FIRST TERM EXAMINATION [FEB. 2019]
SIXTH SEMESTER [B.TECH]
INFORMATION THEORY AND CODING
[ETEC-304]

Time : 1.5 hrs. **M.M. : 30**

Note: Q. 1. is compulsory. Attempt any two questions from the rest.

Q.1. (a) DMS emits two symbols with probabilities $P(x_1) = \frac{5}{7}$, $P(x_2) = \frac{2}{7}$ respectively. Find self-information in decits and nats. **(2)**

Ans.

$$P(x_1) = \frac{5}{7},\ P(x_2) = \frac{2}{7}$$

$$I(s_K) = \log_2 \frac{1}{p(x_k)} \text{ bits } 0.4854 + 1.807$$

$$= 2.2927$$

Self-information in decits is given as

$$I(s_K) = \log_{10} \frac{1}{p(x_k)} = \log_{10} \frac{7}{5} + \log_{10} \frac{7}{2} = 0.6902 \text{ decits}$$

Self-information in nats is given as

$$I(s_K) = \log_e \frac{1}{p(x_k)} = \log_e \frac{7}{5} + \log_e \frac{7}{2} = 1.5892 \text{ nats}$$

Q. 1. (b) Show that the mutual information is I(X; Y) ≥ 0. **(2)**

Ans. We know that-

$$p(x_j, y_k) = p(x_j / y_k)\, p(y_k)$$

$$\therefore \quad p(x_j / y_k) = \frac{\rho(x_j, y_k)}{p(y_k)} \qquad ...(1)$$

substituting eq. (1) in eq. of $I(x; y)$, we get-

$$I(x; y) = \sum_{j=0}^{J-1} \sum_{k=0}^{k-1} p(x_j, y_k) \log_2 \left[\frac{p(x_j, y_k)}{p(x_j), p(y_k)} \right]$$

$$\geq 0 \qquad \left[\because \sum_{k=0}^{k-1} p_k \log_2 \left(\frac{q_k}{p_k} \right) \leq 0 \right]$$

with equality, if and only if,

$$p(x_j, y_k) = p(x_j)\, p(y_k) \quad \forall\ j \text{ and } k.$$

$$\therefore \quad I(x; y) \geq 0$$

Hence, proved.

Q. 1. (c) State and discuss the necessary and sufficient condition for existence of an instantaneous binary code. **(2)**

Ans. Instantaneous code: Prefix codes are distinguished from other uniquely decodable codes by the fact that the end of a code word is alwyas recognizable. Hence, the decoding of a prefix can be accomplished as soon as the binary sequence representing a source symbol is fully received. For this reason, prefix codes are also referred to as instantancous codes.

Give a DMS of entropy H(s), a prefix code can be constructed with an averge code word length $\bar{L}$, which is bounded as follows-

$$\boxed{H(S) \le \bar{L} < H(S) + 1} \quad ...(1)$$

The left-hand bound of eq. (1) is satisfied with equality under the condition that symbol s_k is emitted by the source with probability-

$$p_k = 2^{-l_k}$$

Then,

$$\sum_{k=0}^{k-1} 2^{-l_k} = \sum_{k=0}^{k-1} p_k = 1$$

Under this condition, the kraft McMillan inequality of eq. (1) tells us that we can construct a prefix code, such that the length of the code word assigned to source symbol s_k is $-\log_2 p_k$. For such a code, the average code word length is.

$$\bar{L} = \sum_{k=0}^{k-1} \frac{l_k}{2^{l_k}}$$

and the corresponding entropy of the source is-

$$...(2)$$

$$H(S) = \sum_{k=0}^{k-1} \left(\frac{1}{2^{l_k}}\right) \log_2(2^{l_k}) = \sum_{k=0}^{k-1} \frac{l_k}{2^{l_k}} \quad ...(3)$$

Hence, from eq. (2) & (3), it is found that the prefix code is matched to the source is that $\bar{L} = H(S)$.

- But to match the prefix code to an arbitrary DMS, we use the extended source.

Let $\bar{L}_n$ denote the average codeword length of the extended prefix code. For a uniquely decodable code. $\bar{L}_n$ is the smallest possible.

From eq. (2), we deduce that-

$$H(S^n) \le \bar{L}_n < H(S^n) + 1$$

$$n\,H(S) \le \bar{L}_n < n\,H(S) + 1$$

$$H(S) \le \frac{\bar{L}_n}{n} < H(S) + \frac{1}{n} \quad ...(4)$$

In the limit, as 'n' approaches infinity, the lower & upper bounds in eq.(4) converge, as-

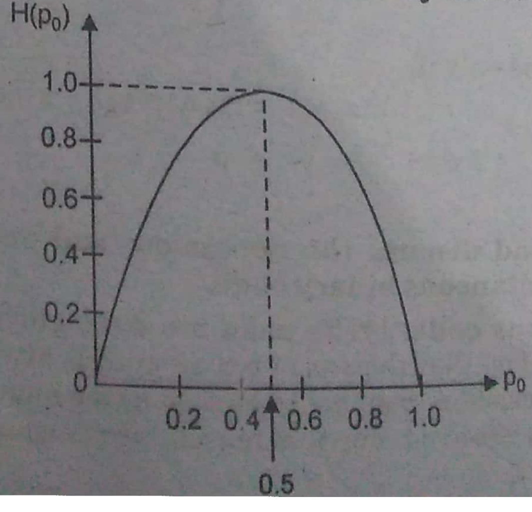
$$\boxed{\lim_{n \to \infty} \frac{1}{n} \bar{L}_n = H(S)} \quad ...(5)$$

$\therefore$ The average codeword length of an extended prefix code can be made as small as the entropy of the source provided the extended code has a high order, in accordance with the source-coding

Drawback $\rightarrow$ Increased decoding complexity.

Q. 1. (d) If p denotes probability of some event then plot the amount of information and the average information gained by the occurrence of this event for $0 \le p \le 1$. (2)

Ans.



Q. 1. (e) Model the Discrete memoryless Channel (DMC) in terms of probability transition matrix. **(2)**

Ans. Discrete Memoryless Channels: A discrete memoryless channel is a statistical model with an input X and an output Y that is a noisy version of X (both X and Y are random variables). Every unit of time, the channel accepts an input symbol X selected from an alphabet x and, in response, it emits an output symbol Y from an alphabet y.

- The channel is said to be 'discrete' when both of the alphabets x and y have finite sizes.
- It is said to be 'memoryless' when the current output symbol depends only on the current input symbol and not any of the previous ones.

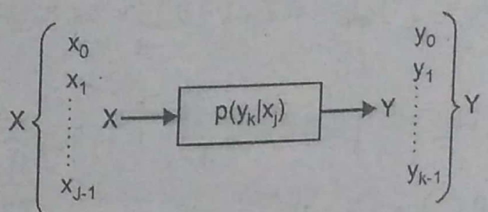


Fig. 1. Discrete Memory channel

The Discrete Memory less channel, as shown in fig. 1, is described in terms of an input alphabet.

$$X = \{x_0, x_1, \ldots\ldots, x_{J-1}\}$$

an output alphabet,

$$Y = \{y_0, y_1, \ldots\ldots y_{k-1}\}$$

and a set of transition probabilities

$$p(y_k | x_j) = P(Y = y_k | X = x_j), \ \forall : j \text{ and } k.$$

Also, we have

$$0 \le p(y_k | x_j) \le 1, \ \forall j \text{ and } k.$$

- Convenient way of describing a discrete memoryless channel

$$P = \begin{bmatrix} p(y_0|x_0) & p(y_1|x_0) & \cdots\cdots & p(y_{k-1}|x_0) \\ p(y_0|x_1) & p(y_1|x_1) & \cdots\cdots & p(y_{k-1}|x_1) \\ \vdots & \vdots & & \vdots \\ p(y_0|x_{J-1}) & p(y_1|x_{J-1}) & \cdots\cdots & p(y_{k-1}|x_{J-1}) \end{bmatrix}$$

The J-by-K matrix P is called the channel matrix or transition matrix or transition matrix.

