

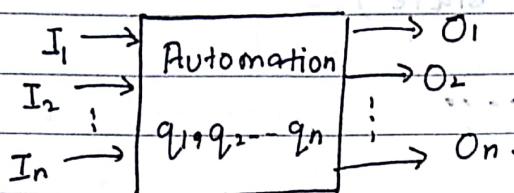
Theory Of Computation

Page No.

Date: / /

→ (TOC)

- Physical system in which energy, material & information are transformed, transmitted & used for performing some function without participation of man.



$\Sigma \rightarrow$ finite & Non empty set of symbols.

- Basic Concept :- ① alphabet - $\{0, 1\}$.

$\{A, B, \dots, Z, a, b, \dots, z\}$.

- ② String - $\{0, 1\}$

$$\begin{aligned}
 \text{imp} \quad \Sigma^+ &= \bigcup_{i=1}^{\infty} \Sigma^i \\
 * &= \bigcup_{i=0}^{\infty} \Sigma^i = \Sigma \\
 &= \bigcup_{i=0}^{\infty} \Sigma^i = \{\Sigma\}
 \end{aligned}$$

- DFA :- finite state Determinate state finite automata?

= state 1st $M = \{Q, \Sigma, \delta, q_0, F\}$.

Q = set of states.

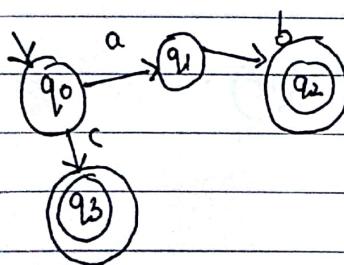
Σ = set of input alphabets.

δ = transition function.

q_0 = initial state.

F = set of final states.

$\delta: (Q \times E) \rightarrow Q$.



$$\begin{aligned}
 \delta(q_0, a) &= q_1 \\
 \delta(q_0, c) &= q_3
 \end{aligned}$$

(Task → create NFA & DFA).

Page No.

Date: / /

→ DFA \Rightarrow if $\delta: (\Omega \times E) \rightarrow Q^1$ (only single 0/p).

→ NFA \Rightarrow if $\delta: (\Omega \times E) \rightarrow 2^Q$ (Power set).

(Non-Determinant Finite State).

PROPERTIES...

① $|\lambda| = 0$.

② $\lambda w = w\lambda$.

③ $|a| = 1$.

④ $|wta| = |w| + 1$.

⑤ $a^R = a$.

⑥ $(wa)^R = a \cdot w^R$.

⑦ $\delta(q, \lambda) = q$.

state can be changed only by an I/P symbol.

⑧ $(wa)^R = q w^R$.

⑨ $\delta(q, a w) = \delta(\delta(q, a), w)$.

⑩ $\delta(q, wa) = \delta(\delta(q, w), a)$.

I/P.

$\rightarrow (110101) \rightarrow$ data is acceptable

q₀ = initial state =

final state.

⑤ 0 1
q₀ ————— q₂ q₁

$\delta(q_0, 1101, 1)$

$\delta(q_1, 10101)$

$\delta(q_0, 0101)$

q₂ q₀ q₀ q₀ $\delta(q_2, 101)$

$\delta(q_0, 01)$

$\delta(q_2, 1)$

$\delta(q_0)$

10/01/18

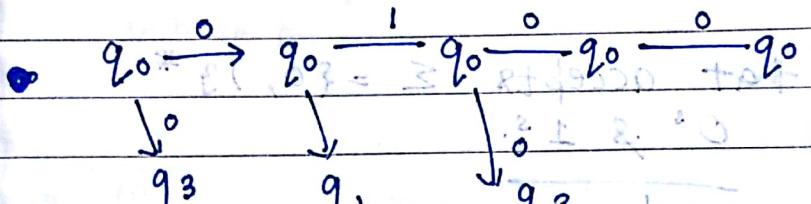
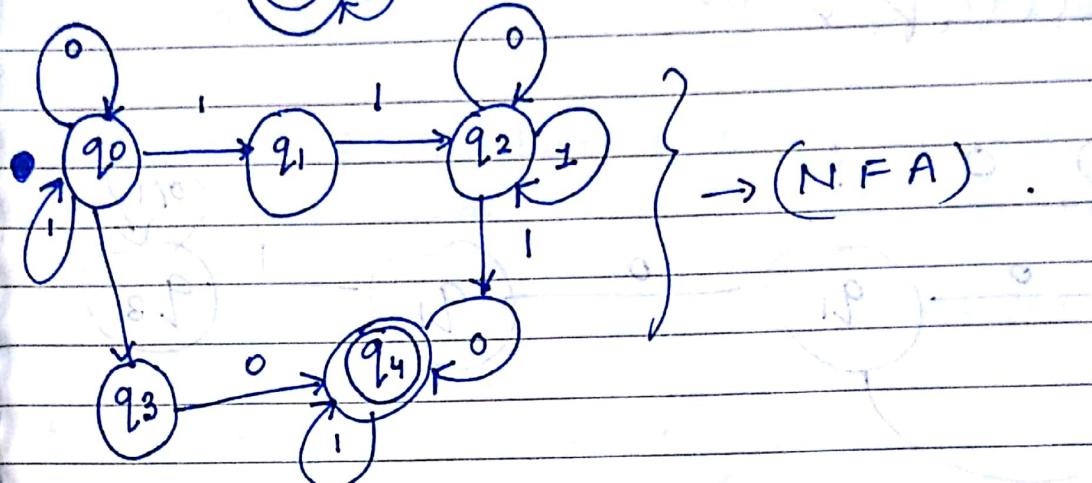
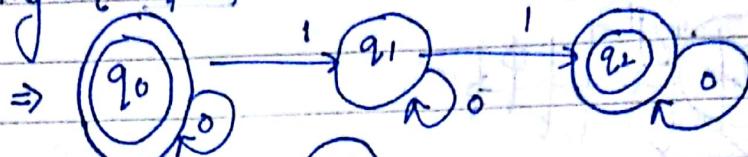
TOC

Page No.

Date: 10/01/18

→ Design a DFA that accepts all strings of $\Sigma = \{0, 1\}$ with even no. of one's.

→ String = $\{0, 11, 00, 1111\}$.



$$\delta(q_0, 0100) = \{q_0, q_2, q_4\}.$$

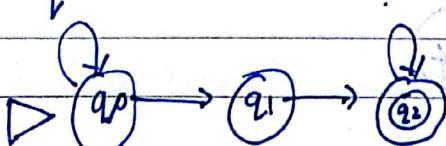
NFA :-

→ $\alpha \times (\Sigma \cup \lambda) \rightarrow 2^Q$ is the transition function.

→ Back-track is allowed.

→ double move allowed.

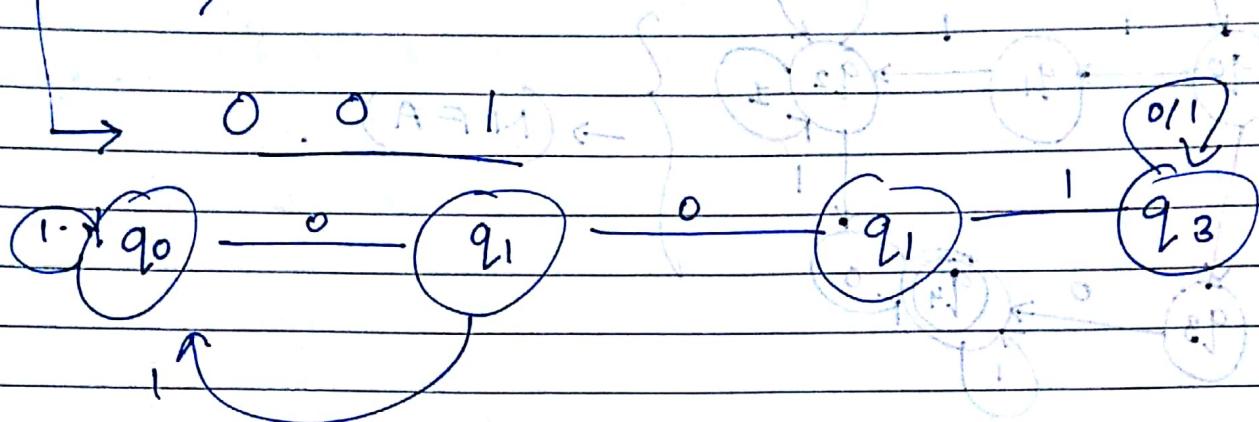
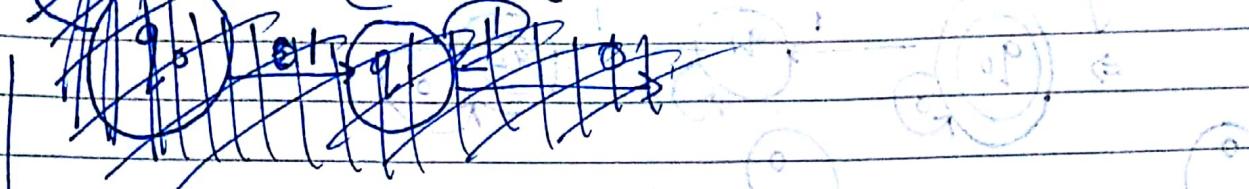
q (lambda - closure); — without giving any input to move to next state!



Design a DFA that accepts all strings of $\Sigma = \{0, 1\}^*$ with 001 as substring.

