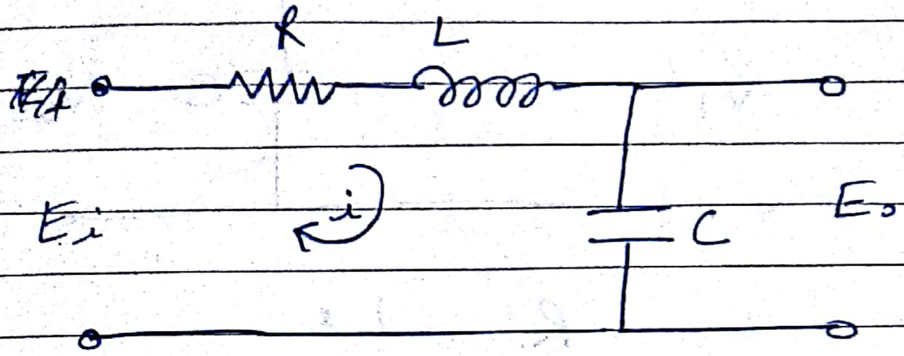
R – R
L – Ls
C – $\frac{1}{Cs}$

Page No.
Date

17/03/18

Control Systems

Ex. 1.2



$$E_i = iR + L\frac{di}{dt} + \frac{1}{C}\int (i)\,dt$$

$$E_i(s) = RI(s) + sLI(s) + \frac{I}{Cs}$$

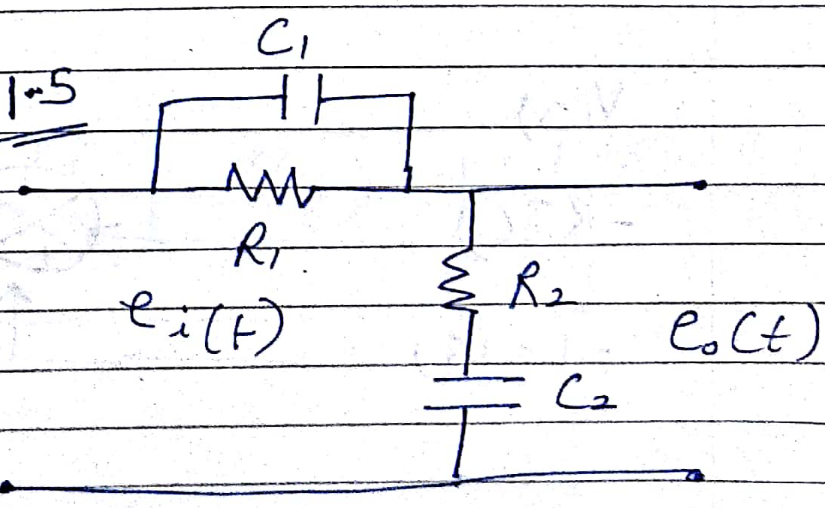
$$E_o(s) = \frac{I}{Cs}$$

$$\frac{E_o}{E_i} = \frac{I}{\cancel{Cs}} \times \frac{\cancel{Cs}}{CsRI(s) + Cs^2LI(s) + I}$$

$$= \frac{\cancel{I}\ 1}{CsR\cancel{I(s)} + Cs^2L\cancel{I(s)} + 1}$$

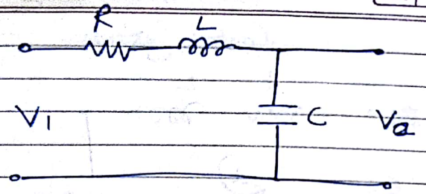
HW
1.49

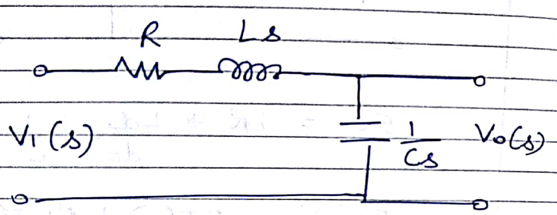
HW → Eg. 1.5



Find TF.

1.29

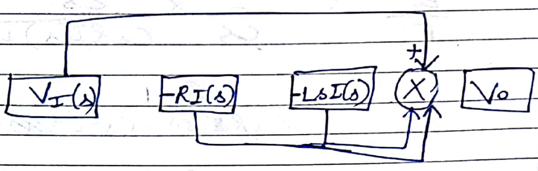


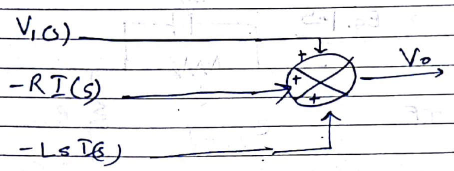


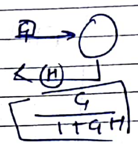
$$V_1(s) = IR + I(s)Ls + \frac{I(s)}{Cs}$$

$$V_1(s) = IR + LsI(s) + V_o(s)$$

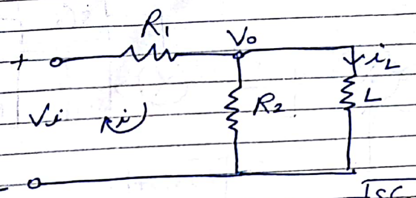
$$V_o = V_I(s) - IR - LsI(s)$$





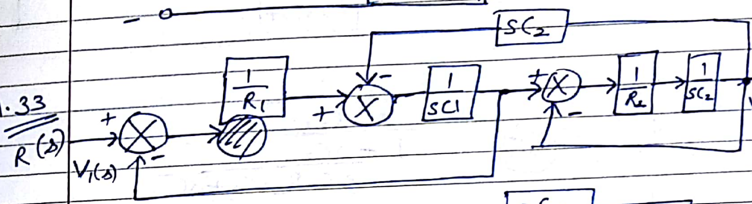


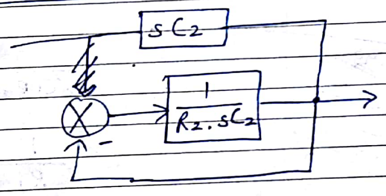
1.31



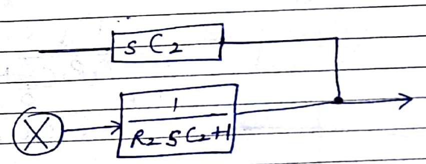
Convert into Block diag. & reduce to find T.F

