RELACIÓN 3 MAXIMA

EJERCICIO~1

$\longrightarrow$ nodos:makelist(i/8, i, 0, 8);

(nodos) $$[0, \frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{5}{8}, \frac{3}{4}, \frac{7}{8}, 1]$$

$\longrightarrow$ f(x):=sin(x) - 2*x;

(% o6) $$\mathrm{f}(x) := \sin(x) - 2x$$

$\longrightarrow$ imagenes:makelist(f(nodos[i+1]), i, 0, 8);

(imagenes)
$$[0, \sin\left(\frac{1}{8}\right) - \frac{1}{4}, \sin\left(\frac{1}{4}\right) - \frac{1}{2}, \sin\left(\frac{3}{8}\right) - \frac{3}{4}, \sin\left(\frac{1}{2}\right) - 1, \sin\left(\frac{5}{8}\right) - \frac{5}{4}, \sin\left(\frac{3}{4}\right) - \frac{3}{2}, \sin\left(\frac{7}{8}\right) - \frac{7}{4}, \sin(1) - 2]$$

$\longrightarrow$ l(i,x):=product((x-nodos[j])/(nodos[i]-nodos[j]),j,1,i-1)*product((x-nodos[j])/(nodos[i]-nodos[j]),j,i+1,9);

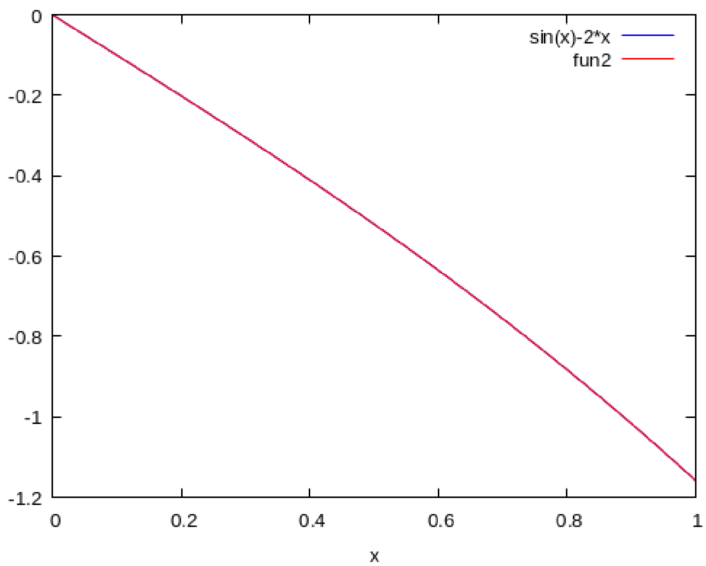
(% o8) $$\mathrm{l}(i,x) := \prod_{j=1}^{i-1} \frac{x - nodos_j}{nodos_i - nodos_j} \prod_{j=i+1}^{9} \frac{x - nodos_j}{nodos_i - nodos_j}$$

$\longrightarrow$ p(x):=sum(imagenes[i]*l(i, x), i, 1, 9);

(% o9) $$\mathrm{p}(x) := \sum_{i=1}^{9} imagenes_i \, \mathrm{l}(i,x)$$

$\longrightarrow$ wxplot2d([f(x), p(x)], [x,0,1])$

(% t10)



EJERCICIO~2

$\longrightarrow$ N:9;

(N) $$9$$

$\longrightarrow$ nodos:makelist(i/8, i, 0, 8);

(nodos) $$[0, \frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{5}{8}, \frac{3}{4}, \frac{7}{8}, 1]$$

$\longrightarrow$ f(x):=sin(x) - 2*x;

(% o19) $$\mathrm{f}(x) := \sin(x) - 2x$$

$\longrightarrow$ imagenes:makelist(f(nodos[i+1]), i, 0, 8);

(imagenes)
$$[0, \sin\left(\frac{1}{8}\right)-\frac{1}{4}, \sin\left(\frac{1}{4}\right)-\frac{1}{2}, \sin\left(\frac{3}{8}\right)-\frac{3}{4}, \sin\left(\frac{1}{2}\right)-1, \sin\left(\frac{5}{8}\right)-\frac{5}{4}, \sin\left(\frac{3}{4}\right)-\frac{3}{2}, \sin\left(\frac{7}{8}\right)-\frac{7}{4}, \sin(1)-2]$$

