———————————————————————————EJERCICIO~7——————————————————
———————————— Spline lineal

$\longrightarrow$ f(x):=1/(1+25*x^2);

(%o1) $$\mathrm{f}(x) := \frac{1}{1+25x^2}$$

$\longrightarrow$ P:makelist(-1+2*i/5, i, 0, 5);

(P) $$[-1, -\frac{3}{5}, -\frac{1}{5}, \frac{1}{5}, \frac{3}{5}, 1]$$

$\longrightarrow$ imagenes:makelist(f(P[i]), i, 1, 6);

(imagenes) $$[\frac{1}{26}, \frac{1}{10}, \frac{1}{2}, \frac{1}{2}, \frac{1}{10}, \frac{1}{26}]$$

$\longrightarrow$ B(i, x):=if i=1 then(if (P[1]<=x and x<=P[2]) then (P[2] - x)/(P[2]-P[1]) else 0) else if (1<i and i<6) then(if (P[i-1]<x and x<P[i]) then (x-P[i-1])/(P[i]-P[i-1])else if (P[i]<x and x<P[i+1]) then (P[i+1] - x)/(P[i+1]-P[i])else 0) else if i=6 then(if (P[5]<=x and x<=P[6]) then (x-P[5])/(P[6] - P[5]) else 0) else 0;

(%o4)

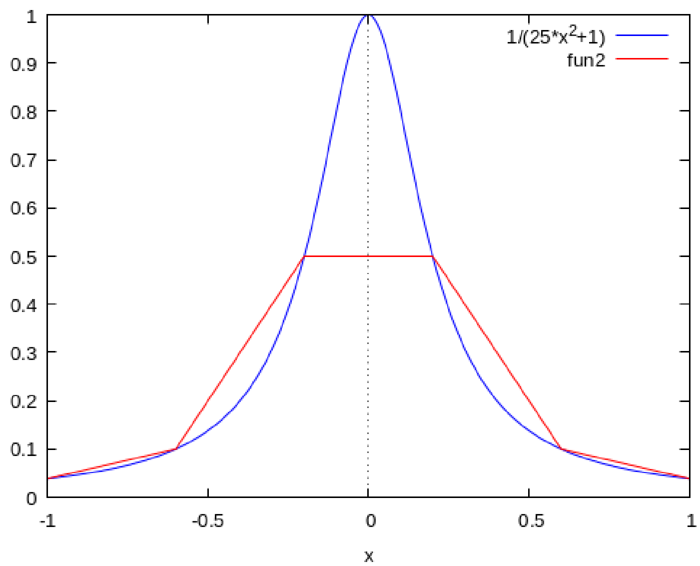
$$\mathrm{B}(i,x) := \mathrm{if}\ i=1\ \mathrm{then}\ \mathrm{if}\ P_1 < \ = x and x < \ = P_2\ \mathrm{then}\ \frac{P_2 - x}{P_2 - P_1}\ \mathrm{else}\ 0\ \mathrm{else}\ \mathrm{if}\ 1 < \ i$$
$$and i < \ 6\ \mathrm{then}\ \mathrm{if}\ P_{i-1} < \ x and x < \ P_i\ \mathrm{then}\ \frac{x - P_{i-1}}{P_i - P_{i-1}}\ \mathrm{else}\ \mathrm{if}\ P_i < \ x and x < \ P_{i+1}\ \mathrm{then}\ \frac{P_{i+1} - x}{P_{i+1} - P_i}\ \mathrm{else}\ 0$$
$$\mathrm{else}\ \mathrm{if}\ i=6\ \mathrm{then}\ \mathrm{if}\ P_5 < \ = x and x < \ = P_6\ \mathrm{then}\ \frac{x - P_5}{P_6 - P_5}\ \mathrm{else}\ 0\ \mathrm{else}\ 0$$

$\longrightarrow$ s_lineal(x):=sum(imagenes[i]*B(i, x), i, 1, 6);

(%o5) $$s_{lineal}(x) := \sum_{i=1}^{6} imagenes_i\, \mathrm{B}(i,x)$$

⟶ wxplot2d([f(x), s_lineal(x)], [x,-1,1])$

(% t6)



Spline cúbico

⟶ h:2/5;

(h) $$\frac{2}{5}$$

⟶ A:genmatrix(lambda([i,j], if i=j then 2 else if i=j+1 or j=i+1 then 1/2 else 0), 6, 6);

(A) $$\begin{pmatrix} 2 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 2 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 2 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 2 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 2 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 2 \end{pmatrix}$$

⟶ A[1, 2]:0;

(% o9) $$0$$

⟶ A[6, 5]:0;

(% o10) $$0$$

→ b:makelist(0, i, 1, 6);

(b) $$[0,0,0,0,0,0]$$

→ for i:2 thru 5 do b[i]:(imagenes[i+1]-2*imagenes[i]+imagenes[i-1])*3/h^2;

(% o12) $$done$$

→ c:invert(A).b;

