

Writing up solutions

Use **complete sentences** whenever possible.

Lesson 1: Good sample solutions

- Use complete sentences.
- Assume I know little context about the problem.
- Your written explanation read aloud to someone else over the phone should be understandable (assuming the other person has a copy of the accompanying pictures or diagrams you have drawn).

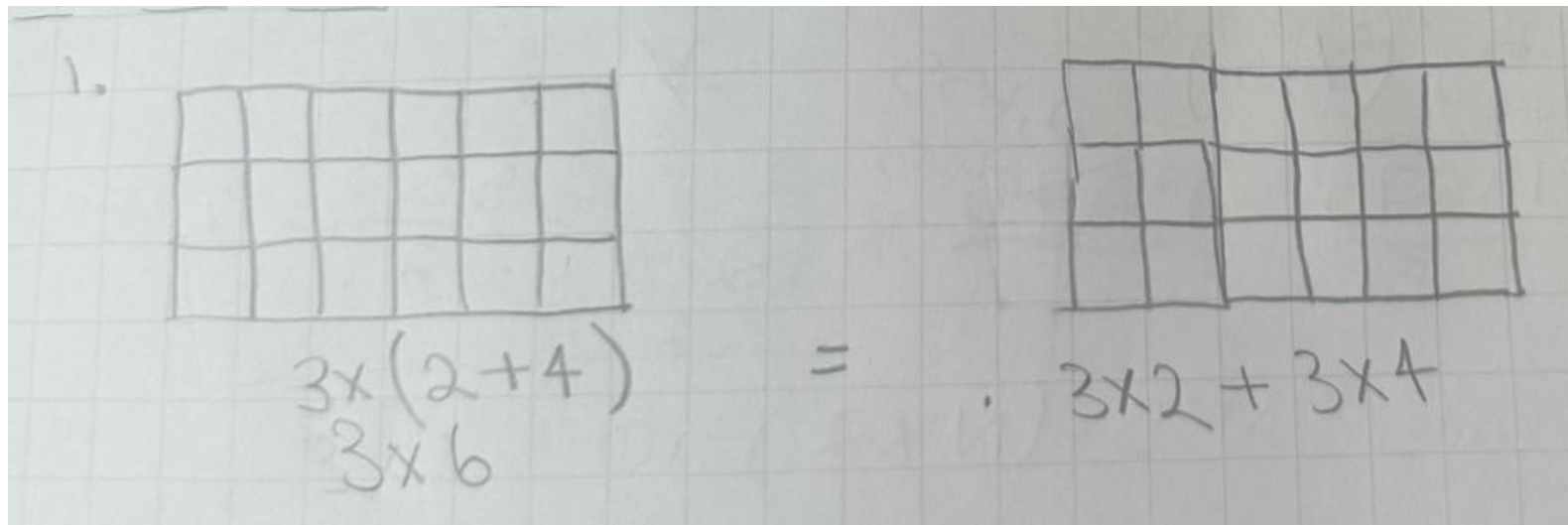
Lesson 1: Good sample solutions

2: You can compute $1 + 2 + 3 \dots + 99 + 100$ by visualizing it as that number of boxes stacked next to each other to form a staircase. The reason for this is because it is not a perfect triangle. Then, if you mirror that staircase and place it on top of the previous staircase upside down, you get a rectangle that is 100 squares by 101 squares. Then, to find the sum of the equation above, we do the formula of $\frac{1}{2} \times \text{base} \times \text{height}$, in this case being $\frac{1}{2} \times 100 \times 101$. This leaves us with an answer of 5050.

3: In order to estimate this, we can round 9,999 up to 10,000. Multiplied by 73, this gives us the estimated answer of 730,000. Then, to find the actual answer, we can multiply 73 by the number we had to add to 9,999 to get to 10,000, in this case being 1. So 1×73 is 73, and we can then subtract 73 from 730,000 to get 729,927.

4: Multiplying a 3 digit number such as 327 by 7, 11, then 13 gives us the answer of 327,327. The reason the number repeats like that is because $7 \times 11 \times 13$ is equal to 1001. When broken apart, it can be seen as $327 \times 1000 + 327 \times 1$, or $327,000 + 327$.

Lesson 1: Good sample solutions



I interpret 3×6 as 3 rows by 6 columns. For $3 \times 2 + 3 \times 4$, I put 3 rows by 2 columns right next to 3 rows by 4 columns to show they're being added to each other.