- Each row of the channel matrix P corresponds to a fixed channel input.
- Each column of the channel matrix P corresponds to a fixed channel output.

from the row of the channel matrix, we have one property-

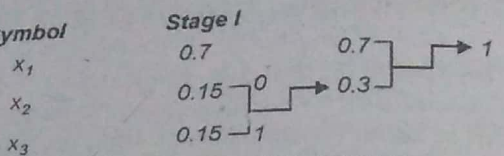
$$\sum_{k=0}^{k-1} p(y_k|x_j) = 1, \ \forall j$$

Q. 2. (a) State and discuss the source coding theorem. **(5)**

Ans. Refer Q.1(a) End Term Examination 2019.

Q. 2. (b) Consider DMS with symbols probabilities $P(x_1) = 0.7$, $P(x_2) = 0.15$, $P(x_3) = 0.15$. Apply Huffman Coding and find the average code word length. Consider the second order extended source and apply Huffman Coding to the extended source. **(5)**

Ans.



Symbol	Probability	Codeword	l_k
x_1	0.7	0	1
x_2	0.15	10	2
x_3	0.15	11	2

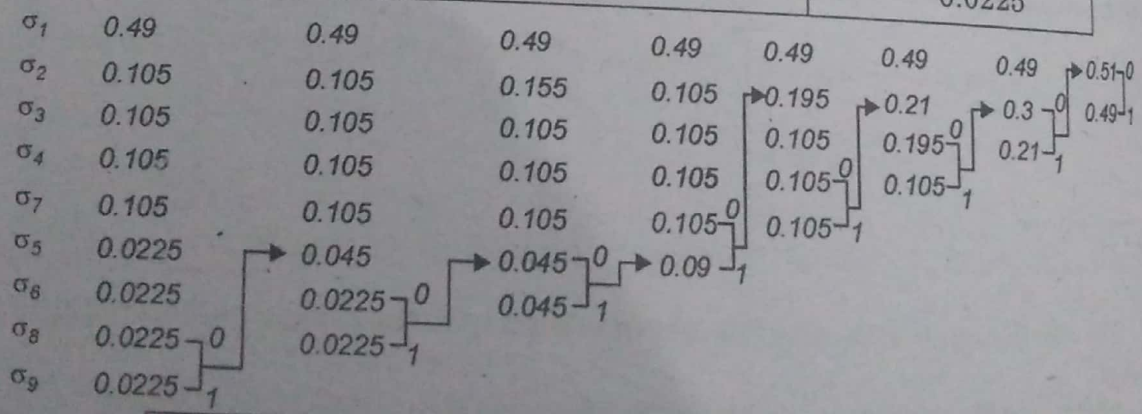
$$\bar{L} = \sum_{k=1}^{k} p_k l_k = \sum_{k=1}^{3} p_k l_k = 0.7 \times 1 + 0.15 \times 2 + 0.15 \times 2 = 1.3 \text{ bits/symbol}$$

$$H(s) = \sum_{k=1}^{3} p_k \log_2 \left(\frac{1}{p_k}\right) = 0.7 \log_2 \frac{1}{0.7} + 0.15 \log_2 \frac{1}{0.15} + 0.15 \log_2 \frac{1}{0.15}$$

$$= 0.7 \times 0.514 + 2 \times 0.15 \times 2.737 = 1.181 \text{ bits/symbol}$$

2nd order Extension

News ymbols	Extended symbols	StageI
σ_1	x_1x_1	0.49
σ_2	x_1x_2	0.105
σ_3	x_1x_3	0.105
σ_4	x_2x_1	0.105
σ_5	x_2x_2	0.0225
σ_6	x_2x_3	0.0225
σ_7	x_3x_1	0.105
σ_8	x_3x_2	0.0225
σ_9	x_3x_3	0.0225



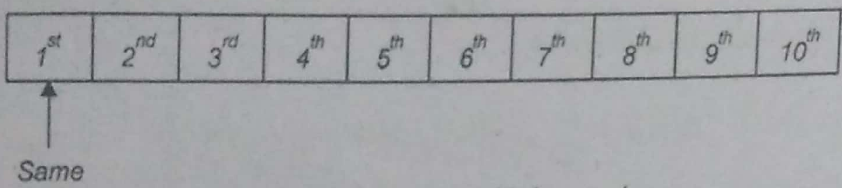
	Carried	l_k
σ_1	0	1
σ_2	001	3
σ_3	010	3
σ_4	011	3
σ_5	000100	6
σ_6	000101	6
σ_7	0000	4
σ_8	000110	6
σ_9	000111	6

$$\bar{L}_2 = \sum_{k=1}^{9} p(\sigma_k) l_k$$

$= 0.49 \times 1 + 0.105 \times 3 \times 0.105 \times 3 + 0.105 \times 3 + 0.0225 \times 6 + 0.0225 \times 6 + 0.105 \times 4 + 0.0225 \times 6 + 0.0225 \times 6 = 2.395$ bits/symbol

Q. 3. (a) A data frame consists of 10 fields. First field in each frame is same. The remaining field in each frame can be filled by any of 32 symbols with equal probability. Find average rate of information if 500 frames are transmitted in every second. (5)

Ans.



K = 32 symbols (with equal probability), $r_s = 500$ frames/sec.

Since the symbols are produced in a data frame of 10 fields, the aveage-information or total entropy of a frame is–

$H = H_1 + H_2 + H_3 + H_4 + H_5 + H_6 + H_7 + H_8 + H_9 + H_{10}$

It is given that the first symbol in each frame is always same, i.e. $p_1 = 1$

$$H_1 = p_1 \log_2 \frac{1}{p_1} = 0$$

Since, the 2^{nd}, 3^{rd},...., 10^{th} positions are occupied by any of 32 symbols with equal prob., i.e., $p_2 = p_3 .... = p_{10} = \frac{1}{K} = \frac{1}{32}$

$\therefore H_2 = H(S)_{max} = \log_2 K = \log_2 32 = 5$ bits/symbol.

Similarly, $H_3 = H_4 ....H_{10} = 5$ bits/symbol.

$\Rightarrow$ $H = 0 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 45$ bits/frame

The average rate of imformation or entropy rate is

$$R_5 = r_s . H = 45 \times 500 = 22{,}500 \text{ bits/sec}$$

Q. 3. (b) Define : Lossless Channels and Deterministic Channel. (5)

Ans. Lossless Channel

A channel is lossless if H(X/Y) = 0 for all input distributions. A lossless channel is characterized by the fact that the input is determined by the output and hence no transmission errors can occur. Equivalently, the values of Y may be partitioned into disjoint sets B_1, B_2,....B_m such that

$P\{Y \in B_i | X = x_i\} = 1$ (i = 1,....m)

The structure of a loss less channel is shown in fig. 1.

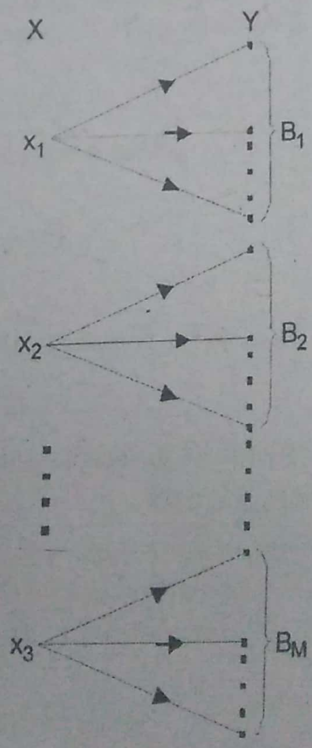


Fig.1. Lossless Channel

Deterministic Channel

–A channel is deterministic if $p(y_k / x_j) = 1$ or 0 for all j, k i.e., if Y is determined by X, on equivalently H(Y/X) = 0 for all input distributions.

