Circuits & Systems

Systems

(i) Single Inp. Single Output (SISO)
(ii) Multiple Inp. Mul. Out. (MIMO)

(I) Those systems that follow homogeinity & superpositions are linear system.

Those that dont follow are non-linear.

(II) Time-Varying & Time-invarying

(III) Continuous & Discrete Time Systems (CTS & DTS)

(IV) Instantaneous & Dynamic Systems.
Those who work in current state.

$$y(t) = 3x(t) - x^2(t)$$

$$v(t) = \frac{1}{C} \int_0^t i\omega \, dt + V_c(t)$$

(V) Casual Systems : Based on present & past values.

Non-Casual :

(VI) Invertibility & Inverse System

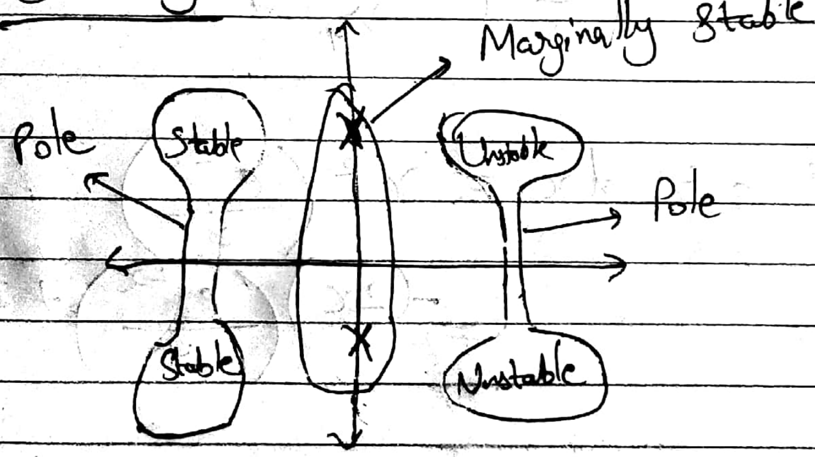
$$x(t) \xrightarrow{x(t)} \boxed{\text{System}} \xrightarrow{y(t)} \boxed{\text{Inverse System}} \xrightarrow{x(t)} x(t)$$

$y(t)$

Transfer F^n

$$\text{gain} = \lim_{s \to 0} \left(\frac{O/P}{I/P} \right)$$

(VII) Stability



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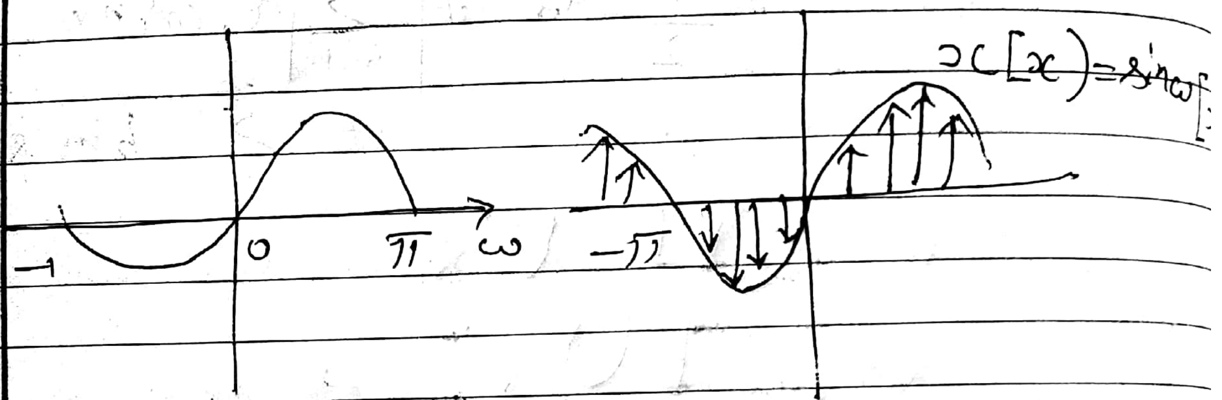
Circuit Theory

Interconnection of Systems

Signals

(1) Continuous Time & Discrete Time Signal



Even & Odd Signals

$x(-t) = x(t)$

$\rightarrow t^2, t^4, \cos t, \sin^2 t$

odd $x(-t) = -x(t)$

$\rightarrow t, t^3, \sin t$

Periodic / Unperiodic

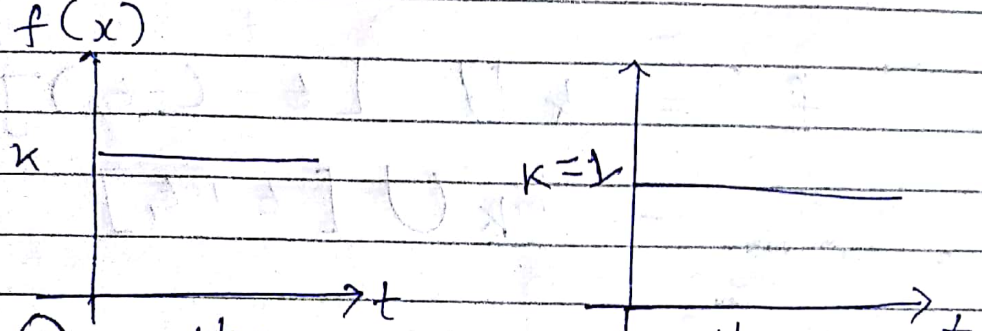
Periodic $\sin t, \cos t, \sin t/2$

Unperiodic $\rightarrow e^t, t^2, t \ldots$

Standard signals

(1) Step Signals

$$f_s(t) = \begin{cases} 0 ; & t < 0 \\ k ; & t > 0 \end{cases}$$ [where k is the amplitude of step signal]



