Unit 2 Q·M (Part a)
Cont.

Physics.
II Sem.
1st Sem.
3rd Sel.
Dr. Pragati.

Chapter-4a **Quantum Mechanics** (Unit II)

part-a

1. **de-Broglie hypothesis** *for wave-particle Duality:*
   - *derivation of De Broglie wavelength (Non-relativistic case)*
   - *Davisson and Germer experiment*
2. ***Wave Packet***, *Phase and group velocities,*
3. ***Uncertainty principle*** *and*
   - *its Explanation by Experiment*
   - *Simple applications of uncertainty Principle*
4. ***Wave function*** *and its physical significance*
   - *Conditions of an acceptable wave function*

PPT slides presented by: Dr. Pragati sharma
Applied Physics-I

Chapter-4b **Quantum Mechanics** (Unit-II)
to be given later

Part-b

5. ***Postulates*** *of Quantum mechanics,*
   - *Wave function & Probability density function*
   - *Quantum Mechanical operators*
   - *Expectation value.*
6. ***Schrodinger's Wave Equation;***
   - *Free particle Schrodinger equation*
   - *Time dependent Schrodinger equation*
   - *Time independent Schrodinger equation*
7. ***"Particle in a box"*** *(1-D) – its solution by Sch. Wave Eq.*

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Applied Physics-I

Classical mechanics

describes the motion of particle in macroscopic scale.

It cannot explain:

- *The motion of electrons, protons etc.,*
- *Stability of atoms,*
- *Spectral distribution of black body radiation,*
- *Origin of discrete spectra of atoms,*
- *Phenomena like photoelectric effect, Compton's effect, Raman effect*

Inefficiency of classical mechanics led to the development of quantum mechanics.

3

Quantum mechanics

describes the motion & interaction of particles at microscopic scale. Development of quantum mechanics took place in two stages, which is the basis of Quantum Wave Mechanics

***First stage** involves* *Wave → Particle*

- *Max Plank's hypothesis,* $E=h\nu$

  "Light is a Wave, can act as Particle"

***Second stage** involves* *Moving particle → Wave*

- *deBroglie's hypothesis of matter wave,* $\lambda=h/p$

  "Material particle in motion possesses a wave like nature"

- *Heisenberg's Uncertainty Principle,* $\Delta x \Delta p \geq \frac{h}{4\pi}$

4

In response to dual characteristic of light by Plank's theory,

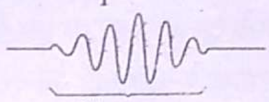
de Broglie introduced the hypothesis of matter wave that

"Every moving particle such as electron, proton etc., possesses a wave like nature". The wave associated with moving particle is known as de- Broglie waves, the wavelength of which is given by

$$\lambda_d = \frac{h}{p}$$ *where*

'h' is Planck's constant, 6.63×10^{-34} Js and 'p' particle's momentum.

de Broglie wave for a moving particle is Represented as Wave packet i.e. group of waves"



Wave group

5

Derivation of de-Broglie equation: *Consider the case of photon*

If photon is considered as wave of frequency ν, then its energy is $E = h\nu$

If it is considered as particle of mass m, then its energy is $E = mc^2$

$$h\nu = mc^2$$

Photon has no rest mass (m_o=0), but has a momentum, $p = mc$

$$h\nu = pc$$

$$\frac{hc}{\lambda} = pc$$

$$\lambda = \frac{h}{p}$$

de-Broglie assumed that this equation is a general formula applicable to any moving particle having momentum, i.e. to both photons & material particles.

