

Everyone says happiness starts from 'H'  
How come my starts from 'V'.

Neelgagan

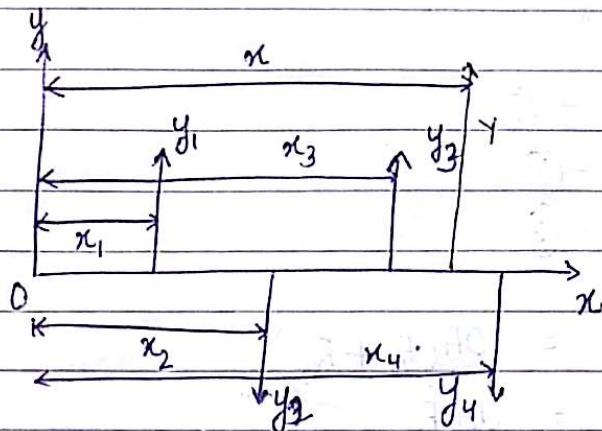
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Page No.

17/1/16

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General case of Parallel forces in a plane →



$$\gamma = y_1 - y_2 + y_3 - y_4$$

$$\gamma = \sum y_i$$

OY

$$\therefore \gamma \cdot x = y_1 x_1 - y_2 x_2 + y_3 x_3 - y_4 x_4$$

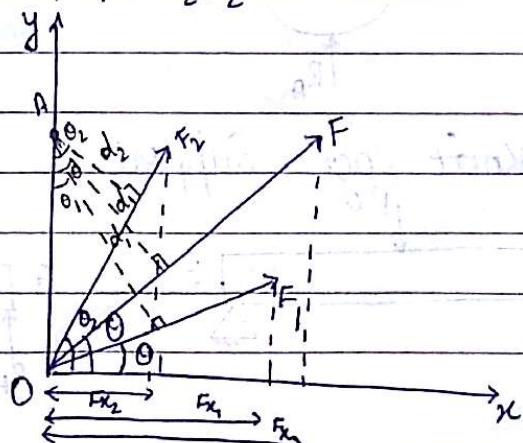
$$n = \frac{\sum y_i x_i}{\gamma} = \frac{\sum y_i x_i}{\sum y_i}$$

Principle of Moments / Varignon's Theorem →

Moment of a force about any axis is equal to the sum of moment of its components abt the same axis.

$$F \rightarrow F_1, F_2$$

$$F \cdot d = F_1 d_1 + F_2 d_2$$



$$F_x = F \cos \theta$$

$$F_{x_1} = F_1 \cos \theta_1$$

$$F_{x_2} = F_2 \cos \theta_2$$

Moment abt. A

$$F.d = F.OA \cos\theta$$

$$= OA \cdot F \cos\theta$$

$$= OA \cdot F_x$$

$$Fd = OA F_x - ①$$

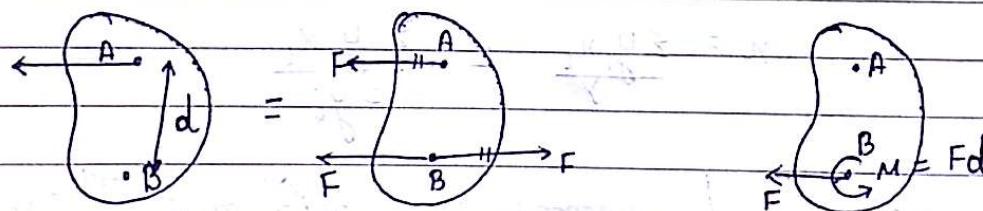
$$F_1 d_1 = OA \cdot F_{x_1} - ②$$

$$F_2 d_2 = OA \cdot F_{x_2} - ③$$

$$② + ③$$

$$F_1 d_1 + F_2 d_2 = OA(F_{x_1} + F_{x_2}) \\ = OA F_x$$

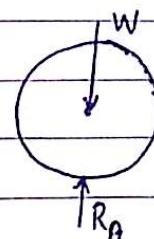
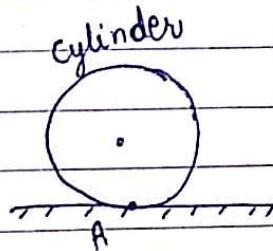
Resolution of a Force into a force and couple



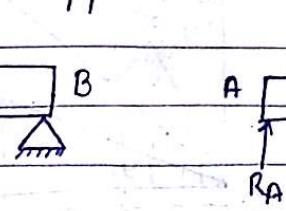
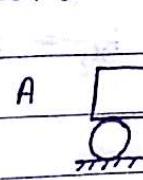
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Supports and Reactions

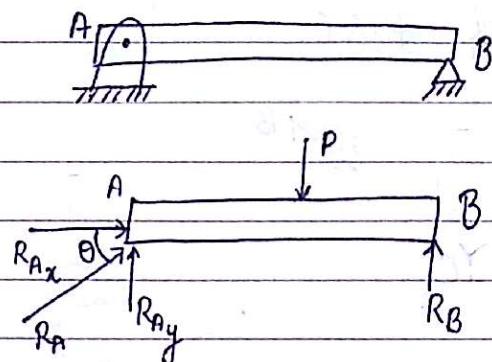
① Frictionless Support



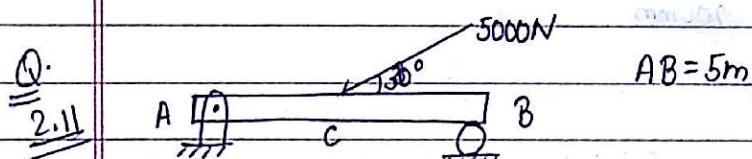
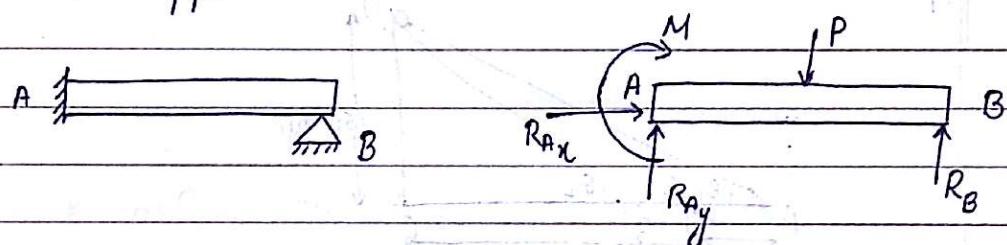
② Roller and Knife edge Support



(3) Hinge / Pin Joint Support →



(4) Built-in Support



Sol:-

$$\sum F_x = 0 \quad R_{A_x} = 5000 \times \frac{\sqrt{3}}{2} = 2500\sqrt{3} N = 4330 N$$

$$\sum F_y = 0 \quad R_{A_y} + R_B = 2500 N$$

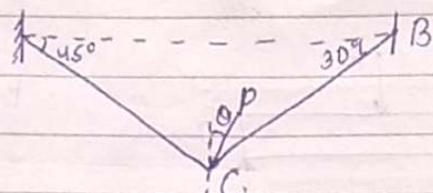
$$\sum M_A = 0 \quad -2.5 \times 2500 + R_B \times 5 = 0 \quad R_B = 4506.8 N$$

$$R_B = \frac{6250}{5} = 1250 N$$

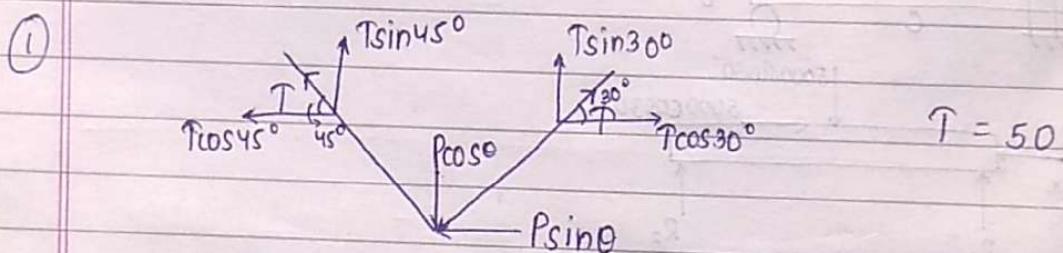
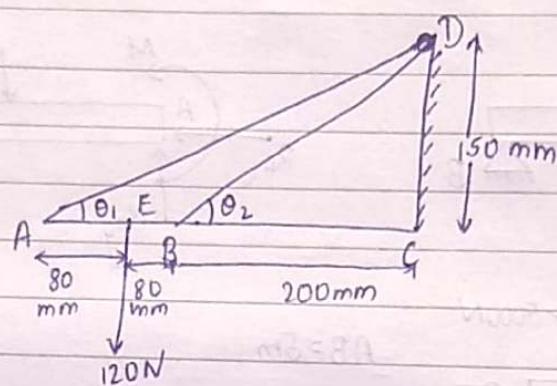
$$R_{A_y} = 2500 - 1250 = 1250 N$$

$$R_{A_y} = 2500 - 1250 = 1250 N$$

Q.1 Force P is applied at point C of the cable AB ACB. If tension in each part is 50N. Find out the value of P & θ.



Q.2 Determine tension in cable ADB & reaction at pt. C



$$Ps \sin \theta = 50 \cos 30^\circ - 50 \sin 45^\circ \quad \text{--- (1)}$$

$$Ps \cos \theta = 50 \sin 45^\circ + 50 \sin 30^\circ \quad \text{--- (2)}$$

(1)/(2)

$$\begin{aligned} \tan \theta &= \frac{\sqrt{2}/2 - 1/\sqrt{2}}{1/\sqrt{2} + 1/2} \\ &= \frac{\sqrt{6} - 2}{2 + \sqrt{2}} \\ &= 0.13 \end{aligned}$$

$$\theta = 7.4^\circ$$

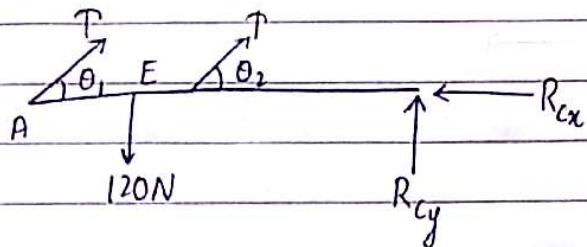
$$P = 61.69 \text{ N}$$

$$(2) \quad \theta_1 = \tan^{-1} \frac{150}{360}$$

$$= 22.6^\circ$$

$$\theta_2 = \tan^{-1} \frac{150}{200}$$

$$= 36.87^\circ$$



$$T \cos \theta_1 + T \cos \theta_2 = R_{cx}$$

$$T(\cos \theta_1 + \cos \theta_2) = R_{cx}$$

$$1.72 T = R_{cx}$$

$$T(\sin \theta_1 + \sin \theta_2) + R_{cy} = 120$$

$$0.98 T + R_{cy} = 120$$

Moment abt. pt. C = 0

$$T \cos \theta_2 \times 200 - 120 \times 280 + T \sin \theta_1 \times 360 = 0$$

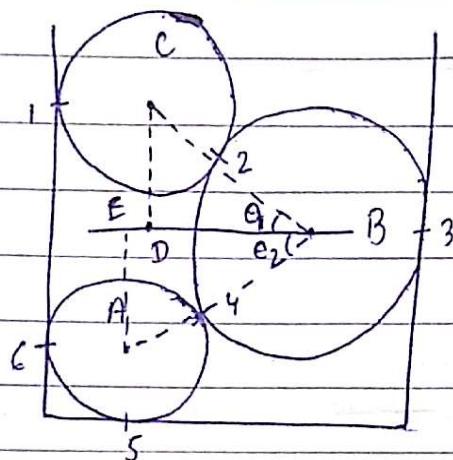
$$T(120 + 138.35) = 33600$$

$$T = 130.056 N$$

$$R_{cx} = 223.6 N$$

$$R_{cy} = -7.4 N$$

(3)



$$r_A = 4\text{cm} \quad w_A = 15$$

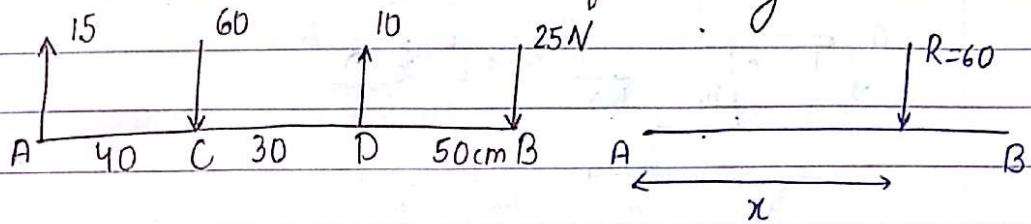
$$r_B = 6 \quad w_B = 40$$

$$r_C = 5 \quad w_C = 20\text{kg}$$

$$\cos \theta_1 = \frac{BD}{11} = \frac{7}{11}$$

$\leftarrow$   $18\text{cm}$   $\rightarrow$

① (a) Find out the resultant of the system.

