Circuits & Systems

- **Signal** is a physical quantity that varies with time, space or any other independent variable (s).

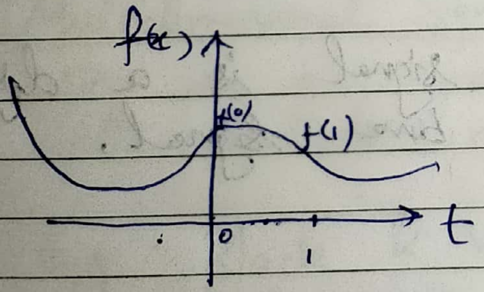
Example — Acoustic / Speech Signal (Time dependant)

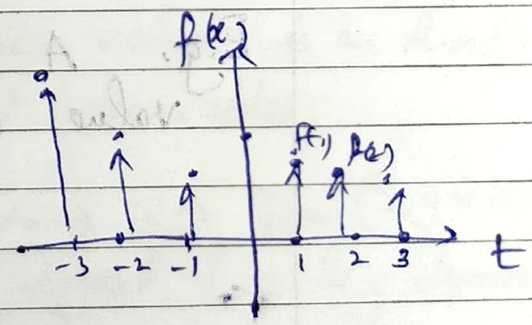
A picture can be represented by two spacial variables ; x & y. (Space Dependant)

- **Systems** are used to process / modify / extract additional info. from signals. It processes a set of inputs to yield another set of outputs.

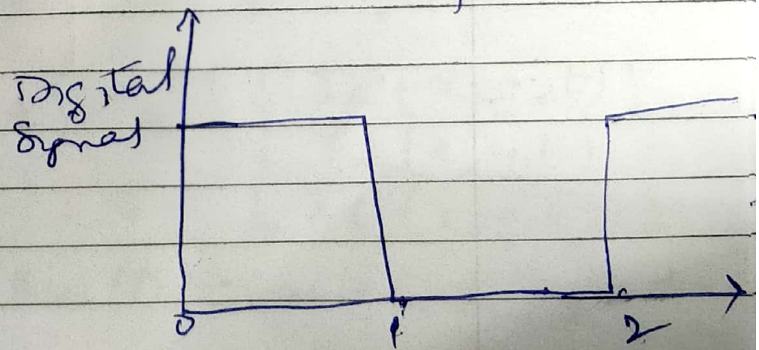
- Classification of Signals

(1) Continuous Time v/s Discrete time signal





$f(1.5) = ND$



- Continuous Signal
It is an analog signal defined for continuous values of independent variable (time).

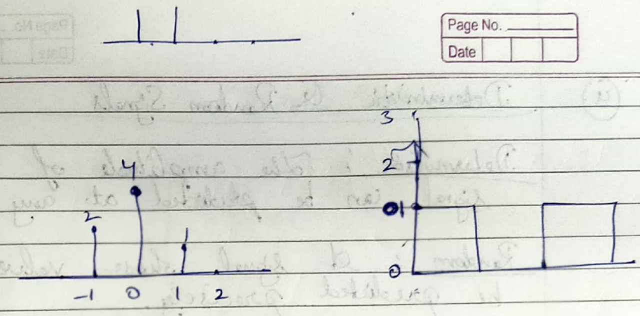
- Discrete time signal is defined only at discrete time instants. Amplitude of the DTS in b/w those instants is N.D.

(2) Continuous Valued & Discrete Valued Signals

- Continuous Valued : It means that a signal can take all possible values on a infinite range ($-\infty$ to $+\infty$).

- Discrete Valued Signal : It is a signal that takes on values from a finite set of possible values.

E.g. A digital signal is a digital value discrete time signal.

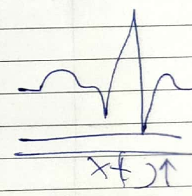


(3) Multichannel & Multidimensional Signals

Multichannel : When a signal is generated by multiple sources.

$$X(t) = \begin{cases} x_1(t) \\ x_2(t) \\ x_3(t) \end{cases}$$

eg. ECG Signal.



Multidimensional Signal
It is a signal whose value is a function of multiple inddependent variables.

E.g. Black & white TV → 3D Signal (x, y & brightness)
Picture is a 2-D signal (x & y dependent)

Colour - TV picture can be represented as

$$I(x,y,t) = \begin{cases} I_R(x,y,t) \\ I_B(x,y,t) \\ I_G(x,y,t) \end{cases}$$

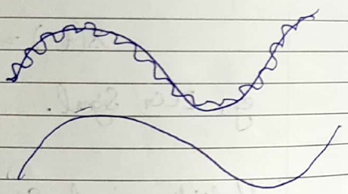
R + G + B → Multi chanel + Multi dimensional

4. ## Deterministic & Random Signals

Deterministic: The amplitude of the signal can be predicted at any time.

Random: A signal whose values can't be predicted precisely.

E.g. Noise Signal



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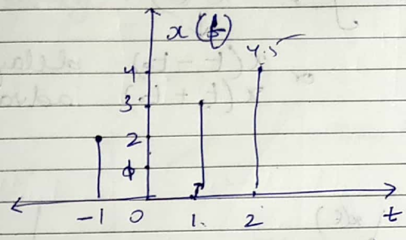
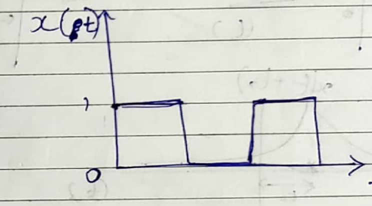
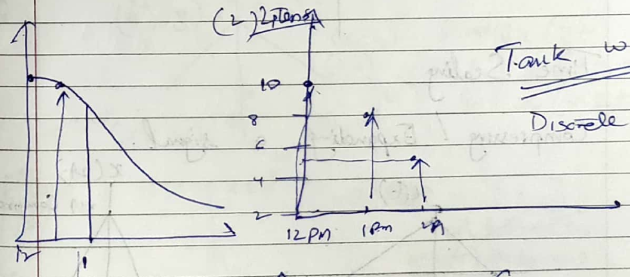
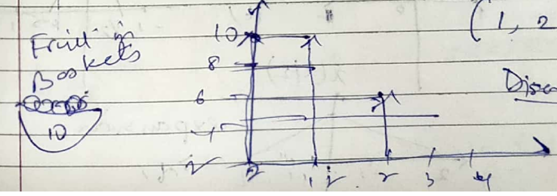
Circuits & Systems









