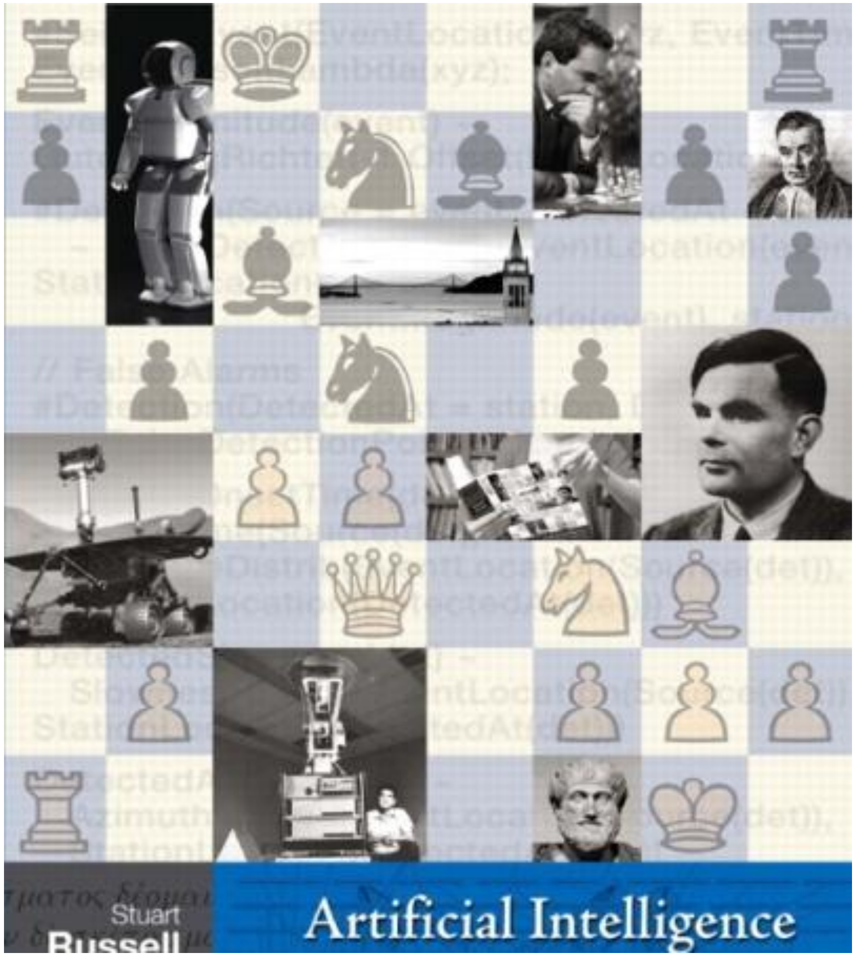
Lecture 4: Informed Search Algorithms
Russell and Norvig Chapter 4



CS-4820/5820

Tu/Th 12:15 PM-1:30 PM

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Disclosure

- Part of materials being used in this session were created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley

Announcement

- What will be covered till midterm:
  - Local search algorithms, Chapter 4, Sections 3-4
  - Contraint Satisfaction Problems, Chapter 5
  - First-order logic, Chapter 8
  - Inference in first-order logic, Chapter 9

  - We might also cover Chapters 6 (Game Playing) and 7 (Local Agents) but might trade this for more in-depth coverage of search optimization (not covered in Russell and Norvig book)

Outline

- Best-first search
- Greedy best-first search
- A^* search
- Heuristics
- Local search algorithms
- Hill-climbing search
- Simulated annealing search
- Local beam search

Review: Tree search

```
function TREE-SEARCH(problem, fringe) returns a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST[problem] applied to STATE(node) succeeds return node
        fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

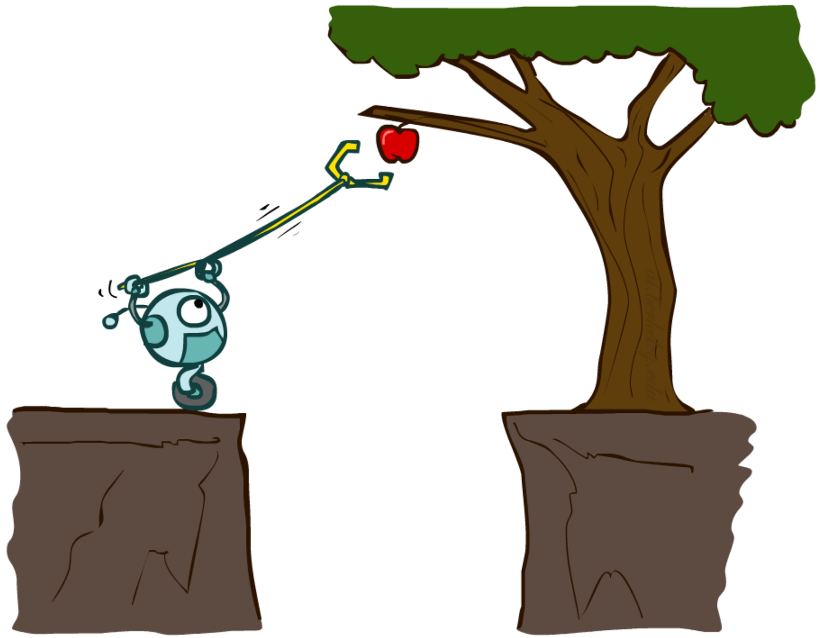
- A search strategy is defined by picking the order of node expansion

Planning Agents

- Planning agents:
  - Ask “what if”
  - Decisions based on (hypothesized) consequences of actions
  - Must have a model of how the world evolves in response to actions
  - Must formulate a goal (test)
  - Consider how the world WOULD BE

- Optimal vs. complete planning

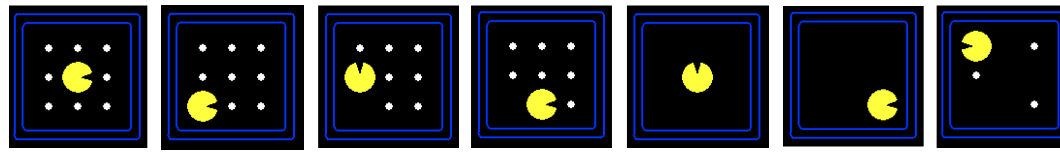
- Planning vs. replanning



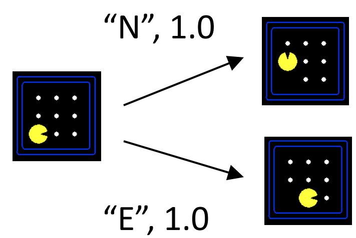
Search Problems

Search Problems

- A search problem consists of:
  - A state space

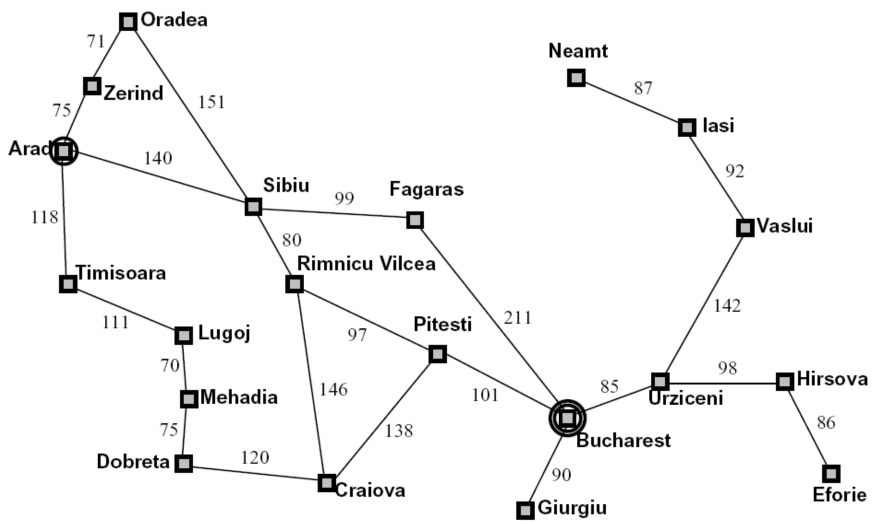


  - A successor function (with actions, costs)



  - A start state and a goal test
- A solution is a sequence of actions (a plan) which transforms the start state to a goal state

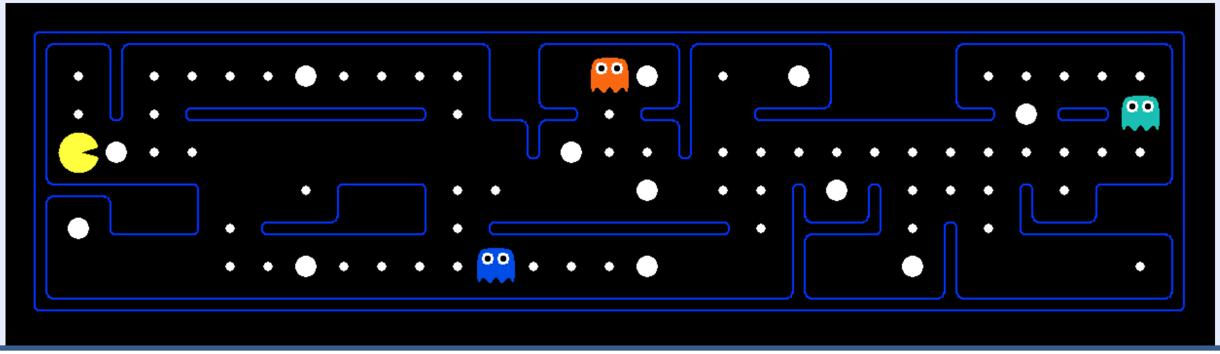
Example: Traveling in Romania



- State space:
  - Cities
- Successor function:
  - Roads: Go to adjacent city with cost = distance
- Start state:
  - Arad
- Goal test:
  - Is state == Bucharest?
- Solution?

What's in a State Space?

The world state includes every last detail of the environment

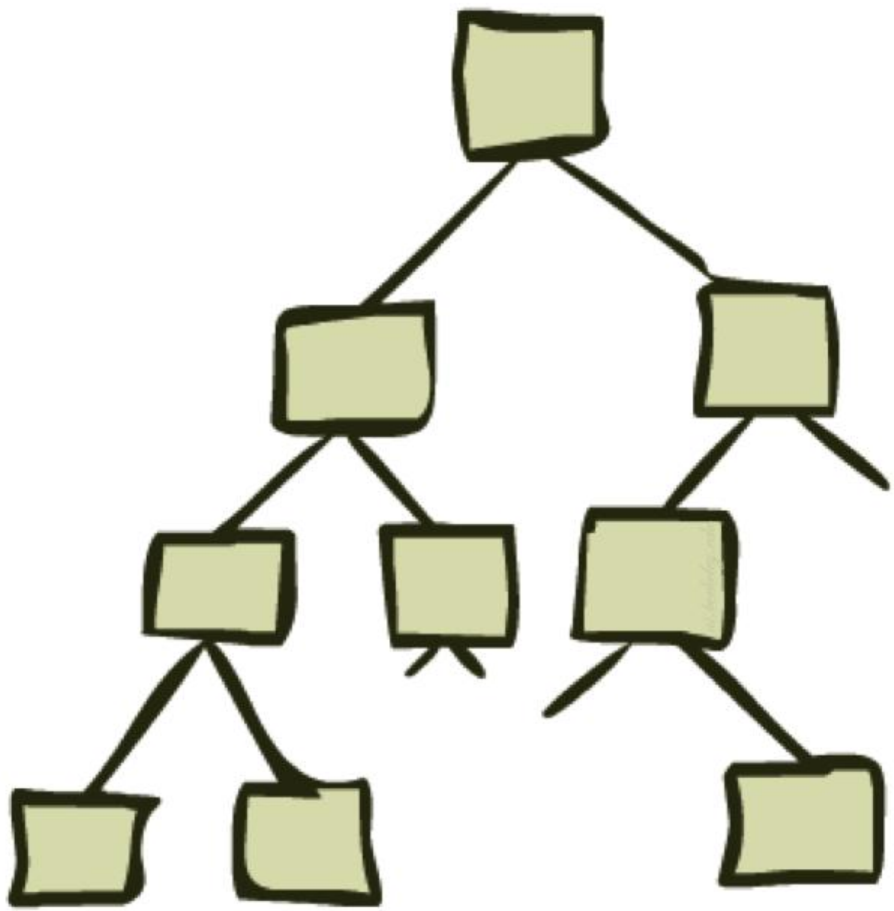


A search state keeps only the details needed for planning (abstraction)

- Problem: Pathing
  - States: (x,y) location
  - Actions: N,S,E,W
  - Successor: update location only
  - Goal test: is (x,y)=END

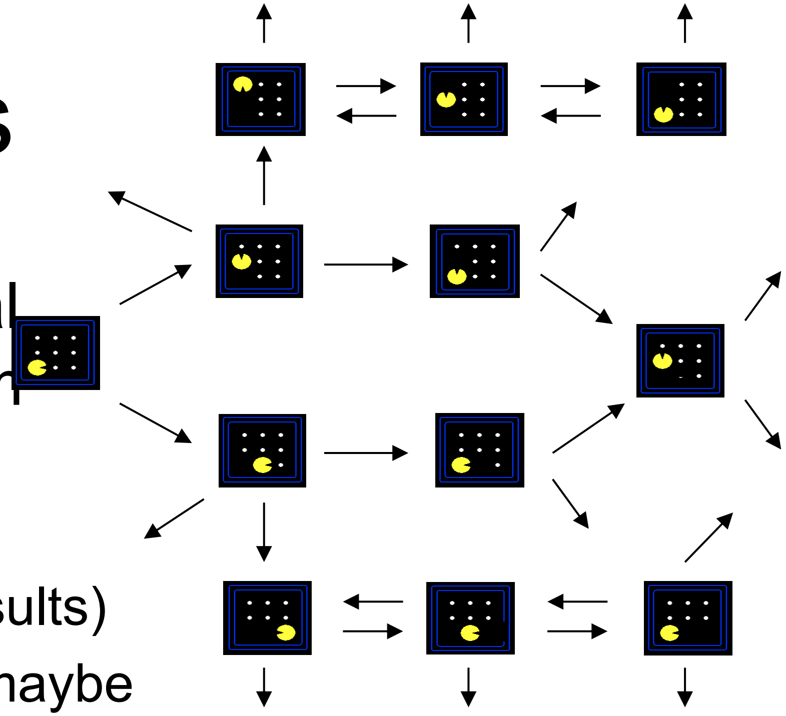
- Problem: Eat-All-Dots
  - States: {(x,y), dot booleans}
  - Actions: N,S,E,W
  - Successor: update location and possibly a dot boolean
  - Goal test: dots all false

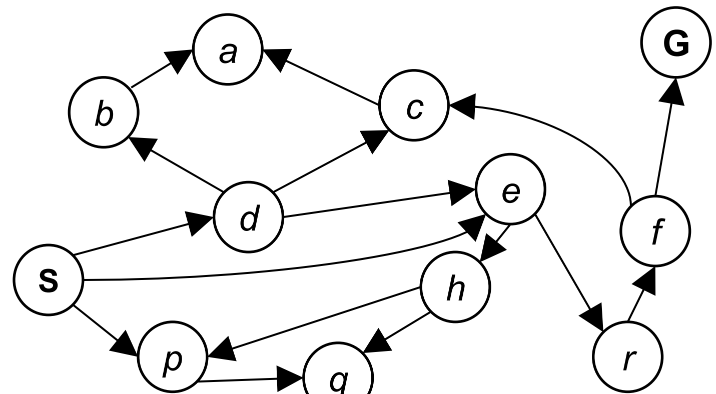
State Space Graphs and Search Trees



State Space Graphs

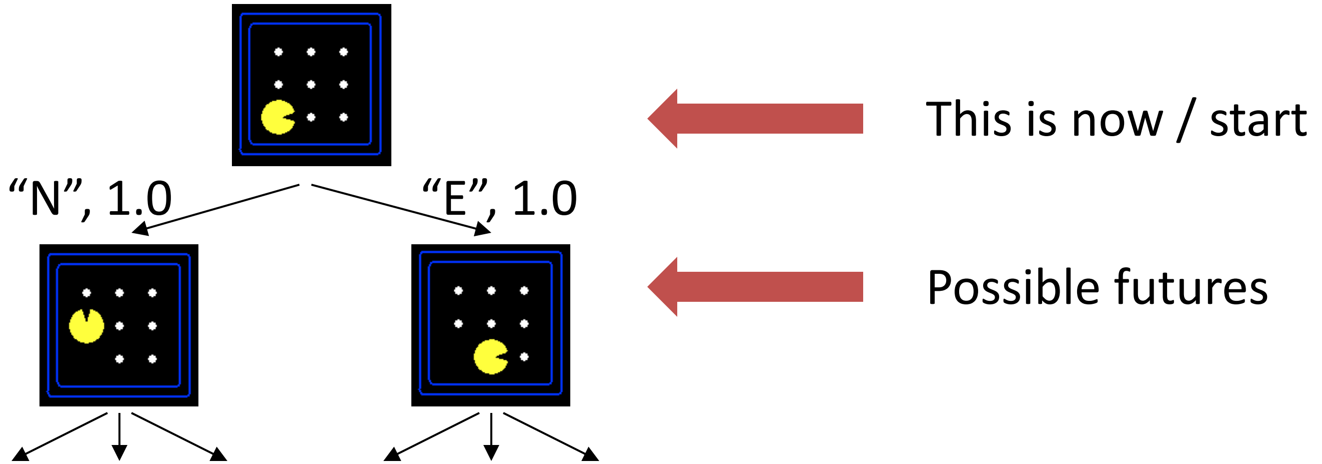
- State space graph: A mathematical representation of a search problem
  - Nodes are (abstracted) world configurations
  - Arcs represent successors (action results)
  - The goal test is a set of goal nodes (maybe only one)
- In a state space graph, each state occurs only once!
