Mealy and Moore Models T.O.C

The finite automata have binary output i.e o/p is 1 if input string is accepted, otherwise o/p is 0. Mealy and moore models are the one in which output can be chosen from some other alphabet.

A Moore machine is a six-tuple $(Q, \Sigma, \Delta, \delta, \lambda, q_0)$ where

1. Q is finite set of states
2. Σ is input alphabet
3. Δ is output alphabet mapping
4. δ is transition function $\Sigma \times Q$ into Q.
5. λ is output function mapping Q into Δ
6. q_0 is initial state.

A Mealy machine is 6-tuple $(Q, \Sigma, \Delta, \delta, \lambda, q_0)$ where all symbols are same as Moore except λ which maps $Q \times \Sigma$ into Δ.

Note:
1. The difference between moore and mealy m/c is that in moore m/c, o/p ~~depends~~ depends only on states, but in mealy m/c, o/p depends both on states and input alphabet.
2. A finite automaton can be converted to a Moore machine by introducing $\Delta = \{0, 1\}$ and defining $\lambda(q) = 1$ if $q \in F$ and $\lambda(q) = 0$ if $q \notin F$.

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$Q_1 = \{q_3, q_4\}$ $Q_2 = \{q_0, q_1, q_2, q_5, q_6, q_7\}$

$\pi_0 = \{q_3, q_4\}$ $\{q_0, q_1, q_2, q_5, q_6, q_7\}$
$(5,6)$ $(7,6)$ $(1,2)$ $(4,3)$ $(4,3)$ $(3,6)$ $(6,6)$ $(4,6)$

$\pi_1 = \{q_3, q_4\}$ $\{q_0, q_6\}$ $\{q_1, q_2\}$ $\{q_5, q_7\}$
$(5,6)$ $(7,6)$ $(1,2)$ $(6,6)$ $(4,3)$ $(4,3)$ $(3,6)$ $(4,6)$

$\pi_2 = \{q_3, q_4\}$ $\{q_0\}$ $\{q_6\}$ $\{q_1, q_2\}$ $\{q_5, q_7\}$

$\pi_3 = \{q_3, q_4\}$ $\{q_0\}$ $\{q_6\}$ $\{q_1, q_2\}$ $\{q_5, q_7\}$

As $\pi_2 = \pi_3$, we stop.

The minimum state automaton M' is :-

$M' = (Q', \{a, b\}, \delta', \{q_0\}, \{q_3, q_4\})$

Q' are all subsets of π_3 and

δ

State/Σ	a	b
$[q_0]$	$[q_1, q_2]$	$[q_1, q_2]$
$[q_1, q_2]$	$[q_3, q_4]$	$[q_3, q_4]$
$[q_3, q_4]$	$[q_5, q_7]$	$[q_6]$
$[q_5, q_7]$	$[q_3, q_4]$	$[q_6]$
$[q_6]$	$[q_6]$	$[q_6]$

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(14)

Moore machine

Present state	Next state δ		Output
	q=0	q=1	λ
→ q_0	q_3	q_1	0
q_1	q_1	q_2	1
q_2	q_2	q_3	0
q_3	q_3	q_0	0

For input string 0111, Transition of states is

$q_0 \xrightarrow{0} q_3 \xrightarrow{1} q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_2$

o/p is: 0 0 0 1 0

For input string ∧(empty string), o/p is $\lambda(q_0)$. (here $\lambda(q_0) = 0$)

Mealy machine

Present state	Next state			
	q=0		q=1	
	state	o/p	state	output
→ q_1	q_3	0	q_2	0
q_2	q_1	1	q_4	0
q_3	q_2	1	q_1	1
q_4	q_4	1	q_3	0

For input string 0011, Transition of states is

$q_1 \xrightarrow{0} q_3 \xrightarrow{0} q_2 \xrightarrow{1} q_4 \xrightarrow{1} q_3$

o/p is 0100

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Note: (1) In Mealy machine, we get an output of 1 when input is 1. But in Moore, we get output $\lambda(q_0)$ for input 1.

(2) For Moore machine, if input string is of length n, the output string is of length n+1. But in mealy machine if input string is of length n, output string is also of length n.

Procedure for transforming a mealy into a Moore machine

Q: Consider the Mealy machine described by the following transition table. Construct a Moore machine which is equivalent to the Mealy machine.

Mealy Machine

Present state	Next state			
	Input a=0		Input a=1	
	state	output	state	output
→ q_1	q_3	0	q_2	0
q_2	q_1	1	q_4	0
q_3	q_2	1	q_1	1
q_4	q_4	1	q_3	0

Steps

(1) We look into next state column for all states, say q_i and determine the no. of different outputs associated with q_i.

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output 1 is associated with q_1
" 0+1 are " " q_2
" 0 is " " q_3
" 0+1 are " " q_4

(2) If q_i is associated with more than one output say n, then we split q_i into n different states.

Eg. we split q_2 into q_{20} + q_{21}
and we split q_4 into q_{40} and q_{41}

(3) The pair of states and outputs in the next state are rearranged as:-

Moore machine

Present state	Next state a=0	Next state a=1	Output
→ q_1	q_3	q_{20}	1
q_{20}	q_1	q_{40}	0
q_{21}	q_1	q_{40}	1
q_3	q_{21}	q_1	0
q_{40}	q_{41}	q_3	0
q_{41}	q_{41}	q_3	1

Here we observe that initial state q_1 is associated with output 1. This means that with input 1, we get an output of 1 if m/c starts at state q_1. Thus this Moore m/c accepts a null sequence which is not accepted by Mealy m/c. As in Mealy m/c, if input 1, then output is 1. To overcome this, either we neglect the response of Moore m/c to input 1 or we must add [illegible] starting state [illegible] to [illegible]

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are same as q_1, but output is 0.

Moore machine

Present state	Next state $a=0$	Next state $a=1$	Output
→ q_0	q_3	q_{20}	0
q_1	q_3	q_{20}	1
q_{20}	q_1	q_{40}	0
q_{21}	q_1	q_{40}	1
q_3	q_{21}	q_1	0
q_{40}	q_{41}	q_3	0
q_{41}	q_{41}	q_3	1

Procedure for transforming a Moore machine into a Mealy Machine

(1) We have to define the output function λ for mealy machine as a function of present state and input symbol, we define λ' by:-

$$\lambda'(q, a) = \lambda(\delta(q, a))$$ for all states q and all input symbols a.