Wavelength associated with de-Broglie's matter wave: $\lambda_d = \frac{h}{p}$

➤ *For a massless particle, like Photon* $\lambda_d = \frac{h}{mc}$

➤ *For material particle of mass m, moving with* v *velocity,* $\lambda_d = \frac{h}{mv}$

- *For particles moving with velocity* ($v \ll c$), *the Non-Relativistic expressions for de-Broglie wave for various situations are*

1) *If material particle has kinetic energy,* $K = \frac{1}{2} m v^2$

$$\lambda_d = \frac{h}{\sqrt{2mk}} \qquad (1)$$

2) *If particle of charge q is accelerated through a pot. difference V volts, their kinetic energy is given by* $k = qV$

$$\lambda_d = \frac{h}{\sqrt{2mqV}} \qquad (2)$$

7

3) *When a material particles are in thermal equilibrium at temperature T, they posses a Maxwell's distribution of velocities & their average kinetic energy is given by* $K = \frac{3}{2} kT$

$$\lambda_d = \frac{h}{\sqrt{3mkT}} \qquad (3)$$

- *For particles moving with velocity* ($v \lesssim c$) or having a kinetic energy ($K \lesssim E_0$), *the Relativistic expressions for de-Broglie wave*

$$\lambda_d = \frac{hc}{pc} = \frac{hc}{\sqrt{E^2 - E_0^2}} = \frac{hc}{\sqrt{(k+E_0)^2 - E_0^2}}$$

$$\lambda_d = \frac{hc}{\sqrt{k(k+2E_0)}}$$

$$\lambda_d = \frac{hc}{\sqrt{k(k+2m_0c^2)}} \qquad \text{(4) refer Appendix}$$

8

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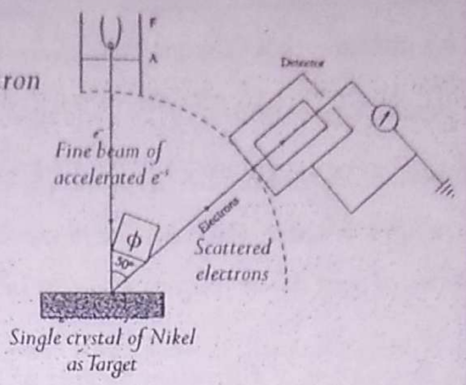
Davisson-Germer Experiment

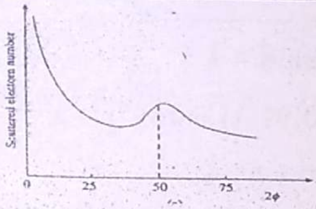
to establish wave nature of moving electron



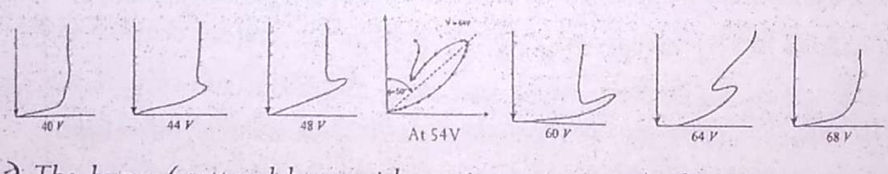
In the experiment a fine beam of accelerated electrons is made to incident normally on the target of nickel crystal. As the electrons hit the crystal, they get scattered by the atoms of the crystal in different direction. The electrons scattered at different positions, are detected by a rotating detector placed on a circular scale. A sensitive galvanometer connected to the detector, records the current (i) and hence the intensity (I) of scattered beam.

9

The intensity of scattered beam is measured for different angles of scattering ϕ w.r.t. the incident beam.



The experiment was performed at different applied voltages such as 40V, 44V, 48V, 54V, 60V, 64V & 68V. Polar curves are plotted between the intensity of scattered beam angle of scattering, ϕ for different accelerating potentials.



→ *The hump (scattered beam with maximum intensity) always occur in the curve along $\phi = 50°$ & but it becomes most pronounced at 54eV, while at rest of the potentials (i.e. at electrons energy other than this), the hump diminishes.*

⇒ *This indicates that "the incident electrons are found to be scattered the most at accelerating potential of 54V, along $\phi = 50^\circ$ w.r.t. the incident beam"*

⇒ *If moving electron behaves as wave, then the appearance of maximum in a particular direction could be due to the constructive interference of electrons diffracted from different layers of atoms in the crystal.*