- We can rarely build this full graph in memory (it's too big), but it's a useful idea





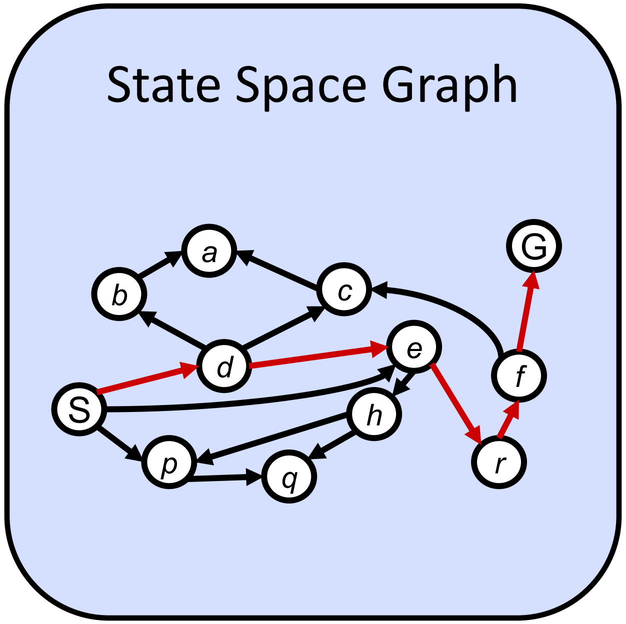
Tiny search graph for a tiny search problem

Search Trees



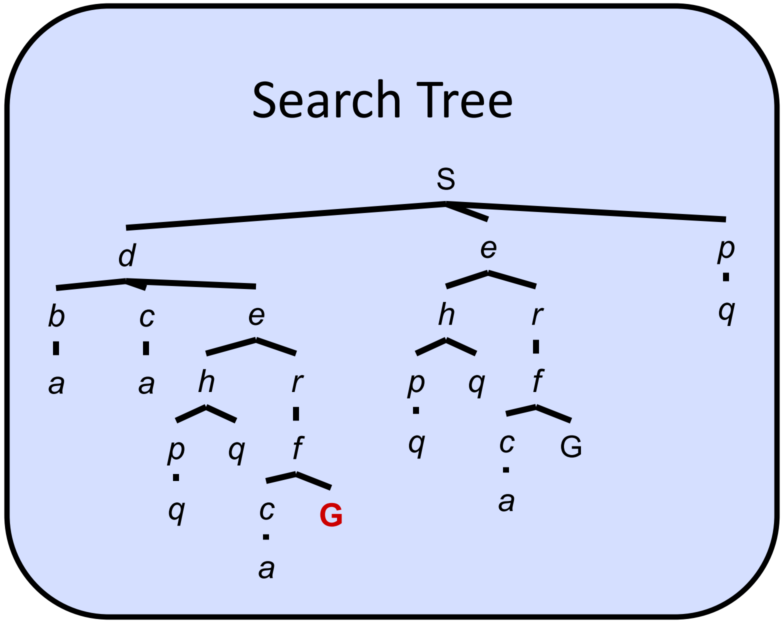
- What is a search tree?
  - A "what if" tree of plans and their outcomes
  - The start state is the root node
  - Children correspond to successors
  - Nodes show states, but correspond to PLANS that achieve those states
  - For most problems, we can never actually build the whole tree

State Space Graphs vs. Search Trees



Each NODE in the search tree is an entire PATH in the state space graph.

We construct both on demand – and we construct as little as possible.



What is the difference between tree search and graph search?

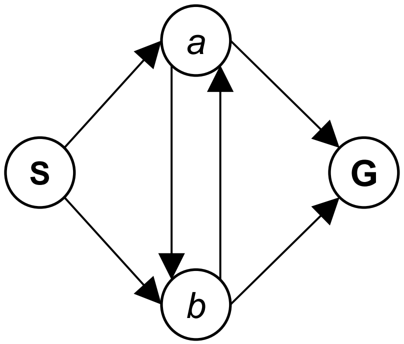
What is the difference between tree search and graph search?

- In graph search, we use a list (*closed list*). This list holds all the nodes that are already visited/expanded. This prevent the method from reexploring the same node.

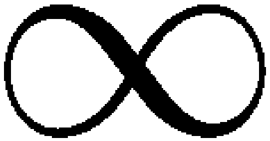
- In tree search, we don't use a "closed list". This allows the same node to be visited multiple times and can result in a tree that may have multiple occurrence of the same node

State Space Graphs vs. Search Trees

Consider this 4-state graph:



How big is its search tree (from S)?



Important: Lots of repeated structure in the search tree!

General Tree Search

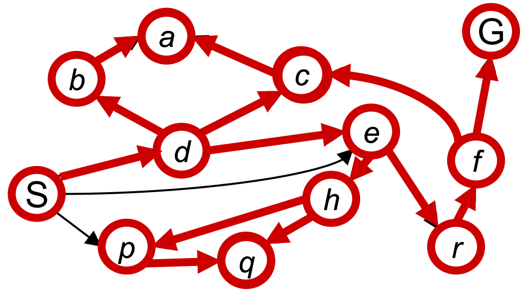
```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
   initialize the search tree using the initial state of problem
   loop do
      if there are no candidates for expansion then return failure
      choose a leaf node for expansion according to strategy
      if the node contains a goal state then return the corresponding solution
      else expand the node and add the resulting nodes to the search tree
   end
```

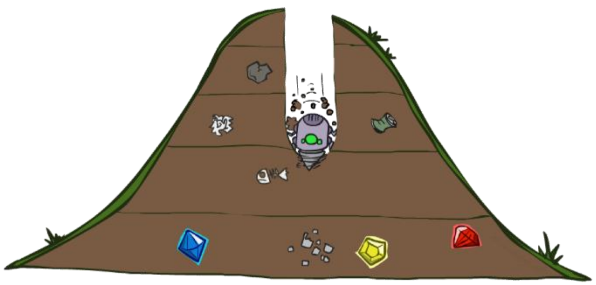
- Important ideas:
  - Fringe
  - Expansion
  - Exploration strategy
- Main question: which fringe nodes to explore?

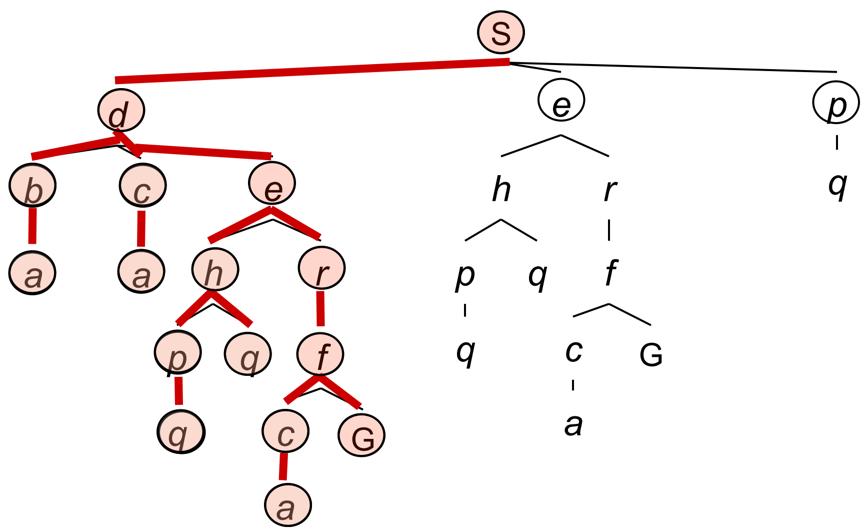
Depth-First Search (DFS)

Strategy: expand a deepest node first

Implementation: Fringe is a LIFO stack



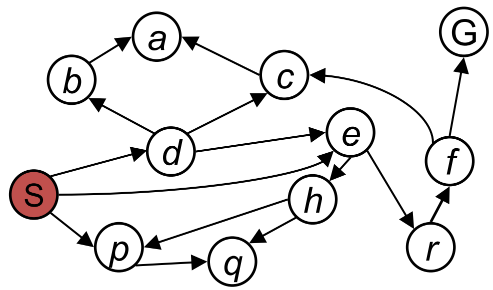


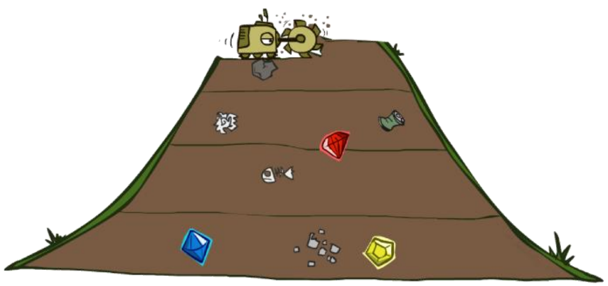


Breadth-First Search

Strategy: expand a shallowest node first

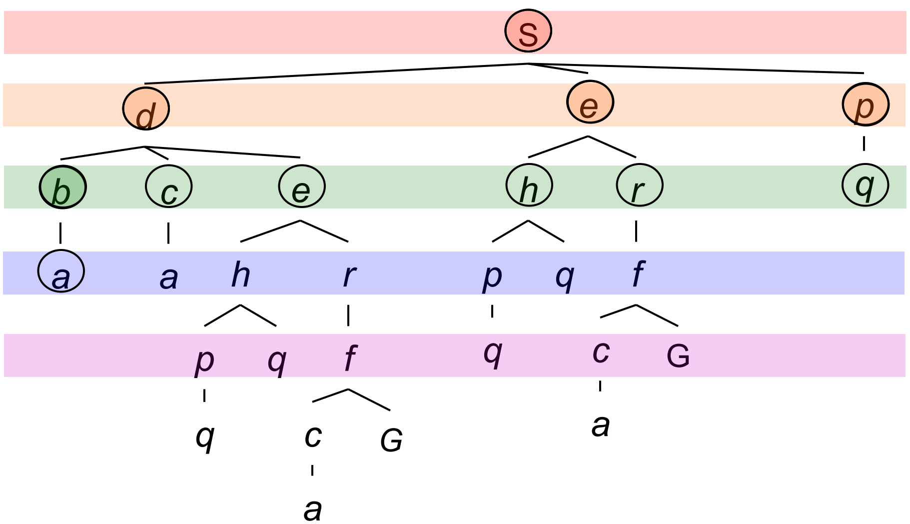
Implementation: Fringe is a FIFO queue



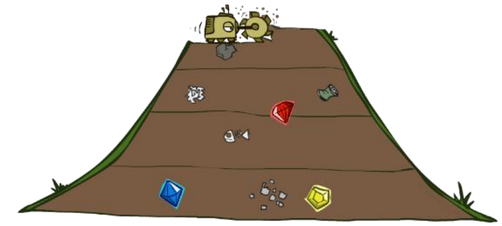


Search

Tiers

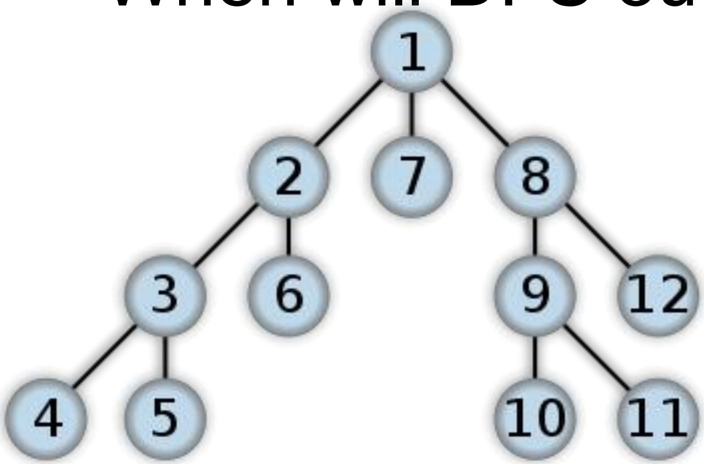


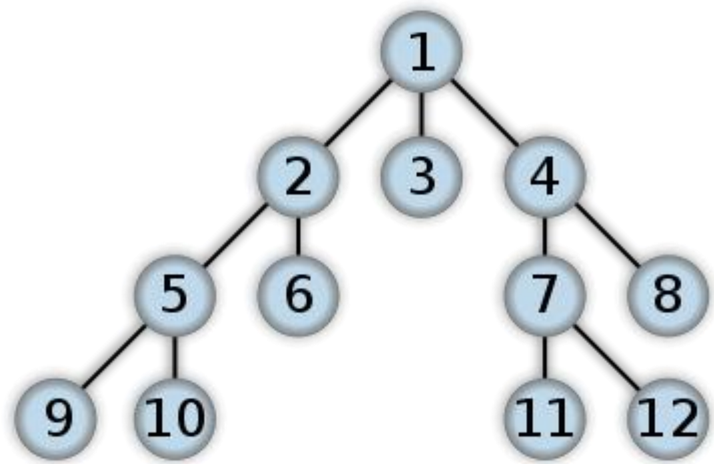
Quiz: DFS vs BFS



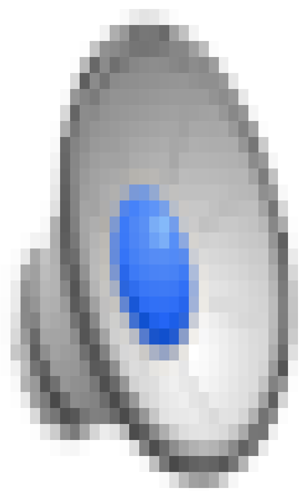
- When will BFS outperform DFS?

