22/11/17

Unit 4

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- Positive Real Functions

Conditions for PR function –

(i) All the quotient terms of the function are real.

(ii) All the poles & zeroes of the function should lie on the left half of the s-plane.

(iii) Real part of the function should be ≥ 0 for all values of ω.

Q21.16 Check whether $f(s) = \frac{s+2}{s+1}$ is PR fn. or not.

$$Re\left[F(j\omega)\right] = \frac{(j\omega+2)}{(j\omega+1)} \times \frac{(-j\omega+1)}{(-j\omega+1)}$$

$$= \frac{(-j\omega)^2 + j\omega - 2j\omega + 2}{(j\omega)^2 + \cancel{j\omega} - \cancel{j\omega} + 1}$$

$$= \frac{(j\omega)^2 - j\omega + 2}{(j\omega)^2 + 1}$$

$$= Re\left[\frac{\omega^2 - j\omega + 2}{\omega^2 + 1}\right]$$

$$= \frac{\omega^2 + 2}{\omega^2 + 1}$$

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$\therefore$ F(s) is a PR function.

Q $Z(s) = \frac{4s+1}{s+2}$

Pole $\rightarrow s = -2$

Zero $\rightarrow s = \frac{-1}{4}$

$$Re\left[Z(j\omega)\right] = Re\left[\frac{4j\omega+1}{j\omega+2} \times \frac{-j\omega+2}{-j\omega+2}\right]$$

$$= \frac{4(j\omega)^2 + 8j\omega - j\omega + 2}{(j\omega)^2 + \cancel{2j\omega} - \cancel{2j\omega} + 4}$$

$$= \frac{4(j\omega)^2 + 7j\omega + 2}{(j\omega)^2 + 4}$$

$$= Re\left[\frac{4\omega^2 + 7j\omega + 2}{\omega^2 + 4}\right]$$

$$= \frac{4\omega^2 + 2}{\omega^2 + 4}$$

$Re\left(Z(j\omega)\right] \geq 0$; for all ω

$\therefore$ Z(s) is a PR function

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$$f(s) = \frac{s^2+10s+4}{s+2}$$

Q.

Pole $\rightarrow$ $s = -2$

Zero $\rightarrow$ $s^2+10s+4 = 0$

$\Rightarrow s^2+2s+8s+4 = 0$

$\Rightarrow s(s+2)+4(s+2) = 0$

$\Rightarrow (s+4)(s+2) = 0$

$\Rightarrow s = -4$ or $s = -2$

$$Re[f(j\omega)] = \frac{(j\omega)^2+10(j\omega)+4}{(j\omega)+2} \times \frac{-j\omega+2}{-j\omega+2}$$

$$= \frac{-(j\omega)^3+2(j\omega)^2+10(j\omega)^2+20j\omega \; 4j\omega+8}{(j\omega)^2+4}$$

$$= Re\left[\frac{-(j\omega)^3+12(j\omega)^2+16j\omega+8}{(j\omega)^2+4}\right]$$

$\boxed{j \times j = -1}$

$$= \frac{\cancel{j\omega^3}+12\omega^2+16\omega+8}{\omega^2+4}$$

$$\frac{-\omega^2+10j\omega+4}{j\omega+2} \times \frac{-j\omega+2}{-j\omega+2}$$

$$\cancel{j}\omega^3+(-2\omega^2)-10j^2\omega^2+2\cancel{0j}\omega$$
$$-4j\omega+8$$

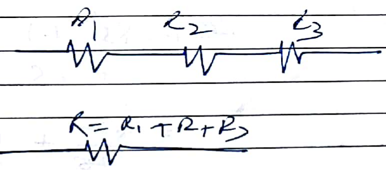
$$\frac{8\omega^2+8}{\omega^2+4}$$

$Re[F(j\omega)] \geq 0$ for all ω.

$\therefore F(s)$ is a PR function.

Q. 21.21 $\quad Y(s) = \dfrac{s^2+2s+20}{s+10}$ (PR)

Network Synthesis



Conditions —

(i) Numerator should have the highest power of s.