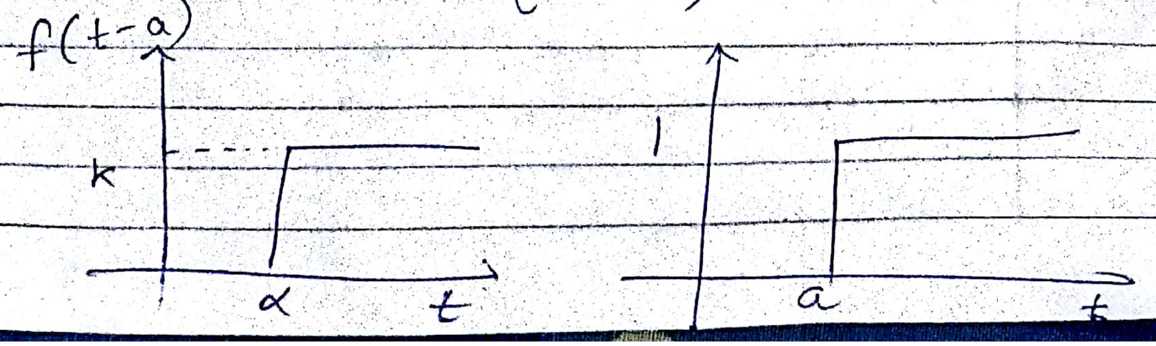
(1) step   unit-step

$$v(t) = \begin{cases} 0 , & t < 0 \\ 1 , & t > 0 \end{cases}$$

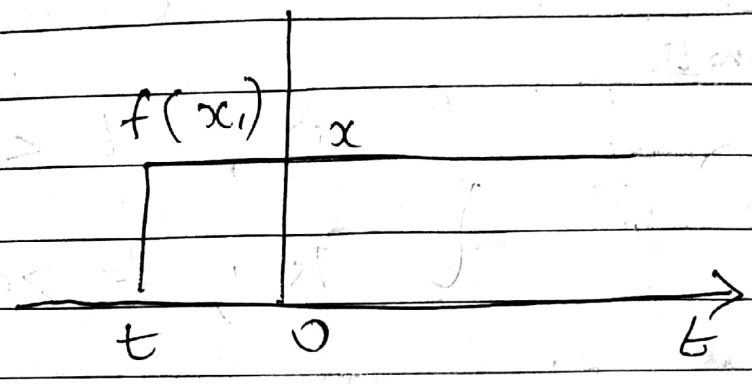
Shifting the signal

$$f_s(t-a) = \begin{cases} 0 , & t < a \\ k , & t > a \end{cases}$$

$$u(t-a) = \begin{cases} 0 , & t < a \\ 1 , & t > a \end{cases}$$



Q. Express the given waveform :

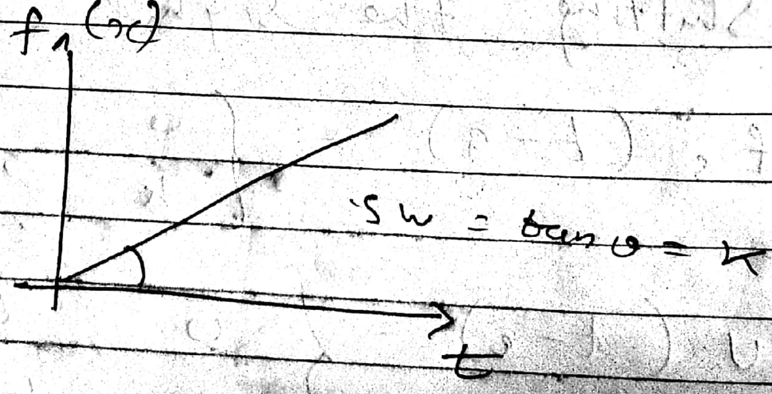


$$f = kU[t-(-t_1)]$$

$$= kU[t+t_1]$$

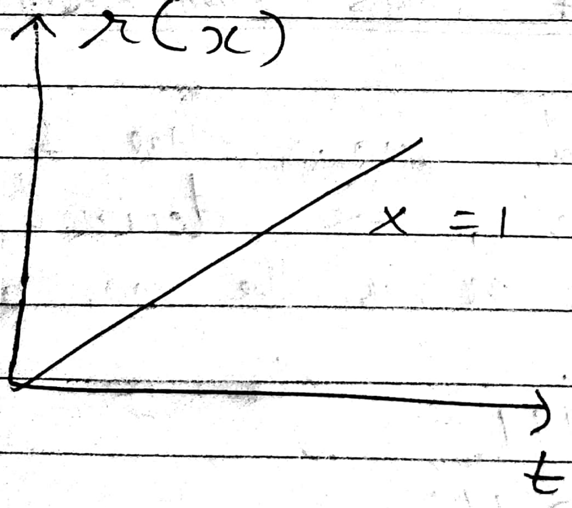
• Ramp Signal

$$f_r(t) = \begin{cases} 0, & t<0 \\ kt, & t \ge 0 \end{cases}$$



when $k = 1$ [Unit step sig]