- When will DFS outperform BFS?





Video of Demo Maze Water DFS/BFS (part 1)

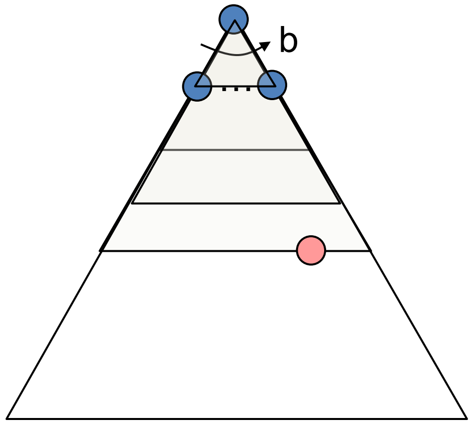


Video of Demo Maze Water DFS/BFS (part 2)

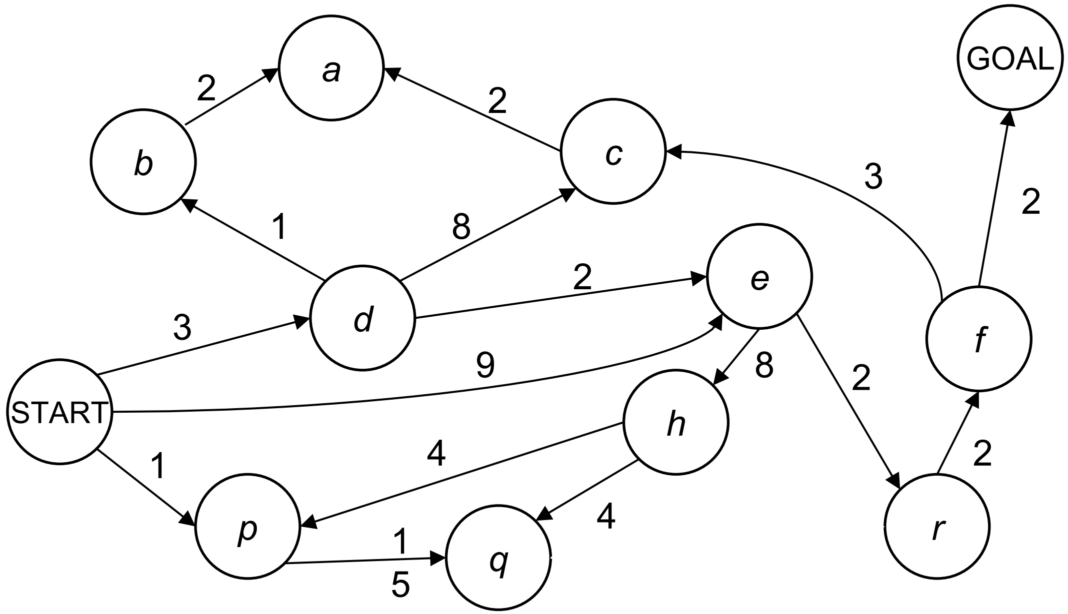
Iterative Deepening

- Idea: get DFS's space advantage with BFS's time / shallow-solution advantages
  - Run a DFS with depth limit 1. If no solution…
  - Run a DFS with depth limit 2. If no solution…
  - Run a DFS with depth limit 3. …..

- Isn't that wastefully redundant?
  - Generally most work happens in the lowest level searched, so not so bad!



Cost-Sensitive Search

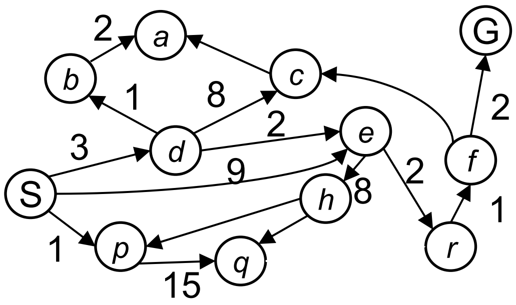


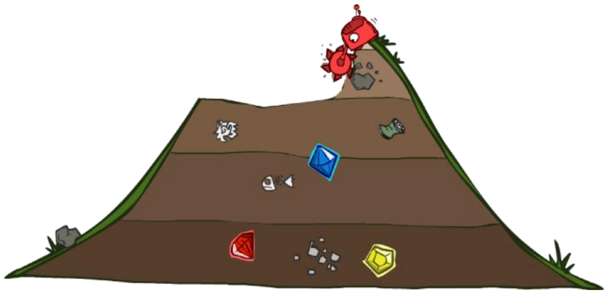
BFS finds the shortest path in terms of number of actions. It does not find the least-cost path. We will now cover a similar algorithm which does find the least-cost path.

Uniform Cost Search

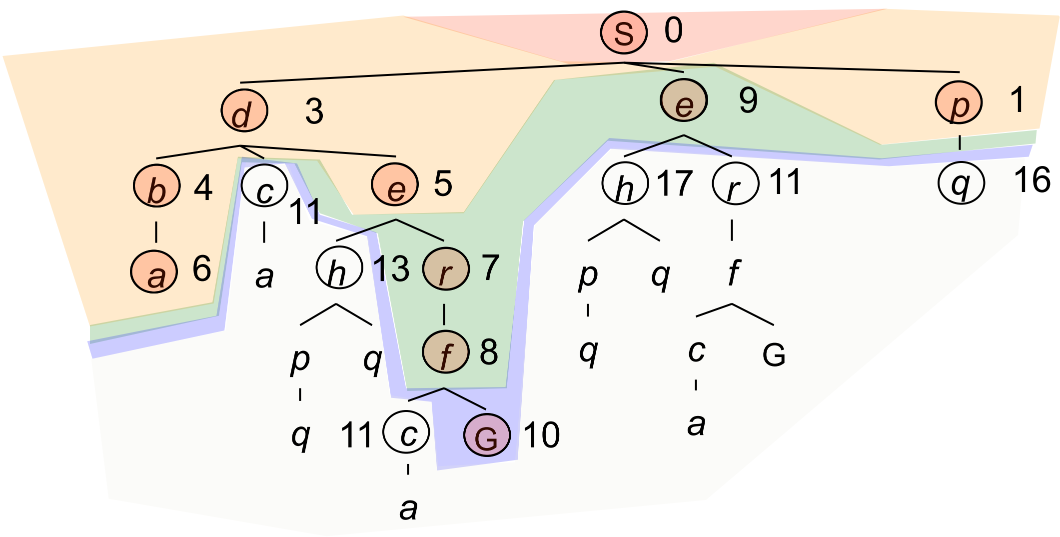
Strategy: expand a cheapest node first:

Fringe is a priority queue (priority: cumulative cost)



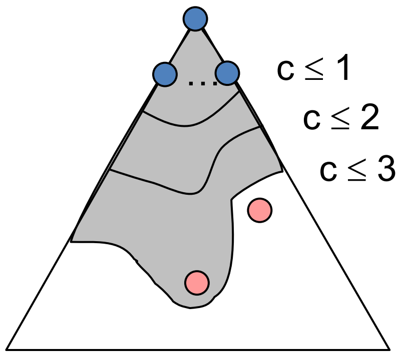


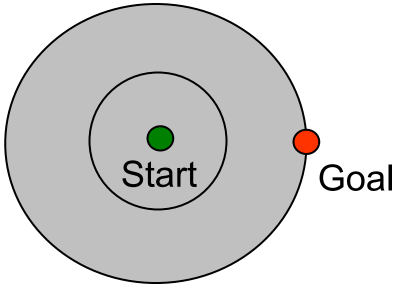
Cost contours



Uniform Cost Issues

- Remember: UCS explores increasing cost contours
- The good: UCS is complete and optimal!
- The bad:
  - Explores options in every “direction”
  - No information about goal location
- We’ll fix that soon!

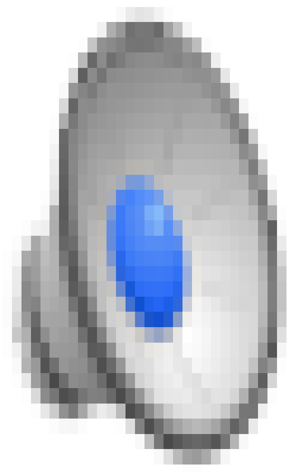




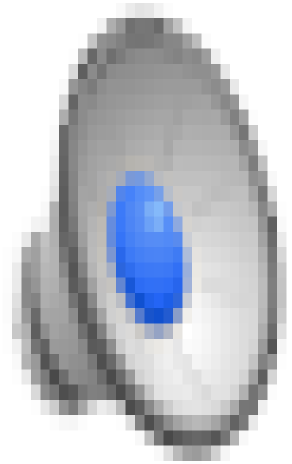
Video of Demo Empty UCS

Video of Demo Maze with Deep/Shallow Water --- DFS, BFS, or UCS? (part 1)

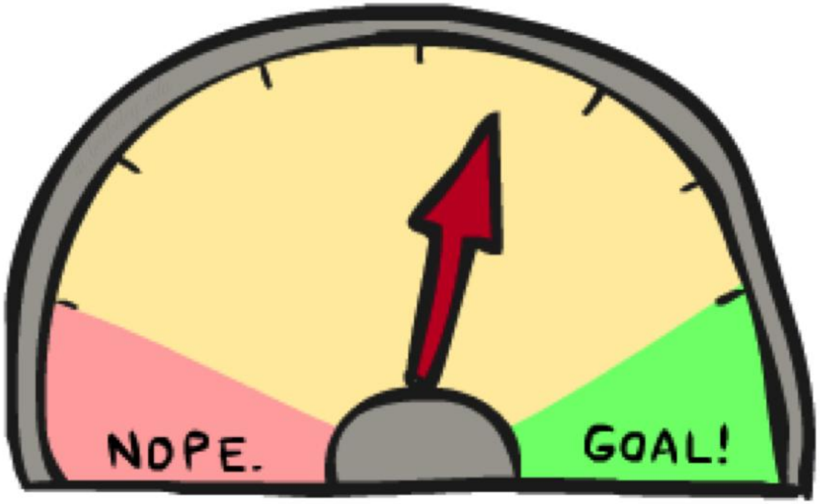
Video of Demo Maze with Deep/Shallow Water --- DFS, BFS, or UCS? (part 2)



Video of Demo Maze with Deep/Shallow Water --- DFS, BFS, or UCS? (part 3)



Informed Search



Best-first search

- Idea: use an evaluation function *f(n)* for each node
  - estimate of "desirability“

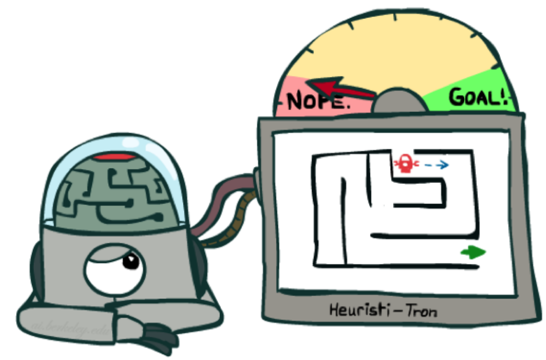
→Expand most desirable unexpanded node

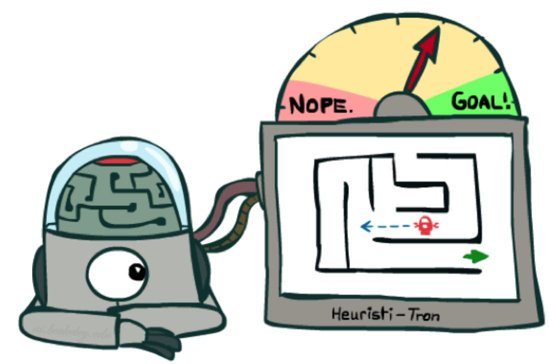
- <u>Implementation</u>:
  Order the nodes in fringe in decreasing order of desirability

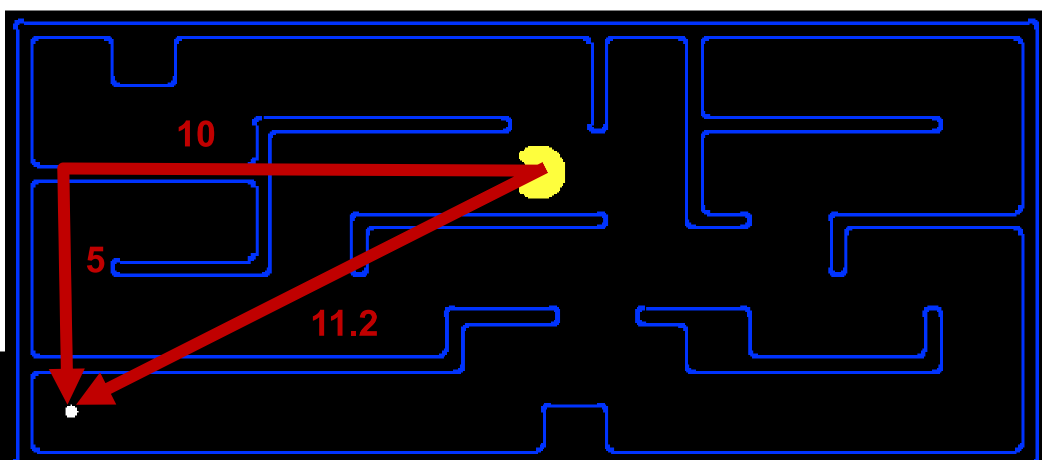
- Special cases:
  - greedy best-first search
  - A^* search

Search Heuristics

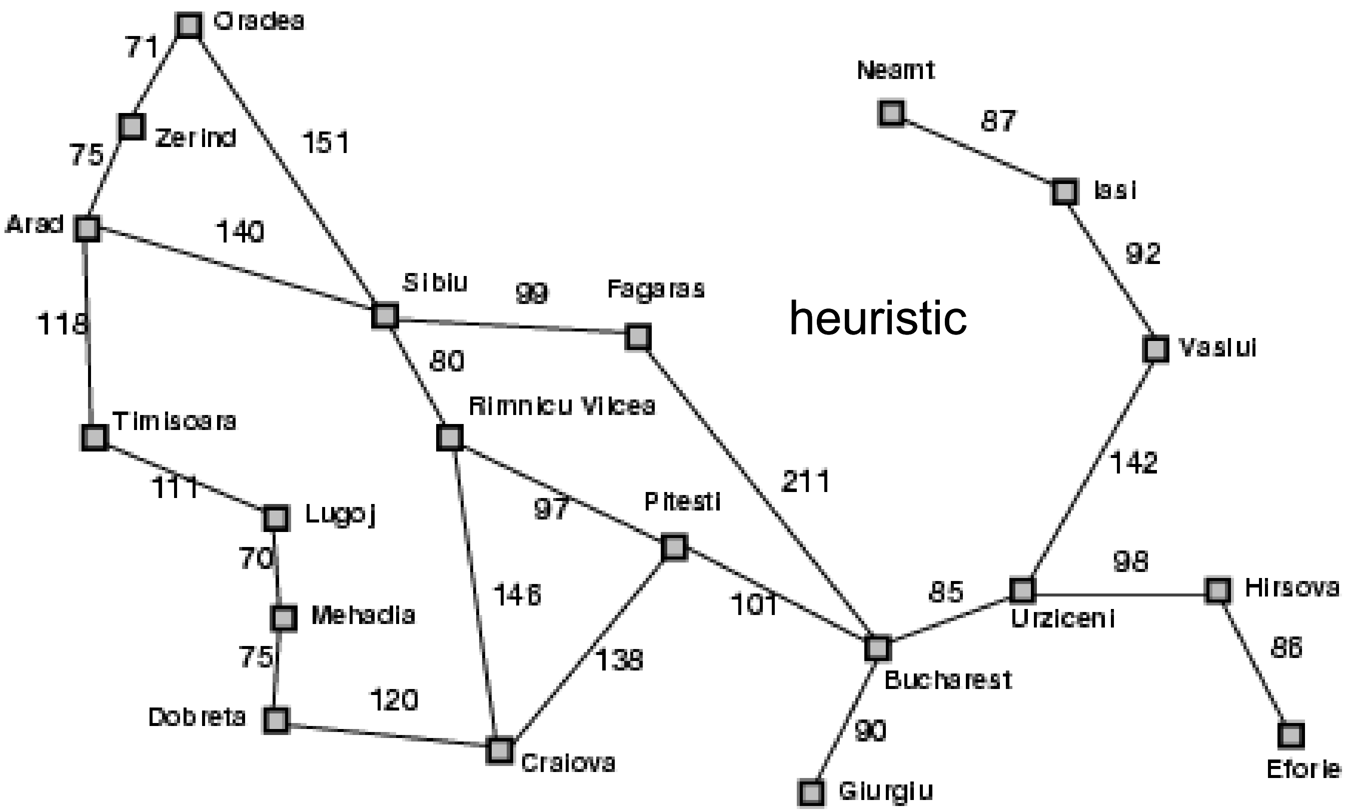
- A heuristic is:
  - A function that *estimates* how close a state is to a goal