(ii) eg: $Z(s) = \dfrac{s^2+1}{s+1}$ ✓

$Y = \frac{1}{Z(s)} = \dfrac{s+1}{s^2+1}$

$Y(s) = \dfrac{s^2+1}{s+1}$

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(ii) If we are taking the Z eqn., then, s represents a series inductor and $1/s$ represents a Capacitor.

If we are taking the Y equation, then, s represents capacitance & $1/s$ represents inductor.

(iii) Quotient of s represents the value of the inductor or capacitor.

Eg: $Z(s) = 4s + \frac{1}{2s}$

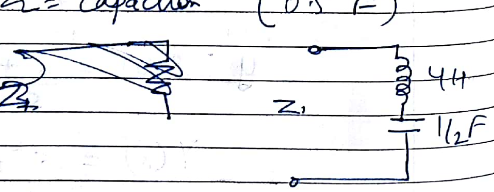
$$Z(s) = \frac{8s^2+1}{2s}$$

$$= \frac{8s^2}{2s} + \frac{1}{2s}$$

$$Z(s) \Rightarrow 4s + \frac{1}{2s}$$

$$= Z_1 + Z_2$$

Z_1 = Inductor (4 H)
Z_2 = Capacitor (0.5 F)



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Q. $Z(s) = \frac{s^3+4s}{s^2+2}$. Realize the network.

$$s^2+2 \overline{) s^3+4s} (s$$
$$s^3+2s$$
$$2s \overline{) s^2+2} (\frac{1}{2}s$$
$$s^2$$

$$Z(s) = s + \frac{2s}{s^2+2}$$

$$= Z_1 + Z_2$$

$$\boxed{Z_1 = 1H}$$

$$Z_2 = \frac{2s}{s^2+2}$$

$$Y_2 = \frac{s^2+2}{2s}$$

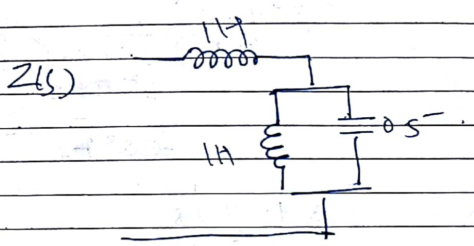
$$Y_2 = \frac{s^2}{2s} + \frac{2}{2s} = \frac{s}{2} + \frac{1}{s}$$

$$= Y_{21} + Y_{22}$$

=

$Y_{21} = 0.5\,F$ Cap.

$Y_{22} = 1\,H$.



21.25 Q: $Z(s) = \dfrac{s^4 + 10s^2 + 7}{s^3 + 2s}$

$s^3+2s\,)\; s^4 + 10s^2 + 7\,(\; s$
$s^4 + 2s^2$
$8s^2 + 7\,)\; s^3 + 2s\,(\; s/8$
$s^3 + \frac{7}{8}s$

$2 - \frac{7}{8}$

$\frac{16-7}{8}$

$\frac{9}{8}s\,)\; s^3 + 2s\,(\; \frac{8}{9}s^2$
s^3
$2.\frac{1}{4}$

$\frac{9}{8}s\,)\; 8s^2 + 7\,(\; \frac{64}{9}s$
$8s^2$
7

$Z(s) = s + \dfrac{8s^2+7}{s^3+2s}$

$= Z + 1/Y$

$\boxed{Z = 1\,H} = L_1$

$Y = \dfrac{s^3+2s}{8s^2+7}$

$= \dfrac{s}{8} + \dfrac{\left(\frac{9}{8}\right)s}{8s^2+7}$

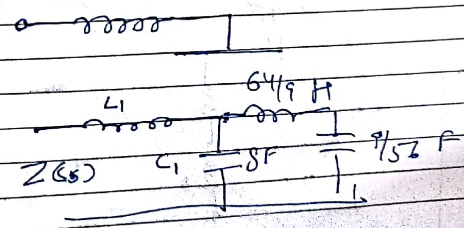
$= Y_1 + 1/Z_1$

where $\boxed{Y_1 = 0.125\,F} = C_1$

$Z_1 = \dfrac{8s^2+7}{(9/8)s}$

$= \dfrac{64s}{9} + \dfrac{7}{9/8\,s}$ — $9/56\,F$

$Z_1 = 7.11\,H + 6.22\,F$



21.29 $Y(s) = \frac{4s^2+6s}{s+1}$

Realize the network

$$s+1 \overline{)\,4s^2+6s\,}(4s$$
$$\cancel{4s^2}+4s$$
$$2s \overline{)\,\cancel{s}+1\,}(\tfrac{1}{2}$$
$$\cancel{s}$$
$$1$$

