Page No.
Date

Unit 2 Laplace Transform of Signals

- Bilateral (Double sided) Laplace Transform is used to find system characteristics.

E.g. causality, frequency response, stability.

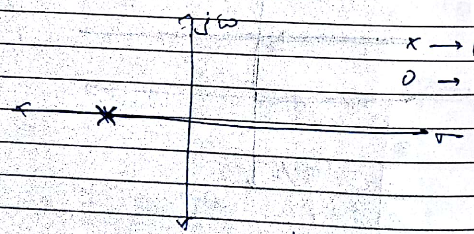
- Unilateral laplace is used for solving the differential eqns when the initial equations are known.

Laplace
$$\boxed{H(s) = \int_{-\infty}^{\infty} h(t).e^{-st} dt}$$

Inverse laplace
$$\boxed{h(t) = \frac{1}{2\pi . j} \int_{-\infty}^{\infty} H(s).e^{st} dt}$$

S-Plane — Real (s) σ
Imag (s) $j\omega$

$$\boxed{s = \sigma + j\omega}$$



x → Poles
o → holes/Z

$H(s) \leftrightarrow \frac{1}{s+2}$

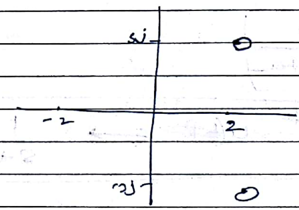
$e^{-2t} \longleftrightarrow \frac{1}{s+2}$

Page No.
Date

$$X(s) = \frac{s+2}{(s+3)(s+4)}$$

$s = 2 + 3j$ – Zero

Roots of Numerator – Zeros / holes – o
Roots of Denominator – Poles – x



[splane]

Q. Find the laplace transform of $x(t) = e^{-at}.u(t)$

$$X(s) = \int_{-\infty}^{\infty} x(t).e^{-st}.dt$$

$$= \int_{-\infty}^{\infty} e^{-at}.e^{-st}.u(t)dt$$

$$= \int_{-\infty}^{0} u(t).e^{-at}.e^{-st}dt + \longleftarrow X(s)\ \text{zero}$$

$$+ \int_{0}^{\infty} u(t).e^{-at}.e^{-st}dt$$

$$X(s) = \int_{0}^{\infty} e^{-at}.e^{-st}dt.$$

$$= \int_0^\infty e^{-(s+a)t}\, dt$$

$$= \left[\frac{-1}{s+a} \cdot e^{-(s+a)t} \right]_0^\infty$$

$$= \cancel{0/H} \; \frac{-1}{s+a} \left[e^{-(s+a)t} \right]_0^\infty$$

$$= \frac{-1}{s+a} \left[0 - 1 \right]$$

$$\boxed{X(s) = \frac{1}{s+a}}$$

$$\boxed{e^{-at} \cdot u(t) \longleftrightarrow \frac{1}{s+a}}$$

Q. $x(t) = -e^{-at}\, u(-t)$

$$X(s) = \int_{-\infty}^{\infty} x(t) \cdot e^{-st}\, dt$$

$$= -\int_{-\infty}^{\infty} e^{-at} \cdot e^{-st} \cdot u(-t)\, dt$$

$$= -\int_{-\infty}^{0} e^{-at} \cdot e^{-st} \cdot u(-t)\, dt$$

$$- \int_0^\infty e^{-at} \cdot e^{-st} \cdot u(-t) \cdot dt$$

$$= -\int_{-\infty}^{0} e^{-at} \cdot e^{-st} \cdot \cancel{u(-t)} \cdot dt \; - 0$$

$$= -\int_{-\infty}^{0} e^{-(s+a)t}$$

$$= \frac{1}{s+a} \left[e^{-(s+a)t} \right]_{-\infty}^{0}$$

$$= \frac{1}{s+a} \left[1 - 0 \right]$$

$$= \frac{1}{s+a}$$

$$\boxed{-e^{-at}\, u(-t) \longleftrightarrow \frac{1}{s+a}}$$

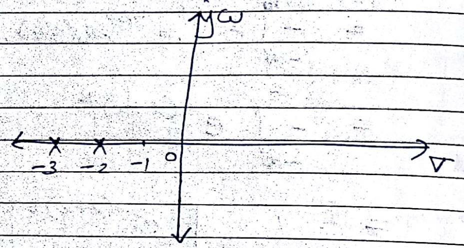
Q. $x(t) = e^{-2t} \cdot u(t) - e^{-3t} \cdot u(t)$
find Laplace Transform & ROC.