  - Designed for a particular search problem
  - Examples: Manhattan distance, Euclidean distance for pathing







Romania with step costs in km



heuristic

Straight–line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

h(x)

Greedy best-first search

- Evaluation function $f(n) = h(n)$ (heuristic)

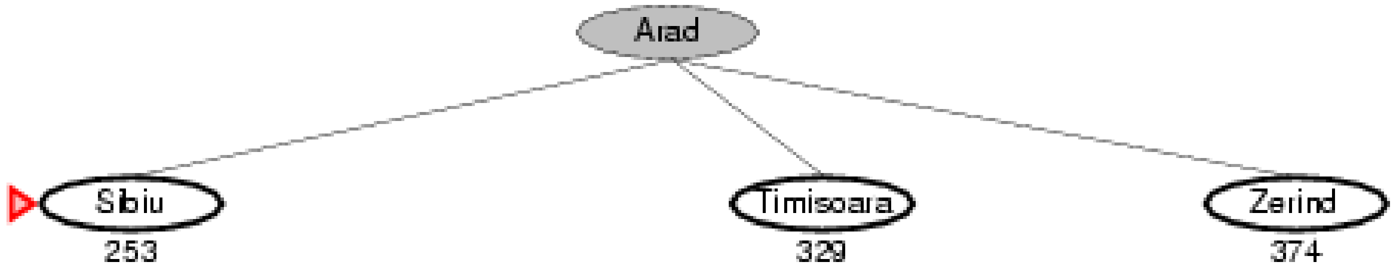
  $= \text{estimate of cost from } n \text{ to } goal$

- e.g., $h_{SLD}(n)$ = straight-line distance from n to Bucharest

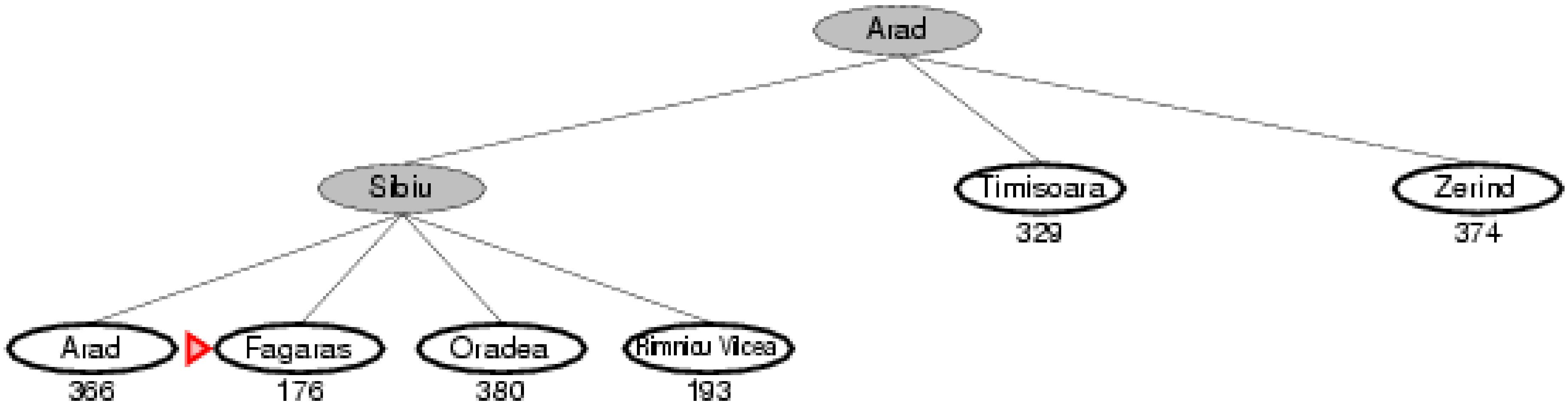
- Greedy best-first search expands the node that appears to be closest to goal

Greedy best-first search example

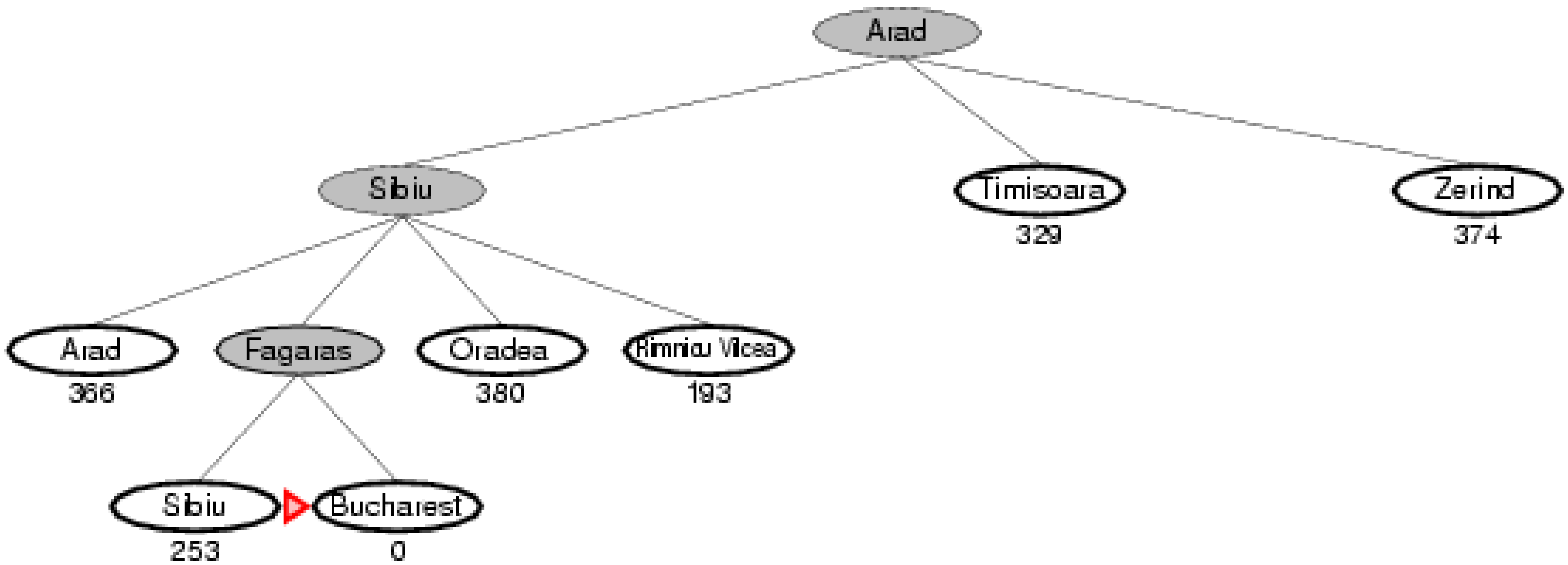
Greedy best-first search example



Greedy best-first search example

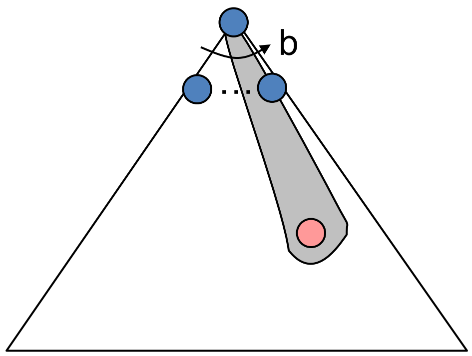


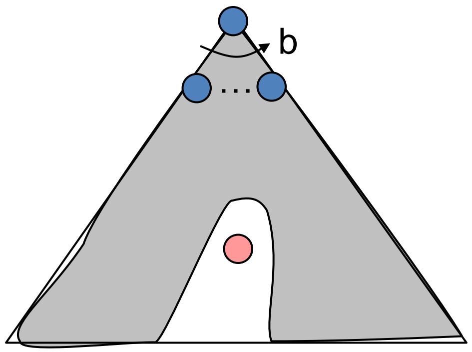
Greedy best-first search example



Greedy Search

- Strategy: expand a node that you think is closest to a goal state
  – Heuristic: estimate of distance to nearest goal for each state
- A common case:
  – Best-first takes you straight to the (wrong) goal
- Worst-case: like a badly-guided DFS





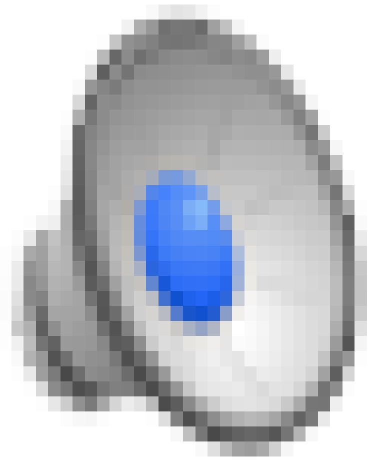
Properties of greedy best-first search

- Complete? No – can get stuck in loops, e.g., Iasi → Neamt → Iasi → Neamt →

Complete in finite space with repeated-state checking

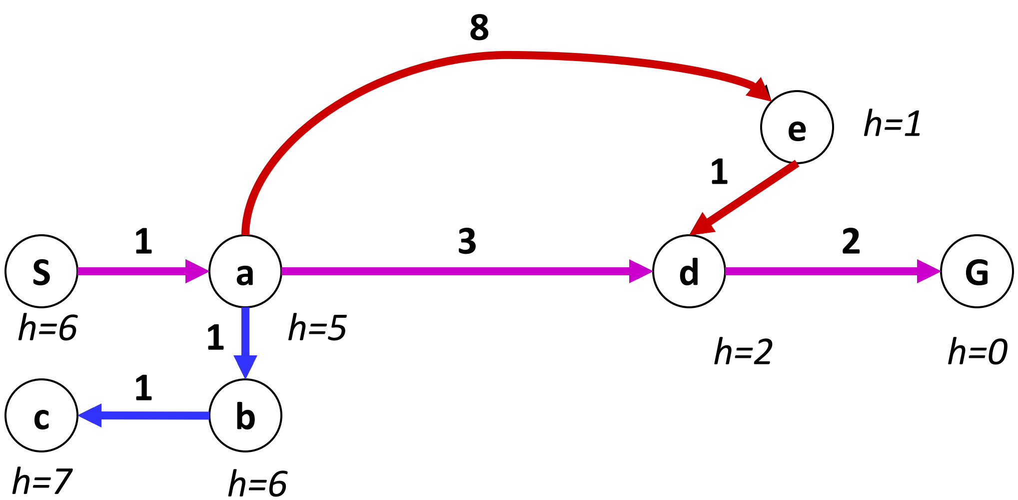
- Time? $O(b^m)$, but a good heuristic can give dramatic improvement
- Space? $O(b^m)$ -- keeps all nodes in memory
- Optimal? No

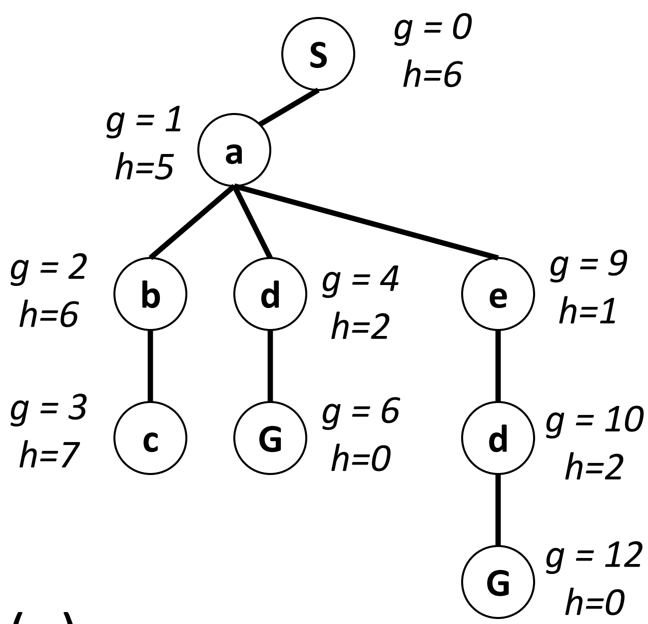
Video of Demo Contours Greedy (Pacman Small Maze)



Combining UCS and Greedy

- Uniform-cost orders by path cost, or *backward cost* g(n)
- Greedy orders by goal proximity, or *forward cost* h(n)





- A* Search orders by the sum: f(n) = g(n) + h(n)

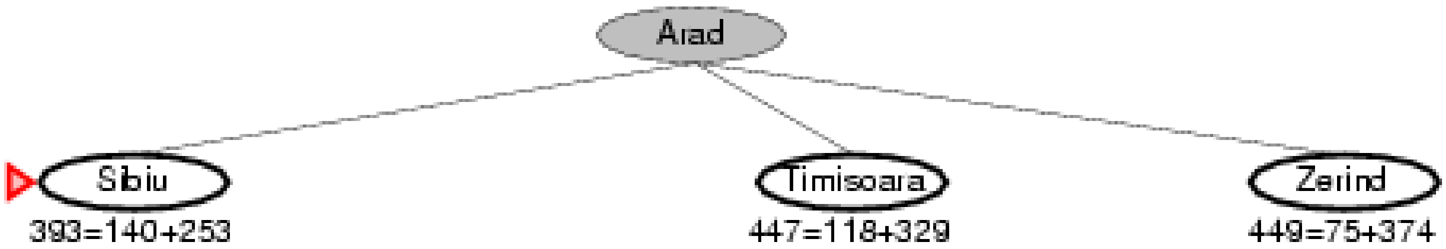
Example: Teg Grenager

A* search

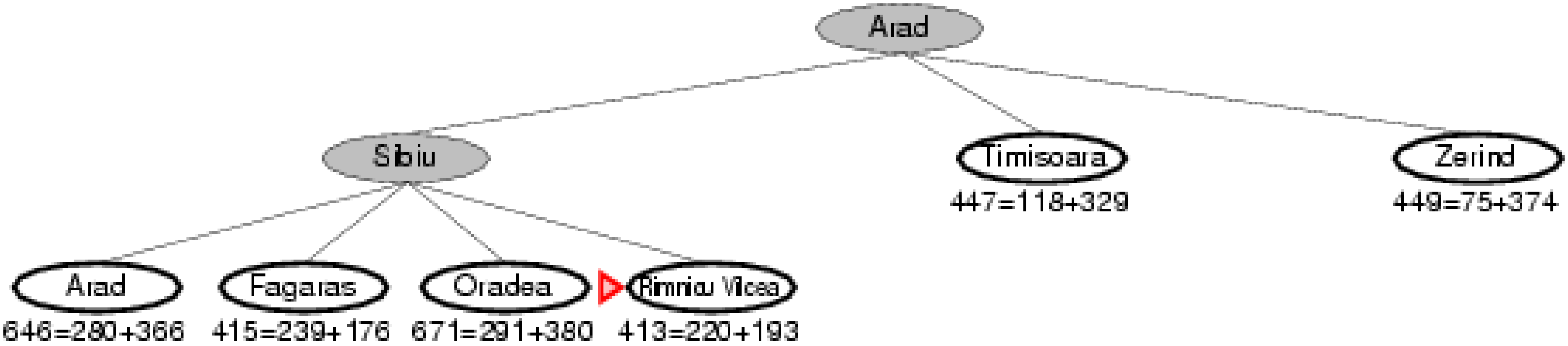
- Idea: avoid expanding paths that are already expensive
- Evaluation function $f(n) = g(n) + h(n)$
- $g(n)$ = cost so far to reach n
- $h(n)$ = estimated cost from n to goal
- $f(n)$ = estimated total cost of path through n to goal
- A* search uses an **admissible** heuristic
- i.e., $h(n) \leq h^*(n)$ where $h^*(n)$ is the true cost from n.