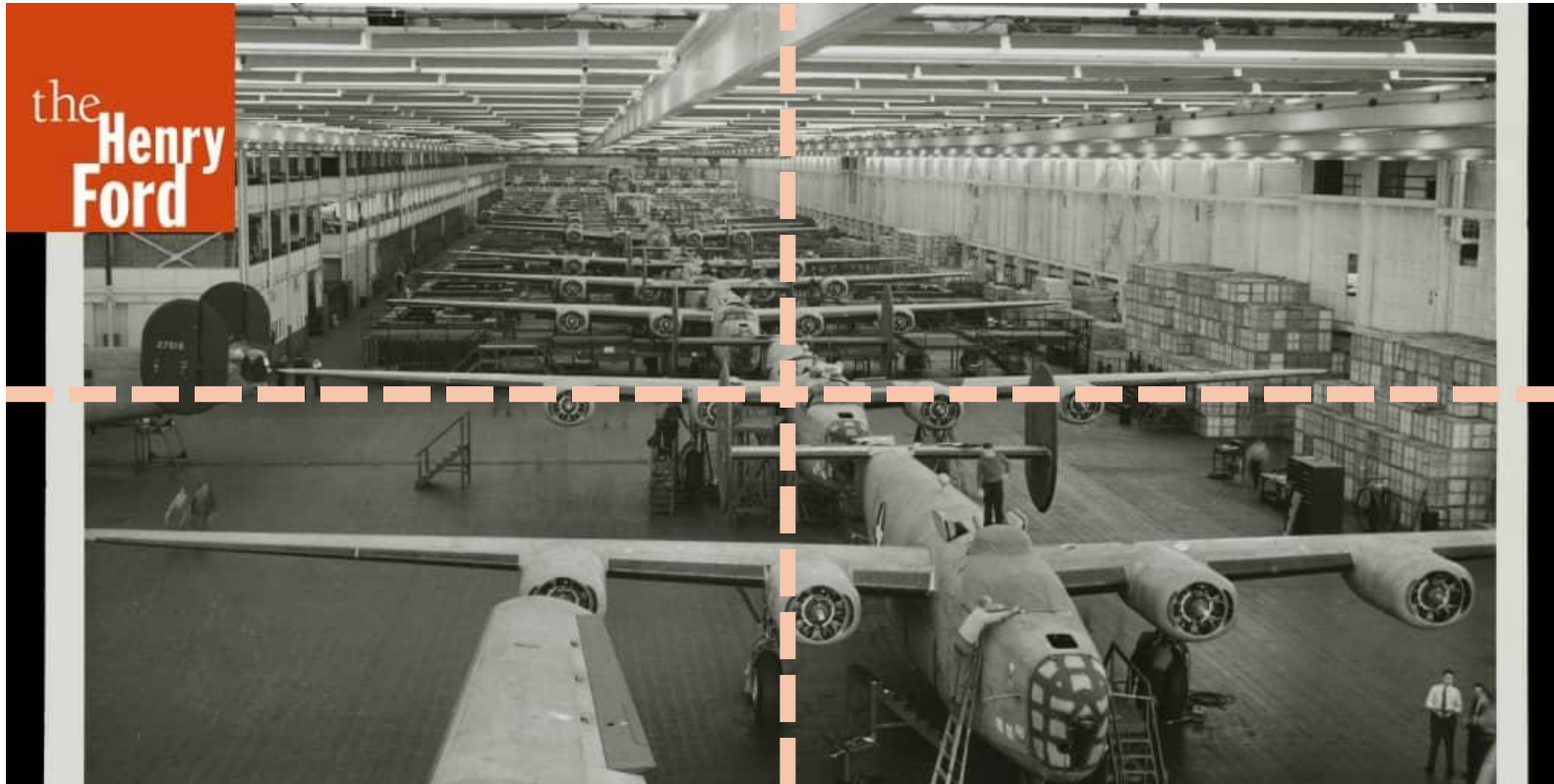
Lesson 6

Viewing point and viewing distance

The viewing point of an uncropped photo is directly in the center

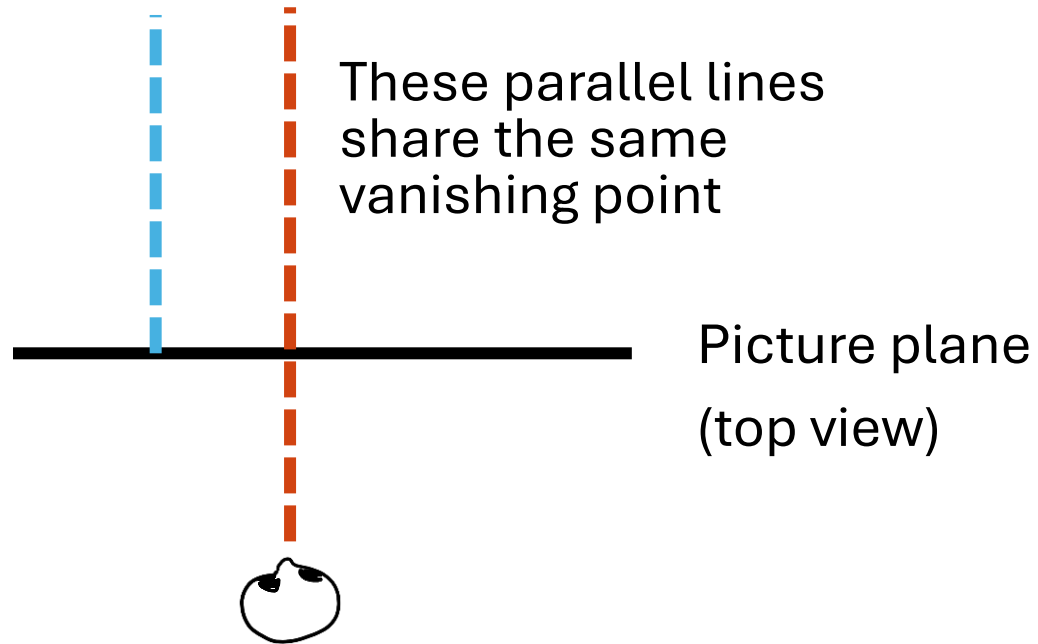


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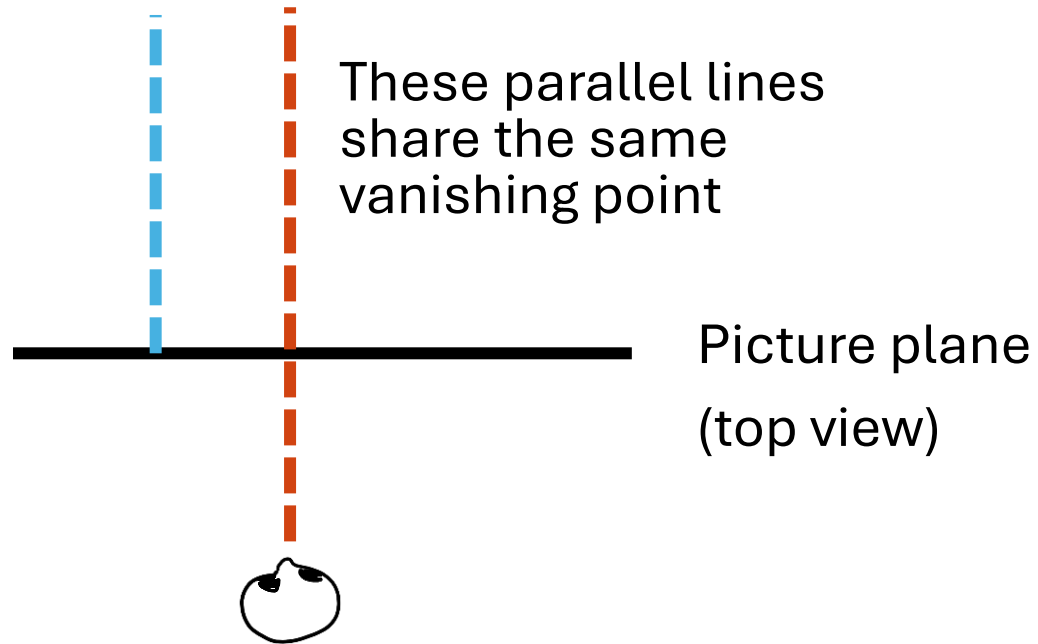


If this photo is uncropped, then the camera is tilted down below the horizon.