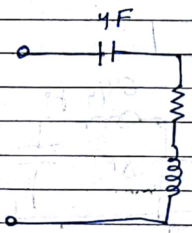
$Y(s) = 4s + \frac{2s}{s+1}$

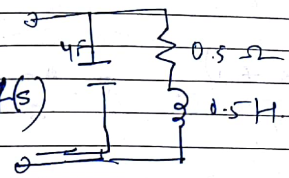
$= Y_1 + Y_2 \rightarrow 1/Z_2$

where $Y1 = 4F$

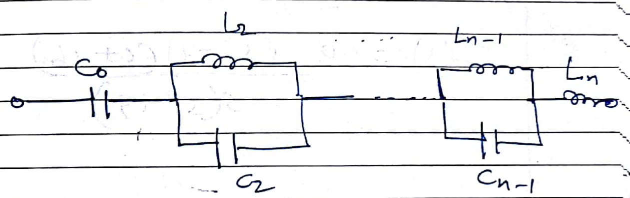
$Z_2 = \frac{1}{2} + \frac{1}{2s}$

$4s + \frac{2s}{s+1+}$





Foster Form



Fosters 1st Form.

$C_0 = \frac{1}{A_0}$

$L_n = L_\infty = H$

$L_2 = \frac{2A_2}{\omega_2^2}$ $\quad L_n = \frac{2A_n}{\omega_n^2}$

$C_2 = \frac{1}{2A_2}$ $\quad C_n = \frac{1}{2A_n}$

$$\frac{(s^2+2s+2)}{(s+1)^2} = \frac{A_0}{s} + \frac{A_2}{s+1} + \frac{A_3}{(s+1)^2} + Hs$$

Q $$Z(s) = 10\,\frac{(s^2+4)(s^2+16)}{s(s^2+9)}$$

$\omega = 0$ ✓ pole exist

$\therefore C_0$ exists

$\omega = \infty$ ✓ pole exist

L_∞ exists

$$Z(s) = \frac{A_0}{s} + \frac{A_2}{s+j3} + \frac{A_2^*}{s-j3} + Hs$$

$$A_0 = \left.\frac{10(s^2+4)(s^2+16)}{s^2+9}\right|_{s=0}$$

$$A_0 = \frac{10\times 4\times 16}{9} = 71.11$$

$$C_0 = \frac{1}{71.11} = 0.0141\ F$$

~~C_2~~ =

$$A_2 = \left.\frac{10(s^2+4)(s^2+16)}{s(s-j3)}\right|_{s=-j3}$$

$$= \frac{10((-j3)^2+4)((-j3)^2+16)}{-j3(-j3-j3)}$$

$$= \frac{10(-9+4)(-9+16)}{-3j(-6j)}$$

$-3\times -6\times j\times j$

$= 18\times j^2$

$= -18$

$$= \frac{350}{18} = 19.45$$

$-3j\times -6j$

$= -3\times -6\times j^2$

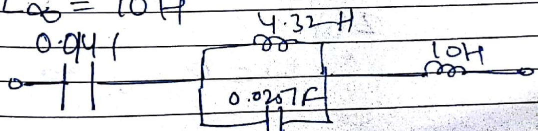
$= 18\times j^2$

$= 18\times -1 = -18$

$$C_2 = \frac{1}{2A_2} = \frac{1}{2\times 19.45} = 0.0257F$$

$$L_2 = \frac{2\times 19.45}{3^2} = 4.32$$

$L_\infty = 10H$



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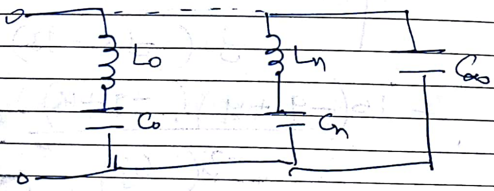
HW
21.37 $Z(s) = \frac{2\cancel{8}(s^2+9)(s^2+16)}{s(s^2+4)}$

Find Fosters' First Form.