(2) The transition function is same as that of given Moore machine.

2. Construct a Mealy machine which is equivalent to the following Moore machine:-

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Moore machine

Present state	Next state		Output
	a=0	a=1	
→ q_0	q_3	q_1	0
q_1	q_1	q_2	1
q_2	q_2	q_3	0
q_3	q_3	q_0	0

Mealy machine

Present state	Next state			
	a=0		a=1	
	state	o/p	state	o/p
→ q_0	q_3	0	q_1	1
q_1	q_1	1	q_2	0
q_2	q_2	0	q_3	0
q_3	q_3	0	q_0	0

Note: We can reduce the no. of states in any model by considering states with identical transitions. If 2 states have identical transitions, then we can delete one of them

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Q. Mealy Machine

Present state	Next state			
	a = 0		a = 1	
	state	output	state	output
→ q_1	q_1	0	q_2	0
q_2	q_1	0	q_3	1
q_3	q_1	0	q_3	1

Here q_2 and q_3 are identicals so we can delete one of them say q_3, we get: (and replace q_3 by q_2)

Present state	Next state			
	a = 0		a = 1	
	state	output	state	output
→ q_1	q_1	0	q_2	0
q_2	q_1	0	q_2	1

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Grammar

A Grammar G_1 is defined as quadruple

$$G_1 = (V, T, S, P)$$

where

V is finite set of variables

T is finite set of Terminal symbols

$S \in V$, is start variable

P is set of production rules / substitution rules.

Start Variable usually occurs on the L.H.S. of the topmost rule. We use a grammar to describe a language by generating each string of that language in the following manner;

1. Write down the start variable.
2. Find a variable that is written down and a rule that starts with that variable. Replace the written down variable with the R.H.S. of that rule.
3. Repeat step 2 until no variables remain.

Rules are of form $x \to y$

where $x \in (V \cup T)^+$ ~~left~~ / usually not null string

$y \in (V \cup T)^*$

we write $x \Rightarrow y$

or "x derives y"

"y is derived from x"

If $w_1 \Rightarrow w_2 \Rightarrow w_3 \cdots \Rightarrow w_n$

we say that w_1 derives w_n

& write $w_1 \overset{*}{\Rightarrow} w_n$

* indicates that an unspecified no. of steps can be taken to derive w_n from w_1.

Also $w \overset{*}{\Rightarrow} w$

Let $G = (V, T, S, P)$ be a grammar

$L(G) = \{ w \in T^* : S \overset{*}{\Rightarrow} w \}$

is the language generated by G

If $w \in L(G)$, then the sequence

$S \Rightarrow w_1 \Rightarrow w_2 \Rightarrow \cdots \Rightarrow w_n \Rightarrow w$

is a derivation of sequence w.

The strings $S, w_1, w_2, \cdots w_n$ which contain variables as well as terminal symbols are called sentential forms of the derivation.

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~~Q. $G = (V, T, S, P)$~~

Q. Find language generated by:

$G = (\{S, A\}, \{a, b\}, S, P)$

$S \rightarrow Ab$

$A \rightarrow aAb$

$A \rightarrow \lambda$

~~$A \Rightarrow \lambda$~~

~~$A \Rightarrow aAb \Rightarrow ab$~~

~~$A \Rightarrow aAb \Rightarrow aabb$~~

$S \Rightarrow Ab \Rightarrow b$

~~$S \Rightarrow b$~~

$S \Rightarrow Ab \Rightarrow aAbb \Rightarrow abb$

$S \Rightarrow Ab \Rightarrow aAbb \Rightarrow aaAbbb$
$\Rightarrow aabbb$

$L(G) = \{b, abb, aabbb, \ldots\}$

$L(G) = \{a^n b^{n+1} : n \geq 0\}$

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Q: Find language.

$G_1 = (\{A, S\}, \{a, b\}, S, P_1)$

with P_1 as

$S \to aAb \mid \lambda$

$A \to aAb \mid \lambda$

$S \to aAb$

$S \to \lambda$

$A \to aAb$

$A \to \lambda$

$L(G_1) = \{\lambda, ab, aabb \ldots\}$

$S \Rightarrow \lambda$

$A \Rightarrow aAb \Rightarrow ab$

~~$S \Rightarrow aAb$~~ $A \Rightarrow aAb \Rightarrow aaAbb$

$\Rightarrow aabb$

$L = \{a^n b^n : n \geq 0\}$

For a language, there can be 2 grammars but for a grammar cannot have 2 languages.

Ambiguous grammar → to derive a string more than 1 form exist

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Q: Find language

$G = (\{S\}, \{a, b\}, S, P)$

with P given by:

$S \to asa \mid bsb \mid \lambda$

① $S \to asa$ $L(G) = \{\lambda, aa,$

② $S \to bsb$ $bb, aaaa,$

③ $S \to \lambda$ $bbbb \ldots\}$

Using ③

$S \Rightarrow \lambda$

② $S \Rightarrow aSa \Rightarrow aa$

① $S \Rightarrow bSb \Rightarrow bb$

$S \Rightarrow bSb \Rightarrow baSab \Rightarrow baaab$

$S \Rightarrow aSa \Rightarrow abSba \Rightarrow abba$

$S \Rightarrow aSa \Rightarrow aaSaa \Rightarrow aaaaa$

$S \Rightarrow bSb \Rightarrow bbSbb \Rightarrow bbbbb$

$\Rightarrow$

$L = \{w.w^R : w \in \{a, b\}^*\}$

$*$ indicates that string w consists of symbol a & b

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Q.
$G = (\{S\}, \{a, b\}, S, P)$

with P defined by:

① $S \to aSb$

② $S \to \lambda$

$S \Rightarrow aSb$
$\Rightarrow aaSbb$
$\Rightarrow a^2Sb^2$
$\Rightarrow a^nSb^n$
$\Rightarrow a^nb^n$

③ $S \Rightarrow \lambda$

④ $aSb \Rightarrow ab$ (using ② put this $S = \lambda$)