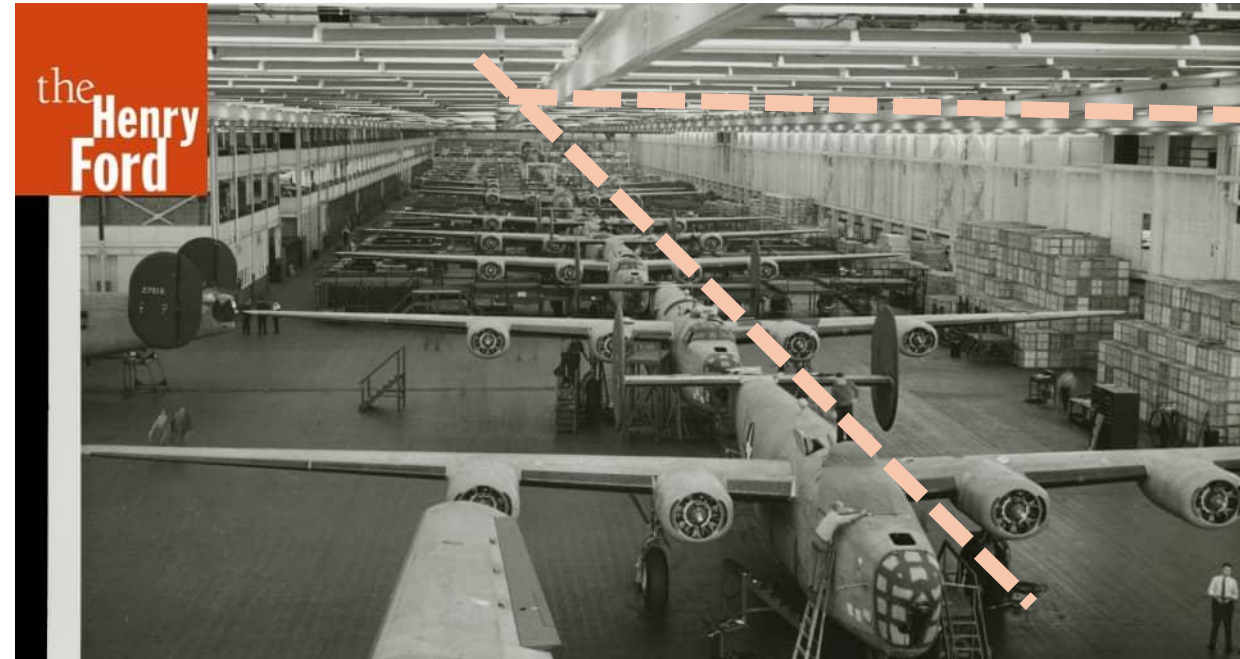
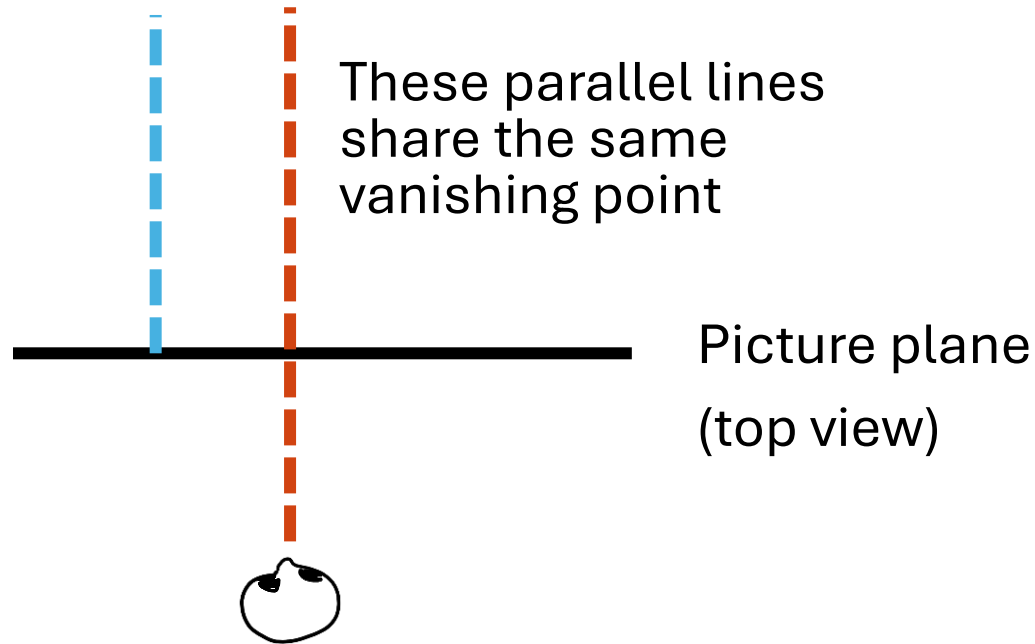
The viewing point of an observer perpendicular to a picture plane is at the vanishing point of lines perpendicular to picture plane.