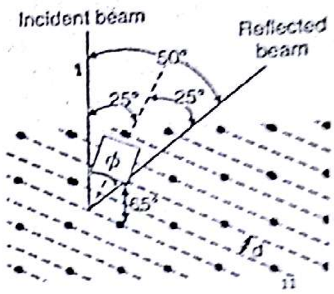
From Bragg's law of diffraction for first order

$$2d\sin\theta = n\lambda$$

where d is interplaner spacing for Ni crystal

$$d = 0.91 Å$$

θ is glancing angle, the angle between the scattered beam of electron and planes of atoms of the crystal, $\theta = \frac{1}{2}(180^\circ - 50^\circ) = 65^\circ$



$$2d\sin\theta = \lambda \; ;$$

$$2(0.91 Å)\sin(65^\circ) = \lambda$$

$$\lambda = 1.65 Å \qquad (1)$$

From de-Broglie hypothesis, the wavelength of Electron $\lambda = \frac{h}{p}$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}} = \frac{12.25}{\sqrt{V}} = \frac{12.25}{\sqrt{54}} = 1.67 Å \quad — \quad (2)$$

The theoretical value of wavelength of electron from de-Broglie's hypothesis is

$\lambda = 1.66 Å$ *at operating voltage of 54V,*

Result: *The close agreement between the experimental value of wavelength for electron (from Bragg's diffraction formula) & theoretical values (from de-Broglie hypothesis of matter wave) shows that electrons have wave like nature & hence confirms the de-Broglie hypothesis of matter wave.*

12

Properties of de-Broglie's matter wave are:

1) Wavelength associated with de-Broglie's matter wave is $\lambda_m = \frac{h}{mv}$

2) If $v=0$, $\lambda=\infty$!! Matter wave is associated with moving particles only

3) For a particle, of larger mass m, moving with velocity v, a shorter wavelength λ is associated.

4) matter wave are neither electromagnetic nor acoustic wave, but just a hypothetical matter wave. It is just a wave model to describe a moving matter in microscopic world.

5) It is also known as pilot wave.

6) The velocity of matter wave depends on the wavelength, even if the particle is moving in vacuum. Whereas all electromagnetic waves travel with same velocity, independent of the wavelength.

13

8) It exhibits diffraction phenomena as any other waves.

9) Matter wave for moving particle is represented as Wave packet i.e. equivalent to a group of waves.

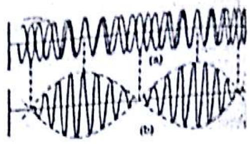
9) The wave nature of matter introduces an uncertainty in the position of the particle because the moving particle cannot be available at fixed point. Therefore de-Broglie waves are called probability waves. The amplitude of the wave reveals the probability of finding the particle in space at a particular time. A larger amplitude means larger probability at that position.

14

Phase velocity & Group velocity in Wave Mechanics

Phase velocity $\quad v_p = \frac{\omega}{k} = \frac{E}{p}$

Group velocity $\quad v_g = \frac{d\omega}{dk} = \frac{dE}{dp}$



General relationship between v_p and v_g

$$v_g = v_p + k\frac{dv_p}{dk}$$

$$v_g = v_p - \lambda\frac{dv_p}{d\lambda}$$

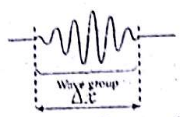
Concept of Wave packet for de Broglie wave of moving particle

Phase velocity of de-Broglie wave has no physical meaning.

$$v_p = \frac{\omega}{k} = \frac{E}{p} = \frac{mc^2}{mv_g} = \frac{c^2}{v_g} \quad \text{!! Unexpected result as } v_p \ngtr c$$

Group velocity of de-Broglie wave represents the particle's velocity, v

$$v_g = \frac{d\omega}{dk} = \frac{d\omega/dv}{dk/dv} = v$$

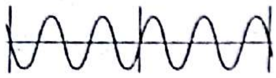


"Material particle in motion is equivalent to a group of waves"

Concept of Phase velocity

In Wave Mechanics

It is defined as the rate at which planes of constant phase advances in space. It simply represents the velocity of individual wave.