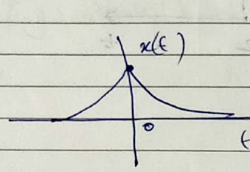
(1, 2, . . . 10)

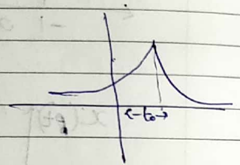
Discrete Values

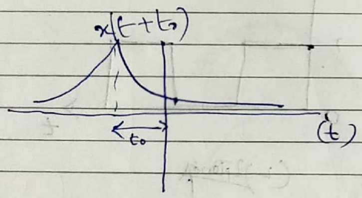
Page No.
Date

- **Transformations**

1. **Time Shifting** & $x(t)$ $\downarrow$ $x(t-t_0)$ delay or $x(t+t_0)$ advancing

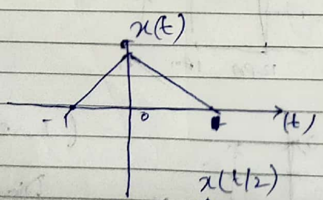


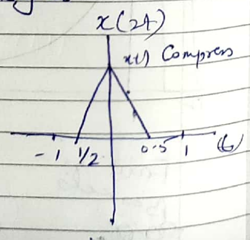


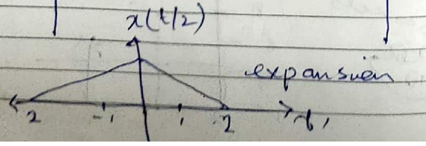


2. **Time Scaling**

Compressing / Expanding a signal.





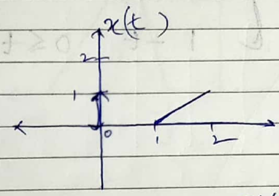


Time Shifting → Scaling → Reversing

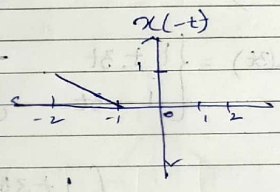
Page No.
Date

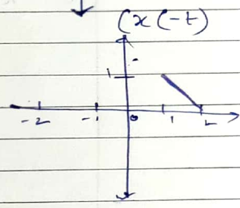
3. **Time Reversal**

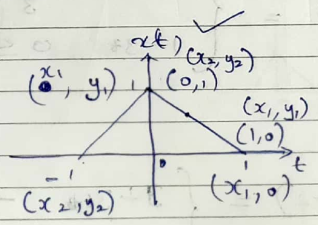
Reversing / Mirroring Operation of time. (wrt origin)



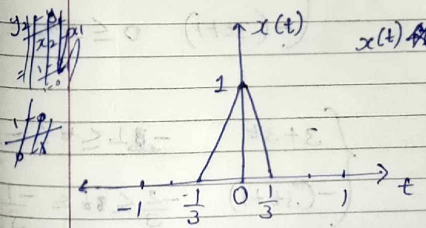
Time Reversal





Q. 

Find (i) $x(3t)$
(ii) $x(3t+2)$
(iii) $x(-2t-1)$



$x(t) = [1, 1]$

~~$x(t) + t = 0$~~

$x(t) = -t + 1$

$[0 \le t \le 1]$

$x(t) = t + 1$
$(-1 \le t \le 0)$

$$x(t) = \begin{cases} 1+t & -1 \le t \le 0 \\ 1-t & 0 \le t \le 1 \end{cases}$$

(i) $x(3t)$

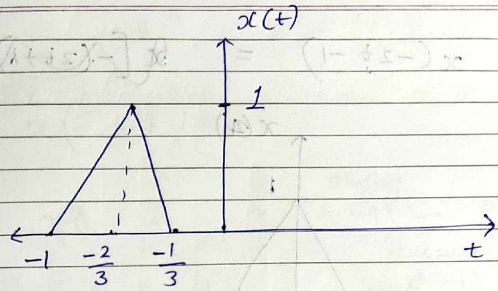
$$x(3t) = \begin{cases} 1+3t & -1 \le 3t \le 0 \\ 1-3t & 0 \le 3t \le 1 \end{cases}$$

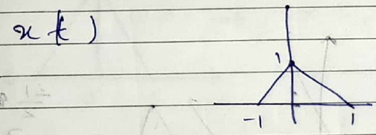
$$x(3t) = \begin{cases} 1+3t & \frac{-1}{3} \le t \le 0 \\ 1-3t & 0 \le t \le \frac{1}{3} \end{cases}$$

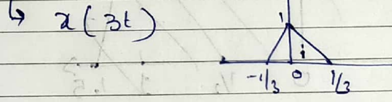
(ii) $x(3t+2)$

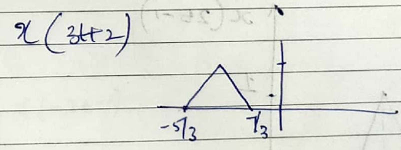
$$x(3t+2) = \begin{cases} 3+3t & -1 \le 3t+2 \le 0 \\ -(3t+1) & 0 \le 3t+2 \le 1 \end{cases}$$

$$x(3t+2) \begin{cases} 3+3t & -1 \le t \le \frac{-2}{3} \\ -(3t+1) & \frac{-2}{3} \le t \le \frac{-1}{3} \end{cases}$$