$\longrightarrow$ w(i, x):=if i=1 then 1 else product(x-nodos[j], j, 1, i-1);

(% o21) $$\mathrm{w}(i, x) := \text{if } i = 1 \text{ then } 1 \text{ else } \prod_{j=1}^{i-1} x - nodos_j$$

$\longrightarrow$ difer: genmatrix(lambda([i,j], 0), N, N);

(difer)
$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$\longrightarrow$ for i:1 thru N do difer[i, 1]:imagenes[i];

(% o23) $\quad done$

$\longrightarrow$ for i:2 thru N do (for j:i thru N do difer[j, i]: (difer[j, i-1] - difer[j-1, i-1])/(nodos[j]-nodos[j-i+1]));

(% o24) $\quad done$

$\longrightarrow$ p(x):=sum(difer[i, i]*w(i, x), i, 1, 9);

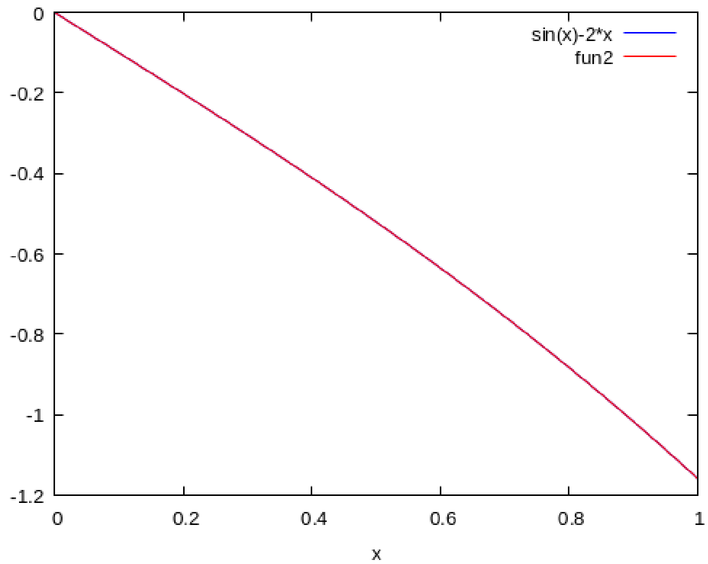
(% o26)
$$\mathrm{p}(x) := \sum_{i=1}^{9} \mathit{difer}_{i,i}\, \mathrm{w}(i, x)$$

$\longrightarrow$ float(expand(p(x)));

(% o27)
$$1.18287365467040210^{-5}\, x^8 - 2.20308210600705910^{-4}\, x^7 + 2.2417555328502210^{-5}\, x^6$$
$$+ 0.0083195917127341 09x^5 + 5.11623886723100410^{-6}\, x^4 - 0.1666677845536526x^3$$
$$‘ + 1.29241286117576210^{-7}\, x^2 - 1.000000005932017x$$

$\longrightarrow$ wxplot2d([f(x), p(x)], [x,0,1])$

(% t28)



EJERCICIO 3

$\longrightarrow$ f(x):=7.21*cos(2*x/%pi);

(% o1) $$\mathrm{f}(x) := 7.21\cos\left(\frac{2x}{\pi}\right)$$

$\longrightarrow$ nodos:makelist(1-2*j/21, j,0, 21);

(nodos)

$$[1, \frac{19}{21}, \frac{17}{21}, \frac{5}{7}, \frac{13}{21}, \frac{11}{21}, \frac{3}{7}, \frac{1}{3}, \frac{5}{21}, \frac{1}{7}, \frac{1}{21}, -\frac{1}{21}, -\frac{1}{7}, -\frac{5}{21}, -\frac{1}{3}, -\frac{3}{7}, -\frac{11}{21}, -\frac{13}{21}, -\frac{5}{7}, -\frac{17}{21}, -\frac{19}{21}, -1]$$

$\longrightarrow$ imagenes:makelist(f(nodos[i]), i, 1, 22);