From the definition

$$(kx - \omega t) = Constant$$

$$\frac{d}{dt}(kx - \omega t) = 0$$

$$k\frac{dx}{dt} - \omega = 0$$

$$\frac{dx}{dt} = \frac{\omega}{k}$$

$$v_p = \frac{\omega}{k} = \frac{E}{p}$$

$$v_p = \frac{\omega}{k} = \frac{E}{p}$$

Where

$E = \hbar\omega$ *from Plank's*

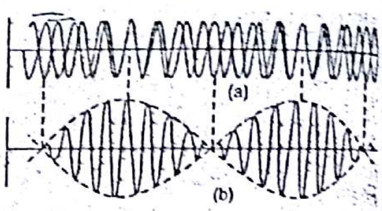
$p = \hbar k$ *from de-Broglie*

17

Concept of wave packet and Group velocity

In Wave Mechanics

A **wave packet** is a group of waves each with slightly different frequency. In a medium, the velocities of its different components are different. The observed velocity, however, is the Group velocity.

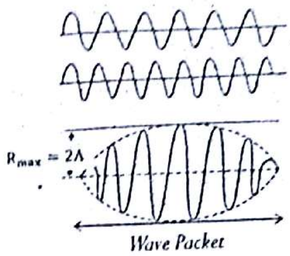


Group velocity is the resultant velocity of a **wave packet**. The velocity with which the maximum amplitude (or envelope) of the group advances in space. In other words it represents the velocity with which the information (or energy) is conveyed along the wave. It is given by

$$v_g = \frac{d\omega}{dk} = \frac{dE}{dp}$$

Derivation:

Consider two SHW of same amplitude A travelling simultaneously along x-direction, differing in angular frequencies by amount $\Delta\omega=(\omega_1-\omega_2)$ *and wave number by amount* $\Delta k=(k_1-k_2)$



Their instantaneous displacement is y_1 & y_2

$$y_1 = A\cos(\omega_1 t - k_1 x) \text{ and}$$
$$y_2 = A\cos(\omega_2 t - k_2 x)$$

Their superposition will give: $y = y_1 + y_2$

$$y = A\cos(\omega_1 t - k_1 x) + A\cos(\omega_2 t - k_2 x)$$

$$y = 2A\cos\left[\frac{(\omega_1+\omega_2)t}{2} - \frac{(k_1+k_2)x}{2}\right]\cos\left[\frac{(\omega_1-\omega_2)t}{2} - \frac{(k_1-k_2)x}{2}\right]$$

$$y = 2A\cos\left[\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right]\cos(\omega t - kx)$$

$$y = R\cos(\omega t - kx)$$

This represents the resultant wave having Angular frequency ω, *Propagation constant* k *and Amplitude R as*

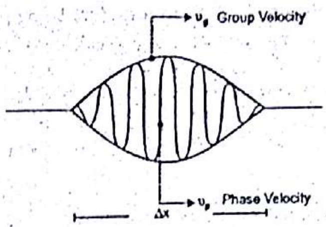
$$\omega = \frac{\omega_1+\omega_2}{2} \ \& \ k = \frac{k_1+k_2}{2}$$

$$R = 2A\cos\left[\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right]$$

The amplitude of the resultant wave is modulated both in space and time by a slowly varying envelops of frequency $\frac{\Delta\omega}{2}$ and *Propagation constant* $\frac{\Delta k}{2}$. The velocity with which the envelop moves represents the *group velocity* of wave packet v_g given by

$$v_g = \lim_{\Delta k \to 0}\frac{\Delta\omega}{\Delta k} = \frac{d\omega}{dk}$$

General relationship between v_p and v_g *In Wave Mechanics*



Phase velocity, $v_p = \frac{\omega}{k}$

Group velocity, $v_g = \frac{d\omega}{dk} = \frac{d}{dk}(kv_p)$

$$v_g = v_p + k\frac{dv_p}{dk}$$

$$= v_p - \lambda\frac{dv_p}{d\lambda}$$

In a dispersive medium, where v_p is frequency dependent, $\frac{dv_p}{d\lambda} \neq 0$

$$v_g < v_p$$

1

However, in a non-dispersive medium, where v_p is constant, $\frac{dv_p}{d\lambda} = 0$

$$v_g = v_p$$

21

Q: An electron has a de Broglie wavelength of 2 pm ($=2\times10^{-12}$ m). Find its kinetic energy and the Phase and Group velocities of its de-Broglie waves? [$K=E-E_0=292$keV, $v_p=1.3c$; $v_g = 0.77c$