$$X(s) = \frac{1}{s+2} - \frac{1}{s+3}$$

$$= \frac{s+3-s-2}{(s+2)(s+3)}$$

$$= \frac{1}{(s+2)(s+3)}$$

$s = -2 \qquad s = -3$



Q. $x(t) = -e^{-2t} \cdot u(-t) + e^{-3t} u(-t)$

$$X(s) = \frac{1}{s+2} + X\left[e^{-3t} u(-t)\right]$$

$$= \int_{-\infty}^{\infty} e^{-3t} \cdot e^{-st} \cdot u(-t)$$

$$= \int_{-\infty}^{0} e^{-3t} \cdot e^{-st}$$

$$= \int_{-\infty}^{0} e^{-(s+3)t}$$

$$= \frac{-1}{s+3} \left[e^{-(s+3)t}\right]_{-\infty}^{0}$$

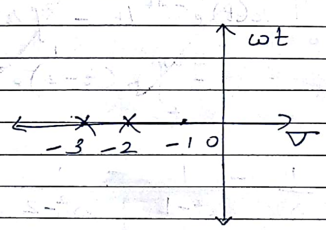
$$= \frac{-1}{s+3} (1-0)$$

$$= \frac{-1}{s+3}$$

$$\therefore X(s) = \frac{1}{s+2} - \frac{1}{s+3}$$

$$= \frac{s+3-s-2}{(s+2)(s+3)}$$

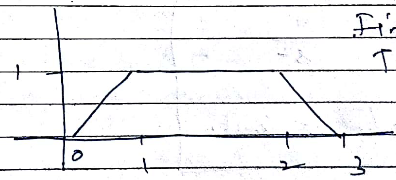
$$= \frac{1}{(s+2)(s+3)}$$



$$x(t) = -e^{-2t} \cdot u(-t) - \left(-e^{-3t} u(-t)\right)$$

$$\frac{1}{s+2} - \frac{1}{s-3}$$

Q. Find laplace Transform.



Page No.
Date

$$x(t) = r(t) - r(t-1) - r(t-2) + r(t-3)$$

$$X(s) = \int_{-\infty}^{\infty} x(t) \cdot e^{-st} dt$$

$$= \int_{-\infty}^{\infty} \Big[r(t) e^{-st} - r(t-1) e^{-st} - r(t-2) e^{-st} \Big] dt$$

$$= \int_{0}^{1} r(t) e^{-st} dt - \int_{1}^{2} r(t-1) e^{-st} dt - \int_{2}^{\infty} r(t-2) e^{-st} dt$$

$$= \frac{1}{s^2} - \frac{1}{s^2-1} - \frac{1}{s^2-2} + \frac{1}{s^2-3}$$

Unit – $\frac{1}{s}$

Ramp – $\frac{1}{s^2}$

Impulse → Const.

Q. R-L

$R = 2\Omega$

$L = 10H$

$V = 100V$

i? at $t = 5$

$T = \frac{L}{R} = \frac{10}{2}$

$T = 5$ sec

$$i = I(1 - e^{-t/T})$$

$$= \frac{100}{2}(1 - e^{-5/5})$$

$$= 50(1 - e^{-1})$$

$$\boxed{i = 31.60 \text{ A}}$$

Q. $R = 25\Omega$

$L = 5H$

$V = 100V$ at $t = 0$

Find - (i) i and V across R and L

(ii) i after $t = 0.5$

(iii) t at which voltage drop across R & L is same.

$\frac{25t}{5}$ $\frac{t}{T} = \frac{R}{L}$

(i). $$i = I(1 - e^{-t/T})$$

$$= \frac{100}{25}(1 - e^{Rt/L})$$

$$i = 4(1 - e^{-5t}) \text{ A}$$

$$V_o = 100(1 - e^{-5t}) \text{ V}$$

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$$\boxed{\frac{d}{dt} e^{-at} = a e^{-at}}$$