(imagenes)

$$[7.21\cos\left(\frac{2}{\pi}\right), 7.21\cos\left(\frac{38}{21\pi}\right), 7.21\cos\left(\frac{34}{21\pi}\right), 7.21\cos\left(\frac{10}{7\pi}\right), 7.21\cos\left(\frac{26}{21\pi}\right), 7.21\cos\left(\frac{22}{21\pi}\right),$$

$$7.21\cos\left(\frac{6}{7\pi}\right), 7.21\cos\left(\frac{2}{3\pi}\right), 7.21\cos\left(\frac{10}{21\pi}\right), 7.21\cos\left(\frac{2}{7\pi}\right), 7.21\cos\left(\frac{2}{21\pi}\right), 7.21\cos\left(\frac{2}{21\pi}\right),$$

$$7.21\cos\left(\frac{2}{7\pi}\right), 7.21\cos\left(\frac{10}{21\pi}\right), 7.21\cos\left(\frac{2}{3\pi}\right), 7.21\cos\left(\frac{6}{7\pi}\right), 7.21\cos\left(\frac{22}{21\pi}\right), 7.21\cos\left(\frac{26}{21\pi}\right),$$

$$7.21\cos\left(\frac{10}{7\pi}\right), 7.21\cos\left(\frac{34}{21\pi}\right), 7.21\cos\left(\frac{38}{21\pi}\right), 7.21\cos\left(\frac{2}{\pi}\right)]$$

$\longrightarrow$ imag_pert:makelist(imagenes[i]+10^(-3)*(-1)^(i-1), i, 1, 22);

(imag_ pert)

$$[7.21\cos\left(\frac{2}{\pi}\right)+\frac{1}{1000}, 7.21\cos\left(\frac{38}{21\pi}\right)-\frac{1}{1000}, 7.21\cos\left(\frac{34}{21\pi}\right)+\frac{1}{1000}, 7.21\cos\left(\frac{10}{7\pi}\right)-\frac{1}{1000},$$
$$7.21\cos\left(\frac{26}{21\pi}\right)+\frac{1}{1000}, 7.21\cos\left(\frac{22}{21\pi}\right)-\frac{1}{1000}, 7.21\cos\left(\frac{6}{7\pi}\right)+\frac{1}{1000}, 7.21\cos\left(\frac{2}{3\pi}\right)-\frac{1}{1000},$$
$$7.21\cos\left(\frac{10}{21\pi}\right)+\frac{1}{1000}, 7.21\cos\left(\frac{2}{7\pi}\right)-\frac{1}{1000}, 7.21\cos\left(\frac{2}{21\pi}\right)+\frac{1}{1000}, 7.21\cos\left(\frac{2}{21\pi}\right)-\frac{1}{1000},$$
$$7.21\cos\left(\frac{2}{7\pi}\right)+\frac{1}{1000}, 7.21\cos\left(\frac{10}{21\pi}\right)-\frac{1}{1000}, 7.21\cos\left(\frac{2}{3\pi}\right)+\frac{1}{1000}, 7.21\cos\left(\frac{6}{7\pi}\right)-\frac{1}{1000},$$
$$7.21\cos\left(\frac{22}{21\pi}\right)+\frac{1}{1000}, 7.21\cos\left(\frac{26}{21\pi}\right)-\frac{1}{1000}, 7.21\cos\left(\frac{10}{7\pi}\right)+\frac{1}{1000}, 7.21\cos\left(\frac{34}{21\pi}\right)-\frac{1}{1000},$$
$$7.21\cos\left(\frac{38}{21\pi}\right)+\frac{1}{1000}, 7.21\cos\left(\frac{2}{\pi}\right)-\frac{1}{1000}]$$

$\longrightarrow$ maximo:apply(max, makelist(abs(imagenes[i]-imag_pert[i]), i, 1, 22));

(maximo) $$0.001$$

$\longrightarrow$ l1(i, x):=product((x-nodos[j])/(nodos[i]-nodos[j]), j, 1, i-1)*product((x-nodos[j])/(nodos[i]-nodos[j]), j, i+1, 22);

(% o6) $$\mathrm{l1}\left(i,x\right):=\prod_{j=1}^{i-1}\frac{x-\mathit{nodos}_j}{\mathit{nodos}_i-\mathit{nodos}_j}\prod_{j=i+1}^{22}\frac{x-\mathit{nodos}_j}{\mathit{nodos}_i-\mathit{nodos}_j}$$

$\longrightarrow$ p1(x):=sum(imagenes[i]*l1(i, x), i, 1, 22);

(% o7) $$\mathrm{p1}(x):=\sum_{i=1}^{22}\mathit{imagenes}_i\,\mathrm{l1}\left(i,x\right)$$

$\longrightarrow$ l2(i, x):=l1(i, x);