$\frac{-1}{3} + 2$

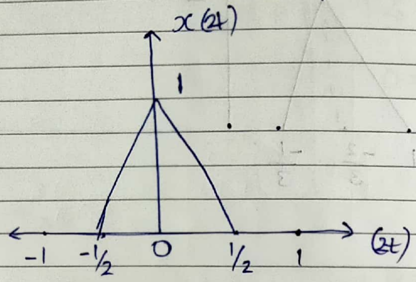
$\frac{-1+6}{3}$

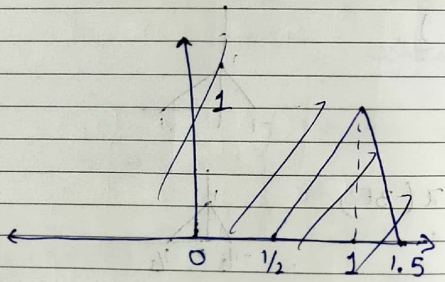
$5/3$

$\frac{1}{3} + 2$

$x(-2t-1) = x[-(2t+1)]$

(C)



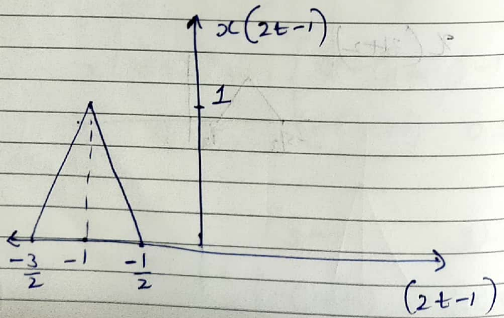


$-\frac{1}{2}+1$

$-\frac{3}{2}$

$\frac{1}{2}+1$

$-\frac{1}{2}$



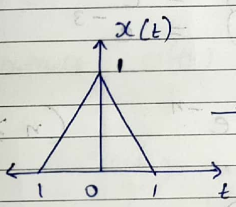
$x(-2t-1)$

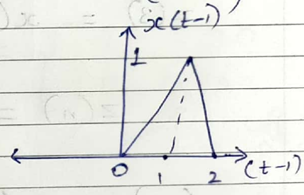
$x(t) \rightarrow x(t-1) \rightarrow x(2t-1)$

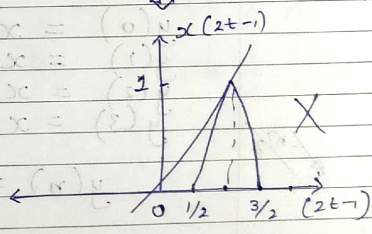
Shifting Scaling

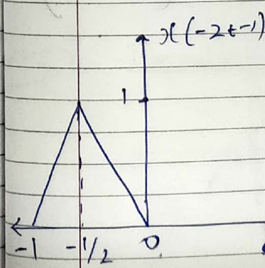
reversing

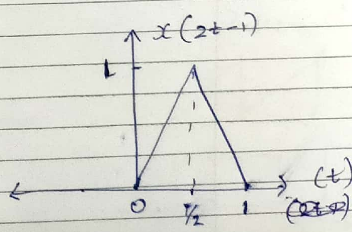
$x(-2t-1)$











Q. If $x(n) = e^{-\frac{n}{2}}$, find
(i) $x(2n) = z(n)$
(ii) $x(n^2)$

(i)
$z(0) = x(0) = 1$
$z(1) = x(2) = e^{-1}$
$z(2) = x(4) = e^{-2}$
$z(3) = x(6) = e^{-3}$

$$z(n) = e^{-n} \quad (n \geq 0)$$

(ii) $x(n^2) = y(n)$

$y(0) = x(0) = 1$
$y(1) = x(1) = e^{-1/2}$
$y(2) = x(4) = e^{-2}$
$y(3) = x(9) = e^{-9/2}$

$e^{-n^2/2}$

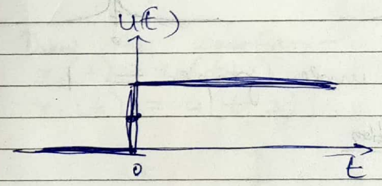
$$y(n) = e^{-\frac{n^2}{2}} \quad (0 \leq n)$$

• Step, Ramp & Impulse Functions

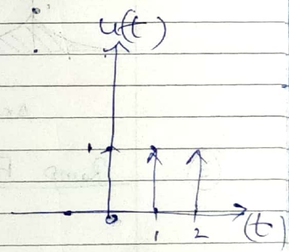
(1) Step function (Unit)

Continuous $u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \\ 1/2 & t = 0 \end{cases}$

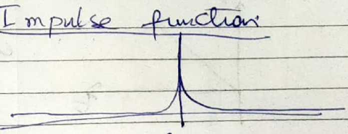
Discrete $u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$



Continuous

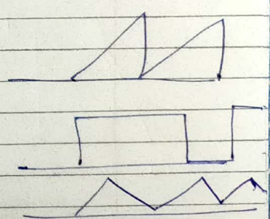


(2) Impulse function



Impulse

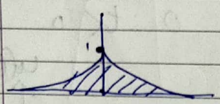
Pulses

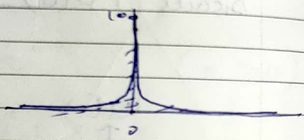


unit $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}$

Unit Impulse

Impulse funct

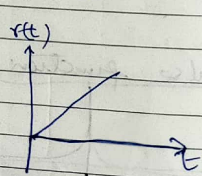




Area under graph = 1

(3) Ramp Function

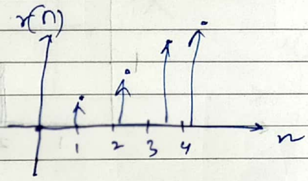
$$r(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$