– A channel represented by a channel matrix with one and only one non-zero element in every row is known as Deterministic channel. For *e.g.*.

$$\begin{array}{c|ccc} & y_0 & y_1 & y_2 \\ \hline x_0 & 1 & 0 & 0 \\ x_1 & 1 & 0 & 0 \\ x_2 & 0 & 1 & 0 \\ x_3 & 0 & 1 & 0 \\ x_4 & 0 & 1 & 0 \\ x_5 & 0 & 0 & 1 \end{array}$$

The equivocation is given by

$$H(Y/X) = \sum_{j=0}^{5}\sum_{k=0}^{2} P(x_j, y_k)\log_2\left[\frac{1}{P(y_k / x_j)}\right]$$

$$= \sum_{j=0}^{5}\sum_{k=0}^{2} P(x_j)\left\{P(y_k / x_j)\log_2\left[\frac{1}{P(y_k / x_j)}\right]\right\} = 0$$

The mutual information of a deterministic channel is

$$I(X; Y) = H(Y) - H(Y/X) = H(Y) - 0 = H(Y)$$

The channel capacity C is given by

$$C = \text{Max}\,\{I(X; Y)\}$$
$$= \text{Max}\,\{H(Y)\} = H(Y)_{max}$$
$$= \log_2 K$$
$$= \log_2 3 \qquad [\because 0 \le H(s) \le \log_2 K]$$

Q. 4. (a) Show that the capacity of BSC channel is

$$C = 1 + p\log_2 p + (1-p)\log_2(1-P) \qquad (5)$$

Ans. Consider the binary symmetric channel as desecribed by the transition probability diagram. This diagram is uniquely defined by the conditional probability of error p.

Find the entropy and channel capacity.

The entropy H(X) is maximized when the channel input probability.

$$p(x_0) = p(x_1) = \frac{1}{2}$$

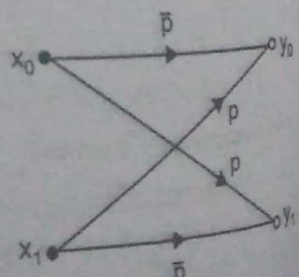
(Here symbol x_o corresponds to 0 & symboly x_1 corresponds to 1)

$$\therefore \quad H(p) = p\log_2\left(\frac{1}{p}\right) + (1-p)\log_2\left(\frac{1}{1-p}\right)$$

$$= -p\log_2 p - (1-p)\log_2(1-p)$$

Similarly, the mutual information $I(x\,;y)$ is maximized.

$$\therefore \quad C = I(x\,;y)\,|_{p(x_0)=p(x_1)=\frac{1}{2}}$$



From the given diagram, we have-

$p(y_0|x_0) = 1-p \qquad p(y_1|x_0) = p$

$p(y_0|x_1) = 1-p \qquad p(y_1|x_1) = 1-p$

We know that-

$$I(x;y) = H(x) - H(x/y)$$

$$= \sum_{j=0}^{J-1} p(x_j)\log_2\left[\frac{1}{p(x_j)}\right] - \sum_{k=0}^{K-1}\sum_{j=0}^{J-1} p(x_j, y_k)\log_2\left[\frac{1}{p(x_j|y_k)}\right] \quad (J = K = 2)$$

$= \sum_{j=0}^{1}$

Also from the dia

$p(x_0|y_0) = 1-p$

$p(x_0|y_1) = p$

We know that-

$\therefore \quad I(x;y) =$

$\Rightarrow \quad C =$

Q. 4. (b) Gi

Find H(X), H(

Ans.

(i) Find

$$= \sum_{j=0}^{1} p(x_j)\log_2\left[\frac{1}{p(x_j)}\right] - \sum_{k=0}^{1}\sum_{j=0}^{1} p(y_k/x_j)p(x_j)\log_2\left[\frac{1}{p(x_j|y_k)}\right]$$

Also from the diagram, we have-

$p(x_0|y_0) = 1-p$ $\qquad$ $p(x_1|y_0) = p$

$p(x_0|y_1) = p$ $\qquad$ $p(x_1|y_1) = 1-p$

We know that-

$$\therefore \quad I(x;y) = p(x_0)\log_2\left[\frac{1}{p(x_0)}\right] + p(x_1)\log_2\left[\frac{1}{p(x_1)}\right]$$

$$- \left\{p(y_0|x_0)\cdot p(x_0)\log_2\left[\frac{1}{p(x_0/y_0)}\right] + p(y_0|x_1)\cdot p(x_1)\log_2\left[\frac{1}{p(x_1|y_0)}\right]\right.$$

$$\left. + p(y_1|x_0)\cdot p(x_0)\log_2\left[\frac{1}{p(x_0|y_1)}\right] + p(y_1|x_1)\cdot p(x_1)\log_2\left[\frac{1}{\rho(x_1|y_1)}\right]\right\}$$

$$\Rightarrow \quad C = \frac{1}{2}\log_2(2) + \frac{1}{2}\log_2(2) - \left\{(1-p)\cdot\frac{1}{2}\log_2\left(\frac{1}{1-p}\right) + \right.$$

$$\left. p\cdot\frac{1}{2}\log_2\left(\frac{1}{p}\right) + p\cdot\frac{1}{2}\log_2\left(\frac{1}{p}\right) + (1-p)\cdot\frac{1}{2}\log_2\left(\frac{1}{1-p}\right)\right\}$$

$$= 1 - \left\{(1-p)\log_2\left(\frac{1}{1-p}\right) + p\log_2\left(\frac{1}{p}\right)\right\}$$

$$= 1 - \{-(1-p)\log_2(1-p) - p\log_2(p)\}$$

$$\boxed{C = 1 - H(p)}$$

Q. 4. (b) Given Channel matrix, $P(Y|X) = \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.3 & 0.2 & 0.5 \\ 0.1 & 0.4 & 0.5 \end{bmatrix}$, $P[X] = \left[\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right]$.

Find H(X), H(Y), H(X,Y), H(Y/X), H(X/Y). **(5)**

Ans.

$$P(Y/X) = \begin{array}{c c} & \begin{array}{ccc} y_0 & y_1 & y_2 \end{array} \\ \begin{array}{c} x_0 \\ x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.3 & 0.2 & 0.5 \\ 0.1 & 0.4 & 0.5 \end{bmatrix} \end{array}$$

$$p(x_0) = \frac{1}{2}, p(x_1) = \frac{1}{4}, p(x_2) = \frac{1}{4}$$

(i) Finding H(X)

$$H(X) = \sum_{j=0}^{J-1} p(x_j)\log_2\left[\frac{1}{p(x_j)}\right] = \sum_{j=0}^{2} p(x_j)\log_2\left[\frac{1}{p(x_j)}\right]$$

$$= p(x_0)\log_2\left[\frac{1}{p(x_0)}\right] + p(x_1)\log_2\left[\frac{1}{p(x_1)}\right] + p(x_2)\log_2\left[\frac{1}{p(x_2)}\right]$$

$$= \frac{1}{2}\log_2 2 + \frac{1}{4}\log_1 4 + \frac{1}{4}\log_2 4 = 1.5 \text{ bits/symbol}$$