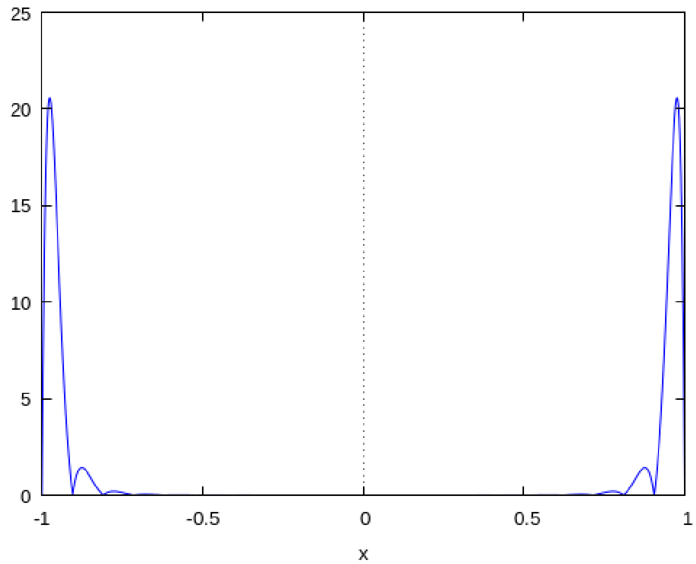
(% o8) $$\mathrm{l2}\left(i,x\right):=\mathrm{l1}\left(i,x\right)$$

$\longrightarrow$ p2(x):=sum(imag_pert[i]*l2(i, x), i, 1, 22);

(% o9) $$\mathrm{p2}(x):=\sum_{i=1}^{22}\mathit{imag\_pert}_i\,\mathrm{l2}\left(i,x\right)$$

→ wxplot2d([abs(p1(x) - p2(x))], [x,-1,1])$

(% t10)



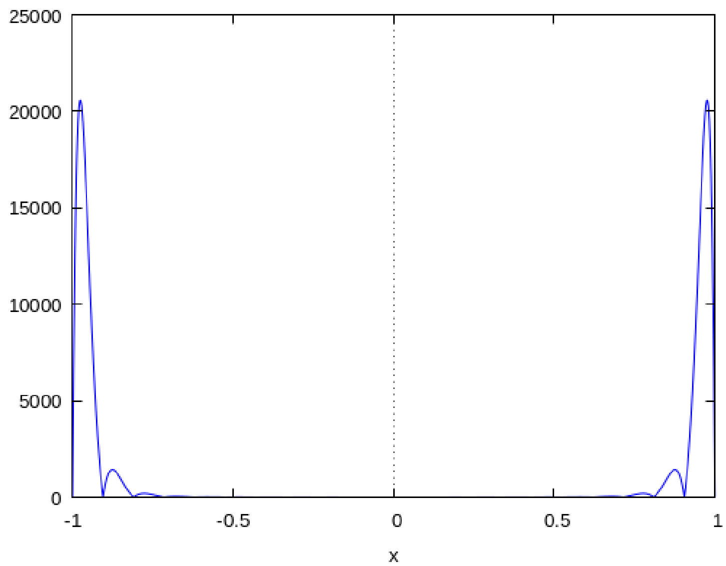
Con este gráfico podemos observar que la distancia entre los dos polinomios es elevada en números cercanos a 1, con un valorligeramente mayor que 20. Por tanto, el problema no es estable.

→ lebesgue(x):=sum(abs(l1(i, x)), i, 1, 22);

(% o11) $$\mathrm{lebesgue}(x) := \sum_{i=1}^{22} |\mathrm{l1}\,(i, x)|$$

$\longrightarrow$ wxplot2d([lebesgue(x)], [x,-1,1])$

(%t12)



Con este gráfico, lo que observamos es que la constante de Lebesgue es mayor que 2000. Al ser este valor tan elevado, elcondicionamiento también es malo.