- Foster's Second Form



$L_n = \frac{1}{2B_n}$

$C_n = \frac{2B_n}{W_n^2}$

$= \frac{2B_1 s}{s} + \frac{2B_2 s}{(s+2)^2}$

Q 21.36 $Y(s) = \frac{s(s^2+9)}{10(s^2+4)(s^2+25)}$

$Z(s)$.

$W = 0, \infty$

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$Y(s) = \frac{2B_1 s}{s^2+4} + \frac{2B_2 s}{s^2+25}$

$B_1 = \left. \frac{1}{10} \frac{s(s^2+9)}{(s-2j)(s^2+25)} \right|_{s=-2j}$

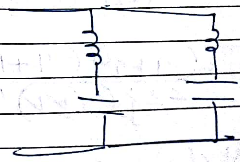
$= +\frac{1}{84}$

$B_2 = \frac{1}{26.25}$.

$L_1 = \frac{84}{2\times} = 42\ H$

$C_1 = \frac{2}{84 \times 2^2} = \frac{1}{168}$

$L_2 \Rightarrow C_2 \Rightarrow \cancel{2}$.



21.38

$$Z(s) = \frac{8(s^2+4)(s^2+25)}{s(s^2+16)}$$

1st & 2nd.

21.37

$$Z(s) = \frac{2(s^2+9)(s^2+16)}{s(s^2+4)}$$

$$= \frac{A_0}{s} + \frac{A_2}{s+2j} + \frac{A^*}{s-2j}$$

$\omega = 0$ $\quad \omega = 2$

Finding A_0,

$$A_0 = \frac{2(9)(\cancel{16})^4}{\cancel{4}}$$

$$\boxed{A_0 = 72}$$

For A_1,

$$A_2 = \frac{2((-2j)^2+9)((-2j)^2+16)}{(-2j)((-2j)^2+4)}$$

$$= \frac{2(4+9)(4+16)}{\cancel{-2j}\cancel{(4+4)}(-2j)^3+4(-2j)}$$

$$= \frac{2((4\times(-1))+9)(-4+16)}{-2j(\cancel{\quad})(-2j)(-2j)}$$

$$= \frac{2(\cancel{45})(12)}{8j^2} = \frac{120}{-8} = -15.$$

$\therefore$ Foster's Form doesn't exist.

21.38 Foster's First Form

$$Z(s) = \frac{8(s^2+4)(s^2+25)}{s(s^2+16)}$$

$$= \frac{A_0}{s} + \frac{A_1}{s+4j} + \frac{A_*}{s-4j}$$

$$A_0 = \frac{2\cancel{8}(4)(25)}{\cancel{16}\,\cancel{4}}$$

$$\boxed{A_0 = 50}$$

$$A_2 = \frac{8((-4j)^2+4)((-4j)^2+25)}{-4j(-4j-4j)}$$

$$= \frac{8(-16+4)(-16+25)}{-4j(-8j)}$$

$$= \frac{\cancel{8}(\cancel{-12}^{3})(9)}{\cancel{+32}\,\cancel{4}}$$

$$\boxed{A_2 = 27}$$

$$C_0 = \frac{1}{A_0} \quad \therefore \boxed{C_0 = 0.02}$$

$$\boxed{L_\infty = H = 8}$$

$$L_2 = \frac{2A_2}{\omega_2^2} \Rightarrow \boxed{L_2 = 3.375} \qquad C_2 = \frac{1}{2A_2} \quad \therefore \boxed{C_2 = 0.0185}$$

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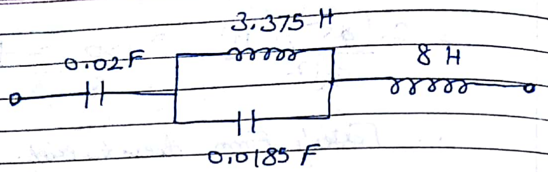