continues time

discrete time

$$r(n) = \begin{cases} n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

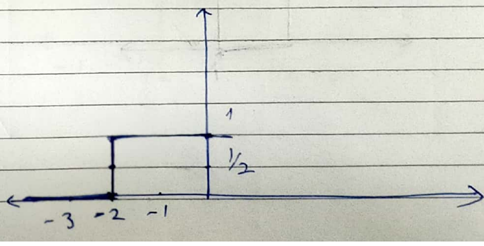


Q. Draw the waveform —

(a) $x_1(t) = u(t+2)$

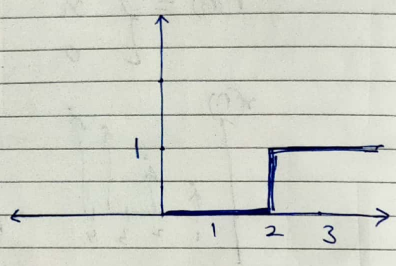
(b) $x_2(t) = u(t-2)$

$$u(t+2) = \begin{cases} 1 & t+2 > 0 \text{ or } t > -2 \\ 0 & t < -2 \\ 1/2 & t = -2 \end{cases}$$



(ii)

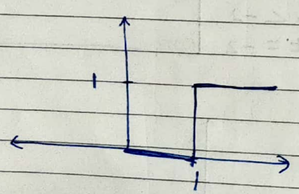
$$u(t-2) = \begin{cases} 1 & t > 2 \\ 0 & t < 2 \\ 1/2 & t = 2 \end{cases}$$



Q. $u(-2t-1)$

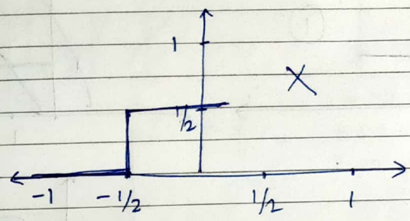
$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \\ 1/2 & t = 0 \end{cases}$$

$$u(t-1) = \begin{cases} 1 & t > 1 \\ 0 & t < 1 \\ 1/2 & t = 1 \end{cases}$$

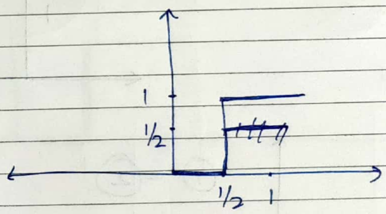


$u(t-1)$

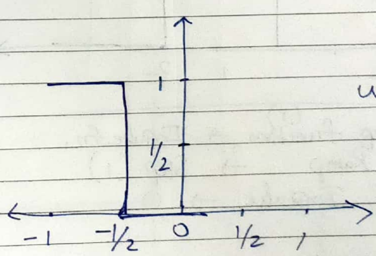
$$u(-2t-1) = \begin{cases} 1 & t < -\frac{1}{2} \\ 0 & t > -1/2 \\ 1/2 & t = -1/2 \end{cases}$$



$$u(2t-1) = \begin{cases} 1 & t > 1/2 \\ 0 & t < 1/2 \\ 1/2 & t = 1/2 \end{cases}$$



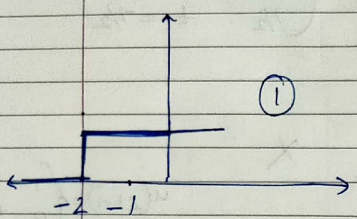
$u(2t-1)$

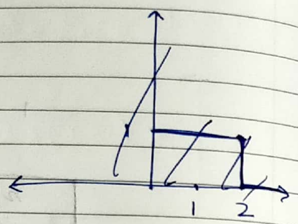


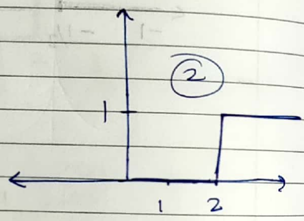
$u(-2t-1)$

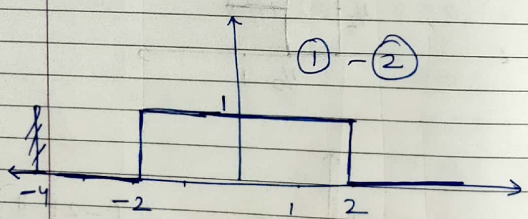
Q. $f(t) = x(1)t - x(2)t$

$2u(t+2) \quad u(t-2)$









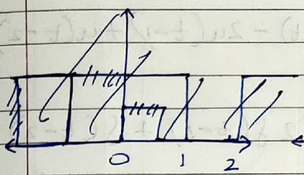
Derivatives

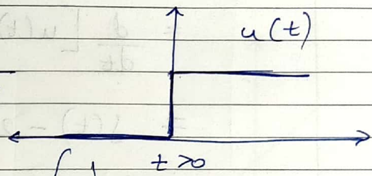
Step function → Impulse fn. (1)

Ramp → Step (1)

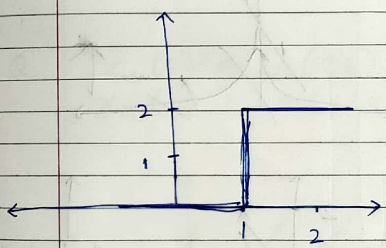
Impulse → 0

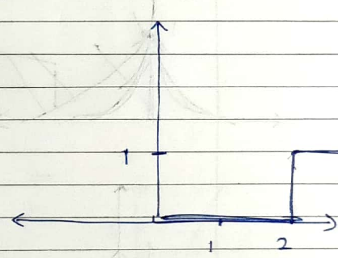
Q. $G(t) = u(t) - 2u(t-1) + u(t-2)$

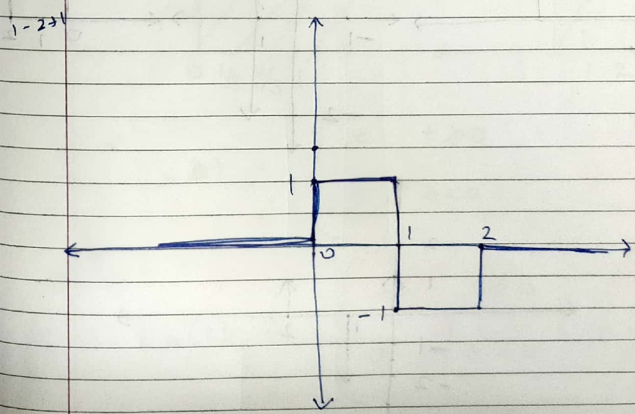




$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \\ 1/2 & t = 0 \end{cases}$$