1.33



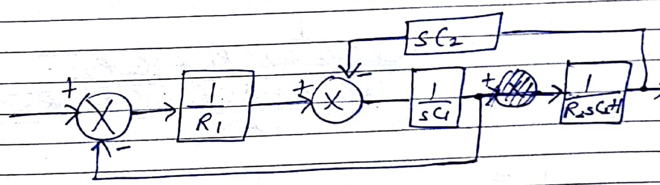


$$\frac{\frac{1}{R_2.sC_2}}{1+\frac{1}{R_2.sC_2}}$$



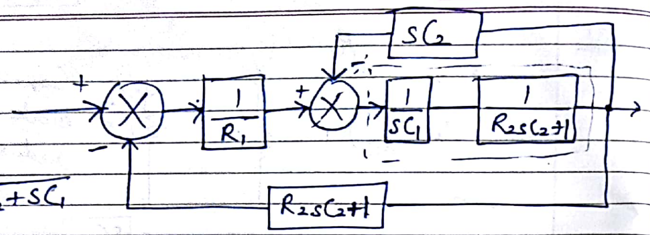
$$\frac{\frac{1}{R_2sC_2}}{\frac{R_2.sC_2+1}{R_2.sC_2}}$$

$$\frac{1}{R_2.sC_2+1}$$



Scanned by CamScanner

Page No.
Date



$\frac{1}{sC_1R_2sC_2+sC_1}$

$$\frac{\frac{1}{sC_1R_2sC_2+sC_1}}{1+\frac{sC_2}{sC_1R_2sC_2+sC_1}}$$

1.40

$\frac{\frac{1}{s}}{1+\frac{1}{s}} = \frac{1}{s+1}$

$$= \frac{1}{s^2C_1R_2C_2+sC_1+sC_2}$$

$$\times \frac{1}{R_1}$$

$$= \frac{\frac{1}{s^2C_1R_1R_2C_2+sC_1R_1+sC_2R_1}}{1+\frac{R_2sC_2+1}{s^2C_1C_2R_1R_2+sC_1R_1+sC_2R_1}}$$

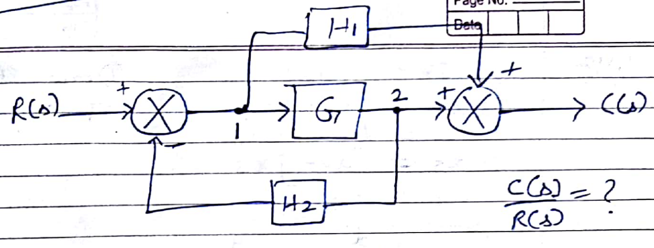
$$= \frac{\frac{1}{s^2C_1C_2R_1R_2+sC_1R_1+sC_2R_2}}{\frac{s^2C_1C_2R_1R_2+sC_1R_1+sC_2R_2+sR_2C_2+1}{}}$$

$$= \frac{1}{s^2C_1C_2R_1R_2+sC_1R_1+2sC_2R_1+sC_2R_2+1}$$

Pg 56

Page No.
Date

1.37



$\frac{C(s)}{R(s)} = ?$

- Signal Flow Graph

Mason's Formula

T.F. = Transmittance

$$\boxed{T = \frac{\sum g_k \Delta_k}{\Delta}}$$

$g_k \rightarrow$ Gain of fwd. path.

Δ_k = Part of Δ not touching the k^{th} fwd. path.

$1-(0)$

$\Delta \rightarrow 1 -$ [sum of all individual loop gains]

$+$ [sum of all product of 2 non-touching loops]

$-$ [sum of all product of 3 non-touching loops]

1.42

$x_2 = x_1 + a x_5$

$x_3 = b x_2 + c x_4$

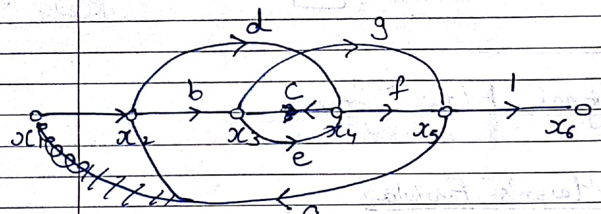
$x_4 = d x_2 + e x_3$

$x_5 = f x_4 + g x_3$

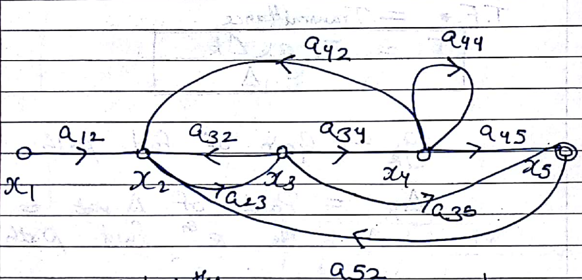
$x_6 = x_5$

Draw SFG

TF = ?



1.43



Gain of fwd. paths

$g_1 = a_{12}\, a_{23}\, a_{34}\, a_{45}$ | $\Delta_1 = 1 - 0$

$g_2 = a_{12}\, a_{23}\, a_{35}$ | $\Delta_2 = (1 - a_{44})$

Gain of individual Loops

$L_1 = a_{32}\, a_{23}$

$L_2 = a_{23}\, a_{35}\, a_{52}$

$L_3 = a_{23}\, a_{34}\, a_{45}\, a_{52}$

$L_4 = a_{44}$

$L_5 = a_{23}\, a_{34}\, a_{42}$

Gain of 2-non touching Loops

~~$L_1 L_4$~~

$L_2 L_4$

$L_1 L_4 = a_{32}\, a_{23}\, a_{44}$

$L_2 L_4 = a_{23}\, a_{35}\, a_{52}\, a_{44}$

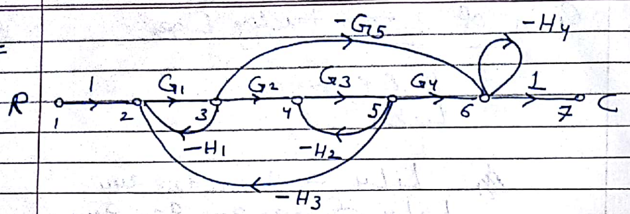
$$T = \frac{g_1 \Delta_1 + g_2 \Delta_2}{\Delta}$$

$$T = \frac{(a_{12}\, a_{23}\, a_{34}\, a_{45}) + [(a_{12}\, a_{23}\, a_{35}) - (a_{12}\, a_{23}\, a_{35}\, a_{44})]}{1 - (a_{32} a_{23} + a_{23} a_{35} a_{52} + a_{23} a_{34} a_{45} a_{52} + a_{44} + a_{23} a_{34} a_{42}) + (a_{32} a_{23} a_{44} + a_{23} a_{35} a_{52} a_{44})}$$