$\longrightarrow$ chebyshev:makelist(cos(((2*i+1)*%pi)/44), i, 0, 21);

(chebyshev)

$$[\cos\left(\frac{\pi}{44}\right), \cos\left(\frac{3\pi}{44}\right), \cos\left(\frac{5\pi}{44}\right), \cos\left(\frac{7\pi}{44}\right), \cos\left(\frac{9\pi}{44}\right), \frac{1}{\sqrt{2}}, \cos\left(\frac{13\pi}{44}\right), \cos\left(\frac{15\pi}{44}\right), \cos\left(\frac{17\pi}{44}\right), \cos\left(\frac{19\pi}{44}\right), \cos\left(\frac{21\pi}{44}\right), \cos\left(\frac{23\pi}{44}\right), \cos\left(\frac{25\pi}{44}\right), \cos\left(\frac{27\pi}{44}\right), \cos\left(\frac{29\pi}{44}\right), \cos\left(\frac{31\pi}{44}\right), -\frac{1}{\sqrt{2}}, \cos\left(\frac{35\pi}{44}\right), \cos\left(\frac{37\pi}{44}\right), \cos\left(\frac{39\pi}{44}\right), \cos\left(\frac{41\pi}{44}\right), \cos\left(\frac{43\pi}{44}\right)]$$

$\longrightarrow$ l3(i, x):=product((x-chebyshev[j])/(chebyshev[i]-chebyshev[j]), j, 1, i-1)*product((x-chebyshev[j])/(chebyshev[i]-chebyshev[j]), j, i+1, 22);

(%o14)

$$\mathrm{l3}\left(i,x\right) := \prod_{j=1}^{i-1} \frac{x - \mathit{chebyshev}_j}{\mathit{chebyshev}_i - \mathit{chebyshev}_j} \prod_{j=i+1}^{22} \frac{x - \mathit{chebyshev}_j}{\mathit{chebyshev}_i - \mathit{chebyshev}_j}$$

$\longrightarrow$ p3(x):=sum(f(chebyshev[i])*l3(i, x), i, 1, 22);

(%o15)

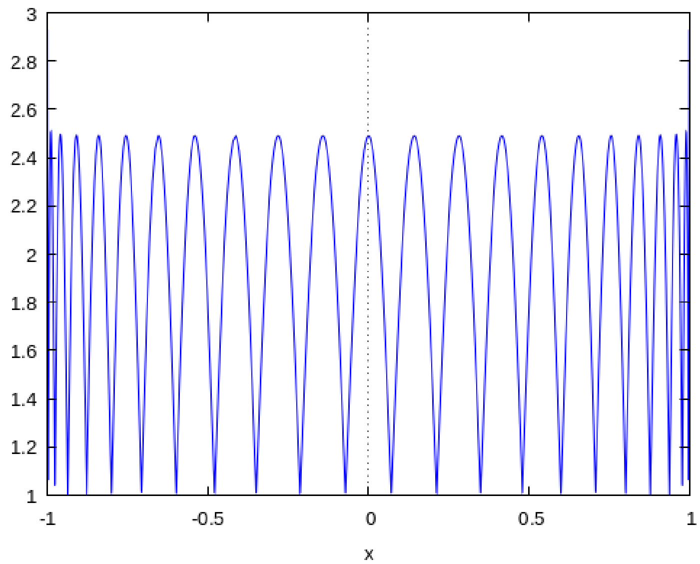
$$\mathrm{p3}(x) := \sum_{i=1}^{22} \mathrm{f}\left(\mathit{chebyshev}_i\right) \mathrm{l3}\left(i,x\right)$$

$\longrightarrow$ lebesgue2(x):=sum(abs(l3(i, x)), i, 1, 22);

(% o17) $$\mathrm{lebesgue2}(x) := \sum_{i=1}^{22} |\mathrm{l3}\,(i, x)|$$

$\longrightarrow$ wxplot2d([lebesgue2(x)], [x,-1,1])$

(% t18)



De aquí deducimos que la constante de Lebesgue es, aproximadamente, 2.5, de donde el condicionamiento de este problemaes bueno. EJERCICIO∼6

$\longrightarrow$ P:[0.4, 0.5, 2.34, 3.45, 4.567, 5.081, 5.26];

(P) $$[0.4, 0.5, 2.34, 3.45, 4.567, 5.081, 5.26]$$

$\longrightarrow$ f(x):=1-x^2/20.78;

(% o52) $$\mathrm{f}(x) := 1 - \frac{x^2}{20.78}$$

$\longrightarrow$ imagenes:makelist(f(P[i]), i, 1, 7);

(imagenes)

$$[0.9923002887391723, 0.9879692011549567, 0.7364966313763235, 0.4272136669874879,$$
$$-0.003729018286814156, -0.2423754090471608, -0.331453320500481]$$

