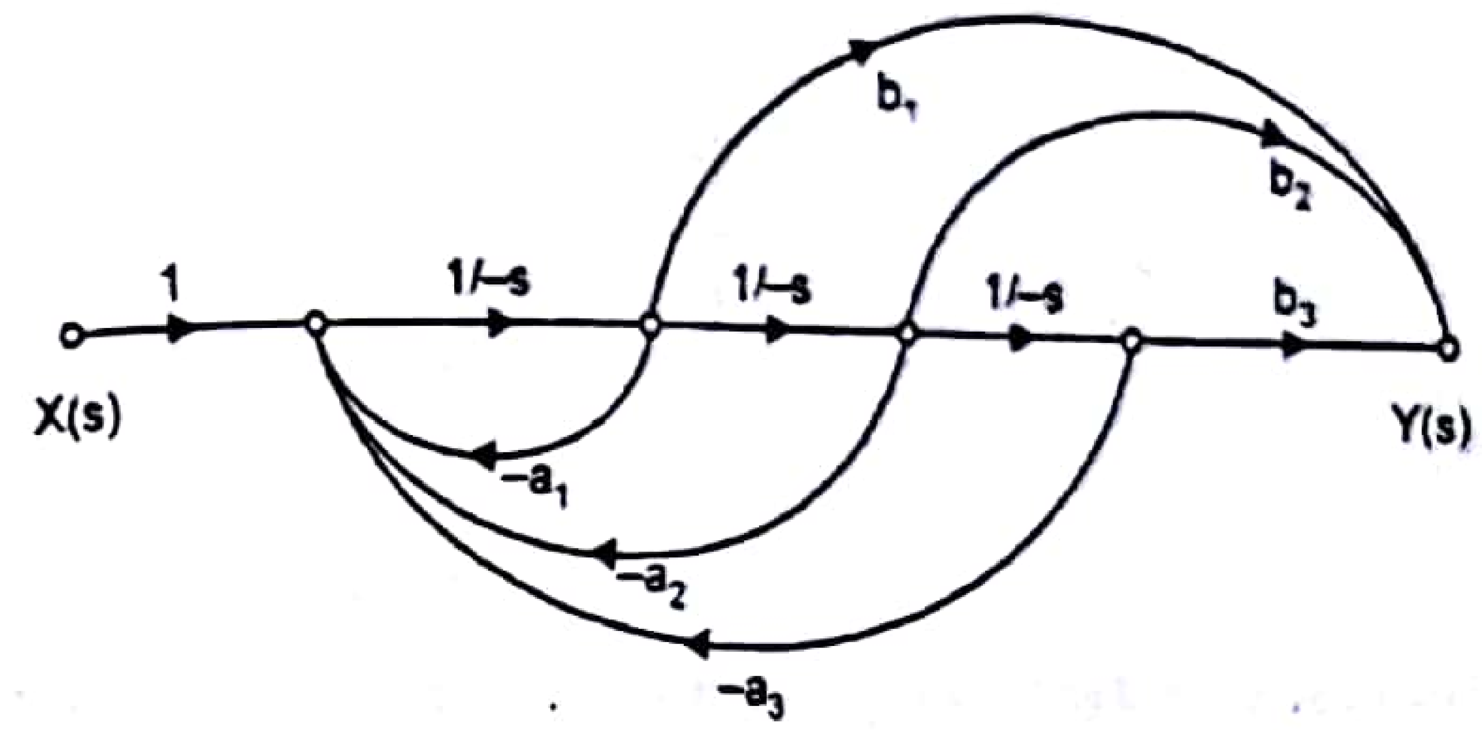




let $\dfrac{G_1(G_2G_3+G_4)}{1+(G_2G_3+G_4)H_2} = x$ and $\dfrac{H_1G_2}{G_2G_3+G_4} = y$

$$\frac{C}{R} = \frac{\dfrac{x}{1+xy}}{1+\left(\dfrac{x}{1+xy}\right)\times 1} = \frac{x}{x+1+xy} \quad \text{(Ans.)}$$

Q.3. (a) Obtain the transfer function $\dfrac{Y(s)}{X(s)}$ of SFG shown: (8)



Ans. There are three forward paths between R and C. The forward path gains are as follows:

Forward path $y_1-y_2-y_3-y_6-y_7$, path gain

$$p_1 = 1.\frac{1}{s}.b_1.1 = \frac{b_1}{s}$$

Forward path $y_1-y_2-y_3-y_4-y_6-y_7$,

$$p_2 = 1\cdot\frac{1}{s}\cdot\frac{1}{s}.b_2.1 = \frac{b_2}{s^2}$$

Forward path $y_1-y_2-y_3-y_4-y_5-y_6-y_7$,

$$p_3 = 1\cdot\frac{1}{s}\cdot\frac{1}{s}\cdot\frac{1}{s}.b_3.1 = \frac{b_3}{s^3}$$

The loop gains are

$$L_1 = \frac{1}{s}.(-a_1) = \frac{-a_1}{s}$$

$$L_2 = \frac{1}{s}.\frac{1}{s}(-a_2) = \frac{-a_2}{s^2}$$

$$L_3 = \frac{1}{s}.\frac{1}{s}.\frac{1}{s}(-a_3) = \frac{-a_3}{s^3}$$

All the loops touch that is, there are no nontouching loops. The graph determinant is, therefore,

$$\Delta = 1-(L_1+L_2+L_3)$$

$$= 1-\left(\frac{-a_1}{s}-\frac{a_2}{s^2}-\frac{a_3}{s^3}\right)$$

$$= 1+\frac{a_1}{s}+\frac{a_2}{s^2}+\frac{a_3}{s}$$

$$\Delta_1 = 1-0 = 1$$

$$\Delta_2 = 1-0 = 1$$

$$\Delta_3 = 1-0 = 1$$

Applying Mason's rule, the transfer function is

$$\frac{C(s)}{R(s)} = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3)$$

$$= \frac{\frac{b_1}{s}.1 + \frac{b_2}{s^2}.1 + \frac{b_3}{s^3}.1}{1 + \frac{a_1}{s} + \frac{a_2}{s_2} + \frac{a_3}{s^3}}$$

$$= \frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$$

Q.3.(*b*) Find the transfer function of an armature controlled D.C. motor. (4.5)

Ans. Armature Controlled d.c. motor

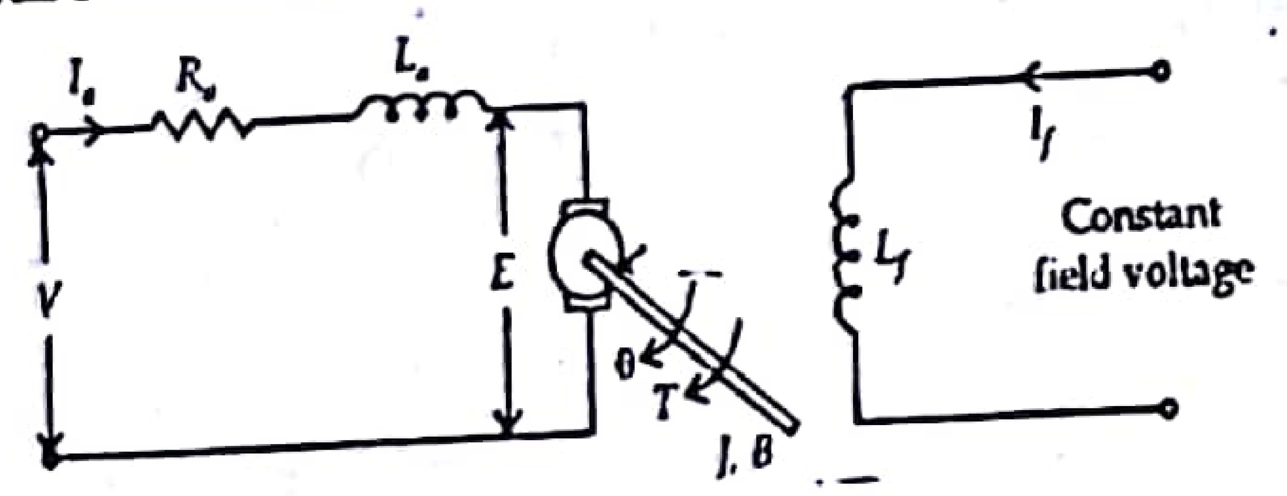


Fig.1.

Consider the armature controlled d.c. motor and assume that the demagnetizing effect of armature reaction is neglected, magnetic circuit is assumed linear and field voltage is constant i.e. I_f = constant.

Let R_a = Armature resistance

L_a = Armature self inductance caused by armature flux

i_a = armature current

i_f = field current

E = Induced e.m.f in armature

V = Applied voltage

T = Torque developed by the motor

θ = Angular displacement of the motor shaft

J = Equivalent moment of inertia of motor shaft & load referred to the motor

B = Equivalent coefficient of friction of motor and load referred to the motor.

Apply *KVL* in armature circuit

$$V = R_a i_a + L\frac{di_a}{dt} + E \qquad ...(1)$$

Since, field current I_f is constant, the flux ϕ will be constant

When armature is rotating, an e.m.f is induced

$$E \alpha \phi\omega$$

$$E = K_b\omega$$

or,

$$E = K_b\frac{d\theta}{dt}$$

Where,

ω = angular velocity

K_b = back e.m.f constant

Now, the torque T delivered by the motor will be the product of armature current and flux

$$T \alpha \phi i_a$$

$$T = K i_a \qquad ...(3)$$

where K = motor torque constant

The dynamic equation with moment of inertia and coeff. of friction will be

$$T = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} \qquad ...(4)$$

Take the laplace transform of equation (1) (2) (3) and (4).

$$V(s) - E(s) = I_a(s)(R_a + SL_a)$$

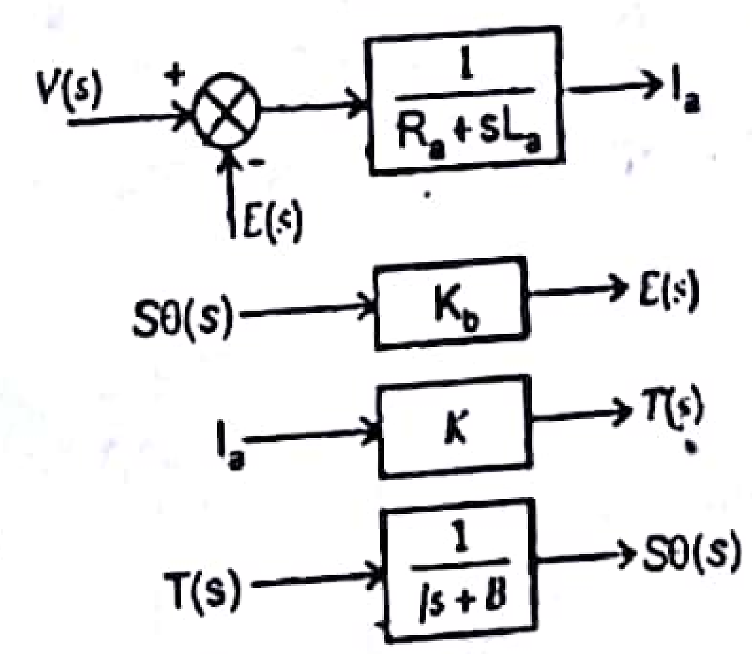
$$E(s) = K_b S\theta(s)$$

$$T(s) = KI_a(s)$$

$$T(s) = (s^2 J + sB)\,\theta(s)$$

$$T(s) = (SJ + B)\,S\theta(s)$$

The block diagram for each equation



Combine all four block diagrams

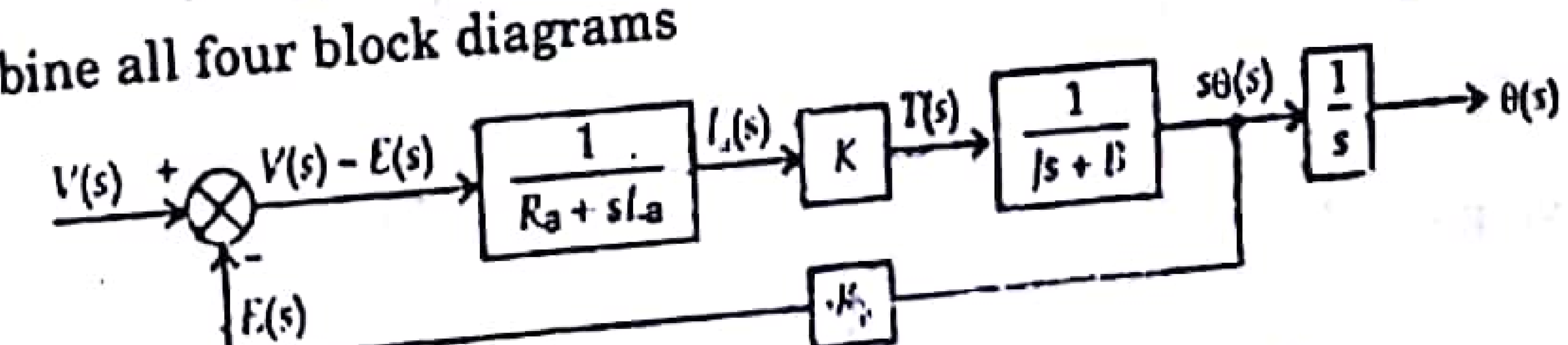
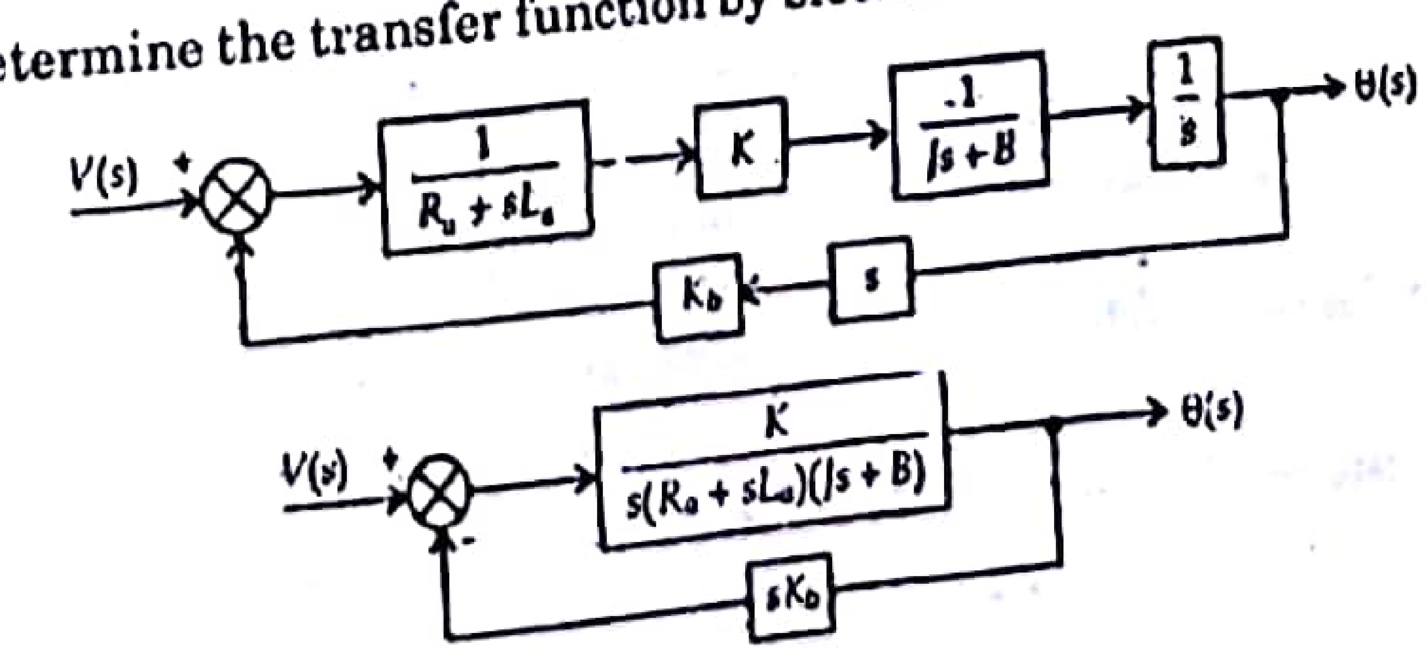


Fig. 2. Block diagram of armature controlled.

Now determine the transfer function by block reduction method.



$$\frac{\theta(s)}{V(s)} = \frac{K}{(R_a + sL_a)(Js + B)s + KK_b s} \quad ...(5)$$

Equation (5) can be written as

$$\frac{\theta(s)}{V(s)} = \frac{K}{R_a\left(1 + s\frac{L_a}{R_a}\right)sB\left(1 + s.\frac{J}{B}\right) + KK_b s}$$

Put $\frac{L_a}{R_a} = \tau_a$ time constant of armature circuit

$\frac{J}{B} = \tau_m$ = mechanical time constant

equation (5) becomes

$$\frac{\theta(s)}{V(s)} = \frac{K}{sR_aB(1+s\tau_a)(1+s\tau_m) + KK_b s} \quad ...(6)$$

From the block diagram (2) it is clear that it is a closed loop system. The effect of the back e.m. f is represented by the feedback signal proportional to the speed of the motor.

UNIT-II

Q.4. (a) For the system given by $\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$. ...(6)

Obtain the unit ramp response and steady state error.

Ans.

We have, $$C(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}R(s) \quad ...(1)$$

If the input is a unit ramp function

$$r(t) = t, R(s) = \frac{1}{s^2}$$

Therefore, $$C(s) = \frac{\omega_n^2}{s^2(s^2 + 2\zeta\omega_n s + \omega_n^2)} \quad ...(2)$$

Resolving Eq. (2) into partial fractions

$$\frac{\omega_n^2}{s^2(s^2 + 2\zeta\omega_n s + \omega_n^2)} = \frac{A_1}{s} + \frac{A_2}{s^2} + \frac{A_3 s + A_4}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad ...(3)$$

Therefore $$\omega_n^2 = A_1 s(s^2 + 2\zeta\omega_n s + \omega_n^2) + A_2(s^2 + 2\zeta\omega_n s + \omega_n^2) + (A_3 s + A_4)s^2 \quad ...(4)$$

Equating the coefficients of s^3 on both sides, we get

$$A_1 + A_3 = 0 \quad ...(5)$$

Equating the coefficients of s^2 on both sides gives

$$2\zeta\omega_n A_1 + A_2 + A_4 = 0 \quad ...(6)$$

Equating the coefficients of s on both sides

$$A_1\omega_n^2 + 2\zeta_n\omega A_2 = 0 \quad ...(7)$$

Equating the coefficients of the terms without s on both sides,

$$A_2\omega_n^2 = \omega_n^2 \quad ...(8)$$

From Eq. (8)

$$A_2 = 1 \quad ...(9)$$

From Eqs. (7) and (9)

$$A_1\omega_n^2 + 2\zeta\omega_n.1 = 0$$

Therefore, $$A_1 = -\frac{2\zeta}{\omega_n} \quad ...(10)$$

From Eqs. (5) and (10)

$$A_3 = -A_1 = \frac{2\zeta}{\omega_n} \quad ...(11)$$

Substituting the values of A_1 and A_2 in Eq. (6), we get

$$2\zeta\omega_n\left(-\frac{2\zeta}{\omega_n}\right) + 1 + A_4 = 0$$

Therefore, $$A_4 = -1 + 4\zeta^2 \quad ...(12)$$

Substitution of the value of A_1, A_2, A_3 and A_4 in Eq. (3) gives

$$C(s) = \frac{-2\zeta}{\omega_n s} + \frac{1}{s^2} + \frac{\frac{2\xi}{\omega_n}s - (1 - 4\xi^2)}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad ...(13)$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = (s + \zeta\omega_n)^2 + \omega_n^2 - \zeta^2\omega_n^2$$

$$= (s + \zeta\omega_n)^2 + \omega_n^2(1 - \zeta^2)$$

$$\beta = \sqrt{1 - \zeta^2} = \sin\phi \quad ...(14)$$

Therefore, $$s^2 + 2\zeta\omega_n s + \omega_n^2 = (s + \zeta\omega_n)^2 + (\beta\omega_n)^2 \quad ...(15)$$

Therefore Eq. (13) becomes

$$C(s) = \frac{-2\zeta}{\omega_n s} + \frac{1}{s^2} + \frac{\frac{2\zeta}{\omega_n}s - (1 - 4\zeta^2)}{(s + \zeta\omega_n)^2 + (\beta\omega_n)^2}$$

$$= \frac{-2\zeta}{\omega_n s} + \frac{1}{s^2} + \frac{\frac{2\zeta}{\omega_n}(s + \zeta\omega_n) - 2\zeta^2 - 1 + 4\zeta^2}{(s + \zeta\omega_n)^2 + (\beta\omega_n)^2}$$

$$= \frac{-2\zeta}{\omega_n s}\cdot\frac{1}{s^2} + \frac{1}{s^2} + \frac{2\zeta}{\omega_n}\cdot\frac{(s + \zeta\omega_n)}{(s + \zeta\omega_n)^2 + (\beta\omega_n)^2}$$

$$- \frac{(1 - 2\zeta^2)\cdot(\beta\omega n)}{\beta\omega_n[(s + \zeta\omega_n)^2 + (\beta\omega_n)^2]}$$

Taking the inverse Laplece of Eq. 16. We obtain.

$$c(t) = t - \frac{2\zeta}{\omega_n} + e^{-\zeta\omega_n t}\left[\frac{2\zeta}{\omega_n}\cos\beta\omega_n t - \frac{(1-2\zeta^2)\sin\beta\omega_n t}{\beta\omega_n}\right]$$

Q.4.(b) The open loop transfer function of a servo system with unity feedback is $(s) = \frac{10}{s(0.1s+1)}$. Find the static error constant and obtain the steady state error of the system with input of: $r(t) = A_0 + A_1 t + \frac{A_2}{2}t^2$ **(6.5)**

