Situation Calculus Temporally Lifted Abstractions for Program Synthesis

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Motivation

Objective: We address the program synthesis task to create concrete programs from high-level (HL) abstractions of data structure behavior.

Tools: We use nondeterministic SitCalc for the specifications, ConGolog programs to map HL into LL, and LTL for trace constraints and goals.

Overview: We consider a single (propositional) HL action theory/model with incomplete information and a concrete LL action theory with several models and complete information. We formally prove that if an agent has a strategy to achieve a goal under LTL constraints at the HL, then there exists a refinement of the strategy at the LL.

Nondeterministic SitCalc, ConGolog and LTL

A Nondeterministic Basic Action Theory (NDBAT) is an extension of classical BATs handling nondeterministic actions. For each agent action $A(\vec{x})$, the environment selects a reaction e to produce the system action $A(\vec{x}, e)$.

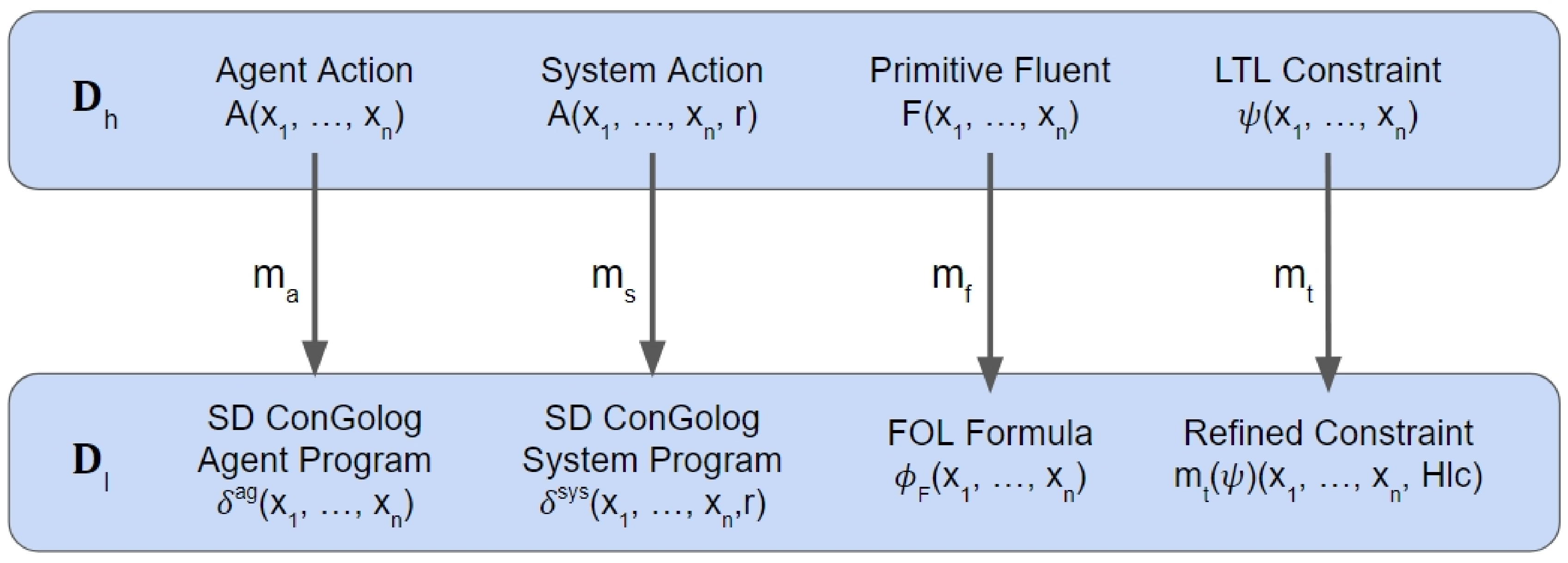
ConGolog is a HL programming language that supports complex action sequences, whose constructs include sequential execution, nondeterministic choice, variable binding, iteration, and interleaving.

LTL is a formalism for expressing temporal properties of reactive systems. LTL synthesis involves generating a controller that satisfies a LTL goal and it can be exploited in the context of generalized planning problems.