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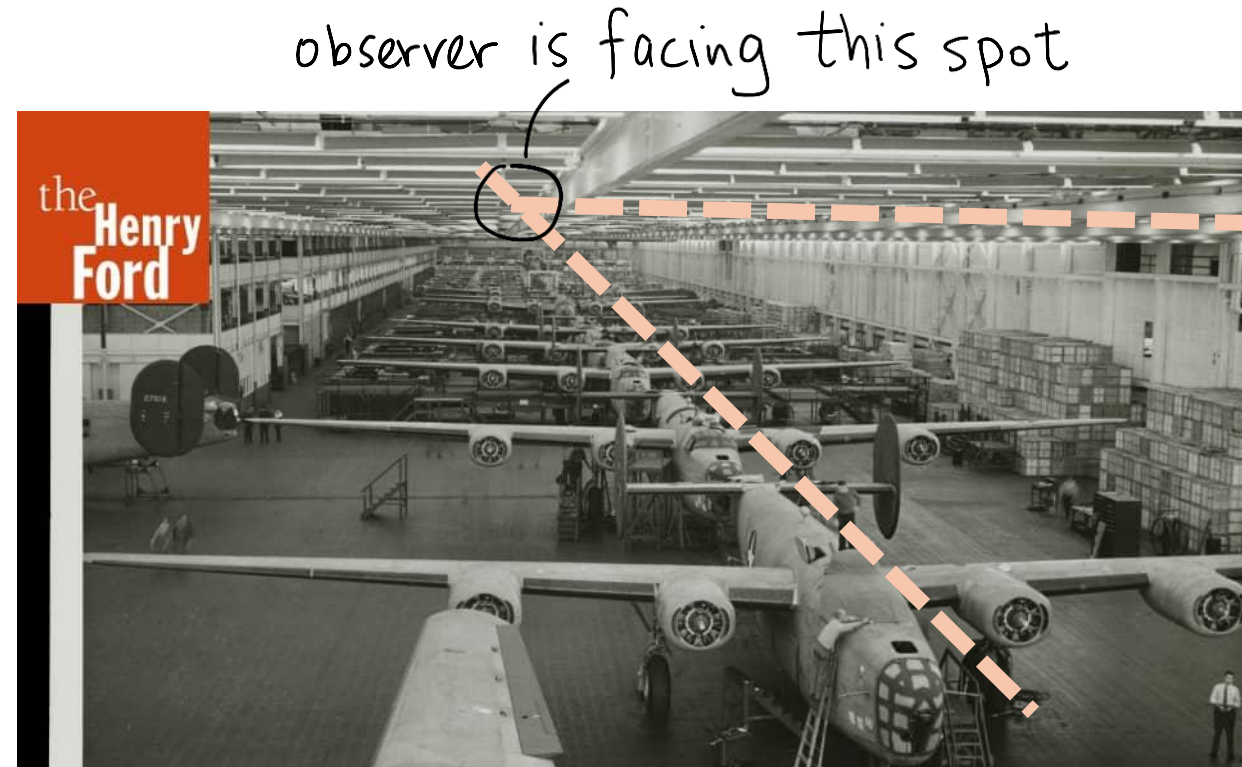
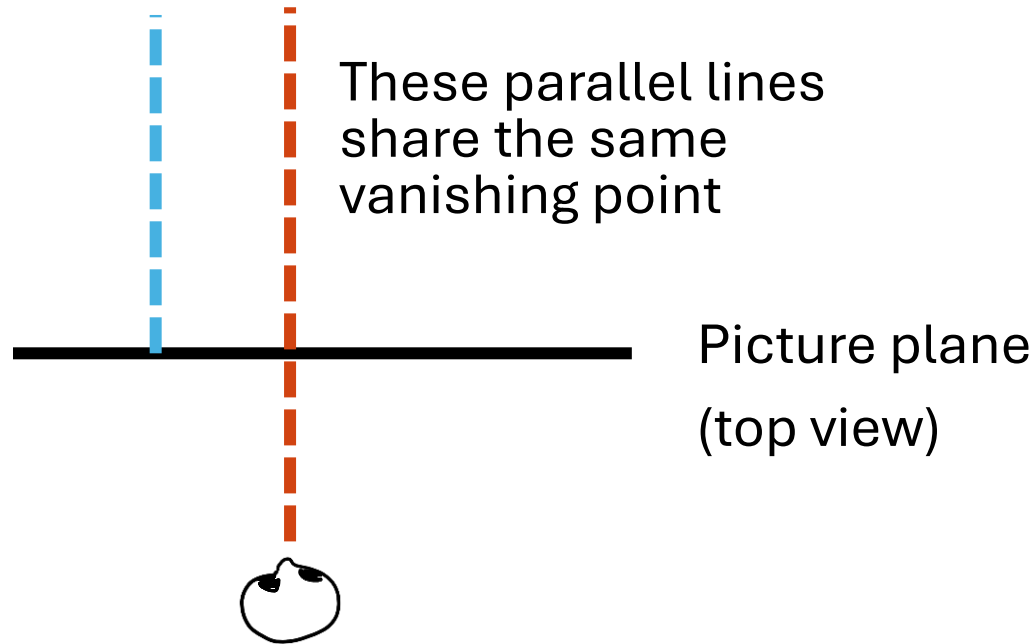


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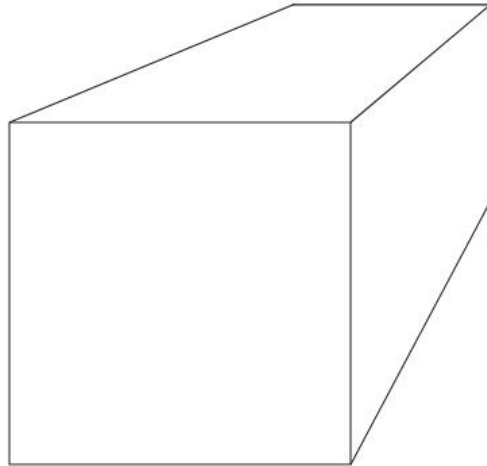
Directly in front of the observer's view
(vs camera's downward-tilted view)

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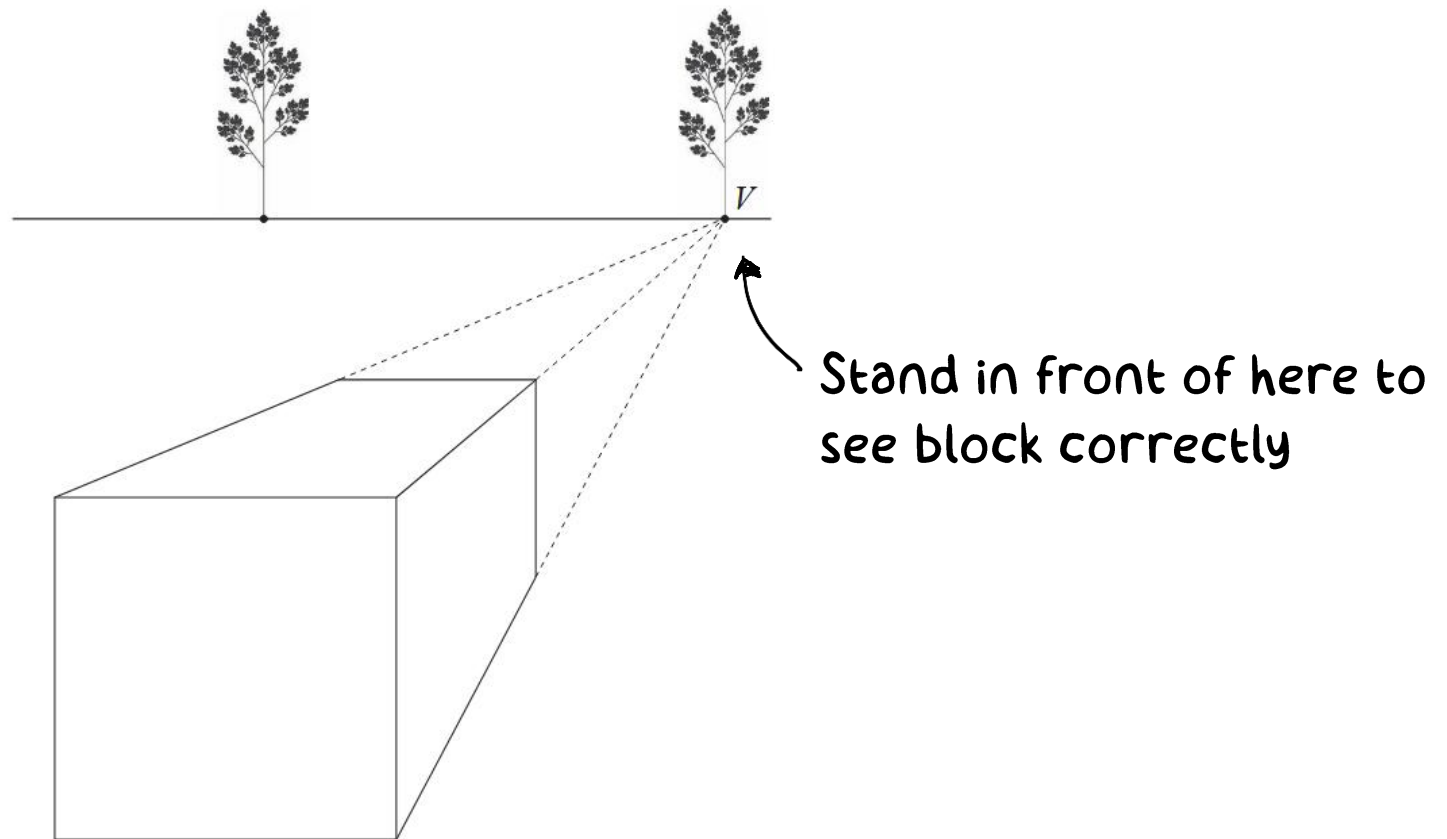