Hint: First step is to find pc

$$pc = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{2\times10^{-12}\times1.6\times10^{-19}} = 620\text{ keV . sec}$$

pc is comparable to the rest mass energy of electrons

Relativistic expression for kinetic energy is used $K=E-E_0$

$$K=\sqrt{(pc)^2+E_0{}^2}-E_0=\sqrt{(620\text{keV})^2+(511\text{keV})^2}-511\text{eV}$$

$$K=803\text{ keV}-511\text{ keV}=292\text{ keV}$$

$$v_g = v = c\sqrt{1-\frac{E_0{}^2}{E^2}}$$

$$v_g = \frac{c^2}{v}$$

Phase velocity v_p *(of de-Broglie wave)*

Phase velocity is the rate at which planes of constant phase advances in space.

$$v_p = \frac{\omega}{k} = \frac{E}{p} \quad \textit{by definition}$$

Angular frequency, $\omega = \frac{E}{\hbar} = \frac{mc^2}{\hbar}$

Propagation constant, $k = \frac{p}{\hbar} = \frac{mv}{\hbar}$

Where

$E = mc^2$

$p = mv$

$$v_p = \frac{mc^2}{mv} = \frac{c^2}{v};$$ Unexpected result !! as $v_p \ngtr c$

➢ *If* $v_p = c^2/v$, *the de-Broglie wave associated with the moving particle would travel faster than particle itself, leaving the particle far behind, which is unrealistic. So moving particle cannot be equivalent to a single wave train. Phase velocity of de-Broglie wave has no physical meaning*23

Group velocity v_g *(of de-Broglie wave)*

v_g *is defined as the resultant velocity with which the maximum amplitude (or the envelope) of the group advances in space.*

$$v_g = \frac{d\omega}{dk} = \frac{d\omega/dv}{dk/dv} \quad \textit{by definition}$$

If angular frequency ω *& propagation constant* k *of de-Broglie wave are associated with the body of mass* m, *moving with velocity,* v

where m is relativistic mass, $m = \frac{m_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$ then

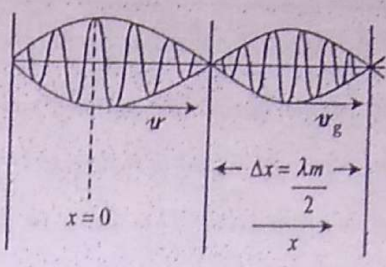
$$\omega = \frac{E}{\hbar} = \frac{mc^2}{\hbar} \qquad \omega = \frac{1}{\hbar}\frac{m_0c^2}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$$

$$k = \frac{p}{\hbar} = \frac{mv}{\hbar} \qquad k = \frac{1}{\hbar}\frac{m_0c^2}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$$

24

$$\frac{d\omega}{dv} = \frac{1}{\hbar}\frac{m_0 v}{\left(1-\frac{v^2}{c^2}\right)^{3/2}} \;\&$$

$$\frac{dk}{dv} = \frac{1}{\hbar}\frac{m_0}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}$$



Thus de-Broglie group velocity

$$v_g = \frac{d\omega/dv}{dk/dv} = v$$

➢ *de-Broglie wave group associated with a moving particle travels with the same velocity as the particle, hence "Material particle in motion is equivalent to a group of waves"*