Page No.
Date

$$-5 \times \left(\frac{-1}{5}\right) \cdot 4\left(1 - e^{-5t}\right)$$

$$5 \times \left(\frac{-1}{5} \cdot \left(-e^{-5t}\right)\right)$$

$$= e^{-5t}$$

$$5 \times \frac{d}{dt}\left(4\left(1-e^{-5t}\right)\right)$$

$$V_L \Rightarrow 5 \times \frac{d}{dt}\left(4 - 4e^{-5t}\right)$$

$$= 5 \times \left(0 + 20 e^{-5t}\right)$$

$$= 100 e^{-5t}\ V$$

(ii) i after $t = 0.5s$

$$i = 4\left(1 - e^{-5t}\right)$$

$$= 4\left(1 - e^{-0.5 \times 5}\right)$$

$$i = 3.67\ A$$

(iii) V_1 at $V = 50$

$$100 e^{-5t} = 50$$

$$e^{-5t} = \frac{1}{2}$$

Page No.
Date

$$e^{5t} = 2$$

$$5t = 0.6931$$

or $t = \dfrac{0.6931}{5}$

$$t = 0.138\ sec$$

8.11 L-R

$V = 200V$

$R = 20\Omega$

$L = 0.2H$

$V = ?$ at (i) $t = 0$ (ii) $t = 0.02$

Ans. $V = iR + L\dfrac{di}{dt}$

$$200 = 20i + 0.2 \frac{di}{dt}$$

$$i = I\left(1 - e^{-t/T}\right) \qquad T = \frac{L}{R}$$

$$= \frac{200}{20}\left(1 - e^{-tR/L}\right) \qquad = 0.01$$

$$= 10\left(1 - e^{0}\right)$$

$$i = 0$$

$$i = 10\left(1 - e^{-0.02 \times 20/0.2}\right)$$

$$= 10\left(1 - e^{-2}\right)$$

$$i = 8.64\ A$$

Voltage Drop $= 200 - 172.93 = 27.07\ V$

$$\frac{di}{dt} = \frac{200 - 172.93}{0.2} = 135.35\ A/sec$$

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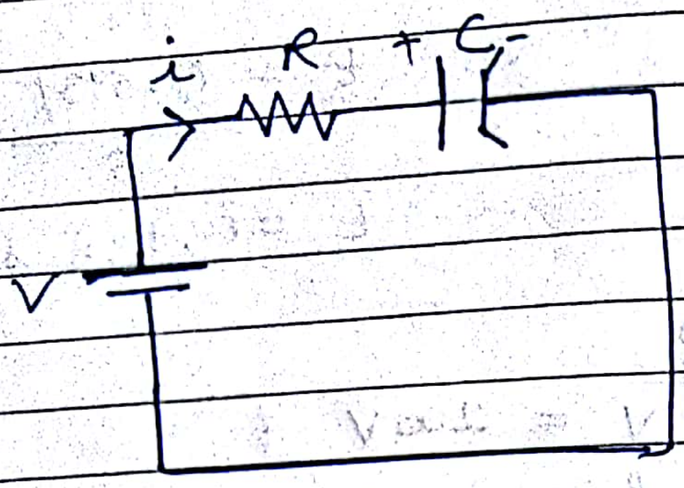
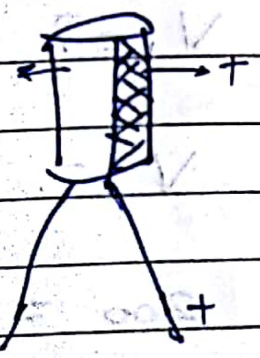
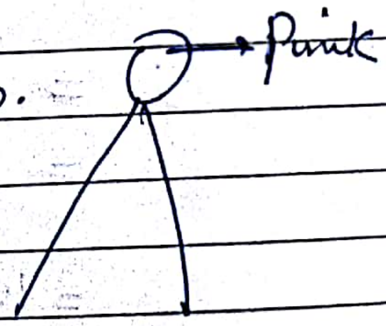
Transient Response ...



Paper Capacitor (unidirectional)



Ceramic Cap. Bi directional



$$\boxed{V = iR + \frac{1}{C}\int i\,dt}$$

$$0 = R\frac{di}{dt} + \frac{i}{C}$$

Page No. Date

Q. $R = 2000\ k\Omega = 2000000\ \Omega$

$C = 5\ \mu F = 5\times10^{-6} F$

$t = ?$ $\quad T = R.C = 10$ sec

$q = (50\%) Q$

$q = Q.(e^{-t/T})$

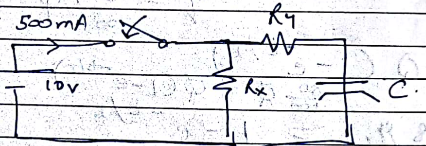
$\frac{50}{100} \cancel{Q} = \cancel{Q}.e^{-t/10}$

$0.5 = e^{-t/10}$

$\cancel{-}\frac{t}{10} = \cancel{-}0.6931$

$\Rightarrow t = 6.931$ sec

Q//



$t = ?$ when $i = 500\ mA = 500\times10^{-3} A$

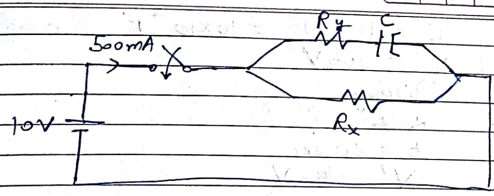
$R_x = 50\ \Omega$

$R_y = 70\ \Omega$

$C = 100\ \mu F = 100\times10^{-6} F$

$V = 10V$

Page No. Date



$i_y = I e^{-t/T}$ $\quad T = R.C = 70\times100\times10^{-6} = 7\times10^{-3}$ sec

$V_x = R_x.500 = 50\times500\times10^{-3} = 25\ A$

$i_x = \frac{V_x}{R_x} = \frac{25}{50} = 0.5\ A$

Total current $= I_x + I_y = 500\ mA$

$I_x = \frac{10}{50} = 200\ mA$

$I_y = 300\ mA \quad 0.3\ A$

$0.3 = \frac{10}{70} e^{-t/(7\times10^{-3})}$

$2.1 = e^{-t/(7\times10^{-3})}$ $\quad e^{t/(7\times10^{-3})} = 0.4761$

$0.74 = \frac{-t}{7\times10^{-3}}$ $\quad \frac{t}{7\times10^{-3}} = -0.7419$

-5.18×10^{-3} $\quad t =$

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$$V = iR + L.\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

$$0 = R.\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C}$$

$$\frac{R}{L}.\frac{di}{dt} + \frac{d^2i}{dt^2} + \frac{i}{L.C} = 0$$

$$\underline{I} \quad \left(\frac{R}{L}P + P^2 + \frac{1}{LC}\right)i = 0$$

$$\mathcal{L} = \frac{-b \pm \sqrt{b^2 - 4AC}}{2a}$$

$$i(t) = K_1 e^{-P_1 t} + K_2 e^{-P_2 t}$$

P_1, P_2 are roots of $\underline{I}$

wkt. at $t = 0$,

$i = 0$

$\Rightarrow \quad k_1 + k_2 = 0$

$k_1 = -k_2$ —— (2)

At $t = 0$,

$V = L \frac{di}{dt}$ $\quad [i = 0]$

Page No.
Date

$$5k_1 = 20$$

$$\Rightarrow \boxed{k_1 = 4}$$

$$\boxed{k_2 = -4}$$

$$i(t) = 4e^{4.791t} - 4e^{-208t}$$

- <u>Sinusoidal Response of RLC Circuit</u>

ϕ is the angle b/w voltage & current or the reactance & resistance.

$$\omega L = X_L = 2\pi f . L$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f . C}$$