$$r(t) = \begin{cases} 0, & t < 0 \\ t, & t \geq 0 \end{cases}$$



9/8/17

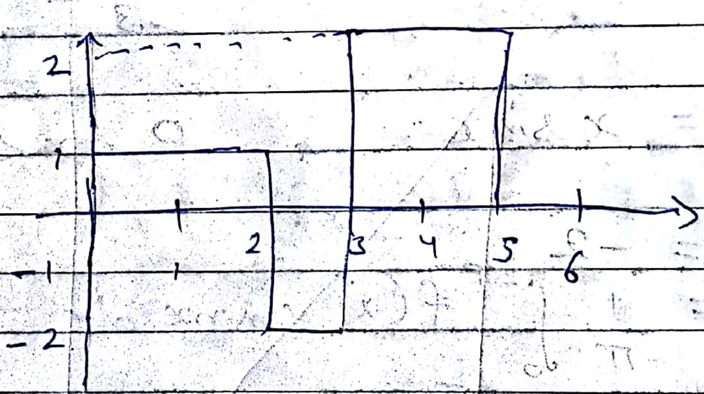
- Direct formula (or KM formula)

$$f(t) = \sum_{T=-\infty}^{\infty} (A_f - A_i)\, U(t-T)$$

$T \rightarrow$ time instant
$f(t) \rightarrow$ function
$A_f \rightarrow$ final value
$A_i \rightarrow$ initial value

$$f(t) = \sum_{T=0}^{\infty} (A_f - A_i)\, U(t-T)$$

Q. Synthesise the waveform.



$$f(t) = G_{0,2}(t) + (-2)G_{2,3}(t) + 2G_{3,5}(t)$$

$$= 1[u(t) - u(t-\cancel{8}2)] + (-2)[u(t-2) - u(t-3)] + 2[u(t-3) - u(t-5)]$$

$$= u(t) - 3u(t-2) + 4u(t-3) - 2u(t-5)$$

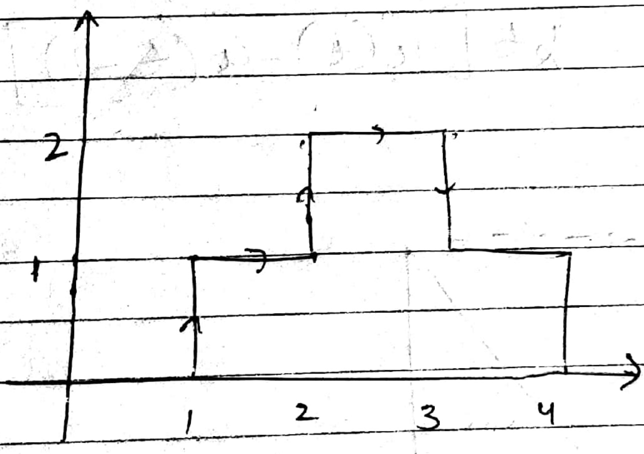
Direct Value,

$$f(t) = \sum_{T=0}^{\infty} (A_f - A_i)\, u(t-T)$$

$$= (1-0)\,u(t-0) + (-2-1)\,u(t-2) + (2-(-2))\,u(t-3) + (0-2)\,u(t-5)$$

$$= u(t) - 3u(t-2) + 4u(t-3) - 2u(t-5)$$

Q.



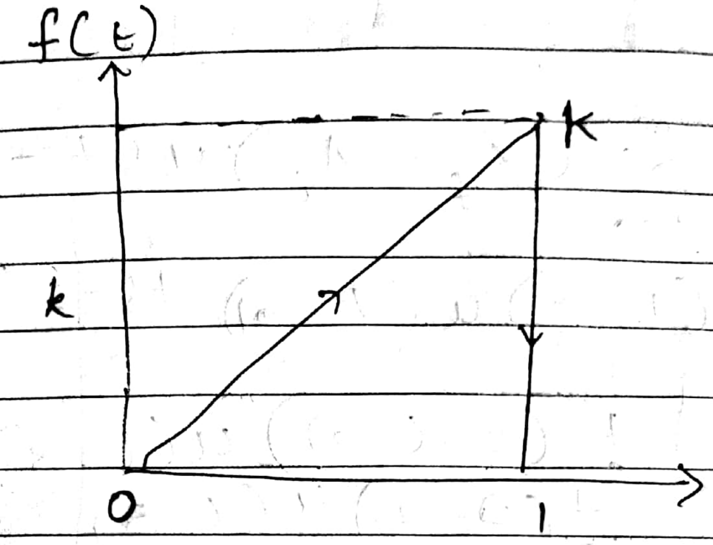
$$f(t) = \sum_{T=0}^{\infty} (A_f - A_i)\, u(t-T)$$

$$= (1-0)\,u(t-1) + (2-1)\,u(t-2) + (2-1)\,u(t-3) + (0-0)\,u(t-4)$$

$$= u(t) - \underline{u} + u(t) - \underline{2u} + u(t) - \underline{3u} - u(t) - \underline{4u}$$

$$= 5u(t) - 10u$$

Q.