(2) Finding H(Y)

$$P(Y) = P(X).\,P(Y/X)$$

$$= \begin{bmatrix}0.5 & 0.25 & 0.25\end{bmatrix}\begin{bmatrix}0.4 & 0.3 & 0.3\\ 0.3 & 0.2 & 0.5\\ 0.1 & 0.4 & 0.5\end{bmatrix}$$

$$= \begin{bmatrix}0.3 & 0.3 & 0.4\end{bmatrix}$$

$$H(Y) = \sum_{k=0}^{K-1} p(y_k)\log_2\left[\frac{1}{p(y_k)}\right]$$

$$= \sum_{k=0}^{2} p(y_k)\log_2\left[\frac{1}{p(y_k)}\right]$$

$$= p(y_0)\log_2\left[\frac{1}{p(y_0)}\right] + p(y_1)\log_2\left[\frac{1}{p(y_1)}\right] + p(y_2)\log_2\left[\frac{1}{p(y_2)}\right]$$

$$= 0.3\log_2\frac{1}{0.3} + 0.3\log_2\frac{1}{0.3} + 0.4\log_2\frac{1}{0.4}$$

$$= 1.571 \text{ bits/symbol}$$

(3) Finding H(X, Y)

$$P(X, Y) = P(X)_d\, P(Y/X)$$

$$= \begin{bmatrix}0.5 & 0 & 0\\ 0 & 0.25 & 0\\ 0 & 0 & 0.25\end{bmatrix}\begin{bmatrix}0.4 & 0.3 & 0.3\\ 0.3 & 0.2 & 0.5\\ 0.1 & 0.4 & 0.5\end{bmatrix}$$

$$= \begin{bmatrix}0.2 & 0.15 & 0.15\\ 0.075 & 0.05 & 0.125\\ 0.025 & 0.1 & 0.125\end{bmatrix}$$

$$H(X,Y) = \sum_{j=0}^{J-1}\sum_{k=0}^{K-1} p(x_j, y_k)\log_2\left[\frac{1}{p(x_j, y_k)}\right]$$

$$= 0.2\log_2\left[\frac{1}{0.2}\right] + 0.15\log_2\left[\frac{1}{0.15}\right] + 0.15\log_2\left[\frac{1}{0.15}\right] + 0.075\log_2\left[\frac{1}{0.075}\right] + 0.05$$
$$\log_2\left[\frac{1}{0.05}\right] + 0.125\log_2\left[\frac{1}{0.125}\right] + 0.025\log_2\left[\frac{1}{0.025}\right] + 0.1\log_2\left[\frac{1}{0.1}\right]$$
$$+ 0.125\log_2\left[\frac{1}{0.125}\right]$$

$$= 0.2 \times 2.322 + 0.15 \times 2.737 \times 2 + 0.075 \times 3.737 + 0.05 \times 4.322 + 0.125 \times 3 \times 2 + 0.025 \times 5.322 + 0.1 \times 3.322$$

$$= 2.997125 \text{ bits/symbol}$$

$$\approx 3 \text{ bits/symbol}$$

(4) Finding H(Y/X)

$$H(Y/X) = \sum_{k=0}^{K-1}\sum_{j=0}^{J-1} p(x_j, y_k)\log_2\left[\frac{1}{p(y_k|x_j)}\right]$$

$$= -P(X,Y).\log_2(P(Y/X)]$$

To obtain elements of [H(Y/X)], multiply the corresponding elements of P(X,Y) and $\log_2$ [P(Y/X)], and add them, also,

$$H(Y/X) = -\begin{bmatrix} 0.2 & 0.15 & 0.15 \\ 0.075 & 0.05 & 0.125 \\ 0.025 & 0.1 & 0.125 \end{bmatrix}\begin{bmatrix} -1.322 & -1.737 & -1.737 \\ -1.737 & -2.322 & -1 \\ -3.322 & 1.322 & -1 \end{bmatrix}$$

$$= -(-1.322 \times 0.2 - 1.737 \times 0.15 - 1.737 \times 0.15 - 1.737 \times 0.075$$
$$-2.322 \times 0.05 - 1 \times 0.125 - 3.322 \times 0.025 - 1.322 \times 0.1 - 1 \times 0.125$$
$$= 1.497 \text{ bits/symbol.}$$

(5) Finding H(X/Y)

To obtain [P(X/Y)] divide the columns of P(X,Y) by $p(y_0)$ $p(y_1)$ and $p(y_2)$, respectively.

$$P(X/Y) = \begin{bmatrix} \frac{0.2}{0.3} & \frac{0.15}{0.3} & \frac{0.15}{0.4} \\ \frac{0.075}{0.3} & \frac{0.05}{0.3} & \frac{0.125}{0.4} \\ \frac{0.025}{0.3} & \frac{0.1}{0.3} & \frac{0.125}{0.4} \end{bmatrix}$$

$$= \begin{bmatrix} 0.67 & 0.5 & 0.375 \\ 0.25 & 0.167 & 0.3125 \\ 0.083 & 0.33 & 0.3125 \end{bmatrix}$$

$$H(X/Y) = \sum_{j=0}^{J-1}\sum_{k=0}^{K-1} P(x_j, y_k)\log_2\left[\frac{1}{P(x_j / y_k)}\right]$$

$$= -P(X,Y)\log_2[P(X|Y)]$$

$$= -\begin{bmatrix} 0.2 & 0.15 & 0.15 \\ 0.075 & 0.05 & 0.125 \\ 0.025 & 0.1 & 0.125 \end{bmatrix}\begin{bmatrix} -0.578 & -1 & -1.415 \\ -2 & -2.582 & -1.678 \\ -3.6 & -1.6 & -1.678 \end{bmatrix}$$

$$= -(-0.578 \times 0.2 - 1 \times 0.15 - 1.415 \times 0.15 - 2 \times 0.075 - 2.582 \times 0.05 - 1.678 \times 0.125$$
$$- 3.6 \times 0.025 - 1.6 \times 0.1 - 1.678 \times 0.125)$$

$$= 1.42645 \text{ bits/symbols.}$$

MID TERM EXAMINATION [FEB. 2018]
SIXTH SEMESTER [B.TECH]
INFORMATION THEORY AND CODING [ETEC–304]

Time : 1.5 hrs. M.M. : 30

Note: Q. 1 is compulsory. Attempt any 2 of the rest.

Q. 1. Answer the following briefly.

(a) Define Differential Entropy for continuous ensembles. **(2)**

Ans. Definition:

Let X be any random variable with a probability density function $f_x(x)$. Then differential entropy can be stated as

$$h(x) = -\int_x f(x) \log f(x) dx$$

Q. 1. (b) State the Source Coding, Theorem **(2)**

Source coding theorem gives a relation between $\bar{L}$ (average code length) and entropy. As per this theorem $\bar{L} \geq H(x)$.

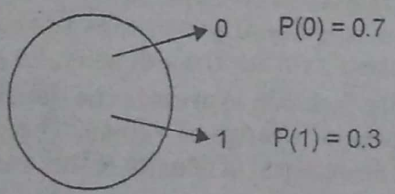
Ans.

$$\bar{L} = \sum_{k=0}^{k-1} P_k l_k \qquad ...(1)$$

$$\eta = \frac{L_{min}}{\bar{L}} \qquad ...(2)$$

$$\eta = \frac{H(f)}{\bar{L}} \qquad ...(3)$$

Q. 1. (c) Consider a binary system emitting two symbols [0,1] with probabilities 0.7 and 0.3 respectively. Find the information conveyed by bits '0' and '1' and the entropy of the system. **(2)**



Ans. Binary System

$$I(0) = \text{information conveyed by '0'}$$

$$= \log_2 \frac{1}{P(0)} = \log_2 \frac{1}{0.7} = 0.52 \text{ bits}$$

$$I(1) = \text{information conveyed by '1'}$$

$$= \log_2 \frac{1}{P(1)} = \log_2 \frac{1}{0.3} = 1.74 \text{ bits}$$

$$\text{Entropy} = \sum_{i=1}^{2} P_i \log \frac{1}{P_i}$$

$$= P_1 \log \frac{1}{P_1} + P_2 \log \frac{1}{P_2}$$

Here

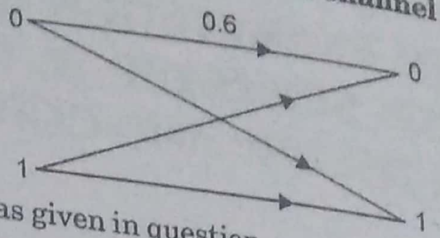
$$P_1 = P(0) = 0.7$$

$$P_2 = P(1) = 0.3$$

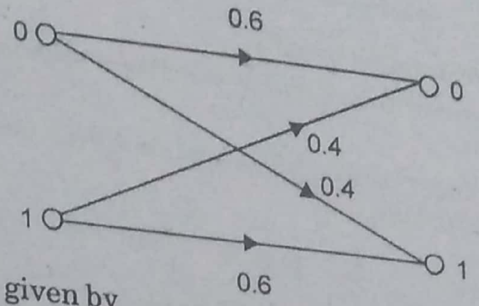
So Entropy = 0.7 (0.52) + 0.3 (1.74)

$$= 0.364 + 0.522 = 0.886$$

Q. 1. (d) For the BSC shown below, write the channel matrix. (2)



Ans. Channel is shown as given in question



The channel matrix is given by

$$P(Y/X) = \begin{bmatrix} p(y_1/x_1) & p(y_2/x_4) \\ p(y_1/x_2) & p(y_2/x_2) \end{bmatrix} = \begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{bmatrix}$$

Q. 1. (e) State the properties of PDF of a random variable. (2)

Ans. Probability Density Function

The probability density function is the probability function which is defined for the continuous random variable. The probability density function is also called the probability distribution function or probability function. It is denoted by $f(x)$.