Scanned by CamScanner

Page No.
Date

1.44



Gain of fwd. paths

$g_1 = G_1 G_2 G_3 G_4$
$g_2 = G_1 G_5$

Individual Loop Gains

$L_1 = -G_1 H_1$
$L_2 = -G_3 H_2$
$L_3 = -G_1 G_2 G_3 H_3$
$L_4 = -H_4$

Two non-touching Loops

$L_1 L_2 = G_1 H_1 G_3 H_2$
$L_1 L_4 = G_1 H_1 H_4$
$L_2 L_4 = G_1 G_2 G_3 H_3 H_4$
$L_3 L_4 = G_1 G_2 G_3 H_3 H_4$

Three Non-touching

$L_1 L_2 L_4 = -G_1 H_1 G_3 H_2 H_4$

Page No.
Date

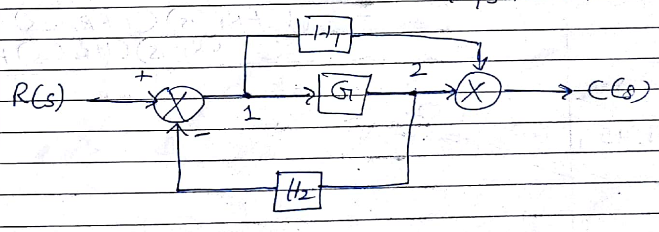
$\Delta_1 = 1 - (0) = 1$
$\Delta_2 = 1 + G_3 H_2$

$$\boxed{\frac{C}{R} = \frac{g_1 \Delta_1 + G_2 \Delta_2}{\Delta}}$$

$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1 L_2 + L_1 L_4 + L_2 L_4 + L_3 L_4) - (L_1 L_2 L_4)$

$$\Rightarrow \frac{C}{R} = \frac{G_1 G_2 G_3 G_4 + G_1 G_5 (1 + G_3 H_2)}{1 + G_1 H_1 + G_3 H_2 + G_1 G_2 G_3 H_3 + H_4 + G_3 G_1 H_1 H_2 + G_1 H_1 H_4 + G_3 H_2 H_4 + G_1 G_2 G_3 H_3 H_4 + G_1 G_3 H_1 H_2 H_4}$$

1.37



Page No.
Date

1.45

$$Z_1 = \frac{R_1 \cdot \frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} = \frac{R_1}{R_1C_1s+1}$$

$$Z_2 = R_2 + \frac{1}{sC_2} = \frac{R_2sC_2+1}{sC_2}$$

$$\frac{E_o}{E_i} = \frac{Z_2(s)}{Z_1(s)+Z_2(s)}$$

$$= \frac{(1+R_2sC_2)/sC_2}{\frac{R_1}{C_1R_1s+1} + \frac{R_2C_2s+1}{sC_2}}$$

$$= \frac{(1+R_1C_1s)(1+R_2C_2s)}{(1+R_1C_1s)(1+R_2C_2s)+R_1C_2s}$$

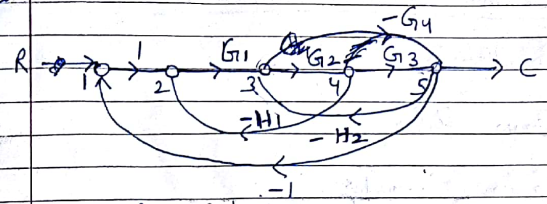
HW

1.45, 1.50, 1.52

Page No.
Date

18/3/18

1.47



Gain of all fwd. paths

$$g_1 = G_1G_2G_3$$
$$g_2 = -G_1G_4$$

Gain of Individual Loops

$$L_1 = -G_1G_2G_3\cancel{H_2}$$
$$L_2 = -G_1G_2H_1$$
$$L_3 = -G_2G_3H_2$$
$$L_4 = G_4H_2$$

~~Gain of all single non-touching loops~~

$$\Delta_1 = 1-(0) = 1$$
$$\Delta_2 = 1$$

$$T = \frac{g_1\Delta_1 + g_2\Delta_2}{\Delta}$$

$$\Delta = 1 + (G_1G_2G_3 + G_1G_2H_1 + G_2G_3H_2 - G_4H_2)$$

$$T = \frac{G_1G_2G_3 - G_1G_4}{1+(G_1G_2G_3+G_1G_2H_1+G_2G_3H_2-G_4H_2)}$$

Scanned by CamScanner

Page No.
Date

Control System Components

(i) Potentiometer

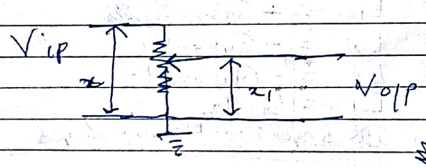
(ii) Servo Motor

(iii) Tacho Generator

(iv) Stepper Motor

(v) Synchro

(i)