We impose LTL trace constraints for filtering world histories in NDBATs, leveraging on the axiomatization for infinite paths and the special symbol $Holds(\psi, p)$ (meaning that a constraint ψ holds on a path p).

Refinement Mapping m

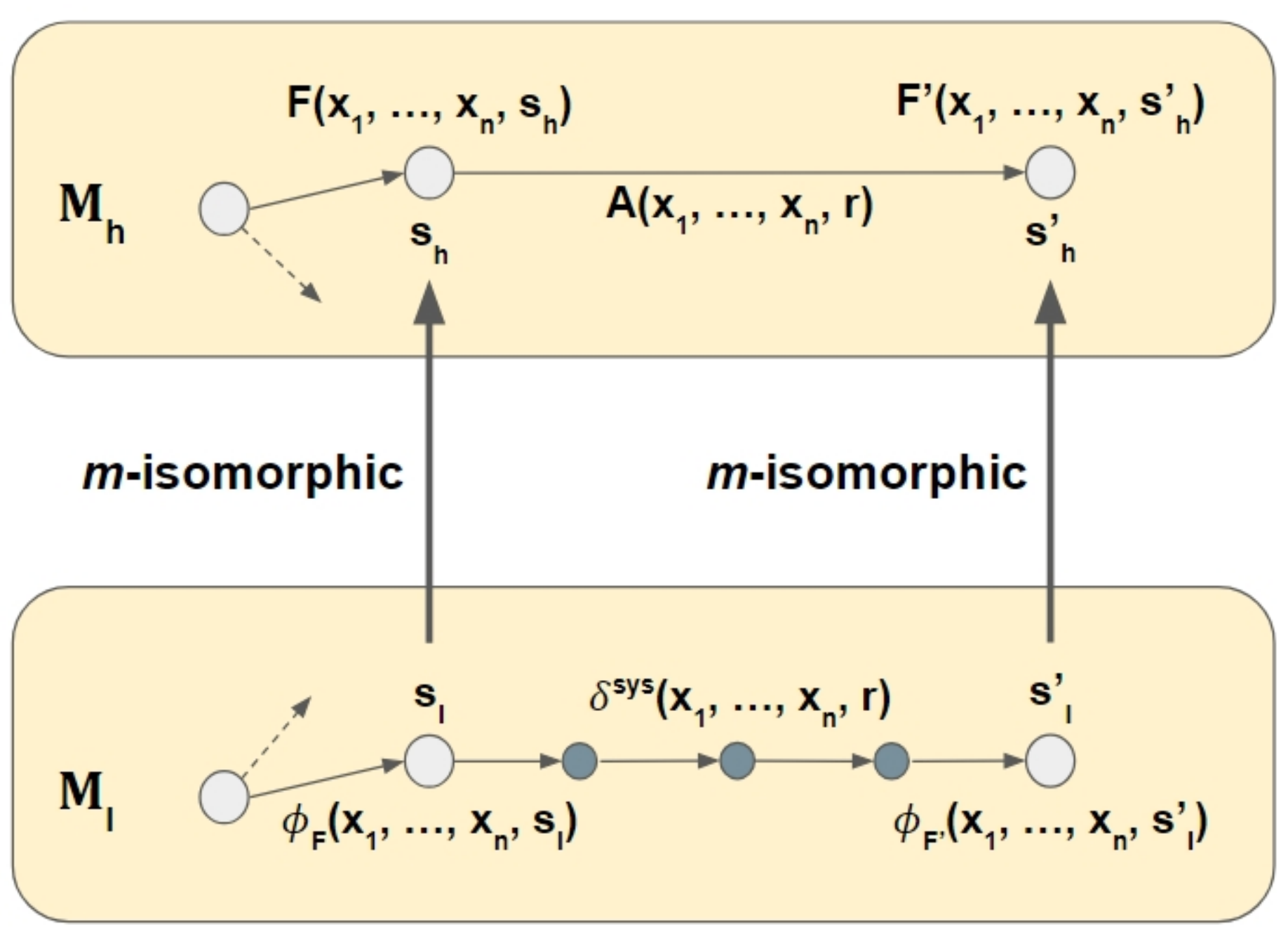
An NDBAT refinement mapping m is a tuple $\langle m_a, m_s, m_f, m_t\rangle$. In defining m_t to map HL constraints into LL ones, we suppose that the LL theory tracks when refinements of HL actions end using a state formula $Hlc(s)$, meaning that a HL action has just completed in situation s.