Parallel R-C

$$I_R = \frac{V}{R}$$

$$I_C = \frac{V}{\frac{1}{j\omega C}} = V.j\omega C$$

$$\theta = \tan^{-1}(\omega RC)$$

- $\boxed{\text{RLC Circuit}}$

~~Ro~~ Series

$$V = IZ = I\left(\frac{1}{j\omega C} + j\omega L + R\right)$$

$$\theta = \tan^{-1}\left[\left(\omega L - \frac{1}{j\omega C}\right)/R\right]$$

Parallel

$$\theta = \tan^{-1}\left(\frac{\omega C - \frac{1}{\omega L}}{1/R}\right)$$

$\downarrow$

Phase $\angle$

Page No. ____ Date ____

(ii) $I = \frac{V}{Z} = \frac{100}{5.08\angle 38.14}$

$I = 19.685\,(-\angle 38.14)$

(iii) $V_R = I.R$

$= +19.685\angle -38.14 \times 4$

$= +78.74\angle -38.14\ V$

$V_L = 19.685\angle -38.14\,(100\pi \times 10^{-2})$

$= 61.842\angle -38.14\ V$

$90 - 38.14$

$(\because R \perp X_{L\,or\,C}$, $\phi = 90^{\circ})$

$= 61.842\angle 51.86$

(iv) $P.F = \cos(\phi)$

$= \cos(-38.13)$

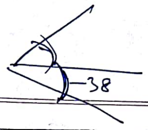
$= 0.786$

$= 78.6\,\%$

(v) Real Power $= P = IV\cos(\phi)$

$= 19.685 \times 100 \times 0.786$

$= 1547.241\ W$



Page No. ____ Date ____

(vi) Total Power $= I.V = 100 \times 19.685$

$= 1968.5\ W$

$\boxed{\text{H.W Eg. } 4.3,\ 4.4,\ 4.6}$

4.2 $R = 20\ \Omega$

$L = 60\ mH$

$\phi = 60^{\circ}$

$f = ?$

$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$

$\tan(60) = \omega \cdot \frac{60 \times 10^{-3}}{20}$

$\omega = \frac{\sqrt{3} \times 20}{0.06}$

$\omega = 577.350\ rad/sec$

$= 2\pi f$

$\Rightarrow f = \frac{577.35}{2\pi}$

$\boxed{f = 91.88\ Hz}$

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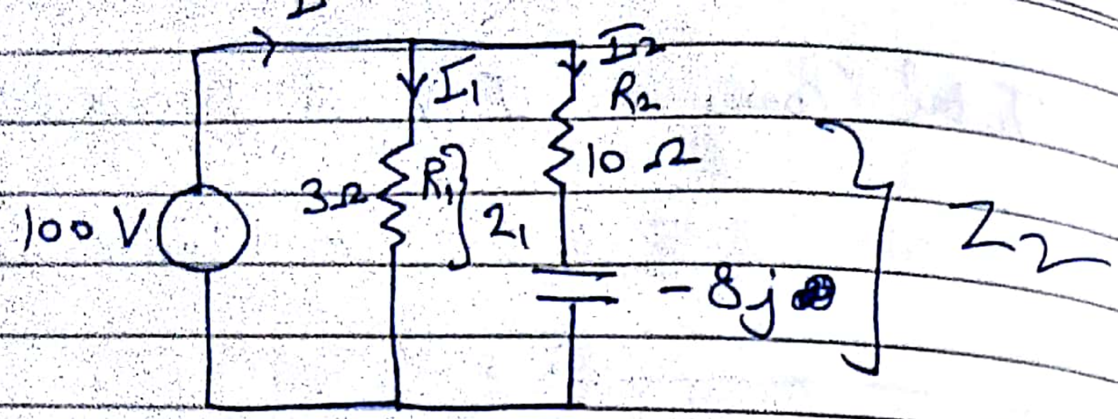
4.25



$I = ?$ $I_1 = ?$ $I_2 ?$

$R_{total} = ?$

$Z_1 = 3$

$Z_2 = 10 - 8j$

$|Z_2| = \sqrt{8^2 + 10^2} \cdot \tan^{-1}(X_c/R)$

8/10

$= \sqrt{64+100}\ \tan^{-1}(\ )$

$= 12.80 \times \tan^{-1}(0.8)$

$= 12.80 \angle 38.65$

$Z_1 = 3 \angle 0^\circ$

$I_1 = \frac{V}{Z_1} = \frac{100}{3} = 33.3V\angle 0$

$I_2 = \frac{V}{Z_2} = \frac{100}{12.80 \angle 38.65} = 7.81 \angle -38.6$

Page No.
Date

E4.4

$I = 1A$
$V = 10V$
$f = 50\ Hz$
$R = 5\Omega$

$L = ?$
$P.F = ?$

$\omega = 2\pi f = 2\pi \times 10 = 20\pi$

$Z = \sqrt{R^2 + (\omega L)^2}$

$\frac{V}{I} = \sqrt{R^2 + (\omega L)^2}$

$10 = \sqrt{25 + 400\pi^2 L^2}$

$100 = 25 + 400\pi^2 L^2$

$\Rightarrow L^2 = \sqrt{\frac{75}{2500\pi^2}} = \sqrt{\frac{75}{4\pi^2 \times 50^2}} = 0.027\ H$

$\Rightarrow L = 0.027\ H$

$PF \Rightarrow \cos\phi = \frac{R}{Z}$

$= \frac{5}{10} = \frac{1}{2} = 0.5$

$X_L = \omega L$
$X_C = \frac{1}{\omega C}$

4.6

$R = 4\Omega$
$L = 0.01H$
$f = $ (i) 100 Hz (ii) 500 Hz
$Z = ?$

$Z_{100} = R + jX_L$
$= 4\Omega + j\omega L$
$= 4\Omega + 2\pi \times 100 \times 0.01$
$Z_{100} = 4 + j6.28$

$Z_{100} = \sqrt{R^2 + (\omega L)^2} \angle \tan^{-1}\frac{\omega L}{R}$

$= \sqrt{16 + (2\pi)^2} \angle \tan^{-1}\left(\frac{2\pi}{4}\right)$

$= 7.44 \angle 57.51$

$Z_{500} = R + jX_L$
$= 4 + j\omega L = 4 + j(500 \times 2\pi \times 0.01)$

$\omega = 2\pi f = 1000\pi$
$X_L = 10\pi$

$= 4 + 10\pi j$
$= 4 + 31.4j$

$Z_{100} = \sqrt{16 + 100\pi^2} \angle \tan^{-1}\left(\frac{10\pi}{4}\right)$

$= 31.66 \angle 82.74$

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Q4.33 $f = 50\ Hz$

$V = 311 \sin\omega t$

R–L series 

Steady state I/V/ Values

$R = 5\Omega$

$L = 0.02\ H$

(i) $I_{rms} = ?$

(ii) ϕ

(iii) $i = ?$

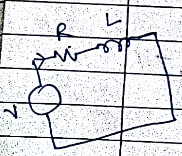
(iv) V across each ckt. element.