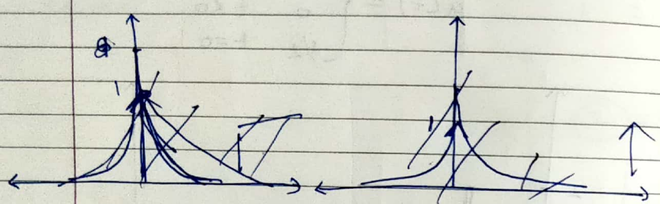


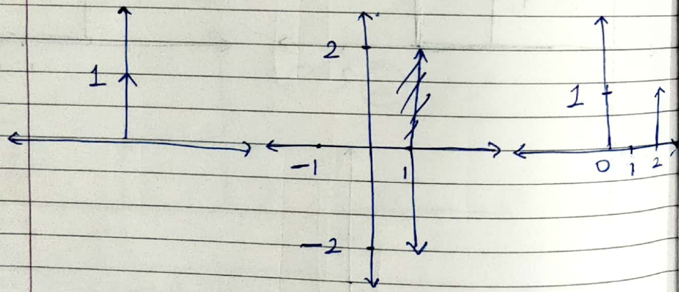


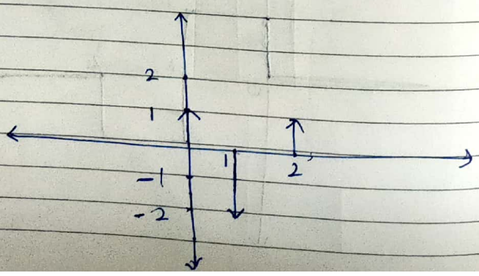
Q. $h(t) = \frac{d(g(t))}{dt}$

$$= \frac{d}{dt}\left[u(t) - 2u(t-1) + u(t-2)\right]$$

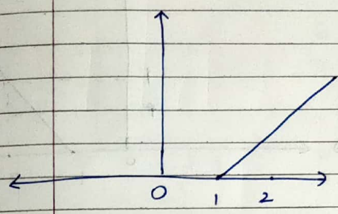
$$= \delta(t) - 2\delta(t-1) + \delta(t-2)$$

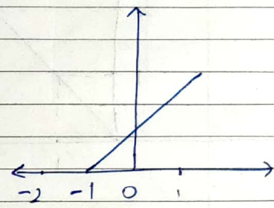




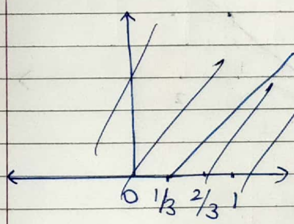


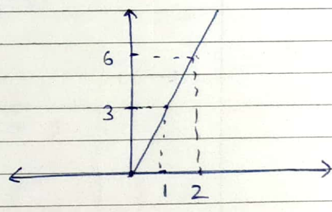
Q. $x_1(t) = r(t-1)$
$x_2(t) = r(t+1)$





Q. $x_3(t) = r(3t)$
$x_4(t) = r(-3t)$

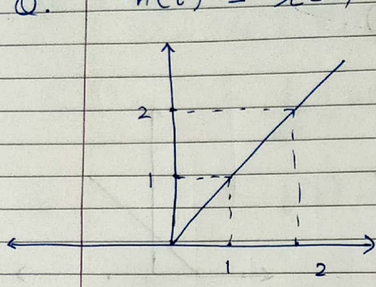


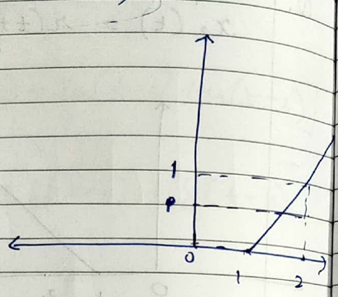


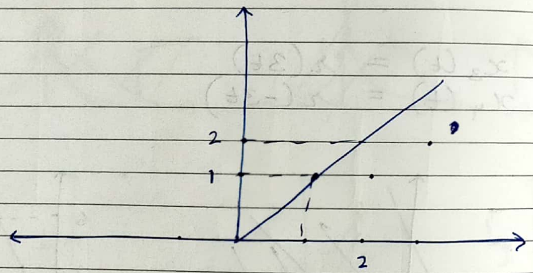
$$r(3t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

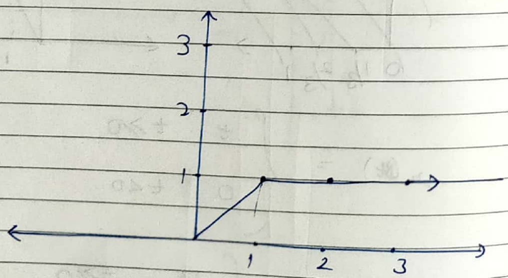
$$r(t) = \begin{cases} 3t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Q. $h(t) = r(t) - r(t-1)$