Ans. Given that $G(s) = \frac{10}{s(0.1s+1)}$ and $H(s) = 1$

Static error constant

$$K_p = \underset{s\to 0}{Lt}\, G(s)H(s) = \underset{s\to 0}{Lt}\frac{10}{s(0.1s+1)} = \infty$$

$$K_v = \underset{s\to 0}{Lt}\, sG(s)H(s) = \underset{s\to 0}{Lt}\, s\times\frac{10}{s(0.1s+1)} = \underset{s\to 0}{Lt}\frac{10}{(0.1s+1)} = 10$$

$$K_a = \underset{s\to 0}{Lt}\, s^2G(s)H(s) = \underset{s\to 0}{Lt}\, s^2\times\frac{10}{s(0.1s+1)} = 0$$

Steady state error with input of $r(t) = A_0 + A_1 t + \frac{A_2}{2}t^2$

$$G(s) = \frac{10}{s(0.1s+1)} \text{ and } H(s) = 1$$

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)H(s)} = \frac{1}{1+\frac{10}{s(0.1s+1)}\times 1} = \frac{s(0.1s+1)}{s(0.1s+1)+10}$$

$$= \frac{0.1s^2+s}{0.1s^2+s+10} = \frac{s+0.1s^2}{10+s+0.1s^2}$$

Dividing numerator by denominator

$$\frac{E(s)}{R(s)} = 0.1s - 0.001s^3 + 0.0001s^4 - \ldots$$

$$E(s) = 0.1sR(s) - 0.001s^3R(s) + 0.0001s^4R(s)$$

Taking Inverse Laplace transform

$$e(t) = 0.1\,\dot{r}(t) - 0.001\,\dddot{r}(t) + 0.0001\,\ddddot{r}(t) \quad \ldots(1)$$

Compare it with

$$e(t)|_{t\to\infty} = r(t)C_0 + \dot{r}(t)C_1 + \frac{\ddot{r}(t)}{2!}C_2 + \frac{\dddot{r}(t)}{3!}C_3 + \frac{\ddddot{r}(t)}{4!}C_4 + \ldots\ldots$$

We have

$$C_0 = 0, C_1 = 0.1, C_3 = 5\times 10^{-4}, C_4 = 4.16\times 10^{-6}$$

Given that

$$r(t) = A_0 + A_1 t + \frac{A_2}{2}t^2$$

$$\dot{r}(t) = A_1 + A_2 t$$

$$\ddot{r}(t) = A_2$$

$$\dddot{r}(t) = 0$$

$$\ddddot{r}(t) = 0$$

From equation (1)

$$e(t) = 0.1(A_1 + A_2 t) - 0.001(0) + 0.0001(0)$$

$$\therefore \quad e(t) = 0.1A_1 + 0.1A_2 t$$

Steady state error $e_{ss} = \underset{t\to\infty}{Lt}\, e(t) = \infty$

Q.5.(a) A unity feedback system is characterized by an open-loop transfer function $(s) = \frac{k}{s(s+10)}$. Determine the gain K so that system will have a damping ratio of 0.5. For this value of K determine the settling time, peak overshoot and time to peak overshoot for a unit step input. **(6.5)**

Ans. Given that $G(s) = \frac{K}{s(s+10)}$ and $H(s) = 1$

characteristic equation is

$$1 + G(s)H(s) = 0$$

$$\Rightarrow 1 + \frac{K}{s(s+10)}\times 1 = 0 \Rightarrow s^2 + 10s + k = 0 \quad \ldots(1)$$

Comparing eqn (1) with $s^2 + 2\zeta w_n s + w_n^2 = 0$

We have, $2\zeta w_n = 10$

Given that $\zeta = 0.5$

so, $2\times 0.5\times w_n = 10 \therefore w_n = 10$

Also we have $k = \omega_n^2 = (10)^2 = 100$

Setting time $t_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.5\times 10} = 0.8$

Q.5.(b) Write short note on PID controller. **(6)**

Ans. The combination of proportional integral and derivation control action is called PID control action and the controller is called three action controller. Mathematically

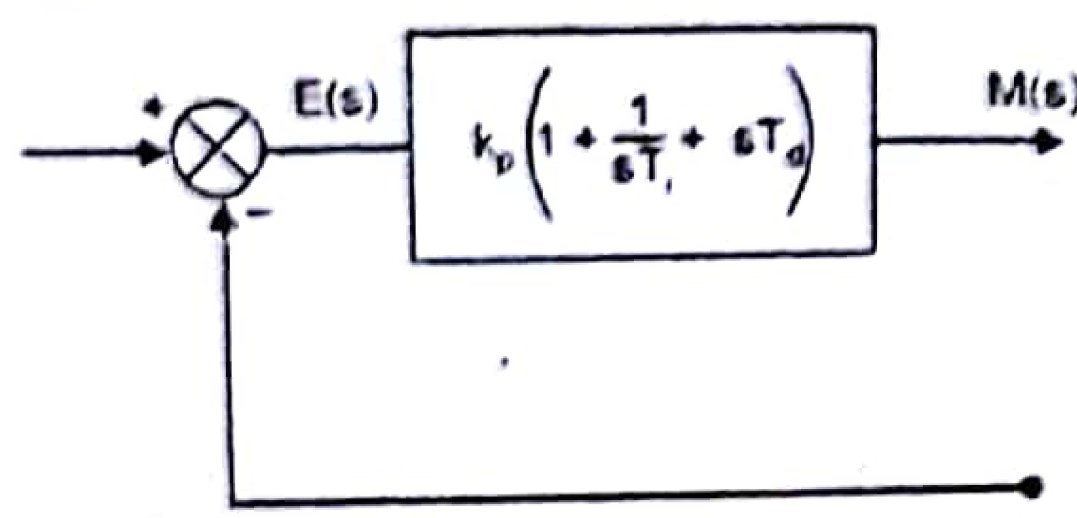
$$m(t) = K_p e(t) + K_p\frac{1}{T_i}\int_0^t e(t)dt + K_pT_d\frac{d}{dt}e(t) \quad \ldots(1)$$

Taking Laplace tranform both sides,

$$M(s) = K_p E(s) + \frac{K_p}{sT_i} E(s) + K_p T_d \, sE(s) \quad ...(2)$$

$$\frac{M(s)}{E(s)} = K_p \left(1 + \frac{1}{sT_i} + sT_d\right) \quad ...(3)$$

The block diagram is



where K_p = proportional gain

T_i = integral time

T_d = derivative time.

Let actuating error signal is given by $e = At$, where 'A' is a constant and 't' is time

From eqn (1)

$$m(t) = K_p At + \frac{K_p}{T_i}\int_0^t At\,dt + K_p T_d \frac{d}{dt} At$$

$$m(t) = K_p A\left[t + \frac{t^2}{2T_i} + T_d\right]$$

UNIT-II

Q.6. Sketch the bode plot of the transfer function $(s) = \dfrac{200(s+2)}{s(s^2+10s+100)}$ **(12.5)**

Ans. Refer Q. no. 3 Important questions

Q.7. (a) Explain relative stability and its significance. **(6.5)**

Ans. Relative Stability Analysis:

By relative stability we mean how close the system is to instability. The degree or extent of the system is called relative stability. It is a quantitative measure of how, fast the system transients die out in a system,

The Routh criterion ascertains the absolute stability of a system by deter-mining whether any of the roots of the characteristic equation lies in the right half of the s-plane. In order to know how close the system is to instability, we should know how far from the $j\omega$-axis is the pole closest to it. This can be obtained from the Routh criterion by shifting the vertical axis in the S-plane to obtain the P-plane as shown in Fig,

Let the p-plane axis be on the left side of the S-plane axis and the horizontal distance between the axes is α. Hence compared to s-plane, the origin of the p-plane will be $(-\alpha, 0)$. If the origin is shifted from the s-plane to the p-plane, the value of p will be $(s + \alpha)$ or $s = p - \alpha$.

Fig. Shift of the axis to the left by α.

Hence, in the polynomial $\Delta(s)$ we replace s by $(p - \alpha)$ we get a new polynomial $\Delta(p)$ Now we apply the Routh stability criterion to this new polynomial in p to know how many hoots $\Delta(p)$ has in the right half of the p-plane. The number of changes of sign in the first column of the array developed for the polynomial in p is equal to the number of roots that are located to the right of the vertical line $s = -\alpha$.

In other words, if the new characteristic polynomial satisfies the Routh criterion, it implies that all the roots of the original characteristic polynomial are more negative titan $-\alpha$. This is also the number of roots of $\Delta(s)$ located towards the right of the line $s = -\alpha$ in the s-plane.

Q.7. (b) What is the minimum/non minimum phase system? Explain **(6)**

Ans. Refer Q.1(*ii*) Second Term Examination 2016.

UNIT-IV

Q.8. The open loop transfer function of UFB system is given $(s) = \dfrac{K}{s(s+1)}$

It is desired to have the velocity error constant $K_v = 12\ \text{sec}^{-1}$ and phase margin as 40°. Design a lead compensator to meet the above specifications. (12.5)

Ans. Given that

$$G(s) = \frac{K}{s(s+1)}$$

$$K_v = 12\ \text{sec}^{-1}$$

and

$$\text{P.M} = 40°$$

We have

$$K_v = \underset{s\to 0}{Lt}\, sG(s) = \underset{s\to 0}{Lt}\, s \times \frac{K}{s(s+1)} = 12$$

$$\Rightarrow \boxed{K = 12}$$

$$G(s) = \frac{12}{s(s+1)} = \frac{12}{s(1+s)}$$

Put $s = j\omega$

$$G(j\omega) = \frac{12}{j\omega(1+j\omega)}$$

$$|G(j\omega)| = \frac{12}{\omega\sqrt{1+\omega^2}} \text{ and } \angle G(i\omega) = -90° - \tan^{-1}\omega$$

The gain crosover frequency is obtained as

$$40 \log \omega g_c = 20 \log 12 = 21.58$$

$$\log_{10} \omega_{g_c} = 0.539 \therefore \omega_{g_c} = 3.46$$

The angle contribution when $\omega = \omega_{g_c}$ is

$$\phi = -90° - \tan^{-1} 3.46 = -163.89$$

Hence the phase margin of the uncompensated system threfore is

$$\phi_m = 180° - 163.89° = 16.11°$$

The required phase margin is 40°. The phase angle deficiency is 40° – 16.11, 23.89° and an allowance of 10° is added to compensate for the decrease in phase margin when the lead network is inserted and ω_{g_c} is shifted to the right. The angle deficiency now becomes 23.89° + 10° = 33.89°. This is taken as θ_m = 33.89°. The attenuation factor as

$$\alpha = \frac{1-\sin\theta_m}{1+\sin\theta_m} = \frac{1-\sin(33.89°)}{1+\sin(33.89°)} = 0.295$$

when the lead network is inserted, the gain crossover frequency is shifted to the right and becomes $\omega_{g_c} = \omega_m$. The amount of attenuation required is

$$20 \log \sqrt{\alpha} = 20 \log \sqrt{0.295} = -5.3\text{dB}$$

Corresponding to – 5.3dB attenuation ω_m is calculated as

$$40\log \frac{\omega_m}{3.46} = 5.3 \Rightarrow \omega_m = 4.7$$

We have,

$$\frac{1}{T} = \omega_n \sqrt{\alpha} = 4.7\sqrt{0.295} = 2.55$$

$$\therefore T = 0.391$$

$$\frac{1}{\alpha T} = \frac{1}{0.295 \times 391} = 0.653$$

The T.F. of the compensator is

$$G_c(s) = \frac{\left(s+\frac{1}{T}\right)}{\left(s+\frac{1}{\alpha T}\right)} = \frac{s+2.55}{s+8.65}$$

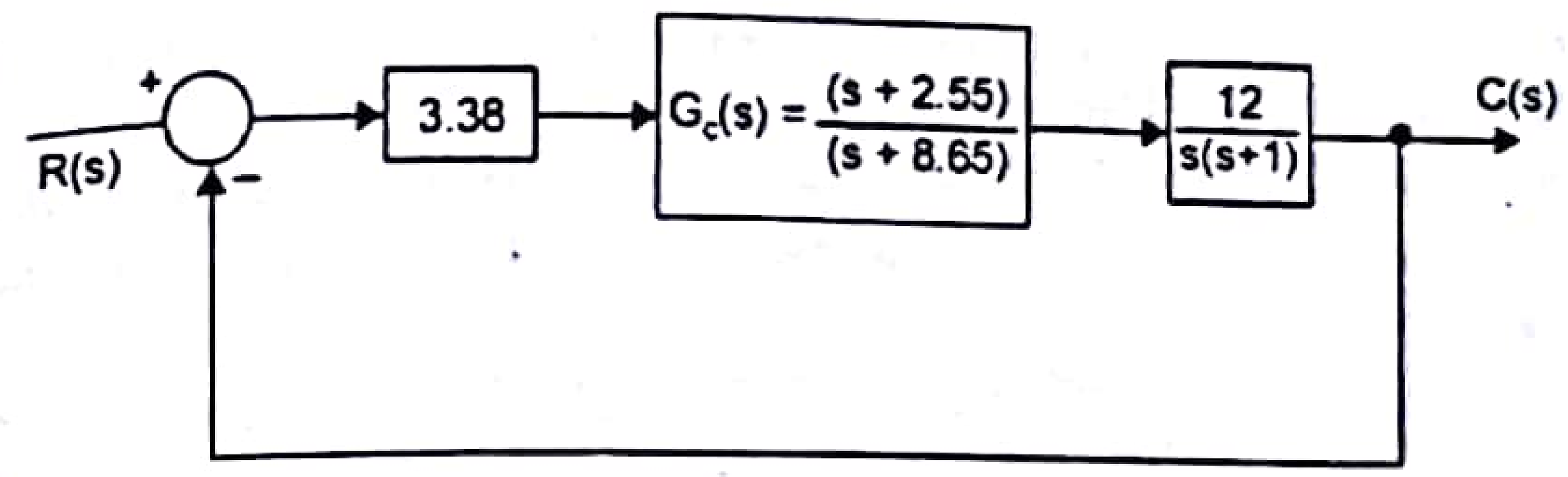
The forward path T.F. of the compensated system now becomes

$$G_C(s)G(s) = \frac{(s+2.55)}{s(s+8.65)} \frac{12}{(s+1)}$$

The overall system gain is reduced by a factor α. Hence an amplifier with gain of

$$\frac{1}{\alpha} = \frac{1}{0.295} = 3.38$$

Is to be introduced in the forward path. The block diagram of the compensated system is shown below



Q.9. Write short note on:

(a) Routh Hurwitz stability criterion **(6.5)**

Ans. Routh's Stability Criterion:

A necessary condition for stability is that all the roots of the characteristic equation have negative real parts, which in turn requires that all the $[a_i]$ be positive.

A *necessary* (but not sufficient) *condition* for stability is that all the coefficients of the characteristic polynomial be positive.

If any of the coeffrients are missing (are zero) or are negative, then the system will have poles located outside the LHP.

A system is stable if and only if all the elements in the first column of the Routh array are positive.

This is necessary and sufficient condition for stability.

The stability of the system can be indicated from Routh's array.

Routh's stability criterion *states that the number of roots of the characteristic equation with positive real parts is equal to the number of changes in sign of the coefficients of the first column of the Routh array.*

This routh's criterion requires that there be no changes in sign in the first column for a stable system.This requirement is both necessary and sufficient any change of sign in the first column of the Routh's array indicates :

(*i*) the system is unstable.

(*ii*) the number of changes of sign gives the number of roots in right-half of *s*-plane. A pattern of +,– , + is counted as *two* sign changes : one change from + to – and another from - to +.

It is to be noted that exact values of the terms in the first column need not be known, instead, only the signs are needed.

If any of the coefficients of the characteristic polynomial is zero or negative in the presence of at least one positive coefficient, there is a root or roots that are imaginary or that have positive real parts. In such a case the system is unstable. If we are interested in only absolute stability, there is no need to follow the procedure further to generate the Routh's array.

Routh's criterion can be applied to the denominator of a transfer function to determine whether the system is stable. It can also be used to study the effect of parameter variation on system stability.

Q.9.(b) Root Locus applications **(6)**

Applications of Root Locus method

(*i*) Absolute and relative stability can be determined using root locus.

(*ii*) For the given system, the gain of the system can be determined for the system to be marginally stable. The corresponding frequency of oscillation can be determined.

(*iii*) For a given value of the system gain K, the closed loop pole locations can be determined.

(*iv*) For the required damping ratio of the dominant closed loop system, the closed loop pole locations can be determined.

(*v*) For the given gain of the system, time domain specifications can be determined.

(*vi*) From root locus the frequency domain specifications such as phase margin and gain margin can be obtained.

(*vii*) The root locus shows clearly the contribution of each open loop pole or zero to the locations of the closed loop poles which ultimately decide the absolute and relative stabilities.

(*viii*) It is also used in the design of compensators in the time domain.

FIRST TERM EXAMINATION [FEB. 2017]
FOURTH SEMESTER [B.TECH.]
CONTROL SYSTEMS [ETEE-212]

Time : 1½ hrs. **M.M. : 30**

Note: *Q. No. 1 is compulsory. Attempt any two questions from the rest. Assume missing data.*

Q.1. (*a*) Classify different types of control system. Explain the difference between open loop and closed loop control system. **(2)**

Ans. Open loop and Closed loop Systems.

An open loop system is one in which the output is independent on input, but controlling action or input is totally independent of the output or changes in output of the system.

A system in which the controlling action or input is somehow dependent on the output or changes in output is called closed-loop system.

Q.1. (*b*) Define steady state error and explain the different static error coefficients. **(2)**

Ans. Steady State Error (e_{ss}): It is the difference between actual output and desired output as time 't' tends to infinityy $e_{ss} = \lim_{t \to \infty}[r(t) - c(t)]$

$$G(s)\,H(s) = \frac{K(1+sT_1)(1+sT_2)}{s(1+sT_a)(1+sT_b)}$$

$$K_p = \lim_{s\to 0} G(s)H(s) = \lim_{s\to 0}\frac{K(1+sT_1)(1+sT_2)......}{s(1+sT_a)(1+sT_b).....}$$

$\boxed{K_p = \infty}$ static error coefficients.

$$\text{Steady state error} = e_{ss} = \frac{1}{1+K_p} = 0$$

$$\boxed{e_{ss} = 0}$$

Q.1. (*c*) Define Delay time, rise time, peak time and Maximum Overshoot. **(2)**

Ans. t_d – *Delay* time: The time that the system output response takes for the step input to reach 50% of its final value is called delay time.

t_r – Rise time: The rise time is the time required for the response to rise from 10% to 90% or 0% to 100% of its final value.

For underdamped second order systems, the 0% to 100% rise time is normally used. For overdamped systems, the 10% to 90% rise time is commonly used.

t_p – Peak Time: The time needed to reach the maximum overshoot is called peak time.

m_p – Maximum overshoot: The maximum positive deviation of the output with respect to its desired value is known as maximum overshoot.

Q.1. (*d*) Explain the principle and application of Tachometer **(2)**

Ans. Tachometers are electromechanical devices, which transforms the mechanical energy into electrical energy. In tachometers its magnetic flux is constant and induced e.m.f. is proportional to the speed of the shaft i.e. angular speed.

Further Tachometers are of two types :-

(*i*) D.C. Tachometer (*ii*) A.C. Tachometer

Application

(*i*) To measure the rotation speed of any mechanical object and provide accurate reading on a digital LCD display.

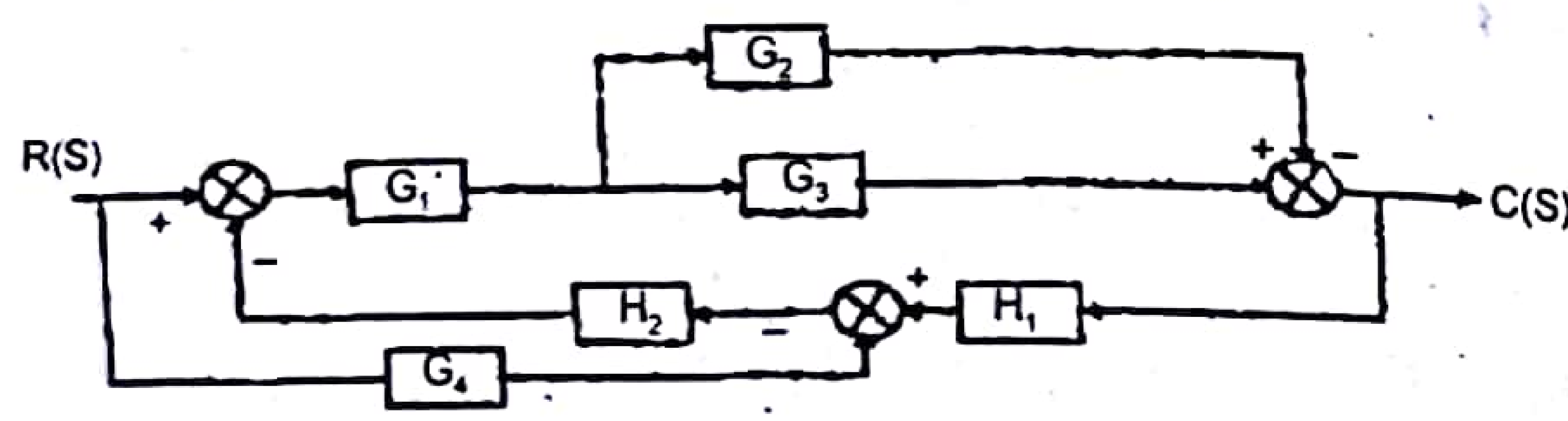
(*ii*) Airplanes

(*iii*) Laser Instruments

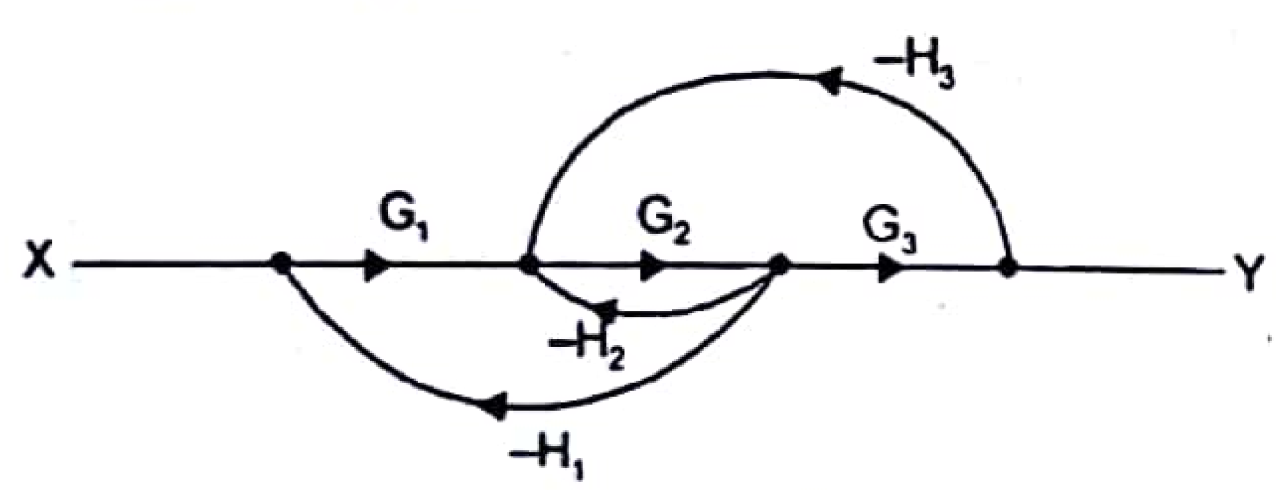
Q.1. (*e*) Classify different types of controller. (2)

Ans. Refer Q.9. of End Term Examination 2017.