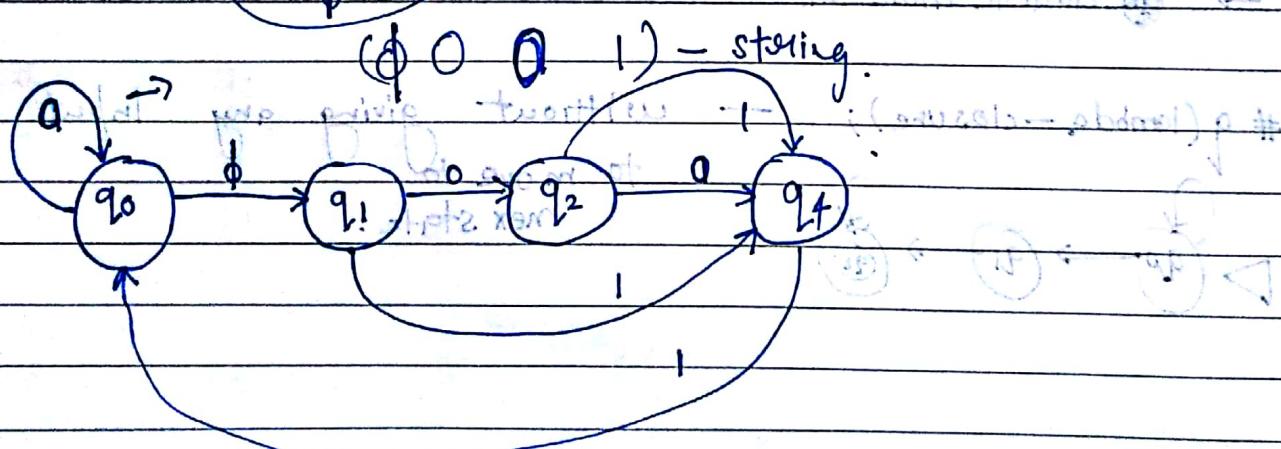
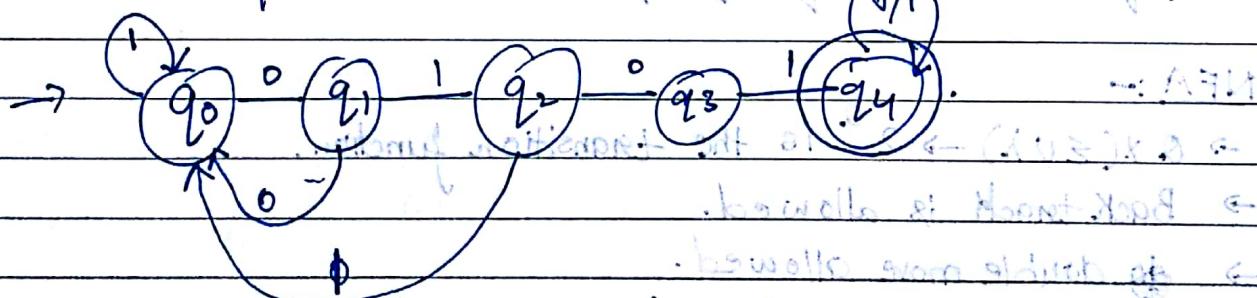
Page No. _____

$$\Rightarrow (\lambda)001(\lambda)^* \quad (0/1) \quad (0/1) \rightarrow (100010.01)$$



Design a DFA that accepts $\Sigma = \{0, 1\}^*$ even no. of 0's & 1's.

$$\Rightarrow (0101).$$



11/01/18

TOC

Page No.

Date: / /

Let $M = \{\emptyset, \leq, \delta, q_0, F\}$

be a NFA accepting L .

We construct $M' = \{\emptyset', \leq', \delta', q_0', F'\}$.

where,

$$\emptyset' = 2^\emptyset$$

$$q_0' = [q_0]$$

δ' is set of all subset of \emptyset containing F .

$$\delta'([q_1, q_2, \dots, q_i], a) = \delta(q_1, a) \cup \delta(q_2, a) \cup \dots \cup \delta(q_i, a)$$

Construct a DFA equivalent to;

$$M = \{[q_0, q_1], \{0, 1\}, \delta, q_0, F\}$$

where δ :

$$\begin{array}{ccc} q & 0 & 1 \\ \rightarrow (q_0) & q_0 & q_1 \end{array} \quad \text{where } \delta([q_0, q_1], 0) = [q_0] \text{ and } \delta([q_0, q_1], 1) = [q_1]$$

$$q_1 \quad q_1 \quad q_0, q_1$$

$$\begin{array}{ccc} 0' = q_0 & \phi = \emptyset & 0 \\ q' & q_0' = q_0 & q_0, q_0' \\ \phi & q_0 \cdot q_1 & q_0, q_1 \end{array} \quad F' = q_0, q_0q_1$$

$$q_0q_1, q_0q_1 \quad q_0q_1, q_0q_1$$

A Φ is \emptyset state $\emptyset\Phi$.

$$\begin{aligned} \delta([q_0, q_1], 0) &= \delta(q_0, 0) \cup \delta(q_1, 0) \\ &= q_0 \cup q_1 \\ &= q_0 \cdot q_1 \end{aligned}$$

P.T.O.

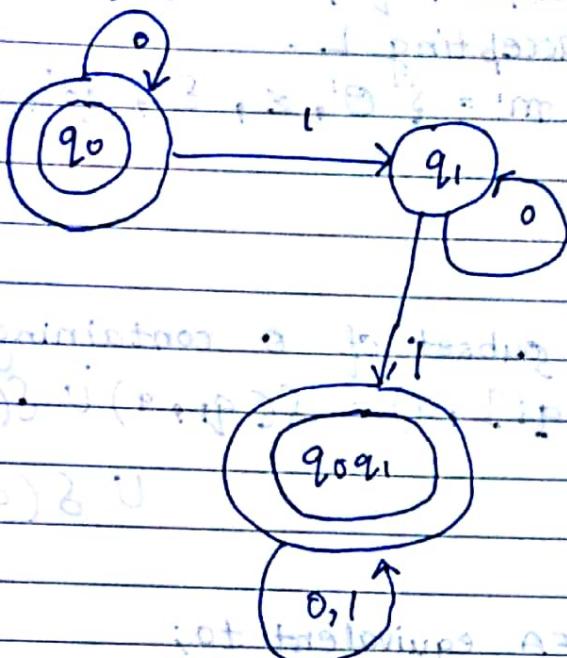
$$\delta((q_0, q_1), 1) = \delta(q_0, 1) \cup \delta(q_1, 1)$$

$$= q_1 \cup [q_0, q_1].$$

Page No. _____
Date: / /

$$= q_0 q_1.$$

Hence; DFA;



• NFA; $(Q) (S) (\Sigma) ^\circ (\delta) (q_0) (F)$

$$m = \left(\{q_0, q_1, q_2\}, \{a, b, c\}, \delta, q_0, \{q_0\} \right)$$

δ is;

(S)	(a)	(b)	(c)
q_0	$q_0 q_1$	q_2	
q_1	q_0	q_1	$q_0 q_1$
q_2	\emptyset	q_0, q_1	$q_1 q_2$

$$\begin{aligned}
\emptyset' &= \emptyset \\
q_0' &= q_0 \\
q_1' &= q_1 \\
q_2' &= q_2 \\
q_0 q_1' &= q_0 q_1 \\
q_1 q_2' &= q_1 q_2 \\
q_0 q_1 q_2' &= q_0 q_1 q_2
\end{aligned}$$

• 3 states in NFA.

Hence, 8 states in DFA. $= (2, 6, 8) / 2$

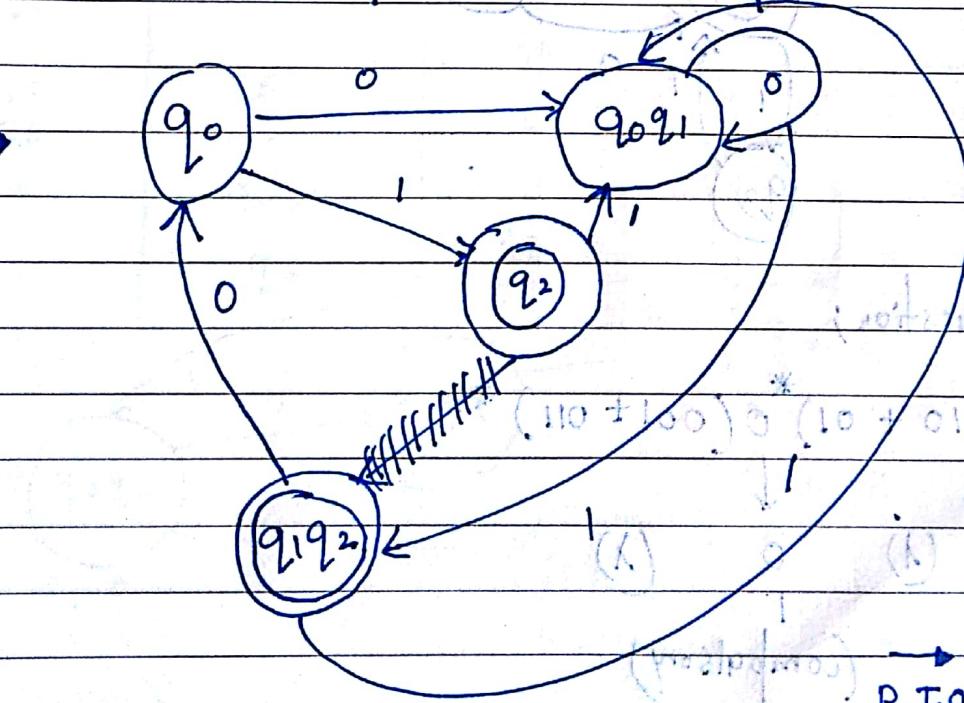
THEOREM

Page No.