- (Also require $h(n) \geq 0$, so $h(G) = 0$ for any goal G.)
- E.g., $h_{SLD}(n)$ never overestimates the actual road distance
- Theorem: A* search is optimal

A* search example

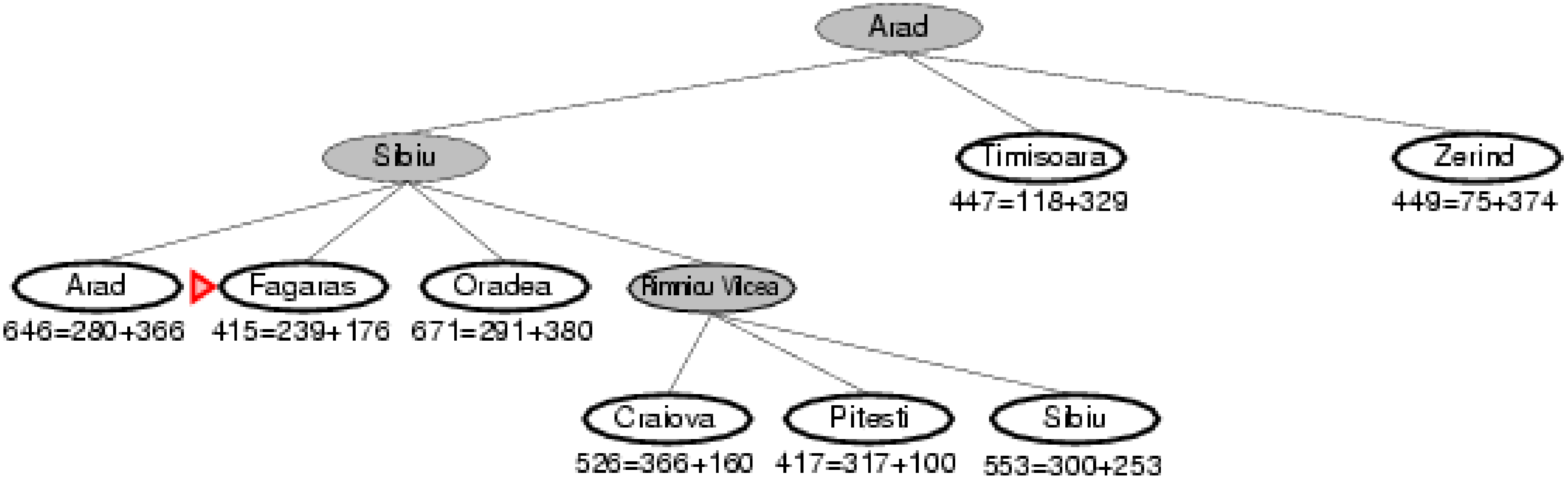
A^* search example



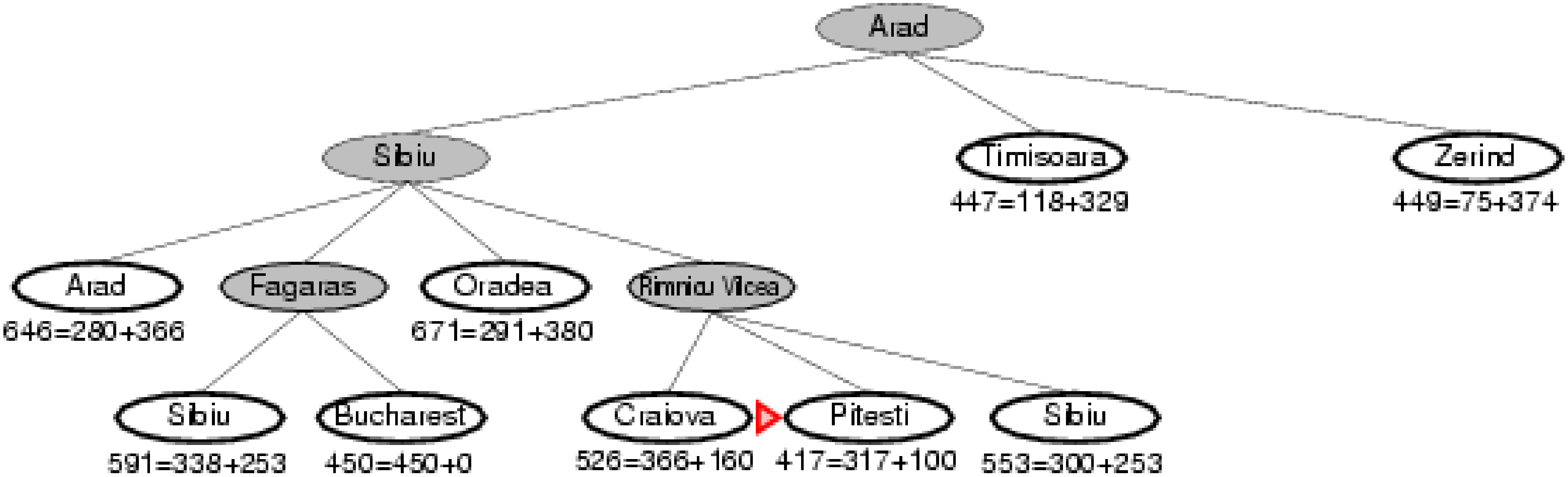
A* search example



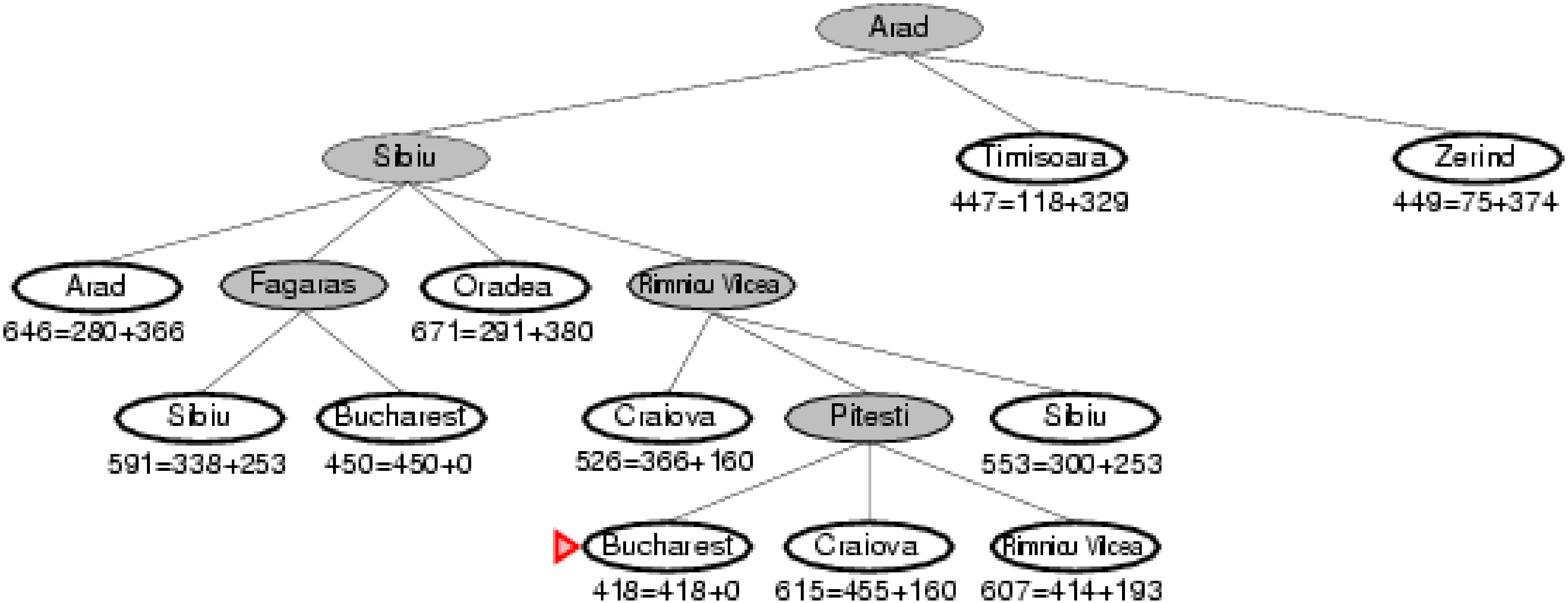
A* search example



A* search example

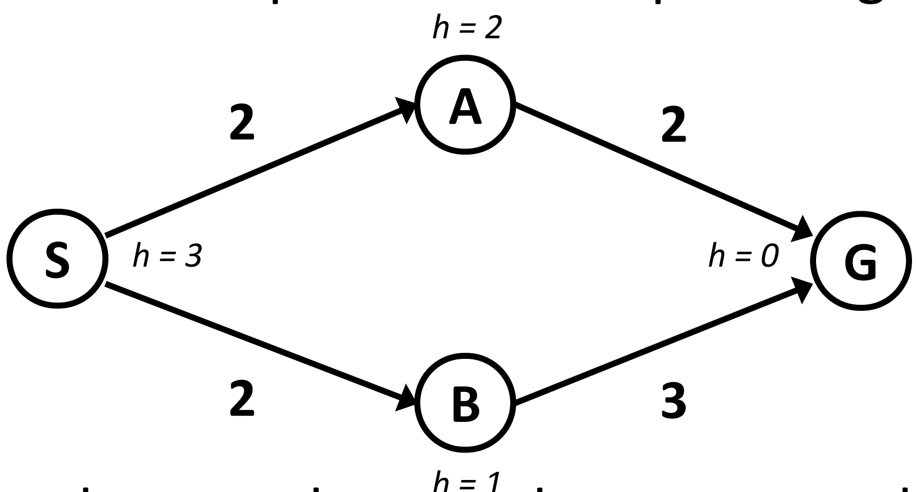


A* search example



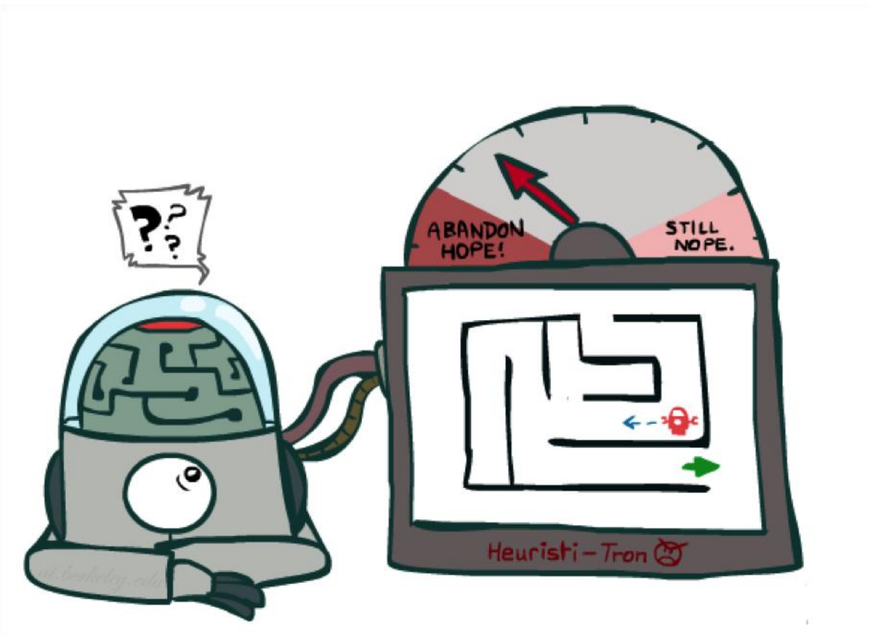
When should A* terminate?