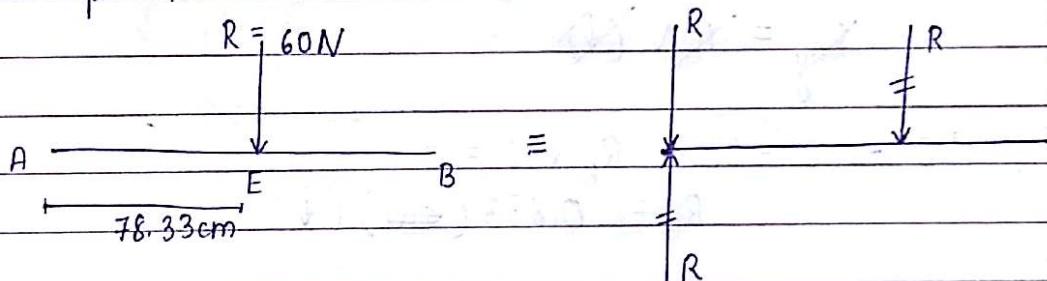


Sol<sup>n</sup>-

$$\begin{aligned} F_{\text{net}} &= 15 + 10 - 60 - 25 \\ &= -60 \text{ N} \\ &= 60 \text{ N} (\downarrow) \end{aligned}$$

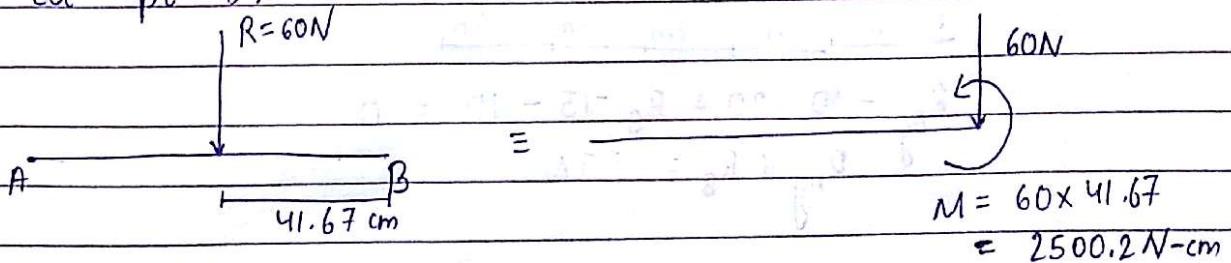
$$\begin{aligned} -60x &= -60 \times 40 + 10 \times 70 - 25 \times 120 \\ x &= 78.33 \text{ cm} \end{aligned}$$

(b) Reduce the system into a single force and moment at pt. A.

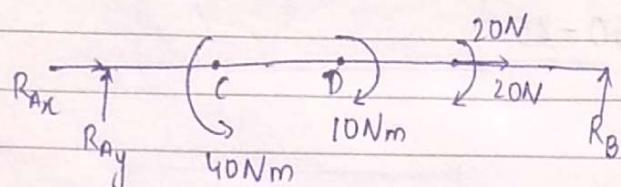
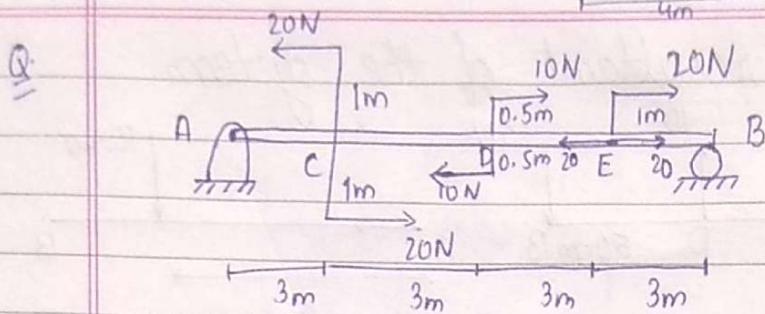
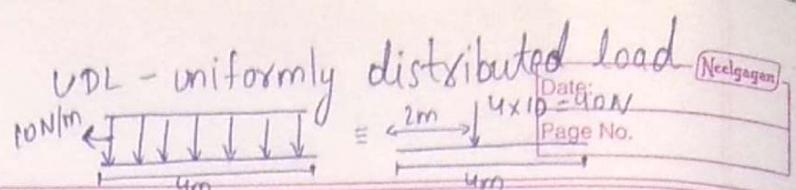


$$\begin{aligned} M &= F \times d = 60 \times 78.33 \\ &= 4699.8 \text{ N-cm} \\ &= 47 \text{ N-m (CCW)} \end{aligned}$$

Reduce the system into a single force & moment at pt. D.



$$\begin{aligned} M &= 60 \times 41.67 \\ &= 2500.2 \text{ N-cm} \end{aligned}$$



$$R_{A_x} = 20N$$

$$R_{A_y} + R_B \neq$$

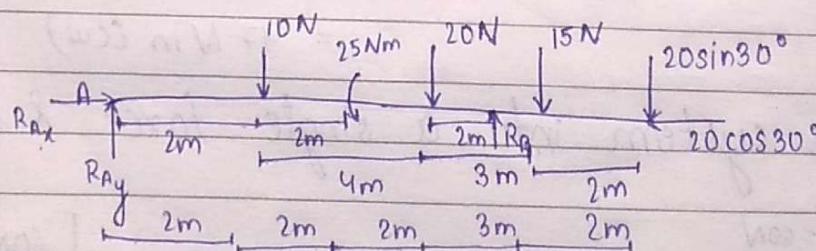
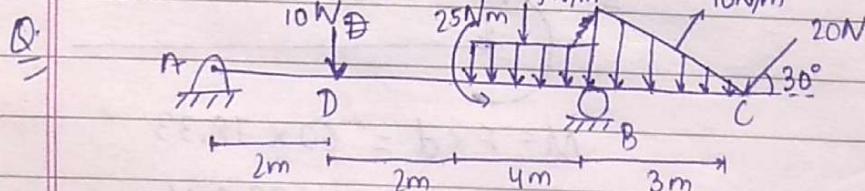
$$\cancel{40 \times 3 + 10 \times 6 + 20 \times 4} \neq R_B \times 12 \Rightarrow R_B = 10N (\uparrow)$$

$$R_{A_y} = 10N (\downarrow)$$

$$40 - 10 - 20 + R_B \times 12 = 0$$

$$R_B = 0.833 (\leftarrow) (\downarrow)$$

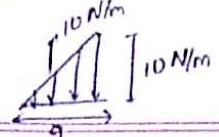
$$R_{A_y} = 0.833 (\uparrow)$$



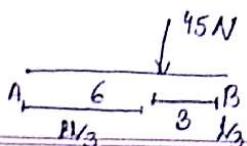
$$R_{A_y} - 10 - 20 + R_B - 15 - 10 = 0$$

$$R_{A_y} + R_B = 55N$$

UVL  $\rightarrow$  Uniformly Varying Load



$$F = \frac{1}{2} \times 10 \times 9 = 45N$$



Date: \_\_\_\_\_  
Page No.: \_\_\_\_\_  
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$$R_{Ax} = 20 \cos 30^\circ$$

$$= 20 \times \frac{\sqrt{3}}{2}$$

$$= 17.32 N (\rightarrow)$$

$$\sum M_A = 0$$

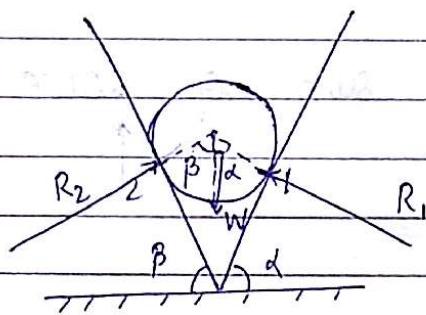
$$-10 \times 2 + 25 - 20 \times 6 + R_B \times 8 - 15 \times 9 - 10 \times 11 = 0$$

$$R_B = 45 N (\uparrow)$$

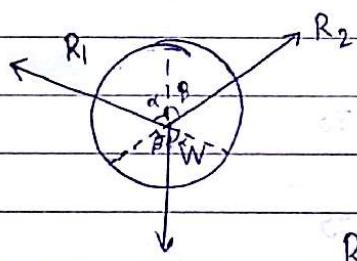
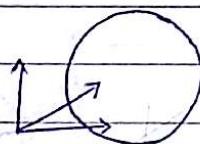
$$R_{Ay} = 55 - 45$$

$$= 10 N (\uparrow)$$

Q:



Find reaction at 1 & 2.



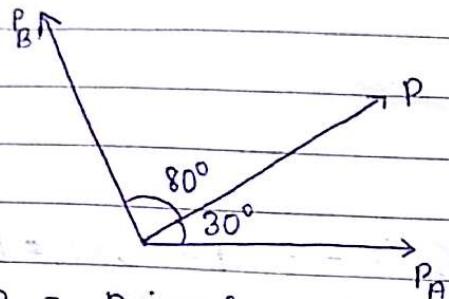
$$\frac{R_1}{\sin(180-\beta)} = \frac{R_2}{\sin(180-\alpha)} = \frac{W}{\sin(\alpha+\beta)}$$

$$R_1 = \frac{W \sin \beta}{\sin(\alpha+\beta)} \quad R_2 = \frac{W \sin \alpha}{\sin(\alpha+\beta)}$$

10/1/17

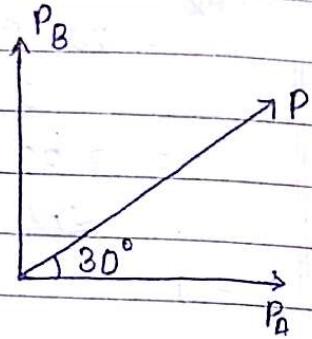
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Date:  
Page No.

## Force System



$$P_A = \frac{P \sin 30^\circ}{\sin(30^\circ + 80^\circ)}$$

$$P_B = \frac{P \sin 80^\circ}{\sin(30^\circ + 80^\circ)}$$



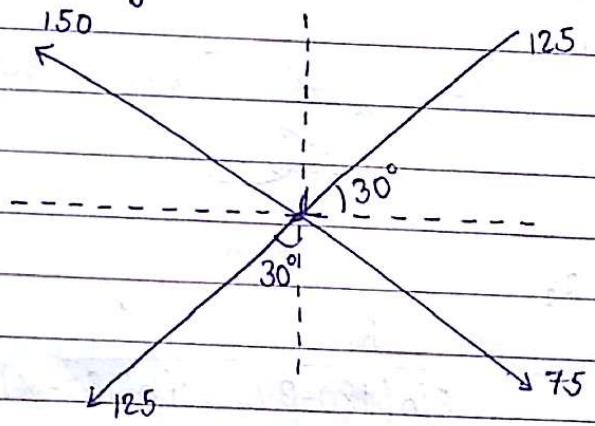
$$P_A = \frac{P \sin 30^\circ}{\sin(90^\circ)} = P \sin 30^\circ$$

$$P_B = \frac{P \sin 60^\circ}{\sin 90^\circ} = P \sin 60^\circ$$

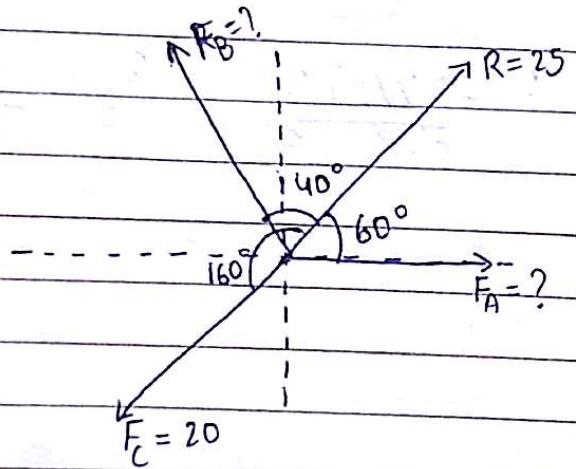
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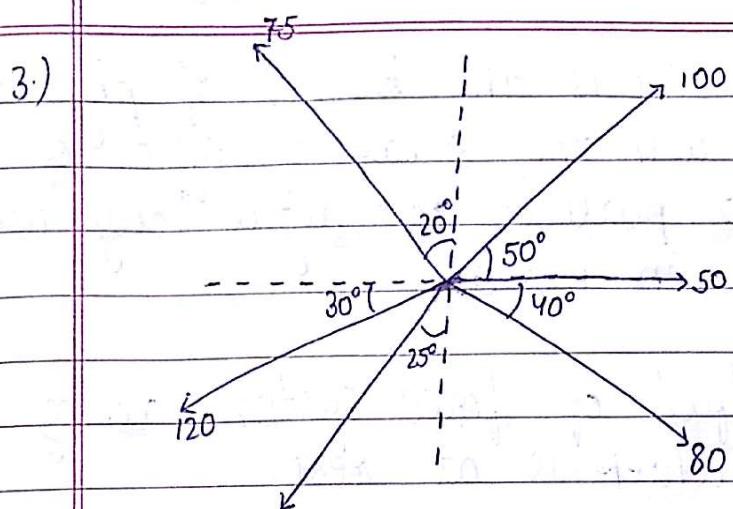
Determine the algebraic sum of force components in x & y direction

1.)