For every NFA, there exist a DFA which simulates the behaviour of NFA i.e. if L is accepted by NFA then there exist a DFA which also accepts (L).

for DFA:

	(3)	(0)	(1)
1.	\emptyset	\emptyset	\emptyset
2.	q_0	$q_0 q_1$	q_2
3.	$q_0 q_1$ q_2	\emptyset	$q_0 q_1$
4.	$q_0 q_1$	$q_0 q_1$	$q_1 q_2$
5.	$q_1 q_2$	q_0	$q_0 q_1$

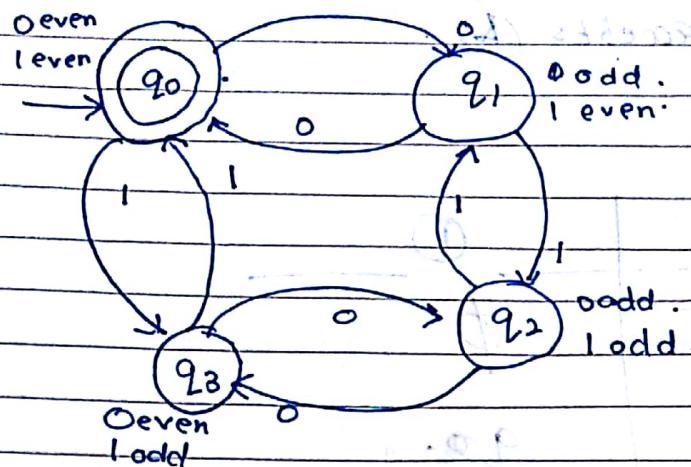


P.T.O.

important;

$$\leq = (0, 1)$$

All strings with even No. of 0's & 1's.



1. 0

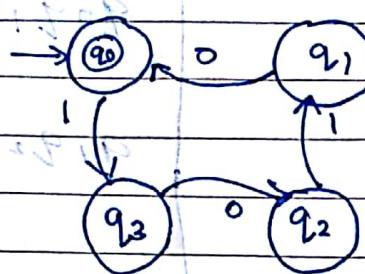
dot means 10} - Both.

1/0 OR 1+0..

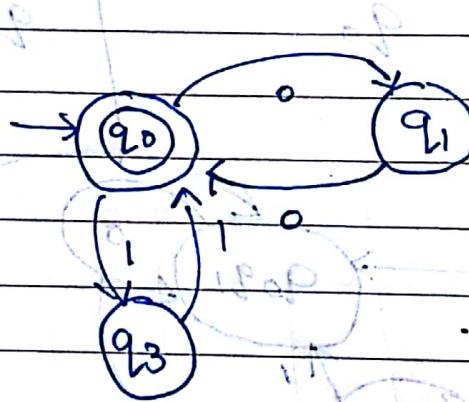
means can be 1 or
can be 0.

for 1010 11001010;

(1) 1010



(2) 1100



Last Sashional question;

$$*(10 + 01) \circ (001 + 011)*$$

(1) 0 (X)

(compulsory)

101001 0 001
(1) (2) (3)

16/01/18

...TOC...

Page No. 81

Date: 1/1/18

#	State	0	1
	q_0	$q_1 q_2 q_3 q_4 q_5 q_6 q_7$	q_6
	q_1	$q_2 q_3 q_4 q_5 q_6 q_7$	q_2
	q_2	q_0	$q_2 q_3 q_4 q_5 q_6 q_7$
	q_3	q_2	q_6
	q_4	q_7	q_5
	q_5	q_2	$q_6 q_7$
	q_6	q_6	q_4
	q_7	q_6	$q_2 q_3$

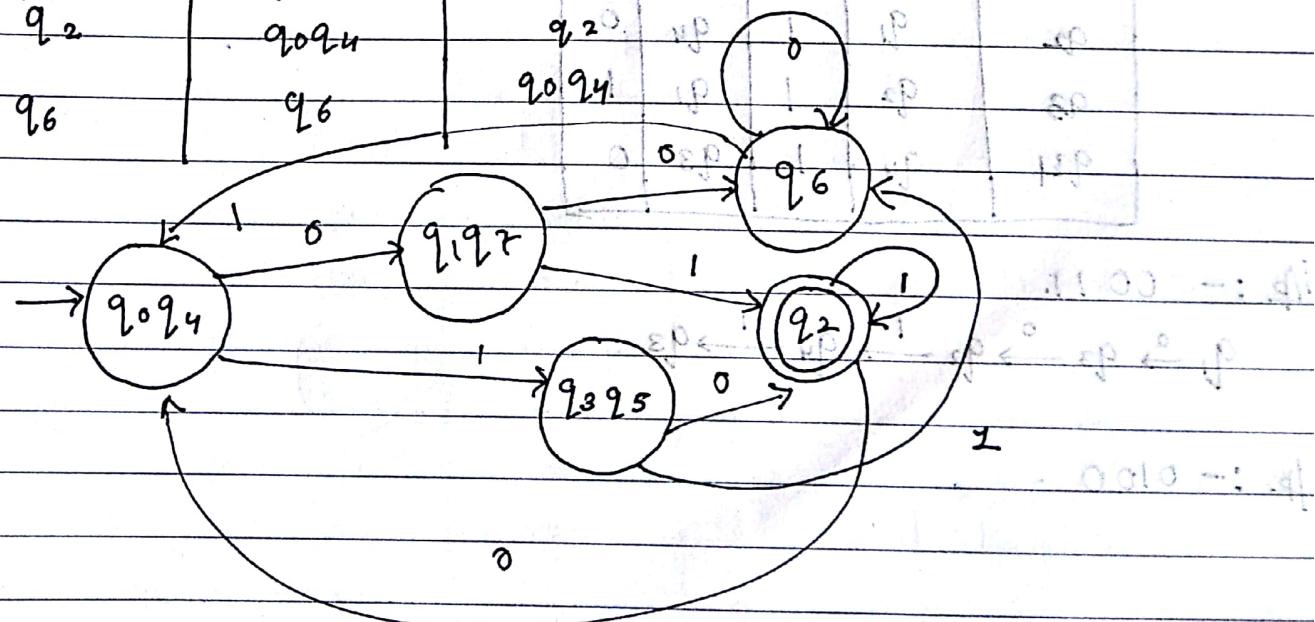
$$T_0 = \{q_2\} \{q_0 q_1 q_3 q_4 q_5 q_6 q_7\}$$

$$T_1 = \{q_2\} \{q_3 q_5\} \{q_0 q_4 q_6\} \{q_1 q_7\}$$

$$T_2 = \{q_2\} \{q_3 q_5\} \{q_1 q_7\} \{q_0 q_4\} \{q_6\}$$

$$T_3 = \{q_2\} \{q_3 q_5\} \{q_1 q_7\} \{q_0 q_4\} \{q_6\}$$

State	0	1
$\rightarrow q_0 q_4$	$q_1 q_7$	$q_3 q_5$
$q_1 q_7$	q_6	q_2
$q_3 q_5$	q_2	q_6
q_2	$q_0 q_4$	q_2
q_6	q_6	$q_0 q_4$



Page No.

Date: / /

Question :-

Design a DFA that accepts all strings of $\Sigma(a, b)$ that (i) starts with a ?

(ii) end with a .

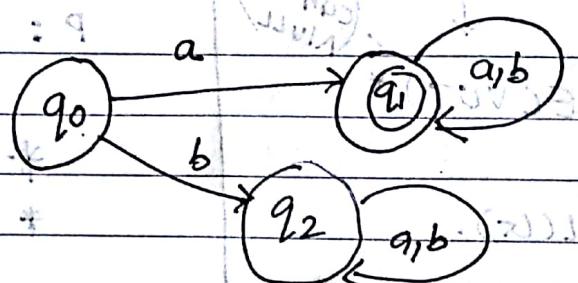
(iii) start with $a \&$ end with b .

(iv) start with $a \&$ end with a .

(v) neither start with a nor end with b .

Answer :-

(i) minimum = a .



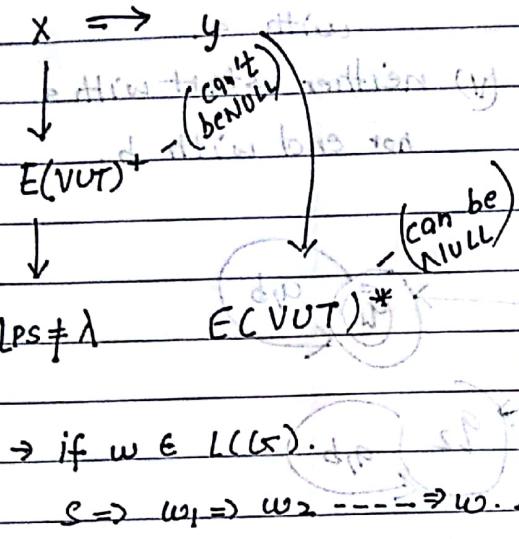
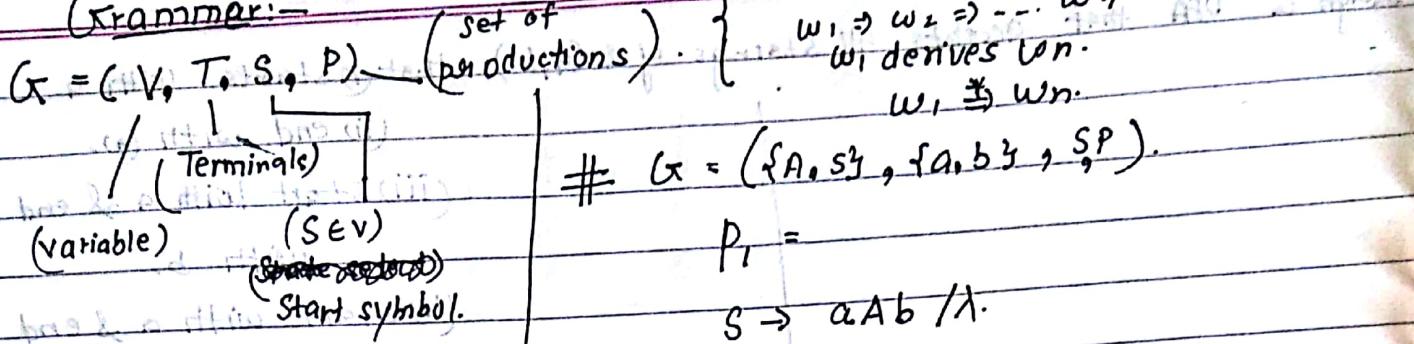
(ii) $(a + b)^*$

also - min. no.