Q.2. (*a*) Draw the signal flow graph and evaluate the closed loop transfer function of the system whose block diagram is shown below. (5)



Ans. Equivalent signal flow graph.



$$L_1 = -G_2H_2;\ L_2 = -G_2G_3H_3;\ L_3 = -G_1G_2H_1$$

$$\Delta = 1-(L_1+L_2+L_3)$$

$$\Delta = 1+G_2H_2+G_2G_3H_3+G_1G_2H_1$$

Forward paths are :-

$$P_1 = G_1G_2G_3$$

$$\Delta_1 = 1$$

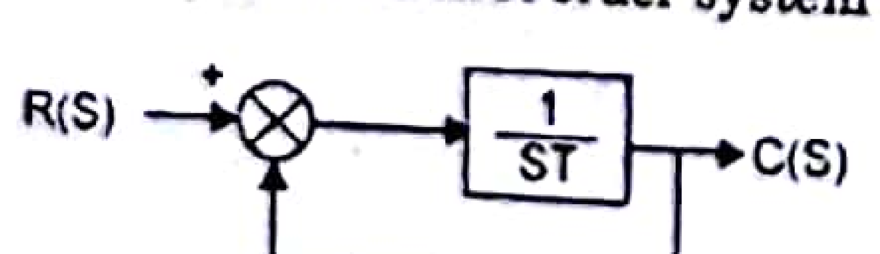
The Transfer function is,

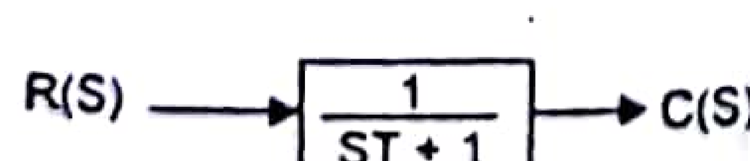
$$\frac{X}{Y} = \frac{P_1\Delta_1}{\Delta}$$

$$\boxed{\frac{Y}{X} = \frac{G_1G_2G_3}{1+G_2H_2+G_2G_3H_3+G_1G_2H_1}}$$

Q.2. (*b*) Derive the expression and draw the response of first order system with unit step input. (5)

Ans. Time Response of a first order system with unit step input.





$$G(S) = \frac{1}{ST}$$

$$H(S) = 1$$

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)H(S)} = \frac{1/ST}{1+\frac{1}{ST}1}$$

$$= \frac{1/ST}{(ST+1)/ST} = \frac{1}{ST+1}$$

$$\therefore \quad \frac{C(S)}{R(S)} = \frac{1}{ST+1}$$

$$\therefore \quad \frac{C(S)}{1} = \frac{1}{ST+1}R(S)$$

For Unit Step Function

$$\therefore \quad R(S) = \frac{1}{S}$$

So,

$$C(S) = \frac{1}{S(ST+1)}$$

$$C(S) = \frac{1}{S} - \frac{T}{1+ST}$$

$$C(S) = \frac{1}{S} - \frac{1}{S+\frac{1}{T}}$$

$$L^{-1}C(S) = L^{-1}\left\{\frac{1}{S}\right\} - L^{-1}\left\{\frac{1}{S+\frac{1}{T}}\right\}$$

$$C(t) = 1-e^{-t/T}$$

When $t = T$

$$C(t) = 1-e^{-T/T}$$

$$C(t) = 1-e^{-1} = 0.632 \text{ or } 63.2\ \%$$

where 'T' is Time constant.

Q.3. (*a*) Derive the expression for maximum overshoot for a 2nd order system with unit step input. (5)

Ans. The output of the system is given by

$$C(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n+\omega_n^2}R(s)$$

For a Unit-step input $\quad r(t) = 1$ and $R(s) = \frac{1}{s}$

Therefore,
$$C(s) = \frac{\omega_n^2}{\left(s^2 + 2\xi\omega_n s + \omega_n^2\right)s} \quad ...(1)$$

$$= \frac{1}{s} - \frac{s + 2\xi\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

or
$$C(s) = \frac{1}{s} - \frac{s + 2\xi\omega_n}{\left(s^2 + \xi\omega_n\right)^2 + \omega_d^2} - \frac{\xi w_n}{(s + \xi\omega_n)^2 + \omega_d^2} \quad ...(2)$$

We know that,
$$= L^{-1}\left[\frac{s + \xi\omega_n}{(s + \xi\omega_n)^2 + \omega_d^2}\right] = e^{-\xi\omega_n t}\cos\omega_d t \quad ...(3)$$

$$L^{-1}\left[\frac{\omega_d}{(s + \xi\omega_n)^2 + \omega_d^2}\right] = e^{-\xi\omega_n t}\sin\omega_d t \quad ...(4)$$

Taking the inverse Laplace transform of Eq.(2), we get
$$c(t) = L^{-1}C(s) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi\omega_n}{\omega_d}e^{-\xi\omega_n t}\cos\omega_d t \quad ...(5)$$

Since $\omega_d = \omega_n\sqrt{1-\xi^2}$

Eq. (5) may be written as
$$c(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi}{\sqrt{1-\xi^2}}e^{-\xi\omega_n t}\sin\omega_d t$$

$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\left(\sqrt{1-\xi^2}\right)\cos\omega_d t + \xi\sin\omega_d t\right] \quad ...(6)$$

Put $\sqrt{1-\xi^2} = \sin\phi$

Therefore, $\cos\phi = \xi$ and $\tan\phi = \frac{\sqrt{1-\xi^2}}{\xi}$

Therefore Eq. (6) can be written as
$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin\phi\cos\omega_d t + \cos\phi\sin\omega_d t\right] \quad ...(7)$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t + \phi) \quad ...(8)$$

$$\tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{\sqrt{1-\xi^2}}{\xi}$$

$$\phi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\text{ radians} \quad ...(9)$$

Equation (8) can be written as
$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\left[\omega_d t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right] \text{ for } t \geq 0 \quad ...(10)$$

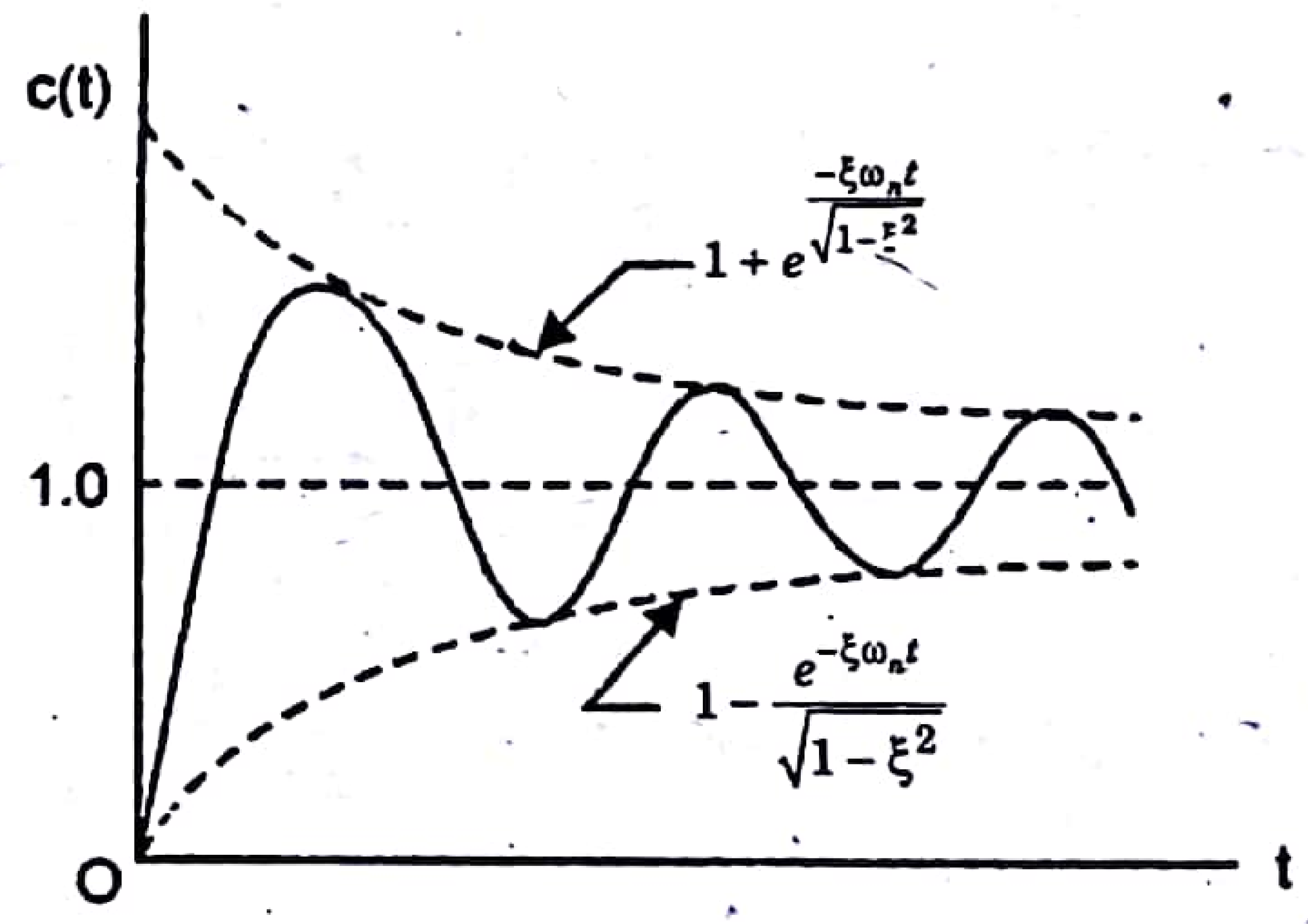
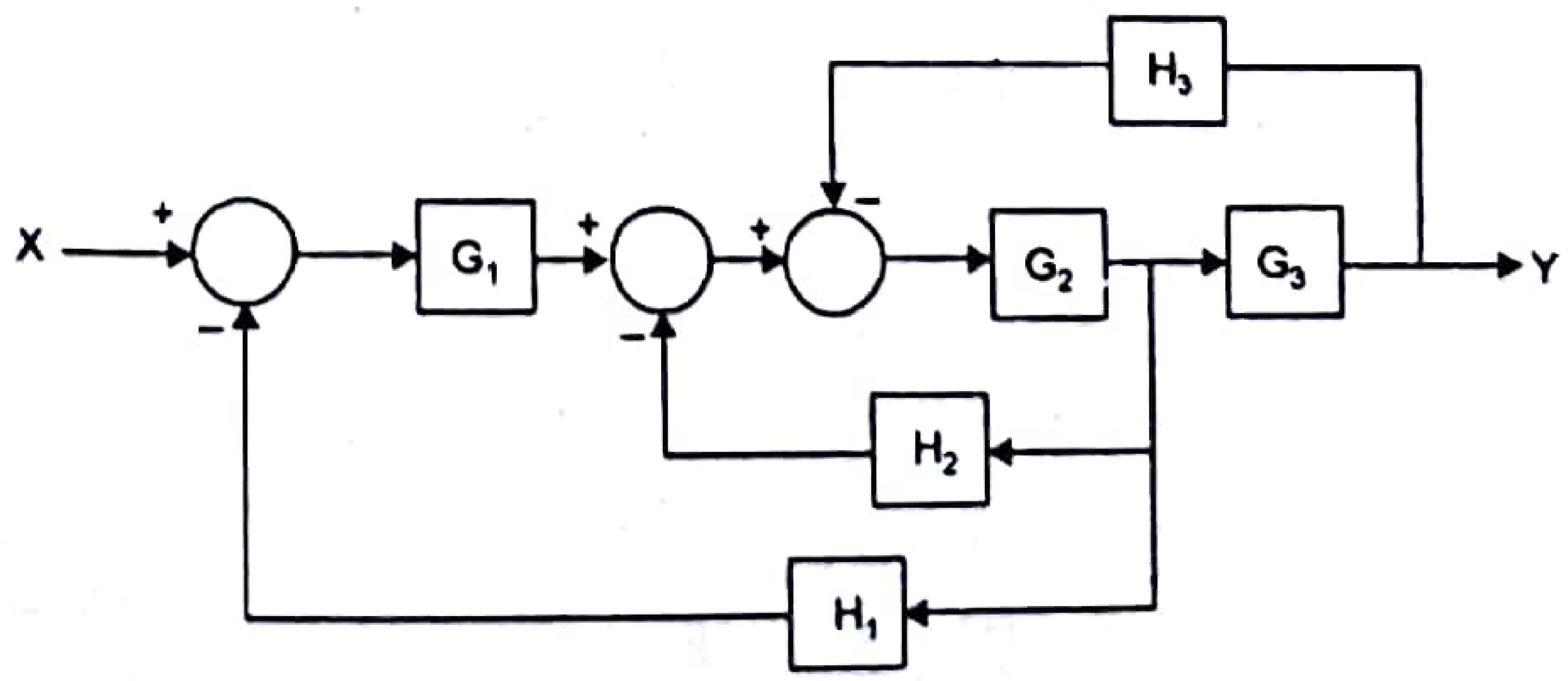


Fig. Unit-step response of a second-order system

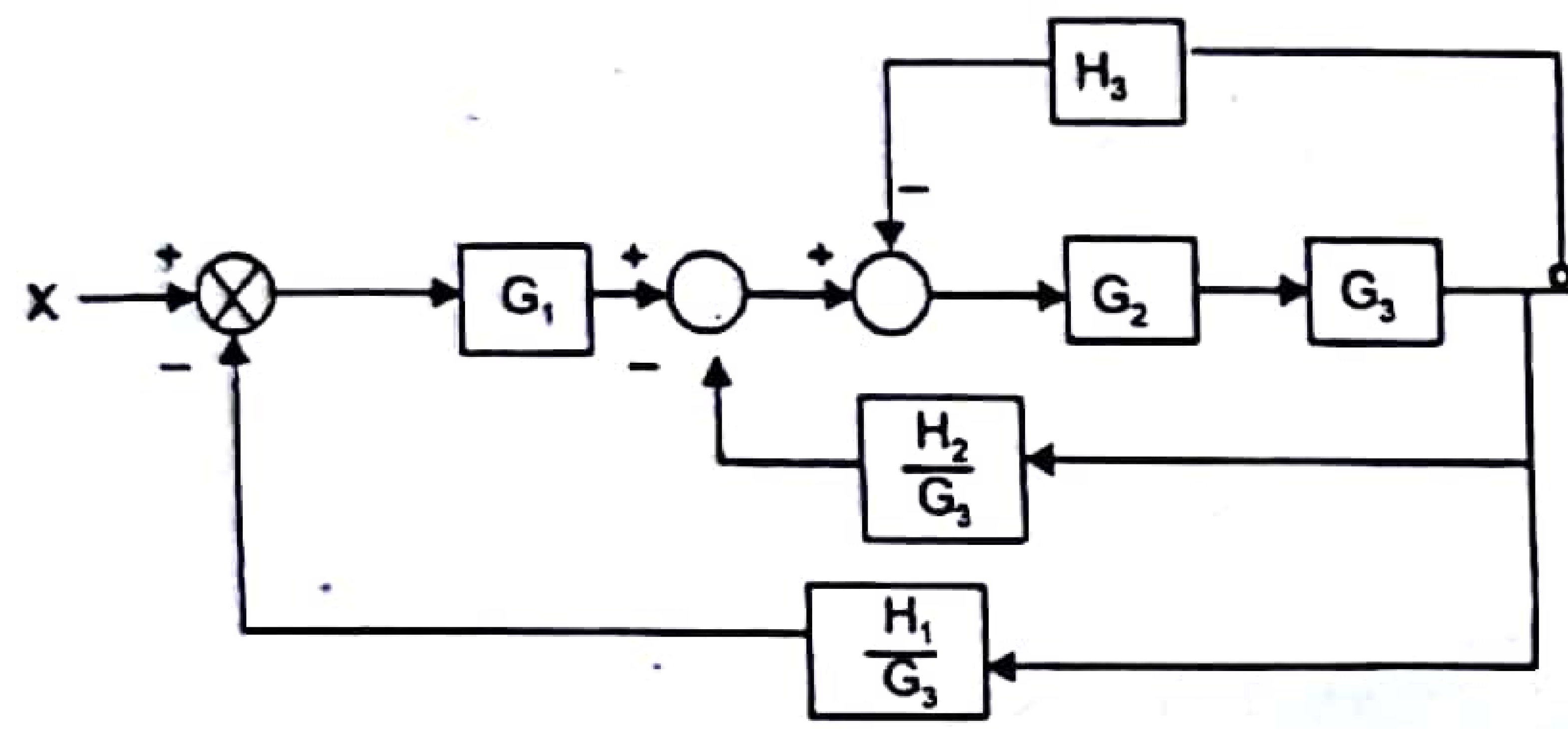
Figure shows the response c(t) of Eq. (10) for $0 < \xi < 1$. It is a sinusoid decaying on an exponetial envelope.

For a step of A units, the expression for c(t) is given by
$$c(t) = A\left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t + \phi)\right] \quad ...(11)$$

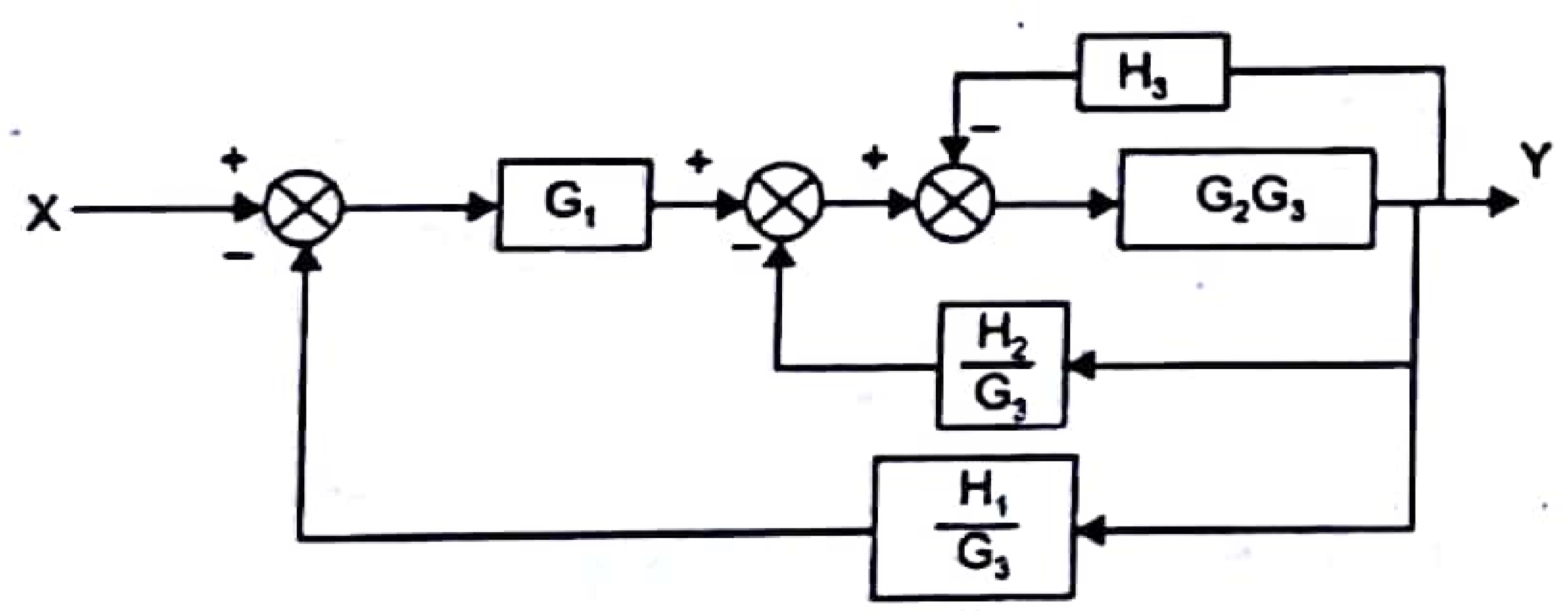
Q.3. (*b*) Derive the Transfer function of the block diagram which is shown below.