Probability density function or PDF is said to be the density of a continuous random variable. It is defined as a function which represents the likelihood that this random variable takes on some given value.

Sometimes, the probability density function is called "probability distribution function" or simply "probability function". But, among statisticians and probabilists, these terms are usually not considered standard. It is also known as "probability mass function" rather than density, for a discrete random variable.

The probability density function expresses the density of some continuous random variable lying between a certain range of values. The probability density function is represented in the form of an integral of density of this variable density over given range.

This implies that the probability density function is defined by the area between the density function curve and horizontal axis within the given range on vertical axis. Thus, the PDF is positive everywhere. Also, the integral of PDF over whole given area is equal to one.

Conditions for a valid probability density function:

Let X be the continuous random variable with a density function $f(x)$. Therefore,

- $f(x) \geq 0$ for all x.

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

Example:

Check whether the given probability density function is valid or not.

The probability density function is,

$F(x) = 4x^3$ then total probability of this function can be obtained as follow:

$$\int_{-\infty}^{\infty} f(x)dx = \int_{0}^{1} 4x^3 dx$$

$$= 4\left[\frac{x^4}{4}\right]_0^1 = [x^4]_0^1 = 1^4 - 0 = 1$$

There are the following properties if this function:

- The formula for E(X) is $E(X) = \int_{-\infty}^{\infty} xf(x)dx.$
- The formula for $E(X^2)$ is $E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)dx.$
- Formula for $P(a < X < b)$ is $P(a < X < b) = \int_a^b f(x)dx.$
- Formula for cumulative distribution function is $F(X) = \int_{-\infty}^{x} f(z)dz.$

Some of the common PDFs are as given below:

Distribution	Probability density function
Uniform	$f(x) = \frac{1}{b-a} \quad a < x < b$
Exponential	$f(x) = \lambda e^{-\lambda x} \quad x > 0$
Normal	$f(x) = \frac{1}{\sigma(2\pi)} e^{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad -\infty < x < \infty$

Q. 2. (a) Prove that the mutual information is always a non-negative entity, clearly indicating the condition under which it would be zero. **(4)**

Ans. By definition

$$I(X;Y) = -\sum_{x \in X}\sum_{y \in Y} P(x,y)\log\left(\frac{p(x)p(y)}{P(x,y)}\right)$$

Now the negative of logarithm is little bit

Complex and $\sum_{x \in X}\sum_{y \in Y} P(x,y) = 1$

So we can write $I(X;Y) \geq -\log\left(\sum_{x \in X}\sum_{y \in Y} P(x,y)\frac{P(x)P(y)}{P(x,y)}\right)$

$$= -\log\left(\sum_{x \in X}\sum_{y \in Y} P(x)P(y)\right) = 0$$

Hence we can write $I(X;Y) \geq 0$

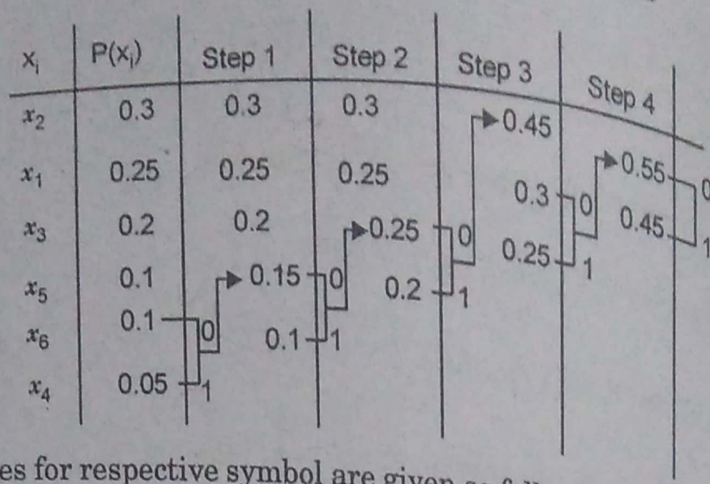
Which proves that mutual information is non-negative in nature.

Q. 2. (b) For a DMS with 6 possible symbol X_i, i = 1, 2, 3, 4, 5, 6; the occurrence probabilities are given in the table below:

X_1	X_2	X_3	X_4	X_5	X_6
0.25	0.3	0.2	0.05	0.1	0.1

Construct the Huffman tree and hence determine the Huffman Code for each of the possible symbols. Also determine the code efficiency. **(6)**

Ans. For the given probabilities, the Huffman tree is obtained by arranging given probabilities in decreasing order



Huffman codes for respective symbol are given as follow:

X	Code(C_i)	length of code $n(C_i)$
x_1	01	2
x_2	00	2
x_3	11	2
x_4	1001	4
x_5	101	3
x_6	1000	4

Code effeciency $$\eta = \frac{H(x)}{L\log_2 2} = \frac{H(X)}{L}$$

$$H(X) = -\sum_{i=1}^{6} P(x_i)\log P(x_i)$$

$$= -(0.25\log_2(0.25)+0.3\log(0.3)+0.2\log(0.2)$$
$$+0.05\log(0.05)+0.1\log(0.1)+0.1\log(0.1))$$

Here base to log is '2' $$= +(0.15+0.16+0.14+0.065+0.2)$$

$$H(X) = 0.715$$

$$\bar{L} = \sum_{i=1}^{6} n_i p(x_i)$$
$$= 2(0.25)+2(0.3)+2(0.2)+0.05(4)+3(0.1)+4(0.1)$$
$$= 2.4$$

Coding efficiency $$= \frac{H(X)}{L}$$

$$= \frac{0.715}{2.4} = 0.2979 = 29.79\%$$

Q. 3. (a) State the Kraft McMillian Inequality and prove its necessary and sufficient condition. (4)

Ans. As per kraft Mc-Millan inequality

$$\sum_{i=1}^{r} S^{-ni} \leq 1$$

$$2^{-1}+2^{-2}+2^{-3}+2^{-3} \leq 1$$

$$\sum_{i=1}^{j-1} S^{n_j-n_i}$$

$$S^{n_j} > \sum_{i=1}^{j-1} S^{n_j-n_i}, \forall j = 2,3, \ldots r$$

$$S^{n_j} \geq \sum_{i=1}^{j-1} S^{n_j-n_i} + 1 = \sum_{i=1}^{j} S^{n_j-n_i} \forall j = 1,2,\ldots r$$

$$1 \geq \sum_{i=1}^{j} S^{-n_i}, \forall j = 1,2,\ldots r$$

$$\sum_{i=1}^{r} S^{-n_i} \leq 1$$

Q. 3. (b) Channel matrix for a channel is given as follows: **(6)**

$$P\left(\frac{Y}{X}\right) = \begin{bmatrix} 0.5 & 0.1 & 0.4 \\ 0.2 & 0.3 & 0.5 \\ 0.3 & 0.6 & 0.1 \end{bmatrix}$$

Also, the input probabilities of the channel are:

$$P(X) = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

Find: (i) H(X) (ii) H(Y) (iii) H(X, Y) (iv) H(Y/X)