$(a + b)^*$ - accept half

18/01/18 ... TQ ...

Grammar:-



find lang; $G = (\{S\}, \{a, b\}, S, P)$.

$$\begin{aligned} s &\Rightarrow ab \\ A &\Rightarrow aAb \\ A &\leftarrow \lambda \end{aligned} \quad \text{--- (P).}$$

$\rightarrow \{b, abb, aabb, \dots\}$.

$$L(G) = \{a^n b^{n+1} : n \geq 0\}.$$

$\rightarrow G = (\{A, S\}, \{a, b\}, S, P_1)$.

$$\# P_1 \Rightarrow S \rightarrow \lambda \quad A \rightarrow aAb$$

$$S \rightarrow aAb \quad A \rightarrow \lambda.$$

Here, $S \rightarrow \lambda$

$$S \rightarrow aAb \Rightarrow ab.$$

$$S \rightarrow aAb$$

$$aabb \rightarrow aabb.$$

$$L = \{a^n b^n : n \geq 0\}.$$

x derives y
 y is derived from x .

Page No. _____
 Date: / /

$w_1 \rightarrow w_2 \dots \rightarrow w_n$.
 w_i derives lan.
 $w_i \Rightarrow w_j$.

$$\# G = (\{S\}, \{a, b\}, S, P).$$

$$P_1 =$$

$$S \rightarrow aAb / \lambda.$$

$$A \rightarrow aAb / \lambda.$$

$$\# G = (\{S\}, \{a, b\}, S, P).$$

$$P: S \rightarrow aSa / bSb / \lambda.$$

* CAPS-Variable — P

* Small-terminals — including {NULL}.

$$\# G = (\{S\}, \{a, b\}, S, P).$$

$$P: S \rightarrow aSa / bSb / \lambda.$$

$$aSa - aa$$

$$bSb - bb$$

$$aSSa - aaaa$$

$$bSSb - bbbb$$

$$aSa = abba.$$

$$absba = ab aaba.$$

$$h = \{ww^R\}.$$

$$\rightarrow w \in (a, b)^*$$

$$\text{Q1} \# G = \{ (S), (a, b), S, P \} \rightarrow \text{Sol: } L = \{ a^n b^n : n \geq 0 \}$$

Page No. _____

Date: / /

2. Find grammar for $L = \{ a^n b^{n+1} : n \geq 1 \}$.

$$\# G = \{ (S), (a, b), S, P \}$$

$$P: S \rightarrow ss / aSb / bSa / \lambda$$

$$(3) \rightarrow G = \{ (S), (a, b), S, P \}$$

$$S \rightarrow \lambda$$

$$S \rightarrow aSb \rightarrow ab \rightarrow aSb$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ aSb & & abab \\ aabb & & \end{array}$$

$$S \rightarrow bSa \rightarrow ba \rightarrow bSa$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ bSa & & baba \\ \downarrow & & \\ bbaa & & \end{array}$$

$$S \rightarrow ss \rightarrow assb \rightarrow ab \rightarrow baa$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ aaabb & & bbbbaa \\ \downarrow & & \\ bbbbaa & & \end{array}$$

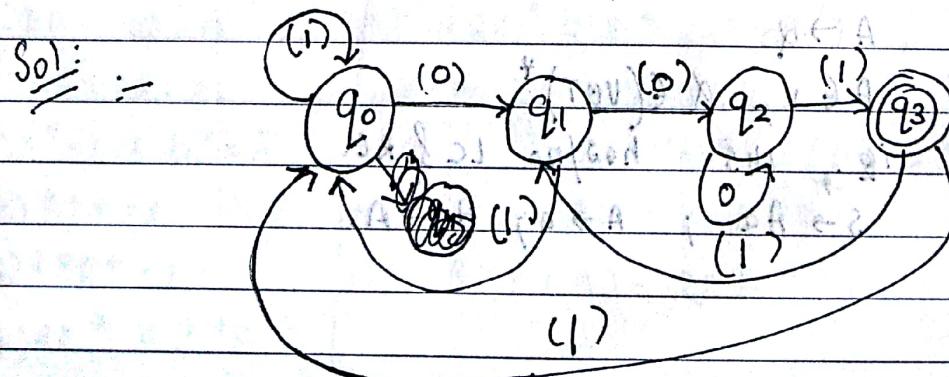
$$\therefore L = \{ w \in (a, b)^*: n_a = n_b \}$$

$$(2) L = \{ a^n b^{n+1} : n \geq 1 \}$$

→ $S \rightarrow Ab$

$$A \rightarrow aAb / ab$$

Q1: Draw DFA that accept $\Sigma = (0, 1)^*$ ending with 001



... Arden's Theorem

24/01/18

Page No. : 89

Date : 1/1/18

Theorem :- Let P & Q be two regular expressions over Σ .

If P doesn't contain λ then the following

cgn :-

$$R = Q + RP \dots \text{ (A)}$$

can be written as $R = QP^*$.

Proof :- To prove this theorem, we put the value of R in cgn.

(A) Hence, we get

$$Q + RP = Q + (Q + RP)P.$$

$$= Q + QP + RPP.$$

$$= Q + QP + QP^2 + QP^3 + QP^4 + \dots$$

$$= Q(1 + P + P^2 + P^3 + \dots) + RP^4.$$

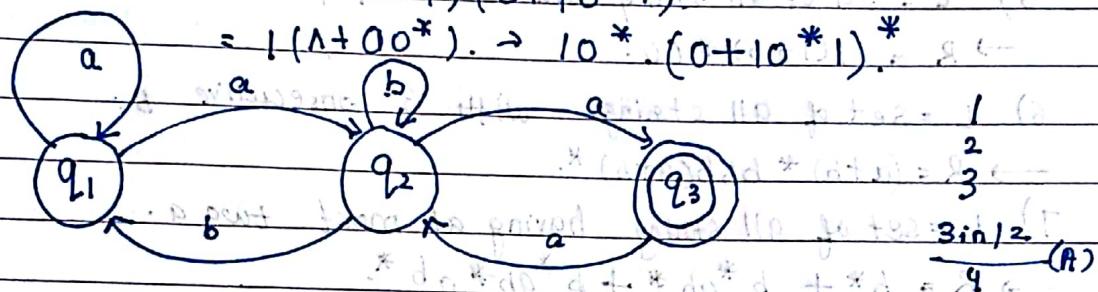
$$\# \text{ Prove } := (1 + 00^* 1) + (1 + 00^* 1)(0 + 10^* 1)(0 + 10^* 1) = Q^* 1 (0 + 10^* 1)^*.$$

$$(1 + 00^* 1) = (1 + (0 + 10^* 1))^* (0 + 10^* 1).$$

$$= (1 + 00^* 1)(0 + 10^* 1)^*.$$

$$= 1(1 + 00^*). \rightarrow 10^*. (0 + 10^* 1)^*.$$

Sashni :-



$$\text{Sol} \therefore q_1 = q_1 a + q_2 a + \lambda. \quad \text{using this, initial fa to q2} = 1. \quad (1)$$