(c) $$\begin{pmatrix} 0 \\ \frac{1020}{247} \\ -\frac{945}{247} \\ -\frac{945}{247} \\ \frac{1020}{247} \\ 0 \end{pmatrix}$$

→ alpha:makelist((imagenes[i+1]-imagenes[i])/h - (h/6)*(c[i+1,1]-c[i,1]), i, 1, 5);

(alpha) $$[-\frac{30}{247},\frac{378}{247},0,-\frac{378}{247},\frac{30}{247}]$$

→ beta:makelist(imagenes[i]-c[i, 1]*h^2/6, i, 1, 5);

(beta) $$[\frac{1}{26},-\frac{5}{494},\frac{1487}{2470},\frac{1487}{2470},-\frac{5}{494}]$$

→ s(i, x):=c[i,1]*(P[i+1]-x)^3/(6*h) + c[i+1, 1]*(x-P[i])^3/(6*h) + alpha[i]*(x-P[i]) + beta[i];

(% o16) $$\mathrm{s}\left(i,x\right):=\frac{c_{i,1}\left(P_{i+1}-x\right)^3}{6h}+\frac{c_{i+1,1}\left(x-P_i\right)^3}{6h}+alpha_i\,\left(x-P_i\right)+beta_i$$

→ p(i, x):= if P[i]<=x and x<P[i+1] then s(i, x) else 0;

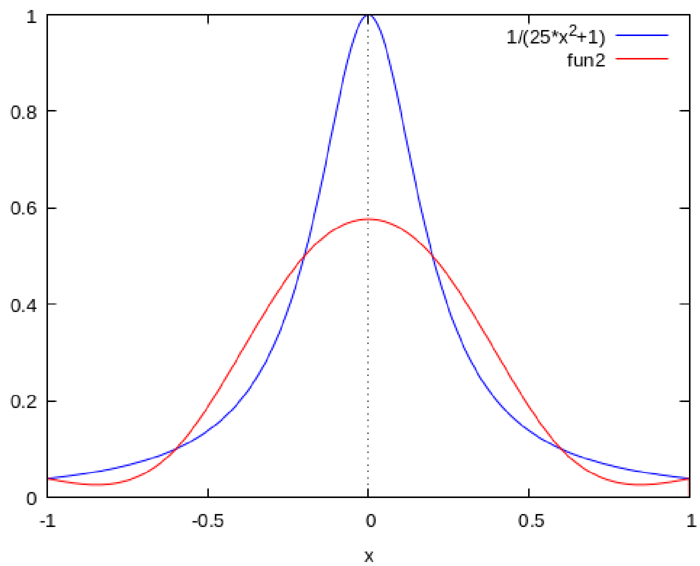
(% o17) $$\mathrm{p}\left(i,x\right):=\mathrm{if}\; P_i< =x and x< \; P_{i+1}\;\mathrm{then}\,\mathrm{s}\left(i,x\right)\mathrm{else}\,0$$

→ s_cubic(x):=sum(p(i, x), i, 1, 5);

(% o18) $$s_{cubic}(x):=\sum_{i=1}^{5}\mathrm{p}\left(i,x\right)$$

→ wxplot2d([f(x), s_cubic(x)], [x,-1,1])$

(% t19)



Lo que dice el principio de mínima energía es que la integral definida entre a y b, de la segunda derivada del spline cúbico queacabamos de calcular al cuadrado, es menor o igual que la integral definida entre a y b, de la segunda derivada del polinomiointerpolador, al cuadrado. Veamos que esto ocurre. Calculemos primero el polinomio interpolador (de Lagrange) de f.

→ l(i,x):=product((x-P[j])/(P[i]-P[j]),j,1,i-1)*product((x-P[j])/(P[i]-P[j]),j,i+1,6);

(% o20) $$\mathrm{l}\left(i,x\right):=\prod_{j=1}^{i-1}\frac{x-P_j}{P_i-P_j}\;\prod_{j=i+1}^{6}\frac{x-P_j}{P_i-P_j}$$

→ p(x):=sum(imagenes[i]*l(i, x), i, 1, 6);

(% o21) $$\mathrm{p}(x):=\sum_{i=1}^{6} \mathit{imagenes}_i\,\mathrm{l}\left(i,x\right)$$

→ float(expand(p(x)));

(% o22) $1.201923076923076x^4 - 1.73076923076923x^2 + 0.5673076923076923$

→ integral_interp:float(integrate((diff(p(x), x, 2))^2, x, -1, 1));

(integral_ interp) 40.60650887573964

Calculamos ahora la integral del spline

$\longrightarrow$ integral_s:sum(integrate((diff(s(i, x), x, 2))^2, x, P[i], P[i+1]), i, 1, 5);

(integral_ s) $$\frac{47010}{3211}$$

$\longrightarrow$ float(%);

(% o25) 14.64029897228277

Y está claro que se cumple el principio de mínima energía.