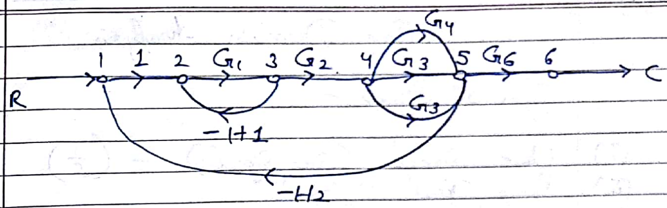


$$V_{out} = V_{ip} \times \frac{x_1}{x}$$

$$V_{out} \propto x_1$$

Page No.
Date

1.45



$g_1 = G_1 G_2 G_3 G_6$ $\quad \Delta_1 = 1-0 = 1$

$g_2 = G_1 G_2 G_4 G_6$ $\quad \Delta_2 = 1-0 = 1$

$g_3 = G_1 G_2 G_3 G_6$ $\quad \Delta_3 = 1-0 = 1$

Individual Loop Gains

$L_1 = -G_1 H_1$

$L_2 = -G_1 G_2 G_3 H_2$

$L_3 = -G_1 G_2 G_4 H_2$

$L_3 = -G_1 G_2 G_5 H_2$

Two non-touching

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4)$$

$$\frac{C}{R} = \frac{g_1 \Delta_1 + g_2 \Delta_2}{\Delta}$$

$$= \frac{G_1 G_2 G_3 G_6 + G_1 G_2 G_4 G_6 + G_1 G_2 G_5 G_6}{1 + G_1 H_1 + G_1 G_2 G_3 H_2 + G_1 G_2 G_4 H_2 + G_1 G_2 G_5 H_2}$$

25/3/18

Time Domain Analysis

Types of Signals –

(i) Unit Signal (Step Signal) – $\left(\frac{1}{s}\right)$

(ii) Ramp Signal – $\left(\frac{1}{s^2}\right)$

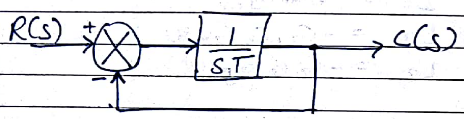
(iii) Impulse Signal – (1)

Types of Response –

(i) Steady State Response

(ii) Transient Response.

• Time Response of I[st] Order System –



$$T = \frac{C(s)}{R(s)} = \frac{\frac{1}{ST}}{1+\frac{1}{ST}}$$

$$= \frac{\frac{1}{ST}}{\frac{ST+1}{ST}}$$

$$= \frac{1}{1+ST}$$

• Step Function

$$\frac{C(s)}{1/s} = \frac{1}{ST+1}$$

$$= \frac{1}{S(ST+1)}$$

$$C(s) = \frac{A}{S} + \frac{B}{ST+1}$$

$$\frac{1}{\cancel{S(ST+1)}} = \frac{A(ST+1)+Bs}{\cancel{S(ST+1)}}$$

$$1 = A(ST+1) + Bs$$

Put $s = 0$

$$\boxed{A = 1}$$

$$1 = (T+1) + B$$

$$\boxed{B = -T}$$

$$C(s) = \frac{1}{S} + \frac{(-T)}{ST+1}$$

$$\cancel{c(t) = \frac{1}{t} + \frac{-T}{T+1}}$$

$$c(t) = u(t) - \frac{1}{\frac{ST+1}{T}}$$

$$= u(t) - \frac{T}{s+\frac{1}{T}}$$

$$= u(t) - \cancel{T} e^{-t/T}$$

$$\boxed{c(t) = 1 - e^{-t/T}}$$

Time constant

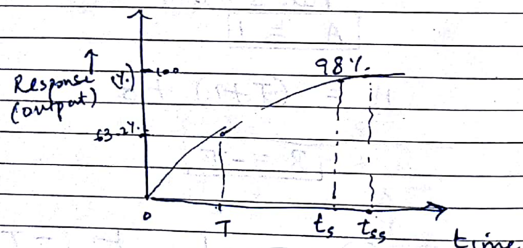
$$T = t$$

$$= 1 - e^{-T/T}$$

$$= 1 - e^{-1} =$$

$$= 0.632$$

$= 63.2\%$ of final value



$$\boxed{t_s = 4T}$$

$t_{0 - t_s}$ = Transient time

t_s onwards = Steady state time

Ramp

$$R(s) = \frac{1}{s^2}$$

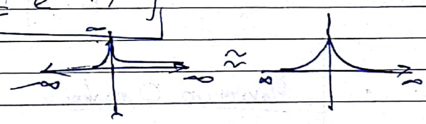
$$\boxed{C(t) = t - T + T e^{-t/T}}$$

Impulse

$$R(s) = 1$$

$$i \begin{cases} \infty & t = 0 \\ 0 & 0 < t < 0 \end{cases}$$

$$\boxed{c(t) = \frac{1}{T} e^{-t/T}}$$



- Time Response for IInd Order

$$\boxed{s^2 + 2\xi \omega_n s + \omega_n^2}$$

$\omega \rightarrow$ Angular freq.

$\xi \rightarrow$ Damping Ratio

$\boxed{P\,98}$

$\xi = 0 \rightarrow$ undamped system

$\xi = 0 - 1 \rightarrow$ Underdamped

$\xi > 1 \rightarrow$ Over damped

$\xi = 1 \rightarrow$ Critically Damped

Scanned by CamScanner

Page No. ______
Date

→ Definitions on Pg. 91

- **Rise Time**

$$t_r = \frac{\pi - \tan^{-1}\left(\sqrt{1-\xi^2}/\xi\right)}{\omega_n\sqrt{1-\xi^2}}$$

- **Peak Time**

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

- **Maximum Overshoot**

$$m_p\,(\%m_p) = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} \times 100$$

- **Settling Time**

$$t_s = \frac{4}{\xi\omega_n}$$

Page No. ______
Date

Eg 2.1 Unit Step fn.

$$\xi = 0.5$$

$$\omega_n = 6 \text{ rad/sec}$$

$t_r = ?$ $t_p = ?$ $T_s = ?$ $M_p = ?$

$$t_r = \frac{\pi - \tan^{-1}\left(\sqrt{1-0.5^2}/0.5\right)}{6\sqrt{1-0.5^2}}$$

$$= \frac{\pi - 60\,20}{\frac{6}{3} \times \frac{\sqrt{3}}{2}} = \cancel{\frac{2\pi}{\sqrt{3}}}\ \cancel{1.04}\text{ sec}$$

$$= \frac{\pi - 1.04}{6 \times \frac{\sqrt{3}}{2}} = \frac{2.101}{3\sqrt{3}}$$

$$= 0.404 \text{ sec}$$

$$t_p = \frac{\pi}{6\sqrt{1-0.5^2}} = \frac{\pi}{3\sqrt{3}} = 0.60 \text{ sec}$$

$$\%M_p = e^{\frac{-\pi \times 0.5}{\sqrt{1-0.5^2}}} \times 100$$

$$= 0.163 \times 100$$

$$= 16.3\%$$

$$t_s = \frac{4}{0.5 \times 6}$$

$$= 1.33 \text{ sec}$$

[illegible]