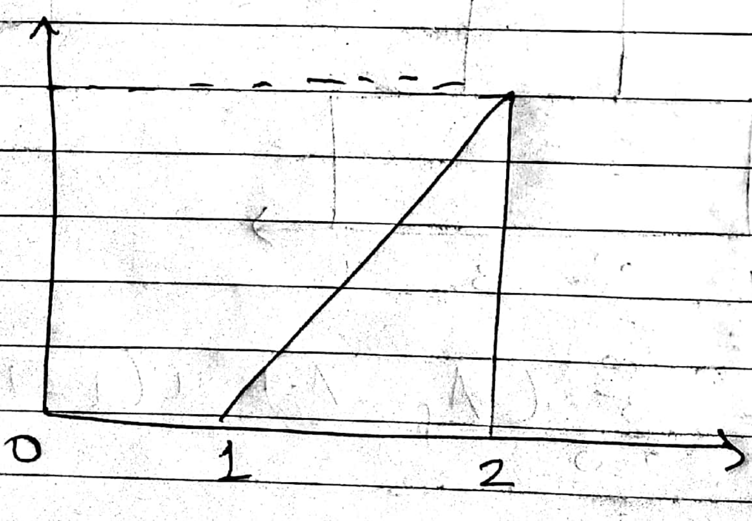


$\cancel{f(t) = (1-k)\,u(t-0)}$

$\cancel{= u - u(k)}$

$$f(t) = kt\left[u(t) - u(t-1)\right]$$

Q.



$$f(t) = k(t-1)\left[u(t-1) - u(t-2)\right]$$

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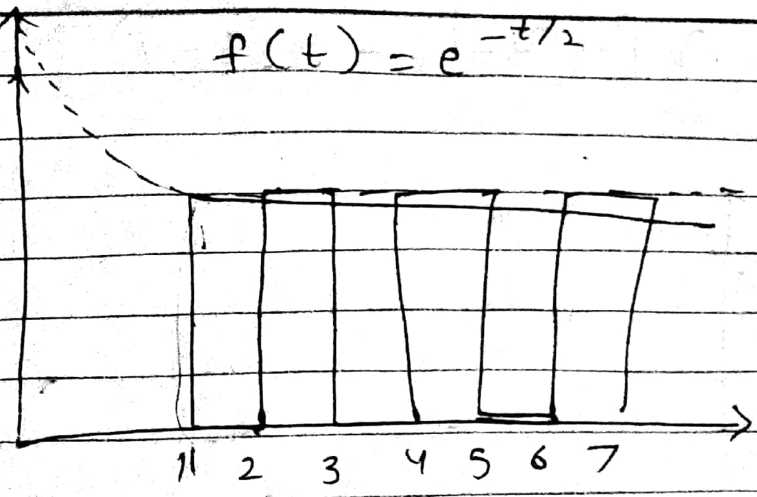
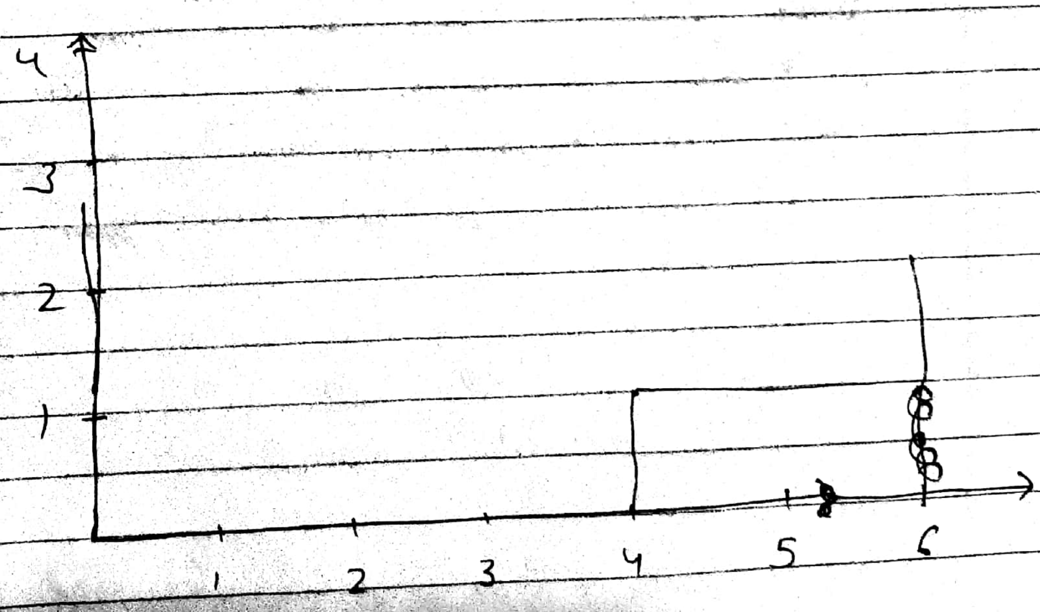
Q. 