Ans. Given channel matrix is

$$P\left(\frac{Y}{X}\right) = \begin{bmatrix} 0.5 & 0.1 & 0.4 \\ 0.2 & 0.3 & 0.5 \\ 0.3 & 0.6 & 0.1 \end{bmatrix}$$

$$P(X) = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

Since $$P(Y/X) = \frac{P(X,Y)}{P(X)}$$

$$P(X, Y) = P(X)\, P(Y/X)$$

$$P(X) = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$

$$P(X, Y) = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 0.5 & 0.1 & 0.4 \\ 0.2 & 0.3 & 0.5 \\ 0.3 & 0.6 & 0.1 \end{bmatrix} = \begin{bmatrix} \frac{0.5}{3} & \frac{0.1}{3} & 0.4/3 \\ \frac{0.2}{3} & 0.1 & \frac{0.5}{3} \\ 0.1 & 0.2 & \frac{0.1}{3} \end{bmatrix}$$

$$P(Y) = [0.234 \quad 0.433 \quad 0.332]$$

$$H(X) = -\sum_{i=1}^{3} P(x_i)\log P(x_i)$$

$$= -\left(\frac{1}{3}\log\frac{1}{3} + \frac{1}{3}\log\frac{1}{3} + \frac{1}{3}\log\frac{1}{3}\right)$$

$$= -\log\frac{1}{3} = \frac{0.477}{0.301} = 1.59$$

$$H(Y) = -(0.234\log(0.234) + 0.433\log(0.433) + 0.332\log(0.332))$$

$$= 0.234\,(0.63) + 0.433\,(0.364) + 0.332\,(0.478)$$

$$= \frac{0.464}{0.301} = 1.54$$

$$H(X, Y) = \sum_{i=1}^{3}\sum_{j=1}^{3} P(x_i\, y_j)\log(P(x_i\, y_j))$$

$$= 0.166\log(0.166) + 0.03\log(0.03) + 0.133\log(0.133) + 0.06\log(0.06) + 0.1\log(0.1) + 0.166\log(0.166) + 0.1\log(0.1) + 0.2\log(0.2) + 0.133\log(0.133)$$

$$= 0.129 + 0.045 + 0.116 + 0.073 + 0.1 + 0.1 + 0.129 + 0.14 + 0.045 = 0.927$$

and with base 2 to log, $\quad H(X, Y) = \dfrac{0.927}{0.301} = 3.0797$

Similarly H(X/ Y) and H(Y/X) can be obtained

Q. 4. (a) Describe the use of channel coding theorem in Digital Communication Systems. If the source entropy is 1 bit per source symbol, determine the condition for which the code exists with minimum probability of error. (5)

Ans. Here source entropy is given to be as $H(x)$ and as per the given conditions the entropy is 1 bit per symbol so

$$H(x) = 1$$

As per channel coding theorem $R \le C$ so

for minimum probability of error

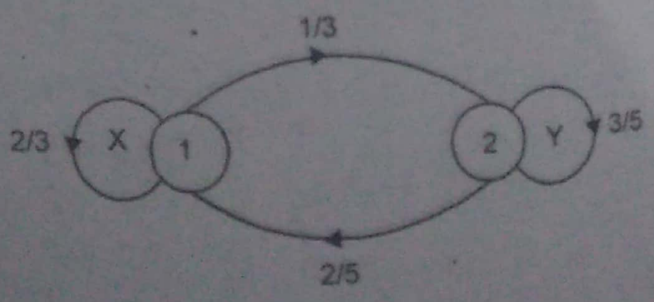
$$H(x)T_S < CT_c$$

Hence

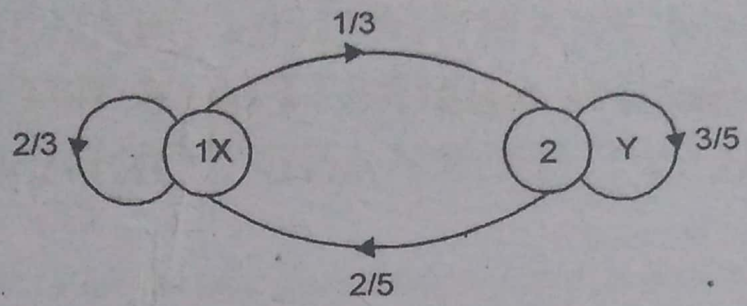
$$T_s < CT_c$$

which is the required condition for minimum possible P_e

Q. 4. (b) For the Markow Source shown in the figure below, find i. State probabilties ii. State Entropies. iii. Source Entropy (5)



Ans. The given fig can be drawn as follow



Here in the given model, there are two states X and Y

'X' generates two state having following probabilities:

$$P(X_1) = \frac{2}{3} \text{ and } P(X_2) = \frac{1}{3}$$

'Y' generates two states having following probabilities:

$$P(Y_1) = \frac{3}{5} \text{ and } P(Y_2) = \frac{2}{5}$$

Entropy of state 'Y' is given by;

$$\begin{aligned} H(Y) &= -(P(Y_1) \log P(Y_1) + P(Y_2) \log P(Y_2)) \\ &= -(0.6 \log (0.6) + 0.4 \log (0.4)) \\ &= 0.133 + 0.159 \\ &= 0.292 \end{aligned}$$

and with base '2' of log, H(Y) is given by

$$H(Y) = 0.97$$

Entropy of state 'X' is given by:

$$\begin{aligned} H(X) &= -(0.67 \log (0.67) + 0.33 \log (0.33)) \\ &= +[0.1165 + 0.1589] \\ &= 0.2754 \end{aligned}$$

and with base 2, H(X) is given by

$$\frac{0.2754}{0.301} = 0.915$$

FIRST TERM EXAMINATION [FEB. 2017]
SIXTH SEMESTER [B.TECH.]
INFORMATION THEORY & CODING
(ETEC-304)

Time : 1½ hrs. M.M. : 30

Note: Attempt Q.No. 1 which is compulsory and any two more questions from remaining. Each question carries 10 marks.

Q.1. (a) What is the relation between information and uncertainty? (1)

Ans. Let $I(x_i)$ indicate information in event x_i and Let $P(x_i)$ be uncertainty of even x_i then

$$I(x_i) = \log_2 \frac{1}{P(x_i)}$$

More the uncertainty in any event more will be the information in the event.

Q.1. (b) If probability of occurance of any event is P, then information supplied by that event I is given by (1)

(i) I = $\log_{10}$ 1/P hartley **(ii) I = $\log_2$ 1/P decits**

(iii) I = $\log_e$ P nats **(iv) I = $\log_e$ 1/P bits**

Ans.

$$I = \log_{10}\left(\frac{1}{P}\right) \text{ Hartley}$$

Q.1. (c) Channel capacity of a noise free channel having M symbols is given by (1)

(i) $\log_2$ M (ii) M (iii) 2^M (iv) None

Ans. $\log_2$ M

Q.1. (d) The mutual information of a channel with independent input and output is (1)

(i) Zero (ii) Constant (iii) Infinite (iv) Variable

Ans. Zero

Q.1. (e) Find the entropy of equiprobable binary source. (1)

Ans. Let there be 'M' equiprobable symbols of any source 'X' then entropy of source 'X' H(X) is given by

$$H(X) = \sum_{i=1}^{M} P(x_i) \log_2 \frac{1}{P(x_i)}$$

Here

$$P(x_i) = \frac{1}{M}$$

Then

$$H(X) = M\left(\frac{1}{M}\right)\log_2 M$$

$$= \log_2 M$$

Q.1. (f) What do you understand by source efficiency? (1)

Ans. Source efficiency means that how much redundancy have been removed from the source information. This is done with the help of minimum redundant coding.