→ B(i, x):=if i=1 then(if (P[1]<=x and x<=P[2]) then (P[2] - x)/(P[2]-P[1]) else 0) else if (1<i and i<7) then(if (P[i-1]<x and x<P[i]) then (x-P[i-1])/(P[i]-P[i-1])else if (P[i]<x and x<P[i+1]) then (P[i+1] - x)/(P[i+1]-P[i])else 0) else if i=7 then(if (P[6]<=x and x<=P[7]) then (x-P[6])/(P[7] - P[6]) else 0) else 0;

(% o79)

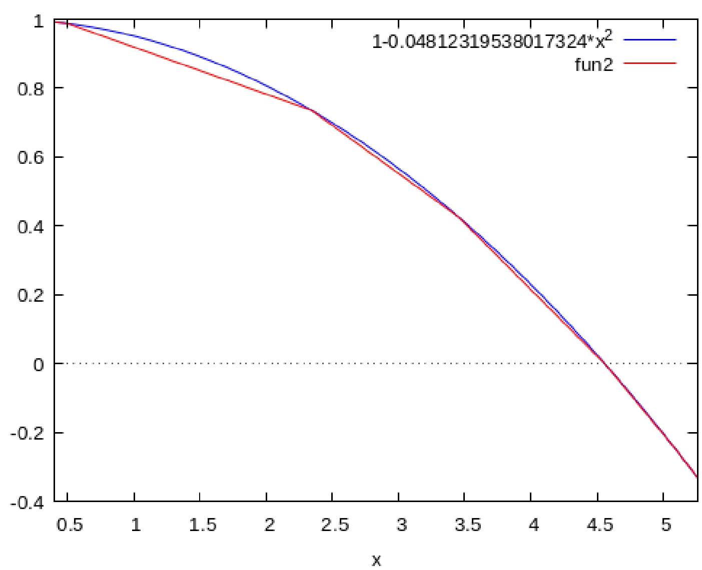
$$\mathrm{B}\left(i,x\right):=\mathrm{if}\ i=1\ \mathrm{then}\ \mathrm{if}\ P_1< =x and x< =P_2\ \mathrm{then}\ \frac{P_2-x}{P_2-P_1}\ \mathrm{else}\ 0\ \mathrm{else}\ \mathrm{if}\ 1< i and i< 7\ \mathrm{then}\ \mathrm{if}\ P_{i-1}< x and x< P_i\ \mathrm{then}\ \frac{x-P_{i-1}}{P_i-P_{i-1}}\ \mathrm{else}\ \mathrm{if}\ P_i< x and x< P_{i+1}\ \mathrm{then}\ \frac{P_{i+1}-x}{P_{i+1}-P_i}\ \mathrm{else}\ 0\ \mathrm{else}\ \mathrm{if}\ i=7\ \mathrm{then}\ \mathrm{if}\ P_6< =x and x< =P_7\ \mathrm{then}\ \frac{x-P_6}{P_7-P_6}\ \mathrm{else}\ 0\ \mathrm{else}\ 0$$

→ s(x):=sum(imagenes[i]*B(i, x), i, 1, 7);

(% o80) $$\mathrm{s}(x):=\sum_{i=1}^{7} imagenes_i\,\mathrm{B}\left(i,x\right)$$

→ wxplot2d([f(x), s(x)], [x,0.4,5.26])$

(% t81)



EJERCICIO 7

→ a:-2.09;

(a) $$-2.09$$

$\longrightarrow$ b:4.56;

(b) $$4.56$$

$\longrightarrow$ h:(b-a)/8;

(h) $$0.8312499999999999$$

$\longrightarrow$ f(x):=log(sqrt(1+abs(x)));

(% o152) $$\mathrm{f}(x) := \log\left(\sqrt{1+|x|}\right)$$

$\longrightarrow$ nodos:makelist(a+i*h, i, 0, 8);

(nodos)
$[-2.09, -1.25875, -0.4275, 0.40375, 1.234999999999999, 2.06625, 2.8975, 3.72875, 4.56]$

$\longrightarrow$ imagenes:makelist(f(nodos[i]), i, 1, 9);

(imagenes)
$[0.564085545454827, 0.4074057814621505, 0.1779623312740141, 0.1695736135352579,$

$0.402120614032766, 0.560227658295281, 0.6801676609748972, 0.7768304484825509, 0.8577990541312456]$

$\longrightarrow$ A:genmatrix(lambda([i,j], if i=j then 2 else if i=j+1 or j=i+1 then 1/2 else 0), 9, 9);