Foster's Second Form

$$Y(s) = \frac{s(s^2+16)}{8(s^2+4)(s^2+25)}$$

$$Y(s) = \frac{2B_1 s}{s^2+4} + \frac{2B_2 s}{s^2+25}$$

$$2B_1 \cancel{s} = \frac{\cancel{s}(s^2+16)}{8(s^2+25)}$$

Put $s = -2j$

$$\therefore B_1 = \frac{1}{16} \cdot \frac{(-4+16)}{(-4+25)}$$

$$\therefore B_1 = \frac{1}{\cancel{16}\,4} \times \frac{\cancel{12}}{\cancel{21}\,7} = \frac{1}{28}$$

$$\Rightarrow \boxed{B_1 = \frac{1}{28} = 0.035}$$

$$2B_2 \cancel{s} = \frac{\cancel{s}(s^2+16)}{8(s^2+4)}$$

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Put $s = -5j$

$$\therefore B_2 = \frac{1}{16} \cdot \frac{(-25+16)}{(-25+4)}$$

$$\Rightarrow B_2 = \frac{1}{16} \times \frac{\cancel{9}\,3}{\cancel{21}\,7} = \frac{3}{112}$$

~~⇒ Foster's Second form doesn't exist.~~

~~21/4/24~~

~~Given $Y(s) = \frac{s^2+2s+20}{s+10}$~~

~~Pole → $s = -10$~~

~~Zeroes → $s^2+2s+20 = 0$~~

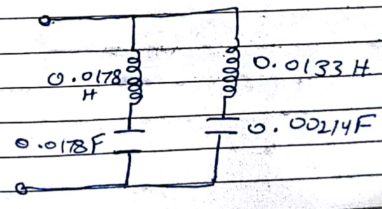
~~$\Rightarrow s = -1 \pm i\sqrt{19}$~~

1/56 3/224

$$L_1 = \frac{1}{2B_1} = 0.0178 \qquad 20/7$$

$$L_2 = \frac{1}{2B_2} = 0.0133 \qquad 18.66$$

$$C_1 = \frac{2B_1}{\omega_1^2} = \frac{1}{14 \times 4} = 0.0178 \qquad C_2 = \frac{2B_2}{\omega_2^2} = \frac{3}{56 \times 25} = 0.002142$$



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21.21

$$Y(s) = \frac{s^2+2s+20}{s+10}$$

Poles $\rightarrow s = -10$

Zeroes $\rightarrow s^2+2s+20 = 0$

$\Rightarrow s = -1 \pm i\sqrt{19}$ — x

$$Re[Y(j\omega)] = \frac{(j\omega)^2+2(j\omega)+20}{j\omega+10} \times \frac{-j\omega+10}{-j\omega+10}$$

$$= \frac{-\omega^2+2j\omega+20}{j\omega+10} \times \frac{-j\omega+10}{-j\omega+10}$$

$$= Re\left[\frac{j\omega^3 - \omega^3 + 2\omega^2 + 10j\omega - 20j\omega + 200}{\omega^2+100}\right]$$

$$= \frac{-\omega^3 + 2\omega^2 + 200}{\omega^2+100} \qquad \frac{-8\omega^2+200}{\omega^2+100}$$

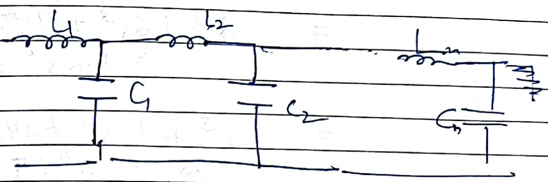
$Re[Y(j\omega)] < 0$ for all $\omega > 5$

$\Rightarrow$ $Y(s)$ is not a PR function.