Ans. ~~I = 311 A~~

$$I_{rms} = \frac{V_{rms}}{\cancel{R}\ \cancel{X}\ \cancel{X_L}\ Z}$$

$\omega = 2\pi f$

$$V_{RMS} = 311/\sqrt{2} = 219.91 \approx 220\ V$$



$$Z = R + jX_L$$

$$X_L = \omega L$$

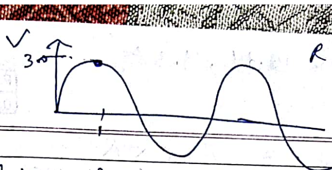
$$\Rightarrow Z = 5 + j(2\pi \times 50 \times 0.02)$$

$$= 5 + 6.28\,j$$

$$Z = \sqrt{R^2 + (\omega L)^2}\ \angle \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$= \sqrt{25 + 4\pi^2}\ \angle \tan^{-1}\left(\frac{2\pi}{5}\right)$$

$$= 8.029 \angle 51.488$$



R L N/2

$$V(1) = 300 \times \sin(90) = 300\ V$$

$$I_{rms} = \frac{220}{8.029 \angle 51.488}$$

$$= 27.40 \angle -51.488\ A$$

✓ Divide angle – sing reverse.

✓ Multiply angle – 90 – ϕ.

(ii) $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = 51.488^\circ$

(iii) $I = I_0 \sin(\omega t + \phi)$

$$= I_{rms} \times \sqrt{2} \sin(\omega t + \phi)$$

$$= 27.40 \angle -51.488 \times \sqrt{2}\ .\sin(100\pi . t + 51.488^\circ)$$

$$= 38.749 \angle \cancel{-51.488}\ \sin(100\pi t + 51.48)$$

$$I(t) = 38.75 \sin(\omega t - 51.48)$$

HW 4.34, 4.37

$V_R = I_{rms} \times R$

$= 27.40 \angle -51.488 \times 5$

$\angle -51.488$

$\Rightarrow V_R = 137 \cancel{\angle -51.488}\ V$

$V_L = I_{rms} \times jX_L$

$= 27.40 \angle -51.488 \times j(100\pi \times 0.02)$

$= 27.40 \angle -51.488 \times j2\pi$

$= 172.15 \angle 38.512\ V$

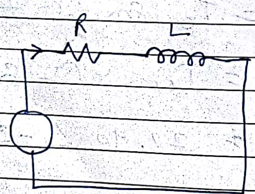
E.4.35

Series RL

$I = 1\cos(314t - 20)$

$V = 10\cos(314t + 10)$

$L = ?$ $R = ?$



$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$

$\tan 30° = \frac{314L}{R}$

$\sqrt{3}R = 942L$

$Z = \sqrt{R^2 + (\omega L)^2}$

$Z = \frac{V_0}{I_0} = \sqrt{R^2 + (\omega L)^2}$

$10 = \sqrt{R^2 + (314L)^2}$

$R^2 + (314L)^2 = 100$

✓Kilo -10^3
✓Mega -10^6
✓nano -10^{-9}
✓milli -10^{-3}
✓micro -10^{-6}
✓pico -10^{-12}

$(314L)^2 = 100 - \left(\frac{942L}{\sqrt{3}}\right)^2$

$98596L^2 = 100 - 295788L^2$

$394384L^2 = 100$

$L = 0.015 = 15\ mH$

$R = \frac{942}{\sqrt{3}} \times 0.015$

$\boxed{R = 8.66\ \Omega}$

Q 4.36 R−C series circuit

$R = 10\ \Omega$

$C = 25\ nF$

$f = 50\ MHz$

max $V_C = 2.5\ V \to$ across C.

$V_c = IX_c = \frac{I}{\omega C}$

$I = V_c \cdot \omega C$

$= 2.5 \times 2\pi \times 50 \times 10^6 \times 25 \times 10^{-9}$

$= 2.5 \times 100\pi \times 25 \times 10^{-3}$

$= 2.5 \times 2.5 \times \pi$

$I_{max} = 19.63\ A$

$I(R + X_c)$

$V_{max} = I_{max} \times Z$

$= I_{max}\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

$= 19.63\sqrt{100 + \left(\frac{1}{2.5\pi}\right)^2} = 196.31\ V$

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$$\overline{\quad R \quad}$$

$$= \tan^{-1}\left(\frac{314 \times 20 \times 10^{-3}}{10}\right)$$

$$\boxed{\phi = 32.12^{\circ}}$$

$$V = I \times Z$$

$$= I \times \sqrt{R^2 + (\omega L)^2} \sin(\omega t + \phi)$$

$$= 10 \times \sqrt{100 + (314 \times 400 \times 10^{-6})^2} \sin(314t + \phi)$$

$$\cancel{= 10 \times 10.0096 \sin(314t + 32.12)}$$

$$\cancel{= 100}$$

$$= 10 \times 11.808 \sin(\;"\;)$$

$$V = 118.08 \sin(314t + 32.12^{\circ})\ V$$

Page No.
Date

$\cancel{\frac{100}{\omega L}}$ $\cancel{+50}$ $50\omega C - \frac{100}{\omega L} = 0$

