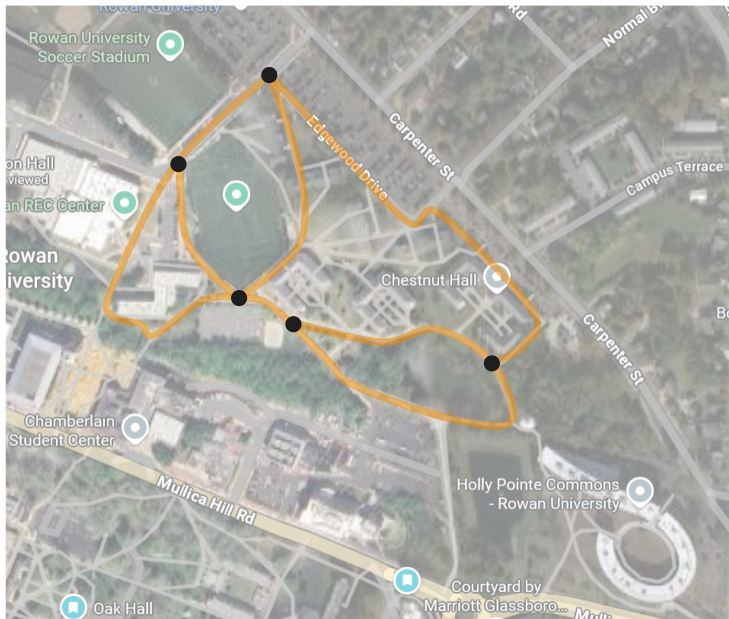# Lesson 9 — Eulerization, TSP, MST

Complete the problems below with proper justification.

## Problems

**Eulerization**

1. Find an Eulerization of the dorm-patrol graph below.



2. To keep grad students from staying late in Robinson Hall, a security guard patrols Robinson Hall. Find the most efficient route for the security guard to walk the three floors (each floor is basically a loop) and the four stairs and return to their starting point, with minimal retracing of their steps. The following shows a graph representation of Robinson Hall:

   https://excalidraw.com/#json=5hTDr7p5ivuSNCSnEABPc,ZE71rn3QH2mH7UTSX2_2jg
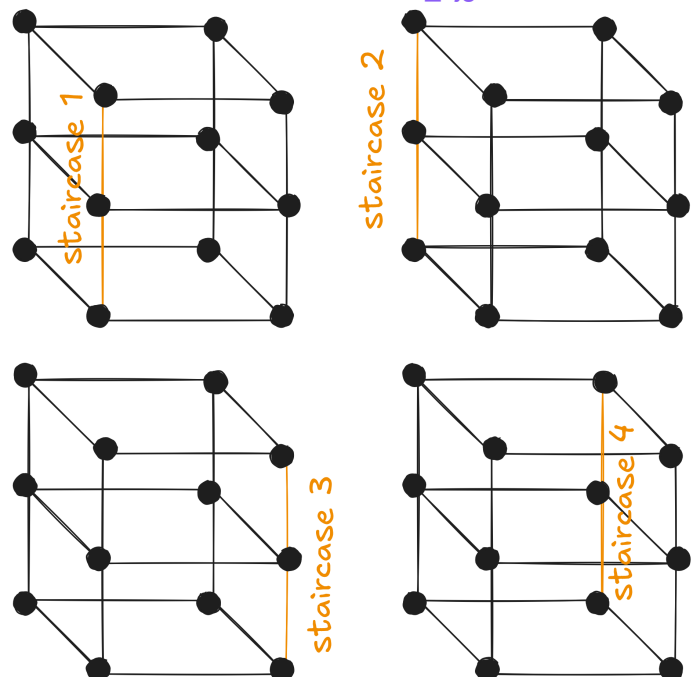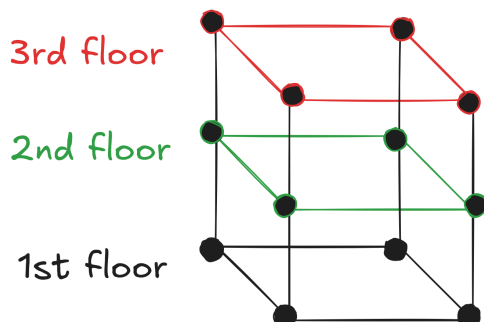
## Travelling salesman problem (TSP)

**35.** The Brute-Force Bandits is a rock band planning a five-city concert tour. The cities and the distances (in miles) between them are given in the weighted graph shown in Fig. 6-43. The tour must start and end at $A$. The cost of the chartered bus in which the band is traveling is $8 per mile.