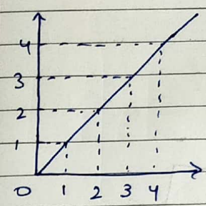




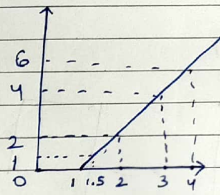


Q. $x_q(t) = h(t) - r(t-1) + r(t-2)$

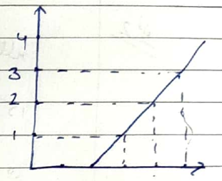
$= r(t) - 2r(t-1) + r(t-2)$



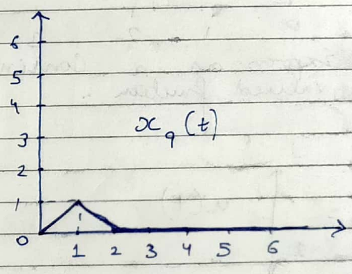
$r(t)$



$2r(t-1)$



$r(t-2)$



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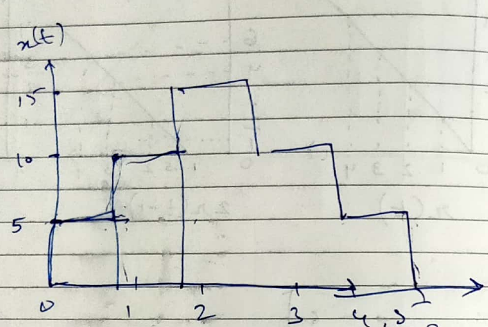
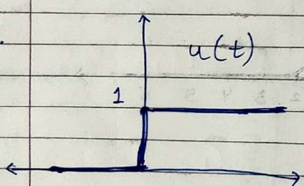
Q1 $x_2(t) = u(t) + r(t) - 2r(t-1) + r(t-2) - u(t-2)$

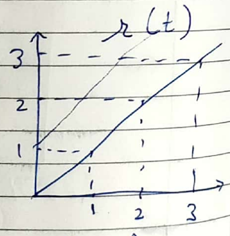
Q2

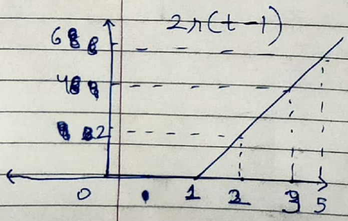


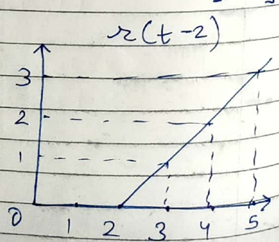
a) Express as a continuous time, discrete valued function.

Ans 1.

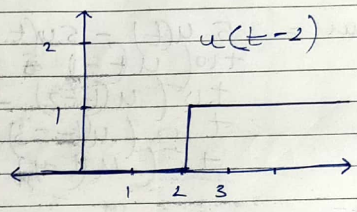


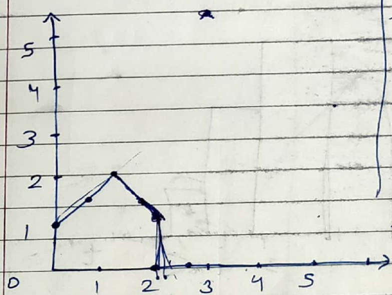






1 + 0.5 + 0 + 0 + 0
1 + 1 − 0 + 0 − 0
1 + 1.5 − 1 + 0 − 0
1 + 2 − 2 + 0 − 1
1 + 2.5 − 3 + 0.5 − 1 =



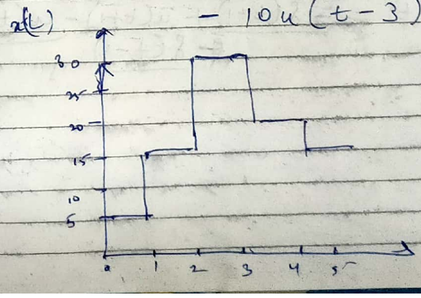


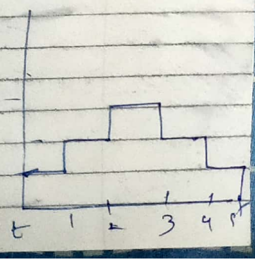
	t	x(t)
1	0	1
2	0.5	1.5
3	1	2
4	1.5	1.5
5	2.0	0
6	2.5	0

Ans 5. $5u(t) + 5u(t-1) + 5u(t-2) - 5u(t-3) - 5u(t-4) - 5u(t-5)$

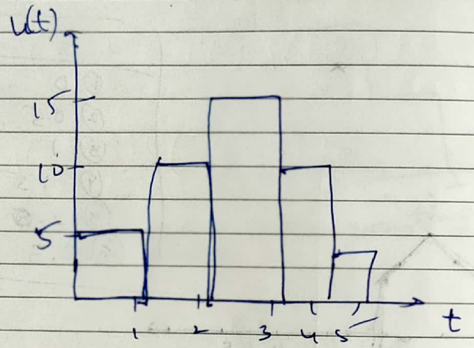
~~Ans 5~~ $5u(t) + 10u(t-1) + 15u(t-2) + 15u(t-3) - 10u(t-3)$

$5u(t) + 10u(t-1) + 15u(t-2) - 10u(t-3) - 5u(t-4) - u(t-5)$





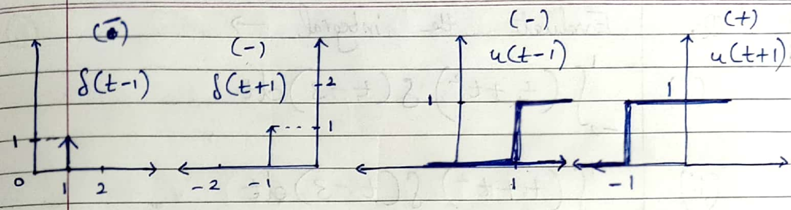
answer = $5\,u(t) - 5u(t-1)$
$+10(u(t-1) - u(t-2))$
$+15(u(t-2) - u(t-3))$
$+10(u(t-3) - u(t-4))$
$+5(u(t-4) - u(t-5))$