2.)





13/01/17

**Engineering Mechanics** - It is the branch of physical science that deals with the study of effect of force system acting on a particle or rigid body which may be at rest or in motion.

**Statics** - Study of effect of force system acting on a particle or body which is at rest.

**Dynamics** - Study of effect of force system acting on a particular rigid body which is at rest.

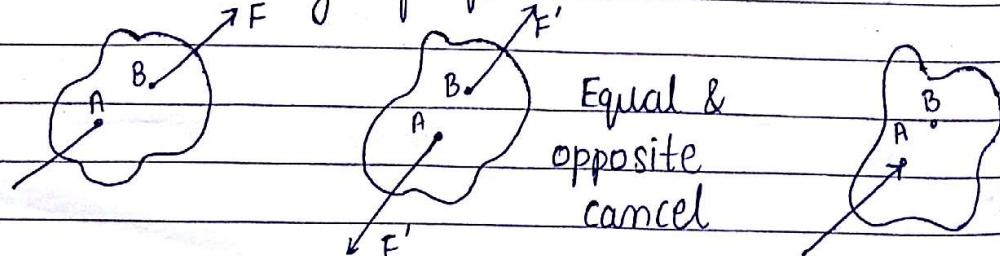
**Kinematics** - Study of effect of force system without reference of cause of motion i.e. relationship b/w velocity, displacement, acceleration are defined.

**Kinetics** - Study of effect of force system with reference to the cause of motion i.e. force and mass are considered.

### Law of Mechanics -

- ① Newton's law of inertia
- ②  $F \propto$  rate of change of momentum
- ③ Equal & opposite reaction
- ④ Newton's gravitation law  $F = \frac{Gm_1 m_2}{r^2}$

- ⑤ Parallelogram law
- ⑥ Transmissibility of forces



The cond<sup>n</sup> of equilibrium of a body will remain unchanged if point of application of a force acting on a rigid body is shifted to any other point acting along its line of action.

7)  $\Delta$  law

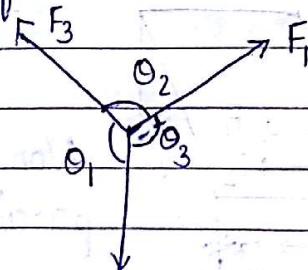
If two forces acting on a pt. are represented by two sides of a triangle taken in order, then their resultant is represented by the 3rd side taken in an opposite order.

8) Polygon law of forces

If n no. of concurrent forces acting on a body are represented in magnitude & dir<sup>n</sup> by the side of a polygon taken in an order, then the resultant is represented in magnitude and direction by the closing side of the polygon taken to in opposite order.

9) Lami's Theorem

If three concurrent forces acting on a body are represented in magnitude and direction by the s having same nature (push or pull) are in equilibrium then each force is proportional to the sine of angle included b/w the other two forces.

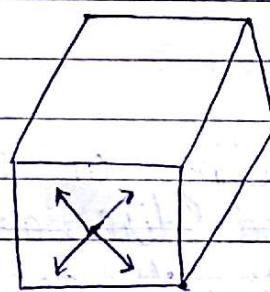
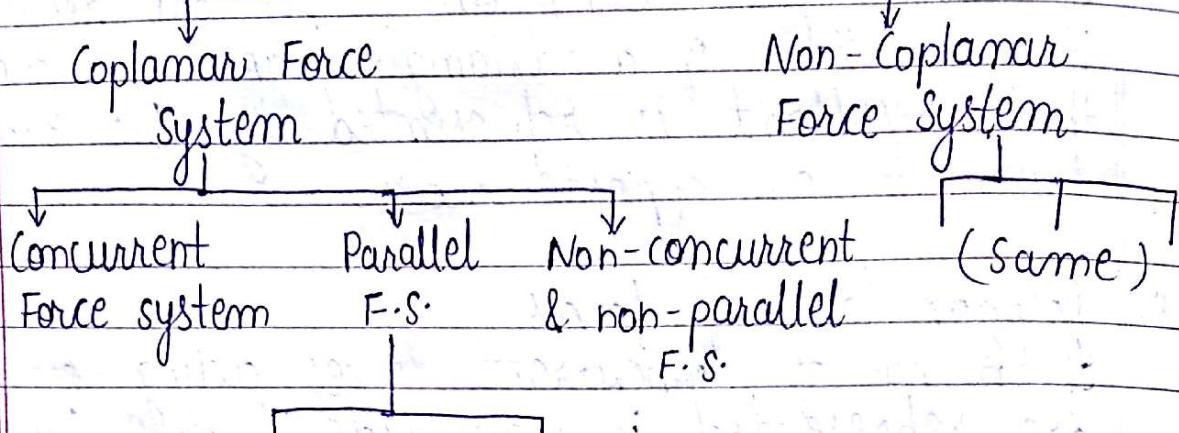


$$\frac{F_1}{\sin \theta_1} = \frac{F_2}{\sin \theta_2} = \frac{F_3}{\sin \theta_3}$$

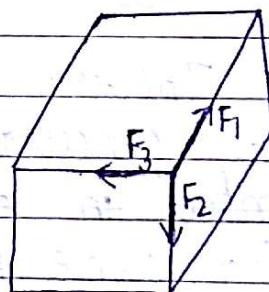
## Force System

It is the combination of different forces acting on a body.

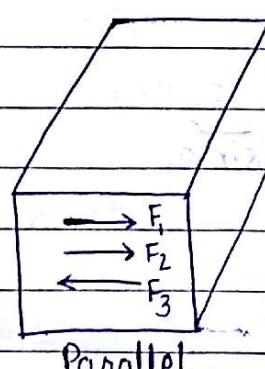
## Force System



Coplanar Concurrent  
F.S.

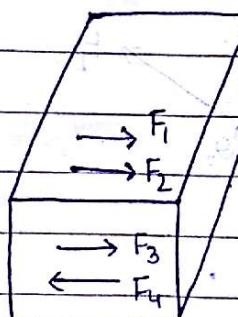


Non-coplanar concurrent  
(meeting at one pt.)  
F.S.

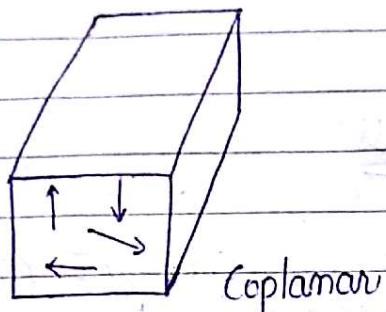


Parallel

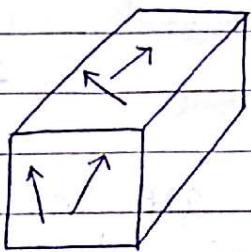
- $F_1$  &  $F_2$  parallel F.S
- $F_2$  &  $F_3$  non-parallel F.S



Non coplanar parallel forces



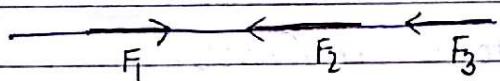
Non-concurrent &  
non-parallel



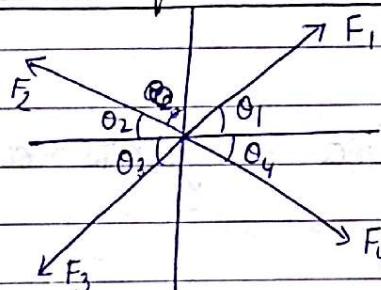
Non-everything

Collinear F.S.

Lie in one line

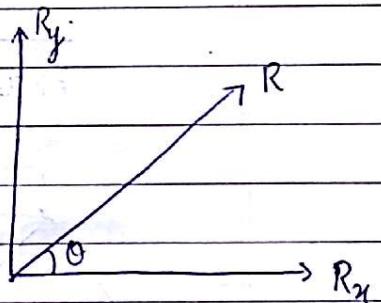


### Resultant of several concurrent Forces



$$R_x = F_{x_1} + F_{x_2} + F_{x_3} + F_{x_4}$$

$$R_y = F_{y_1} + F_{y_2} + F_{y_3} + F_{y_4}$$



$$F_{x_1} = F_1 \cos \theta_1$$

$$F_{x_2} = -F_2 \cos \theta_2$$

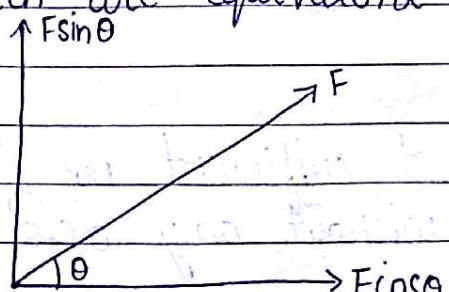
$$F_{x_3} = -F_3 \cos \theta_3$$

$$F_{x_4} = F_4 \cos \theta_4$$

$$R = \sqrt{R_x^2 + R_y^2}$$

### Resolution of Force

Process of breaking the force into a no. of forces which are equivalent to the given force.



24/01/17

Date: \_\_\_\_\_  
Page No. \_\_\_\_\_  
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## Unit-2

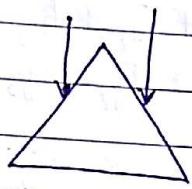
### Truss Truss

The <sup>Truss</sup> are the structures of straight members that are joined together at their ends by riveting & welding.

Load applied at joints only.

Truss is a <sup>true</sup> ~~false~~ force member.

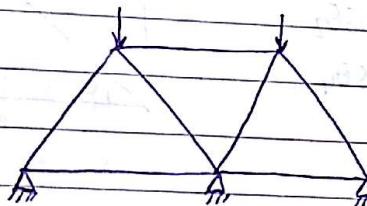
Eg.- Mobile tower, transmission tower



Frame

Load applied on joints as well as members.

3 members



7 Members  
5 joints

Mathematical cond" for perfect truss

$$m = 2j - 3$$

m = No. of member of truss

j = No. of joints

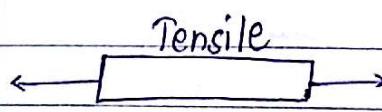
A truss which has just sufficient no. of members to resist the load without any deformation.

- $m > 2j - 3$  Over rigid / Redundant / statically indeterminate. However truss is stable we cannot solve the <sup>-ate</sup> eq<sup>n</sup>s of static eq<sup>m</sup>. Truss contains more member than required.
- $m < 2j - 3$  Under rigid / collapsible condition / Deficient truss. statically determinate, it means eq<sup>n</sup> of static eq<sup>m</sup> are sufficient to determine the axial force in the members of a truss.