Q.1. (g) What do you understand by mutual information? (2)

Ans. Mutual information is given by I(X; Y) and it is infact the amount of information transferred from source 'X' to receiver 'Y'

$$I(X; Y) = H(X) - H(X/Y)$$

Q.1. (h) Explain Source Coding theorem and establish the efficiency of a source encoder in terms of entropy. (2)

Ans. Source Coding Theorem states that for 'N' identically independently distributed random variables each with entropy H(X) can be compressed into more than NH(X) bits with negligible risk to information loss, as $N \to \infty$ but conversely, if they are compressed into fewer than NH(X) bits, it is virtually certain that information will be lost. Mathematically it can be stated as

$$\frac{H(X)}{\log_2 a} \le \bar{L} < \frac{H(X)}{\log_2 a} + 1$$

Q.2. (a) What is entropy? Show that the entropy is maximum when all the symbols are equiprobable. Assume M = 3. (3)

Ans. Entropy
$$H(X) = \sum_{i=1}^{3} P(x_i)\log\frac{1}{P(x_i)}$$

Here no. of symbols are three

Let
$$P(x_1) = \alpha$$
$$P(x_2) = \beta$$

then
$$P(x_3) = 1-(\alpha+\beta)$$

then H(X) can be written as

$$H(X) = \alpha\log_2\frac{1}{\alpha} + \beta\log_2\frac{1}{\beta} + [1-(\alpha+\beta)]\log\frac{1}{[1-(\alpha+\beta)]}$$

To get the maximisation condition of

$$H(X),\ \frac{dH(X)}{dP} = 0$$

But since the given expression of H(X) is a function of α and β so we need to do the partial differentiation of H(X) w.r.t. α and β

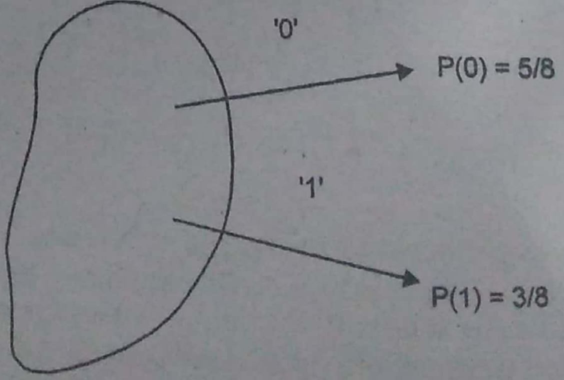
After solving by doing the partial differentiation, we get the probability of any symbol $= \frac{P}{3}$.

Hence probability for each symbol is equally distributed.

Q.2. (b) In a binary PCM if '0' occurs with probability 5/8 and '1' occurs with probability 3/8, then calculate the amount of information in (2)

(i) bits (ii) nats

Ans.



The amount of information can be given by taking the average value

(i)
$$H(X)\big|_{\text{bits}} = P(0)\log_2\frac{1}{P(0)} + P(1)\log_2\frac{1}{P(1)}$$

$$= 5/8\log_2\frac{8}{5}+\frac{3}{8}\log_2\frac{8}{3}$$

$$= \frac{\frac{5}{8}(0.204)+\frac{3}{8}(0.43)}{0.301} = 0.955 \text{ bits}$$

(*ii*)

$$H(X)\Big|_{\text{nats}} = P(0)\log_e\frac{1}{P(0)}+P(1)\log_e\frac{1}{P(1)}$$

$$= \frac{5}{8}\log_e\frac{8}{5}+\frac{3}{8}\log_e\frac{8}{3}$$

$$= \frac{5}{8}(0.47)+\frac{3}{8}(0.89)$$

$$= 0.64 \text{ nats}$$

Q.2. (*c*) A code is composed of dots and dashes. Assume that the dash is 2 times as long as the dot and has one - third the probability of occurance. **(5)**

(*i*) Calculate the information in a dot and that in dash.

(*ii*) Calculate the average information in the dot-dash code.

(*iii*) Assume that a dot lasts for 15 ms and that this same time interval is allowed between symbols. Calculate the average rate of information transmission.

Ans. Given dash duration = 2 times the duration of dot

$$\text{Probability of dash} = \frac{1}{3}$$

$$\text{then probability of dot} = \frac{2}{3}$$

$$\text{dot duration} = 15\times10^{-3} \text{ sec}$$
$$\text{dash duration} = 30\times10^{-3} \text{ sec}$$

(*i*) Information contained in dot and dash

$$H = \frac{1}{3}\log_2(3)+\frac{2}{3}\log_2(3/2)$$
$$= 0.9183 \text{ bits/symbol}$$

(*ii*) Information in dash

$$I = P\log\frac{1}{P}$$

$$= \frac{1}{3}\log_2(3) = \frac{0.159}{0.301} = 0.528$$

Information in dot

$$I = P\log\frac{1}{P}$$

$$= \frac{2}{3}\log_2 3/2 = \frac{0.1174}{0.301} = 0.390$$

(*iii*) Assume that there are total no. of 1200 symbols

$$\text{then no. of dots} = 1200\times P(\text{dot})$$

$$= 1200\times\frac{2}{3} = 800$$

$$\text{and no. of dashes} = 1200 \times \frac{1}{3} = 400$$

Now the total time duration of all the symbols inside the string

T = dot duration + dash duration + (1200 × time between the symbols)

$$= 800 \times 15 \times 10^{-3} + 400 \times 30 \times 10^{-3} + 1200 \times 15 \times 10^{-3}$$
$$= (2800) \times 15 \times 10^{-3}$$
$$= 42000 \times 15 \times 10^{-3} = 630 \text{ sec.}$$

Average symbol rate

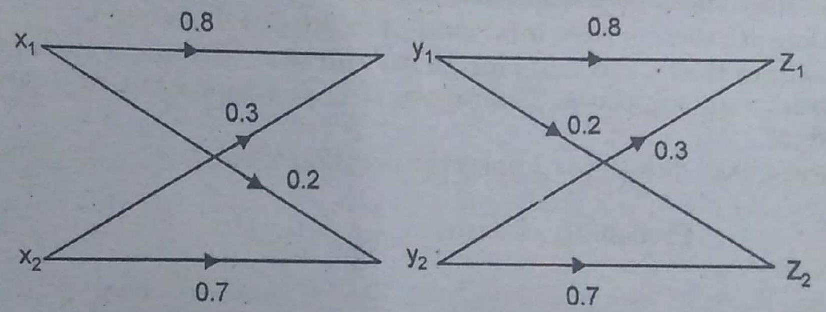
$$r = \frac{1200}{630} = 1.9 \text{ Symbols/sec.}$$

Q.3. (*a*) Two same binary channels are connected in cascade as shown figure 1.

(*i*) Find the overall channel matrix of the resultant channel and draw t resultant equivalent channel diagram.

(*ii*) Find $P(z_1)$ and $P(z_2)$ when $P(x_1) = P(x_2) = 0.5$.

Ans. The given cascaded channel are as follow:



$$P(Y/X) = \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}$$

$$P(Z/Y) = \begin{bmatrix} P(z_1/y_1) & P(z_2/y_1) \\ P(z_1/y_2) & P(z_2/y_2) \end{bmatrix}$$

$$= \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}$$

$$P(Z/X) = [P(Z/Y) \;\; P(Y/X)]$$

$$= \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}\begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}$$

$$= \begin{bmatrix} 0.7 & 0.3 \\ 0.45 & 0.55 \end{bmatrix}$$

$$P(Z) = P(X)\,P(Z/X)$$

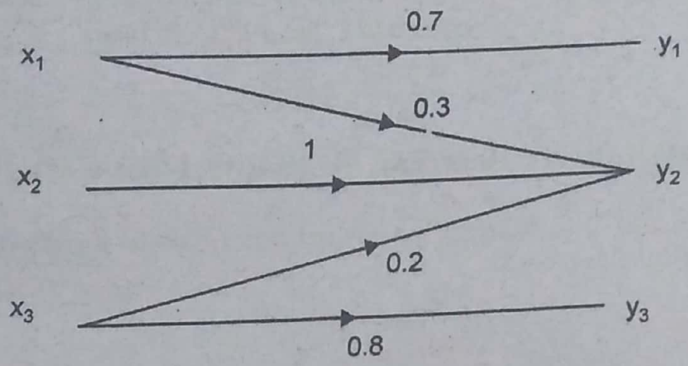
$$= [0.5 \;\; 0.5]\begin{bmatrix} 0.7 & 0.3 \\ 0.45 & 0.55 \end{bmatrix}$$

$$= [0.35 + 0.225 \;\; 0.15 + 0.275]$$

$$P(Z) = [0.575 \quad 0.425]$$
$$P(Z_1) = 0.575$$
$$P(Z_2) = 0.425$$

Q.3. (*b*) A discrete source emits messages x_1, x_2 and x_3 with the probabilities 0.3, 0.4 and 0.3 respectively. The source is connected to the channel given in figure 2. Calculate the entropies H(X), H(Y), H(Y/X) and H(X/Y). **(6)**