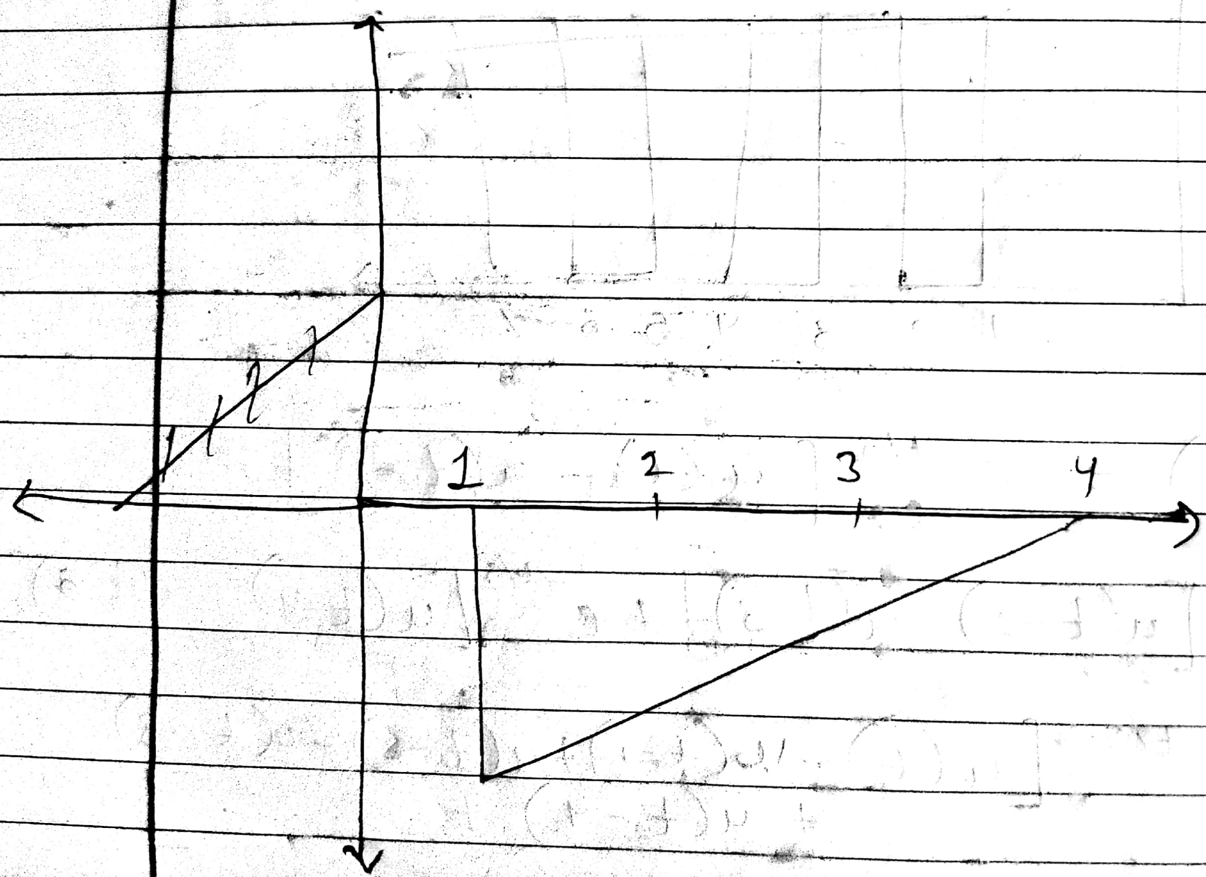
$$f(t) = e^{-t/2}[u(t) - u(t-1)]$$
$$+ e^{-t/2}[u(t-2) - u(t-3)] + e^{-t/2}[u(t-4) - u(t-5)]$$
$$= e^{-t/2}[u(t) - u(t-1) + u(t-2) - u(t-3) + u(t-4) + \dots\dots]$$

Q. Sketch the waveform –

$u(t-6) + u(t-4)$



Q. $(t-4)\left[u(t-1)-u(t-4)\right]$



C & S

① $u(t) =$

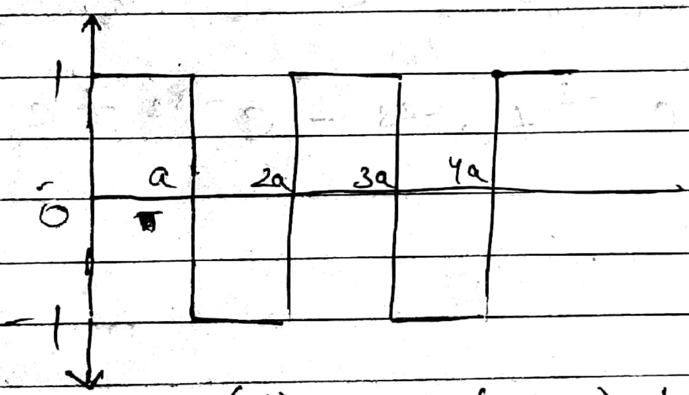
②

③ $K t u(t) = \int_0^\infty k t e^{st} = k \cdot \frac{1}{s^2}$

④ $ke^{-at} = \frac{k}{s+a}$

⑤ $\sin \omega t = \int_0^\infty \frac{1}{2i}\left(e^{i\omega t} - e^{-i\omega t}\right) e^{-st}\, dt$