Directly in front of the observer's view
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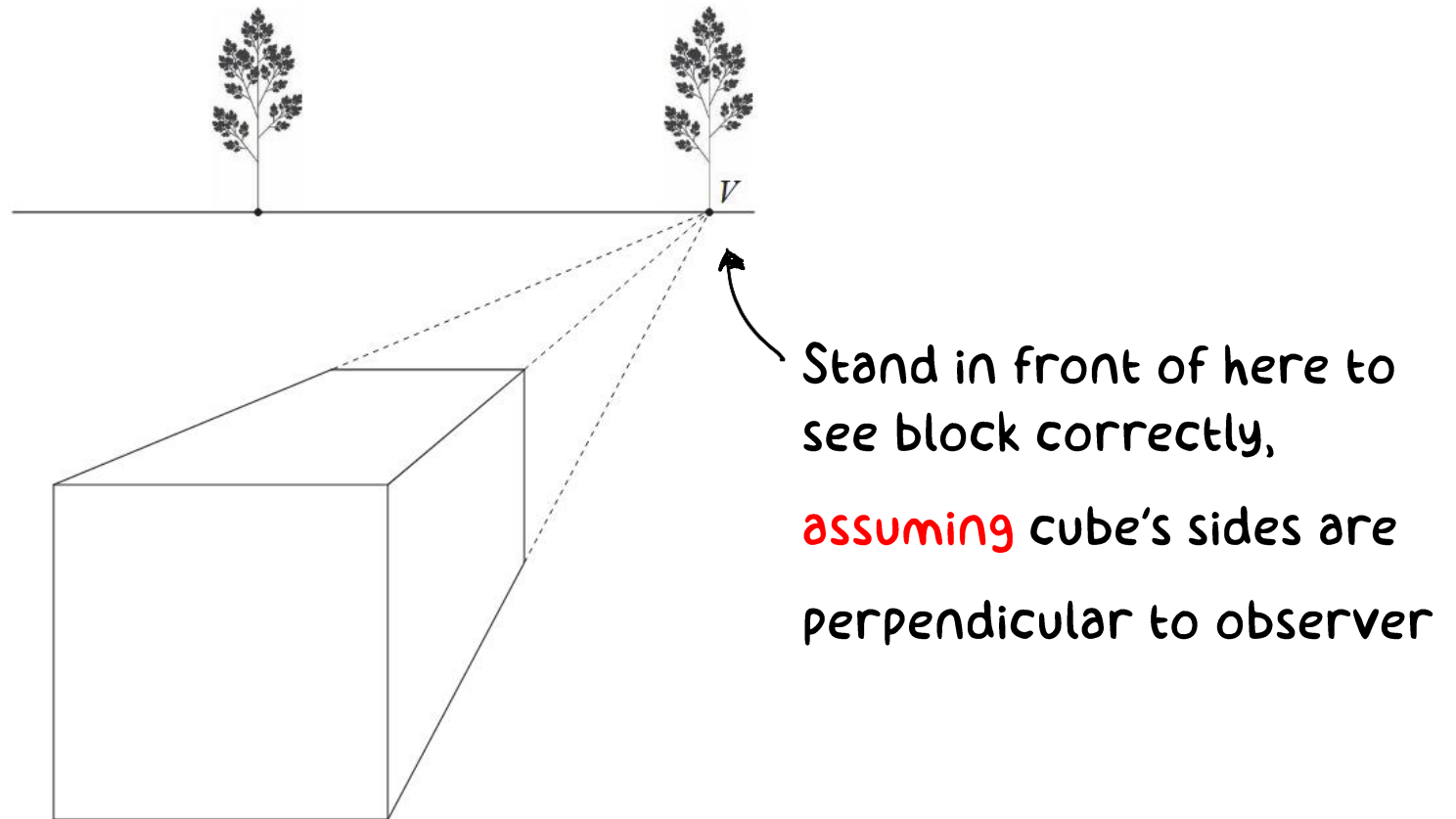
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