   **(a)** Find the nearest-neighbor tour with starting vertex $A$. Give the cost (in $) of this tour.

   **(b)** Find the nearest-neighbor tour with starting vertex $B$. Write the tour as it would be traveled by the band, starting and ending at $A$. Give the cost (in $) of this tour.
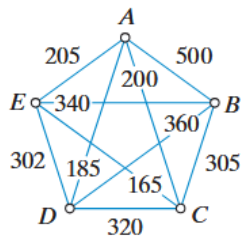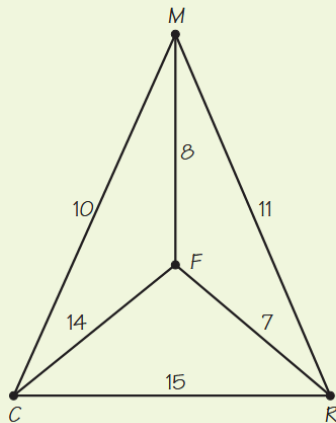


**Figure 6-43**

**42.** After a party at her house, Francine ($F$) has agreed to drive Mary ($M$), Rachel ($R$), and Constance ($C$) home. If the times (in minutes) to drive between her friends' homes are shown below, what route gets Francine back home the quickest?

## Minimum spanning tree (MST)

**45.** The mileage chart in Fig. 7-42 shows the distances between Atlanta, Columbus, Kansas City, Minneapolis, Pierre, and Tulsa. Working directly from the mileage chart use Kruskal's algorithm to find the MST connecting the six cities. (*Hint*: See Example 6.15 for the use of an auxiliary graph.)

**Mileage Chart**

|  | Atlanta | Columbus | Kansas City | Minneapolis | Pierre | Tulsa |
|---|---|---|---|---|---|---|
| **Atlanta** | * | 533 | 798 | 1068 | 1361 | 772 |
| **Columbus** | 533 | * | 656 | 713 | 1071 | 802 |
| **Kansas City** | 798 | 656 | * | 447 | 592 | 248 |
| **Minneapolis** | 1068 | 713 | 447 | * | 394 | 695 |
| **Pierre** | 1361 | 1071 | 592 | 394 | * | 760 |
| **Tulsa** | 772 | 802 | 248 | 695 | 760 | * |

Figure 7-42

**62.** A large company wishes to install a pneumatic tube system that would enable small items to be sent between any of 10 locales, possibly by using relay. If the nonprohibitive costs (in $100) are shown in the accompanying graph model, between which sites should the tube be installed to minimize the total cost?
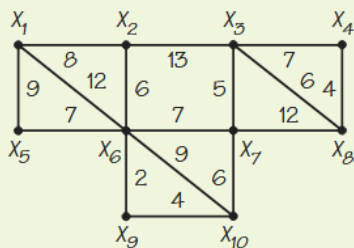


## Minimal spanning tree in society

63. Airline routes roughly resemble spanning trees whose vertices are the destination cities that the airline serves. Explain why the airline routes do not exactly form a spanning tree; that is, why there are circuits within the airline routes. Give two different reasons why this might be the case.

## Trees

64. Draw a tree that has five vertices. Now draw another tree that also has five vertices. Count how many edges each of your trees have. What seems to be the relationship between the number of edges and the number of vertices in a tree?

65. Complete the paragraph by correctly filling in the blanks: If a tree has $E$ many edges, then the tree has _____ many vertices. This is because to draw a tree, the first edge drawn connects _____ (write in a number) vertex/vertices, and every subsequent edge drawn connects _____ (write in a number) new vertex/vertices.