$\omega > 5$

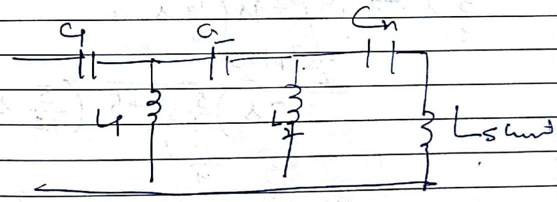
Real [F(s)]

Cauer form.



Cauer 1st form.

$$Z(s) = \cfrac{1}{Y_2(s) + \cfrac{1}{Z_3(s) + \cfrac{1}{Y_4(s) + \dots}}}$$



Q. $$Z_{LC}(s) = \frac{s(s^2+4)(s^2+6)}{(s^2+1)(s^2+5)}$$

$$= \frac{(s^3+4s)(s^2+6)}{s^4+6s^2+5}$$

$$= \frac{s^5+6s^3+4s^3+24s}{s^4+6s^2+5}$$

$$= \frac{s^5+10s^3+24s}{s^4+6s^2+5}$$

$s^4+6s^2+5 \,)\, s^5+10s^3+24s \,(\, s$

$\quad s^5+6s^3+5s$

$\quad 4s^3+19s \,)\, s^4+6s^2+5 \,(\, \frac{s}{4}$

$\quad\quad s^4+\frac{19}{4}s^2$

$\quad\quad \frac{5}{4}s^2+5$

$6-\frac{19}{4}$

$24-19$

$\frac{5}{4}s^2+5 \,)\, 4s^3+19s \,(\, \frac{16}{5}s$

$\quad 4s^3+16s$

$\quad 3s \,)\, \frac{5}{4}s^2+5 \,(\, \frac{5}{12}s$

$\quad\quad \frac{5}{4}s^2$

$\quad\quad 5 \,)\, 3s \,(\, \frac{3}{5}s$

$\quad\quad\quad 3s$

$\quad\quad\quad 0$

$$Z_{LC}(s) = s + \cfrac{1}{\frac{s}{4} + \cfrac{1}{\frac{16s}{5} + \cfrac{1}{\frac{5s}{12} + \cfrac{1}{\frac{3s}{5}}}}}$$

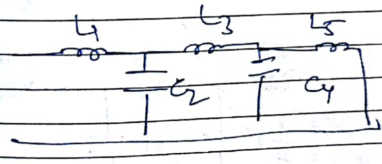
$L_1 = 1\,H$

$C_2 = \frac{1}{4}\,F$

$L_3 = \frac{16}{5}\,H$

$C_4 = \frac{5}{12}\,F$

$L_5 = \frac{3}{5}\,H$



21.42 $Z(s) = \dfrac{s^5+5s^3+3s}{s^4+3s^2+1}$

Find Cauer's I^{st} Form.

$s^4+3s^2+1 \,)\, s^5+5s^3+3s \,(\, s$

s^5+3s^3+s

$2s^3+2s \,)\, s^4+3s^2+1 \,(\, \frac{s}{2}$

s^4+s^2

$2s^2+1 \,)\, 2s^3+2s \,(\, s$

$2s^3+s$

$s \,)\, 2s^2+1 \,(\, 2s$

$2s^2$

$1 \,)\, s \,(\, s$

s

0

$$Z(s) = s + \cfrac{1}{\cfrac{s}{2} + \cfrac{1}{s + \cfrac{1}{2s + \cfrac{1}{s}}}}$$

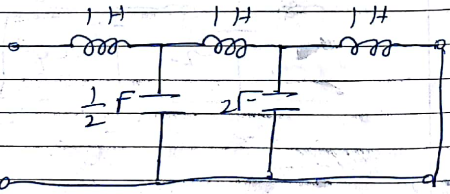
$L_1 = 1\,H$

$C_2 = 1/2\,F$

$L_3 = 1\,H$

$C_4 = 2\,F$

$L_5 = 1\,H$



21.46 $Z(s) = \dfrac{s^4+4s^2+3}{2s^3+3s} = \dfrac{3+4s^2+s^4}{3s+2s^3}$

Cauer II form.