$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aabb$
(using ① $S = aSb$) (using ② $S = \lambda$)

$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaaSbbb \Rightarrow aaabbb$
(sentential form)

$L(G) = \{\lambda, ab, aabb, aaabbb, \ldots\}$

$S \overset{*}{\Rightarrow} aaabbb$

$L(G) = \{a^n b^n : n \geq 0\}$

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Find grammar for:

$L = \{a^n b^{n+1} : n \geq 1\}$

$L = \{abb, aabbb, aaabbbb, \ldots\}$

$G = (\{S, A\}, \{a, b\}, S, P)$

$A \to \lambda$

$S \to aAb$

$A \to Ab$

$S \Rightarrow aAbb \Rightarrow abb$

$S \Rightarrow aAbb \Rightarrow$

$A \to aAb$

$A \to Ab$

$S \to Ab$

$A \to aAb$

$A \to ab$

$S \to Ab \Rightarrow abb$

$S \Rightarrow Ab \Rightarrow$

$aAbb$

$\Rightarrow aaabbb$

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Q: Find language generated by:-

$G = (\{S\}, \{a, b\}, S, P)$

P is given by:

$S \to SS \mid asb \mid bsa \mid \lambda$

$S \to SS$

$S \to asb$

$S \to bsa$

$S \to \lambda$

$L(G) = \{\lambda, ab, ba, abba, abab, \ldots\}$

$S \Rightarrow SS \Rightarrow asbbsa$

$\Rightarrow abba$

$S \Rightarrow asb \Rightarrow ab$

$S \Rightarrow bsa \Rightarrow ba$

~~$a^n b^n$~~

$S \Rightarrow asb \Rightarrow absab$

$\Rightarrow abab$

$L(G) = \{w : n_a(w) = n_b(w), w \in \{a, b\}^*\}$

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epts atleast

a

Chomsky classification of languages (21)

In the definition of grammar (V, T, S, P), V_N and T are sets of symbols and $S \in V_N$. If we want to classify grammars, we have to do it only by considering form of productions.

Chomsky classification of grammars and languages

1) Type 0 grammar (Phase structure grammar)

Is any grammar without any restrictions.

In the production of the form, $\phi A \psi \rightarrow \phi \alpha \psi$, where A is variable, ϕ is left context, ψ the right context and $\phi \alpha \psi$ the replacement string.

eg. $abAbcd \rightarrow abABbcd$.

Here $ab \rightarrow$ left context

$bcd \rightarrow$ right context

$+ \alpha \Rightarrow AB$

eg. $AC' \rightarrow A$.

Here A is left context

Λ is right context

$+ \alpha = \Lambda$

A production without any restriction is called a type 0 production.

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② Type 1 or context sensitive or dependent grammar

- A production of the form $\phi A \psi \to \phi \alpha \psi$ is called type 1 production if $\alpha \neq \wedge$. ie here erasing of A is not permitted.

- A grammar is called type 1 or context sensitive if all its productions are type 1 productions. (The production $S \to \wedge$ is also allowed in a type 1 grammar, but in this case S does not appear on R.H.S of any production)

eg. $aAbcD \to abcDbcD$, $AB \to AbBc$, $A \to abA$

- A language generated by a type 1 grammar is called type 1 or context sensitive language.

③ Type 2 or context free grammar

- A type 2 production is a production of the form $A \to \alpha$ where $A \in V_N$ and $\alpha \in (V \cup T)^*$ ie L.H.S has no left context or right context.

eg. $S \to Aa$, $A \to a$, $A \to \wedge$ are type 2 productions.

- A grammar is called type 2 grammar or context free grammar if it only contains type-2 productions.

- A language generated by a context free grammar is called type-2 language or context free language.

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(4) Type 3 or regular grammar

- A production of the form $A \to a$ or $A \to aB$ where $A, B \in V$ and $a \in T$ is called type 3 production. ($S \to \Lambda$ is allowed here but S should on appear in RHS of any other production.)
- A grammar is called type 3 or regular grammar if all its productions are type 3 productions.

- A language generated by a regular grammar is called type-3 language or regular language

Q: Find the highest type number which can be applied to the following productions:

(a) $S \to Aa$, $A \to c \mid Ba$, $B \to abc$

Here $S \to Aa$, $A \to Ba$ and $B \to abc$ are type 2 and $A \to c$ is type 3.

So, highest type is 2.

(b) $S \to ASB \mid d$, $A \to aA$

$S \to ASB$ is type 2 and $A \to aA$ and $S \to d$ is type 3, so highest type is 2.

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(c) $S \to aS \mid ab$

$S \to aS$ is type 3 and $S \to ab$ is type 2.

So, highest type is 2.

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Regular Expressions

The regular expressions are useful for representing certain sets of strings in algebraic fashion.

The formal definition of regular expression is as:

(1) Any terminal symbol, Λ and ϕ are regular expressions. When we view a in Σ as a regular expression, when denote it by **a**.

(2) The union of 2 regular expressions R_1 and R_2 is $R_1 + R_2$ which is also regular expression

(3) The concatenation of 2 regular expressions R_1 and R_2 is $R_1 R_2$ which is also regular expression.

(4) The iteration (or closure) of a regular expression R, is R^* is also a regular expression.

(5) If R is a regular expression, then (R) is also a regular expression.

(6) The regular expression over Σ are precisely those obtained recursively by application of the rules 1-5 once or several times.

Note: In absence of parentheses, we have the hierarchy of operations as follows:
iterations concatenation and union.

Regular set: Any set represented by a regular expression is called a regular set.