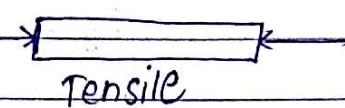
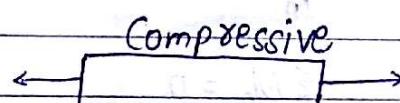
### Assumptions of a perfect truss

- Load applied at joints only negligibly
- Negligible weight of members
- Pin-joint connections
- Joints are frictionless
- Truss is statically determinate

### External Force

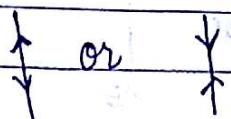


### Internal Force

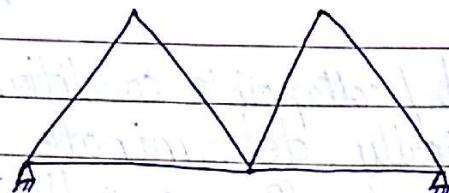


### ① Method of joints

- First of all find out the support r<sub>x</sub>'s  $R_A$  &  $R_B$ .
- Mark the internal forces in the members of the truss.



→ Eq<sup>m</sup> of each joint is considered. If the magnitude of force comes out to be -ve it means assumed dir<sup>n</sup> is simply reversed.



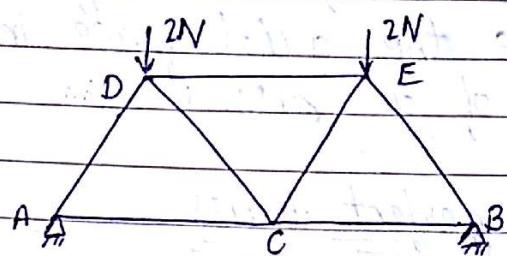
$$m = 2j - 3$$

$$6 = 2 \times 5 - 3$$

$$= 10 - 3$$

$$6 < 7$$

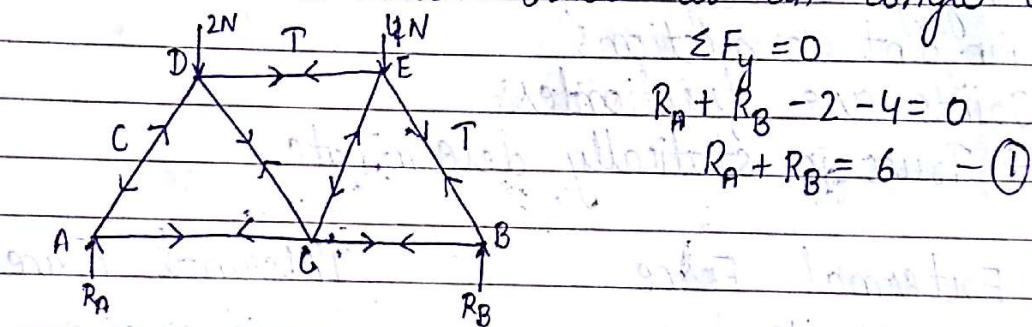
Q.



Find out the magnitude & nature of internal force in member DE.

Length of each member is 3m & every member is inclined to each other at an angle of 60°.

Sol<sup>n</sup>



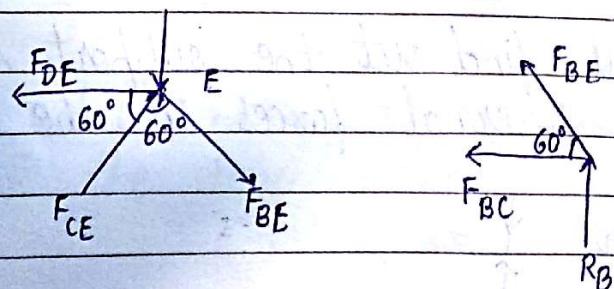
$$\sum M_A = 0$$

$$R_B \times 6 - 2 \times 1.5 - 4 \times 4.5 = 0$$

$$R_B = \frac{21}{6} = \frac{7}{2} = 3.5 N$$

$$R_A = 6 - 3.5$$

$$= 2.5 N$$



$$F_{BE} \sin 60^\circ = R_B$$

$$F_{BE} = R_B \times \frac{2}{\sqrt{3}}$$

$$= 4.04 N$$



$$F_{DE} = F_{CE} \cos 60^\circ + F_{BE} \cos 60^\circ$$

$$F_{DE} = 6.35 N$$

$$F_{CE} \sin 60^\circ = 4 + F_{BE} \sin 60^\circ$$

$$F_{DE} = 1.732$$

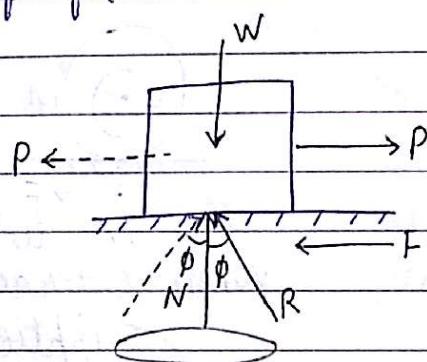
$$F_{CE} \sin 60^\circ = 4 + 3.5$$

$$F_{CE} = 8.66 N$$

## Friction

Friction force

Cone of friction



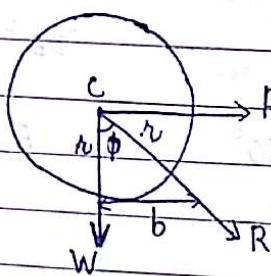
If R is within the cone, it is self locking condition  
i.e. the motion does not occur.

Angle of friction - Angle made by the resultant of normal reaction and frictional force with the normal reaction.

*Makna manha Revital*

Angle of repose - Min<sup>m</sup> angle of inclination of a plane with horizontal surface at which the body will just slide down on it without any external force.

### Rolling friction -



$$\tan \phi = \mu$$

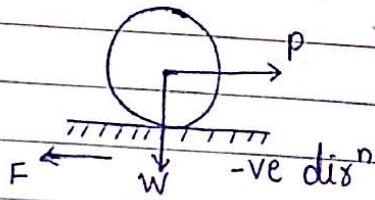
$$\sum M_o = 0$$

$$P \cdot r \cos \phi = W \cdot r \sin \phi$$

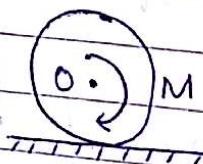
$$\frac{P}{W} = \tan \phi$$

$$\frac{P}{W} = \frac{b}{r} \Rightarrow \tan \phi = \frac{b}{r}$$

- i Force of friction on a wheel  
When a force is applied on a wheel



When ext. force is applied



When external torque is applied

31/01/17

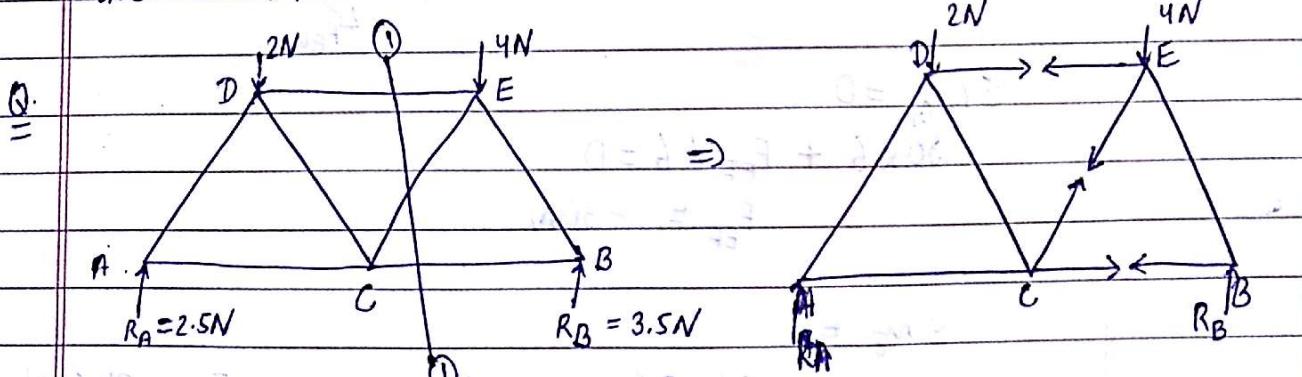
## Truss (cont.)

### Method of section

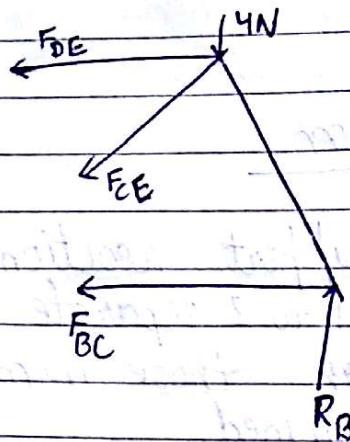
- 1.) Find out the support reaction.
- 2.) Cut the truss into 2 separate portions by passing a section through those members whose forces are to be determined.
- 3.) Internal force of the members which are cut by section become external force.
- 4.) Now every portion of truss is in equilibrium. We can apply eqn of eqm to solve the particular truss.

Note:

- Section should pass through members, not joints.
- A section should cut three members only.
- Consider either right hand portion or left hand portion after cutting.
- While using moment eqn moment can be taken about any point which may or may not lie on the portion of the truss under consideration.
- Do not consider the effects of uncut members in the truss.



Members length = 3m



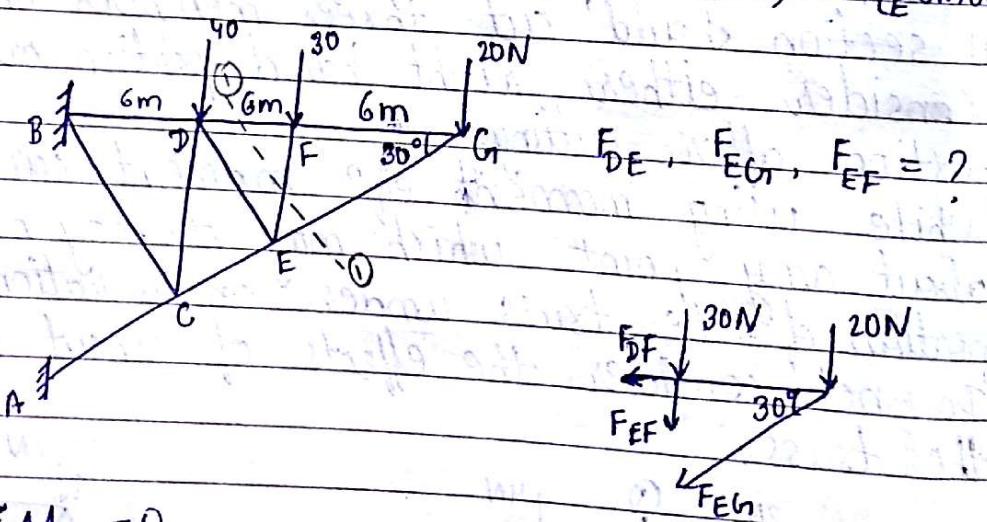
$$\sum M_c = 0$$

$$F_{DE} \times 2.6 - 4 \times 1.5 + 3.5 \times 3 = 0$$

$$F_{DE} = -1.73 N$$

$$\sum M_B = 0$$

$$+ 4 \times 1.5 + F_{DE} (2.6) + F_{CE} \cos 60^\circ (2.6) + F_{CE} \sin 60^\circ (1.5) = 0$$