25

Non-relativistic relationship between v_p & v_g

de-Broglie's particle velocity, $v = v_g$

Phase velocity, $v_p = \nu\lambda$

Wavelength from de-Broglie hypothesis $\lambda = \frac{h}{mv_g}$

Frequency from Planks hypothesis, $\nu = \frac{E}{h} = \frac{mv_g^2}{2h}$

$$v_p = \frac{mv_g^2}{2h} \times \frac{h}{mv_g} \quad \textit{gives}$$

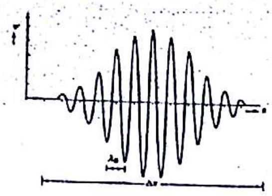
$$v_p = \frac{1}{2}v_g$$

For a non-relativistic free particle, Phase velocity is half of Group velocity

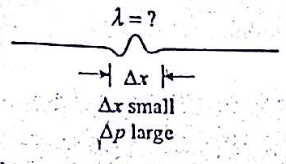
26

A wave nature of moving particle leads to Uncertainty principle

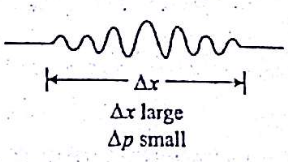
In Quantum mechanics the moving particle is described in terms of a matter wave (or a wave packet). The particle may be located anywhere within the group at a given time. This leads to fundamental limits to the accuracy of measuring the particle's position & momentum (or energy & time).



→ A wave function $\Psi(r,t)$ represents the state of a system, i.e. the amplitude of the matter wave at any point in space. Wave function for a free particle moving in x-direction, is represented mathematically in terms of



Narrower the wave group, more precisely a particle's position can be specified. However, particle's momentum can not be a precise quantity, because in a narrow packet, the wavelength of wave is not well defined (as there are not enough waves to measure λ accurately). And Since $p=h/\lambda$, the particle's momentum can not be a precise quantity.



Wide the wave group, more clearly defined is the wavelength. The particle's momentum $p=h/\lambda$ corresponding to this wavelength is therefore a more precise. But since the width of the group is too wide, it is impossible to say exactly where the particle is at a given time.

28

Heisenberg's Uncertainty principle (1927)

"It is impossible to measure simultaneously both the exact position & exact momentum of an object at the same time" $\Delta x \Delta p \geq \frac{h}{4\pi}$

The product of uncertainty in measuring the position Δx *& momentum* Δp *of a particle, in the same direction at the same instant is approximately equal to Plank's constant,* h $(6.6\times10^{-34}$ J.s)

This holds for other all canonical conjugate pairs also like between

- *Energy & Momentum as:* $\Delta E\, \Delta t \geq \frac{h}{4\pi}$
- *Angular position & Angular momentum as:* $\Delta \phi\, \Delta L \geq \frac{h}{4\pi}$

29

Simple applications of uncertainty Principle

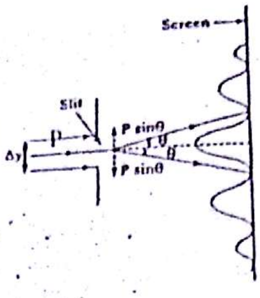
Using uncertainty principle, one can determine

i. *The nonexistence of electron in the nucleus. using* $\Delta x \Delta p_x \geq \frac{h}{4\pi}$
ii. *Existence of neutron, proton, α-particle in the nucleus.*
iii. *Binding energy of electron in an atom.*
iv. *Finite width of spectral lines, using* $\Delta E \approx \frac{h}{\Delta t}$
v. *Strength of nuclear forces, using* $p \approx \frac{h}{4\pi r_0}$

Refer any book for more details
& Numericals

Experiment Explanation of Uncertainty principle

1. Diffraction of electron beam at single slit:



Consider a parallel beam of electron having momentum p, passing through a narrow slit of width Δy. If Δy is comparable to the wavelength of electron beam, it produces a diffraction pattern on the screen.

From the condition of diffraction minima: $\Delta y \sin\theta = \lambda$ (n=1 for first order)

The electron at the slit, could be anywhere within Δy, which defines the uncertainty in the position of electron

$$\Delta y = \frac{\lambda}{\sin\theta} \qquad (1)$$

Electrons at the slit deviate from $-\theta$ to $+\theta$, (for 1st order minima), the y-component of momentum of electron, varies anywhere b/n $p\sin\theta$ & $-p\sin\theta$.