$g_1 = G_2 G_4 G_6$

$G_2 = G_3 G_5 G_7$

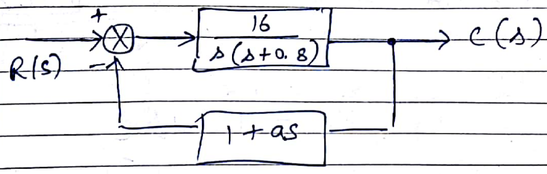
$g_3 = G_2 G_1 G_7$

$g_4 = G_3 G_8 G_6$

$g_5 = -G_2 G_1 H_2 G_8 G_6$

$g_6 = -G_3 G_8 H_1 G_1 G_7$

2.4



$$\frac{C(s)}{R(s)} = \frac{\frac{16}{s(s+0.8)}}{1 + \frac{16(1+as)}{s(s+0.8)}}$$

$$= \frac{\frac{16}{s(s+0.8)}}{\frac{s(s+0.8)+16(1+as)}{s(s+0.8)}}$$

$$= \frac{16}{s(s+0.8)+16(1+as)}$$

$$= \frac{16}{s^2+0.8s+16+16as}$$

$$= \frac{16}{s^2 + \underbrace{s(16a+0.8)}_{2\xi\omega_n} + \underbrace{16}_{\omega_n^2}}$$

$$\Rightarrow \xi\omega_n = 8a + 0.4$$

$$\boxed{\omega_n = 4}$$

$$\Rightarrow \xi = 2a + 0.1$$

$$0.5 = 2a + 0.1$$

$$\Rightarrow \boxed{a = 0.2}$$

Page No.
Date

Rise Time $= t_r = \dfrac{\pi - \tan^{-1}\left(\sqrt{1-\xi^2}/\xi\right)}{\omega_n\sqrt{1-\xi^2}}$

$\xi = 0.5$
$\omega_n = 4$ rad/sec

$t_r = \dfrac{\pi - \tan^{-1}\left(\sqrt{1-0.5^2}/0.5\right)}{4\sqrt{1-0.5^2}}$

$= \cancel{\pi - 0.9686}$

$\dfrac{\pi - \pi/3}{4\sqrt{1-0.5^2}}$

$t_r = 0.604$ sec

Max. Overshoot $= e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} \times 100$

$= e^{-\frac{\pi \times 0.5}{\sqrt{1-0.5^2}}} \times 100$

$= 16.30\%$

HW → 2.5, 2.6, 2.18

Page No.
Date

2.12 $G(s) = \dfrac{12}{s^2+4s+16}$, $H(s) = Ks$

$\xi = 0.8$
$\% MP = ?$ $\dfrac{C}{R} = \dfrac{G(s)}{1+G(s).H(s)}$
$K = ?$

$T = \dfrac{\dfrac{12}{s^2+4s+16}}{1+\dfrac{12}{s^2+4s+16}.Ks}$

$s^2+2\xi\omega_n s+\omega_n^2$

$= \dfrac{\dfrac{12}{s^2+4s+16}}{\dfrac{s^2+4s+16+12Ks}{s^2+4s+16}}$

$= \dfrac{12}{s^2+4s+16+12Ks}$

~~$2\xi\omega_n = 4s$~~ ~~$\omega_n^2 =$~~

$= \dfrac{12}{s^2+(4+12k)s+16}$

$\Rightarrow \xi\omega_n = 2+6k$ $\omega_n = 4$ rad/sec

$\xi = \frac{1}{4}(2+6k)$

$\dfrac{0.8\times4-2}{6} = k$

$\Rightarrow k = \dfrac{1}{5} = 0.2$

$$\% M_p = e^{\frac{-\pi \cdot \varepsilon}{\sqrt{1-\varepsilon^2}}} \times 100$$

$$= 1.51\%$$

2.17 $\boxed{\varepsilon = 0.5}$

$$\frac{\frac{16}{s+0.8}}{1+\frac{16k}{s+0.8}}$$

$$= \frac{1}{s}\,\frac{16}{s+16k+0.8}$$

$$= \frac{\frac{16}{s^2+16sk+0.8s}}{1+\frac{16}{s^2+16sk+0.8s}}$$

$$= \frac{16}{s^2+(16k+0.8)s+16}$$

$$\varepsilon\omega_n = 8k+0.4$$

$$\omega_n = 4$$

$$\Rightarrow \varepsilon = 2k+0.1$$

$$k = \frac{0.5-0.1}{2}$$

$$k = \frac{0.4}{2}$$

$$k = 0.2$$

$$\% M_p = e^{\frac{-\pi\varepsilon}{\sqrt{1-\varepsilon^2}}} \times 100$$

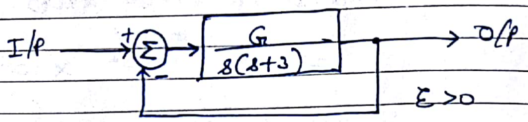
$$= e^{\frac{-\pi\times 0.5}{\sqrt{1-0.5^2}}} \times 100$$