$$q_2 = q_1 a + q_2 b + q_3 b.$$

$$q_3 = q_2 a.$$

$$\text{put (1) in (2); } q_2 = q_1 a + q_2(b + qa).$$

$$\text{Apply Arden's } q_2 = q_1 a(b + qa)^* \quad (4)$$

$$\text{put (4) in (1); } q_1 = q_1 a + q_1 a(b + qa)^* b + \lambda. \quad (1)$$

$$q_1 = \lambda + q_1(a + ab(b + qa)^*).$$

$$\text{Apply Arden's } \therefore q_1 = (a + ab(b + qa)^*). \quad (5)$$

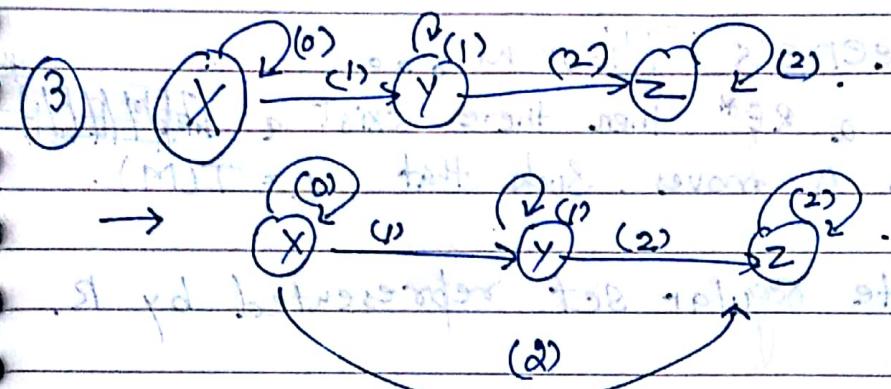
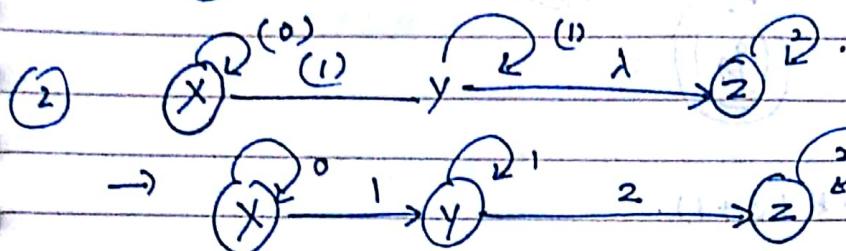
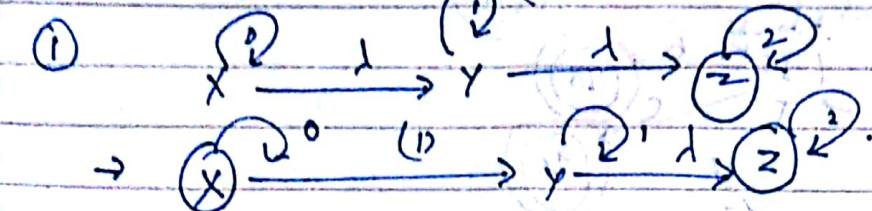
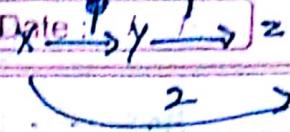
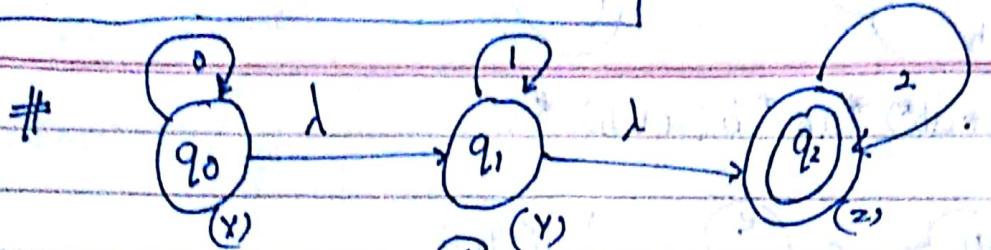
$$\text{Ques. Ans. :- } q_2 \rightarrow [a + ab(b + qa)^*]^* a (b + qa)^*.$$

$$q_3 = q_2 \cdot a.$$

$$1 \rightarrow q_1 = q_1 A + q_2 A + 1.$$

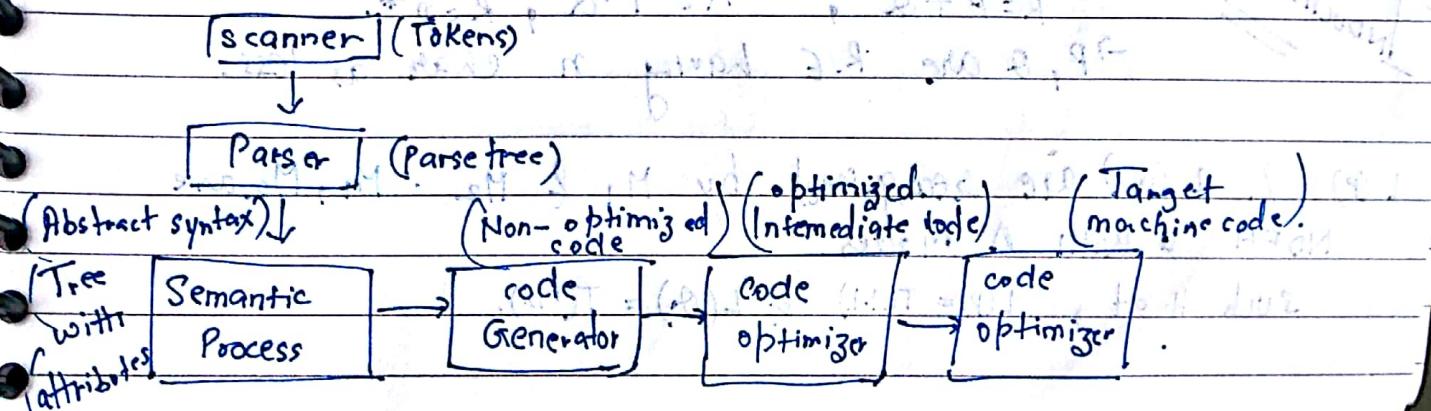
$$2 \rightarrow q_2 = q_1 A + q_2 A + q_3 A.$$

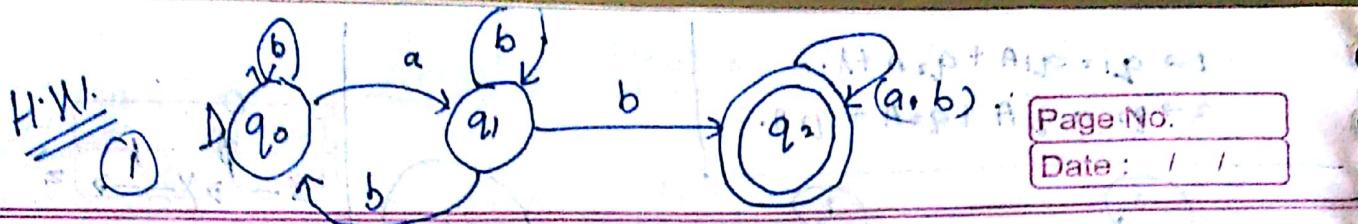
Page No. 1
Date 1/1/2023



- Steps :-
- ① Replace 1 from V_1 to V_2 .
 - ② find all edges starting from V_2 .
 - ③ Duplicate all the edges starting from V_1 without changes the edge ~~levels~~ labels.
 - ④ If V_1 is an initial state make V_2 also initial.
 - ⑤ If V_2 — "final" — make V_1 also final.

• Process of Compilation :-

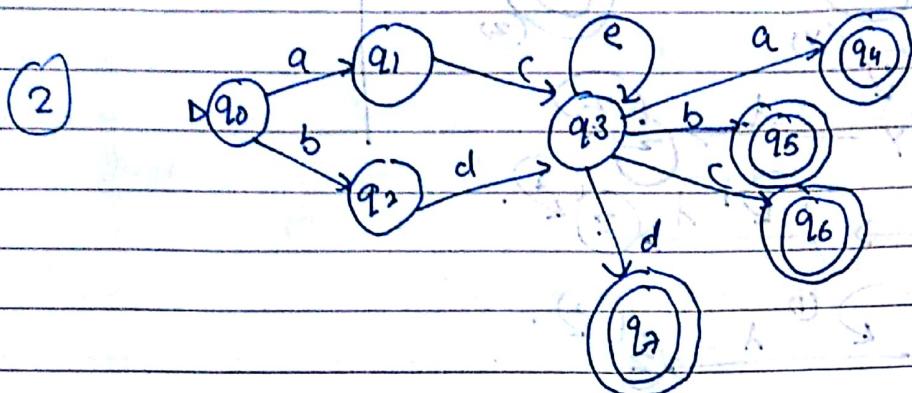




Page No. 5

Date : / /

$$\text{Ans} := (b + ab^*)^* ab^* b(a+b)^*$$



$$\text{Ans} := (ac + bd)e^*(at^b + c + d)$$

~~25/01/18~~ Kleene's Theorem
 \Rightarrow if R is a RE* then there exist a NFA M with N moves. Such that $L = T(M)$.

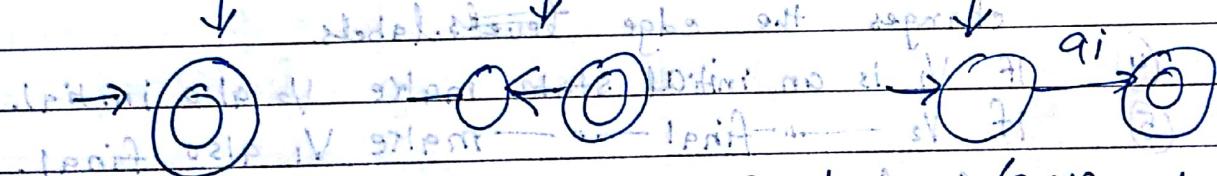
Proof Let $L(R)$ denote regular set represented by R .

Basic Step

\Rightarrow Let no. of chars. in R be 1. Consider $R = \lambda$. Then $L(R) = \{\lambda\}$.

$$R = \lambda \Rightarrow N = 1 \Rightarrow R = \emptyset \Rightarrow R = a_i \quad (a_i \in \Sigma)$$

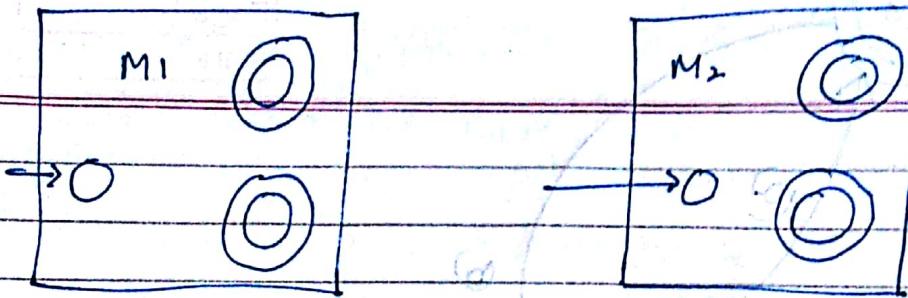
transition N move exists \Rightarrow a_i goes to λ state \Rightarrow $q_0 \xrightarrow{a_i} q_1 \xrightarrow{\lambda} q_0$



Inductive Step :- Let R be R.E. having $(n+1)$ char.
 $R = P + Q$, $R = P \cdot Q$, $R = P^*$.
 $\Rightarrow P, Q$ are R.E. having n char as less.