$$\sum M_G = 0$$

$$30 \times 6 + F_{FE} \times 6 = 0$$

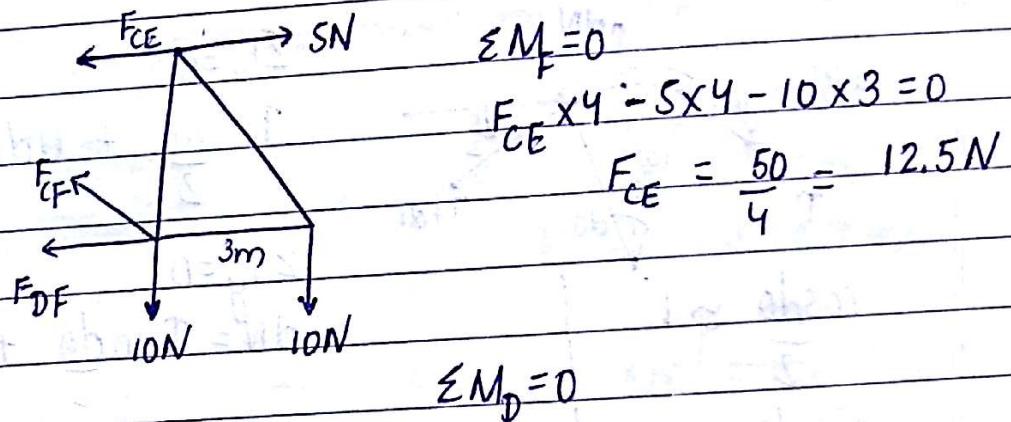
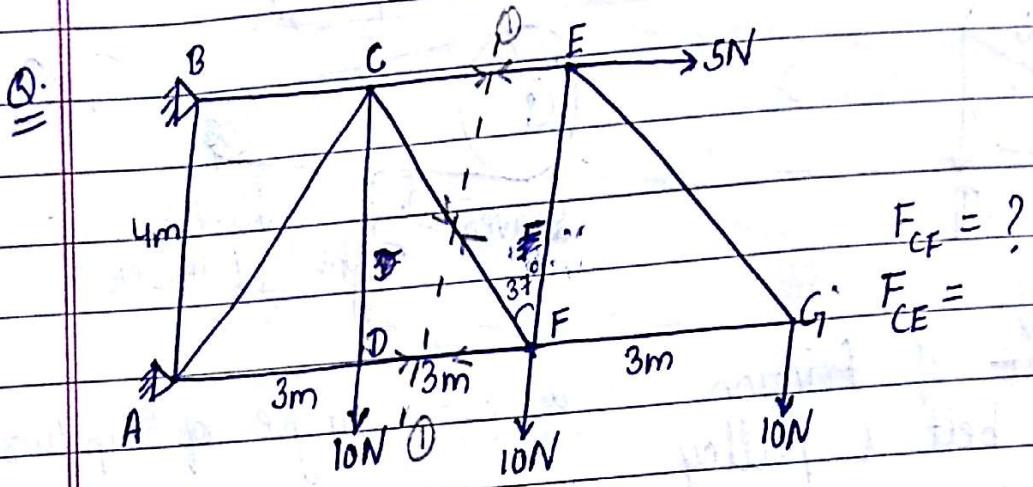
$$\underline{F_{EE}} = -30N$$

$$\sum M_F = 0$$

$$F_{DF} \times \frac{6 \tan 30^\circ - 20 \times 6}{\tan 30^\circ} = 0 \Rightarrow F_{DF} = 34.64N$$

$$F_{EG} \sin 30^\circ + 20 + 30 - 30 = 0$$

$$F_{EG} = -40 \text{ N}$$



$$\sum M_f = 0$$

$$F_{CE} \times 4 - 5 \times 4 - 10 \times 3 = 0$$

$$F_{CE} = \frac{50}{4} = 12.5 \text{ N}$$

$$\sum M_D = 0$$

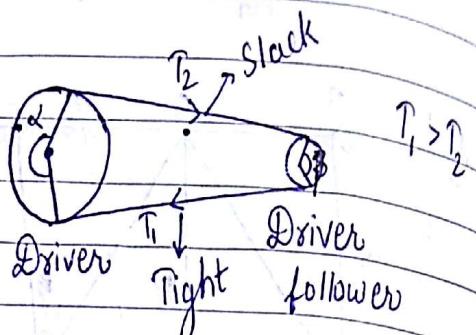
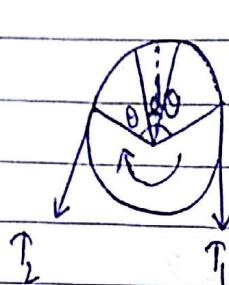
$$\sum F_y = 0$$

$$-10 - 10 + F_{CF} \cos 37^\circ = 0$$

$$F_{CF} = \frac{20}{\cos 37^\circ} = 25.04 \text{ N}$$

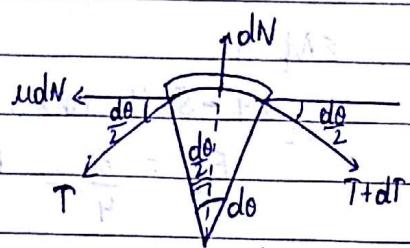
3/2/11

## Belt Friction : Ratio of Tensions



$\mu$  → Angle of friction  
b/w belt & pulley

$\alpha, \beta$  → Angle of lap/wrap



$$\sum F_x = 0$$

$$\frac{T \cos \alpha/2 + udN}{2} = \frac{(T+dT) \cos \alpha/2}{2}$$

$$\sum F_y = 0$$

$$dN = \frac{T \sin \alpha/2}{2} + \frac{(T+dT) \sin \alpha/2}{2} \quad \text{--- (1)}$$

$$\sin \alpha/2 \approx \frac{\alpha}{2}$$

$$T + udN = T + dT$$

$$udN = dT \quad \text{--- (2)}$$

$$dN = \frac{T d\alpha}{2} + \frac{(T+dT) d\alpha}{2}$$

$$dN = T \cdot \frac{d\alpha}{2} + dT \cdot \frac{d\alpha}{2}$$

negligible

$$dN = T d\alpha \quad \text{--- (3)}$$

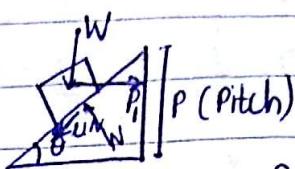
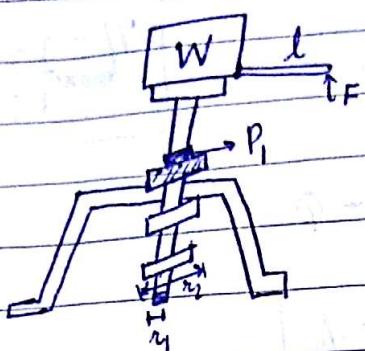
$$\mu \cdot T d\alpha = dT$$

$$\int_{T_1}^{T_2} \frac{dT}{T} = \int_0^{\alpha} \mu \cdot d\alpha \quad \text{--- (4)}$$

$$\ln \frac{T_1}{T_2} = \mu \alpha$$

$$T_1 = T_2 e^{\mu \alpha}$$

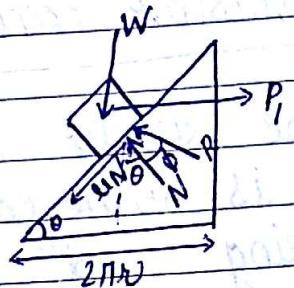
## Friction in a Screw Jack:



$P \rightarrow$  dist. b/w two consecutive threads

$P_1 \rightarrow$  Force applied on thread to raise the load

$r =$  Mean radius of  $r_1$  &  $r_2$



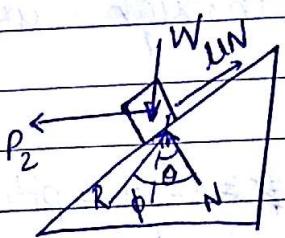
$$\sum F_x = 0 \quad P_1 = R \sin(\theta + \phi)$$

$$\sum F_y = 0 \quad W = R \cos(\theta + \phi)$$

$$P_1 = W \tan(\theta + \phi)$$

$R \rightarrow$  Resultant of Normal  $R_x^n$  & frictional force

To lower the load



$$P_2 = W \tan(\theta - \phi)$$

$$\eta = \frac{\text{Ideal Effort}}{\text{Actual Effort}} = \frac{\tan \theta}{\tan(\theta + \phi)}$$

In case of ideal friction is absent  
 $\phi = 0$

$$\frac{d\eta}{d\theta} = \frac{\sec^2 \theta \tan(\theta + \phi) - \sec^2(\theta + \phi) \tan \theta}{(\tan(\theta + \phi))^2} = 0$$

$$\Rightarrow \sec^2 \theta \tan(\theta + \phi) = \sec^2(\theta + \phi) \tan \theta$$

$$\frac{1}{\cos^2 \theta} \frac{\sin(\theta + \phi)}{\cos(\theta + \phi)} = \frac{1}{\cos^2(\theta + \phi)} \frac{\sin \theta}{\cos \theta}$$

$$2 \sin(\theta + \phi) \cos(\theta + \phi) = 2 \sin \theta \cos \theta$$

$$\sin 2(\theta + \phi) = \sin 2\theta$$

$$2(\theta + \phi) = \pi - 2\theta$$

$$\theta + \phi = \frac{\pi}{2} - \theta$$

$$\eta = \frac{1 - \sin \phi}{1 + \sin \phi}$$

$$\# = \# 2\theta = \frac{\pi}{2} - \phi$$

$$\theta = \frac{\pi - \phi}{2}$$

Self Locking or non-reversible condition

When load  $W$  on the screw remains in place even if effort is removed i.e. effort = 0 that is called self locking cond".

- ①  $\phi' > 0$
- ②  $\eta < 50\%$

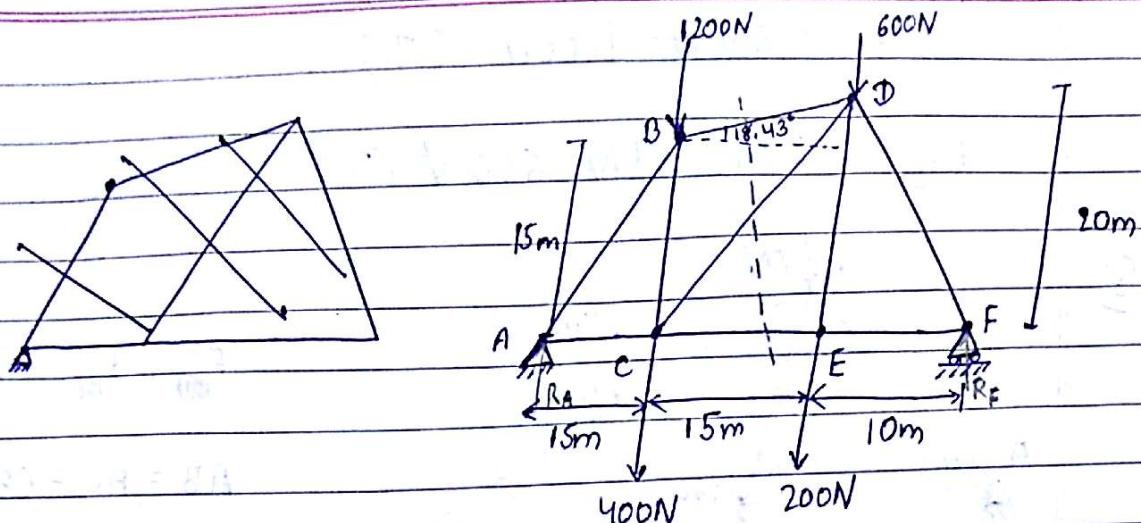
Lead is the axial dist. through which screw advance in one turn.

In single threaded screw, Lead = pitch

In double threaded - After 2 lead = 1 pitch

In triple threaded - 3 lead = 1 pitch

7/02/17



$$F_{BD} =$$

$$F_{CD} =$$

$$F_{CE} =$$

$$\sum F_y = 0$$

$$R_A + R_F = 1200 + 600 + 400 + 200 \\ = 2400 \text{ N}$$

$$\sum M = 0$$

$$-1200 \times 15 - 400 \times 15 - 200 \times 30 - 600 \times 30 + R_F \times 40 = 0$$

$$R_F = 1200 \text{ N}$$

$$R_A + R_F = 2400 \text{ N}$$

$$\Rightarrow R_A = 1200 \text{ N}$$

$$\sum F_y = 0$$

$$-400 + 200 - 1200 + F_{CD} \times \frac{4}{5} + F_{BD} \times 0.316 = 0$$

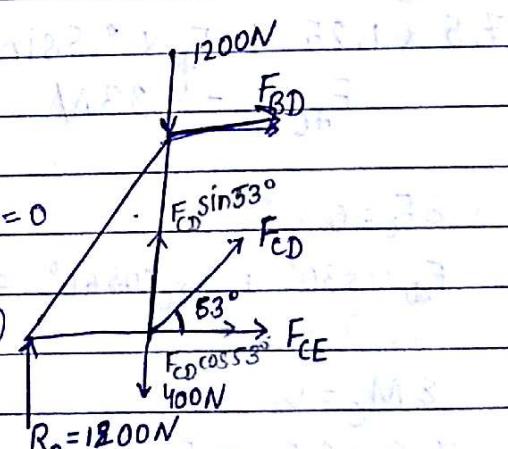
$$F_{CD} \cdot \frac{4}{5} + F_{BD} \sin(18.43^\circ) = 0 \quad \text{--- (1)}$$

$$\sum F_x = 0$$

$$F_{CD} \cos 53^\circ + F_{CE} + F_{BD} \cos 18.43^\circ = 0 \quad \text{--- (2)}$$