$$= 16.30\%$$

$$t_s = \frac{4}{\varepsilon \cdot \omega_n} = \frac{4}{0.5\times 4}$$

$$= \frac{1}{0.5} = 2 \text{ sec}$$

2.5



$\varepsilon > 0$

(a) $$\frac{C(s)}{R(s)} = \frac{\frac{G}{s(s+3)}}{1 + \frac{G}{s(s+3)}}$$

$$= \frac{G}{s^2+3s+G}$$

(b) $s^2 + 2\varepsilon\omega_n s + \omega^2_n = 0$

$\Rightarrow 2\varepsilon\omega_n = 3$ $\qquad \omega_n = \sqrt{G}$

$$\boxed{\varepsilon = \frac{3\sqrt{G}}{2}}$$

2.6

$$G(s) = \frac{10}{(s+2)(s+5)}$$

$$1 + G(s)H(s) = 0$$

$$\Rightarrow 1 + \frac{10}{(s+2)(s+5)} = 0$$

$$s^2 + 7s + 20 = 0$$

Standard eqn. is $s^2 + 2\varepsilon\omega_n s + \omega_n^2 = 0$

$\Rightarrow 2\varepsilon\omega_n = 7$ $\qquad \omega_n^2 = 20$

$\varepsilon = \frac{7}{2\times 4.472}$ $\qquad \omega_n = \sqrt{20} = 4.472$

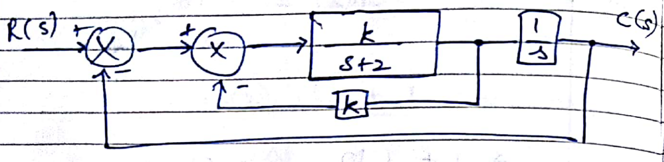
$\varepsilon = 0.782$

$$M_p = e^{-\frac{\pi\varepsilon}{\sqrt{1-\varepsilon^2}}} \times 100$$

$$= e^{-\frac{\pi \times 0.782}{\sqrt{1-0.782^2}}} \times 100$$

$$\boxed{M_p = 1.92\%}$$

2.18



$$\frac{C(s)}{R(s)} = \frac{k}{s^2+(2+Kk)s+k}$$

$$s^2+(2+Kk)s+K=0$$

$$s^2+2\xi\omega_n s+\omega_n^2=0$$

$2\xi\omega_n = 2+Kk$ $\qquad$ $\omega^2 = k$

$\Rightarrow \omega = \sqrt{k}$

$K=16$

$2\times 0.7\times 4 = 2+16k$ $\qquad$ $\Rightarrow \omega = 4$

$\Rightarrow \boxed{k = 0.225}$

2.19

$$c(t) = 1 - e^{-t/T}$$

$t = 60$ sec

$T = ?$

$$c(t) = \frac{98}{100}$$

$$\frac{98}{100} = 1 - e^{-60/T}$$

$$e^{-60/T} = 1-0.98$$

$$e^{-60/T} = 0.02$$

$$-\frac{60}{T}\ln(e) = \ln(0.02)$$

$$\cancel{-}\frac{60}{T} = \cancel{-}3.912$$

$$\Rightarrow T = \frac{60}{3.912}$$

$$\boxed{T = 15.337 \text{ sec}}$$

Error Analysis

Type of Errors

Static Error Coffecients –

(i) Static Position Error (K_p)

$$K_p = \lim_{s \to 0} G(s).H(s)$$

(ii) Steady State Error (e_{ss})

$$e_{ss} = \frac{1}{1 + K_p}$$

(iv) $e_{ss} = \lim_{s \to 0} s.\frac{R(s)}{1+G(s)H(s)}$

[Final Value Theorem]

(ii) Static Velocity Error (K_v)

$$K_v = \lim_{s \to 0} s.G(s).H(s)$$

$$e_{ss} = \frac{1}{K_v}$$

(iii) Static Acceleration Error (K_A)

$$K_A = \lim_{s \to 0} s^2.G(s).H(s)$$

$$e_{ss} = \frac{1}{K_A}$$

Eg 3.1

$$G(s) = \frac{50}{(1+0.1s)(s+10)}$$

$H(s) = 1$.

K_p, K_v, K_A = ?

$$K_p = \lim_{s \to 0} \frac{50}{0.1s^2 + 1.1s + 10}$$

$$K_p = \frac{50}{10} = 5$$

$$K_v = \lim_{s \to 0} s.G(s)H(s)$$

$$= \lim_{s \to 0} s \times \frac{50}{0.1s^2 + 1.1s + 10}$$

$$\cancel{= \lim_{s \to 0} \frac{50}{s\left(0.1s + 1.1 + \frac{10}{s}\right)}}$$

$$\cancel{= \frac{50}{1.1}}$$

$\Rightarrow$ $K_v = 0$

$K_A = 0$

Page No.
Date

3.2

$$G(s) = \frac{5(s^2+2s+100)}{s^2(s+5)(s^2+3s+10)}$$

$H(s) = 1$

$$K_p = \lim_{s\to 0} \frac{5(100)}{\cancel{s^2(s+5)}\,0}$$

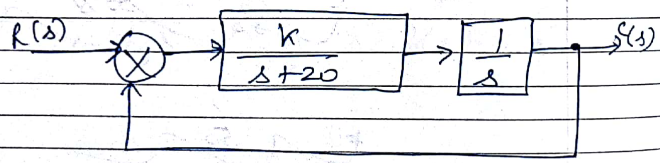
$\boxed{K_p = \infty}$

$\boxed{K_v = \infty}$

$$K_A = \lim_{s\to 0} \cancel{s^2} \times \frac{5(100)}{5(10)}$$

$\boxed{K_A = 10}$

3.3



$$G(s) = \frac{K}{s(s+20)}$$

$H(s) = 1$

$$R(s) = \frac{1}{s^2}$$

$K = ?$

Page No.
Date

(i) $K = 400$

(ii) $e_{ss} = 0.02$

$$e_{ss} = \lim_{s\to 0} \frac{s \cdot R(s)}{1+G(s)H(s)}$$

$$e_{ss} = \lim_{s\to 0} \frac{\frac{s}{s^2}}{1+\frac{k}{s(s+20)}}$$

$$= \lim_{s\to 0} \frac{1}{\cancel{s}} \cdot \frac{\cancel{s}(s+20)}{s(s+20)+k}$$

$$= \frac{20}{20+400}$$

$$e_{ss} = \frac{20}{420} = \frac{1}{21} = 0.047$$

(ii) $e_{ss} = 0.02$

$$0.02 = \lim_{s\to 0} s \cdot \frac{1}{\cancel{s}} \cdot \frac{s(s+20)}{s(s+20)+k}$$

$$0.02 = \frac{20}{20+k}$$

$0.02k + 0.4 = 20$

$k = 980$

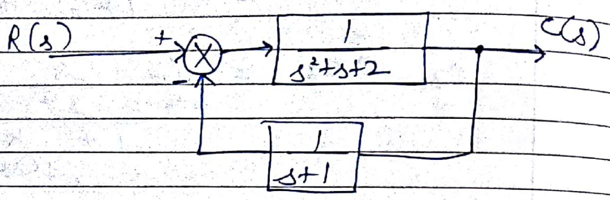
Scanned by CamScanner

Page No.
Date

HW → 3.5, 3.4

~~3.6(a)~~ ~~H(s) = 1~~

3.8



$$G(s) = \frac{1}{s^2+s+2}$$

$$H(s) = \frac{1}{s+1}$$

K_p , e_{ss} = ?