31

So the uncertainty in the momentum of electron along Y

$$\Delta p_y = (p\sin\theta) - (-p\sin\theta)$$

$$\Delta p_y = 2p\sin\theta$$

$p = \frac{h}{\lambda}$, Momentum of de Broglie electron wave

$$\Delta p_y = \frac{2h}{\lambda}\sin\theta \qquad (2)$$

From equation (1) & (2) we have

$$\Delta y \Delta p_y = \frac{\lambda}{\sin\theta} \cdot \frac{2h}{\lambda}\sin\theta$$

$$\Delta y \Delta p_y = 2h$$

$$\Delta y \Delta p_y \approx 2h \quad \text{which is} \geq \frac{h}{4\pi}$$

This is in good argument with uncertainty principle.

32

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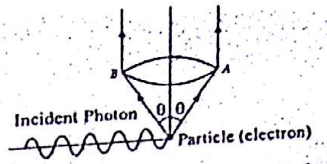
2. Determination of position of electron by γ-Ray microscope:

Consider a process of locating the position of a particle, say electron, by a microscope.



⇒ In order that the electron may be seen through the microscope, the incident photon, after striking the electron, must be scattered into the microscope objective. Let θ be vertical angle of cone of light collected by the microscope.

The smallest distance b/n two points which can be resolved by the microscope, represents the uncertainty in the position of the particle. is given by

$$\Delta x \approx \frac{\lambda}{2\sin\theta}$$ where λ is the wavelength of incident light

Let initial momentum of incident photon be $p = h/\lambda$. After scattering, the x component of momentum varies anywhere between $p\sin\theta$ & $-p\sin\theta$.

33

Since the momentum is conserved in the collision, the uncertainty in the momentum along x is $\Delta p_x = (p\sin\theta) - (-p\sin\theta)$

$$\Delta p_x = 2p\sin\theta$$

$$\Delta p_x = \frac{2h}{\lambda}\sin\theta \qquad (2)$$

From equation (1) & (2) we have

$$\Delta x \Delta p_x = \frac{\lambda}{2\sin\theta} \times \frac{2h}{\lambda}\sin\theta$$

$$\Delta x \Delta p_x = h$$

$$\Delta x \Delta p_x \approx h \qquad \text{which is} \geq \frac{h}{4\pi}$$

This is in good argument with uncertainty principle

34

To prove the nonexistence of electron in the nucleus, using uncertainty principle

⇒ The nuclear radius of any atom is of the order of 0.5×10^{-14}m. If electron is confined inside the nucleus, the uncertainty in the position of electron is

$$\Delta x \approx 1\times 10^{-14}\text{m}$$

⇒ using Uncertainty Principle, $\Delta x \Delta p \geq \frac{h}{4\pi}$,

the minimum uncertainty in the momentum of electron is given by

$$\Delta p_{min} \geq \frac{\hbar}{2\Delta x}$$

$$\Delta p_{min} \geq \frac{1.054\times 10^{-34}}{2\times 1\times 10^{-14}}$$

$$\Delta p_{min} \geq 0.527\times 10^{-20}\text{ kg m/s}$$

35

It means that the total momentum of electron 'p' in the nucleus must be at least of the order of $p \sim \Delta p_x \sim 0.527\times 10^{-20}$kg m. An electron with such a momentum has a kinetic energy $KE = \frac{p^2}{2m}$

$$K = \frac{(0.527\times 10^{-20})^2}{2\times 9.1\times 10^{-31}}\text{ J} = \frac{(0.527\times 10^{-20})^2}{2\times 9.1\times 10^{-31}\times 1.6\times 10^{-19}}\text{ eV}$$

$$K = 95.3\times 10^{6}\text{ eV}$$

$$K \approx 95\text{ MeV}$$

⇒ The electron inside the nucleus can exists only if it possess the energy of the order of 95 MeV. However, the maximum possible kinetic energy of an electron emitted during B-decay in radioactive nuclei is about 4 MeV. Hence, <u>it is concluded that the electron can not reside inside the nucleus.</u>

36

To find the energy of a particle in a Box or ∞ Potential Well, using uncertainty principle