⑥



$u(t) - 2u(t-a) + 2u(t-2a) - 2u(t-3a) + \ldots$

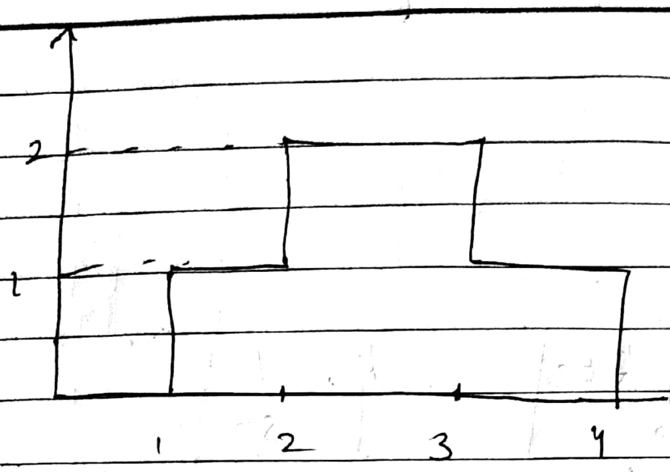
$$= \frac{1}{s}\left[1 - 2e^{-as} + 2e^{-2as} - 2e^{-3as} + \ldots\right]$$

$$= \frac{1}{s}\left[1 - 2e^{-as}\left[1 - e^{-as} + e^{-2as} \ldots\right]\right]$$

$$= \frac{1}{s}\left[1 - 2e^{-as}\right] = \frac{(1-e^{-as})}{s(1+e^{-as})}$$

$$= \frac{e^{as/2} - e^{-as/2}}{s(e^{as/2} + e^{-as/2})}$$

(7)

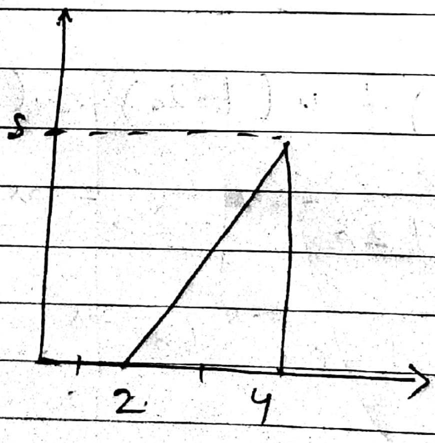


$$u(t-1)+u(t-2)-u(t-3)-u(t-4)$$

$$\Rightarrow \frac{e^{-s}}{s}+\frac{e^{-2s}}{s}-\frac{e^{-3s}}{S}-\frac{e^{-4s}}{s}$$

$$=\frac{1}{s}\left(e^{-s}+e^{-2s}-e^{-3s}+e^{-4s}\right)$$

(8)



$$=\frac{5}{2}r(t-2)-\frac{5}{2}r(t-4)-5u(t-4)$$

$$=\frac{5e^{-2s}}{2s^2}\left(1-e^{-2s}-2se^{-2s}\right)$$

(9) $5\dfrac{(s+3)}{s(s+1)}$

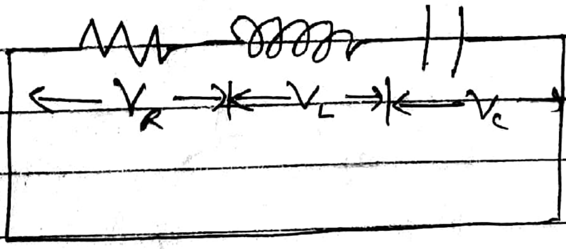
$\lim\limits_{s \to \infty} \left[s \right.$

- **System Modelling in terms of Differential Eqns.**

The process of obtaining the desired mathematical description of the system is called modelling.

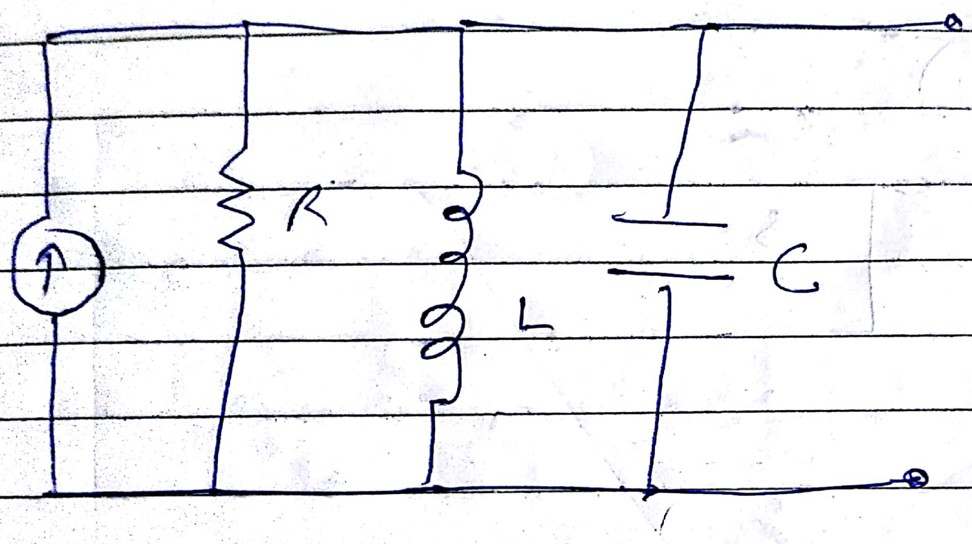
$$\sum_{k=0}^{n} a_k \frac{d^k y(t)}{dt^k} = \sum a_k$$