$$K_p = \lim_{s\to 0} G(s)H(s)$$

$$= \lim_{s\to 0} \frac{1}{(s+1)(s^2+s+2)}$$

$$= \frac{1}{2}$$

$$\boxed{K_p = 0.5}$$

Page No.
Date

$$e_{ss} = \frac{1}{1+K_p}$$

$$e_{ss} = \frac{1}{1+0.5}$$

$$\boxed{e_{ss} = 0.666}$$

3.5

$$G(s) = \frac{108}{s^2(s+4)(s^2+3s+12)}$$

$$r(t) = 2+5t+2t^2$$

$$= \frac{2}{s} + \frac{5}{s^2} + \frac{4}{s^3}$$

$$= \frac{2s^2+5s+4}{s^3}$$

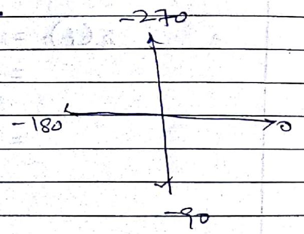
8/4/18

Unit 3

Frequency Domain Analysis

• Polar Plots

1. Find the Transfer fn. $G(s)$.
2. Substitute $s = j\omega$
3. At $\omega = 0$ & $\omega = \infty$ calculate $G(j\omega)$.
4. Calculate phase angle of $j\omega$ at $\omega = 0$ & $\omega = \infty$.
5. Rationalize $G(j\omega)$ & separate the real & imaginary parts.



Eg. $G(s) = \frac{1}{s}$

$G(j\omega) = \frac{1}{j\omega}$

$j\omega = \cancel{\ldots}$

$= \left(\frac{1}{j\omega}\right)^2 \tan^{-1}\left(\left(\frac{1}{j\omega}\right)^2\right)$

$= \frac{1}{\omega} \angle -90$

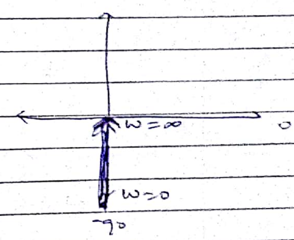
$\lim_{\omega \to 0} \frac{1}{\omega} \angle(-90) = \infty$

$\lim_{\omega \to \infty} G(j\omega) = 0$

$\frac{1}{s(s+1)(s+2)}$

$\rightarrow \frac{1}{j\omega(j\omega+1)}$

$\frac{1}{\omega}$ $\angle -\tan^{-1}\infty - \tan^{-1}\omega, -\tan^{-1}\frac{\omega}{2}$



Ex $\frac{1}{s^2}$

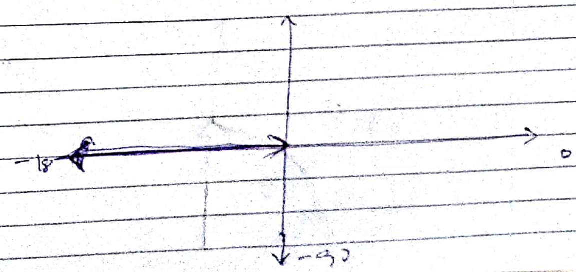
$G(j\omega) = \frac{1}{(j\omega)^2}$

$= \frac{1}{-\omega^2} \angle -\tan^{-1} 180$

$= \frac{1}{-\omega^2} \angle -180$

At $\omega = 0$, $G(j\omega) = -\infty$

$\omega = \infty$, $G(j\omega) = 0$



Page No. ____ Date ____

4.1 Sketch the polar plot for

$$G(s) = \frac{1}{s(s+1)}$$

$$G(j\omega) = \frac{1}{j\omega(j\omega+1)} = \frac{1}{(j\omega)^2 + j\omega}$$

$$= \frac{1}{-\omega^2 + j\omega}$$

$$= \frac{1}{\omega(j\omega+1)} \angle -\tan^{-1}(\infty) - \tan^{-1}\omega$$

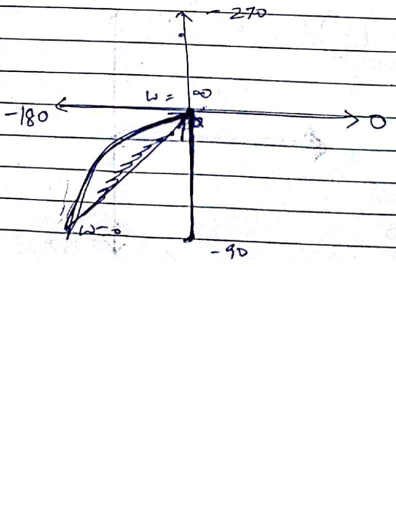
$$\sqrt{(j\omega)^2}$$

$$\frac{1}{\omega\sqrt{(1^2+\omega^2)}} \angle -\tan^{-1}\infty - \tan^{-1}\omega$$

$$= \frac{1}{\omega\sqrt{1^2+\omega^2}} \angle[-90 - \tan^{-1}\omega]$$

at $\omega = 0$, $G(j\omega) = \infty \angle -90^\circ$

$\omega = \infty$, $G(j\omega) = 0 \angle -180^\circ$



Page No. ____ Date ____

HW → 4.2

4.3 $$G(s) = \frac{20}{s(s+1)(s+2)}$$

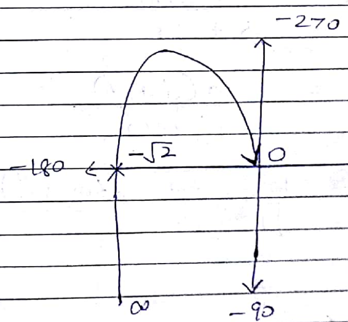
$$G(j\omega) = \frac{20}{j\omega(j\omega+1)(j\omega+2)}$$

$$= \frac{1}{\omega\sqrt{1^2+\omega^2}\sqrt{2^2+\omega^2}} \angle -\tan^{-1}\infty - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$$

$$= \frac{1}{\omega\sqrt{\omega^2+1}\sqrt{\omega^2+2^2}} \angle -90 - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$$