- Should we stop when we enqueue a goal?

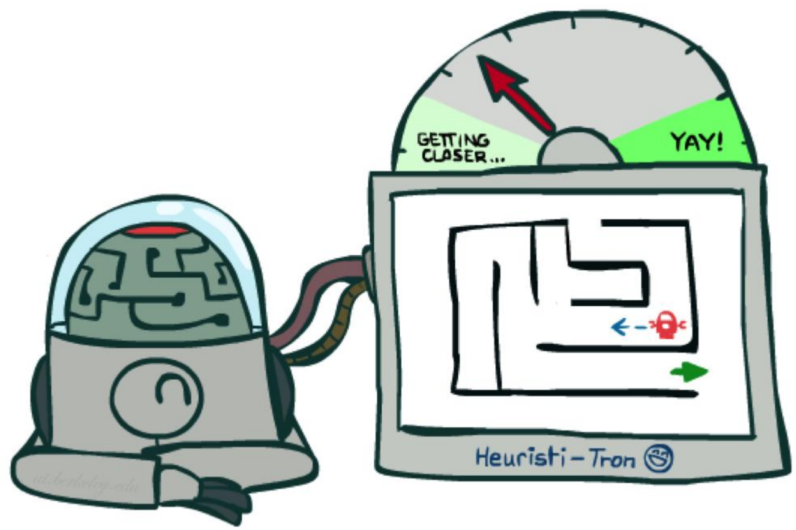


- No: only stop when we dequeue a goal

Idea: Admissibility



Inadmissible (pessimistic) heuristics break optimality by trapping good plans on the fringe



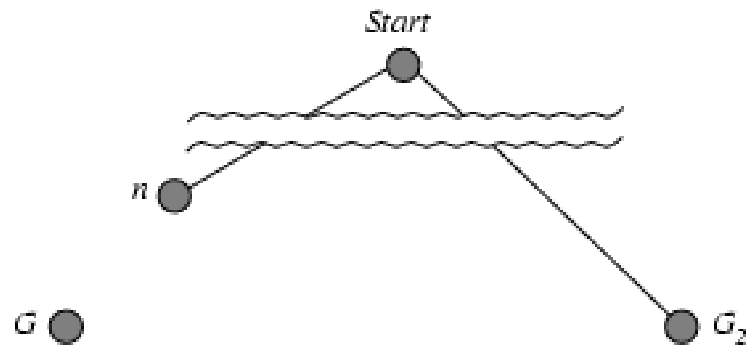
Admissible (optimistic) heuristics slow down bad plans but never outweigh true costs

Admissible heuristics

- A heuristic $h(n)$ is admissible if for every node n, $h(n) \le h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from n.
- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic
- Example: $h_{SLD}(n)$ (never overestimates the actual road distance)
- Theorem: If $h(n)$ is admissible, A^* using TREE-SEARCH is optimal

Optimality of A^* (proof)

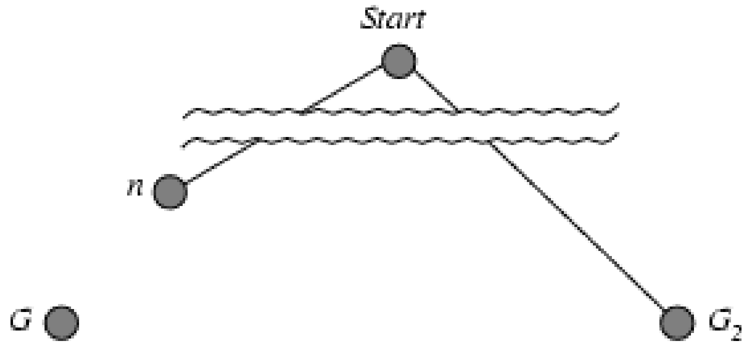
- Suppose some suboptimal goal G_2 has been generated and is in the fringe. Let *n* be an unexpanded node in the fringe such that *n* is on a shortest path to an optimal goal *G*.



$f(G_2) = g(G_2)$	since $h(G_2) = 0$
$g(G_2) > g(G)$	since G_2 is suboptimal
$f(G) = g(G)$	since $h(G) = 0$
$f(G_2) > f(G)$	from above

Optimality of A^* (proof)

- Suppose some suboptimal goal G_2 has been generated and is in the fringe. Let *n* be an unexpanded node in the fringe such that *n* is on a shortest path to an optimal goal *G*.



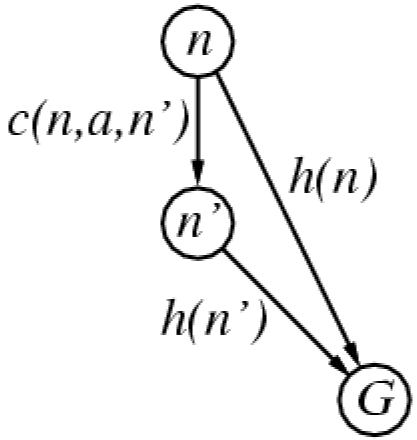
$f(G_2)$	$> f(G)$	from above
$h(n)$	$\leq$ h^*(n)	since h is admissible
$g(n) + h(n)$	$\leq g(n) + h^*(n)$	
$f(n)$	$\leq f(G)$	

Hence $f(G_2) > f(n)$, and A^* will never select G_2 for expansion

Consistent heuristics

- A heuristic is consistent if for every node *n*, every successor *n'* of *n* generated by any action *a*,

  $h(n) \le c(n,a,n') + h(n')$



- If *h* is consistent, we have

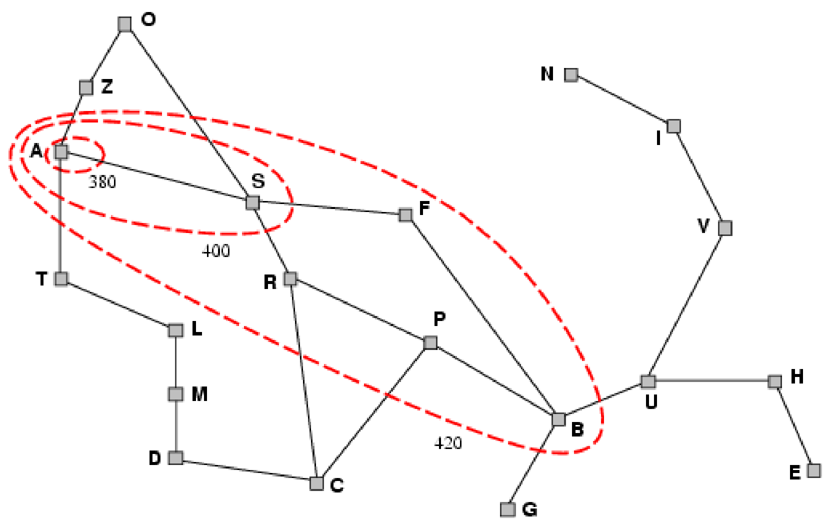
$$
\begin{aligned}
f(n') &= g(n') + h(n') \\
&= g(n) + c(n,a,n') + h(n') \\
&\ge g(n) + h(n) \\
&= f(n)
\end{aligned}
$$

- i.e., *f(n)* is non-decreasing along any path.
- Theorem: If *h(n)* is consistent, **A* using GRAPH-SEARCH is optimal**

Optimality of A*

- A* expands nodes in order of increasing *f* value

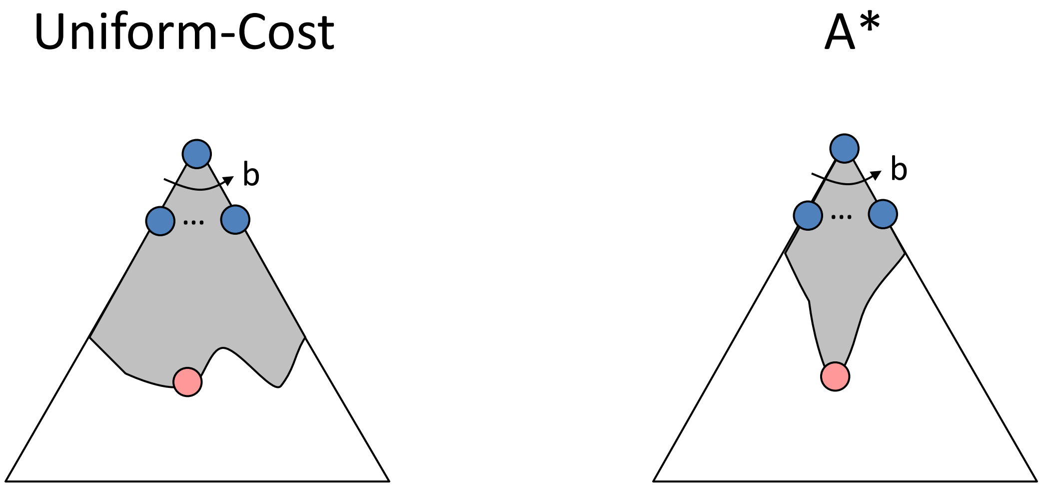
- Gradually adds "*f*-contours" of nodes
- Contour *i* has all nodes with $f=f_i$, where $f_i < f_{i+1}$



Properties of A*

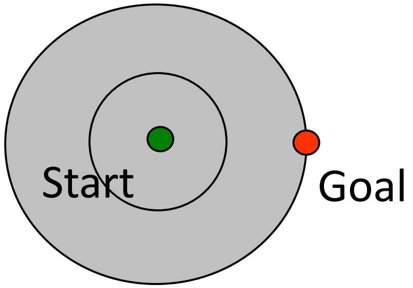
- Complete? Yes (unless there are infinitely many nodes with f ≤ *f(G)*)
- Time? Exponential
- Space? Keeps all nodes in memory
- Optimal? Yes

Properties of A*

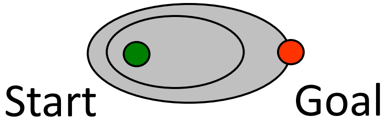


UCS vs A* Contours

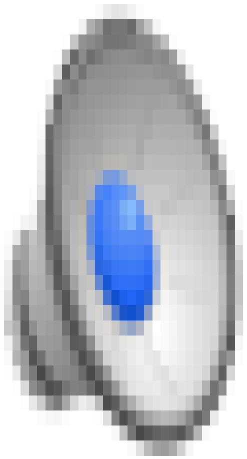
- Uniform-cost expands equally in all "directions"



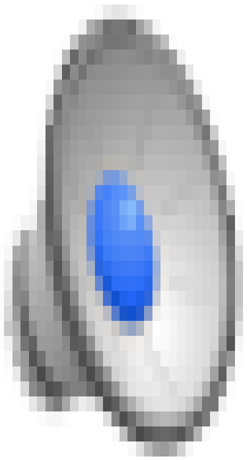
- A* expands mainly toward the goal, but does hedge its bets to ensure optimality