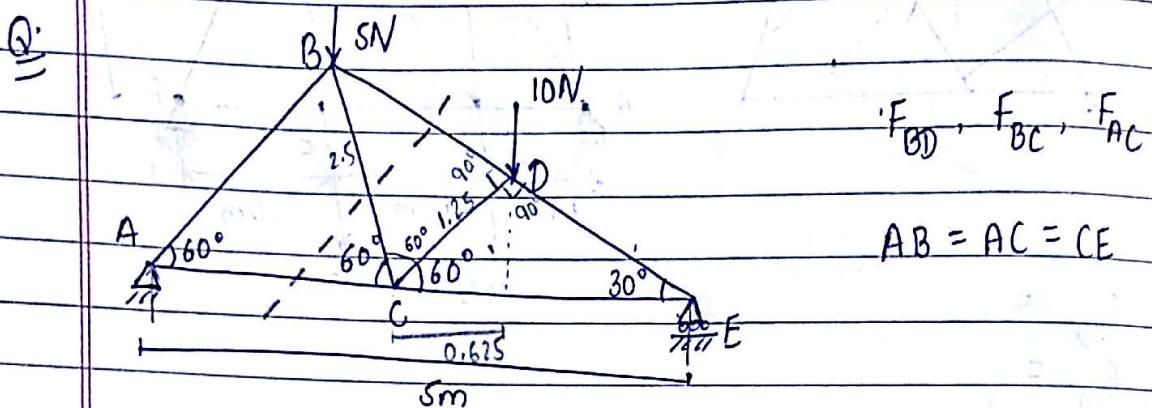
$$\sum M_c = 0$$

$$1200 \times 15 + F_{BD} \cos 18.43^\circ \times 15 = 0 \\ \Rightarrow F_{BD} = -3795.7 \text{ N} - 1265 \text{ N (C)}$$



$$F_{CD} = +1500N \quad 1000N \quad (T)$$

$$F_{CE} = 2698.29 N \quad 600 N (T)$$



$$R_A + R_E - 5 - 10 = 0$$

$$R_A + R_E = 15 N$$

$$t5 \times 1.25 + 31.25 - R_E \times 5 = 0$$

$$R_E = 7.5N$$

$$\Rightarrow R_A = 7.5 N$$

$$\sum M_a = 0$$

$$7.5 \times 1.25 + F_0 \times 2.5 \sin 60^\circ$$

$$F_{AC} = -4.33 N$$

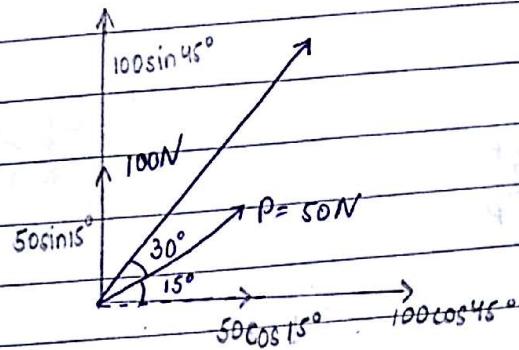
$$\sum F_x = 0$$

$$F_{B_1} \cos 30^\circ + F_{B_2} \cos 60^\circ = 4.33$$

$$\sum M_r = 0$$

$$7.5 \times 2.5 - 5 \times 2.5$$

Ex 2.1

2.2

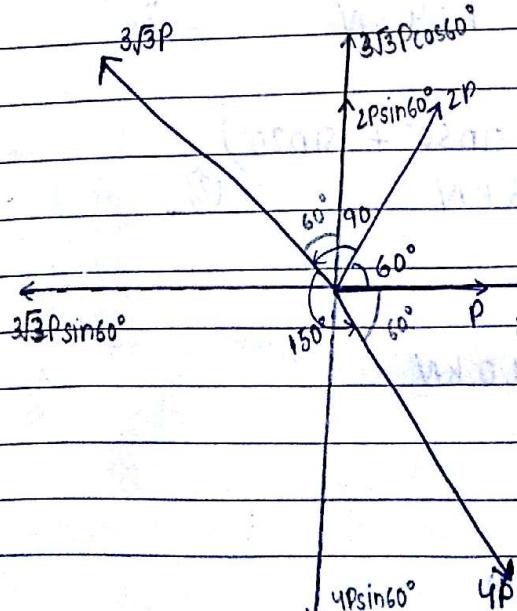
$$\begin{aligned}\sum F_x &= 50\cos 15^\circ + 100\cos 45^\circ \\ &= -83.65 \text{ N} + 119 \text{ N}\end{aligned}$$

$$\begin{aligned}\sum F_y &= 50\sin 15^\circ + 100\sin 45^\circ \\ &= 83.65 \text{ N}\end{aligned}$$

$$\begin{aligned}R &= \sqrt{(F_x)^2 + (F_y)^2} \\ &= \sqrt{(119)^2 + (83.65)^2} \\ &= 145.46 \text{ N}\end{aligned}$$

$$\tan \alpha = \frac{F_y}{F_x} = \frac{0.703}{1} = 0.703$$

$$= 35.1^\circ$$

2.3

$$\begin{aligned}\sum F_x &= P + 2P\cos 60^\circ + 4P\cos 60^\circ \\ &\quad - 3\sqrt{3}P\sin 60^\circ\end{aligned}$$

$$= P \left[ 1 + 2 \times \frac{1}{2} + 4 \times \frac{1}{2} \right]$$

$$- 3\sqrt{3} \times \frac{\sqrt{3}}{2}$$

$$= -P/2$$

$$\begin{aligned}\sum F_y &= 3\sqrt{3}P\cos 60^\circ + 2P\sin 60^\circ \\ &\quad - 4P\sin 60^\circ\end{aligned}$$

$$= \frac{\sqrt{3}P}{2}$$

$$R = \sqrt{\left(\frac{-P}{2}\right)^2 + \left(\frac{\sqrt{3}P}{2}\right)^2}$$

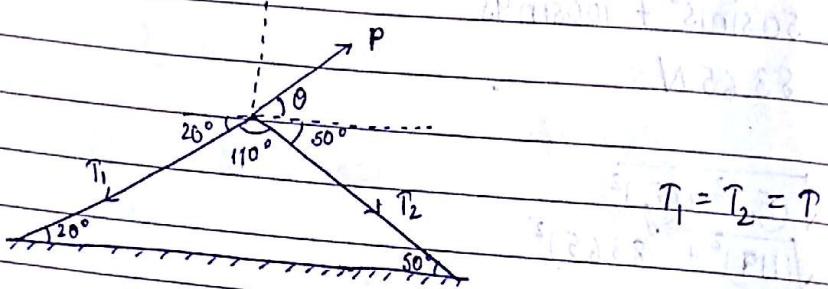
$$= \sqrt{\frac{4P^2}{4}} = P$$

$$\tan \theta = -\frac{\sqrt{3}/2}{1/2} = -\sqrt{3}$$

$$\Rightarrow \theta = \pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3} = 120^\circ$$

2.4



$$\sum F_x = P \cos \theta + T_2 \cos 50^\circ + T_1 \cos 20^\circ = 0$$

$$\Rightarrow P \cos \theta + 2T_1 \cdot 1.58 = 0$$

$$\Rightarrow P \cos \theta = -r$$

$$\sum F_x = 0 \Rightarrow P \cos \theta + T_1 (\cos 50^\circ - \cos 20^\circ) = 0$$

$$P \cos \theta = 1.04 \text{ kN} \quad \text{--- (1)}$$

$$\sum F_y = 0 \Rightarrow P \sin \theta = T_1 (\sin 50^\circ + \sin 20^\circ)$$

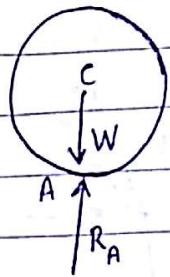
$$P \sin \theta = 3.88 \text{ kN} \quad \text{--- (2)}$$

$$\tan \theta = 3.73$$

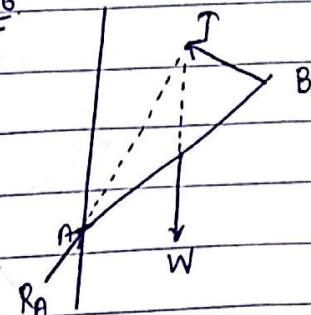
$$\theta = 75^\circ$$

$$P = \frac{3.88}{\sin 75^\circ} = 4.0 \text{ kN}$$

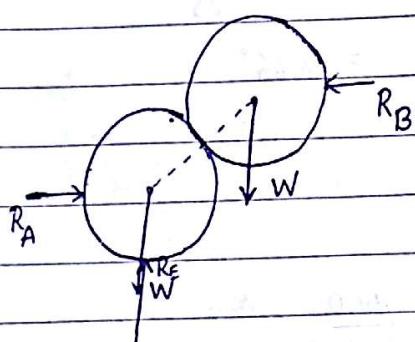
2.5



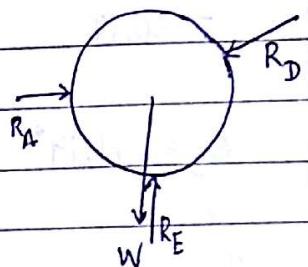
2.6



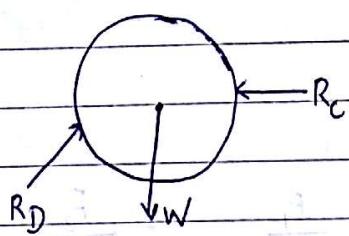
2.7) a)



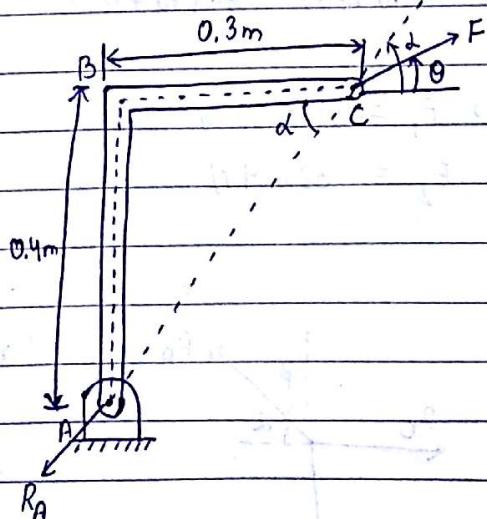
b) P



c)



2.8

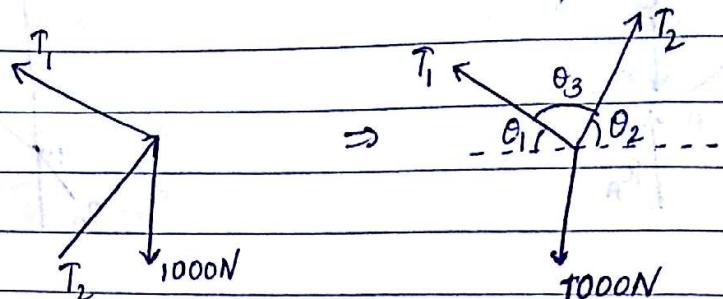


$$\theta = \alpha$$

$$\tan \alpha = \frac{0.4}{0.3} = \frac{4}{3}$$

$$= 53.13^\circ$$

2.9



$$\theta_1 = \tan^{-1} \frac{30}{25}$$

$$= 50.19^\circ$$

$$\theta_2 = \tan^{-1} \frac{20}{25}$$

$$= 38.66^\circ$$

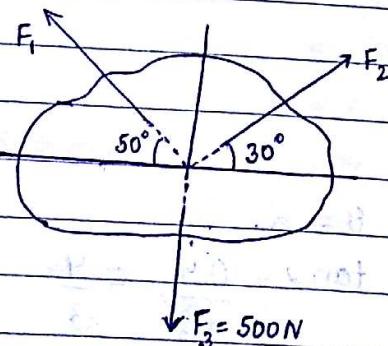
$$\theta_3 = 91.14^\circ$$

$$\frac{T_1}{\sin(90 + \theta_2)} = \frac{T_2}{\sin(90 + \theta_1)} = \frac{1000}{\sin \theta_3}$$