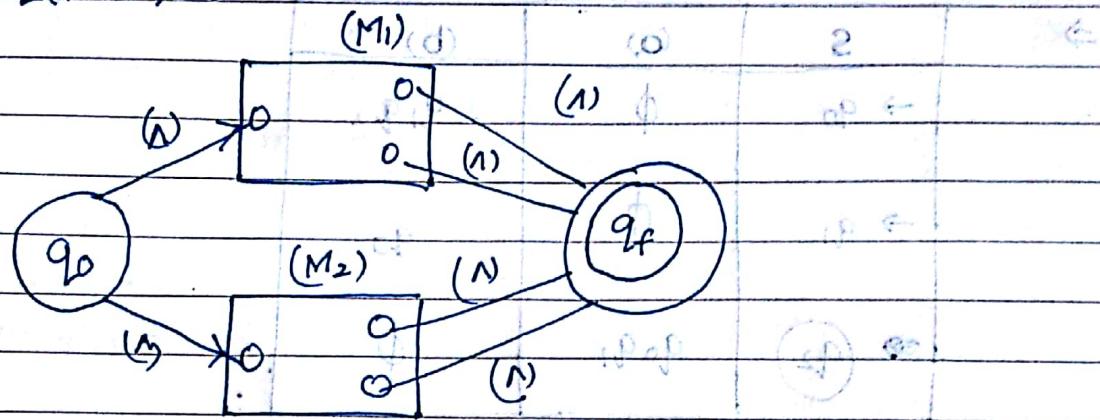
$L(P)$ & $L(Q)$ are recognised by M_1 & M_2 . M_1, M_2 are NFA with N moves.

such that, $L(P) = T(M_1)$ & $L(Q) = T(M_2)$.



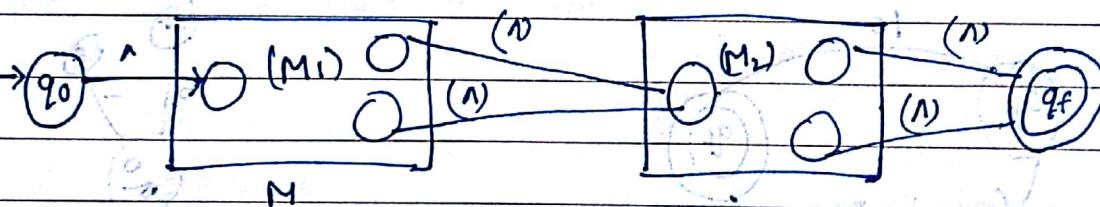
CASE 1 :- $R = P + Q$.

We construct NDFA M with λ moves that accepts $L(P+Q)$:



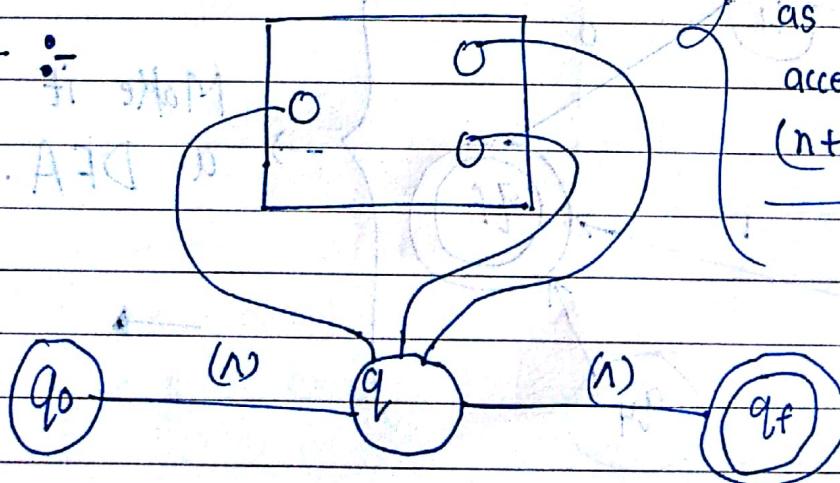
$$T(M) = T(M_1) \cup T(M_2) = L(P+Q).$$

CASE 2 :- $R = P.Q$.

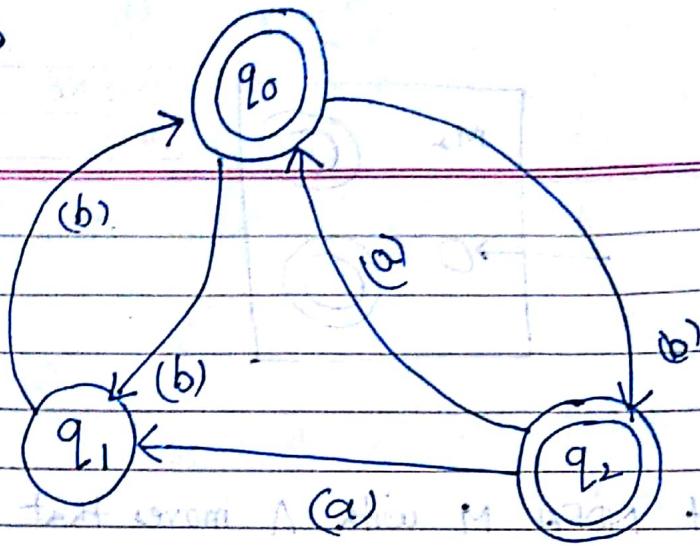


$$\Rightarrow T(M) = T(M_1) + T(M_2) \\ = L(PQ).$$

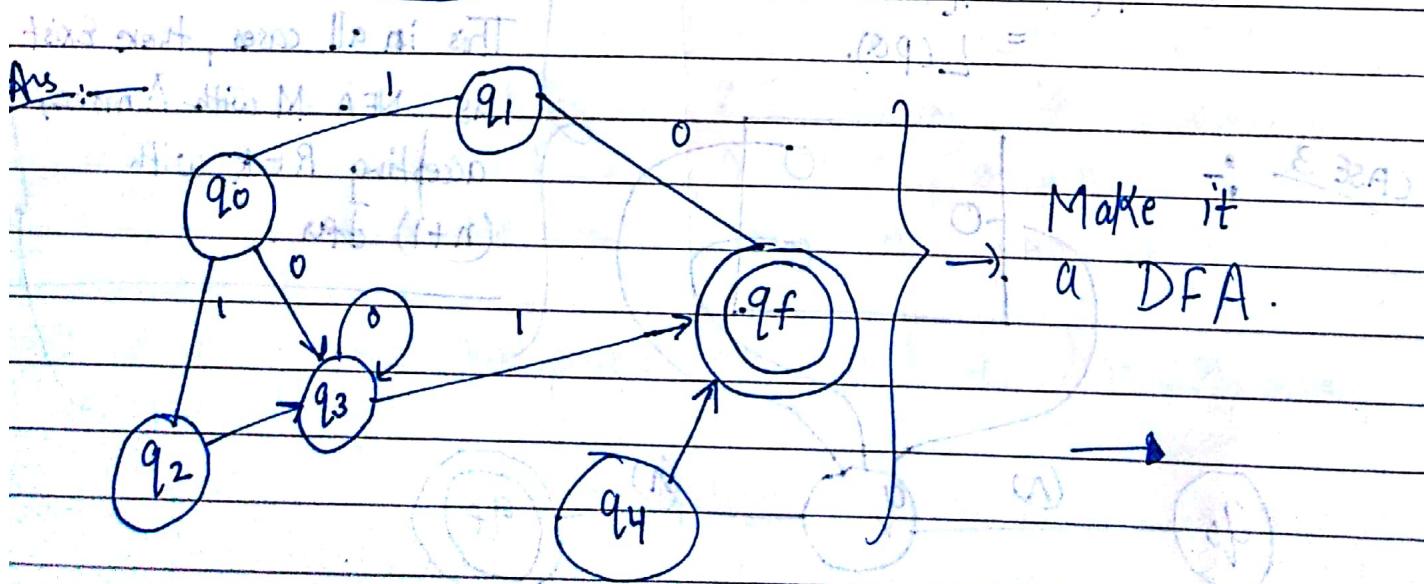
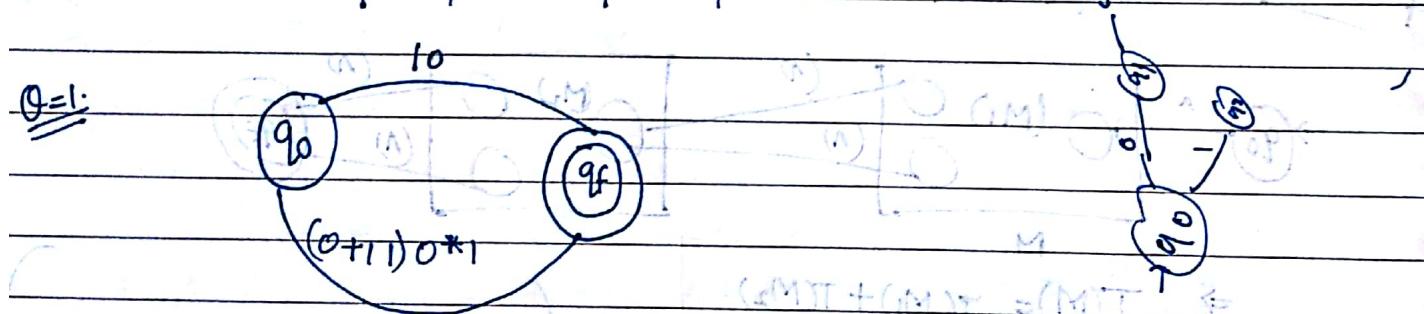
CASE 3 :-



This is in all cases, there Exist as NFA M with λ moves, accepting RER with $(n+1)$ den.