$3s+2s^3 \,)\, 3+4s^2+s^4 \,(\, \frac{1}{s}$

$3+2s^2$

$2s^2+s^4 \,)\, 3s+2s^3 \,(\, \frac{3}{2s}$

$3s+\frac{3}{2}s^3$

$\frac{1}{2}s^3 \,)\, 2s^2+s^4 \,(\, \frac{4}{s}$

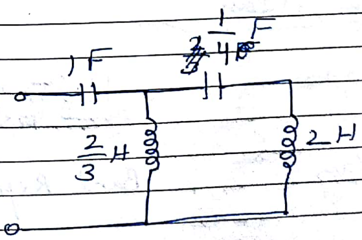
$2s^2$

$s^4 \,)\, \frac{1}{2}s^3 \,(\, \frac{1}{2s}$

$\frac{1}{2}s^3$

0

$$Z(s) = \frac{1}{s} + \cfrac{1}{\cfrac{3}{2s} + \cfrac{1}{\cfrac{4}{s} + \cfrac{1}{\cfrac{1}{2s}}}}$$



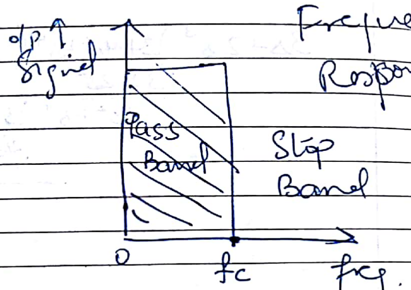
GATE
CMAT
CAT

(MOOCs)

GE education

18.10 HW
21.4.8

Passive Filters



Frequency Response

f_c = cut off frequency

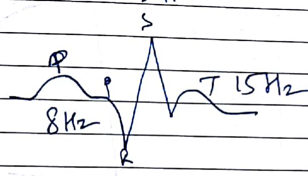
Ideal filters — 100% Pass Band
0% Stop Band