(A) $$\begin{pmatrix} 2 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 2 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 2 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 2 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 2 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 2 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 2 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 2 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 2 \end{pmatrix}$$

$\longrightarrow$ A[1, 2]:0;

(% o156) $$0$$

$\longrightarrow$ A[9,8]:0;

(% o157) $$0$$

$\longrightarrow$ bv:makelist(0, i, 1, 9);

(bv) $[0, 0, 0, 0, 0, 0, 0, 0, 0]$

$\longrightarrow$ for i:2 thru 8 do bv[i]:(imagenes[i+1]-2*imagenes[i]+imagenes[i-1])*3/h^2;

(% o162) $done$

$\longrightarrow$ c:invert(A).bv;

(c)
$$\begin{pmatrix} 0.0 \\ -0.2639287184254621 \\ 0.4238814065967181 \\ 0.4879014754534119 \\ -0.2833551177217908 \\ -8.70132142422697810^{-4} \\ -0.04458261291668436 \\ -0.02292391448997783 \\ 0.0 \end{pmatrix}$$

$\longrightarrow$ alpha:makelist((imagenes[i+1]-imagenes[i])/h - (h/6)*(c[i+1,1]-c[i,1]), i, 1, 8);

(alpha)
$[-0.1519218095946672, -0.3713125567858325, -0.01896113755231057, 0.386606963918338,$

$0.1510680223120995, 0.1503447249687106, 0.1132854279817167, 0.09422992406192267]$

$\longrightarrow$ beta:makelist(imagenes[i]-c[i, 1]*h^2/6, i, 1, 8);

(beta)
$[0.564085545454827, 0.4378005412292599, 0.1291469784010366, 0.1133855328106785,$

$0.4347525715677969, 0.5603278651147297, 0.6853019177449703, 0.7794704297547723]$

$\longrightarrow$ s(i, x):=c[i,1]*(nodos[i+1]-x)^3/(6*h) + c[i+1, 1]*(x-nodos[i])^3/(6*h) + alpha[i]*(x-nodos[i]) + beta[i];

(% o198)
$$\mathrm{s}\left(i,x\right):=\frac{c_{i,1}\left(nodos_{i+1}-x\right)^{3}}{6h}+\frac{c_{i+1,1}\left(x-nodos_{i}\right)^{3}}{6h}+alpha_{i}\,\left(x-nodos_{i}\right)+beta_{i}$$

$\longrightarrow$ p(i, x):= if nodos[i]<=x and x<nodos[i+1] then s(i, x) else 0;

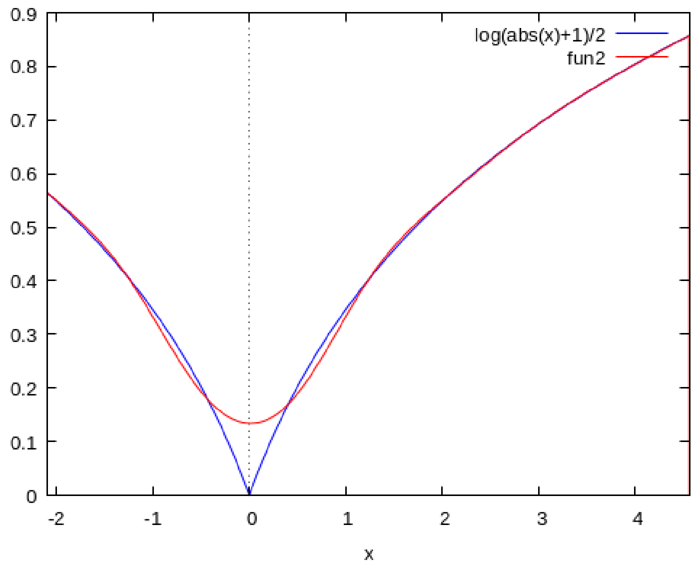
(% o199) $\mathrm{p}\left(i,x\right):=\mathrm{if}\; nodos_{i}<\;=x and x<\;nodos_{i+1}\,\mathrm{then}\,\mathrm{s}\left(i,x\right)\mathrm{else}\,0$

→ s_cubic(x):=sum(p(i, x), i, 1, 8);

(%o200) $$s_{cubic}(x) := \sum_{i=1}^{8} \mathrm{p}\left(i, x\right)$$

→ wxplot2d([f(x), s_cubic(x)], [x,a,b])$

(%t201)



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