$\omega\left(50C - 100L\right) = 0$

$C = 2L = 0$

$\boxed{C = 2L}$

$50\omega C = \cancel{100\omega L}\ \frac{100}{\omega L}$ 2

$\omega^2 LC = 2$ (4)

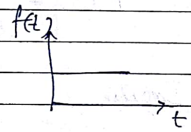
Page No.
Date

Laplace Transform

$$X(s) = \int_{-\infty}^{\infty} f(t).\ e^{-st}\ dt$$

(1) step function

$f(t) = u(t)$



$$F(s) = \int_0^{\infty} u(t)\ e^{-st}\ dt$$

$$= -\frac{1}{s}\left[e^{-st}\right]_0^{\infty}$$

$$= -\frac{1}{s}(0-1) = \frac{1}{s}$$

(2) Exponential function

$f(t) = e^{-at}$

$F(s) = \frac{1}{s+a}$

(3) Sinusoidal

$f(t) = \sin \omega t$

$F(s) = \frac{\omega}{s^2+\omega^2}$

④ $f(t) = \cos \omega t$

$$F(s) = \frac{s}{s^2 + \omega^2}$$

⑤ <u>Ramp</u>

$$f(t) = t$$

$$F(s) = \frac{1}{s^2}$$

⑥ <u>Impulse</u>

$$f(t) = \delta(t)$$

$$F(s) = 1$$

⑦ <u>Parabolic</u>

$$f(t) = t^n$$

$$F(s) = \frac{n}{s^{n+1}}$$

Q. $f(t) = 1 - e^{-at}$

Find $F(s)$.

Ans. $F(s) = \int_{-\infty}^{\infty} e^{-st}\, dt - \frac{1}{s+a}$

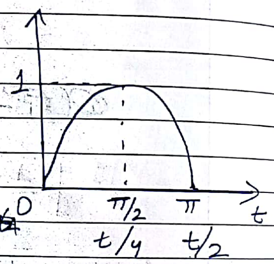
$$= \frac{-1}{s}(0 - 1) - \frac{1}{s+a}$$

$$= \frac{1}{s} - \frac{1}{s+a}$$

s^2

9.11 Find laplace of half cycle sine wave.

$$f(t) = \int_0^{t/2} \sin\omega t \, dt$$

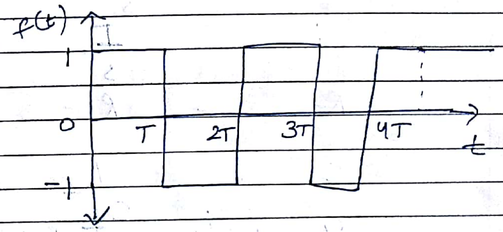


$$F(t) = \int_0^{t/2} \sin\omega t \cdot e^{-st} \, dt$$

$$= \frac{\omega}{\omega^2+s^2}\left[e^{-st/2} - 1\right]_0$$

$$= \frac{\omega}{\omega^2+s^2}\left(e^{-st/2} - 1\right)$$

9.12



$$f(t) = u(t) - 2u(t-T) + 2u(t-2T) - 2u(t-3T) + 2u(t-4T)$$

$$= \frac{1}{s} - \frac{2}{s}e^{-sT} + \frac{2}{s}e^{-2sT} - \frac{2}{s}e^{-3sT} + \frac{2}{s}e^{-4sT}$$

$$= \frac{2}{s}\left[\frac{1}{2} - e^{-sT} + e^{-2sT} - e^{-3sT} + e^{-4sT}\right]$$

$$= \frac{1}{s}\left[1 - 2e^{-sT}\left(1 + e^{sT} - e^{2sT} + e^{3sT}\right)\right]$$

	T	s
1	V	$\frac{V}{s}$
2	$I(t)$	$\frac{I(s)}{s}$
or	i	$\frac{I}{s}$ ✓
3	R	R
4	C	$\frac{1}{Cs}$
5	$V_C = \frac{1}{C}\int i\,dt$	$\frac{I}{Cs} + \frac{V_0}{s}$
6	L	Ls
7	$V_L = L\frac{di}{dt}$	$L\left[sI(s) - I(0)\right]$

Q. $\frac{10^4}{s(s+250)}$ Find L^{-1}

$$= 10^4 \left[\frac{1}{s(s+250)}\right]$$

$$\Rightarrow \frac{10^4}{s(s+250)} = \frac{A}{s} + \frac{B}{s+250} \quad \text{--- (1)}$$

– Partial fraction

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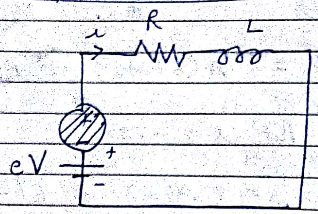
HW → 9.18, 9.19, 9.23

Page No. Date

• Step Response in R-C & R-L



Series R-L

Ex/14 $e u(t) = iR + L\frac{di}{dt}$

Applying L.T.

$$\frac{e}{s} = I(s).R + L[sI(s) - I(0)]$$

$$\frac{e}{s} = I(s)R + LsI(s) - LI(0)$$

$$I(s)[R + Ls] = \frac{e}{s} + LI(0)$$

$$I(s) = \frac{e}{s(R+Ls)} + \underbrace{\frac{LI(0)}{R+Ls}}_{0}$$

$$I(s) = \frac{e}{s(R+Ls)}$$

$$= \frac{e/L}{s(R+Ls)/L}$$

$$\frac{1}{s(s+a)} = a \times e^{-at}$$

Page No. Date

$$I(s) = \frac{e/L}{s\left(\frac{R}{L}+s\right)}$$

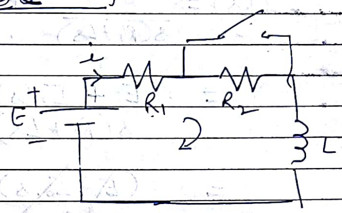
$$= \frac{A}{s} + \frac{B}{\left(\frac{R}{L}+s\right)} = \frac{e/L}{s\left(\frac{R}{L}+s\right)}$$