Ans. The given channel is:



$$P(Y/X) = \begin{bmatrix} P(y_1/x_1) & P(y_2/x_1) & P(y_3/x_1) \\ P(y_1/x_2) & P(y_2/x_2) & P(y_3/x_2) \\ P(y_1/x_3) & P(y_2/x_3) & P(y_3/x_3) \end{bmatrix}$$

$$= \begin{bmatrix} 0.7 & 0.3 & 0 \\ 0 & 1 & 0 \\ 0 & 0.2 & 0.8 \end{bmatrix}$$

$$P(x_1) = 0.3, P(x_2) = 0.4, P(x_3) = 0.3$$

Since $$P(Y/X) = \frac{P(X,Y)}{P(X)}$$

So $$P(X, Y) = P(X).\, P(Y/X)$$

$$= [0.3 \quad 0.4 \quad 0.3] \begin{bmatrix} 0.7 & 0.3 & 0 \\ 0 & 1 & 0 \\ 0 & 0.2 & 0.8 \end{bmatrix}$$

$$= \begin{bmatrix} 0.21 & 0.09 & 0 \\ 0 & 0.4 & 0 \\ 0 & 0.06 & 0.24 \end{bmatrix}$$

$$P(Y_1) = 0.21$$
$$P(Y_2) = 0.55$$
$$P(Y_3) = 0.24$$

$$H(X) = -[P(X_1)\log P(X_1) + P(X_2)\log P(X_2) + P(X_3)\log P(X_3)]$$
$$= -[0.3\log(0.3) + 0.4\log(0.4) + 0.3\log(0.3)]$$
$$= 0.3\,(0.523) + 0.4\,(0.398) + 0.3\,(0.523)$$
$$= 0.6\,(0.523) + 0.1592 = \frac{0.473}{0.301} = 1.53$$

$$H(Y) = -[0.21\log_2(0.21) + 0.55\log_2(0.55) + 0.24\log_2(0.24)]$$

$$= \frac{0.21(0.677) + 0.55(0.26) + 0.24(0.62)}{0.301}$$

$$= \frac{0.43397}{0.301} = 1.44$$

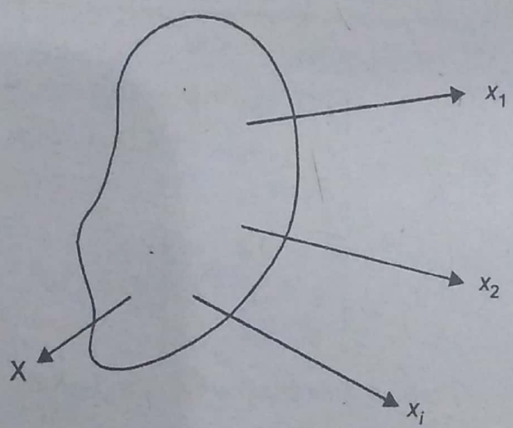
$$H(X, Y) = -[0.21 \log (0.21) + 0.09 \log(0.09) + 0.4 \log (0.4) + 0.06 \log (0.06) + 0.24 \log (0.24)]$$

$$= 0.21 (0.677) + 0.09 (1.046) + 0.4 (0.398) + 0.06 (1.22) + 0.24 (0.62)$$

$$= \frac{0.6175}{0.301} = 2.051$$

Q.4. (a) Explain Kraft-McMillan inequality. What is it's significance for the prefix codes? (2)

Ans. Kraft Mc-Millan inequality is very much useful for the source coding and for discrete memory less sources.



This fig. shows discrete memory less source which generates symbols $\{x_i\}$

Let us assume that these are total of 'm' symbols then this inequality gives the following relation

$$K = \sum_{i=1}^{m} 2^{-n_i} \leq 1$$

The concept of prefix coding is very much clear from the following table:

	Probability	$Code_1$	$Code_{11}$	$Code_{111}$
S_0	0.5	0	0	0
S_1	0.25	1	10	01
S_2	0.125	00	110	011
S_3	0.125	11	111	0111

In the list of $code_1$ and $code_{111}$, the code of one symbol are acting as the prefix of anyone of the other symbol.

While in $code_{11}$ none of the code in the list is acting as the prefix of the other code.

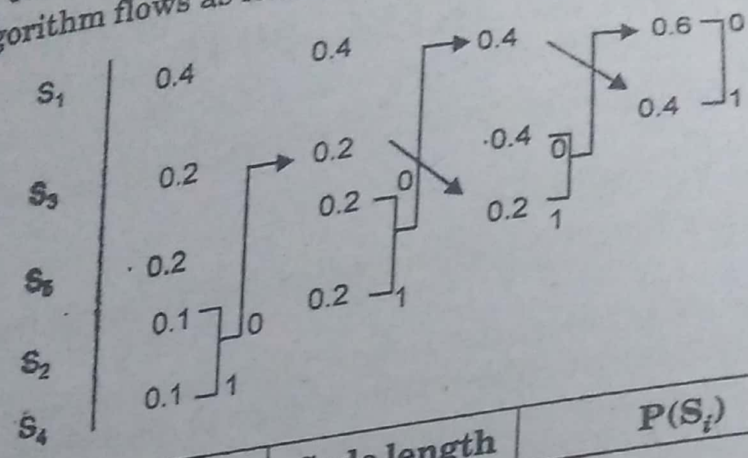
Because of this reason, $code_{11}$ is listed as the prefix coding.

Q.4. (b) What do you understand by Huffman Coding? State briefly. A discrete memoryless source S has an alphabet of five symbols s_1, s_2, s_3, s_4, and s_5 with their probability of occurance 0.4, 0.1, 0.2, 0.1 and 0.2 respectively. Compute the Huffman Code by showing Huffman tree for this source, moving a "combined" symbol as high as possible. Also calculate the efficiency of the code. (6)

Ans.

S_1	S_2	S_3	S_4	S_5
0.4	0.1	0.2	0.1	0.2

Huffman Algorithm flows as follow:



Symbol	Code	Code length	$P(S_i)$
S_1	1	1	0.4
S_2	010	3	0.1
S_3	01	2	0.2
S_4	011	3	0.1
S_5	10	2	0.2

$$H(X) = -[0.4 \log 0.4 + 0.1 \log 0.1 + 0.2 \log 0.2 + 0.1 \log 0.1 + 0.2 \log 0.2]$$
$$= -[0.4 \log 0.4 + 0.2 \log 0.1 + 0.4 \log 0.2]$$
$$= 0.4\,(0.398) + 0.2\,(1) + 0.4\,(0.699)$$
$$= \frac{0.6388}{0.301} = 2.122$$
$$\bar{L} = 1(0.4) + 3(0.1) + 2(0.2) + 3(0.1) + 2(0.2)$$
$$= 0.4 + 0.3 + 0.4 + 0.3 + 0.4$$
$$= 1.8$$

Q.4. (c) Explain Channel Coding theorem. **(2)**

Ans. Channel Coding Theorem: Channel Coding Theorem can be stated in the following two ways

(1) If $R \leq C$

Then there can be some effective error control mechanism by means of which the probability of error can be reduced to zero.

Hence error free transmission is possible in this particular case

(2) If $R > C$

Then chances of error during transmission are very much sure and can not be made to be having zero probability irrespective of any kind of channel coding.

Here C is capacity of channel and R is bit transmission rate.