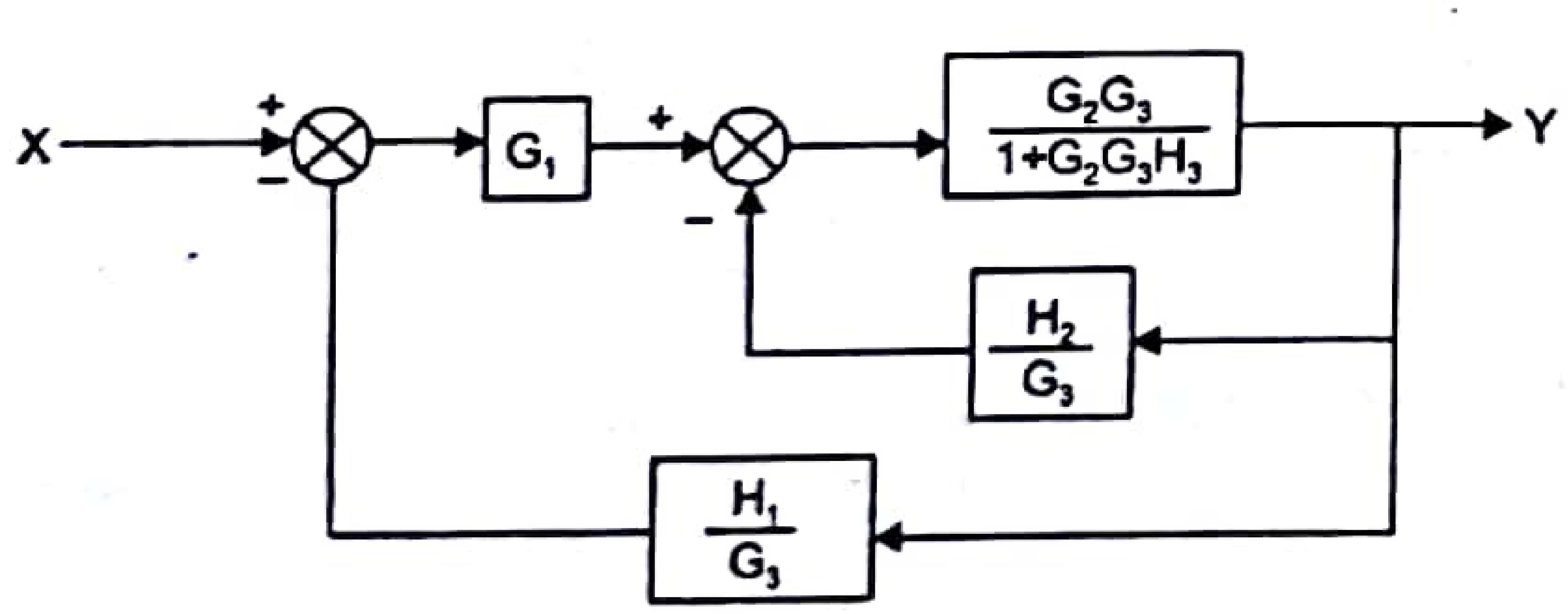
Ans. Step 1



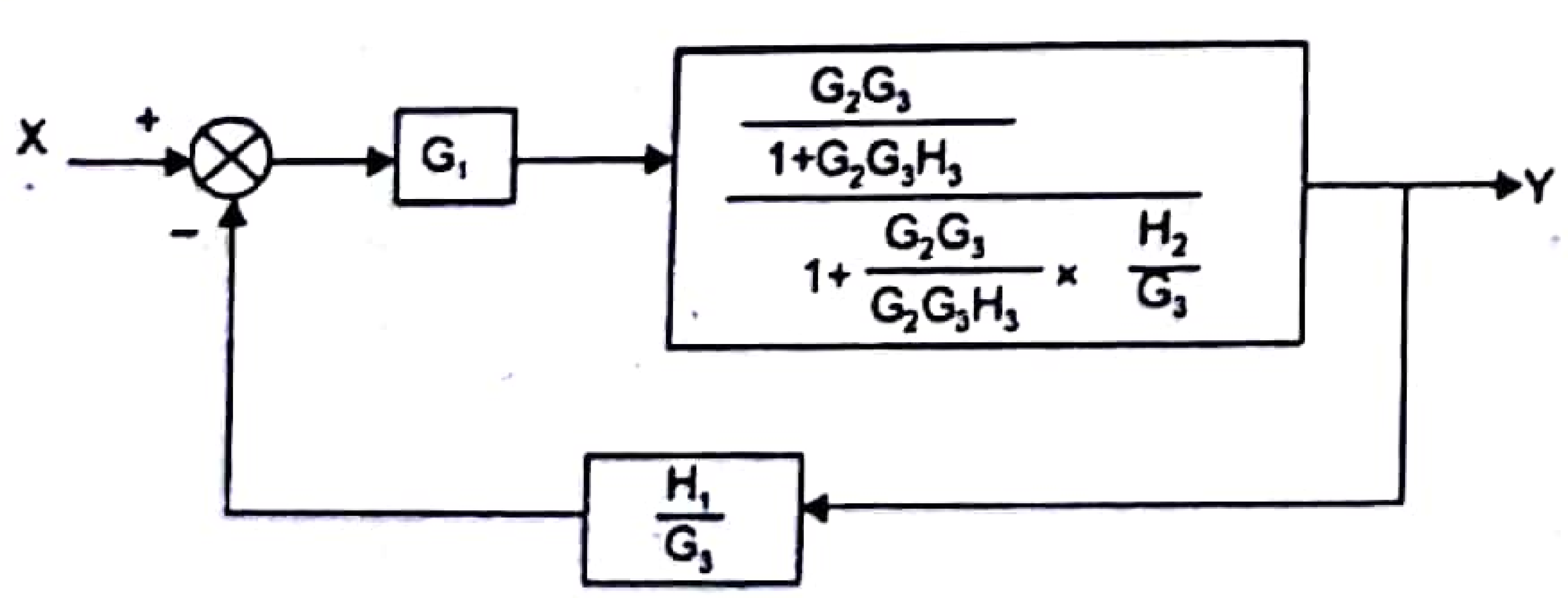
Step 2



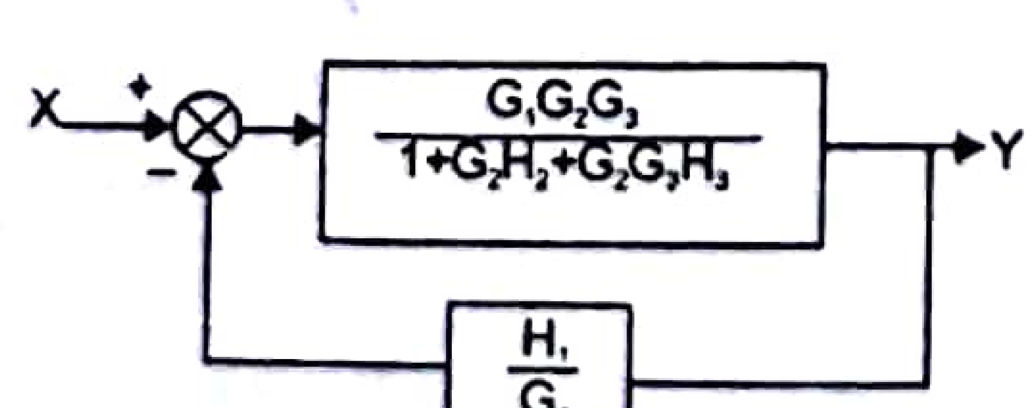
Step 3



Step 4



Step 5



Step 6

$$\frac{Y}{X} = \frac{\dfrac{G_1G_2G_3}{1+G_2H_2+G_2G_3H_3}}{1+\dfrac{G_1G_2G_3}{1+G_2H_2+G_2G_3H_3}\times\dfrac{H_1}{G_3}}$$

$$\boxed{\frac{Y}{X} = \frac{G_1G_2G_3}{1+G_2H_2+G_2G_3H_3+G_1G_2G_3H_1}}$$

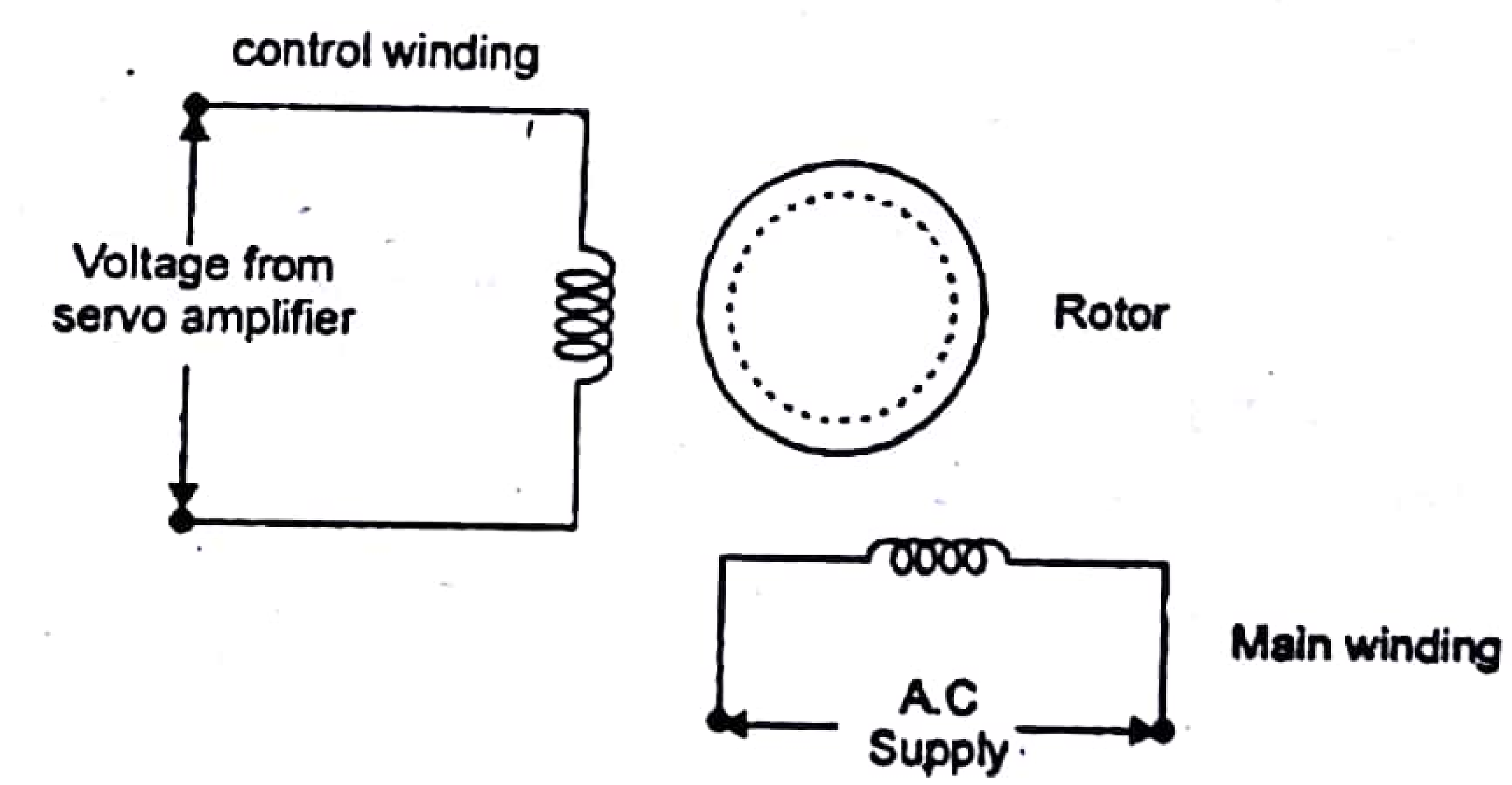
Q.4. Write short notes on the following (Any Two) (5)

Q.4. (a) Servo motors

Ans. Servomotors are two phase induction motor. The stator has two distributed windings. These windings are displaced from each other by 90° electrical.

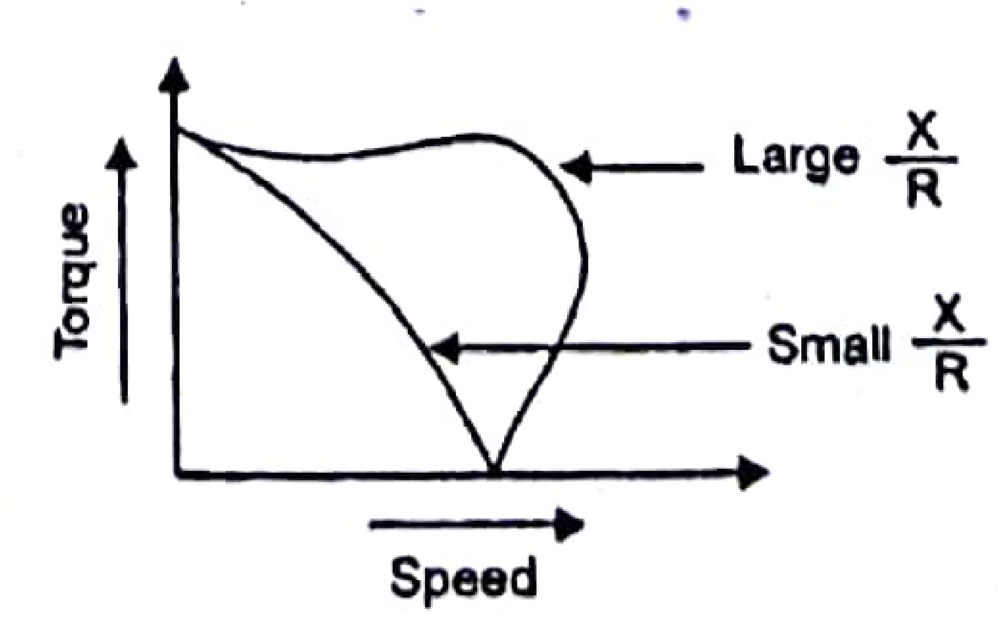
(1) Main winding or Reference winding is excited by constant A.C. voltage.

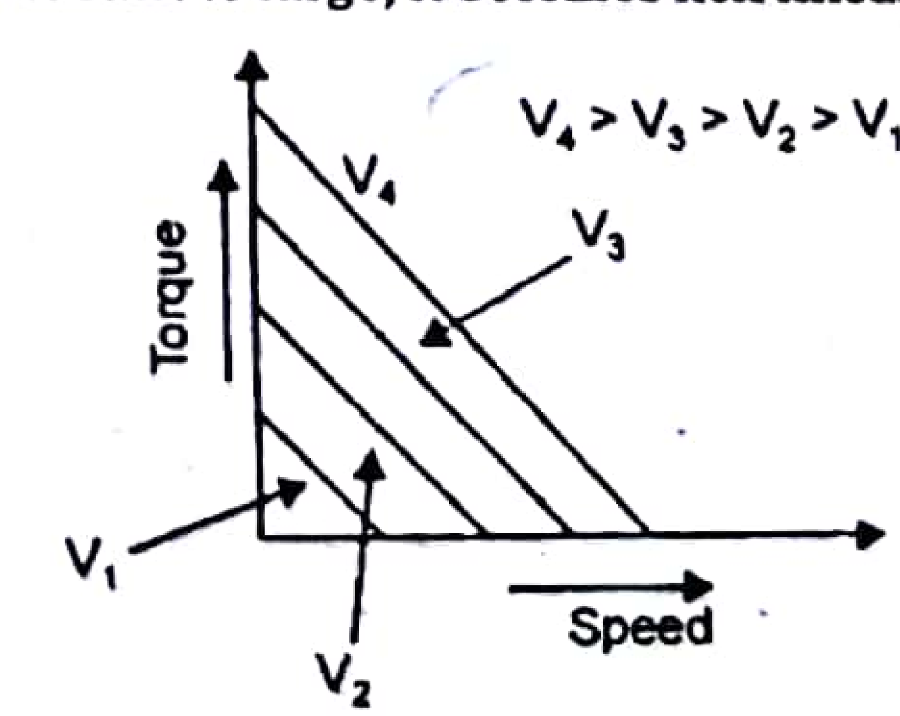
(2) Control winding is excited by variable control voltage of the same frequency as the reference winding, but having a phase displacement of 90° electrical.



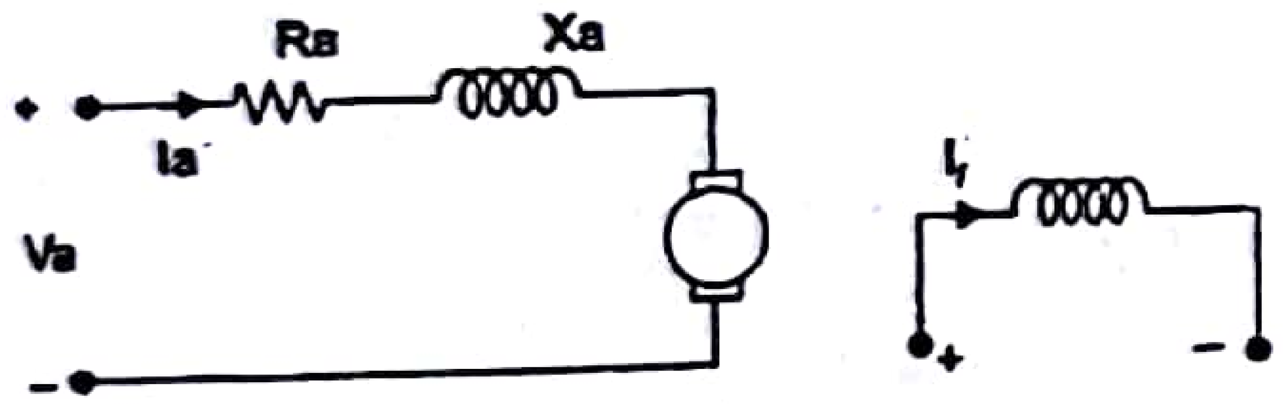
The variable control voltage for control winding is obtained from a servoamplifier. The direction of rotation of the rotor depends upon phase relationship of voltages applied to the two windings. The direction of rotation of the rotor can be reversed by reversing the phase difference between control voltage reference voltage.

Torque-Speed Characteristics:- The torque-speed characteristics of two phase induction motor depends upon the ratio of reactance to resistance. For high resistance & low reactance, the characteristics is linear & as its ratio *is* large, it becomes non linear.

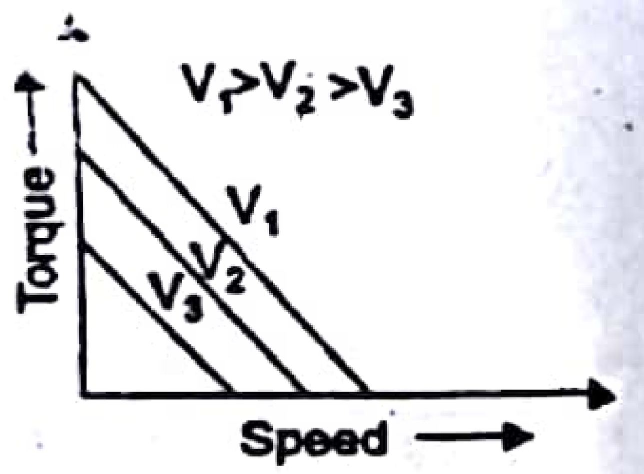




D.C. Servomotors: These motors are separately excited or permanent magnet d.c. servomotors. The armature of d.c. servomotor has a large resistance, therefore torque speed characteristics is linear.



Seperately Excited D.C Servomotor



Torque Speed Characteristics

The d.c. servometer can be controlled from the armature side or from field. In field controlled d.c. servomotor, the ratio of L/R is large, i.e. time constant for field circuit is large. Due to large time constant, the reponse is slow & therefore they are not commonly used.

Q.4. (*b*) Synchros. (5)

Ans. It is a transformer whose primary-to-secondary coupling may be varied by physically changing the relative orientation of the two windings. Synchros are often used for measuring the angle of a rotating machine such as an antenna platform. The primary winding of the transformer, fixed to the rotor, is excited by an alternating current, which by electromagnetic induction, causes currents to flow.

Q.4. (*c*) PI and PID Controllers (5)

Ans. Refer Q.1. (e) of First Term 2017.

END TERM EXAMINATION [MAY-JUNE 2017]
FOURTH SEMESTER [B.TECH.]
CONTROL SYSTEMS [ETEE–212]

Time : 3 hrs. **M.M. : 75**

Note: *Attempt any five questions including Q. No. 1 which is compulsory. Assume missing data if any.*

Q.1. Write short notes on : (5)

(a) Servomechanism

Ans. The servomechanism originated from the words servant (or salve) and mechanism. Here the controlled output is position, speed, acceleration etc. i.e., controlled output is mechanical position or time derivatives of position. Servo voltage stablizer is an example of servo mechanism.A servomechanism may or may not use a servomotor.

Position servomechanisms were first used in military-fire control and marine navigation equipments. Nowadays servomechanisms are used in automatic machine tools, remote control airplanes, automatic navigation systems on boats.

All servomechanisms have a controlled device, a commond device, an error detector, an error signal amplifier. The controlled device generating a signal (such as voltage) called the feedback signal, that represents its current position. This signal is sent to error detector. The error detector compares the feedback signal with the reference input. Any difference of these signals gives the error signal, this error signal sent to an amplifier and the amplified voltage is used to drive the servomotor, which reposition the controlled device.

Q.1. (*b*) Signal flow graph (5)

Ans. (1) Signal flow graph is applicable to linear time-invariant systems.

(2) The signal flow is only along the directions of arrows.

(3) The value of variable at each node is equal to the algebric sum of all signals entering at that node.

(4) The gain of signal flow graph is given by Mason's formula

(5) The signal gets multiplied by the branch gain when it travels along it.

(6) The signal flow graph is not to be the unique property of the system.