$$T_1 = 780$$

$$T_2 = 640$$

2.10

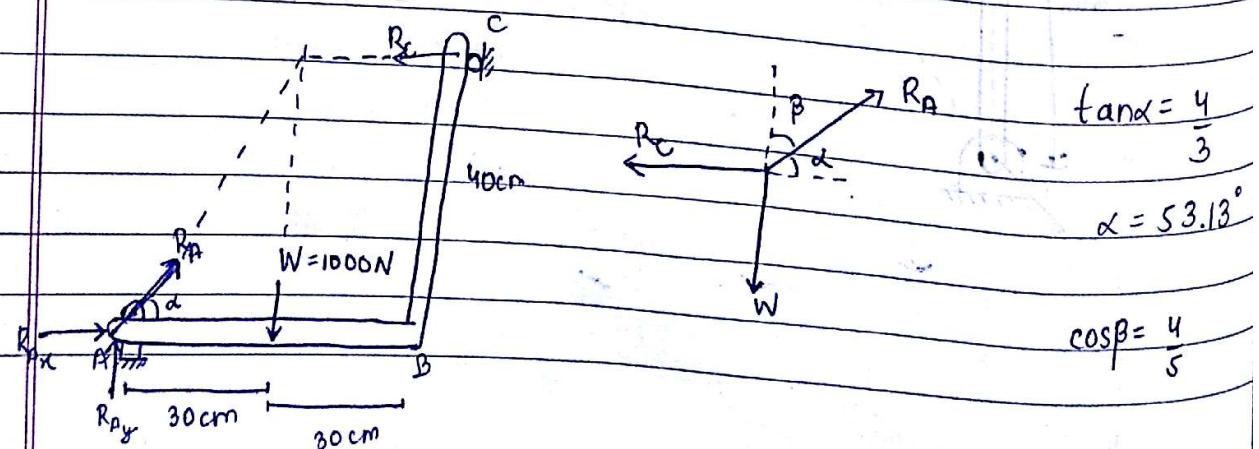


$$\frac{F_1}{\sin(90^\circ + 30^\circ)} = \frac{F_2}{\sin(90^\circ + 50^\circ)} = \frac{500}{\sin 100^\circ}$$

$$\Rightarrow F_1 = 439.6 \text{ N}$$

$$F_2 = 326.4 \text{ N}$$

2.12



$$\tan \alpha = \frac{4}{3}$$

$$\alpha = 53.13^\circ$$

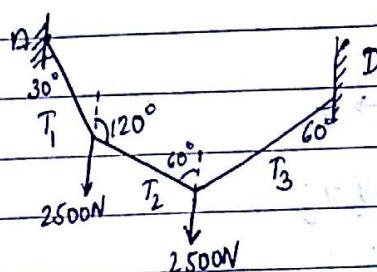
$$\cos \beta = \frac{4}{5}$$

$$\frac{W}{\sin(90 + 36.87)} = \frac{R_A}{\sin 90} - \frac{R_C}{\sin(90 + 53.13)}$$

$$R_A = \frac{1000 \times 5}{4} = 1250 \text{ N}$$

$$R_C = \frac{1000 \times 5}{4} \times \frac{3}{5} \\ = 750 \text{ N}$$

2.13



$$\frac{T_1}{\sin 60^\circ} = \frac{T_2}{\sin(150^\circ)} = \frac{2500}{\sin 150^\circ}$$

$$\Rightarrow T_2 = 2500$$

$$T_1 = 4330.13 \text{ N}$$

$$\frac{T_2}{\sin 120^\circ} = \frac{T_3}{\sin 120^\circ} = \frac{2500}{\sin 120^\circ}$$

$$\Rightarrow T_2 = T_3 = 2500 \text{ N}$$

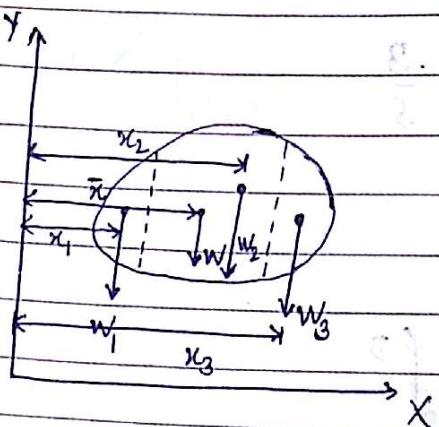
Happy Valentine's  
Day Cadbury ☺

14/2/17

Date: \_\_\_\_\_  
Page No. \_\_\_\_\_

## Centre of Gravity

Centroid: Geometrical centre of plane body  
COG → Centre of Gravity → 3D Bodies



$$W\bar{x} = w_1x_1 + w_2x_2 + w_3x_3$$

$$\bar{x} = \frac{w_1x_1 + w_2x_2 + w_3x_3}{W}$$

(1)

$$\bar{x} = \frac{w_1x_1 + w_2x_2 + \dots + w_nx_n}{w_1 + w_2 + \dots + w_n}$$

$$W = mg$$

$$W = \rho V g$$

$$W = \rho g V$$

$$= \rho g A t$$

$$W_1 = \rho g t_1 A_1$$

$$W_2 = \rho g t_2 A_2$$

⋮

$$W_n = \rho g t_n A_n$$

(2)

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + \dots + A_nx_n}{A_1 + A_2 + \dots + A_n}$$

$$W = \rho A g l$$

$$W_1 = \rho A g l_1$$

$$W_2 = \rho A g l_2$$

!

$$W_n = \rho A g l_n$$

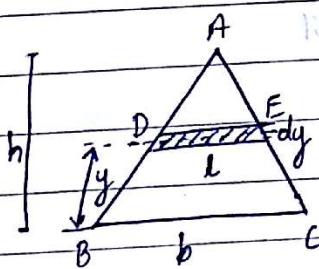
$$(3) \bar{x} = \frac{l_1 x_1 + l_2 x_2 + \dots + l_n x_n}{l_1 + l_2 + \dots + l_n}$$

$$\bar{x} = \frac{\int x dl}{\int dl}, \quad \bar{y} = \frac{\int y dl}{\int dl}$$

$$\bar{x} = \frac{\int x dA}{\int dA}, \quad \bar{y} = \frac{\int y dA}{\int dA}$$

$dA$  - small area under consideration.

(#)



$$\bar{y} = \frac{\int y dA}{\int dA}$$

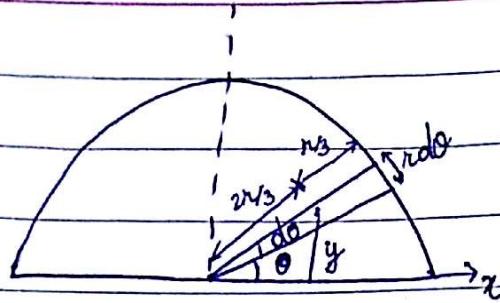
$$\bar{y} = \frac{\int y l dy}{\int l dy}$$

$$\bar{y} = \frac{b}{h} \int_{0}^h (hy - y^2) dy$$

$$= \frac{b}{h} \int_{0}^h (h-y) dy$$

$$= \frac{\frac{hy^2}{2} - \frac{y^3}{3}}{h} \Big|_0^h = \frac{\frac{h^3}{2} - \frac{h^3}{3}}{h^2 - \frac{h^2}{2}} = \frac{\frac{h^3}{2} - \frac{h^3}{3}}{\frac{h^2}{2}} = \frac{h}{3}$$

#



$$\bar{y} = \frac{\int y dA}{\int dA}$$

$$y = \frac{2r \sin \theta}{3}$$

$$dA = \frac{1}{2} r^2 d\theta$$

$$= \frac{\int_0^\pi \frac{2r \sin \theta}{3} \cdot \frac{1}{2} r^2 d\theta}{\int_0^\pi \frac{1}{2} r^2 d\theta}$$

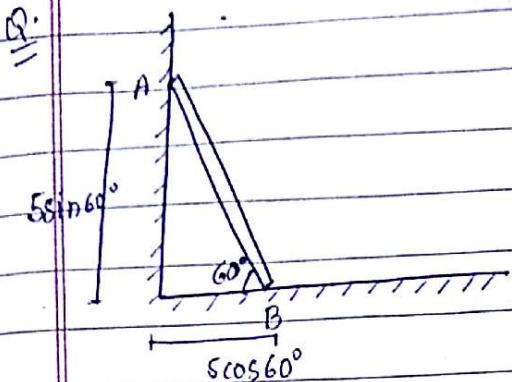
(Semi-circular arc  
or wire)

Disc

$$= \frac{2r}{3} \int_0^\pi \frac{\sin \theta d\theta}{\int_0^\pi d\theta}$$

$$= \frac{2r}{3} \left[ -\cos \theta \right]_0^\pi$$

$$= \frac{4r}{3\pi}$$



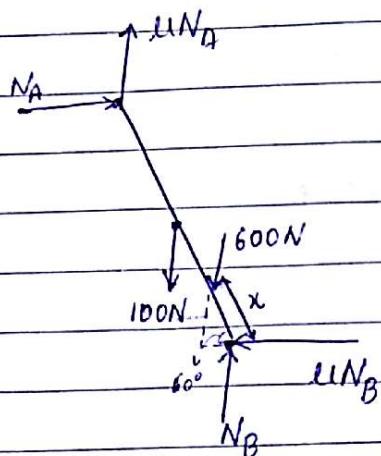
$$AB = 5\text{m}$$

Weight of ladder = 100N

Weight of man = 600N

$$\mu = 0.25$$

Determine how much distance along the ladder a man of 600N can ascend w/o causing it to slip.



$$F_x = 0$$

$$N_A - \mu N_B = 0$$

$$N_A = \mu N_B \quad \text{---(1)}$$

$$F_y = 0$$

$$\mu N_A + N_B - 700 = 0$$

$$\mu N_A + N_B = 700$$

$$N_A = 164.7\text{N}$$

$$\sum M_B = 0$$

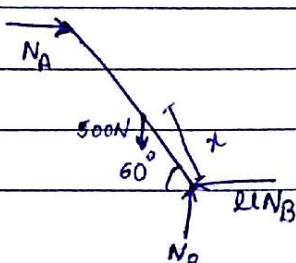
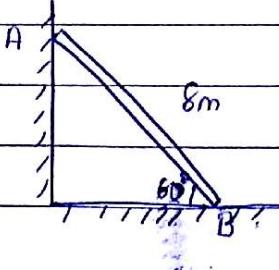
$$N_A \cdot 5\sin 60^\circ + \mu N_A \cdot 5\cos 60^\circ - 100 \cdot 2.5\cos 60^\circ - 600 \cdot x \cos 60^\circ = 0$$

$$\mu (N_B \times 5\sin 60^\circ +$$

$$x = 2.3\text{m}$$

- Q. A weightless ladder is resting against smooth vertical wall. Coeff. of friction is 0.25. A man of weight 500N wants to climb up the ladder w/o slip. A second person of wt. 800N wants to climb the same ladder. Would he climb less than the earlier person? Find the distance covered. L = 8m

Sol<sup>2</sup>



$$N_A = \mu N_B$$

$$N_B = 500 \cancel{x} \text{N}$$

$$N_A = 12.5\text{N}$$

$$N_A \times 8 \sin 60^\circ = 500 \times x \cos 60^\circ$$

$$x = 3.46 \text{ m}$$

For 2nd man

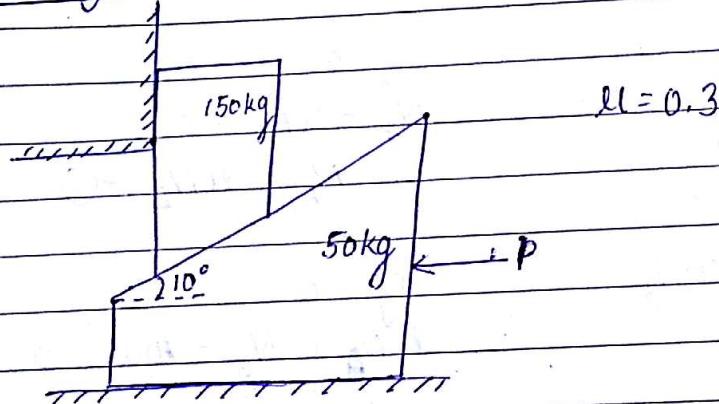
$$N_B = 800 \text{ N} \quad N_A = 200 \text{ N}$$

$$200 \times 8 \sin 60^\circ = 800 \times x \cos 60^\circ$$

$$x = 3.46 \text{ m}$$

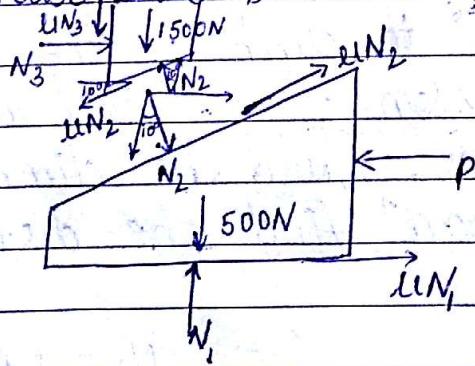
Wedge

(3)



A taper shaped block with very less angle which are used for lifting or holding or shifting the heavy block by very less effort is called wedge.