If for eg. $a, b \in \Sigma$, then (i) **a** denotes the set $\{a\}$
(ii) **a+b** denotes $\{a, b\}$
(iii) **ab** denotes $\{ab\}$
(iv) a^* denotes set $\{\Lambda, a, aa, aaa\}$
(v) $(a+b)^*$ denotes $\{a, b\}^*$

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The set represented by R is $L(R)$
If R_1 and R_2 are any 2 regular expressions.
Then:

(1) $L(R_1 + R_2) = L(R_1) \cup L(R_2)$
(2) $L(R_1 R_2) = L(R_1)\, L(R_2)$
(3) $L(R^*) = (L(R))^* = \bigcup_{n=0}^{\infty} L(R)^n$

Q: Describe the following set by regular expressions:-

(a) $\{101\} \Rightarrow 101$
(b) $\{\wedge, ab\} \Rightarrow \wedge + ab$
(c) $\{\wedge, 0, 00, 000 - - -\} \Rightarrow 0^*$
(d) $\{1, 11, 111, - - - -\} \Rightarrow 1(1)^*$
as concatenation of 1 and any element of 1^*
(e) L_1 is set of all strings of 0's and 1's ending in 00 $\Rightarrow (0+1)^*00$
(f) L_2 is set of all strings of 0's and 1's beginning with 0 and ending with 1
$0(0+1)^*1$

Identities for Regular Expressions.

I_1	$\phi + R = R$
I_2	$\phi R = R\phi = \phi$
I_3	$\wedge R = R \wedge = R$
I_4	$\wedge^* = \wedge$ and $\phi^* = \wedge$
I_5	$R + R = R$
I_6	$R^* R^* = R^*$

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I_7 $RR^* = R^*R$

I_8 $(R^*)^* = R^*$

I_9 $\lambda + RR^* = \lambda + R^*R = R^*$

I_{10} $(PQ)^*P = P(QP)^*$

I_{11} $(P+Q)^* = (P^*Q^*)^* = (P^* + Q^*)^*$

I_{12} $(P+Q)R = PR + QR$ and
$R(P+Q) = RP + RQ$

Theorem (Arden's theorem): Let P and Q be 2 regular expressions over Σ. If P does not contain λ, then the following equation in R, namely $R = Q + RP$ has a unique solution given by $R = QP^*$

Q: Prove $(1+00^*1) + (1+00^*1)(0+10^*1)^*$
$(0+10^*1) = 0^*1(0+10^*1)^*$

$\Rightarrow (1+00^*1)\left(\lambda + (0+10^*1)^*(0+10^*1)\right)$ (using I_{12})

$\Rightarrow (1+00^*1)(0+10^*1)^*$ (using I_9)

$\Rightarrow 1(1+00^*)(0+10^*1)^*$ (using I_{12})

$\Rightarrow 0^*1(0+10^*1)^*$ (using I_9)

$= R.H.S.$

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Finite Automata and Regular Expr

Transition system containing Λ-moves

Transition system can be generalized by permitting Λ-transitions or Λ-moves, these occur whenever input is applied. But it is possible to convert a transition system with Λ-moves into an equivalent transition system without Λ-moves.

Suppose, we want to replace a Λ-move from vertex V_1 to vertex V_2; Then:

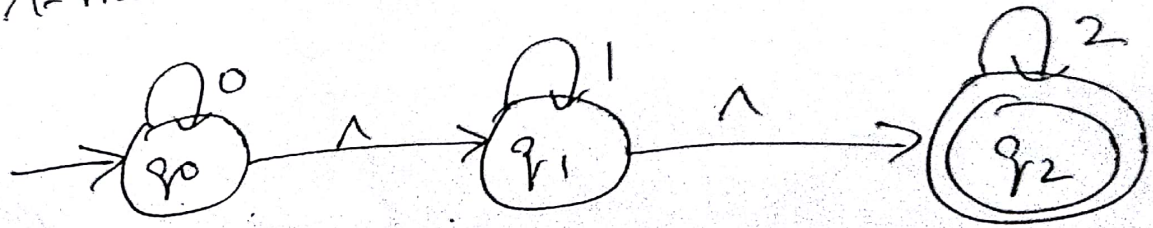
(1) Find all edges starting from V_2.

(2) Duplicate all these edges starting from V_1, without changing the edge labels.

(3) If V_1 is an intial state, make V_2 also intial state.

(4) If V_2 is final state, make V_1 also final state.

Q1- Consider a finite automaton, with Λ-moves Obtain an equivalent automaton without Λ-moves



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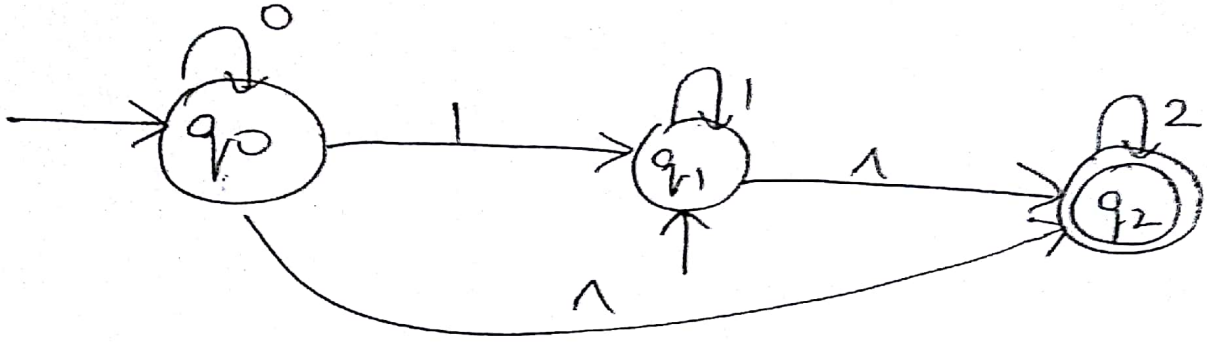
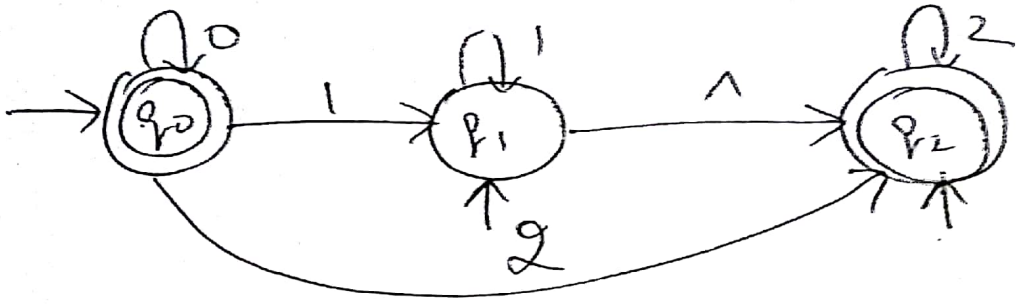
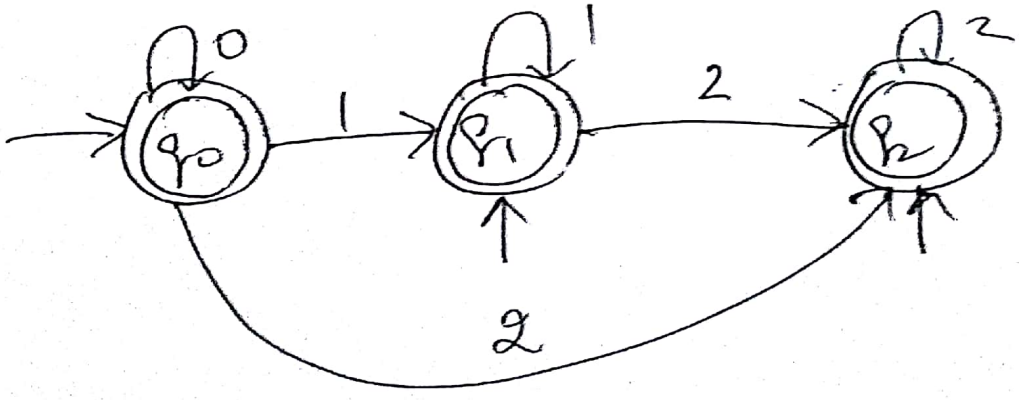
(1) We first remove $\wedge$ from q_0 to q_1