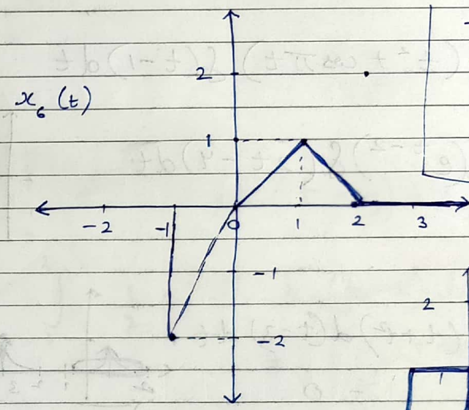


Q. $x_6(t) = \frac{d}{dt} g(t)$

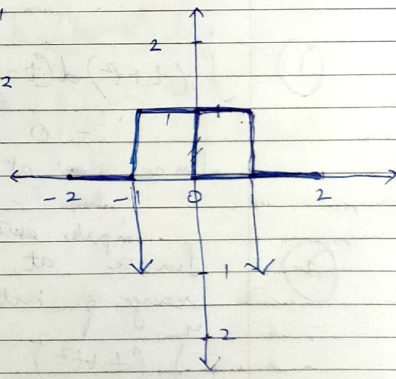
where $g(t) = -u(t+1) + r(t+1) - r(t-1) - u(t-1)$

$\therefore x_6(t) = -\delta(t+1) + u(t+1) - u(t-1) - \delta(t-1)$





-2	0
-1	-2
0	1
1	1
2	0



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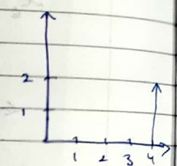
Q. Evaluate the integral →

(i) $\int_{-2}^{1}(t+t^2)\,\delta(t-3)\,dt$

(ii) $\int_{-2}^{4}(t+t^2)\,\delta(t-3)\,dt$

(iii) $\int_{-\infty}^{+\infty}(t^2+\cos\pi t)\,\delta(t-1)\,dt$

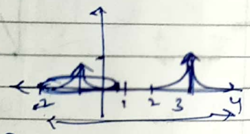
(iv) $\int_{0}^{3}(e^{t-2})\,\delta(2t-4)\,dt$



(1) $\int_{-2}^{1}(t+t^2)\,\delta(t-3)\,dt$

$= 0$

Since ~~a~~ at $t=3$



(2) Since a impulse exists at $t=3$, which is within the range of integral

$\int_{-2}^{4}(t+t^2)\,\delta(t-3) = (t+t^2)_{t=3}$

$= 3+3^2 = \underline{\underline{12}}$

(3) The function is within the range $(-\infty,\infty)$

$\therefore \int_{-\infty}^{\infty}(t^2+\cos\pi t)\,\delta(t-1)\,dt$

$= \int (t^2+\cos\pi t)_{t=1}$

$= (1^2+\cos(\pi))$

$1-1$

$= \underline{\underline{0}}$

(4) $\int_{0}^{3} e^{(t-2)}\,\delta\, 2(t-2))\,dt$

$\int_{0}^{3} \frac{1}{2} e^{(t-2)}\,\delta(t-2)\,dt$

$= \left|\frac{1}{2}e^{(t-2)}\right|_{t=2}$

$= \underline{\underline{\frac{1}{2}}}$

(5) $\int_{-\infty}^{\infty} e^{t}\,\delta\,(2t-2))\,dt$

$= \frac{1}{2}(e^{-t})\int_{-\infty}^{\infty}(t-1)\,dt$

$= \frac{e^{-1}}{2} = \frac{1}{2e}$

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Periodic & Aperiodic Signals

A signal is periodic if

for continuous time signals

$$x(t+T) = x(t) \;\forall\, t$$

for discrete time signals, $x(n+N) = x(n)$ $\forall n$.

Q. Show that $x(t) = e^{j\omega_0 t}$ is periodic. Also find its time period.

$$x(t) = e^{j\omega_0 t}$$

If Periodic

$$x(t) = x(t+T)$$

$$e^{j\omega_0 t} = e^{j\omega_0 (t+T_0)}$$

$$e^{j\omega_0 t} = e^{j\omega_0 t} \cdot e^{j\omega_0 T}$$

$e^{a+b} = e^a \cdot e^b$

$x^{a+b} = x^a \cdot x^b$

$$e^{j\omega_0 T} = 1$$

$$e^{j\omega_0 T} = e^{j2\pi f T}$$

$$f \times T = m$$

IRRATIONAL $\frac{1}{3} = 0.333$ $\frac{3}{10} = 0.3$ - RATIONAL

$$j\omega_0 T = j2\pi m$$

$$\omega_0 T = 2\pi m$$

$$\boxed{T = \frac{2\pi}{\omega_0} \cdot m}$$

Discrete time signals

if $\frac{\omega_0}{2\pi} = \frac{m}{N}$: rational number

signal is periodic

T = LCM of denominator

Q. Find whether the signal is periodic or not. If Periodic, find its time period :

(i) $x_1(t) = je^{j10(t)}$

(ii) $x_3(n) = e^{j7\pi(n)}$

(iii) $x_4(n) = 3e^{j3\pi(n+1/2)/5}$

(iv) $x_5(n) = 3e^{j(3/5)(n+1/2)}$

HW

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Ans (i) $T = \frac{2\pi}{10} = \frac{\pi}{5}$

(ii) $x_3(n) = e^{j7\pi(n)}$

$\omega_0 = 7\pi$

$\frac{\omega_0}{2\pi} = \frac{7\cancel{\pi}}{2\cancel{\pi}} = \frac{7}{2}$ (rational)

∴ The fn. is periodic with

$\boxed{T = 2}$

(iii) $x_4(n) = 3e^{j3\pi(n+1/2)/5}$

$= 3\left[e^{j.3\pi(n)/5} \cdot e^{j.\frac{3\pi}{10}}\right]$

$= 3e^{j\frac{3\pi}{10}}\left(e^{j3\pi n/5}\right)$

$\Rightarrow \omega_0 = \frac{3\pi}{5}$

$\frac{\omega_0}{2\pi} = \frac{3\cancel{\pi}}{10\cancel{\pi}} = \frac{3}{10}$; rational

∴ Given fn. is periodic with

$\boxed{T = 10}$

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Q.
(i) $x(t) = x_1(t) + x_2(t) + x_3(t)$
where T_1 of $x_1(t) = 4$ sec
T_2 of $x_2(t) = 1.25$ sec
T_3 of $x_3(t) = \sqrt{2}$ sec

Ratio of Time periods should be rational.