⇒ *Consider a particle having a mass m in infinite potential well of width L. The maximum uncertainty in position of the particle may be*

$$(\Delta x)_{max} = L$$

⇒ *using Uncertainty Principle,* $\Delta x \Delta p_x \approx \frac{\hbar}{2}$

the minimum uncertainty in the momentum of electron is given by $\Delta p_x \geq \frac{\hbar}{2L}$

⇒ *An electron with such a momentum has a kinetic energy* $KE = \frac{p^2}{2m}$

$$KE = \frac{\hbar^2}{8mL^2}$$

This is the minimum kinetic energy of particle in an infinite potential well of width L

37

Q. A hydrogen atom is 5.3×10^{-11} *m in radius. Use the uncertainty principle to estimate the minimum energy an electron can have in this atom*

3) When a material particles are in thermal equilibrium at temperature T, they posses a Maxwell's distribution of velocities & their average kinetic energy is given by $K = \frac{3}{2}kT$

$$\lambda_d = \frac{h}{\sqrt{3mkT}} \qquad (3)$$

Actually, the kinetic energy of an electron in the lowest energy level of hydrogen atom is 13.6eV

Q. What is the minimum uncertainty in the energy state of an atom if an electron remains in this state for 10^{-8} seconds?

Uncertainty in measurement of time $\Delta t = 10^{-8}\,s$

From energy-time uncertinty relation $\Delta E \Delta t \geq \frac{\hbar}{2}$

or $\Delta E \geq \frac{h}{4\pi \Delta t}$

Putting the values of Δt, and h we get $\Delta E = 0.329 \times 10^{-7}$ eV

Q. Life time of a nucleus in the excited state is 10^{-12}sec. Calculate the probable uncertainty in energy & frequency of γ ray photon emitted by it

➢ The uncertainty in the life time of a nucleus in the excited state is

$\Delta t \approx 10^{-12}$ sec

➢ Using Uncertainty Principle, $\Delta E \Delta t \geq \frac{h}{4\pi}$, or $\Delta E \Delta t \geq \frac{\hbar}{2}$

➢ The minimum uncertainty in energy of γ ray photon emitted by it

$$\Delta E = \frac{\hbar}{2\Delta t} = 1.054 \times 10^{-22}\ \mathrm{J}$$

➢ The minimum uncertainty in Frequency of γ ray photon emitted by it

$$\Delta \nu = \frac{\Delta E}{h} = 1.59 \times 10^{11}\ \mathrm{Hz}$$

Appendix

Relativistic expression for kinetic energy $K = E - E_0$

Relativistic relationship b/n total energy & momentum $E^2 = (pc)^2 + {E_0}^2$

$E_0 = m_0c^2$ Rest mass energy

$E = mc^2$ Total energy

$m = \frac{m_o}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$; Relativistic mass and

m_0 Rest mass of the particle

For $K \ll E_0$ *Relativistic effect may be ignored* (as $v \ll c$)

Relativistic expression for kinetic energy $K = E - E_0$

reduces to the non-relativistic expression as $K = \frac{p^2}{2m_0}$

➢ As long as $K \ll E_0$ (or $v \ll c$)

Relativistic variation of mass can be ignored

Non relativistic expression for de-Broglie wave is used

$$\lambda_d = \frac{h}{\sqrt{2mk}}$$

➢ For $K \lesssim E_0$ (or $v \lesssim c$)

Relativistic variation of mass must be is taken into a/c

Relativistic expression de-Broglie wave is used

$$\lambda_d = \frac{hc}{pc} = \frac{hc}{\sqrt{k(k + 2m_0c^2)}}$$

Particle	${}_ZX^A$	Rest Mass in kg	Rest Mass in a.m.u.	E_0 in MeV	Relative mass
Proton	${}_1H^1$	1.672×10^{-27}	1.007	938.28	$1837m_e$
Neutron	${}_0n^1$	1.674×10^{-27}	1.008	939.57	$1842m_e$
Electron	${}_{-1}e^0$	9.1×10^{-31}	0.00054	0.511	m_e

m_e = 511 KeV