(2) We remove $\wedge$ from q_0 to q_2



(3) We remove $\wedge$ from q_1 to q_2



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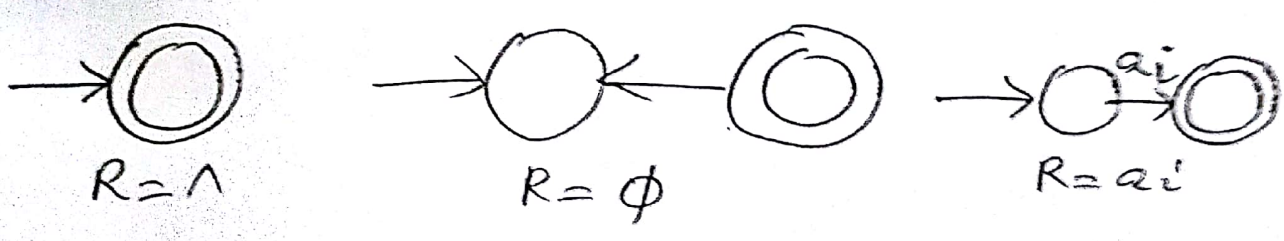
NDFA with Λ-moves and Regular Expressions

Kleene's theorem: If R is a regular expression over Σ representing $L \subseteq \Sigma^*$, then there exists an NDFA M with Λ-moves such that $L = T(M)$.

Proof. Proof by principle of induction on total no. of characters in R. (eg. $\Sigma, \Lambda, \phi, *, +$). (eg. $R = \Lambda + 10^* 11^* 0$, no. of char = 9)
Let $L(R)$ denote the regular set represented by R.

Basic step:- Let the no. of characters in R be 1.
Then $R = \Lambda$ or $R = \phi$ or $R = a_i$, $a_i \in \Sigma$.



Inductive step:- Assume that the theorem is true for regular expressions having n characters. Let R be a regular expression having $n+1$ characters. Then,

$R = P + Q$ or $R = PQ$ or $R = P^*$

according as last operator in R is $+$, product or closure. P and Q are regular expressions

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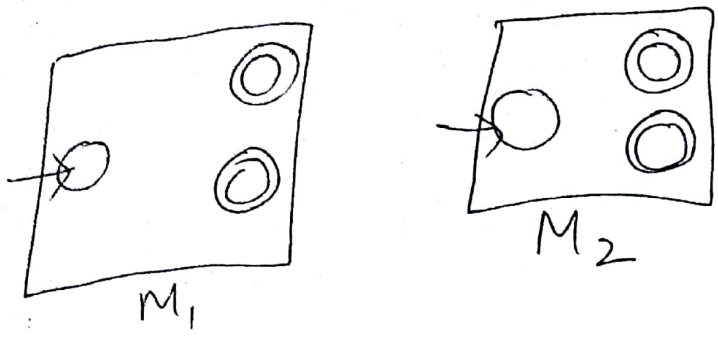
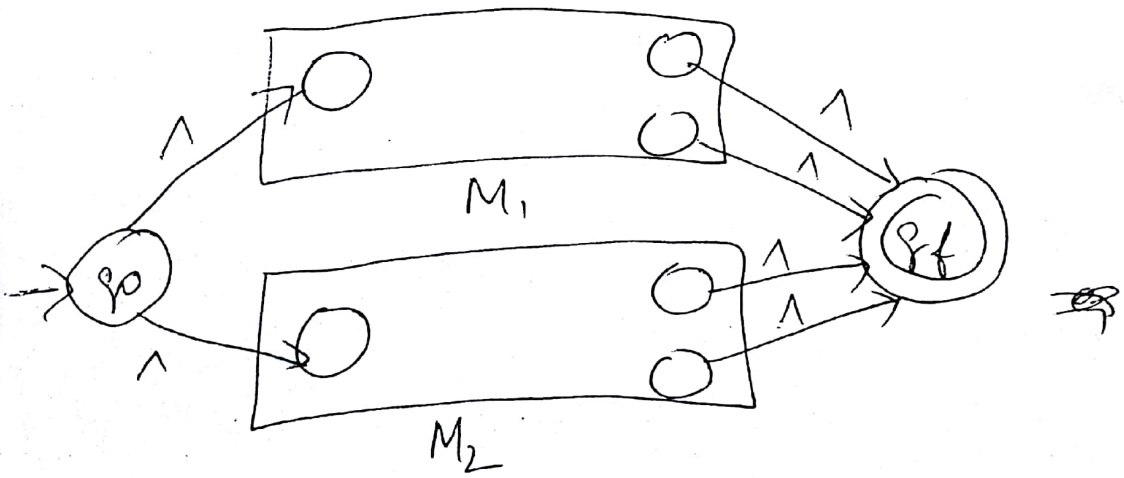
having n characters or less.