- **Electrical System**



$$v(t) = V_R(t) + V_L(t) + V_c(t)$$

$$v(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt$$



$$i(t) = i_R(t) + i_L(t) + i_c(t)$$

$$i(t) = \frac{v(t)}{R} + \frac{1}{L}\int v(t)\,dt + i\,\frac{dv(t)}{dt}$$

Q. $\frac{d^2y}{dt^2}(t) + \frac{3dy}{dt}(t) + 2y(t) = 5v(t)$

$v(t) \rightarrow$ Unit step function

$y(0^+) = -1, \frac{dy}{dt}(0^+) = 2$

Determine $y(t)$ for $t \geq 0$

Sol

$s^2y(s - sy(0^+) - y(0^+) + 3Y(s) - 3Y(0^+) + 2Y(s) = \frac{5}{s}$

Substituting the initial conditions & solving

$s^2Y(s) + s - 2 + 3sY(s) + 3 + 2Y(s) = \frac{5}{s}$

$Y(s) = \frac{-s^2 - s + 5}{s(s+1)(s+2)}$

Using partial fractions,

$Y(s) = \frac{k_1}{s} + \frac{k_2}{s+1} + \frac{k_3}{s+3}$

$k_1 = sY(s)|_{s=0} = \frac{5}{2}$

$k_2 = (s+1)Y(s)|_{s=-1} = -5$

$k_3 = (s+2)Y(s)|_{s=-2} = \frac{3}{2}$

Bunk Pages

$$y(s) = \frac{5}{2s} - \frac{5}{s+1} + \frac{3}{2(s+2)}$$

Taking laplace inverse

$$y(t) = \frac{5}{2} - 5e^{-t} + \frac{3}{2}e^{-2t} \; ; \; t \geq 0$$

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☆ C&S → Chakraborty → Ch 4, 8, 9, 10.

$\boxed{\text{C\&S}}$

• <u>Step response of RL Circuit</u>

$$E\,V(t) = R\,i(t) + L\frac{di}{dt}$$

Laplace

$$\frac{E}{S} = R\,I(s) + L\left[S\,I(s) - i(0)\right]$$

Take $i(0) = 0$

$$\frac{E}{S} = I(s)\left[R + SL\right]$$

$$I(s) = \frac{E/L}{S(S+R/L)} = \frac{A}{S} + \frac{B}{S+R/L}$$

Taking Partial Fractions

$$A = \left[\frac{E/L}{S+R/L}\right]_{S=0} = E/R$$

$$B = \left[\frac{E/L}{S}\right]_{S=-R/L} = -\frac{E}{R}$$

$$I(s) = \frac{E/R}{S} + \frac{(-E/R)}{S+R/L}$$

Taking L^{-1}

$$i(t) = \frac{E}{R} - \frac{E}{R}e^{-R/L\cdot t}$$

Step Response of RL Series circuit

$$E\,u(t) = R\,i(t) + \frac{1}{C}\int i(t)\,dt$$

Laplace

$$\frac{E}{s} = R\,I(s) + \frac{1}{C}\left[\frac{I(s)}{s} + \frac{i(0)}{s}\right]$$

Take $i(0) = \int_{-\infty}^{0} i\,dt = Q_0$

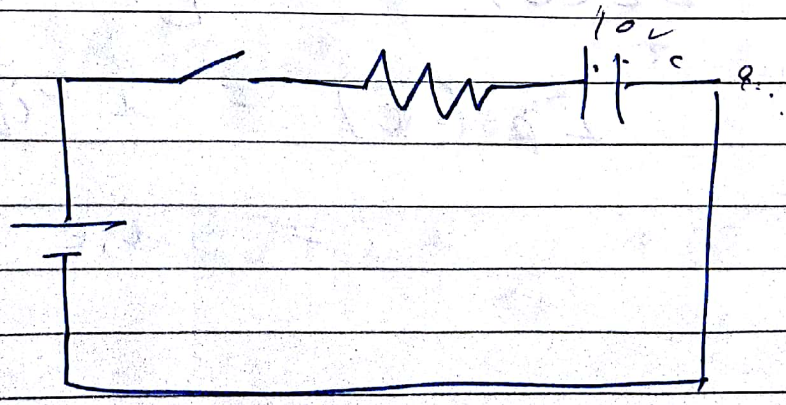
$$\frac{E}{s} = R\,I(s) + \frac{1}{C}\left[\frac{I_s}{s} + \frac{Q_0}{s}\right]$$

$$I_s = \frac{E - \frac{Q_0}{C}}{s\left(R + \frac{1}{C.s}\right)} = \frac{E - Q_0/C}{R}\left(\frac{1}{s + \frac{1}{RC}}\right)$$

$$= \left(\frac{E}{R} - \frac{Q_0}{RC}\right)\left(\frac{1}{s + \frac{1}{RC}}\right)$$