$= \frac{R}{L} \times \frac{E}{L}$ $\quad A = \frac{E}{R} \quad B = -\frac{E}{R}$

$$i(t) = \frac{E}{R} - \frac{E}{R}e^{-\frac{R}{L}t}$$

$$\boxed{V = \frac{Q_0}{C}}$$

9.22



$E = 10V$ $\quad R_1 = 1\Omega$

$L = 1H$ $\quad R_2 = 2\Omega$

$i = ?$

~~$E + iR_1 + iR_2 - i.\frac{Ldi}{dt} = 0$~~

$$+E + iR_1 + iR_2 - i.\frac{Ldi}{dt} = 0$$

$$E\,u(t) = i R_1 + L\frac{di}{dt}$$

$$\frac{E}{s} = I(s).R_1 + L\left[sI(s) - I(0)\right]$$

$$\frac{E}{s} = I(s)R_1 + LsI(s) - LI(0)$$

$$I(s)\left[R_1 + Ls\right] = \frac{E}{s} + LI(0)$$

$$I(s) = \frac{E}{s(R_1+Ls)} + \frac{I(0)}{}$$

$$I(s) = \frac{E}{s(R_1+Ls)} + \frac{LI_0}{R_1+Ls}$$

$$= \frac{E + I_0(s)}{s(R_1+Ls)}$$

$$= \frac{(E - I_0 s)/L}{s\left(\frac{R_1}{L}+s\right)}$$

$$I(s) = \frac{\frac{E}{L} + \frac{I_0 \cdot s}{1}}{s\left(\frac{R_1}{L}+s\right)}$$

$$\frac{A}{s} + \frac{B}{\frac{R_1}{L}+s}$$

$$\frac{}{s\left(\frac{R_1}{L}+s\right)} \quad \frac{}{s} \quad \frac{R_1}{L}+s$$

~~Put s = 0,~~

$$\cancel{0 = As + \frac{BL}{R_1}}$$

$\times s$

$$\frac{\frac{E}{L} + I_0 s}{\frac{R_1}{L}+s} = A + \frac{Bs}{\frac{R_1}{L}+s} \quad \text{—— (1)}$$

Put $s = 0$,

$$\frac{E}{L} \times \frac{L}{R_1} = A$$

$$\boxed{A = \frac{E}{R_1}} = 10$$

$$(1) \times \left(\frac{R}{L}+s\right)$$

$$\Rightarrow \frac{\frac{E}{L} + I_0 \cdot s}{s} = \frac{A}{s}\cdot\left(\frac{R}{L}+s\right) + B$$

Put $s = -R/L$

$$\frac{\frac{E}{L} - I_0\left(\frac{R}{L}\right)}{(R/L)} = A(0) + B$$

Page No.
Date

$$\Rightarrow \quad B = -\left(\frac{E}{\cancel{R}} \times \frac{\cancel{R}}{R}\right) - I_0$$

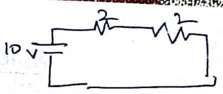
$$\boxed{B = -\left(\frac{E - I_0}{R_1}\right)}$$

$$= -\left(10 - \frac{10}{3}\right)$$

$$\boxed{B = -6.67}$$

$$I(t) = \frac{10}{s} + \frac{(-6.67)}{s + 1}$$

$$= 10u(t) - 6.67\, e^{-t}$$



$$\cancel{I = \frac{10}{2} + \frac{10}{2} = 5 + 5 = 10\,A}$$

$$I = \frac{10}{4} = 2.5\,A.$$

9.18

$$f(s) = \frac{50}{s^2 + 2s + 2}$$

$$= \frac{50}{(s+1)^2 + 1^2}$$

$$f(t) = 50\, e^{-t} \sin t$$

9.19

$$I(s) = \frac{s+1}{s(s^2+4s+4)}$$

$$= \frac{s+1}{s(s+2)^2}$$

$$\frac{s+1}{s(s+2)^2} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{(s+2)^2} \quad \text{--- (1)}$$

$(\times s)$

$$\frac{s+1}{(s+2)^2} = A + \frac{Bs}{s+2} + \frac{Cs}{(s+2)^2}$$

Put $s = 0$

$$\boxed{A = \frac{1}{4}}$$

① $\times (s+2)^2$

$\Rightarrow \cancel{\dfrac{s+1}{s(s+2)}} = \cancel{\dfrac{A(s+2)}{s}} + \cancel{B} + \cancel{\dfrac{C}{s+2}}$

$$\frac{s+1}{s} = \frac{A(s+2)^2}{s} + B(s+2) + C$$

Put $s=-2$

$$A(0) + B(0) + C = \frac{1}{2}$$

$$\Rightarrow \boxed{C = \frac{1}{2}}$$

$$\frac{s+1}{s(s+2)} = \frac{A(s+2)}{s} + B + \frac{C}{s+2}$$

Putting A, C, & $s=1$

$$\frac{2}{3} = \frac{3}{4} + B + \frac{1}{6}$$

$$B + \frac{11}{12} = \frac{2}{3}$$

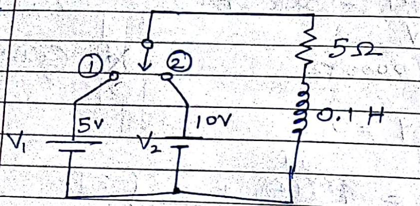
$$B = \frac{2}{3} - \frac{11}{12} = \frac{8-11}{12} = \frac{-3}{12}$$