Q.1. (c) Feedback Compensation (5)

Ans.

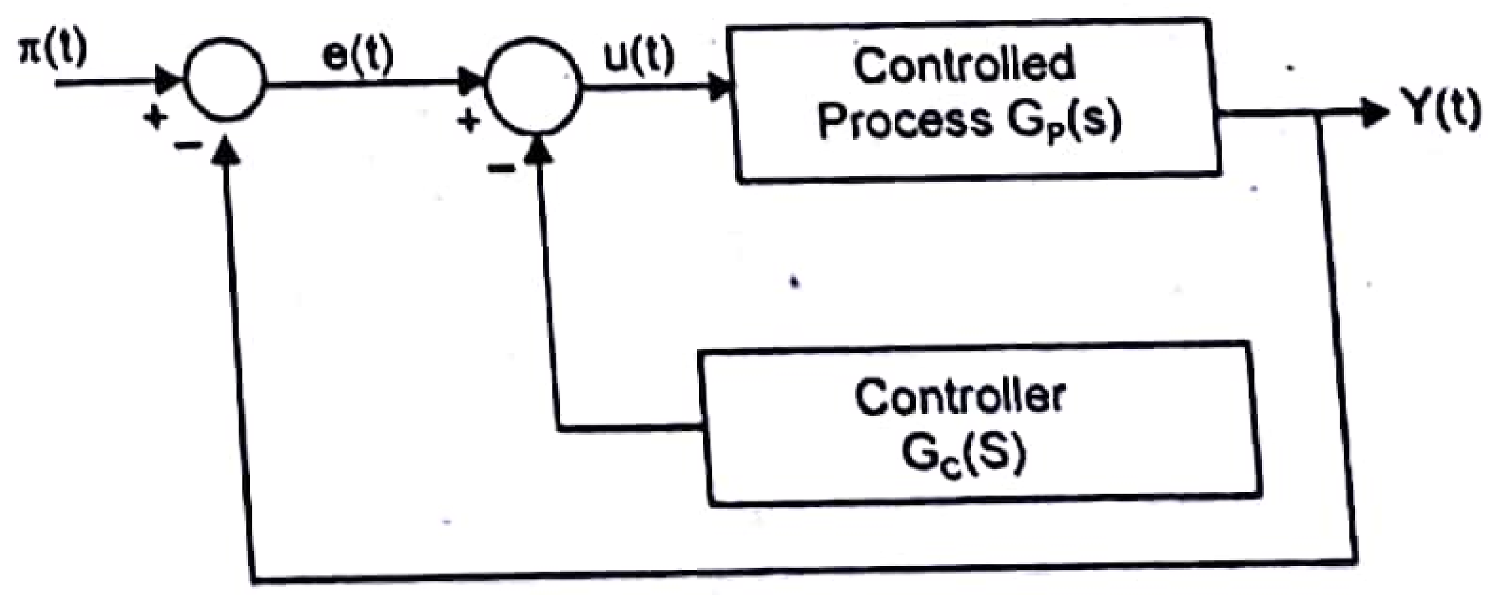


Fig. Feedback compensation

In feedback compensation the controller is placed in the minor feedback path and the scheme is called feedback compensation.

Q.1. (*d*) Magnetic Amplifier. (5)

Ans. The electromagnetic devices used for the amplification of electrical signals which utilizes the magnetic saturation of core principle and certain class of transformer's core non-linear property is called magnetic Amplifier. It is invented in early 1885 and is primarily used in theater lighting and it is designed with basic of design saturable reactor and hence can be used as saturable reactor in electrical machinery.

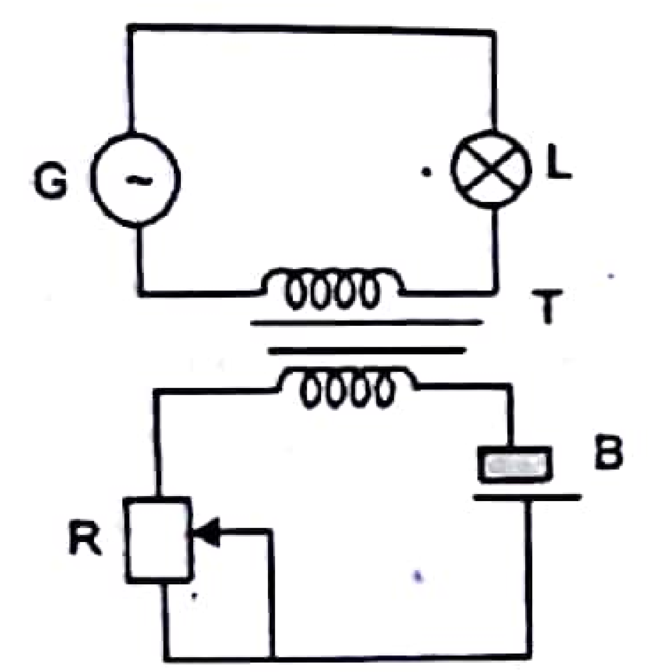


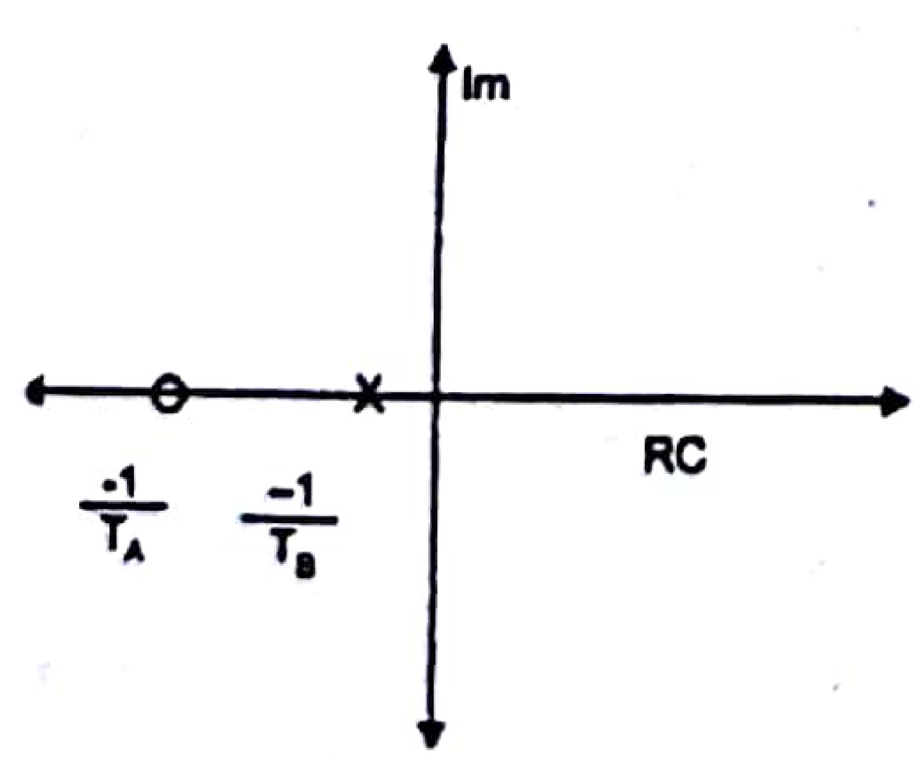
Fig. Magnetic Amplifier

In above Fig. amplifier consists of two cores with control winding and AC winding. By using small DC current fed to control winding the large amount of AC current on AC windings can be controlled and it results in current Amplification.

Q.1. (*e*) Minimum Phase Network. (5)

Ans. Minimum Phase System: The transfer functions having no poles and zeroes in the right half s-plane are called minimum phase transfer functions. System with minimum phase transfer functions are called minimum phase system. The magnitude and phase angle plots of minimum phase systems are uniquely related, that if the magnitude curve is specified for the frequency from zero to infinity, then the phase angle curve is uniquely related.

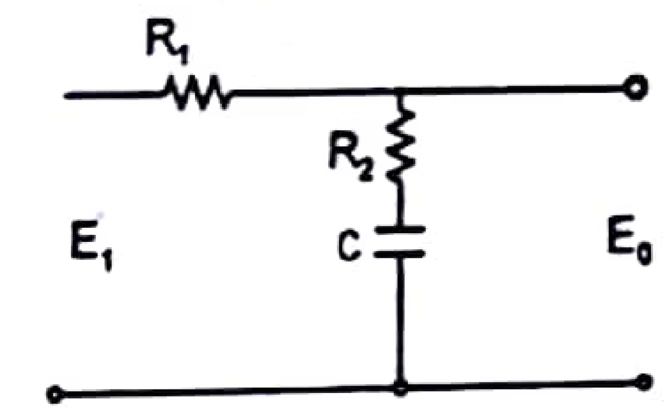
Let
$$G_1(j\omega) = \frac{1+j\omega T_A}{1+j\omega T_B}$$
$$G_2(j\omega) = \frac{1-j\omega T_A}{1+j\omega T_B}$$



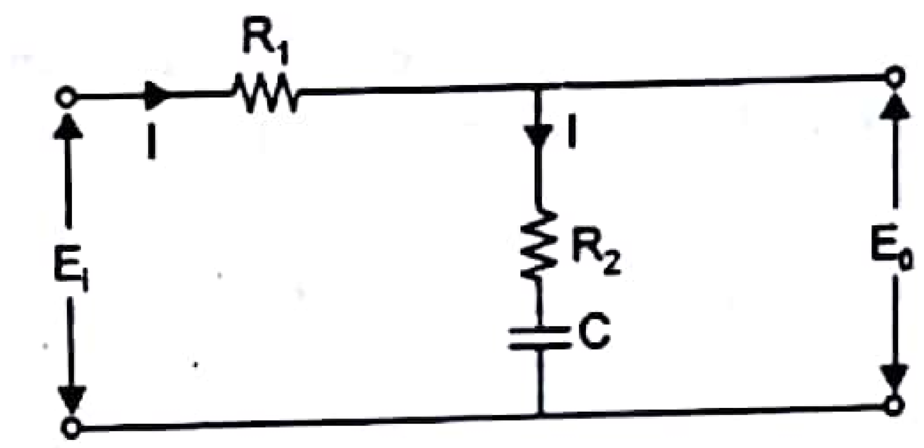
Minimum Phase

Then $G_1(j\omega)$ is a minimum phase transfer function and $G_2(j\omega)$ is a non-minimum phase transfer functions.

Q.2. (*a*) Obtain transfer function for the circuit shown: (5)



Ans. Lag Compensation:



The complex impedances are
$$Z_1(s) = R_1 \text{ and } Z_2(s) = R_2 + \frac{1}{sC} = \frac{1+sR_2C}{sC}$$

Apply KVL at input
$$E_i(s) = I(s)\,(Z_1(s)+Z_2(s))$$

Apply KVL at output
$$E_0(s) = I(s)\,Z_2(s)$$

The transfer function is given by
$$\frac{E_0(s)}{E_i(s)} = \frac{Z_2(s)}{Z_1(s)+Z_2(s)} = \frac{1+sR_2C}{1+s(R_1+R_2)C}$$

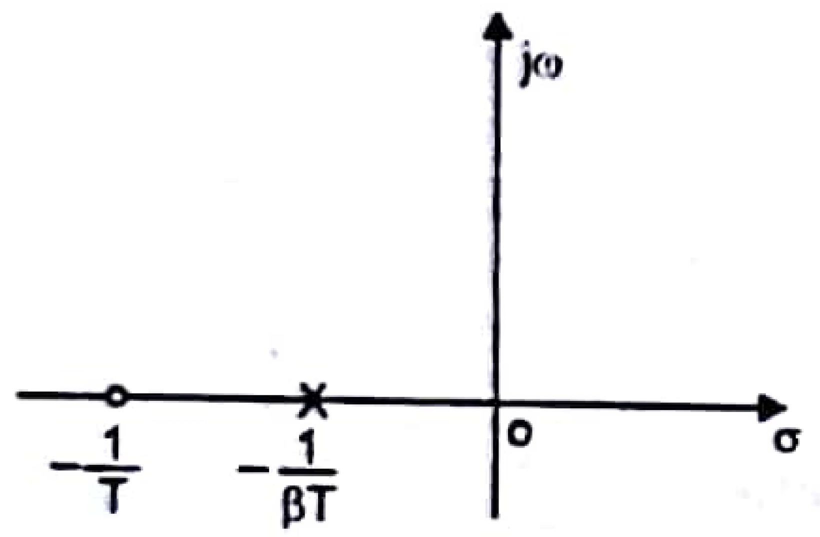
Let
$$R_2C = T, \text{ and } \frac{R_1+R_2}{R_2} = \beta > 1$$

Therefore,
$$\beta T = (R_1+R_2)C$$

and
$$\frac{E_0(s)}{E_i(s)} = \frac{1+sT}{1+s\beta T}; \beta > 1$$
$$= \frac{1}{\beta}\left[\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}\right], \beta > 1$$

Lag compensator has a zero at $s = \frac{-1}{T}$ and a pole at $s = \frac{-1}{\beta T}$

Since $\beta > 1$



Q.2. (b) Derive expression for the output of an undamped unity negative feedback second order system to a unit step input. **(7.5)**

Ans. Refer Q.3. (a) First Term 2017.

Q.3. (a) Sketch polar plot for $G(s) = 3/[(s+0.5)(s+1)]$ and find its intersection with imaginary axis. **(5)**

Ans.
$$G(s) = \frac{3}{(s+0.5)(s+1)}$$

Put $s = jw$

$$\therefore \quad G(jw) = \frac{3}{(0.5+jw)(1+jw)}$$

$$G(jw) = \frac{3}{\sqrt{0.25+w^2}\sqrt{1+w^2}} \underline{\left|-90^\circ - \tan^{-1}\left(\frac{w}{0.25}\right) - \tan^{-1}(w)\right.}$$

Taking the limit for the magnitude of $G(jw)$

$$\lim_{w\to 0} |G(jw)| = \lim_{w\to 0} \frac{3}{\sqrt{0.25+w^2}\sqrt{1+w^2}} = 6$$

$$\lim_{w\to \infty} |G(jw)| = \lim_{w\to \infty} \frac{3}{\sqrt{0.25+w^2}\sqrt{1+w^2}} = 0$$

Taking the limit for the phase angle of $G(jw)$

$$\lim_{w\to 0} \underline{|G(jw)} = \lim_{w\to 0} \underline{\left|-90^\circ - \tan^{-1} w - \tan^{-1}\left(\frac{w}{0.5}\right)\right.} = -90^\circ$$

$$\lim_{w\to \infty} \underline{|G(jw)} = \lim_{w\to \infty} \underline{\left|-90^\circ - \tan^{-1} w - \tan^{-1}\left(\frac{w}{0.5}\right)\right.} = -270^\circ$$

Separating the real and imaginary part of $G(jw)$

$$G(jw) = \frac{3}{(0.5+jw)(1+jw)} = \frac{3}{0.5 + j0.5w + jw - w^2}$$

$$= \frac{3}{0.5 + j\,1.5w - w^2}$$

$$= \frac{30}{5 + j\,15w - 10w^2} = \frac{6}{1 + j\,3w - 2w^2}$$

$$= \frac{6}{(1-2w^2) + j\,3w} \times \frac{(1-2w^2) - j\,3w}{(1-2w^2) - j\,3w}$$

$$= \frac{6[(1-2w^2) - j\,3w]}{(1-2w^2)^2 + 9w^2} = \frac{6 - 12w^2 - j\,18w}{1 + 4w^4 - 4w^2 + 9w^2}$$

$$= \frac{(6-12w^2) - j\,18w}{1 + 5w^2 + 4w^4}$$

$$G(jw) = \frac{(6-12w^2)}{1+5w^2+4w^4} - j\frac{18w}{1+5w^2+4w^4}$$

Equating the imaginary part to zero.

$$\frac{18w}{1+5w^2+4w^4} = 0$$

$$\therefore \quad \boxed{w = 0}$$

Equating real part to zero.

$$\frac{6-12w^2}{1+5w^2+4w^4} = 0$$

$$6 - 12w^2 = 0$$

$$12w^2 = 6$$

$$w^2 = 0.5$$

$$\boxed{w = \pm 0.25}$$

Q.3. (b) Open loop transfer function of a UNFB system is $G(s) = k/[s(s^2+6s+4)]$ where $k = 8$ is forward path gain. Find the value of k for which there is a pair of closed loop poles on jw-axis. Find also the third closed loop pole and gain margin. **(7.5)**

Ans.
$$G(s) = \frac{K}{s(s^2+6s+4)}$$

The characteristic equation is given by.

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K}{s(s^2+6s+4)} = 0$$

$$s^3 + 6s^2 + 4s + k = 0$$

s^3	1	4
s^2	6	K

$$s^1 \quad \frac{24-k}{6}$$

$$s^0 \quad k$$

for Stability

$$k > 0$$

$$\frac{24-k}{6} > 0$$

$$24 - k > 0$$

$$k < 24$$

$$\therefore \text{ Range is } \boxed{0 < k < 24}$$

when $k = 8$, the characteristic equation becomes.

$$s^3 + 6s^2 + 4s + 8 = 0$$

The roots are $\quad S = -5.53 \; -0.23 \pm j\,1.18$

Since, one root is negative and two roots are positive. Therefore one root is lie on negative real axis and two roots are lie on positive real axis.

Damping factor = $\cos\theta$

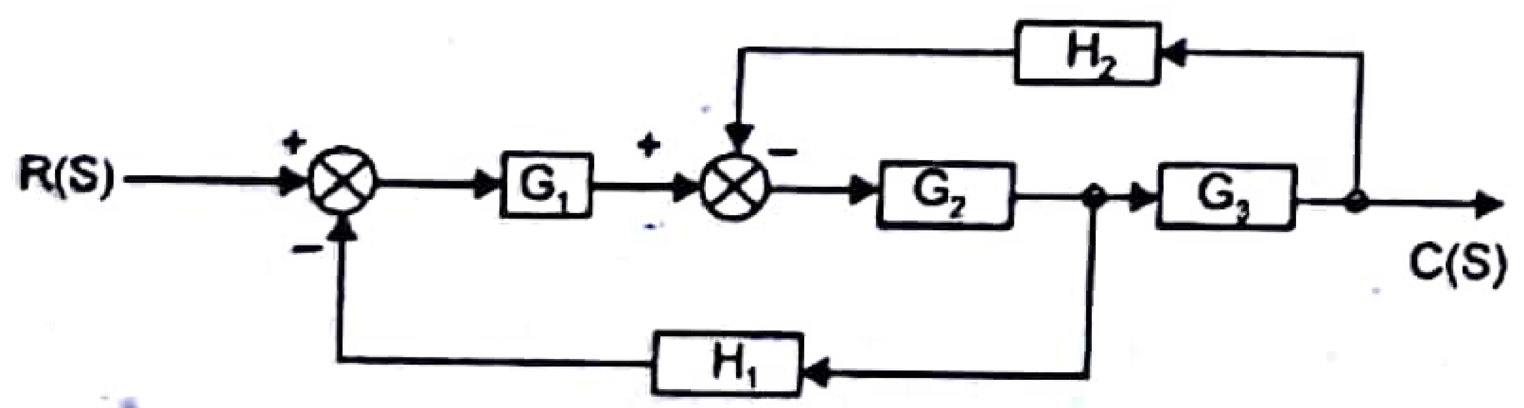
for negative real axis $\theta = 180°$

Damping ratio = $|\cos 180°| = 1$

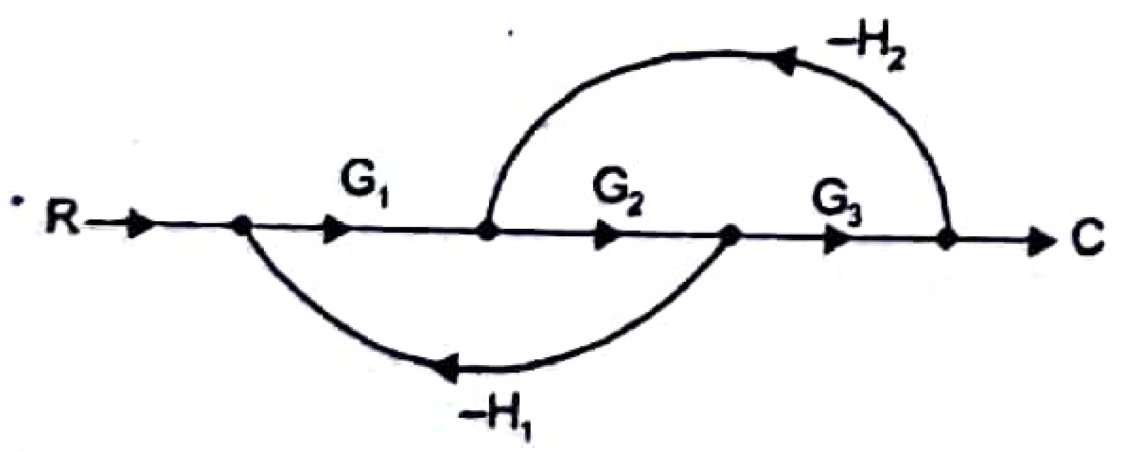
for positive real axis $\theta = 0°$

Damping ratio = $|\cos 0°| = 1$

Q.4. (a) Obtain signal flow graph of the system whose block diagram is shown and from that determine transfer function using Manson's formula. (7.5)



Ans.