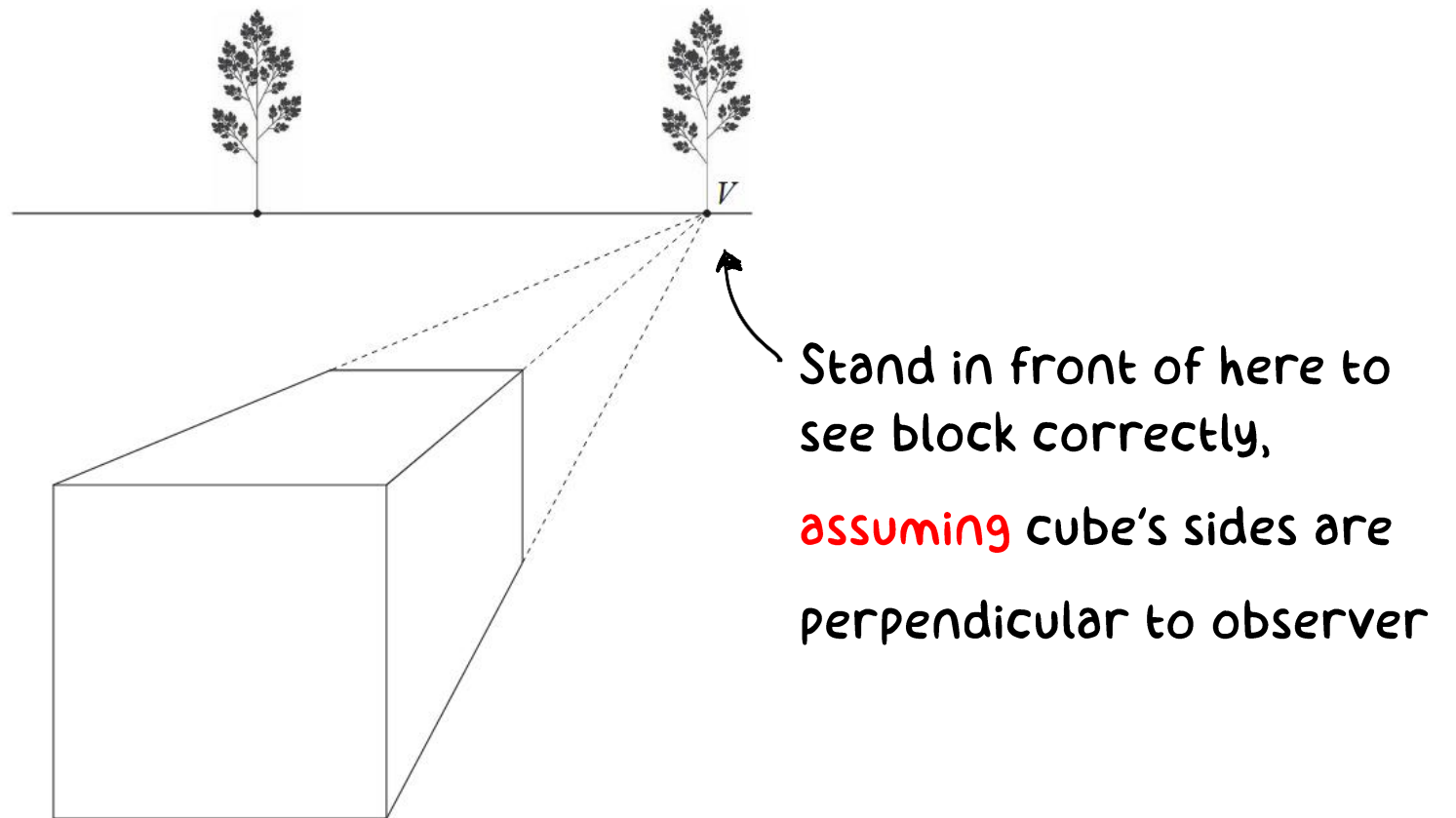
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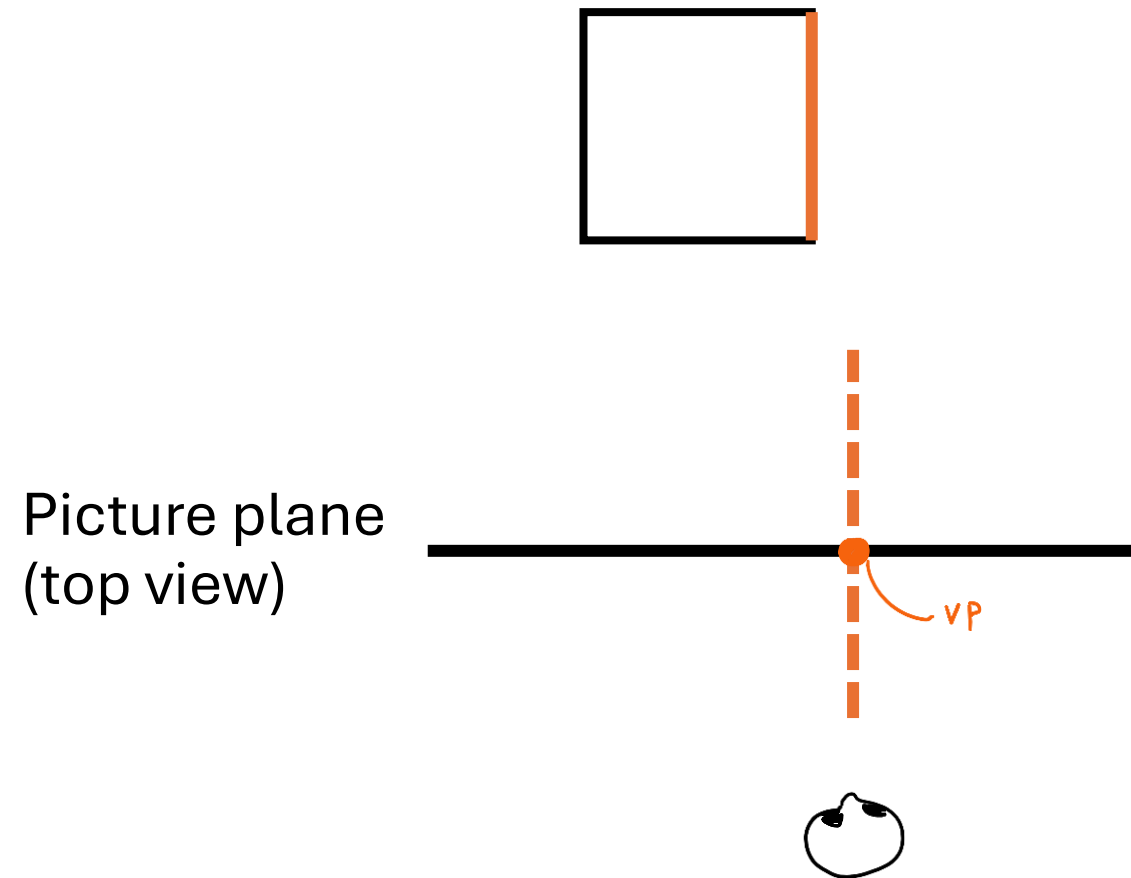
But how close or far to stand?