$L(P)$ and $L(Q)$ are recognized by M_1 and M_2 where M_1 and M_2 are NDFAs with Λ-moves such that $L(P) = T(M_1)$ and $L(Q) = T(M_2)$



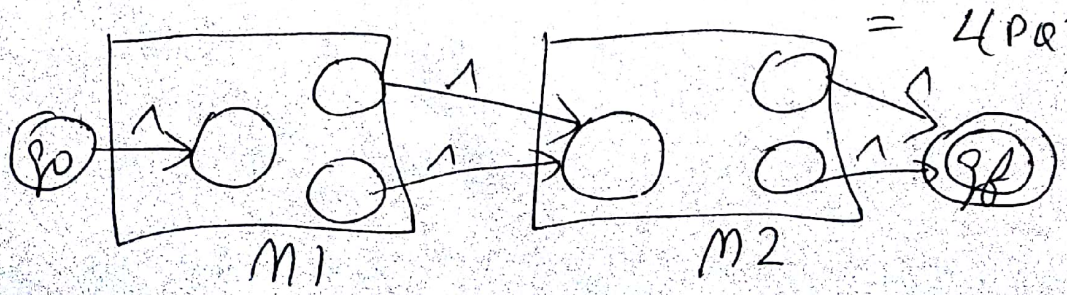
Case ①: $R = P + Q$:- In this case we construct an NDFA M with Λ-moves that accepts $L(P+Q)$ as:



$$T(M) = T(M_1) \cup T(M_2)$$
$$= L(P+Q)$$

Case ② $R = PQ$:-

$$T(M) = T(M_1) \cdot T(M_2)$$
$$= L(PQ)$$



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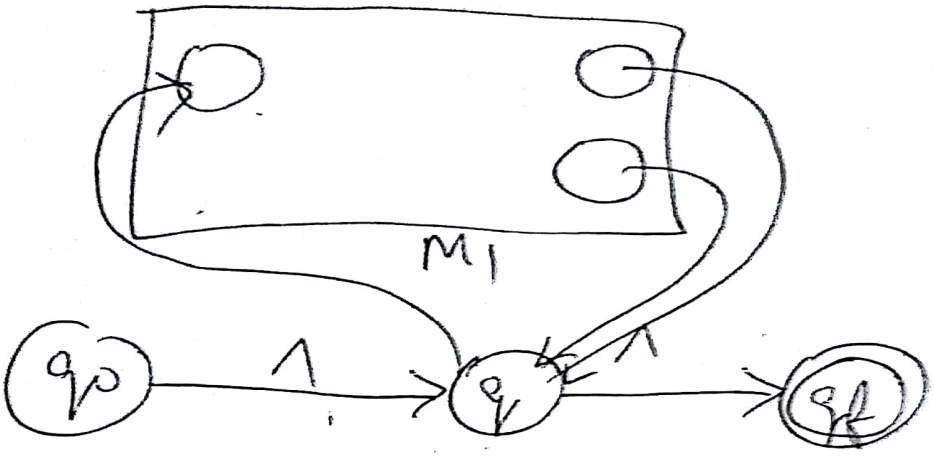
Case (3): $R = P*$



Thus in all cases, there exists an NDFA M with Λ-moves accepting regular expression R with n+1 characters.

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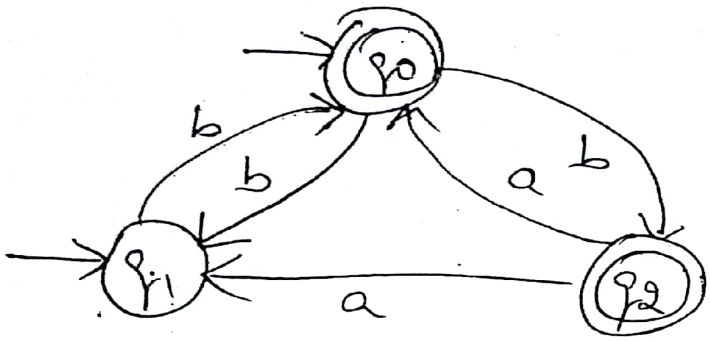
Conversion of NDFA to DFA (Regular Expression)

The construction is similar to the one done earlier, only in the initial step, take set of all initial states.

Q: Find the equivalent DFA:-



δ

State / Σ	a	b
→ q0		q1, q2
→ q1		q0
(q2)	q0, q1	

Equivalent DFA

δ

State / Σ	a	b
[q0, q1]	ϕ	[q0, q1, q2]
ϕ	ϕ	ϕ
[q0, q1, q2]	[q0, q1]	[q0, q1, q2]

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Conversion of a finite automaton to regular expression

(Using Algebraic method using Arden's theorem)

The following assumptions are made regarding the transition system:-

1. The transition graph does not have λ-moves.
2. It has only one initial state, say V_1.
3. Its vertices are $V_1, \ldots V_n$.
4. V_i the r.e. represents set of strings accepted by the system even though V_i is final state.
5. α_{ij} denotes the r.e. representing set of labels of edges from V_i to V_j. When there is no edge $\alpha_{ij} = \phi$.

$$V_1 = V_1\alpha_{11} + V_2\alpha_{21} + \ldots + V_n\alpha_{n1} + \lambda$$

$$V_2 = V_1\alpha_{12} + V_2\alpha_{22} + \ldots + V_n\alpha_{n2}$$

$$V_n = V_1\alpha_{1n} + V_2\alpha_{2n} + \ldots + V_n\alpha_{nn}$$

For getting set of strings recognized by the transition system we have to take union of all V_i's corresponding to final states.