Only one forward is there i.e. $P_1 = G_1G_2G_3$

Two closed loops are

$$L_1 = -G_1G_2H_1$$

$$L_2 = -G_1G_2G_3H_2$$

As all the loops are touching both forward paths, the associated path factors are $\Delta_1 = 1$

The graph determinant is =

$$\Delta = 1 - (L_1 + L_2)$$

$$= 1-(-G_1G_2H_1 - G_1G_2G_3H_2)$$

$$= 1 + G_1G_2H_1 + G_1G_2G_3H_2$$

Applying mason's gain formula the relation C/R is,

$$\boxed{\frac{C}{R} = \frac{G_1G_2G_3}{1 + G_1G_2H_1 + G_1G_2G_3H_2}}$$

Q.4. (b) For a system function G(s) = 4(s + 5)/[s (s + 1)], find the frequency at which (i) phase angle = –124°, (ii) magnitude is unity. (5)

Ans.

$$G(s) = \frac{4(s+5)}{S(s+1)}$$

$$G(jw) = \frac{4(jw+5)}{jw(jw+1)}$$

(i) Phase angle $\phi(w) = \tan^{-1}\left(\frac{w}{5}\right) - 90° - \tan^{-1}(w)$

$$\tan^{-1}\left(\frac{w}{5}\right) - 90° - \tan^{-1}(w) = -124°$$

$$\tan^{-1}\left(\frac{w}{5}\right) - \tan^{-1}(w) = -34°$$

$$\frac{\frac{w}{5} - w}{1 + \frac{w^2}{5}} = \tan(-34°) = \frac{-2}{3}$$

$$\frac{4w}{w^2+5} = \frac{2}{3} \text{ or } w^2 - 6w + 5 = 0$$

$$w = 5 \text{ and } w = 1$$

At the frequency $w = 1$ and at $w = 5$ rad/sec the phase angle is $\phi(w) = -124°$.

(ii)

$$|G(jw)| = \frac{4\sqrt{25+w^2}}{w\sqrt{w^2+1}} = 1$$

$$16(25 + w^2) = w^2(w^2+1)$$

$$w^4 - 15w^2 - 400 = 0$$

$$w^2 = 28.86$$

or

$$w = 5.372 \text{ rad/sec}$$

Q.5. (a) Enumerate steps adopted for sketching Root-Locus graph for a normal second order control system. (6)

Ans. Rules for construction of root locii

Following are the rules to sketch the root locus plot.

Rule 1. The root locus is symmetrical about the real axis.

Rule 2. The root locii starts from an open loop pole with K = 0 e.g For the system having

$$G(s)H(s) = \frac{K(s+3)}{(s+2)} \quad ...(1)$$

Find the starting point of the root locii.

According to the rule the root locii start from s = – 2

3. The root locii will terminate either on an open loop zeros or on infinity with $K = \infty$ e.g., Findind point of the root locii given in eqn. (1) According to the rule the root locii will terminate –3

4. If

$$N = \text{No. of separate locii}$$
$$P = \text{No. of finite poles}$$
$$Z = \text{No. of finite zeros then}$$

other of root locii will be equal to the no. of poles if number of poles are more than number of i.e. P > Z

$$N = P \text{ if } P > Z$$

Z > P, then number of root locii will be equal to the number of zeros.

P = Z, then No. of root locii = Poles = Zeros.

e.g. Find the number of separate root locii for the system given by the eq.

$$P = 1$$
$$Z = 1$$
$$N = 1$$

5. Root Locii On The Real Axis.

Any point on the real axis is a part of the root locus if and only if the number of poles and zeros right is odd.

6. Asymptotes

The branches of root locus tend to infinity along a set of straight line called asymptotes. These Asymptotes making an angle with real axis and is given by

$$\phi = \frac{(2K+1)180°}{P-Z} \text{ where } K = 0, 1, 2,....$$

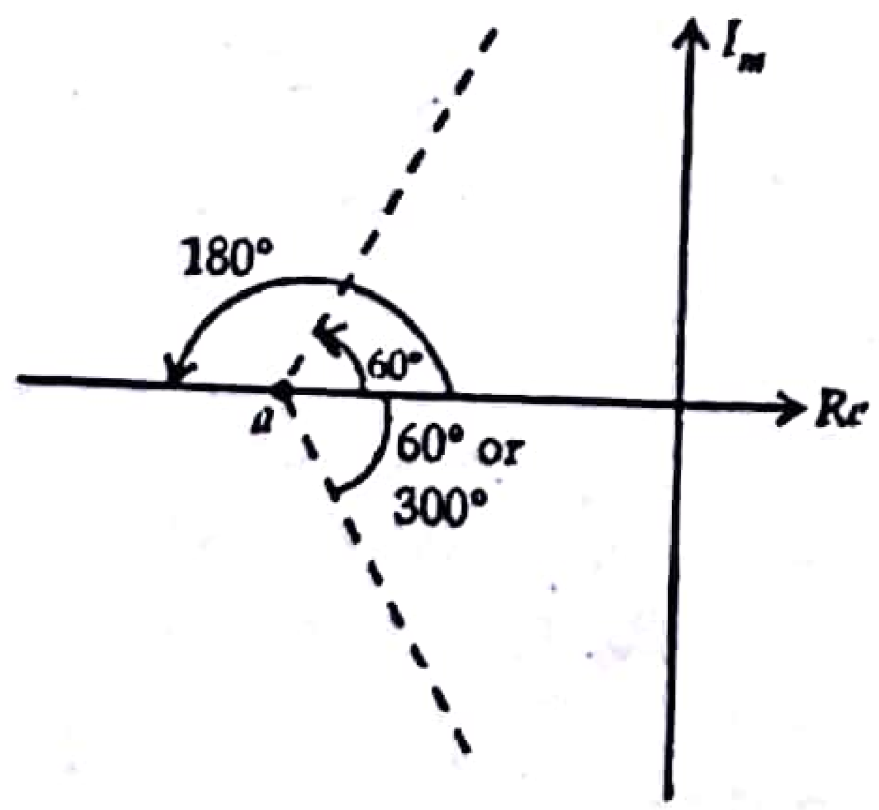


Fig. 1

The total number of asymptotes = P – Z

eg. If.
$$G(s)H(s) = \frac{K}{s(s^2+6s+10)} \quad ...(2)$$

$$P = 3$$
$$z = 0$$

No. of asymtotes = P – Z = 3 – 0 = 3

$$K = 0 \qquad \phi_1 = \frac{(2\times 0+1)180°}{3} = 60°$$

$$K = 1 \qquad \phi_2 = \frac{(2\times 1+1)180°}{3} = 180°$$

$$K = 2 \qquad \phi_2 = \frac{(2\times 2+1)180°}{3} = 300°$$

7. Centroid of Asymptotes

The point of intersection of asymptotes with real axis is called centroid of asymptotes (σ_A) and is given by

$$\sigma_A = \frac{\text{sum of poles} - \text{sum of zeros}}{P-Z}$$

e.g. Find the centroid of asymptotes of the system given by eqn. (2)

Solution : There are three poles at $s_1 = 0, s_2 = -3 + j1, s_3 = -3 - j1$

No. of zeros = 0

$$\therefore \text{centroid} \qquad \sigma_A = \frac{0 - 3 + j1 - 3 - j1 - 0}{3} = -2$$

The centroid is shown Fig. 1 by the Point 'a'.

Rule 8. Angle of Departure & Angle of Arrival of the Root Locii

The angle of departure of the root locus from a complex pole is given by

ϕ_d = 180° – sum of angles of vectors drawn to this pole from other poles + sum of angles vectors drawn to this pole from the zeros.

The angle of arrival at a complex zero is given by

ϕ_a = 180° – sum of angles of vectors drawn to this zero from other zeros + sum of angles of vectors drawn to this zero from poles.

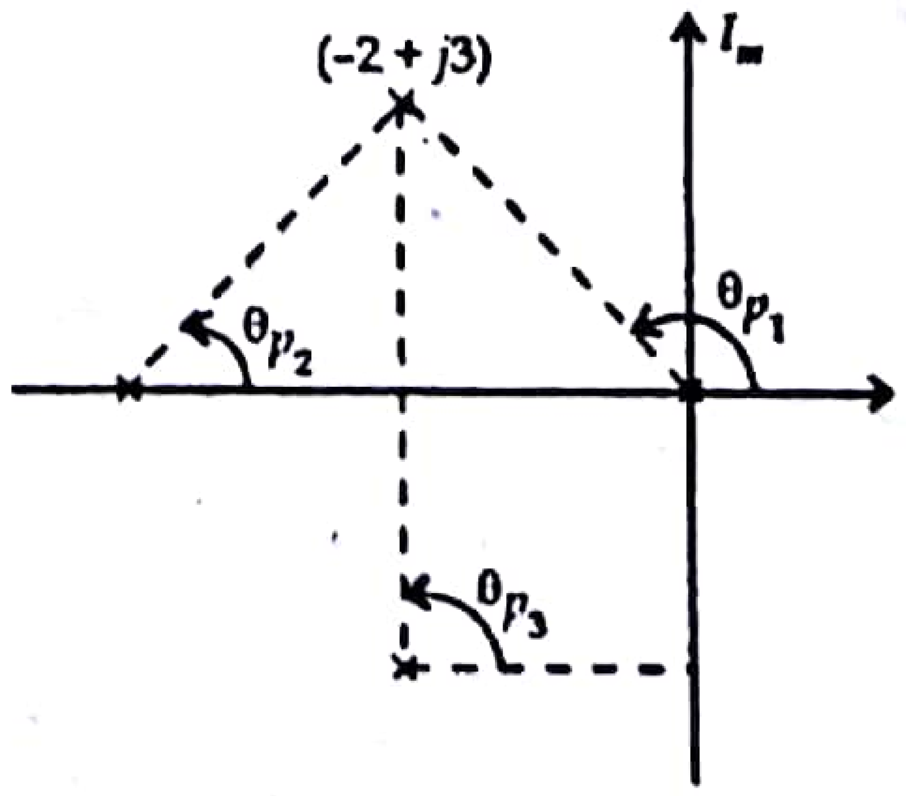


Fig. 2

e.g. For

$$G(s)H(s) = \frac{K}{s(s+6)(s^2+4s+13)}$$

Determine the angle of departure from complex poles.

$$\phi_d = 180° - (\theta_{P_1} + \theta_{P_2} + \theta_{P_3})$$
$$= 180° - (123° + 37° + 90°) = -70°$$

$\therefore$ angle of departure at $(-2 + j3) = -70°$

$\therefore$ angle of departure at $(-2 - j3) = 70°$

Rule 9. Breakaway Point On Real Axis

If the root locus lies between two adjacent open loop poles on the real axis then there will be least one breakaway, because the roots move towards each other as K is increased and meet a point. At this point K is maximum. If we increase the value of K between two poles the root locus breaks in two parts.

Similarly if root locus lies between two adjacent zeros on real axis, then there will be at least one break in point. If the root locus lies between an open loop pole and zero, then there will be no breakaway or break in point or may be both occur.

The break away or break in points can be determined from the roots of

$$\frac{dK}{ds} = 0$$

e.g. If $G(s)H(s) = \dfrac{K}{s(s^2+6s+10)}$ Determine the breakaway point

$$1 + G(s)H(s) = 1 + \frac{K}{s(s^2+6s+10)}$$
$$s(s^2+6s+10) + K = 0$$

or, $$K = -s^3 - 6s^2 - 10s$$

$$\frac{dK}{ds} = -3s^2 - 12s - 10 = 0$$

or, $$3s^2 + 12s + 10 = 0$$

$s_1 = -1.1835$ & $s_2 = -2.815$ are the breakaway points.

Rule 10. The intersection of root locus branches with $j\omega$-axis can be determined through Routh itz criterion.

e.g. If $G(s)H(s) = \dfrac{K}{s(s^2+6s+10)}$. Find the intersection of the root locii with the imaginary axis.

Solution The characteristic eq^n $s^3 + 6s^2 + 10s + K = 0$

s^3	1	10
s^2	6	K
s^1	$\frac{60-K}{6}$	
s^0	K	

Hence, we get a zero row if K = 60

The auxiliary equation $A(s) = 6s^2 + K$

$$6s^2 + K = 0$$
$$6s^2 + 60 = 0$$
$$s = \pm j3.16$$

The root locus branches cross the imaginary axis at $s = \pm j3.16$ for K = 60.

Q.5. (b) Draw Root-Locus for G(s). H(s) = K(s + 3)/[s² (s + 5)]. **(6.5)**

Ans. $$G(s)H(s) = \frac{K(s+3)}{s^2(s+5)}$$

Step 1 Plot the poles and zeros.

Poles are at $S^1 = 0, S^2 = 0, S^3 = -5$

Zeros at $S^4 = -3$

Step 2 The segment between $s = -5$ and $s = -3$ is the part of the root locus.

Step 3 Centroid of Asymptotes

$$\sigma_A = \frac{\text{Sum of poles} - \text{sum of zeros}}{P - Z}$$
$$= \frac{0 + 0 - 5 + 3}{3 - 1}$$
$$= \frac{-2}{2} = -1$$

Step 4 Angle of Asymptotes,

$$\phi = \frac{2K+1}{P-Z}180°$$

$K = 0$ $\quad \phi_1 = 90°$

$K = 1$ $\quad \phi_2 = 270°$

Step 5 Breakaway point of the characteristic eq,

$$1 + \frac{K(s+3)}{s^2(s+5)} = 0$$
$$K = \frac{-(s^3+5s^2)}{(s+3)}$$
$$\frac{dK}{ds} = -\left[\frac{(s+3)(3s^2+10s) - (s^3+5s^2)}{(s+3)^2}\right] = 0$$
$$s(2s^2 + 14s + 30) = 0$$
$$s = 0, s = -3.5 \pm j5.5$$

Breakaway point s = 0 and $-3.5 \pm j5.5$ are neither breakaway point nor breakin point, because the corresponding gain values k becomes complex quantities.

Step 6 Point of intersection of root locii with imaginary axis.

The characteristic eq.

$$s^3 + 5s^2 + sK + 3K = 0$$

Put

$$s = jw$$

$$(jw)^3 + 5(jw)^2 + (jw)K + 3K = 0$$

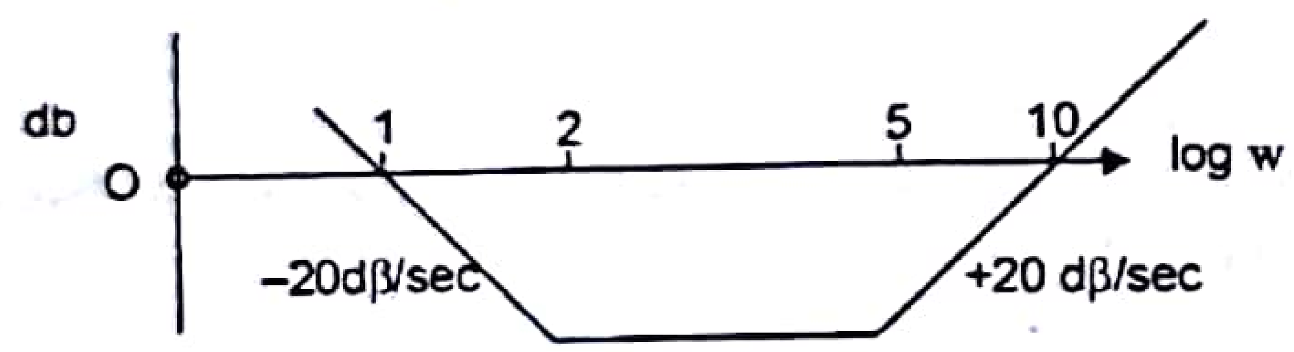
$$-jw^3 - 5(jw)^2 + (jw)K + 3K = 0$$

$$(3k - 5w^2) + jw\,(-w^2 + K) = 0$$

$$w = 0, K = 0$$

Because of double pole at the origin the root locus is tangent to the imaginary axis at $w = 0$.

Q.6. (*a*) Bode's plot for a system is shown here. Obtain its transfer function. (7.5)



Ans. The initial part of the resultant magnitude plot has a slope of $-20dB$/decade. It corresponds to a pole at the origin (i.e. factor $1/jw$). It has a magnitude of $-4db$ at $w = 1$. It corresponds to the open loop gain k, where

$$20 \log k = -4$$

$$\log k = \frac{-4}{20} = -0.2$$

$$k = 0.625$$

At $w = 2.0$, slope changes from –20dB/decade to zero. So a zero factor $(1 + j\,0.5\,w)$ with corner frequency of $w = 5$ has to be added. At $w = 5$, the slope changes from 0 to 20 dB/decade. So a zero factor $(1 + j\,0.2\,w)$ with a corner frequency of $w = 5$ has to be added.

$\therefore$ So the resultant transfer function is

$$G(jw) = \frac{0.625(1 + j\,0.2w)(1 + j\,0.5w)}{jw(1 + j\,0.05w)}$$

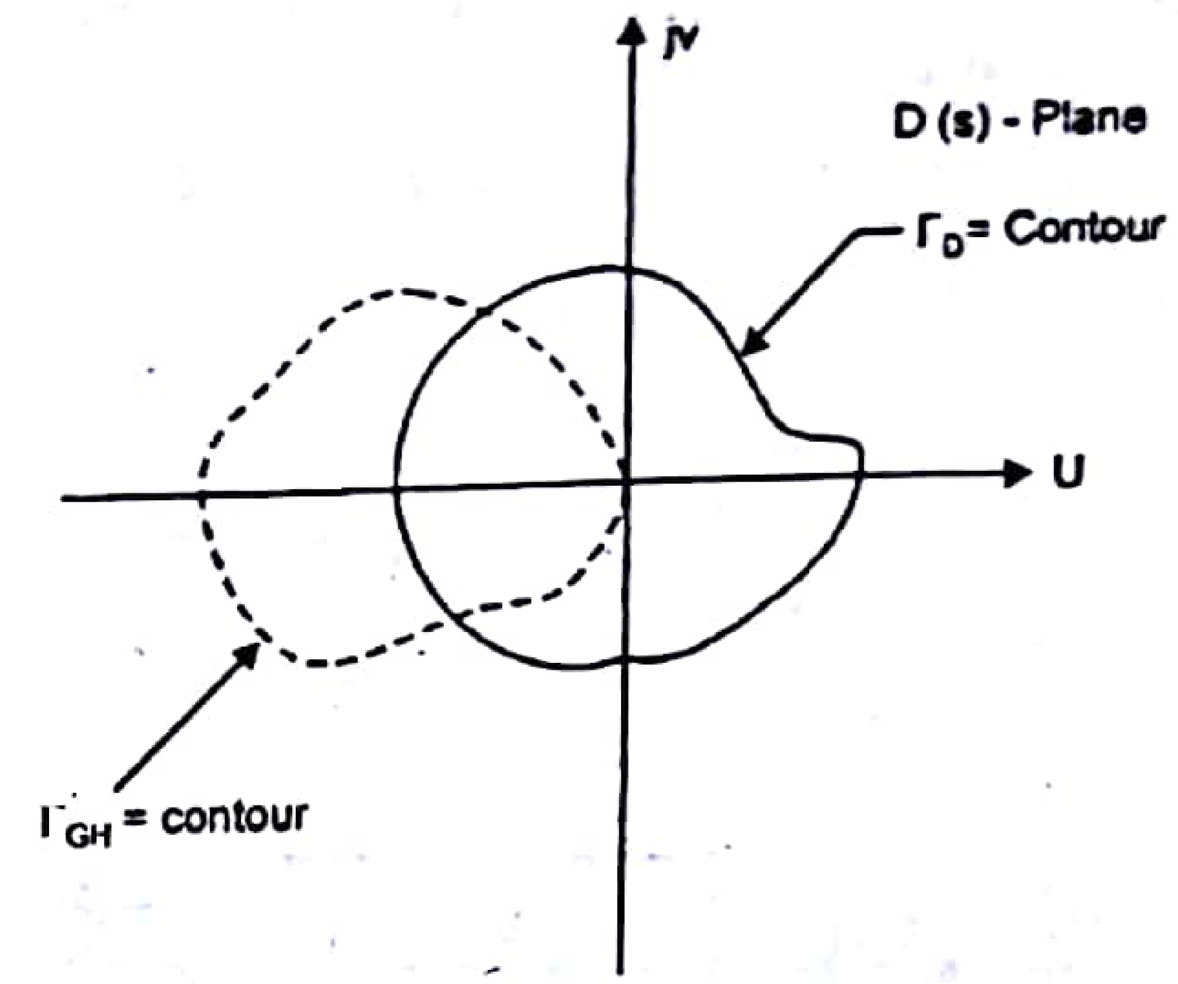
$$\therefore \quad G(s) = \frac{0.625(1 + 0.25)(1 + 0.5s)}{s(1 + 0.05s)}$$

Q.6. (*b*) Give five examples of popular open loop control systems. (5)

Ans. Refer Q.8. (b) of End Term Examination 2017.

Q.7. (*a*) State and example Nyquist's stability criterion as applied to feedback control systems. (7.5)

Ans. A feedback system or closed loop system is stable if the contour Γ_{GH} of the open loop transfer function G(s) H(s) corresponding to the Nyquist contour is the s-plane encircles the point (–1+j0) in clock wise direction & number of counter clockwise encirclement above the (–1+j0) equals to the number of poles of G (s) H (s) in the right half of splane i.e with positive axis. The closed loop system is stable if the contour Γ_{GH} of G (s) H (s) does not pass through or does not encircle (–1+j0) point i.e. net encirclement is zero.



Q.7. (*b*) For a system having G(*s*). H(*s*) = 5/(*s* – 1) investigate, using Nyquist's criterion, if the system is stable. (5)

Ans.

$$G(jw) = \frac{5}{(jw - 1)}$$

The polar plot of G(jw) is a circle with centre at –2.5 on the negative real axis and radius 2.5. As w is increased from $-\infty$ to ∞, the G(jw) locus makes a counter clockwise rotation. In this system P = 1 because there is one pole of G(s) in the right half of s-plane. For the closed loop system to be stable, Z must be equal to zero. There N = Z-P must be equal to –1, or there must be one counter clock wise encirclement of the –1+ j0 point for stability. Thus, for stability K > 1 (which is in our case i.e. 5).