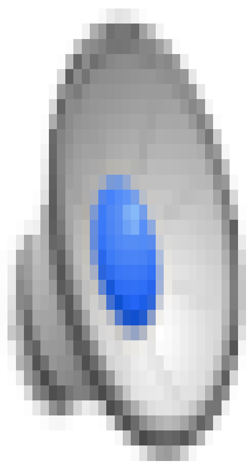
Video of Demo Contours (Empty) -- UCS



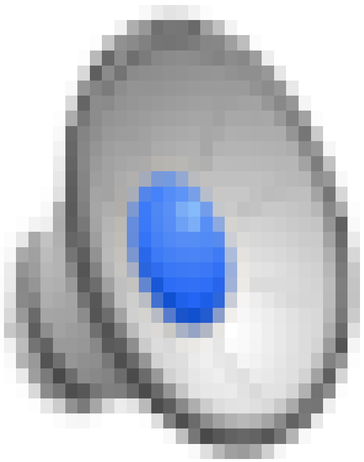
Video of Demo Contours (Empty) -- Greedy



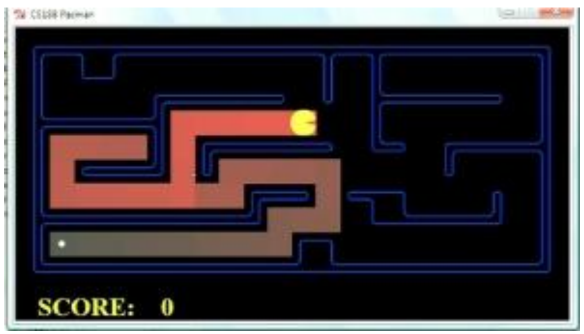
Video of Demo Contours (Empty) – A*



Video of Demo Contours (Pacman Small Maze) – A*



Comparison

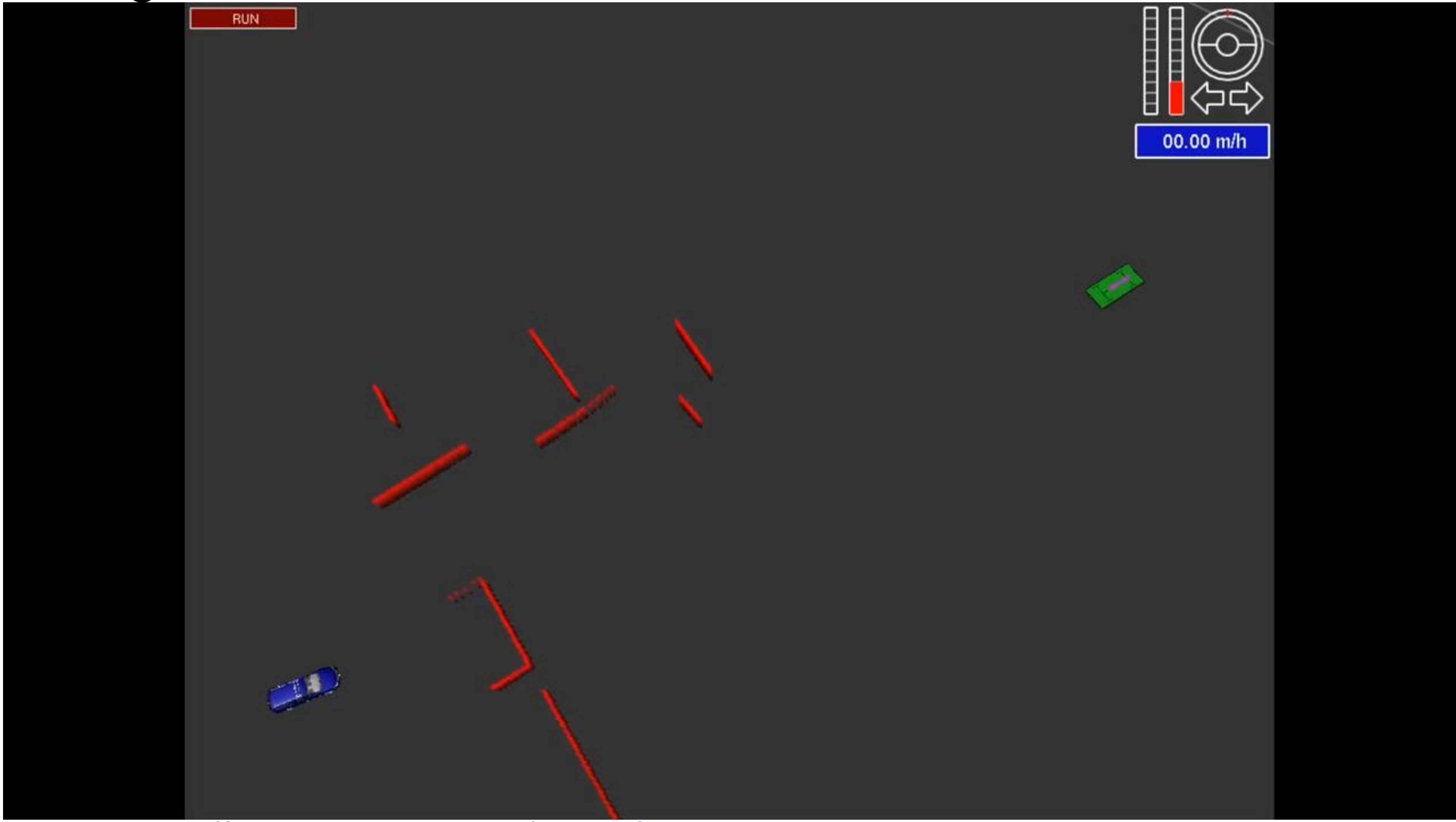


Greedy

Uniform Cost

A*

Self-Driving car: a simplistic application of A* search algorithm

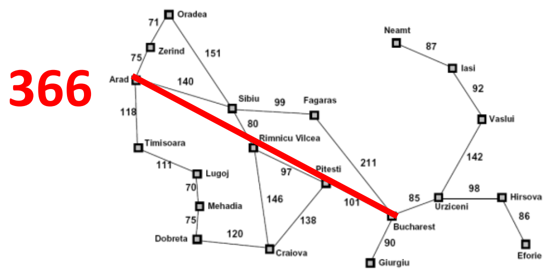


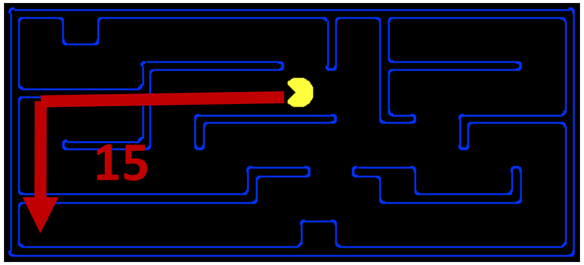
https://www.youtube.com/watch?v=qXZt-B7iUyw

Creating Admissible Heuristics

- Most of the work in solving hard search problems optimally is in coming up with admissible heuristics

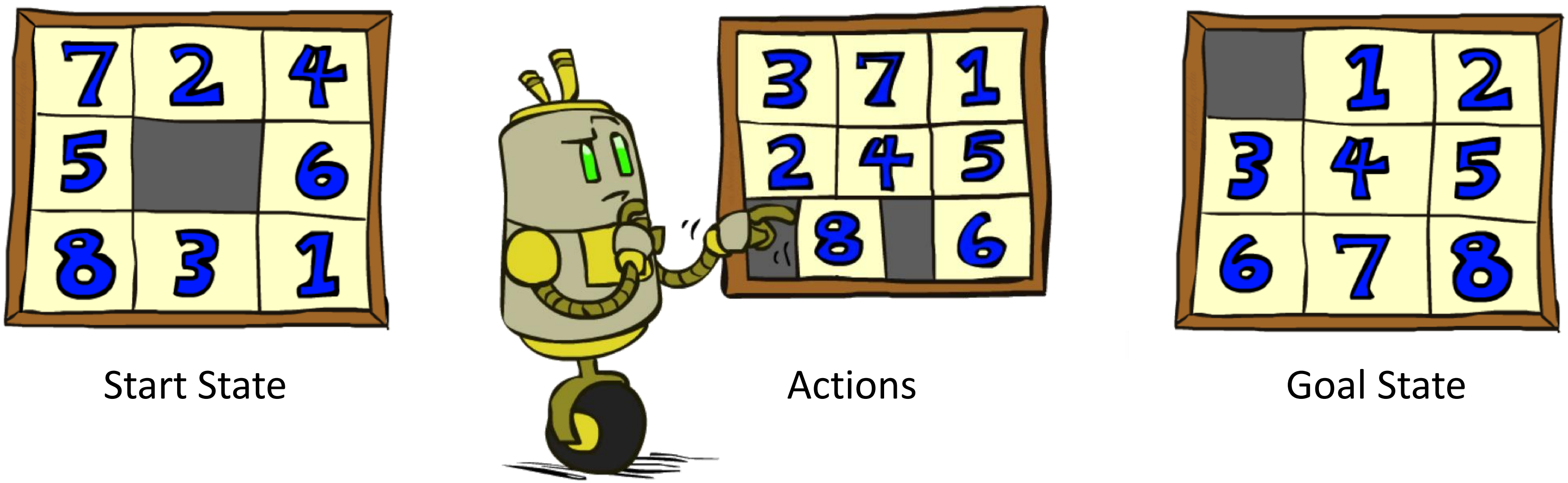
- Often, admissible heuristics are solutions to *relaxed problems,* where new actions are available





- Inadmissible heuristics are often useful too

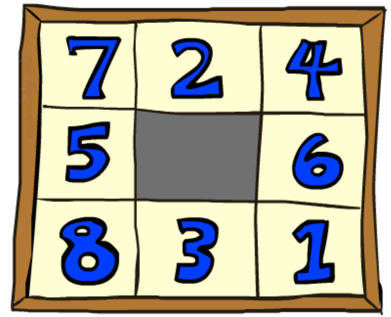
Example: 8 Puzzle



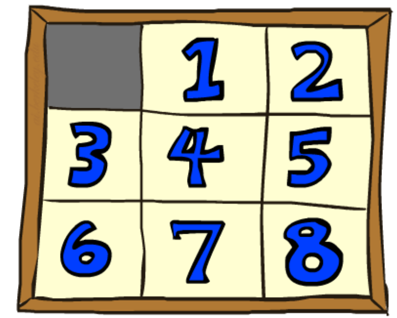
- What are the states?
- How many states?
- What are the actions?
- How many successors from the start state?
- What should the costs be?

8 Puzzle I

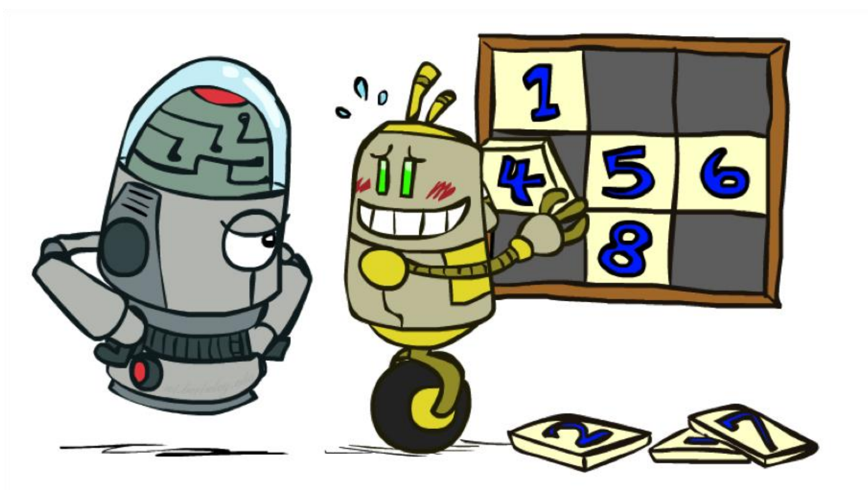
- Heuristic: Number of tiles misplaced
- Why is it admissible?
- h(start) = 8
- This is a *relaxed-problem* heuristic



Start State



Goal State

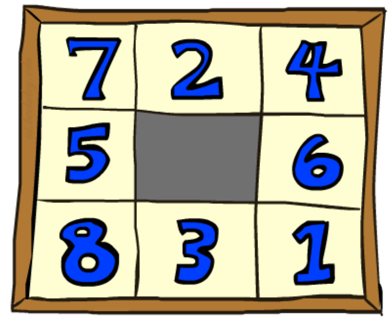


	Average nodes expanded when the optimal path has…		
	…4 steps	…8 steps	…12 steps
UCS	112	6,300	3.6×10^6
TILES	13	39	227

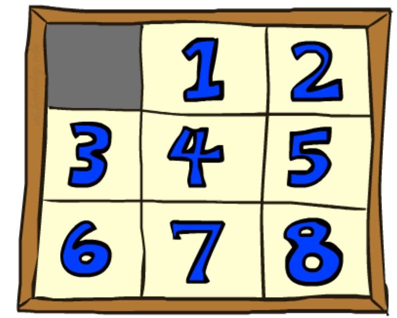
Statistics from Andrew Moore

8 Puzzle II

- What if we had an easier 8-puzzle where any tile could slide any direction at any time, ignoring other tiles?
- Total *Manhattan* distance
- Why is it admissible?
- h(start) = 3 + 1 + 2 + … = 18



Start State

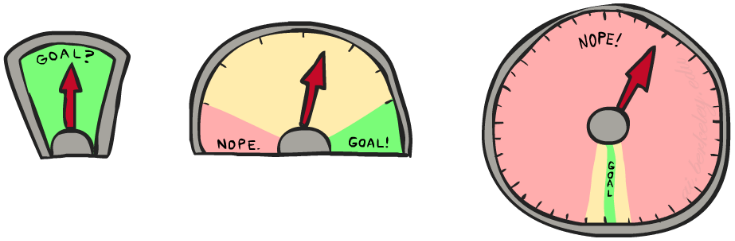


Goal State

	Average nodes expanded when the optimal path has…		
	…4 steps	…8 steps	…12 steps
TILES	13	39	227
MANHATTAN	12	25	73

8 Puzzle III

- How about using the *actual cost* as a heuristic?
  - Would it be admissible?
  - Would we save on nodes expanded?
  - What's wrong with it?



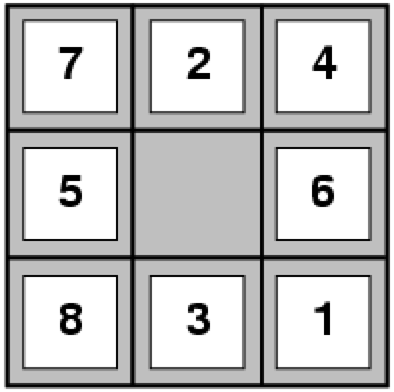
- With A*: a trade-off between quality of estimate and work per node
  - As heuristics get closer to the true cost, you will expand fewer nodes but usually do more work per node to compute the heuristic itself

Admissible heuristics

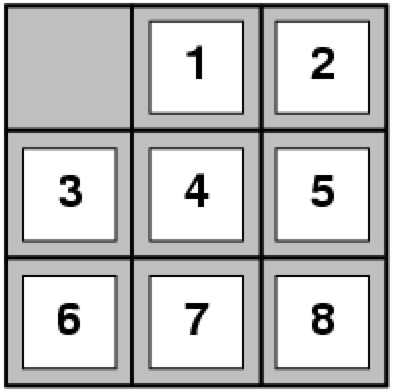
E.g., for the 8-puzzle:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance

(i.e., no. of squares from desired location of each tile)



Start State



Goal State

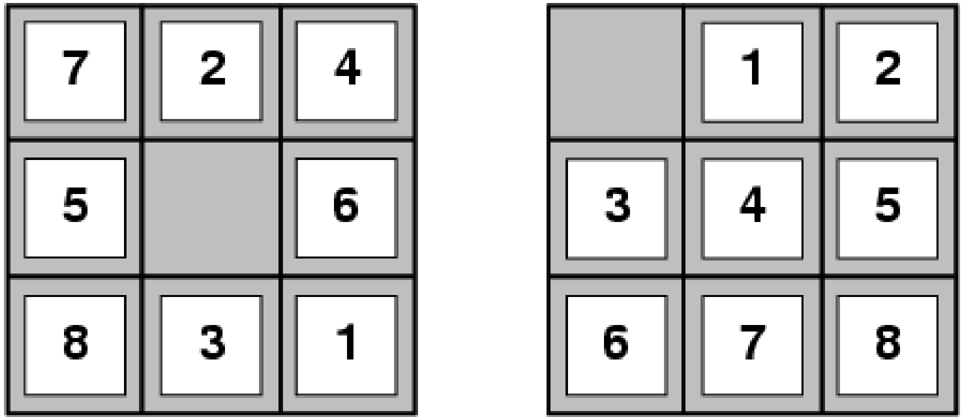
- $h_1(S)$ = ?
- $h_2(S)$ = ?