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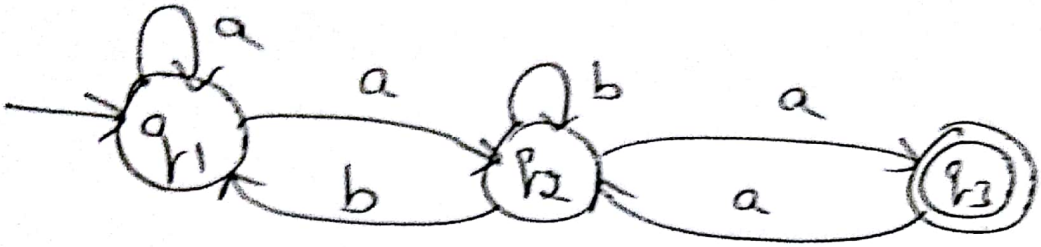
Q: Consider the transition system. Find the regular expression:- (2)



We can apply arden's theorem as graph does not contain any $\wedge$-move and there is only one initial state.

$$q_1 = q_1 a + q_2 b + \wedge \quad \text{— (1)}$$

$$q_2 = q_1 a + q_2 b + q_3 a \quad \text{— (2)}$$

$$q_3 = q_2 a \quad \text{— (3)}$$

Putting (3) in (2)

$$\Rightarrow q_2 = q_1 a + q_2 b + q_2 aa$$

$$\Rightarrow q_2 = q_1 a + q_2 (b + aa)$$

(By applying arden's theorem)

$$\Rightarrow q_2 = q_1 a (b + aa)^* \quad \text{— (4)}$$

Putting (4) in (1)

$$\Rightarrow q_1 = q_1 a + q_1 a (b + aa)^* b + \wedge$$

$$\Rightarrow q_1 = q_1 (a + a(b + aa)^* b) + \wedge$$

(using arden's theorem)

$$\Rightarrow q_1 = \wedge (a + a(b + aa)^* b)^*$$

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$q_2 = q_1 a(b+aa)^* b)^* a(b+aa$

$q_2 = (a+a(b+aa)^* b)^* a(b+aa$

$q_3 = q_2 a$

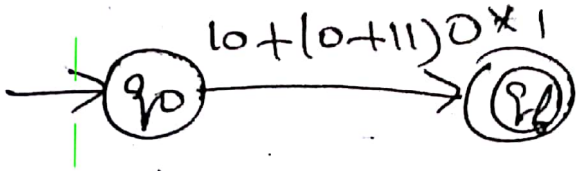
$$q_3 = (a+a(b+aa)^* b)^* a(b+aa)^* a$$

Construction of finite automata equivalent to a regular expression

(1) Construct a transition graph equivalent to the given regular expression using Λ-moves.

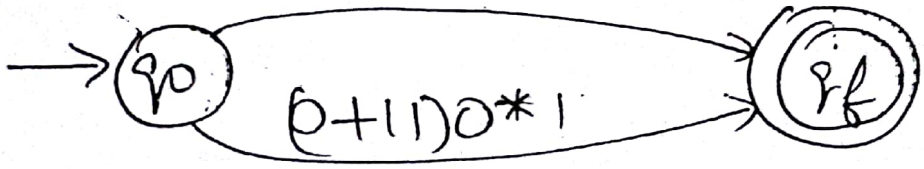
(2) Construct the transition table without Λ-moves and find the equivalent DFA. Reduce the no. of states if possible.

Q: Construct a DFA with reduced states equivalent to r.e. $10+(0+11)0^*1$



We eliminate +

10



We eliminate concatenation and *

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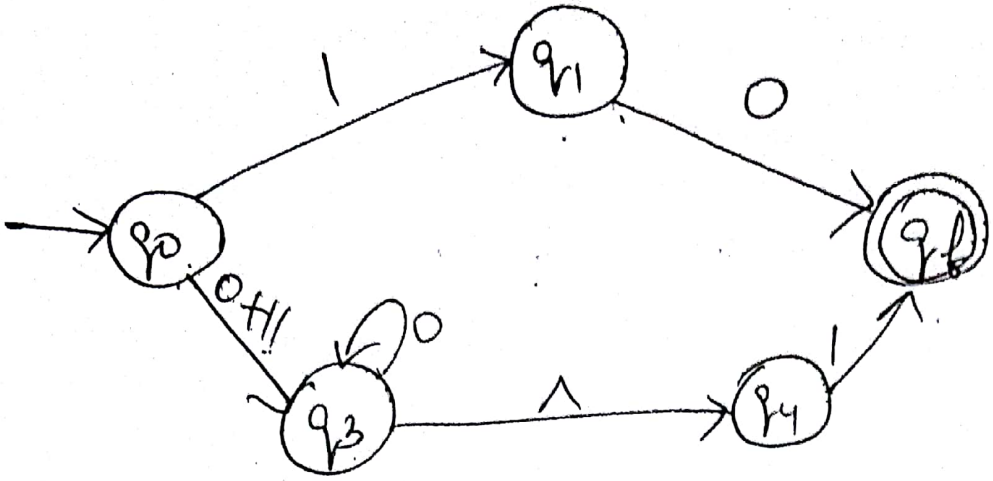
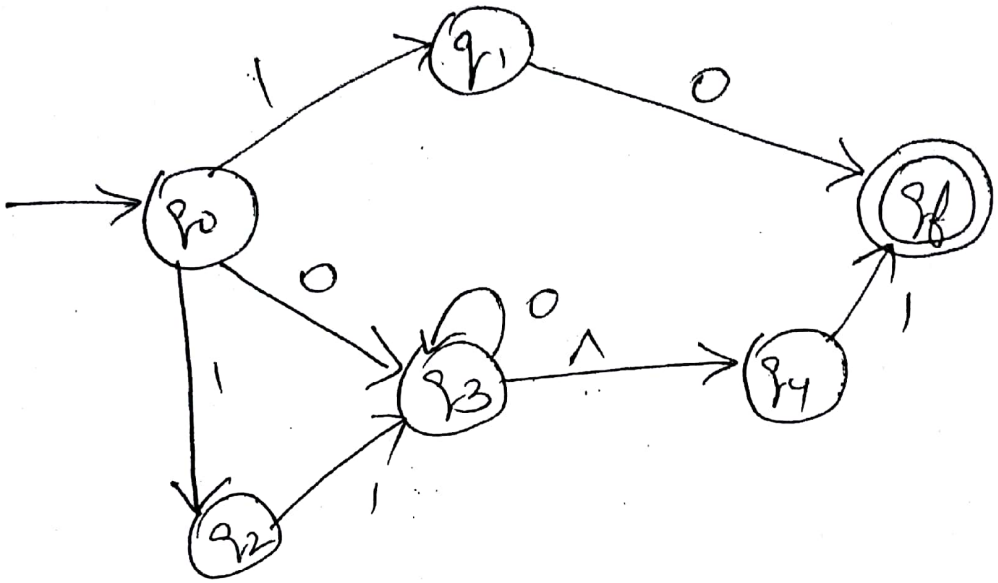
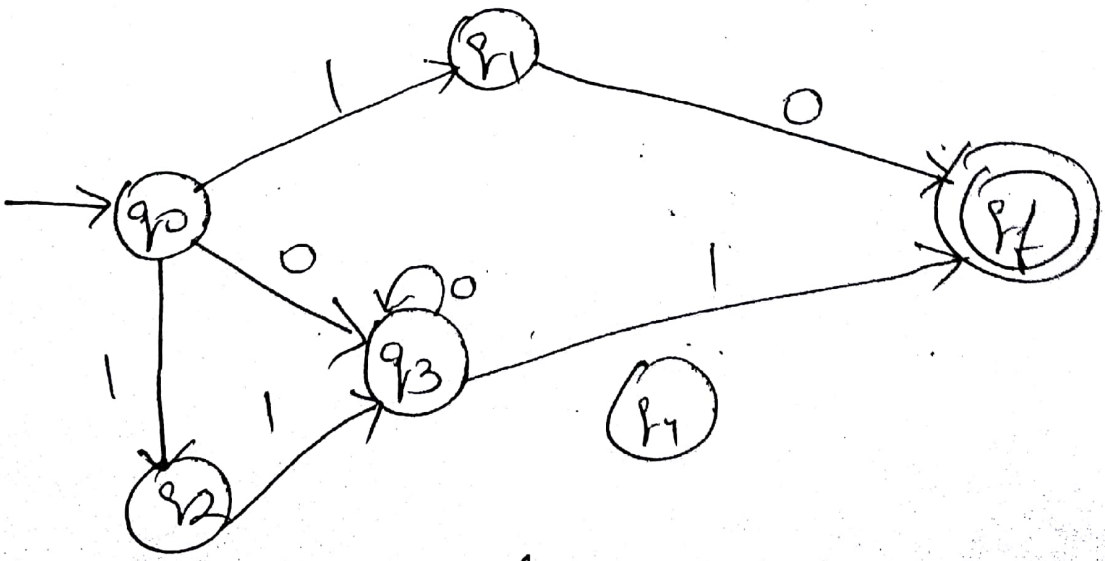