\Rightarrow	s	(a)	(b) (\cap)
$\rightarrow q_0$		\emptyset	$q_1 q_2$
$\rightarrow q_1$		\emptyset	q_0
$\bullet q_2$	$q_0 q_1$	\emptyset	\emptyset
$\rightarrow q_0 q_1$	\emptyset	$q_0 q_1 q_2$	$(MFT + MTT) \cap (MTT)$
$q_0 q_1 q_2$	$q_0 q_1$	$q_0 q_1 q_2$	$q_0 q_1 q_2$
\emptyset	\emptyset	\emptyset	\emptyset



• CONTEXT FREE GRAMMAR

→ A grammar $G = (V, T, P, S)$ is said to be

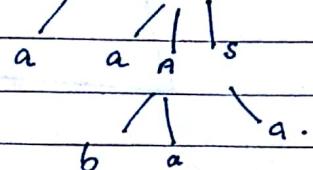
free where, V = finite set of non-terminal, generally represented by capital letters.

Page No. _____
Date: 30/01/18
Context Free Grammar

T = a finite set of terminals, generally represented by small letters.

→ $S \rightarrow aAs/aSS$.

$A \rightarrow sba/ba$.



yield: aabaa.

$S \Rightarrow SS \Rightarrow aS \Rightarrow aaS$.

$\Rightarrow aabaa \Rightarrow aabaa$.

→ $S \rightarrow aAb/a$.

$A \rightarrow sba/ss/ba$.

$S \Rightarrow aabbba$.

• $E \rightarrow E \text{ op } E \mid id$.

$op \rightarrow + \mid - \mid \cdot \mid /$.

→ A context-free grammar G such that some word has two parse trees is known as ambiguous.

Problems

1) Associativity.

2) Precedence.

3)

Sashional

$S \rightarrow OB/IA$.

$A \rightarrow O/OS/IAA$.

$B \rightarrow I/IS/OBB$.

00110101

LMD ; RMD

$S \rightarrow aAS$.
 $aSbAS$.
 $aAbAS$.

$aabbbaa$.

Removal of Ambiguity :-

$E \rightarrow E+E / E * E / id$.

Page No.

Date: 1/1/2023

$S \rightarrow S+A$.

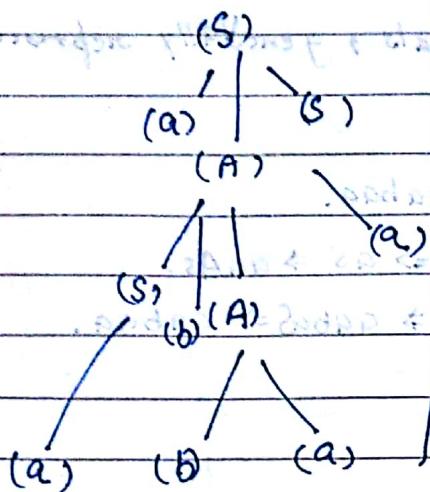
$A \rightarrow$

$E \rightarrow E+T/T$.

$T \rightarrow T * F/F$.

$F \Rightarrow id$.

Q12 :- Simplification of context-free grammar



for LMD :- $S \rightarrow OB$.

$\rightarrow OOB B$.

RMD :-

$S \rightarrow OB$.

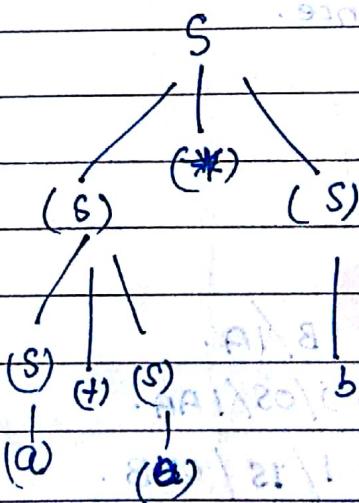
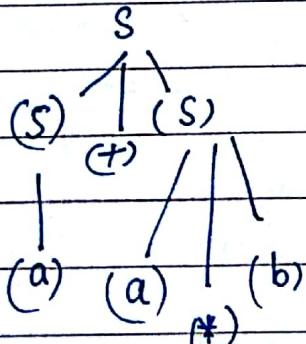
$\rightarrow OOB B$.

Production of S

$S \rightarrow S+S / S*S / a/b . \} \rightarrow$ Prove this to be ambiguous.

O/P $\rightarrow a+a*b . \} -$ (Answer).

Ans :-



06/02/18

$$L = \{a^n b^n ; n \geq 0\}.$$

$$(\{q_0, q_1\}, \{a, b\}, \{z, 1\}; \delta, q_0, z, q_1).$$

δ :-
 (This is wrong)

$$w = aaabb b.$$

$$(q_0, aaabbb, z)$$

$$\delta(q_0, a, z) = (q_0, 1, z)$$

$$+ (q_0, aabbb, 1z).$$

$$\delta(q_0, a, 1) = (q_0, 11) + (q_0, abbb, 11z).$$

$$+ (q_0, bbbb, 111z).$$

$$\delta(q_0, b, 1) = (q_0, 1) + (q_0, bb, 11z).$$

$$+ (q_0, b, 1z).$$

$$\delta(q_0, 1, z) = (q_1, 1) + (q_0, 1, z).$$

$$+ (q_1, 1, 1).$$

$\rightarrow abab \notin L.$

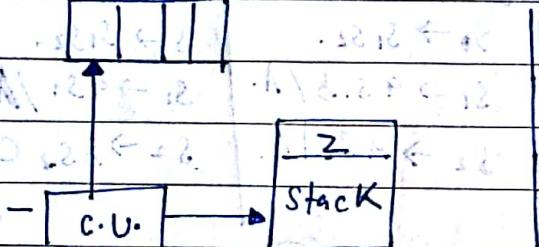
$$(q_0, abab, z) \vdash (q_0, bab, 1z).$$

$$+ (q_0, ab, z) \vdash (q_0, b, 1z).$$

$$+ (q_0, 1, 1, z) \vdash (q_1, 1, 1).$$

accepted.

I/O



$$(q_0, aabb, z).$$

$$+ (q_0, abb, 1z).$$

$$+ (q_0, bb, 11z).$$

$$+ (q_1, b, 1z).$$

$$+ (q_1, 1, z).$$

$$+ (q_2, 1).$$

• correction of this :-

$$\delta: \delta(q_0, a, z) = (q_0, 1z)$$

$$\delta(q_0, a, 1) = (q_0, 11z)$$

$$\delta(q_0, b, 1) = (q_1, 1)$$

$$\delta(q_1, b, 1) = (q_1, 1)$$

$$\delta(q_1, 1, z) = (q_2, 1)$$

$$\delta(q_1, 1, 1) = (q_2, 1)$$

$$\delta(q_2, 1, 1) = (q_2, 1)$$

$$\rightarrow (q_0, abab, z) \vdash (q_0, bab, 1z)$$

$$+ (q_1, ab, z) = \text{HALT}$$

O.1: Design a PDA using $\vdash L = \{a^n b^{2n} : n \geq 0\}$.

Ans :- $(\{q_0, q_1, q_2\}, \{a, b\}, \{\tau, 1\}, \delta, q_0, \tau, q_2)$.

Page No. _____

Date: / /

$$\delta: \delta(q_0, a, \tau) = (q_0, 1z).$$

$$\delta(q_0, a, 1) = (q_0, 1111z).$$

$$\delta(q_0, b, 1) = (q_0, 11z).$$

$$\delta(q_1, b, 1) = (q_1, 1).$$

$$\delta(q_2, 1, z) = (q_2, 1).$$

$$\delta(q_2, 1, z) = (q_2, 1).$$

and

:- $L_1 \cap L_2$.

CFL.

$$G_1 = (V_1 T_1 S_1 P_1) \quad G_2 = (V_2 T_2 S_2 P_2).$$

$$(V_1 \cap V_2) = \emptyset.$$

$$G_3 = (V_3 T_3 S_3 P_3) \quad \text{CFG}.$$

$$L(G_3) = L(G_1) \cup L(G_2).$$

$$V_3 = V_1 \cup V_2 \cup \{S_3\}.$$

$$T_3 = T_1 \cup T_2.$$

$$S_3 = \text{Start sym.}$$

$$P_3 = P_1 \cup P_2 \cup \{S_3 \rightarrow S_1/S_2\}.$$

$$\text{Let } G_4 = (V_4 T_4 S_4 P_4).$$

$$\text{# } L(G_4) = L(G_1) \cup L(G_2).$$

$$V_4 = V_1 \cup V_2 \cup \{S_4\}.$$

$$T_4 = T_1 \cup T_2.$$

$$S_4 = S_1 S_2.$$

$$P_4 = P_1 \cup P_2 \cup \{S_4 \rightarrow S_1 S_2\}.$$

$$\text{Let } G_5 = \{V_5 T_5 S_5 P_5\}.$$

$$L(G_5) = L(G_1) *.$$

$$V_5 = V_1 \cup \{S_5\}.$$

$$T_5 = T_1.$$

$$S_5$$

$$P_5 = P_1 \cup \{S_5 \rightarrow S_1 S_5 / \lambda\}.$$

$$\text{Let } (G_3) = L_1 \cup L_2.$$

$$w \in L_1 \Rightarrow S_3 \rightarrow S_1 \Rightarrow w.$$

$$w \in L_2 \Rightarrow S_3 \Rightarrow S_2 \xrightarrow{*} w.$$

$$\text{if } w \in L(G_3).$$

$$\text{1) either } S_3 \Rightarrow S_1,$$

$$\text{2) or } S_3 \Rightarrow S_2.$$

$$\text{if } S_3 \Rightarrow S_1 \text{ has } V_1 \& (V_1 \cap V_2 + 0).$$

$$\text{then } S_1 \xrightarrow{*} w \text{ involves Prod P only.}$$

$$\text{if } S_3 \Rightarrow S_2 \text{ has } V_2.$$

$$S_2 \xrightarrow{*} w.$$

$$\text{Prod. P}_2 \text{ only.}$$

$$\text{so, either 1 or 2.}$$

② Not closed under intersection & comp.