∴ $y \rightarrow \frac{T_1}{T_2}, \frac{T_1}{T_3}$. Rational

Signal is periodic

$T = \text{LCM of } (T_2 \& T_3)$

$\frac{T_1}{T_3} = \frac{4}{\sqrt{2}} = 2\sqrt{2} \Rightarrow$ Irrational

∴ Given function is aperiodic even if T_1/T_2 is rational.

(ii) $y(t) = y_1(t) + y_2(t) + y_3(t)$
$T_1 = 1.08$, $T_2 = 3.6$, $T_3 = 2.025$

$\frac{T_1}{T_2} = \frac{1.08}{3.6} = 0.3 = \frac{3}{10} \Rightarrow$ rational $\quad \frac{3}{2\times5}$

$\frac{T_1}{T_3} = \frac{1080}{2025} = \frac{\cancel{1080}^{216}}{\cancel{2025}_{405}} = \frac{8}{3\times5} \Rightarrow$ rational

∴ Given fn. is periodic.

with $\boxed{T = 30\times T_1} = 30\times1.08 = 32.4$ sec

③ $y(n) = y_1(n) + y_2(n)$

$T_1 = 90$ sec
$T_2 = 54$ sec

$\frac{T_1}{T_2} = \frac{45}{27} = \frac{5}{3} \Rightarrow$ rational
$\Rightarrow$ periodic

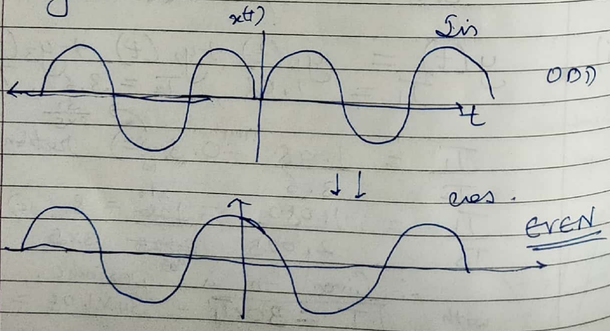
$T = 3 \times 90 = 270$ sec

- Odd & Even Signals

For an even signal

For continuous time, $x(t) = x(-t)$

A signal is called even signal if it is identical with its reflection about the origin.



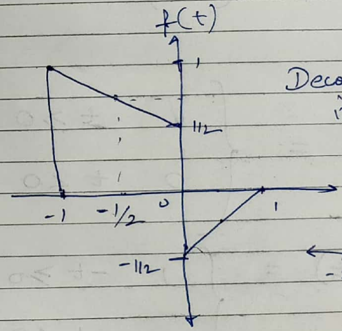
For discrete time signal, signal is even if $x(n) = x(-n)$

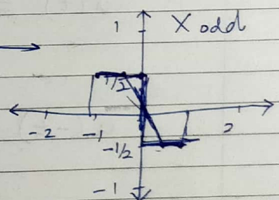
→ How to find odd & even component of a given signal?
For continuous time signal,

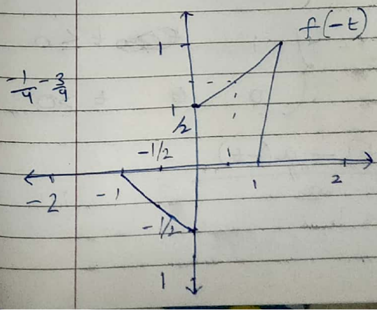
$$x_{odd}(t) = \frac{x(t) - x(-t)}{2}$$

$$x_{even}(t) = \frac{x(t) + x(-t)}{2}$$

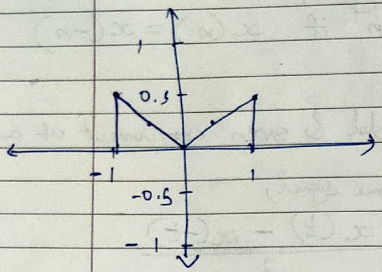
Q Decompose the fn. $f(t)$ into its odd & even parts.







t	x(t)	x(-t)	x odd(t)
-1	1	0	1/2
-0.5	3/4	-1/4	1/2
0			
0.5	-1/4	3/4	-1/2
1	0	1	-1/2



t	x_{even}
-1	$1/2$
-0.5	$1/4$
$+0.5$	$1/4$
1	$1/2$

Q. (i) Find odd & even components of —
$x(t) = e^{-2t}.(\cos t)$

(ii) $u(t)$

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$u(-t) = \begin{cases} 1 & -t \geq 0 \\ 0 & -t < 0 \end{cases}$$

$$= \begin{cases} 1 & t \leq 0 \\ 0 & t > 0 \end{cases}$$

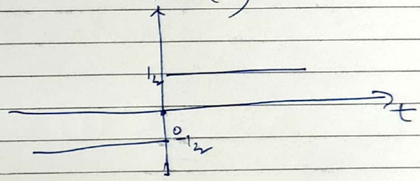
$$u_{odd}(t) = \frac{u(t) - u(-t)}{2}$$

At $t=0$ $u_{odd}(t) = 0$

When $t > 0$, $u_{odd}(t) = \frac{1}{2}$

$t < 0$, $u_{odd}(t) = \frac{-1}{2}$

$u_{odd}(t)$



When $t = 0$,
$u_{even}(t) = 1$

When $t > 0$, $u_{even}(t) = \frac{1}{2}$

When $t < 0$, $u_{even}(t) = \frac{1}{2}$

$u_e(t)$