Admissible heuristics

E.g., for the 8-puzzle:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance

(i.e., no. of squares from desired location of each tile)



Start State Goal State

- $h_1(S)$ = ? 8
- $h_2(S)$ = ? 3+1+2+2+2+3+3+2 = 18

Dominance

- If $h_2(n) \geq h_1(n)$ for all n (both admissible) then h_2 dominates h_1
- h_2 is better for search

- Typical search costs (average number of nodes expanded):

- *d=12* IDS = 3,644,035 nodes
  $A^*(h_1)$ = 227 nodes
  $A^*(h_2)$ = 73 nodes
- *d=24* IDS = too many nodes
  $A^*(h_1)$ = 39,135 nodes
  $A^*(h_2)$ = 1,641 nodes

Relaxed problems

- A problem with fewer restrictions on the actions is called a relaxed problem
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution
- If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution

Local search algorithms

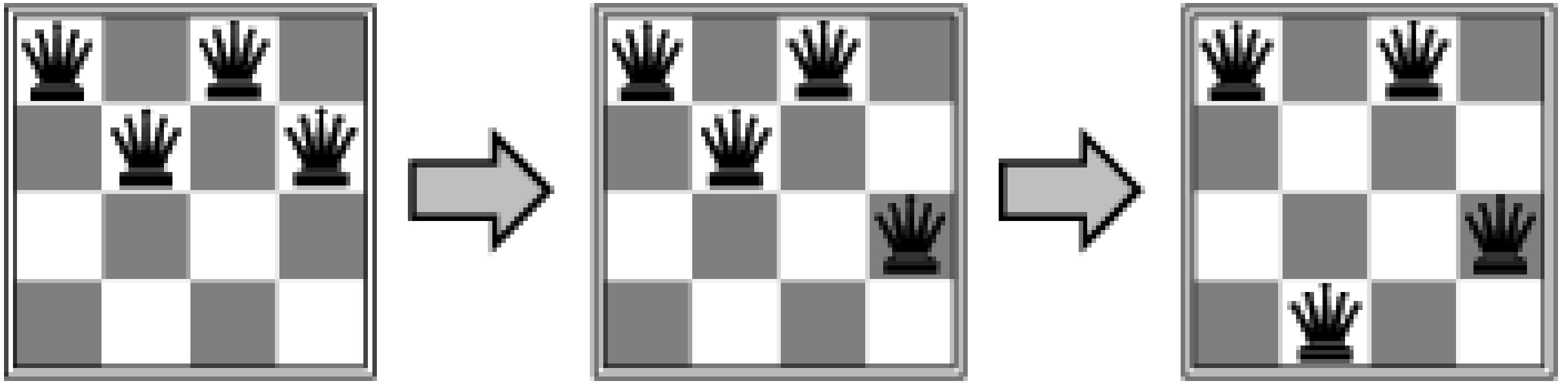
- In many optimization problems, the path to the goal is irrelevant; the goal state itself is the solution

- State space = set of "complete" configurations
- Find configuration satisfying constraints, e.g., n-queens

- In such cases, we can use local search algorithms
- keep a single "current" state, try to improve it

Example: *n*-queens

- Put n queens on an $n \times n$ board with no two queens on the same row, column, or diagonal



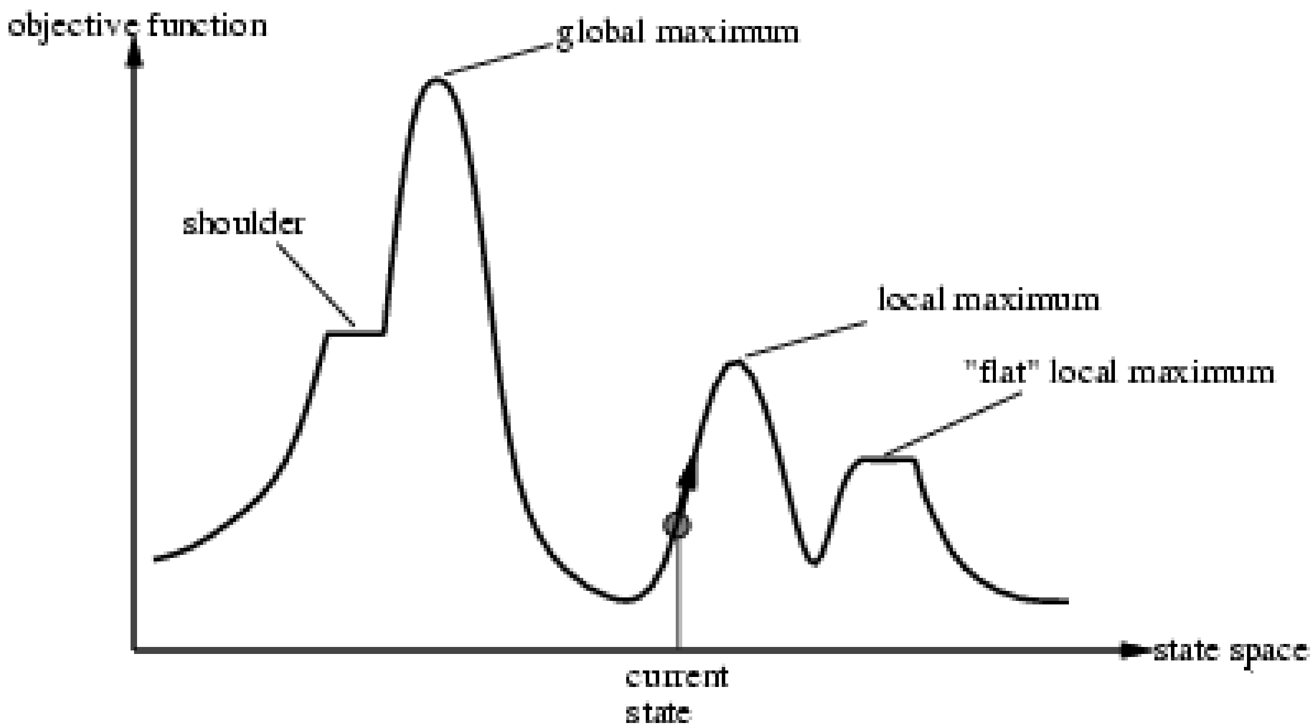
Hill-climbing search

- "Like climbing Everest in thick fog with amnesia"

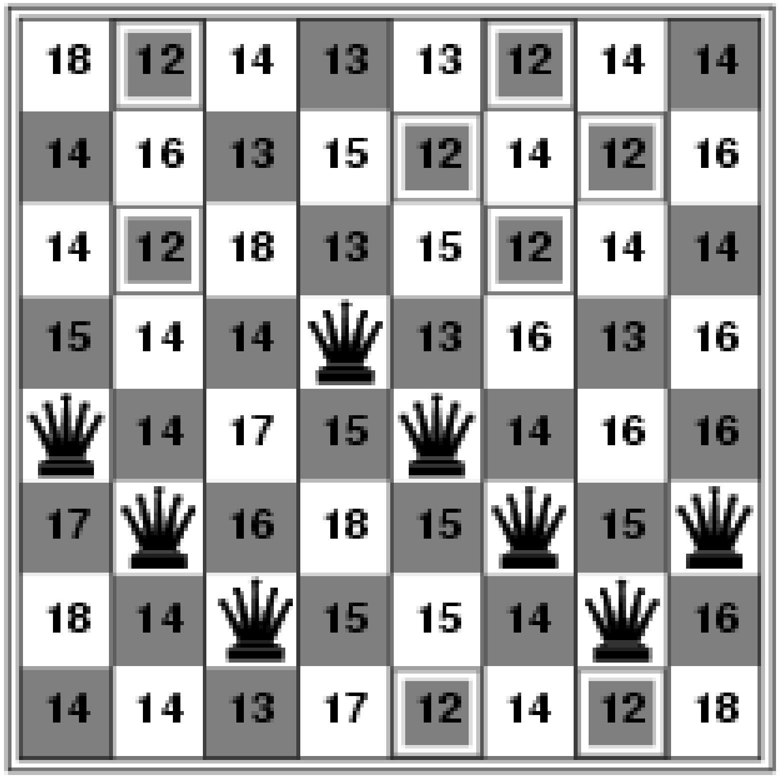
```
function HILL-CLIMBING(problem) returns a state that is a local maximum
    inputs: problem, a problem
    local variables: current, a node
                     neighbor, a node

    current ← MAKE-NODE(INITIAL-STATE[problem])
    loop do
        neighbor ← a highest-valued successor of current
        if VALUE[neighbor] ≤ VALUE[current] then return STATE[current]
        current ← neighbor
```

Hill-climbing search

- Problem: depending on initial state, can get stuck in local maxima

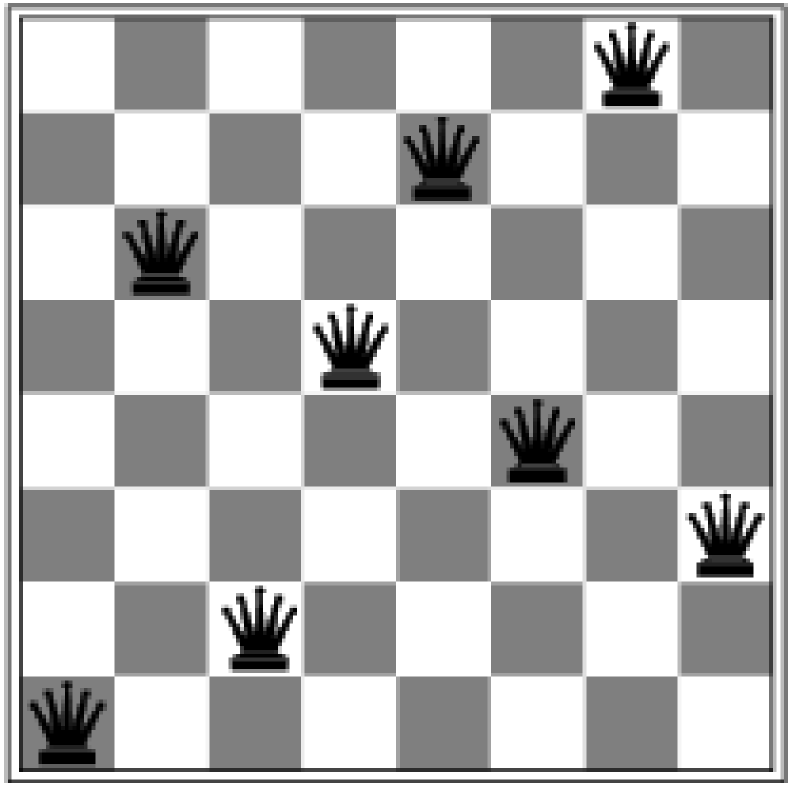


Hill-climbing search: 8-queens problem



- *h* = number of pairs of queens that are attacking each other, either directly or indirectly
- *h = 17* for the above state

Hill-climbing search: 8-queens problem



- A local minimum with $h = 1$

Simulated annealing search

- Idea: escape local maxima by allowing some "bad" moves but gradually decrease their frequency

```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
   inputs: problem, a problem
           schedule, a mapping from time to "temperature"
   local variables: current, a node
                    next, a node
                    T, a "temperature" controlling prob. of downward steps

   current ← MAKE-NODE(INITIAL-STATE[problem])
   for t ← 1 to ∞ do
       T ← schedule[t]
       if T = 0 then return current
       next ← a randomly selected successor of current
       ΔE ← VALUE[next] – VALUE[current]
       if ΔE > 0 then current ← next
       else current ← next only with probability e^(ΔE/T)
```

Properties of simulated annealing search

- One can prove: If *T* decreases slowly enough, then simulated annealing search will find a global optimum with probability approaching 1
- Widely used in VLSI layout, airline scheduling, etc

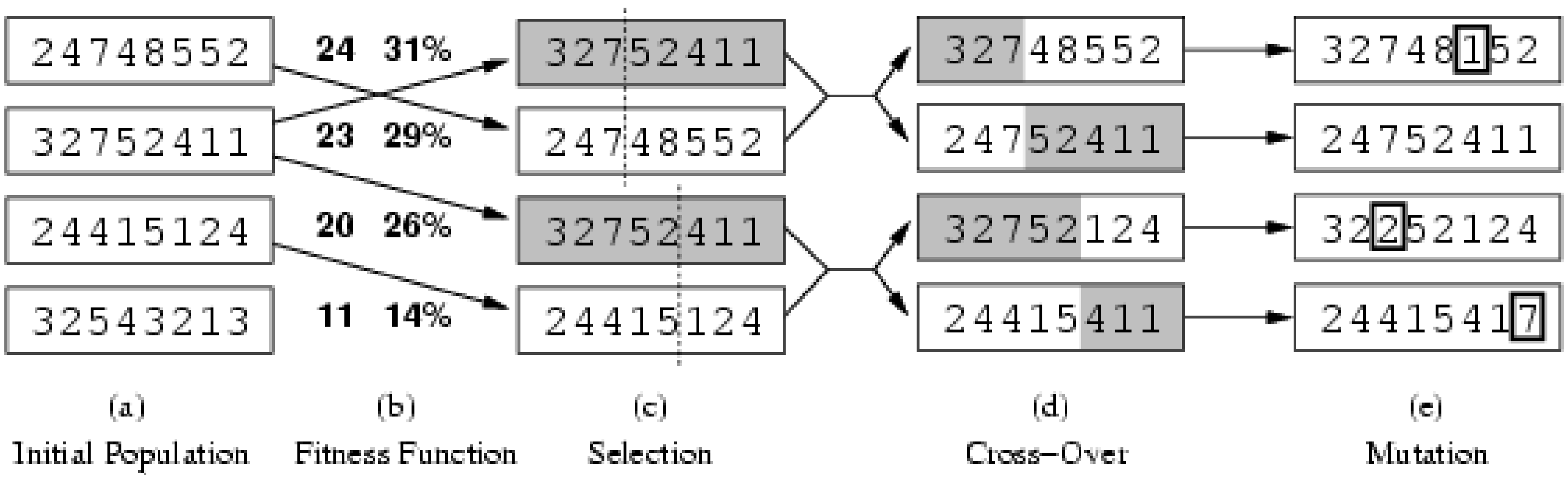
Local beam search

- Use a ***population*** of potential directions rather just one
- Keep track of *k* states rather than just one
- Start with *k* randomly generated states
- At each iteration, all the successors of all *k* states are generated
- If any one is a goal state, stop; else select the *k* best successors from the complete list and repeat.

Genetic algorithms

- A successor state is generated by combining two parent states
- Start with *k* randomly generated states (population)
- A state is represented as a string over a finite alphabet (often a string of 0s and 1s)
- Evaluation function (fitness function). Higher values for better states.
- Produce the next generation of states by selection, crossover, and mutation

Genetic algorithms

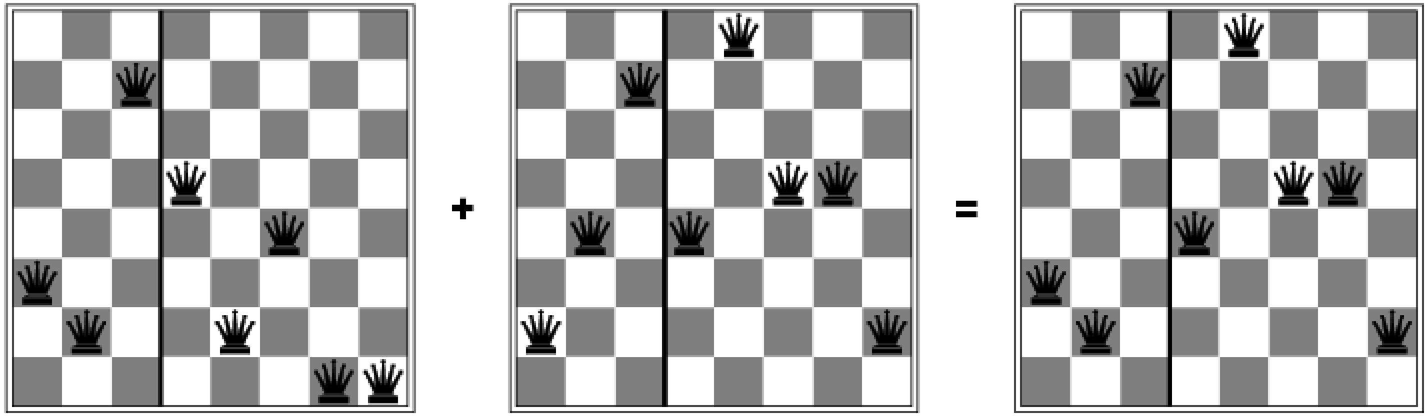


- Fitness function: number of non-attacking pairs of queens (min = 0, max = 8 × 7/2 = 28)

- 24/(24+23+20+11) = 31%

- 23/(24+23+20+11) = 29% etc

Genetic algorithms



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