D_h
Agent Action A(x_1, ..., x_n)
System Action A(x_1, ..., x_n, r)
Primitive Fluent F(x_1, ..., x_n)
LTL Constraint ψ(x_1, ..., x_n)
m_a
m_s
m_f
m_t
D_l
SD ConGolog Agent Program δ^ag(x_1, ..., x_n)
SD ConGolog System Program δ^sys(x_1, ..., x_n, r)
FOL Formula φ_F(x_1, ..., x_n)
Refined Constraint m_t(ψ)(x_1, ..., x_n, Hlc)

m-Simulation

To relate the HL and LL NDBATs, we revisit the notion of bisimulation, sticking to a unidirectional version called m-simulation.

Two situations s_h and s_l are m-isomorphic iff they evaluate all HL fluents the same.



Two models M_h and M_l are m-similar if the initial situations are m-isomorphic and the resulting s'_l after executing $m(A)$ at the LL is always m-isomorphic to the resulting s'_h after executing A at the HL.

Temporally Lifted Abstraction

Definition 1 *Consider an HL NDBAT $\mathcal{D}_h$ equipped with a set of HL state constraint Ψ, a model M_h of $\mathcal{D}_h$, a LL NDBAT $\mathcal{D}_l$ and a refinement mapping m. We say that $(\mathcal{D}_h, M_h, \Psi)$ is a temporally lifted abstraction wrt m if and only if*

- *M_h m-simulates every model M_l of D_l*
- *for every high-level LTL trace constraint $\psi \in \Psi$,*
  $M_h \models \exists p_h.Starts(p_h, S_{0_h}) \wedge Holds(\psi, p_h)$ and
  $D_l \models \forall p_l.Starts(p_l, S_{0_l}) \supset Holds(m_t(\psi), p_l)$

Example: Minimum in a List

Description: We illustrate our framework addressing the problem of finding the minimum value in a singly-linked list.

LL: We represent a list deterministically with predicates identifying the head, each node's value and successor, an iterator and a register. The actions are common methods manipulating the previous elements.

HL: We abstract details using nondeterministic actions $next$ (move cursor), $checkValue$ (compare values), and $update$ (update register). The environment reaction of $next$ tells if the end is reached, and $checkValue$ tells if the current node's value is lower than the registers'.

HL Successor State Axioms:

$$\begin{aligned}
&hasNext(do(a,s)) \equiv \\
&\quad hasNext(s) \wedge a \neq next(End) \\
&lowerThan(do(a,s)) \equiv \\
&\quad a = checkValue(\mathrm{LT}) \vee \\
&\quad lowerThan(s) \wedge \\
&\qquad a \neq checkValue(\mathrm{GEQ}) \wedge \\
&\qquad a \neq update(Success_{HU}) \wedge \\
&\qquad \forall r. a \neq next(r)
\end{aligned}$$

Fragment of Refinement Mapping:

$$\begin{aligned}
&m_s(checkValue(r_h)) = \\
&\quad \textbf{if } \neg\exists c.iterator(c) \\
&\qquad \textbf{then } it\_set(Success_{IC}) \textbf{ else } nil \textbf{ endIf} \\
&\quad (\pi c).[iterator(c)?; \\
&\qquad mark(c, \mathrm{visited}, Success_{MD})]; \\
&\quad \textbf{if } \exists c, v, k, v'.iterator(c) \wedge node(c, v, k) \wedge \\
&\qquad min\_register(v') \wedge v < v' \\
&\qquad \textbf{then } r_h = \mathrm{LT}\,? \textbf{ else } r_h = \mathrm{GEQ}\,? \textbf{ endIf}
\end{aligned}$$

HL LTL Trace Constraint: $(\Box\Diamond doneNext) \rightarrow \Diamond\neg hasNext$
i.e., moving repeatedly to the next node eventually leads to the last one