Elimination of +



Elimination of Λ-moves



We can neglect q_7

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0...0

(2)

State/Σ	0	1
→ q_0	q_3	q_1, q_2
q_1	q_f	ϕ
q_2	ϕ	ϕ q_3
q_3	q_3	q_f
q_f	ϕ	ϕ

(3) The equivalent DFA is:

State/Σ	0	1
→ $[q_0]$	$[q_3]$	$[q_1, q_2]$
$[q_3]$	q_3	q_f
$[q_1, q_2]$	q_f	q_3
$[q_f]$	ϕ	ϕ
ϕ	ϕ	ϕ

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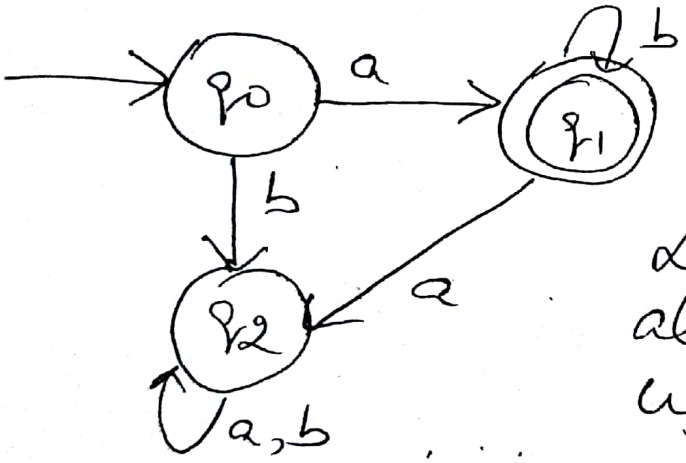
Pumping Lemma

Theorem:

Let $M = (Q, \Sigma, \delta, q_0, F)$ be a finite automaton with n states. Let L be the regular set (lang.) accepted by M. Then, there exists some positive integer m such that for any string $w \in L$ with $|w| \geq m$ and $m \geq n$, can be decomposed as $w = xyz$ with $|xy| \leq n$, $y \neq \Lambda$ and $xy^iz \in L$ for each $i \geq 0$.

Proof:

Eg. $L = \{ab^n : n \geq 0\}$



Let us consider all strings w with $|w| \geq 3$

Now w can be decomposed as

$w = xyz$

where $x = a$
$y = b$
$z = b$

$\left[|xy| = |ab| = 2 \leq 3 \right.$
$\left. |y| = |b| = 1 \geq 1 \right]$

Now, since every $w_i = xy^iz$
$= ab^ib \in L$ for $i > 0$, $\therefore L$ is regular

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Proof: Let $w = a_1 a_2 \dots a_m$, $m \geq n$

$\delta(q_0, a_1 a_2 \dots a_i) = q_i$ for $i = 1, 2 \dots m$;

$Q_1 = \{q_0, q_1 \dots q_m\}$

Here Q_1 is sequence of states in the path with path value $w = a_1 a_2 \dots a_m$. As there are only n distinct states, atleast 2 states in Q_1 must coincide. Let us take them as q_j and q_k $(q_j = q_k)$. Then, $0 \leq j < k \leq n$.

So, w can be decomposed as:-

Let $\underbrace{a_1 a_2 \dots a_j}_{x}$, $\underbrace{a_{j+1} \dots a_k}_{y}$ and $\underbrace{a_{k+1} \dots a_m}_{z}$

As $k \leq n$, $|xy| \leq n$ and $w = xyz$

Path will be:-

It starts with q_0, on applying string x it reaches $q_j (= q_k)$. On applying string y, it comes back to $q_j (= q_k)$. So after application of y^i for $i \geq 0$, automaton remains in same state q_j. On applying z, it reaches q_m.

Hence $xy^i z \in L$.

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