Taking L^{-1}

$$\left(\frac{E}{R} - \frac{Q_0}{RC}\right) e^{-1/RC\,t}$$



Step Response of RL parallel

$$\boxed{R I_R = L \frac{dI_L}{dt}}$$

$$I = I_R + I_L$$

$$RI = I_R . R + RI_L$$

$$RI = L\, dI_L + RI_L$$

Laplace $R \frac{I(s)}{s} = I(s) \frac{R}{s(R+Ls)}$

$$= \frac{I(s)\,(R/L)}{s(R/L+1)} = I(s)\left[\frac{A_0}{S} + \frac{B}{S+}\right.$$

$$A = 1 \,,\; B = -1$$

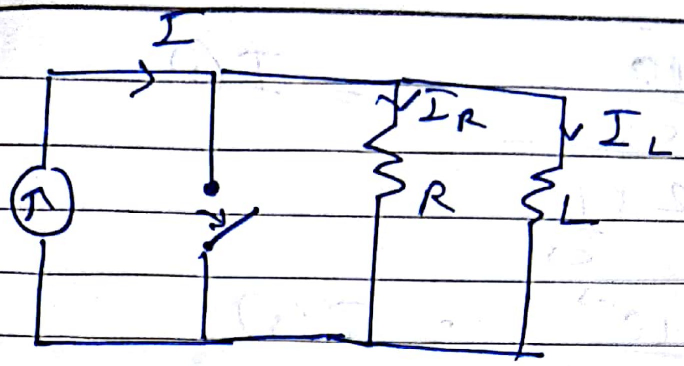
$$I_L = I(s)\left[\frac{1}{s} - \frac{1}{s+R/L}\right]$$

L^{-1}

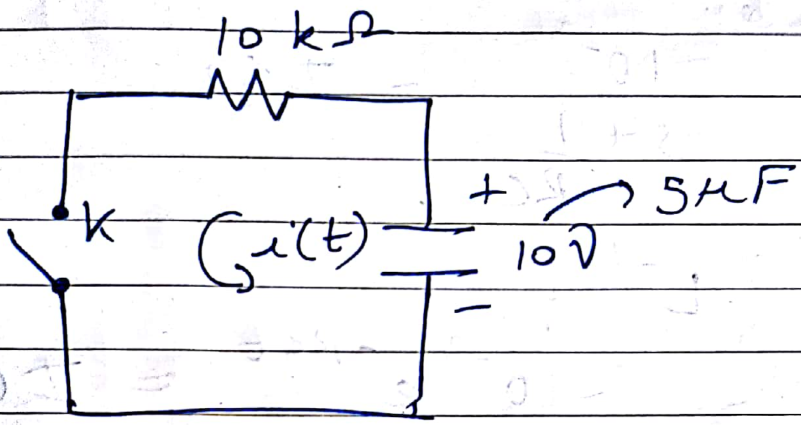
$$I_L(t) = i(t)\left[1 - e^{-R/L\,t}\right]$$

$$I_R(t) = i - i_L$$

$$= i.e^{-R/L\,t}$$



Q.



$q = CV$

$V = IR$

$\frac{10}{10 \times 10^3}$

$10^{-3} = I$

Initial charge = 5μF

discharging current ?

K.V.L.

$$10 = R\,i(t) + \frac{1}{C}\int i(t)\,dt$$

$$\mathcal{L}:\quad 0 = R I(s) + \frac{1}{C}\left[\frac{I(s)}{s} + \frac{10}{s}\right]$$

$$= R I(s) + \frac{1}{5 \times 10^{-6}}\left[\frac{I_s}{s} + \frac{10^{-3}}{s}\right]$$

$$= R I(s) + \frac{I_s}{Cs} + \frac{10^{-3}}{Cs}$$

$$\left[\left(R + \frac{1}{Cs}\right)\right] - \frac{10^{-3}}{Cs} = I(s)$$

Impulse Response of Series RC Network

KVL

$$E\delta(t) = R\,i(t) + \frac{1}{C}\int i(t)\,dt$$

Taking laplace

$$E = R I(s) + \frac{1}{C}\left[\frac{I(s)}{s} + \frac{Q_0}{s}\right]$$

assume $\boxed{Q_0 = 0}$

$$\Rightarrow E = \left[R + \frac{1}{Cs}\right] I(s) = \frac{R}{s}\left(s + \frac{1}{RC}\right) I(s)$$

$$\boxed{RC = T}$$

$$I(s) = \frac{E}{R} \cdot \frac{s}{s + \frac{1}{T}} = \frac{E}{R}\left[1 - \frac{\frac{1}{T}}{s + \frac{1}{T}}\right]$$

Hence L^{-1}

$$i(t) = \left(\frac{E}{R}\delta(t) - \frac{Q_0}{T}\right) e^{-\frac{1}{T}t}$$

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