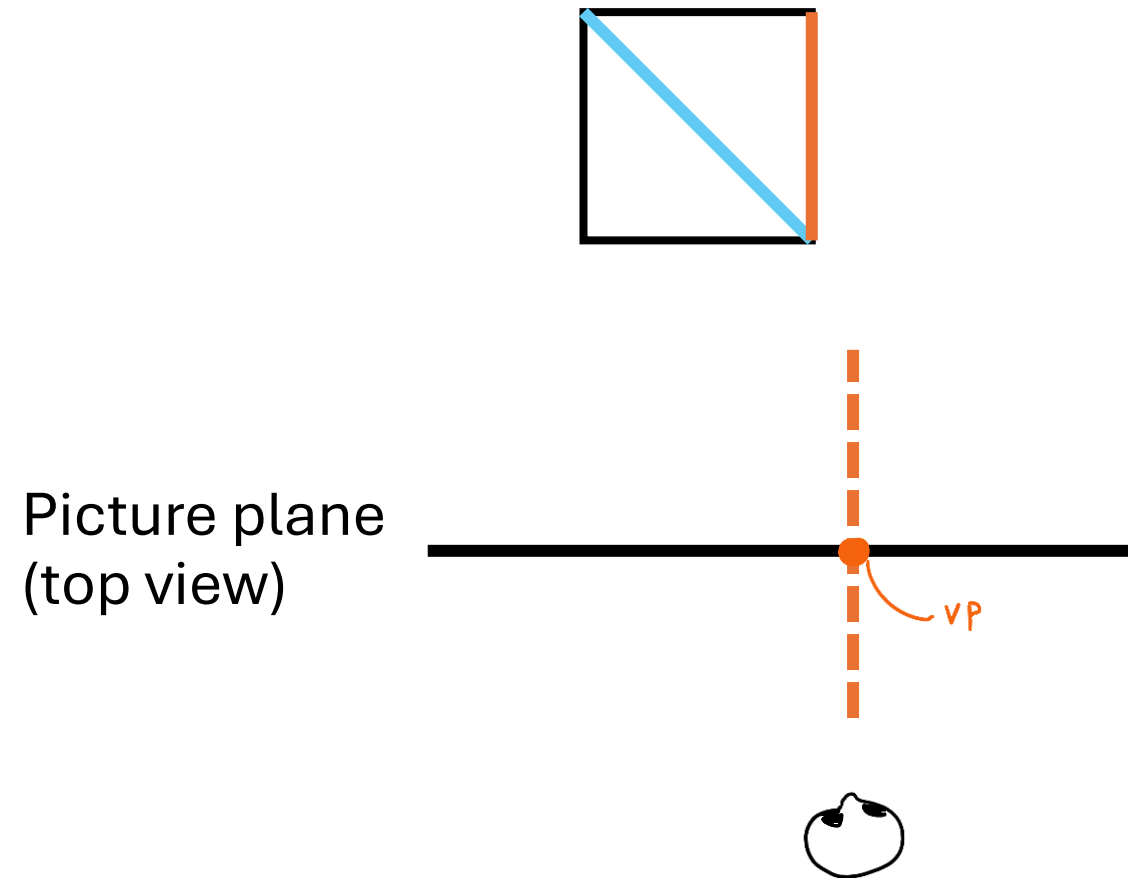
Viewing distance in 1PP

Where to stand to make a rectangular prism look like a **cube**.