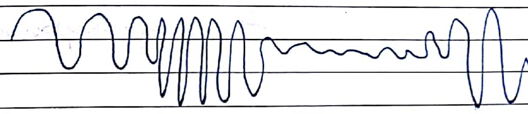
Practical filters

Low pass — High Pass

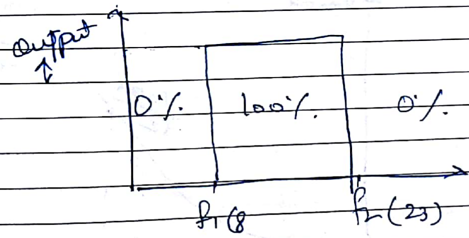
Band pass — Band Reject / Band stop

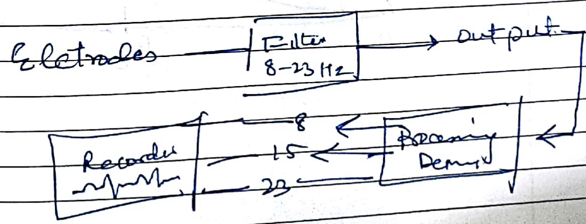
Use of filters



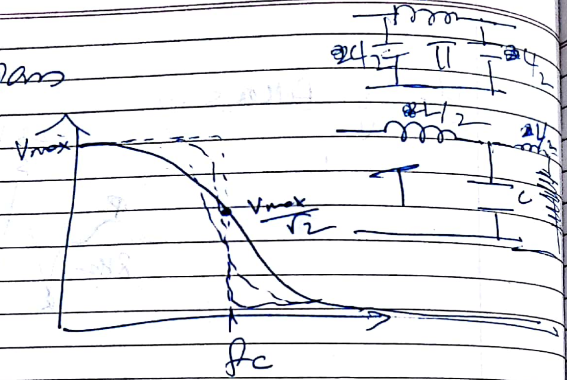


useful part = 8 Hz – 23 Hz



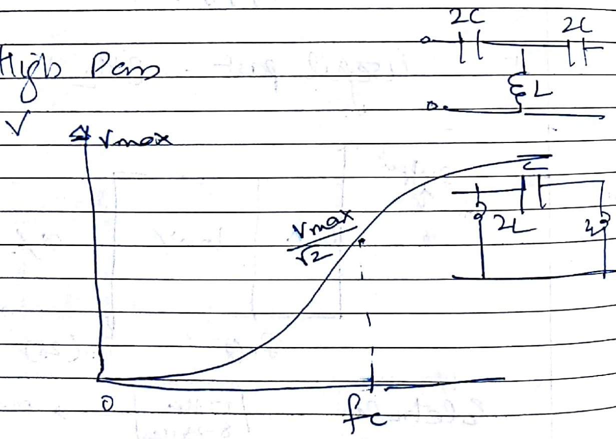


Low pass

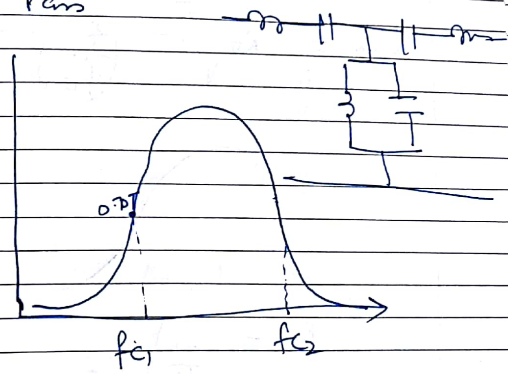


frequency Response of LPF

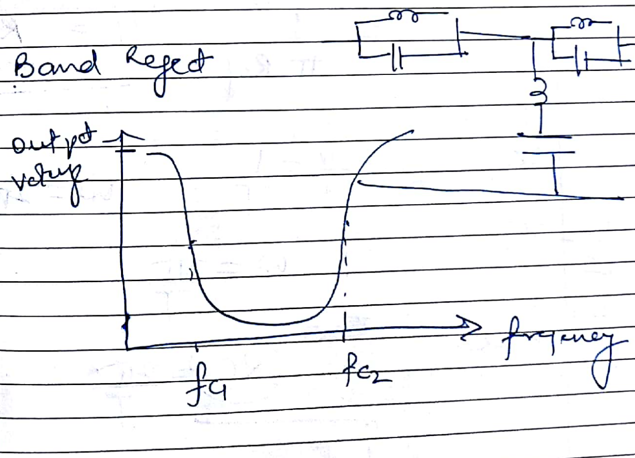
High pass



Band pass



Band Reject



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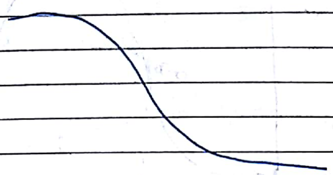
$$Z_o = R_o\sqrt{1-\left(\frac{f}{f_c}\right)^2} \quad \text{LPF}$$



$$f_c = \frac{1}{\pi\sqrt{LC}}$$

$$C = \frac{1}{\pi R_o f} \qquad \boxed{L = \frac{R_o}{\pi f}}$$

$$\frac{1}{LC} \qquad f = \frac{1}{T} \qquad \omega = 2\pi f$$

$$\frac{1}{LC} = 2\pi f$$

$$\omega = \frac{2\pi}{T}$$

$$f = \frac{1}{\cancel{2}\pi LC}$$

$$RC = \frac{2\pi}{T}$$

$$f = \frac{1}{2\pi RCT}$$

$$\boxed{C = \frac{1}{\pi R f}}$$

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18.1 Low pass filter has an impedance R_o. Find characteristic impedance at $0.9\,f_c$.

$$Z_o = R_o\sqrt{1-\left(\frac{f}{f_c}\right)^2}$$

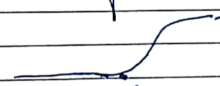
$$= R_o\sqrt{1-(0.9)^2}$$

$$= R_o\sqrt{1-0.81}$$

$$= 0.435\,R_o$$

HPF

$$Z_o = R_o\sqrt{1-\frac{f_c^2}{f^2}}$$



f_c

18.8 LPF $R_o = 500\ \Omega$

$f_c = 5\ \text{kHz}$

$$C = \frac{1}{\pi R_o f_c} = \frac{1}{\pi \cdot 500\times 5000}$$

$$= 1.273\times10^{-7}\ \text{F}$$