$$B = \frac{-1}{4}$$

$RI(t)$

$\& \; I(s)$

Eq. 24



$$10 = 5i + 0.1 \times \frac{di}{dt}$$

Applying L.T,

$u(t) \Rightarrow \frac{1}{s}$

$$\frac{10}{s} = 5\,I(s) + 0.1\left[s\,I(s) - I(0)\right]$$

$$\frac{10}{s} = 5I(s) + 0.1\,s\,I(s) - 0.1\,I(0)$$

$\frac{5}{0.1}$

$$I(s)\left[5 + 0.1\,s\right] = \frac{10}{s} + 0.1\,I(0)$$

$$I(s) = \frac{(10/s) + 0.1\,I(0)}{5 + 0.1\,s}$$

$$= \frac{100/s + I(0)}{50 + s}$$

$$= \frac{100 + sI(0)}{s(s+50)}$$

$$= \frac{100 + s}{s(s+50)}$$

$\cancel{I(t)} \,/\, \neq$

$$\frac{100+s}{s(s+50)} = \frac{A}{s} + \frac{B}{s+50}$$

$$\frac{100+s}{s(\cancel{s+50})} = \frac{A(s+50) + sB}{s(\cancel{s+50})}$$

$$A(s+50) + B(s) = 100+s$$

Put $s = 0$,

$$A(50) + B(0) = 100$$

$$\Rightarrow \boxed{A = 2}$$

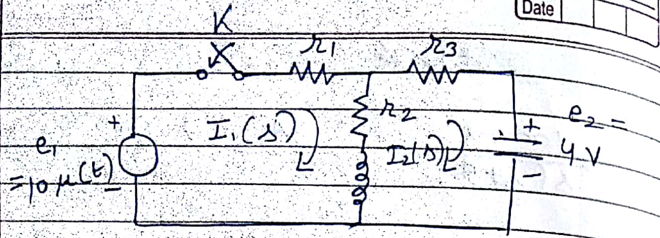
Put $s = -50$,

$$A(0) + B(-50) = 50$$

$$\Rightarrow \boxed{B = -1}$$

$$I(t) = 2\,\cancel{u(t)}\;\cancel{s}\;\cancel{B}\,e^{-50t}$$

E9.27



$r_1 = r_2 = r_3 = 1\,\Omega$

$L = 1H$

$C = 1F$

Ans: $e_1 = r_1(I_1(t)) + r_2(I_1(t) - I_2(t)) + L\frac{d(I_1 - I_2)}{dt}$ —(1)

$+e_2 + r_3 I_2(t) + r_2(I_2(t) - I_1(t)) + L\frac{d(I_2 - I_1)}{dt} + \frac{1}{C}\int I_2 dt = 0$ —(2)

Applying L.T on (1) & (2),

$\frac{e_1}{s} = r_1 I_1(s) + r_2(I_1(s) - I_2(s)) + L[(sI_1(s) - I_1(0)) - (sI_2(s) - I_2(0))]$

$-\frac{e_2}{s} = r_3 I_2(s) + r_2(I_2(s) - I_1(s)) + L[(sI_2(s) - I_2(0)) - (sI_1(s) - I_1(0))] + [\frac{I_2}{s} + V_0]$

$\frac{e_1}{s} = r_1 I_1(s) + r_2 I_1(s) - r_2 I_2(s) + LsI_1(s) - LI_1(0) - Ls(I_2(s)) + LI_2(0)$

$\frac{10}{s} = I_1(s) + I_1(s) - I_2(s) + sI_1(s) - I_1(0) - sI_2(s) + I_2(0)$

$\frac{10}{s} = I_1(s)[2+s] - I_2(s)[1+s]$

$I_1(s)[2+s] = I_2(s)[1+s] + \frac{10}{s}$

$\frac{4}{s} + I_2(s) + I_2(s) - I_1(s) + sI_2(s) - sI_1(s) + \frac{I_2}{s} = 0$

$I_1(s)[1+s] - I_2(s)[2+s+\frac{1}{s}] = \frac{4}{s}$

$I_1(s) = I_2(s)\left(\frac{1+s}{2+s}\right) + \frac{10}{s(s+2)}$

$I_2(s)\left[\left(\frac{1+s}{2+s}\right) + \frac{10}{s(s+2)}\right] - I_2(s)\left[2+s+\frac{1}{s}\right] = \frac{4}{s}$

$\Rightarrow I_2\left[\frac{s+1}{s+2} + \frac{10}{s(s+2)} - \left(s + \frac{1}{s} + 2\right)\right] = \frac{4}{s}$

$$\Rightarrow I_2\left[\frac{s(s+1)+10}{s(s+2)} - \frac{s^2+2s+1}{s}\right] = \frac{40}{s}$$

$$\Rightarrow I_2\left[\frac{s^2+s+10-(s^2+2s+1)(s+2)}{\cancel{s}(s+2)}\right] = \frac{4}{\cancel{s}}$$

$$\Rightarrow I_2\left[\frac{s^2+s+10-(s^3+2s^2+2s^2+4s+s+2)}{s+2}\right] =$$

$$\Rightarrow I_2\left[\frac{-s^3-3s^2-4s+8}{s+2}\right] = 4$$

$$I_2 = -4 \Big/ \left(\frac{s^3+3s^2+4s-8}{s+2}\right)$$

$$I_2 = \frac{-4(s+2)}{s^3+3s^2+4s-8}$$

$$\left[\begin{array}{l} s = -1 \\ s = -2+2i \\ s = -2-2i \end{array}\right]$$

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