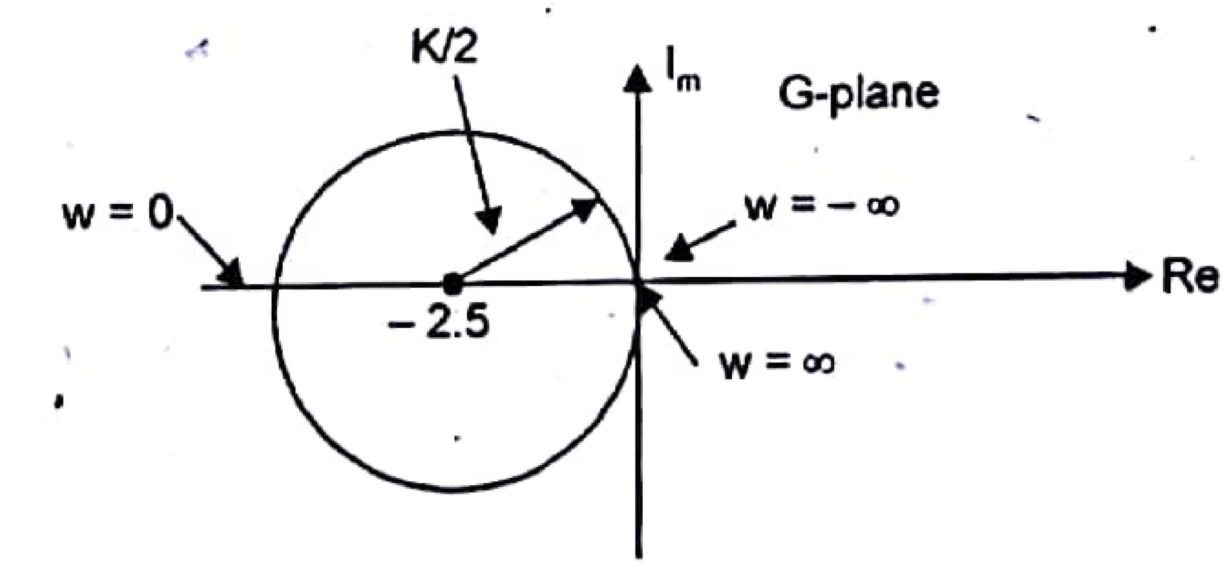


Fig. (a)

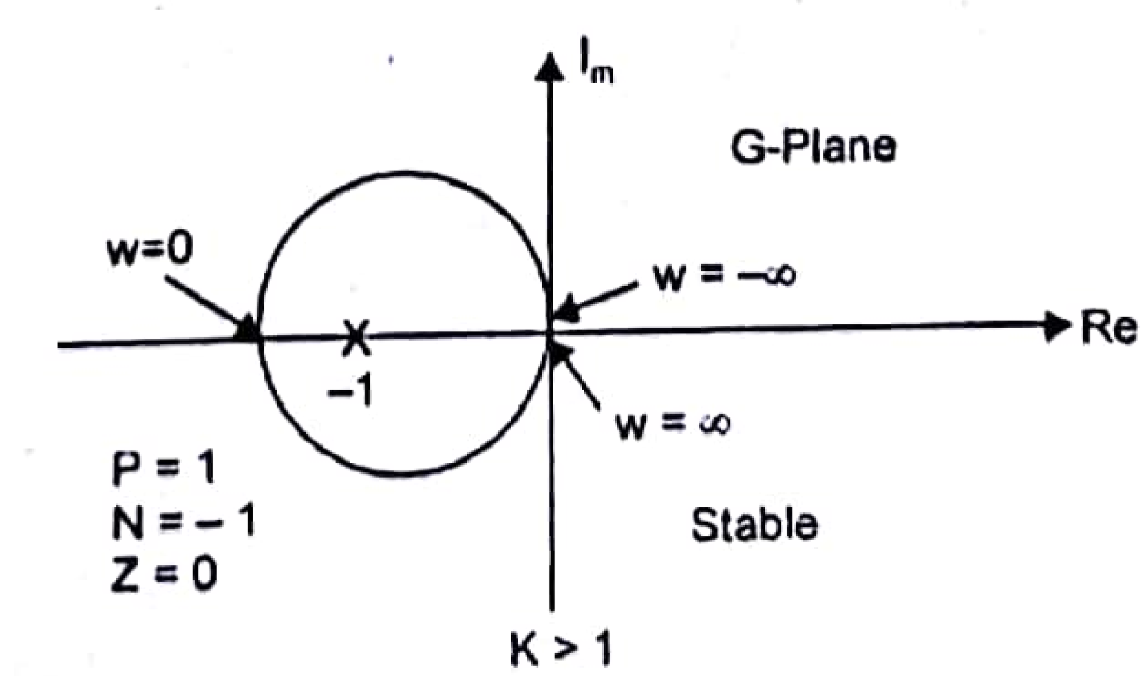


Fig. (b)

Q.8. (a) Explain the co-relation between time and frequency response of a typical second order system. **(5)**

Ans. Correlation between Time and Frequency Response

Consider the second order system

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \qquad ...(1)$$

Put s = jω in equation (1)

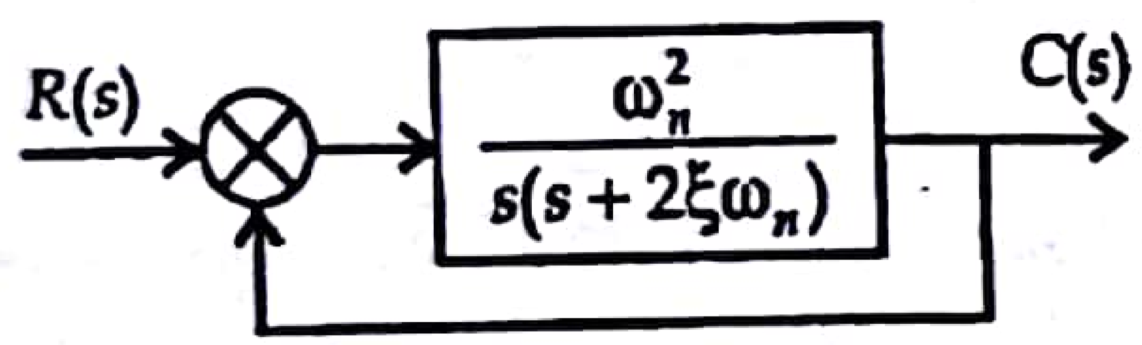


Fig. (1)

$$\frac{C(j\omega)}{R(j\omega)} = \frac{\omega_n^2}{(j\omega)^2 + 2\xi\omega_n(j\omega) + \omega_n^2}$$

$$\frac{C(j\omega)}{R(j\omega)} = \frac{\omega_n^2}{(\omega_n^2 - \omega^2) + j2\xi\omega_n\omega}$$

or
$$\frac{C(j\omega)}{R(j\omega)} = \frac{1}{\left(1 - \frac{\omega^2}{w_n^2}\right) + j2\xi\frac{\omega}{\omega_n}}$$

let
$$\frac{\omega}{\omega_n} = u$$

∴
$$\frac{C(j\omega)}{R(j\omega)} = \frac{1}{(1-u^2) + j2\xi u}$$

$$M(j\omega) = \left|\frac{C(j\omega)}{R(j\omega)}\right| = \frac{1}{\sqrt{(1-u^2)^2 + (2\xi u)^2}} \qquad ...(2)$$

$$\theta = \underline{|M(j\omega)} = -\tan^{-1}\frac{2\xi u}{1-u^2} \qquad ...(3)$$

The steady state output for a sinusoidal input of unit magnitude and variable frequency.

$$C(t) = \frac{1}{\sqrt{(1-u^2)^2 + (2\xi u)^2}}\sin\left[\omega t - \tan^{-1}\frac{2\xi u}{1-u^2}\right]$$

$$\because \; C(t) = y\sin(\omega t + \theta)$$

From equation (2) & (3)

if $u = 0$ $M = 1$ $\theta = 0°$

$u = 1$ $M = \frac{1}{2\xi}$ $\theta = -\pi/2$

$u = \infty$ $M = 0$ $\theta = -\pi$

The magnitude and phase angle characteristic for a certain value ξ is shown in fig. (2).

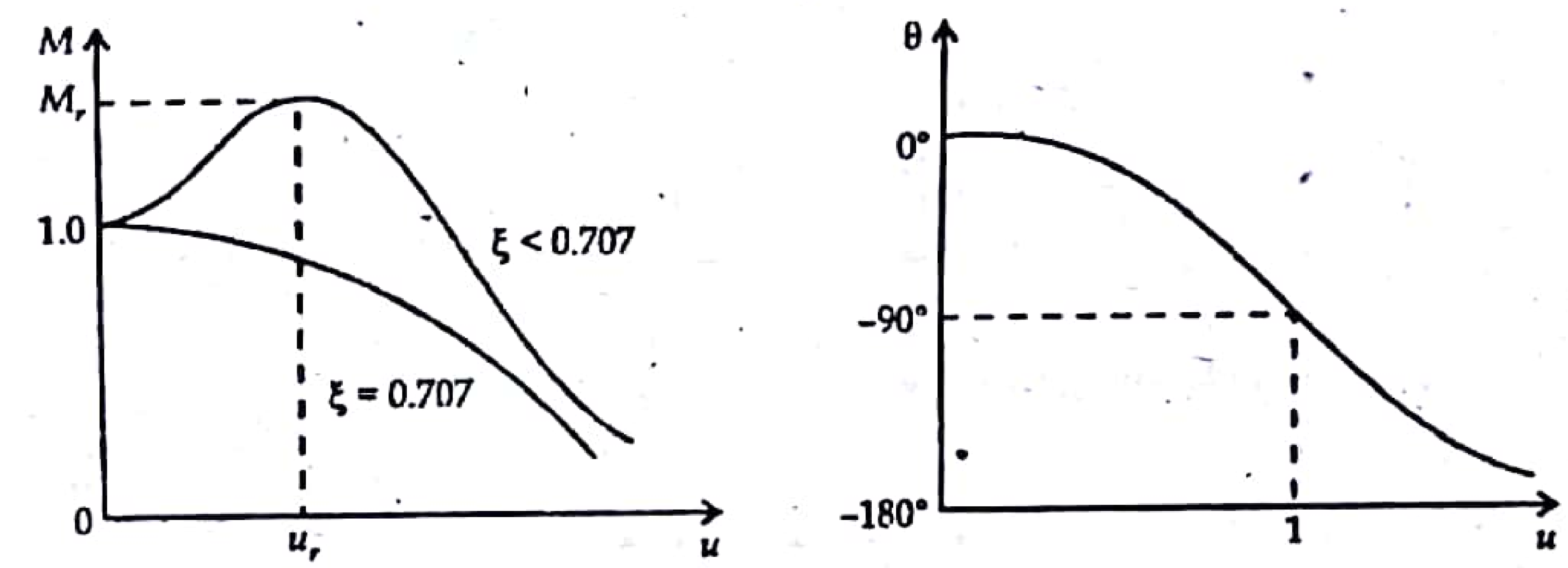


Fig. (2)

The frequency at which M has maximum value is known as resonant frequency (ω_r)

$$\boxed{u_r = \frac{\omega_r}{\omega_n}}$$

where u_r = normalized resonant frequency

Differentiate equation (2) with respect to u and put $u = u_r$ and equate to zero.

$$\frac{d}{du}M\Big|_{u=u_r} = 0$$

$$\frac{d}{du}M\Big|_{u=u_r} = -\frac{1}{2}[(1-u_r^2)^2 + (2\xi u_r)^2]^{-3/2}[4u_r^3 - 4u_r + 8u_r\xi^2] = 0$$

or, $$4u_r^3 - 4u_r + 8u_r\xi^2 = 0$$

or, $$u_r^3 - u_r + 2u_r\xi^2 = 0$$

or, $$u_r = \sqrt{1 - 2\xi^2}$$

But $$u_r = \frac{w_r}{w_n}$$

$$\therefore \quad \frac{\omega_r}{\omega_n} = \sqrt{1-2\xi^2}$$

$$\boxed{\omega_r = \omega_n\sqrt{1-2\xi^2}} \qquad \text{...(4)}$$

Form equation (3)

$$\theta = -\tan^{-1}\frac{2\xi u_r}{1-u^2}$$

$$= -\tan^{-1}\frac{2\xi\sqrt{1-2\xi^2}}{1-1+2\xi^2}$$

$$\boxed{\theta = -\tan^{-1}\frac{\sqrt{1-2\xi^2}}{\xi}} \qquad \text{...(5)}$$

For max. M value of magnitude put $u_r = \sqrt{1-2\xi^2}$ in eqn (2)

$$M_r = \frac{1}{\sqrt{[1-(1-2\xi^2)]^2+4\xi^2(1-2\xi^2)}}$$

$$\boxed{M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}} \qquad \text{...(6)}$$

where M_r is known as resonant peak.

For step response of second order system (for $0 \le \xi \le 1$) maximum overshoot M_p is given by

$$M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \qquad \text{...(7)}$$

and damped frequency of oscillation is given by

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

From equation (4) and (8)

...(8)

$$\frac{\omega_r}{\omega_d} = \frac{\omega_n\sqrt{1-2\xi^2}}{\omega_n\sqrt{1-\xi^2}}$$

$$\omega_r = \omega_d\left[\frac{\sqrt{1-2\xi^2}}{\sqrt{1-\xi^2}}\right] \qquad \text{...(9)}$$

From above expression if the value of ξ is small, the damped frequency ω_d and resonant frequency are nearly same. Hence for large value of ω_r, the time response is faster.

From equation (6) and (7) it is clear that both M_r and M_p are the function of ξ. As ξ increases both M_r and M_p decreases, but the M_p is limited to 1 and M_r having very large values for $\xi < 0.4$ as shown in fig. (2).

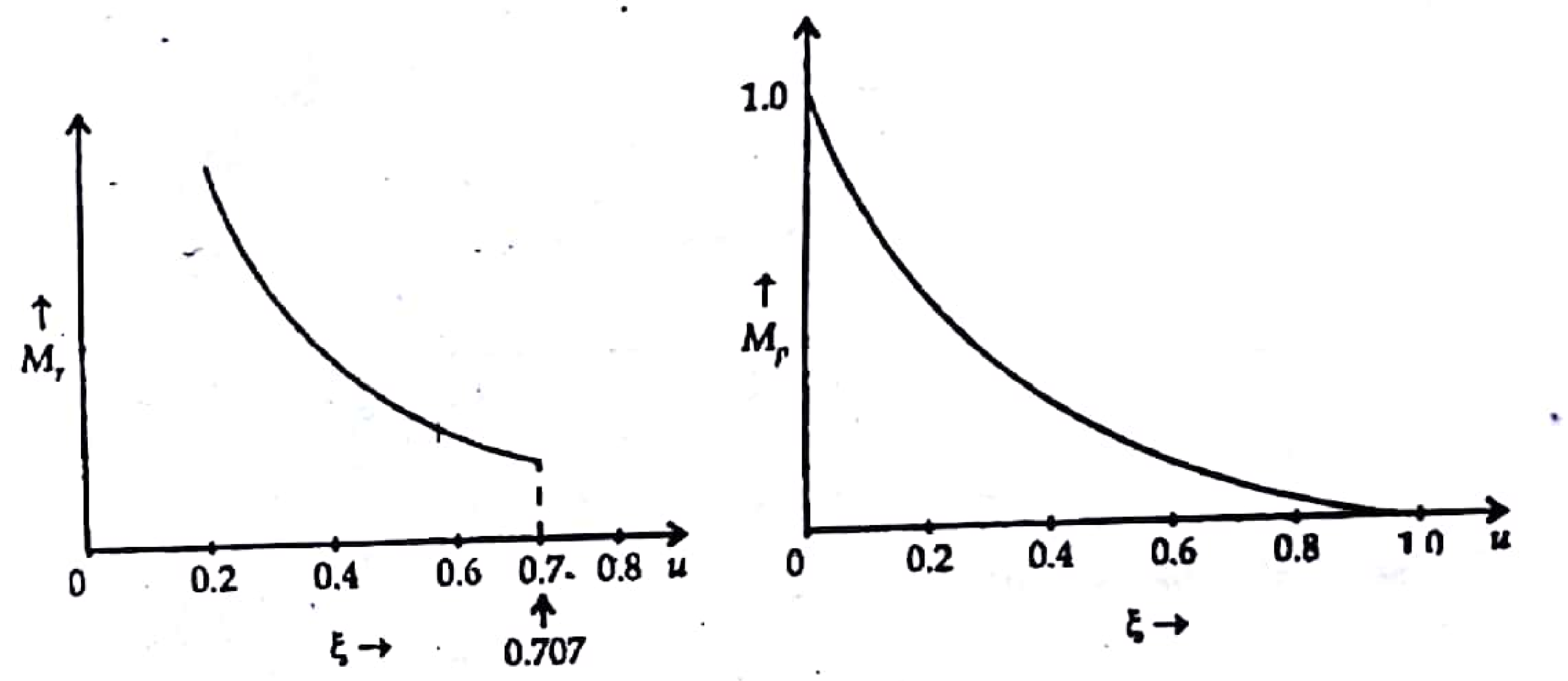


Fig. (3)

The bandwidth is also defined as the range of frequencies over which M is equal or greater than $1/\sqrt{2}$

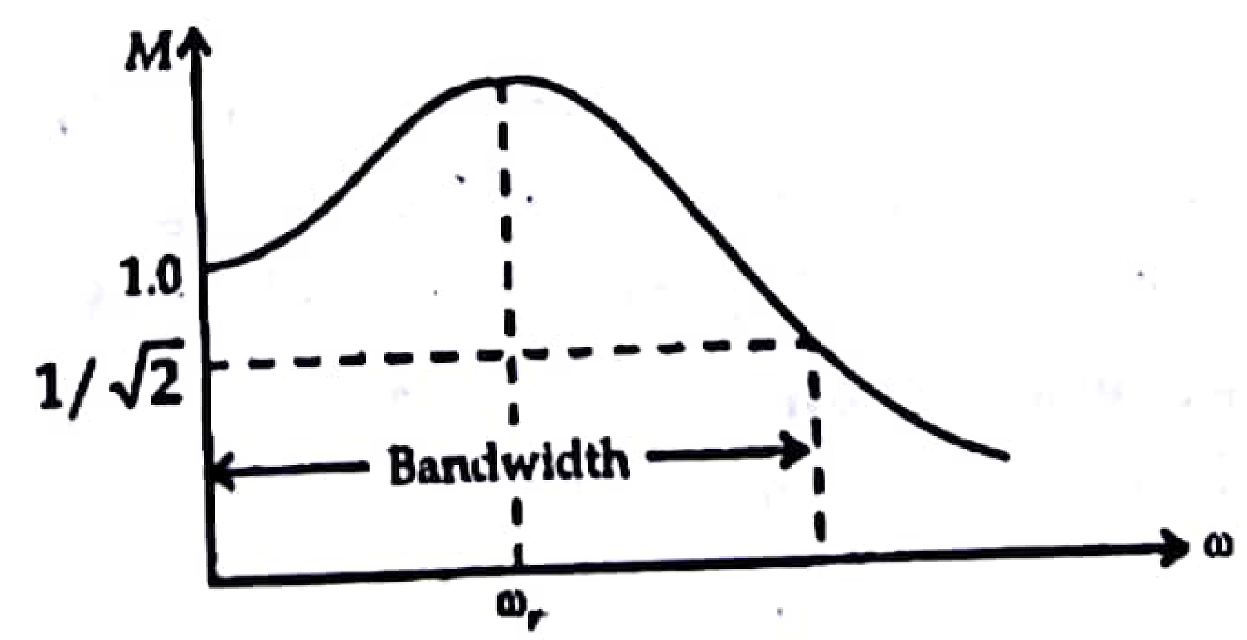


Fig. (4)

From equation no. (2).

$$M = \frac{1}{\sqrt{(1-u^2)^2+(2\xi u)^2}}$$

let u_b = normalized bandwidth = $\frac{\omega_b}{\omega_n}$ then

$$M = \frac{1}{\sqrt{(1-u^2 b)^2 + (2\xi u_b)^2}} = \frac{1}{\sqrt{2}}$$

or,
$$(1-u_b^2)^2 + (2\xi u_b)^2 = 2$$
$$1 + u_b^4 - 2u_b^2 + 4\xi^2 u_b^2 = 2$$
or,
$$u_b^4 - 2u_b^2 + 4\xi^2 u_b^2 - 1 = 0$$
or,
$$u_b^4 - 2u_b^2(1-2\xi^2) - 1 = 0$$
Put
$$u_b^2 = x$$
$$x^2 - 2x(1-2\xi^2) - 1 = 0$$

$\therefore$
$$x = \frac{+2(1-2\xi^2) \pm \sqrt{4(1-2\xi^2)^2 + 4}}{2}$$
$$= 1 - 2\xi^2 \pm \sqrt{1 - 4\xi^2 + 4\xi^4 + 1}$$
$$= 1 - 2\xi^2 \pm \sqrt{2 - 4\xi^2 + 4\xi^4}$$

consider the positive part

$$u_b^2 = 1 - 2\xi^2 + \sqrt{2 - 4\xi^2 + 4\xi^4}$$

$\therefore$
$$\boxed{u_b = \sqrt{1 - 2\xi^2 + \sqrt{2 - 4\xi^2 + 4\xi^4}}} \quad ...(10)$$

But
$$u_b = \frac{\omega_b}{\omega_n}$$

$\therefore$ $\omega_b = u_b \omega_n$ = denormalized bandwidth

$\therefore$
$$\boxed{\omega_b = \omega_n \sqrt{1 - 2\xi^2 + \sqrt{2 - 4\xi^2 + 4\xi^4}}} \quad ...(11)$$

Calculation of Phase Margin

Consider open loop transfer function

$$G(s) = \frac{\omega_n^2}{s(s + 2\xi\omega_n)} \quad ...(12)$$

Put $s = j\omega$

$$G(j\omega) = \frac{\omega_n^2}{j\omega(j\omega + 2\xi\omega_n)} = \frac{\omega_n^2}{-\omega^2 + j2\xi\omega\omega_n}$$

$$|G(j\omega)| = \frac{\omega_n^2}{\sqrt{(-\omega^2)^2 + (2\xi\omega\omega_n)^2}}$$

At gain cross over frequency (ω_1) $|G(j\omega)| = 1$

$\therefore$
$$w_n^2 = \sqrt{(-\omega_1^2)^2 + (2\xi\omega_1\omega_n)^2}$$
$$\omega_1^4 + 4\xi^2\omega_1^2\omega_n^2 - \omega_n^4 = 0$$
Put
$$x = \omega_1^2$$
$$x^2 + (4\xi^2\omega_n^2)x - \omega_n^4 = 0$$
$$x = \frac{-4\xi^2\omega_n^2 \pm \sqrt{(4\xi^2\omega_n^2)^2 + 4\omega_n^4}}{2}$$
$$x = -2\xi^2\omega_n^2 \pm \omega_n^2\sqrt{4\xi^2 + 1}$$
or'
$$\omega_1^2 = -2\xi^2\omega_n^2 \pm \omega_n^2\sqrt{4\xi^2 + 1}$$
or'
$$\omega_1 = \omega_n\sqrt{\sqrt{4\xi^4 + 1} - 2\xi^2}$$
$$\underline{|G(j\omega_1)} = \tan^{-1}\frac{2\xi\omega_1\omega_n}{-\omega_1^2} = +\tan^{-1}\frac{2\xi\omega_n}{\omega_1}$$

Put the value of ω_1

$$\underline{|G(j\omega_1)} = +\tan^{-1}\frac{2\xi\omega_n}{\omega_n\sqrt{\sqrt{4\xi^4 + 1} - 2\xi^2}}$$
$$= +\tan^{-1}\frac{2\xi}{\omega_n\sqrt{\sqrt{4\xi^4 + 1} - 2\xi^2}}$$

$\therefore$ Phase margin

$$\phi = -180 + \tan^{-1}\frac{2\xi}{\sqrt{\sqrt{4\xi^4 + 1} - 2\xi^2}}$$

or'
$$\boxed{\phi = \tan^{-1}\frac{2\xi}{\sqrt{\sqrt{4\xi^4 + 1} - 2\xi^2}}} \quad ...(13)$$

$\because$
$$\tan(180 + \phi) = \tan\phi$$

Q.8. (b) Distinguish between open loop and closed loop control systems. **(7.5)**

Ans. OPEN LOOP CONTROL SYSTEM

The open loop control system is also known as control system without feedback or non feedback control systems. In open loop systems the control action is independent of the desired output. In this system the output is not compared with the reference input.