$\omega = 0 \rightarrow G(j\omega) = \infty \angle -90$

$\omega = \infty \rightarrow G(j\omega) = 0 \angle -270$



$$G(j\omega) = \frac{20}{j\omega(j\omega+1)(j\omega+2)}$$

$$= \frac{20}{[(j\omega)^2 + j\omega](j\omega+2)}$$

$\rightarrow 0$

$$= \frac{20}{(j\omega)^3 + 3(j\omega)^2 + 2j\omega}$$

$$= \frac{20}{-j\omega^3 - 3\omega^2 + 2j\omega}$$

$$\frac{3}{10} \quad \frac{3}{5} + \frac{1}{5}$$

$$\frac{20}{-3\omega^2} + \frac{20}{-j\omega^3 + 2j\omega}$$

Page No.
Date

$$= \frac{20}{j\omega(-\omega^2+2\omega)\,\cancel{8}-3\omega^2}$$

$$-j\left[\frac{20(\omega^3-2\omega)}{(\omega^4+\omega^2)(4+\omega^2)}\right] = 0$$

$$= 20(\omega^3-2\omega) = 0$$

$$\Rightarrow \omega^2 = 2$$

$$\omega = \pm\sqrt{2}$$

Alternative Method for Polar Plots

4.5 Sketch the polar plot for –

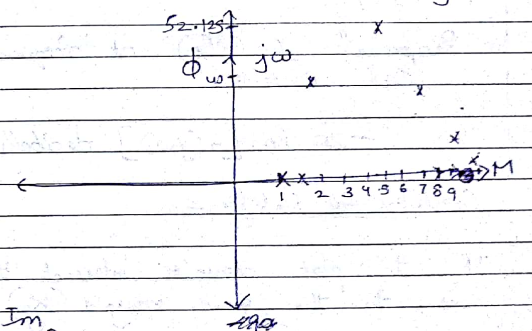
$$G(s) = \frac{10(s+1)}{s+10}$$

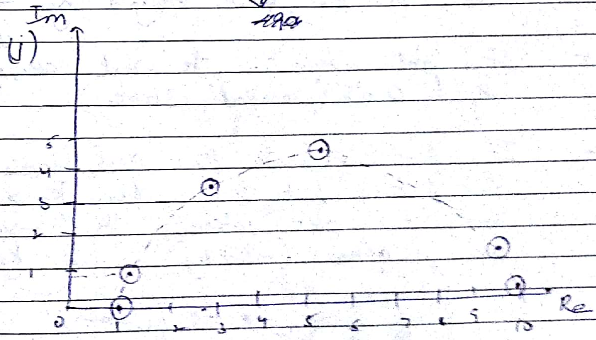
$$G(j\omega) = \frac{10(j\omega+1)}{j\omega+10}$$

$$= \frac{10\sqrt{1^2+\omega^2}}{\sqrt{10^2+\omega^2}} \angle -\tan^{-1}\left(\frac{\omega}{10}\right)+\tan^{-1}\omega$$

Page No.
Date

ω	M	ϕ	$M\cos\phi + jM\sin\phi$
0	1	0	1
1	$\frac{10\sqrt{2}}{\sqrt{101}} = 1.407$	$-5.710+45 = 39.3$	$1.407\cos(39.3)+j\,1.407\sin(39.3) = 1.088+j\,0.89$
5	4.560	52.125	$2.799+j\,3.599$
10	7.106	39.289	$5.499+j\,4.499$
50	9.80	10.164	$9.646+j\,1.729$
100	9.95	5.137	$9.910+j\,0.89$





10/4/18

Bode Plot

Starting Point = 20 log k

Phase Margin

$$\phi_M = 180^\circ + \phi$$

$\phi \rightarrow$ Angle of TF G(s) at gain crossover point.

Gain Margin

Reciprocal of G(s) at frequency at which $\phi = -180^\circ$.

$$-20 \log [G(j\omega)] \text{ decibels.}$$

Conditions for Stability -

- If the plot doesn't intersect the -ve real axis, then the gain margin (K_g) = ∞ (stable) at $(-1+0j)$
- If plot intersects the real axis, then K_g = 0 db (marginally stable)
- If plot intersects the real axis at < (-1 + 0j) but > 0, then K_g > 0 db, and if plot intersects the -ve real axis, after (-1 + 0j), then k_g < 0 db.

[-ve → (Unstable)]

4.9 Bode plot for TF = $G(s) = \dfrac{1000}{(1+0.1s)(1+0.001s)}$

Step 1

Initial slope = 0 db/dec

Starting pt. = 20 ~~dblk~~ log (1000)

~~Gain margin~~ = 20 × 3

= 60 db.

$$\omega_1 = \frac{1}{0.1} = 10 \text{ rad/sec}$$

$$\omega_2 = \frac{1}{0.001} = 1000 \text{ rad/sec}$$

Step 2

After every corner freq., slope of magnitude plot reduces by 20 db/decade.

Page No. ____
Date

4.10 Sketch the Bode Plot of the following TF –

$$G(s) = \frac{1000}{s(1+0.1s)(1+0.001s)}$$

Step 1 Type = 1

Initial slope = −20db

∴ ~~Initial~~ Starting Point = 20 log K

= 20 log(1000)

= 20 × 3

= 60 db

$$\omega_1 = \frac{1}{0.1} = 10 \text{ rad/sec}$$

$$\omega_2 = \frac{1}{0.001} = 1000 \text{ rad/sec}$$

$$\phi_m = -\tan^{-1}(0.1\omega) - \tan^{-1}(0.001\omega) - 90$$

Step 2–4 Graph

Step 5

ω	$-\tan^{-1}(0.1\omega)$	$-\tan^{-1}(0.001\omega)$	R
0	0	0	−90
5	−26.56	−0.286	−116.84
10	−45	−0.572	−135.57
15	−56.30	−0.859	−147.159
20	−63.435	−1.145	−154.58
50	−78.69	−2.862	−171.55
100	−84.289	−5.710	−180.00
200	−87.13	−11.309	−188.439

Scanned by CamScanner