Results about Strategic Reasoning

A strong plan is a strategy ensuring goal achievement regardless of environment reactions. We define $AgtCanForceByIf(Goal, Cstr, f, s)$, meaning that the agent can force a LTL $Goal$ by following strategy f in s if LTL path constraint $Cstr$ holds:

$$\begin{aligned}
&AgtCanForceByIf(Goal, Cstr, f, s) \doteq \\
&\qquad CertExec(f, s) \wedge \forall p.Out(p, f, s) \wedge Holds(Cstr, p) \supset Holds(Goal, p) \\
&AgtCanForceIf(Goal, Cstr, s) \doteq \exists f. AgtCanForceByIf(Goal, Cstr, f, s)
\end{aligned}$$

$CertExec(f, s)$ means that f certainly prescribes executable actions.
$Out(p, f, s)$ means that p is a possible outcome of the agent executing f.

We can prove that if the agent has a strategy to achieve a LTL goal under some constraints at the HL, then there exists a refinement of the strategy to achieve the refinement of the goal at the LL.

Theorem 2 *Let $(\mathcal{D}_h, M_h, Cstr)$ be a temporally lifted abstraction of $\mathcal{D}_l$ wrt refinement mapping m s.t. constraints about actions execution hold[a], and Goal be an LTL goal. If $M_h \models AgtCanForceIf(Goal, Cstr, S_0)$, then there exist a LL strategy f_l such that $\mathcal{D}_l \models AgtCanForceByIf(m(Goal), True, f_l, S_0)$*

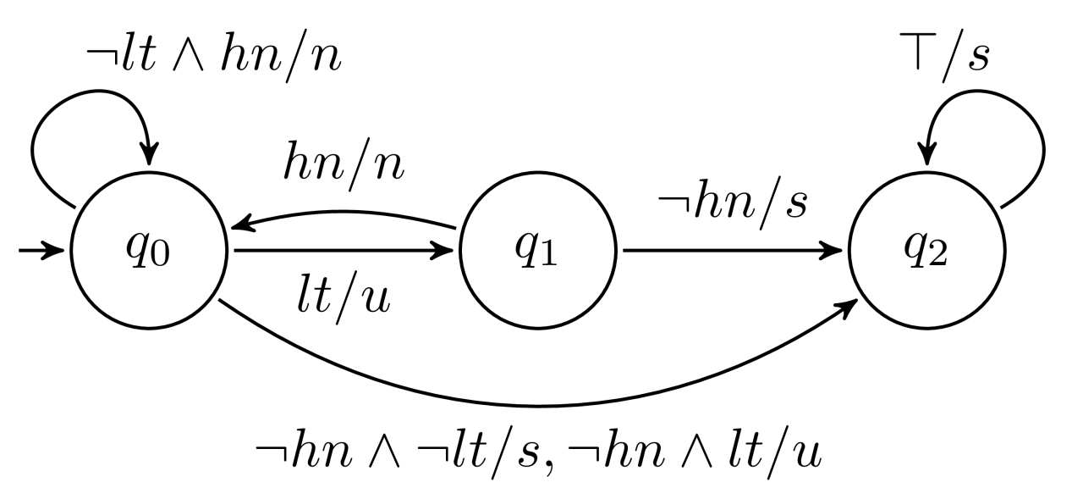
[a]for a comprehensive discussion, please refer to the full paper

Example: Synthesize and Refine a Strategy

HL LTL Goal:

$\Diamond\Box\neg hasNext$
$\Box(doneNext \rightarrow \bigcirc doneCheckValue)$
$\Box(lowerThan \leftrightarrow \bigcirc doneUpdate)$

Controller:



Here is a strategy that guarantees the satisfaction of the goals:

$$f_h(s) = \begin{cases} checkValue & \text{if } doneNext(s) \\ update & \text{if } lowerThan(s) \\ next & \text{if } \neg lowerThan(s) \wedge hasNext(s) \\ stop & \text{otherwise} \end{cases}$$

The previous controller can be generated automatically by an LTL synthesis engine like Strix, and it is perfectly consistent with f_h. By Theorem 2, there must exist a refinement of f_h at the LL.

Conclusion

Future works:

- Create modular libraries of verified abstractions for common data structures
- Explore partial automation of specification generation

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