(Q3) Find the min<sup>n</sup> force applied at wedge to raise the block.



$$F_x = 0$$

$$\mu N_1 + \mu N_2 \cos 10^\circ + N_2 \sin 10^\circ = P$$

$$F_y = 0$$

$$N_1 = 500 + \mu N_2 \cos 10^\circ - \text{②}$$

Block

$$N_3 - \mu N_2 \cos 10^\circ - N_2 \sin 10^\circ = 0$$

$$N_2 \cos 10^\circ = 150g + \mu N_3 + \mu N_2 \sin 10^\circ$$

$$N_3 = \frac{N_2 \cos 10^\circ - 150g - \mu N_2 \sin 10^\circ}{\mu}$$

$$N_2 \left( \frac{\cos 10^\circ}{\mu} - \mu \cos 10^\circ - \sin 10^\circ \right) = \frac{150g}{\mu}$$

$$N_2 = \frac{5000}{2.8 + 2.6399}$$

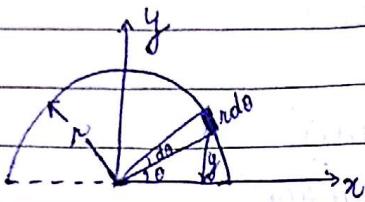
$$= -1777.08N + 1673.78A / 18.56.13$$

$$N_3 = -833.62N + 785.15$$

1

(A)

Semicircular wire/arc



$$dl = r d\theta$$

$$\text{length of wire} = l$$

$$\pi r = l$$

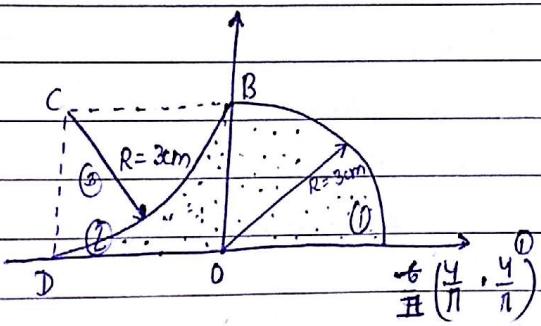
$$r = \frac{l}{\pi}$$

$$\bar{y} = \frac{\int y dl}{\int dl}$$

$$= \frac{\int_0^\pi r \sin \theta r d\theta}{\int_0^\pi r d\theta}$$

$$= \frac{r [-\cos \theta]_0^\pi}{\pi} = \frac{2r}{\pi}$$

(B)

Find CG<sub>x</sub> of shaded portion.

$$\frac{1}{2} \left( \frac{4}{\pi} + \frac{3}{2} \right)$$

$$x_1 = \frac{4}{\pi}, x_2 = -\frac{3}{2}$$

$$y_1 = \frac{4}{\pi}, y_2 = \frac{3}{2}$$

$$A_1 = 7.06, A_2 = 9$$

$$x_3 = -(3 - \frac{4}{\pi})$$

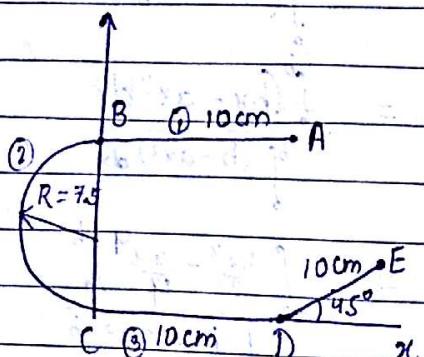
$$y_3 = 3 - \frac{4}{\pi}$$

$$A_3 = 7$$

$$CG_x = \frac{7.06 \times \frac{4}{\pi} + 9 \left( -\frac{3}{2} \right) - 7.06 \left( -3 + \frac{4}{\pi} \right)}{+\frac{4}{\pi} + 9 - 7.06} = 0.3260856$$

$$CG_y = \frac{7.06 \times \frac{4}{\pi} + 9 \times \left( \frac{3}{2} \right) - 7.06 \left( 3 - \frac{4}{\pi} \right)}{+\frac{4}{\pi} + 9 - 7.06} = 1.147$$

Q: A uniform wire is bent into a shape as shown in figure. Calculate the CG of wire.



AB

$$x_1 = 5 \text{ cm} \quad y_1 = +10 \text{ cm} - 15 \text{ cm}$$

$$l_1 = 10 \text{ cm}$$

BC

$$x_2 = -\frac{2r}{\pi} \quad y_2 = 7.5$$

CD

$$l = \pi \times 7.5$$

$$x_3 = 5 \text{ cm} \quad y_3 = 0$$

$$l_3 = 10 \text{ cm}$$

DE

$$x_4 = 10 + 5 \cos 45^\circ \quad y_4 = 5 \sin 45^\circ$$

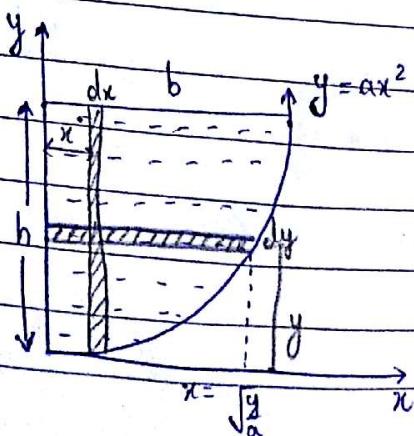
$$l_4 = 10 \text{ cm}$$

$$x_{CG} = \frac{5 \times 10 - 2 \times \frac{(7.5) \times \pi \times 7.5}{\pi} + 5 \times 10 + 10 \times (10 + 5 \cos 45^\circ)}{30 + \pi \times 7.5}$$

$$= 2.29 \text{ cm}$$

$$y_{CG} = \frac{15 \times 10 + 7.5 \times \pi \times 7.5 + 10 \times 5 \sin 45^\circ}{30 + \pi \times 7.5}$$

$$= 6.23 \text{ cm} \quad 6.75 \text{ cm}$$

Q:

$$dA = \sqrt{y} \times dy$$

$$\bar{y} = \frac{\int_0^b y \sqrt{\frac{y}{a}} dy}{\int_0^b \sqrt{\frac{y}{a}} dy} = \frac{\frac{3}{5} \int_0^b y^{3/2} dy}{\frac{2}{3} \int_0^b y^{1/2} dy} = \frac{\frac{3}{5} y^{5/2} \Big|_0^b}{\frac{2}{3} y^{3/2} \Big|_0^b} = \frac{3}{5} \frac{b^{5/2}}{y^{3/2}}$$

$$dA = (b - ax^2) dx$$

$$\bar{x} = \frac{\int_0^b ax^3 dx}{\int_0^b ax^2 dx}$$

$$= \frac{\frac{x^4}{4} \Big|_0^b}{\frac{x^3}{3} \Big|_0^b}$$

$$= \frac{3b}{4}$$

$$\int_0^b (b - ax^2)x dx$$

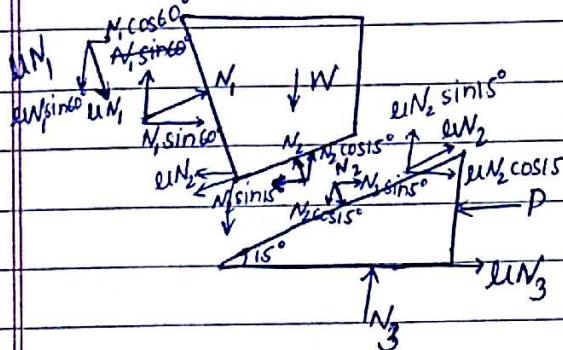
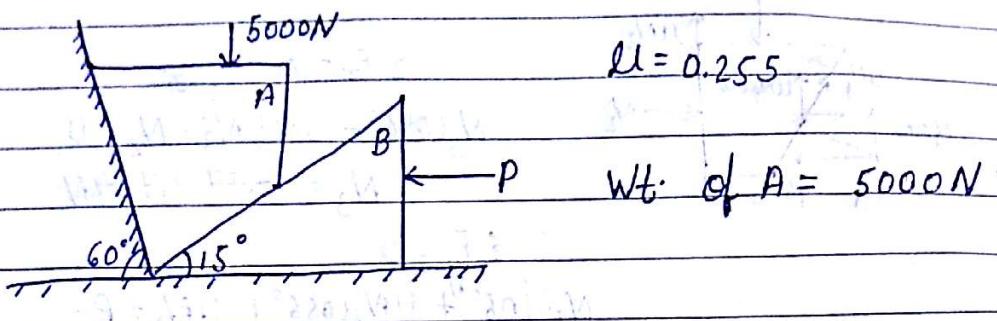
$$= \frac{\int_0^b (bx - ax^3) dx}{\int_0^b (b - ax^2) dx}$$

$$= \frac{\left[ \frac{bx^2}{2} - \frac{ax^4}{4} \right]_0^b}{\left[ bx - \frac{ax^3}{3} \right]_0^b}$$

$$= \frac{\frac{hb^2}{2} - \frac{ab^4}{4}}{\left[ bb - \frac{ab^3}{3} \right]}$$

$$= \frac{3b}{4} \left[ \frac{2b - ab^2}{3b - ab^2} \right]$$

(#)



Block A

$$\sum F_x = 0$$

$$+\mu N_1 \cos 60^\circ + N_1 \sin 60^\circ - \mu N_2 \cos 15^\circ - N_2 \sin 15^\circ = 0$$

$$N_1 = \frac{\mu N_2 \cos 15^\circ + N_2 \sin 15^\circ}{(\sin 60^\circ + \mu \cos 60^\circ)}$$

$$\sum F_y = 0$$

$$\mu N_1 \sin 60^\circ + N_1 \cos 60^\circ + N_2 \cos 15^\circ - W - N_2 \sin 15^\circ = 0$$

$$-N_2 (0.9987) = 5000 - N_2 (1.07359) = 5000$$

$$-N_2 = 5006.3N \quad N_2 = 4657.27N$$

$$\Rightarrow N_1 = 2920N \quad N_1 = 2367.859N \quad (\times)$$

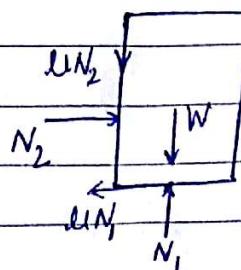
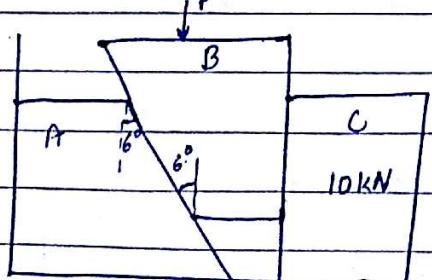
Block B

$$\sum F_x = 0$$

$$N_2 (\sin 15^\circ + \mu \cos 15^\circ) + \mu N_3 = P$$

$$N_3 = N_2 (\cos 15^\circ + \mu \sin 15^\circ) \\ = 4191.2 \quad (\times)$$

Q.

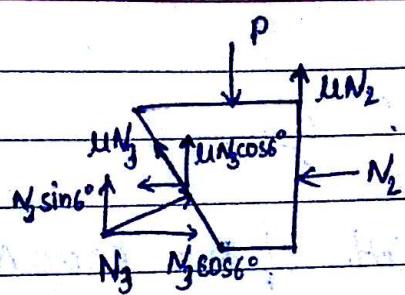


$$N_2 = \mu N_1$$

$$\mu N_2 + W = N_1$$

$$\mu^2 N_1 \\ W = N_1 (\mu^2 - 1) \\ (1 - \mu^2) N_1$$

$$N_2 = 2.67 \text{ KN} \quad N_1 = 10.67 \text{ KN}$$



$$\sum F_x = 0, \sin 6^\circ$$

$$N_3 \cos 6^\circ - 0.25 N_3 - N_2 = 0$$

$$N_3 = 2.57 \text{ kN}$$

$$\sum F_y = 0$$

$$N_3 \sin 6^\circ + \mu N_3 \cos 6^\circ + \mu N_2 = P$$

$$P = 1.64 \text{ kN}$$