The component of the open loop systems are controller and controlled process. The controller may be amplifier, filter etc depends upon the system. An input is applied to the controller and the output of the controller gives to the controlled process & we get the output (desired)..

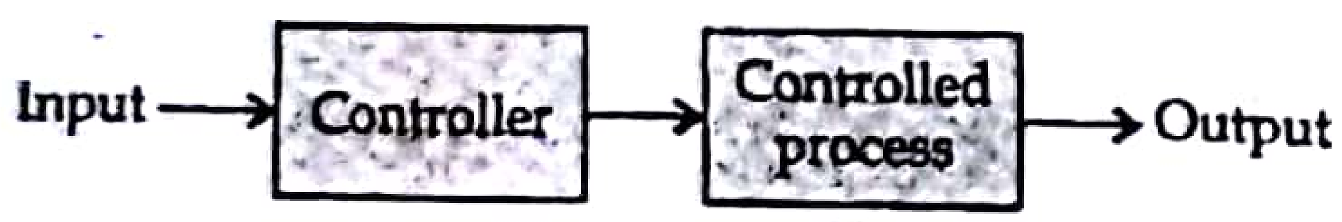


Fig. 1.

Examples :

1. Automatic washing machine is the example of the open loop systems. In the machine the operating time is set manually. 8After the completion of set time the machine will stops, with the result we may or may not get the desired (output) amount of cleanliness of washed cloths because there is no feedback is provided to the machine for desired output.

2. Immersion rod is another example of open loop system. The rod heats the water but how much heating is required is not sense by the rod because of no feedback to the rod.

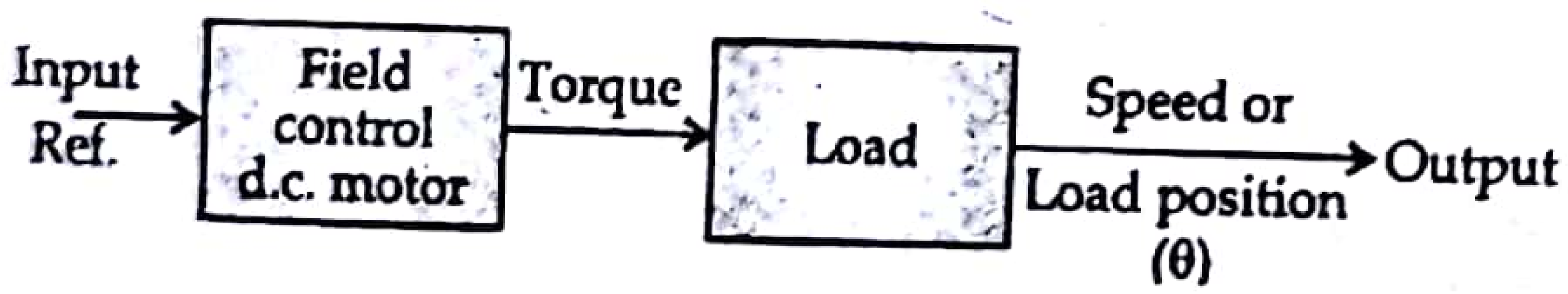


Fig. 2.

3. A field control d.c. motor is the example of open loop system.

4. For automatic control of traffic the lamps of three different colours (red, yellow and green) are used. The time for each lamp is fixed. The operation of each lamp does not depends upon the density of the traffic but depends upon the fixed time. Thus, we can say that the control system which operates on the time basis is open loop system.

Advantages :
1. open loop control systems are simple.
2. open loop control systems are economical.
3. less maintenance is required and not difficult.
4. proper calibration is not a problem.

Disadvantages :
1. Open loop systems are inaccurate.
2. These are not reliable.
3. These are slow.
4. Optimization is not possible.

CLOSED LOOP CONTROL SYSTEM

Closed loop control systems are also known as feedback control systems. In closed loop control systems the control action is dependent on the desired output. If any system having one or more feedback paths forming a closed loop system.

In closed loop systems the output is compared with the reference input and error signal is produced. The error signal is fed to the controller to reduce the error and desired output is obtained.

Error signal

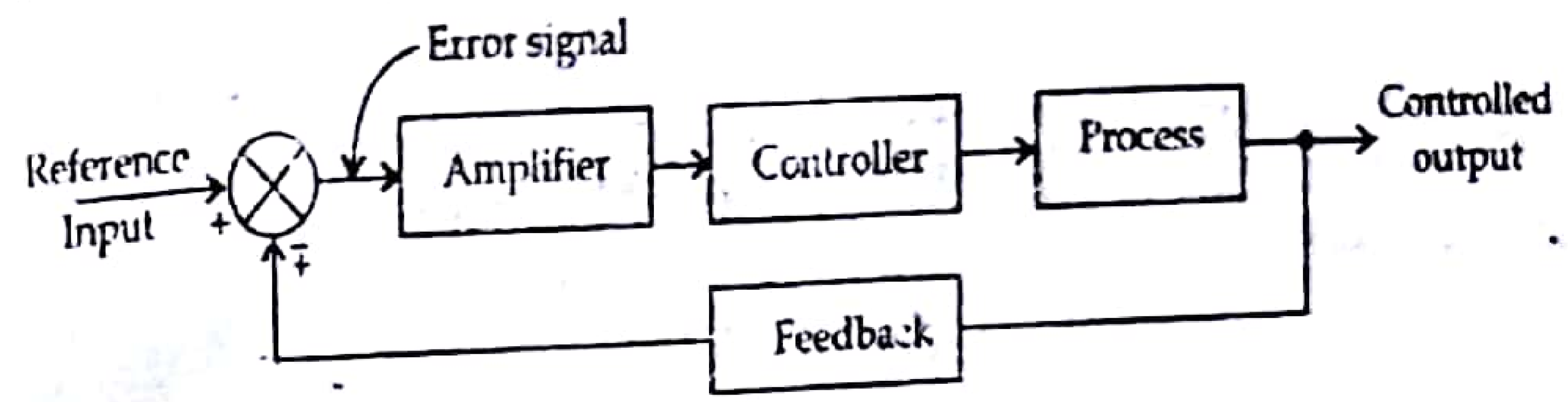


Fig. 3.

Example: In a room we need to regulate the temperature & humidity for comfortable living. Air conditioners are provided with thermostat. By measuring the actual room temperature & compared it with desired temperature, an error signal is produced, the thermostat turns ON the compressor or OFF the compressor. The block diagram is shown in fig. 4.

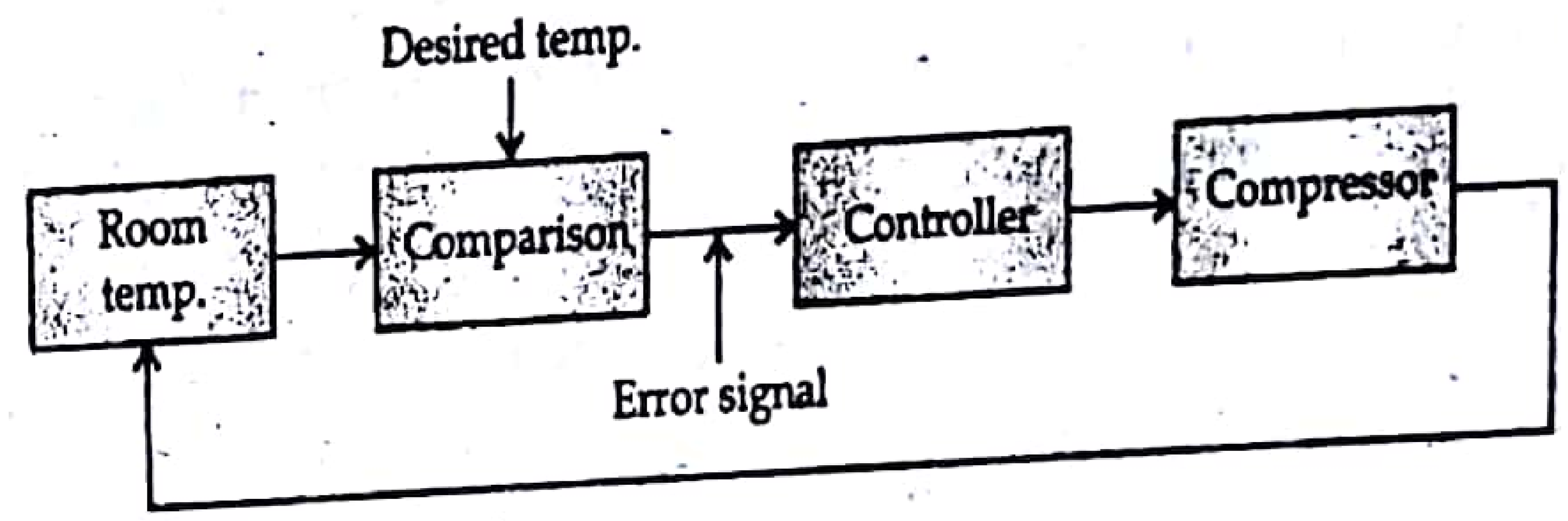


Fig. 4.

Advantages:
1. These systems are more reliable.
2. Closed loop systems are faster.
3. A number of variables can be handled simultaneously.
4. Optimization is possible.

Disadvantages:
1. Closed loop systems are expensive
2. Maintenance difficult.
3. Complicated installation.

COMPARISON BETWEEN OPEN LOOP AND CLOSED LOOP

S.No.	Open Loop system	Closed Loop System
1.	These are not reliable	These are reliable
2.	It is easier to build	It is difficult to build.
3.	If calibration is good/ they perform accurately.	They are accurate because of feedback
4.	Open loop systems are generally more stable.	These are less stable.
5.	Optimization is not possible.	Optimization is possible.

Q.9. Write short note on:

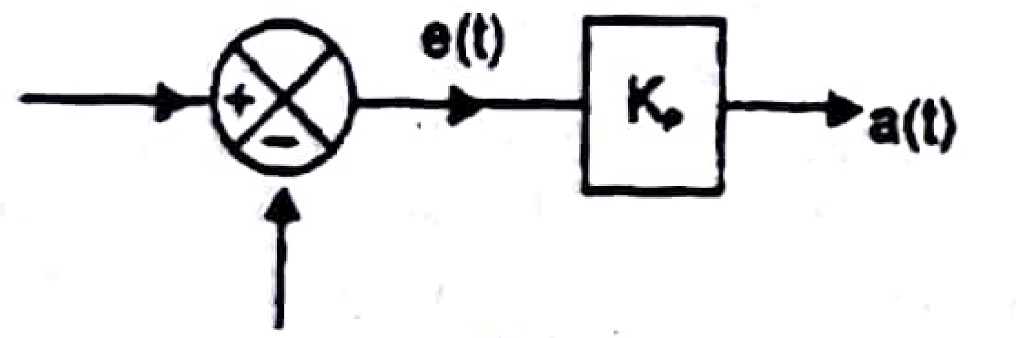
Q.9. (a) PID Controllers **(6)**

Ans. Proportional Controller: This type of controller is used where a deviation is not large or not sudden. Here the output of controller is directly proportional to the error signal.

$$a(t) \propto e(t)$$
$$a(t) = k_p e(t)$$

Taking Laplace

$$A(s) = k_p E(s)$$



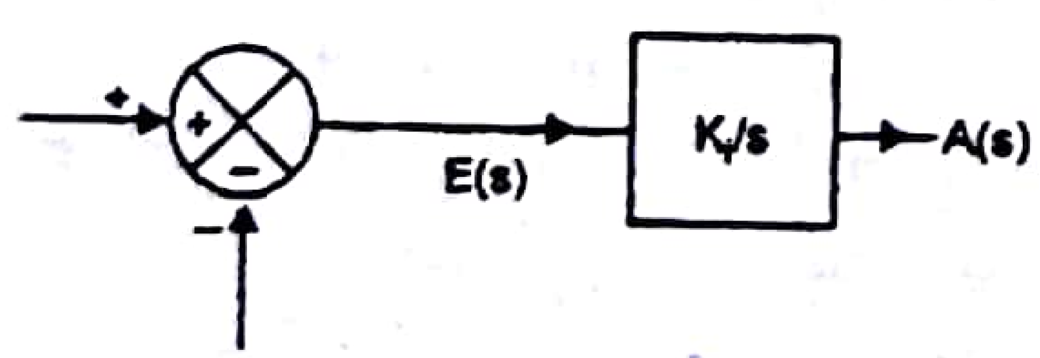
Disadvantage: (a) It causes the offset

(b) It increases the maximum overshoot.

Integral Controller: The term 'integral' is derived from the mathematical consideration of this type of controller. Here the output of the controller is time integral of the error signal. It is also called the reset control.

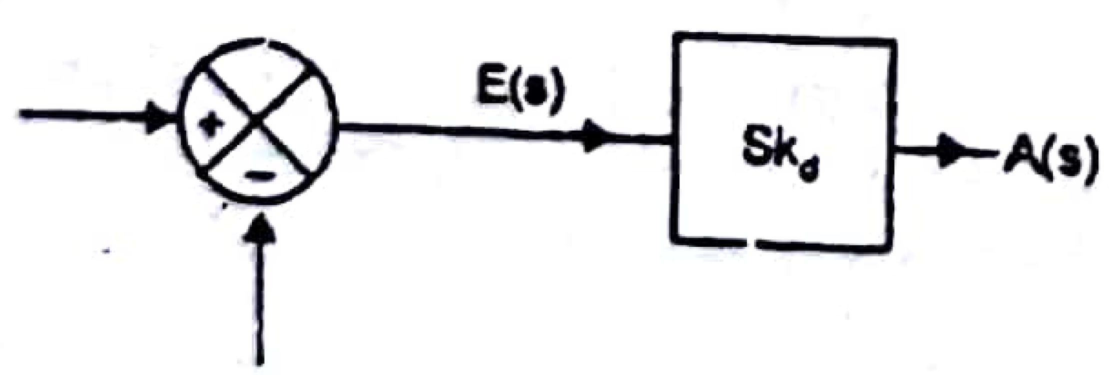
$$a(t) = k_i \int_0^t e(t)dt;$$

$$A(s) = \frac{k_i}{s} E(s); k_i = \text{Integral constant}$$



Derivative Controller: Here the output of the controller is time derivative of the error signal.

$$a(t) = k_d \frac{de(t)}{dt}$$
$$A(s) = k_d SE(s)$$

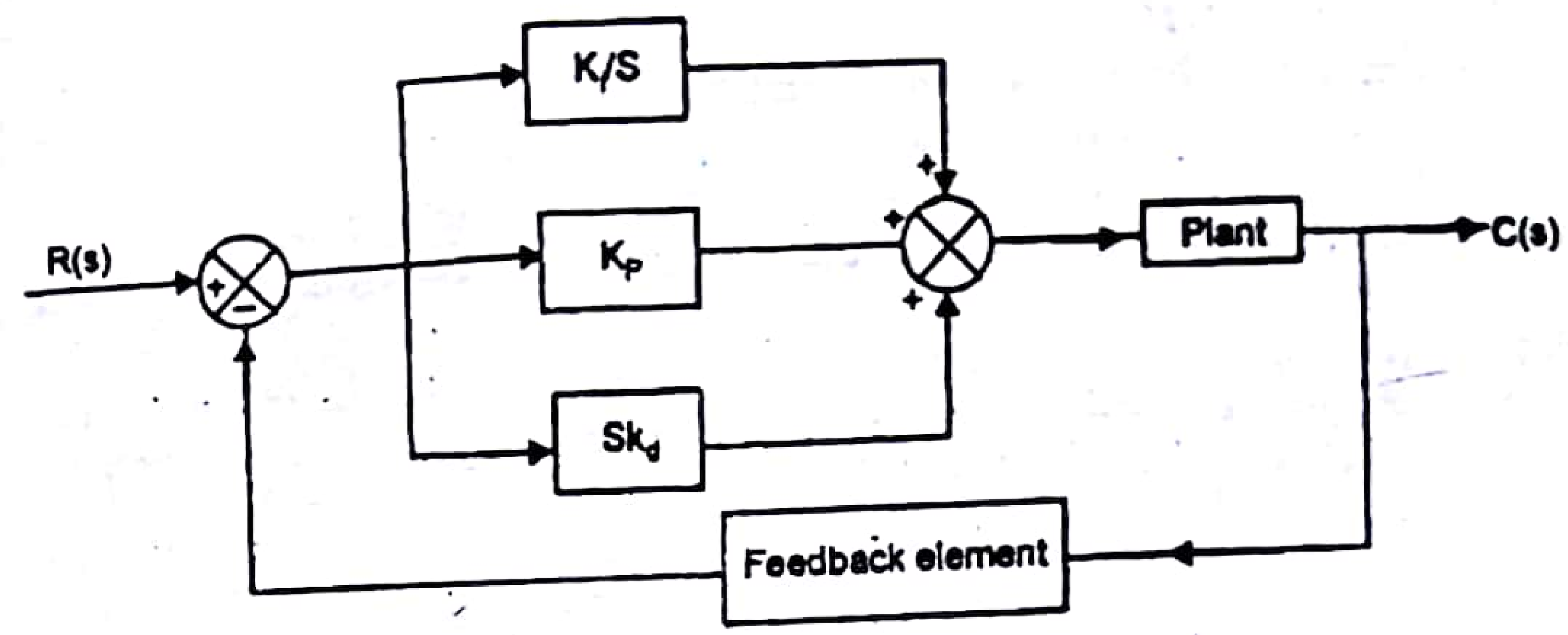


Disadvantage:

(a) It amplifies the noise signal and may cause the saturation effect in the actuator.

(b) It does not improves the steady state error.

P ID Controller:



$$a(t) = k_p e(t) + k_i \int_0^t e(t) + k_d \frac{de(t)}{dt}$$

In S-domain:

$$A(s) = k_p E(s) + \frac{k_i}{s} E(s) + k_d SE(s)$$
$$= \left[k_p + \frac{k_i}{s} + k_d s\right] E(s)$$

When advantages of all individual controller are required by the system, then PID controller is used.

Q.9. (b) Routh Hurwitz Criterion **(6.5)**

Ans. Routh's Stability Criterion:

A *necessary condition for stability* is that all the roots of the characteristic equation have negative real parts, which in turn requires that all the $[a_i]$ be positive.

A *necessary* (but not sufficient) *condition* for stability is that all the coefficients of the characteristic polynomial be positive.

If any of the coefficients are missing (are zero) or are negative, then the system will have poles located outside the LHP.

A system is stable if and only if all the elements in the first column of the Routh array are positive.

This is necessary and sufficient condition for stability.

The stability of the system can be indicated from Routh's array.

Routh's stability criterion states *that the number of roots of the characteristic equation with positive real parts is equal to the number of changes in sign of the coefficients of the first column of the Routh array.*

This routh's criterion requires that there be no changes in sign in the first column for a stable system. This requirement is both necessary and sufficient any change of sign in the first column of the Routh's array indicates :

(i) the system is unstable.

(ii) the number of changes of sign gives the number of roots in right-half of s-plane. A pattern of +,–, + is counted as *two* sign changes : one change from + to — and another from – to +.

It is to be noted that exact values of the terms in the first column need not be known, instead, only the signs are needed.

If any of the coefficients of the characteristic polynomial is zero or negative in the presence of at least one positive coefficient, there is a root or roots that are imaginary or that have positive real parts. In such a case the system is unstable. If we are interested in only absolute stability, there is no need to follow the procedure further to generate the Routh's array.

Routh's criterion can be applied to the denominator of a transfer function to determine whether the system is stable. It can also be used to study the effect of parameter variation on system stability.