* consider a lang: $L_1 = \{a^n b^n c^m : n, m \geq 0\}$.

$$L_2 = \{a^n b^m c^m : n, m \geq 0\}.$$

$$L_1 = S_0 \rightarrow S_1 S_2.$$

$$L_2 = S \rightarrow S_1 S_2.$$

$$S_1 \rightarrow a S_1 b / \lambda.$$

$$S_1 \rightarrow a^2 S_1 / \lambda.$$

$$S_2 \rightarrow a S_2 / \lambda.$$

$$S_2 \rightarrow b S_2 c / \lambda.$$

• compl :-

$$L_1$$

$$L_2$$

$$i.e., L_1 \& L_2 \text{ are CFL.}$$

$$\Rightarrow \bar{L}_1 \cup \bar{L}_2$$

$$\Rightarrow L_1 \cap L_2 \text{ is EFL.}$$

$$\rightarrow L_1 \cap L_2 = \{a^n b^n c^m : n, m \geq 0\}.$$

is not CFL & using P.L.Y.

~~07/02/18~~

PDA

$\lambda; \delta; \Delta$

Page No.

Date: / /

Q=1: Design a PDA accepting set of all string with equal no. of A and B.

Ans: $L = \{a^nb^n : n \geq 0\}$

$\rightarrow (\{q_0, q_1, z\}, \{a, b\}, \{z\}, S, q_0, z, q_1)$.

$\delta: \delta(q_0, a, z) = (q_0, a, z)$

$(q_0, b, z) = (q_1, b, z)$

$(q_0, a, a) = (q_0, aa)$

$(q_0, b, b) = (q_0, bb)$

$(q_0, a, b) = (q_0, ab)$

$(q_0, b, a) = (q_0, ba)$

$(q_0, a, z) = (q_1, a, z)$

e.g. (ddaaababbbaaaa) \rightarrow abab 1

$\xrightarrow{d \ x \ z} \ x \ x \ z$

(Deterministic)

$\delta(q_0, a, z) = (q_0, az)$.

DPDA

$\delta(q_0, b, z) = (q_0, bz)$.

$M = (\emptyset \subseteq \Gamma \cup \delta \cup Q \cup F)$

, $\delta(q_0, a, a) = (q_0, aa)$.

is deterministic: $\forall (q, \alpha) \in M \rightarrow \text{at most one}$

$\delta(q_0, b, b) = (q_0, bb)$.

for every $q \in \emptyset, q \in \Sigma^*$.

$\delta(q_0, a, b) = (q_0, ab)$.

$\delta: \delta(q, \alpha, \beta) \in M \rightarrow \text{at most one}$

$\delta(q_0, b, a) = (q_0, ba)$.

① $\delta(q, a, b)$ contains atmost one element.

$\delta(q_0, \lambda, a) = (q_1, a)$.

② if $\delta(q, \lambda, b)$ is not empty, then,

$\delta(q_0, \lambda, b) = (q_1, b)$.

$\delta(q, c, b)$ must be empty for every $c \in \Sigma$.

A PDA is said to be deterministic.

{on next page} ...

TOPICS LEFT..

PDA TO CFG

CFG TO PDA ✓ (Assignment)

PL

Properties lang.

EX. YACC ✓ (Assignment)

$$\bullet A = \{q, T, VUT, \delta, q_0, S, \phi\}$$

$Q \in \Gamma$ $\delta: q_0 \rightarrow F$

 $A \rightarrow \alpha$.

$$R_1 : \delta(q, \lambda, A) = (q, \alpha).$$

$$R_2 : \delta(q, a, A) = (q, A)$$

for every a in T .

State - state

Terminal - alphabet.

variable + terminal - stack state.

state - initial state.

 ϕ - final state. $S \rightarrow aSbb/a$.

$$M = (Q, \{a, b\}, \{(a, b, S)\}, \delta, q_0, S, F)$$

Page No. / /

Date: / /

S:

$$\delta(q, \lambda, S) = \{(q, aSbb), (q, a)\}.$$

$$\delta(q, a, a) = \{q, \lambda\}.$$

$$\delta(q, b, b) = (q, \lambda).$$

let take any sequence;

$$S \rightarrow aSbb/a.$$

(put aSbb)

$$S \rightarrow aaSbbb.$$

(put a).

$$S \rightarrow aaabbabb.$$

(aaabbabb $\in L$).

$$(q, aaabbabb, S).$$

$$T(q, aaabbabb, S).$$

$$T(q, aaabbabb, aSbb).$$

$$T(q, aabbabb, Sbb).$$

$$T(q, aabbabb, aSbbbbb).$$

$$T(q, abbb, abbb).$$

* PUMPING LEMMA :-

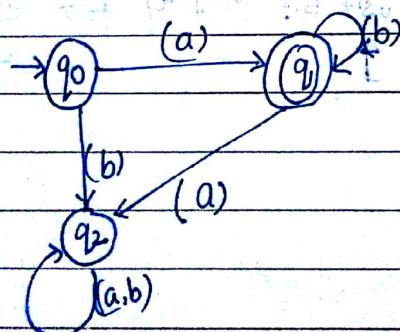
$$T^*(q, \lambda, \lambda).$$

→ Theorem :- $M = (Q, \Sigma; \delta, q_0, F)$, $F \in A$ with n states.Let L be a RS accepted by M .Then, the M at $w \in L$, $|w| \geq m$ & $M \geq n$.can be decomposed into ; $w = xyz$.

$$|xy| \leq n \quad y \neq \lambda.$$

 $xy^i z \in L$ for $i \geq 0$.

$$\text{e.g. : } [ab^n, n > 0].$$



$$w = |w| \geq 3.$$

$$r = abb \Rightarrow w = (a, b, b).$$

$$w = x y z \quad \downarrow \quad \downarrow \quad \downarrow$$

(a) (b) (b)

$$|xy| \leq n \quad 2 \leq 3.$$

$$\Rightarrow |y| = |b| = |z|.$$

$$w = xyz.$$

 $= ab \cdot b \in L \quad \{L \text{ is regular}\}.$

Proof :- $w = a_0, a_1, \dots, a_m, m \geq n$.
 $\delta(a_0, a_1, a_2, \dots, a_i) = q_i$.

Page No. _____
 Date: 1/1
 i = 1, 2, ..., m.

$$\Theta = \{q_0, q_1, \dots, q_m\}.$$

Θ is seq of states in path.
 $w = a_1, a_2, \dots, a_m$.

As only n distinct distinct states atleast 2 states in Θ . consider,

Let them take as q_j & q_{j+1} ($q_j = q_{j+1}$).

\Rightarrow Then, $0 \leq j \leq k \leq n$.

$a_1, a_2, \dots, a_j; a_{j+1}, \dots, a_k$ & a_{k+1}, \dots, a_m .

(X)

(Y)

(Z)

$k \leq n$; $|xy| \leq n$.

• Using final state :- CFG \rightarrow PDA.

If L is λ free then there exist a CFG in GNF form.

$$M = \{(q_0, q_1, q_f), T, \Sigma, \delta\},$$

$$Q \subseteq \Gamma$$

$$\Sigma, q_0, Z, \{q_f\}\}$$

$$\delta, q_0, Z, F$$

$$Z \notin V.$$

$$\delta(q_0, \lambda, Z) = (q_1, S^z).$$

$$\delta(q_1, \lambda, Z) = (q_f, Z).$$

$$\delta(q_1, a, S) = (q_1, A).$$

$$\delta(q_1, a, A) = \{ (q_1, AB), (q_0, \lambda) \}.$$

$$\delta(q_1, b, A) = (q_1, B).$$

$$\delta(q_1, b, B) = (q_1, \lambda).$$

$$\delta(q_1, C, C) = (q_1, \lambda)$$

$$w = aaabc \in L.$$

$$= (q_0, \lambda, aaabc, z).$$

$$\vdash (q_1, aaa, bc, Sz).$$

$$\vdash (q_1, aabc, Az)$$

$$\vdash (q_1, abc, ABCz).$$

→
on
next
Page...

* Whether a^p where p is prime is prove it to be non-prime?

Page No. 103/109

Date: 1/1/2023

→ PL for CFL;

$L \Rightarrow$ CFL.

w ∈ L with $|w| \geq m$. If w is not prime then

$$w = uvxyz$$

$\Rightarrow c_1 vxy \parallel \leq m$ state faithib ~~not~~ is prime. A

($\Rightarrow p = q$) $\Rightarrow p & q$ no split as it is

$\therefore a^p$ is not prime