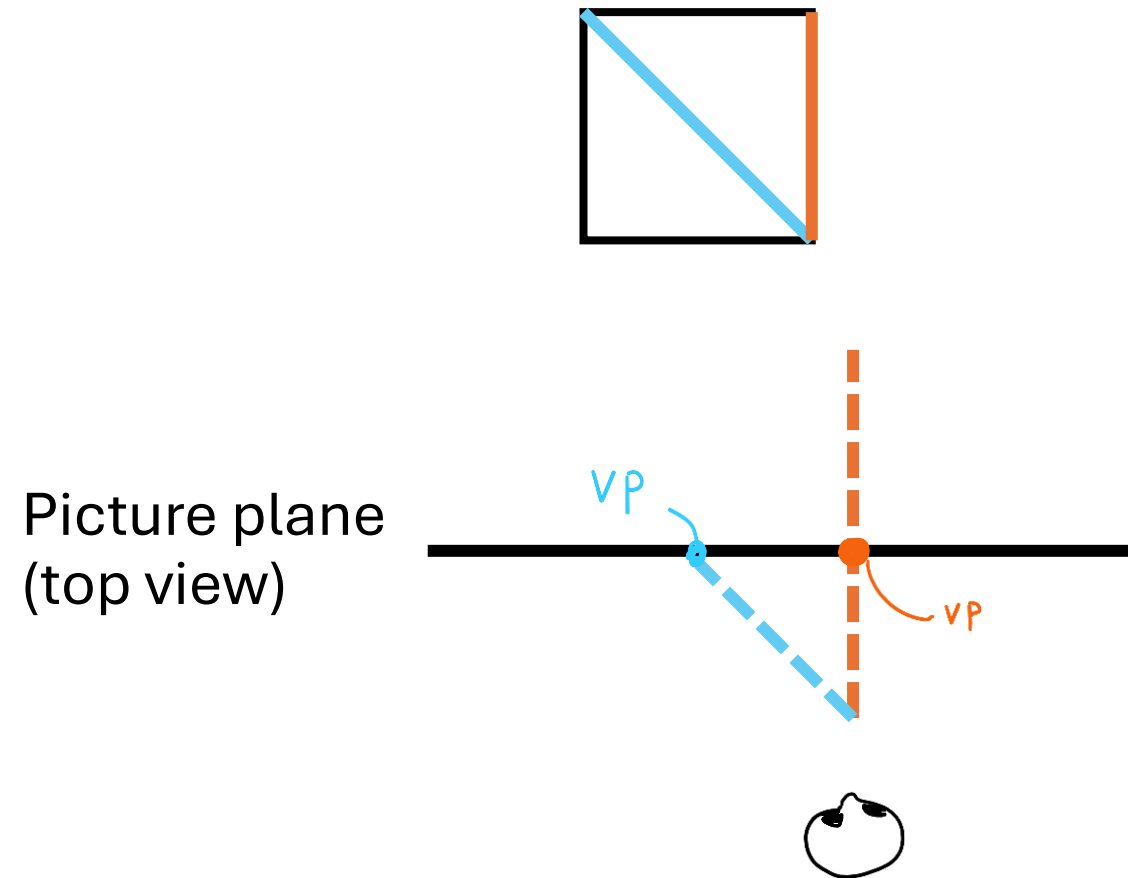
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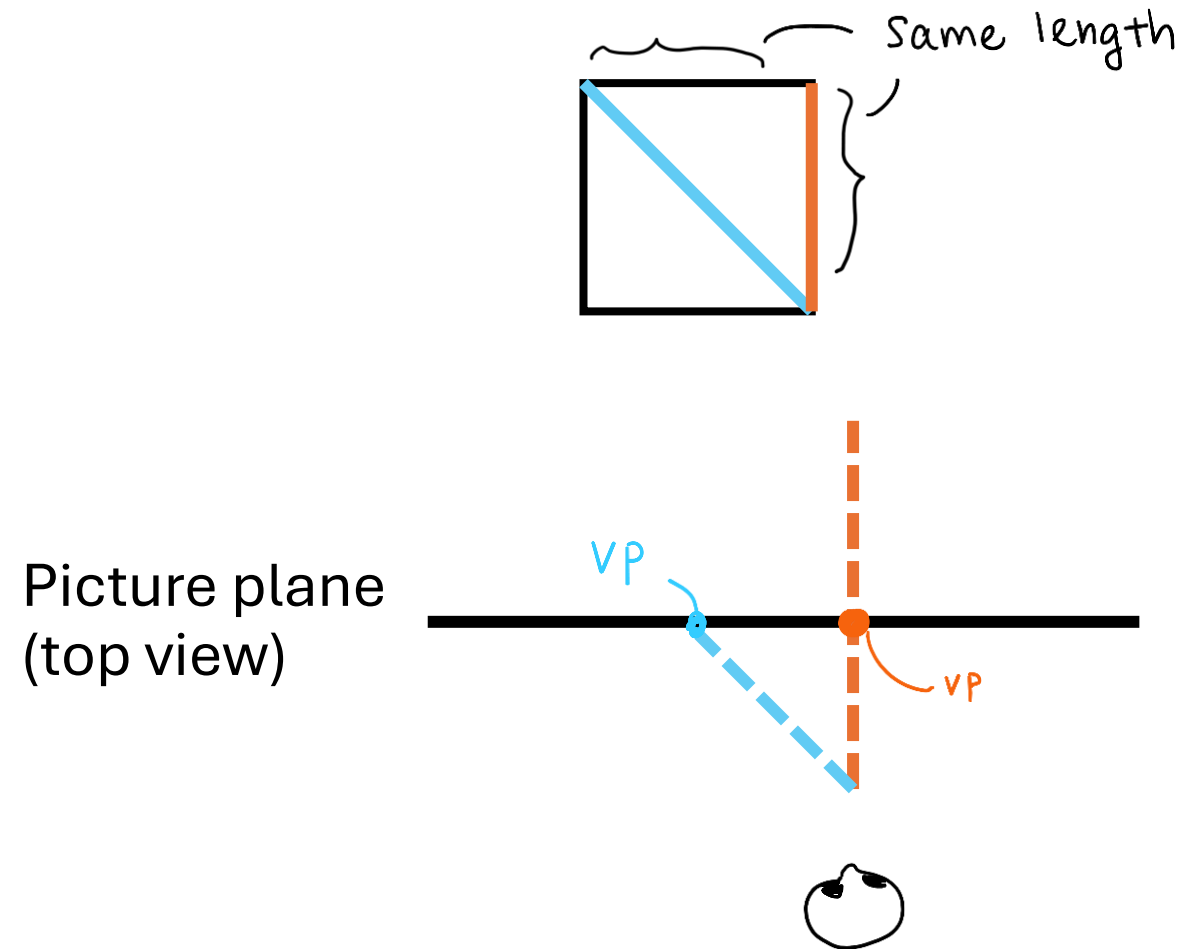
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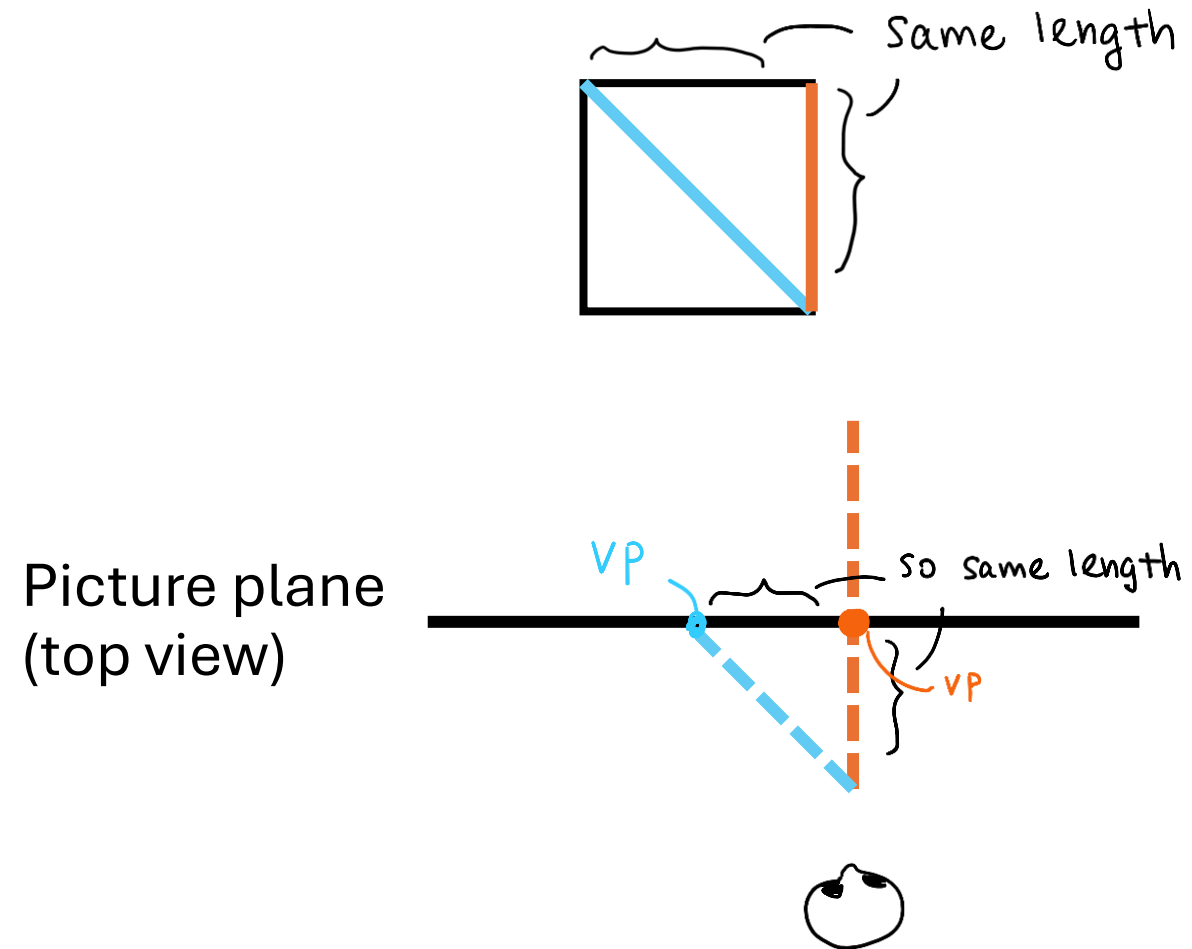
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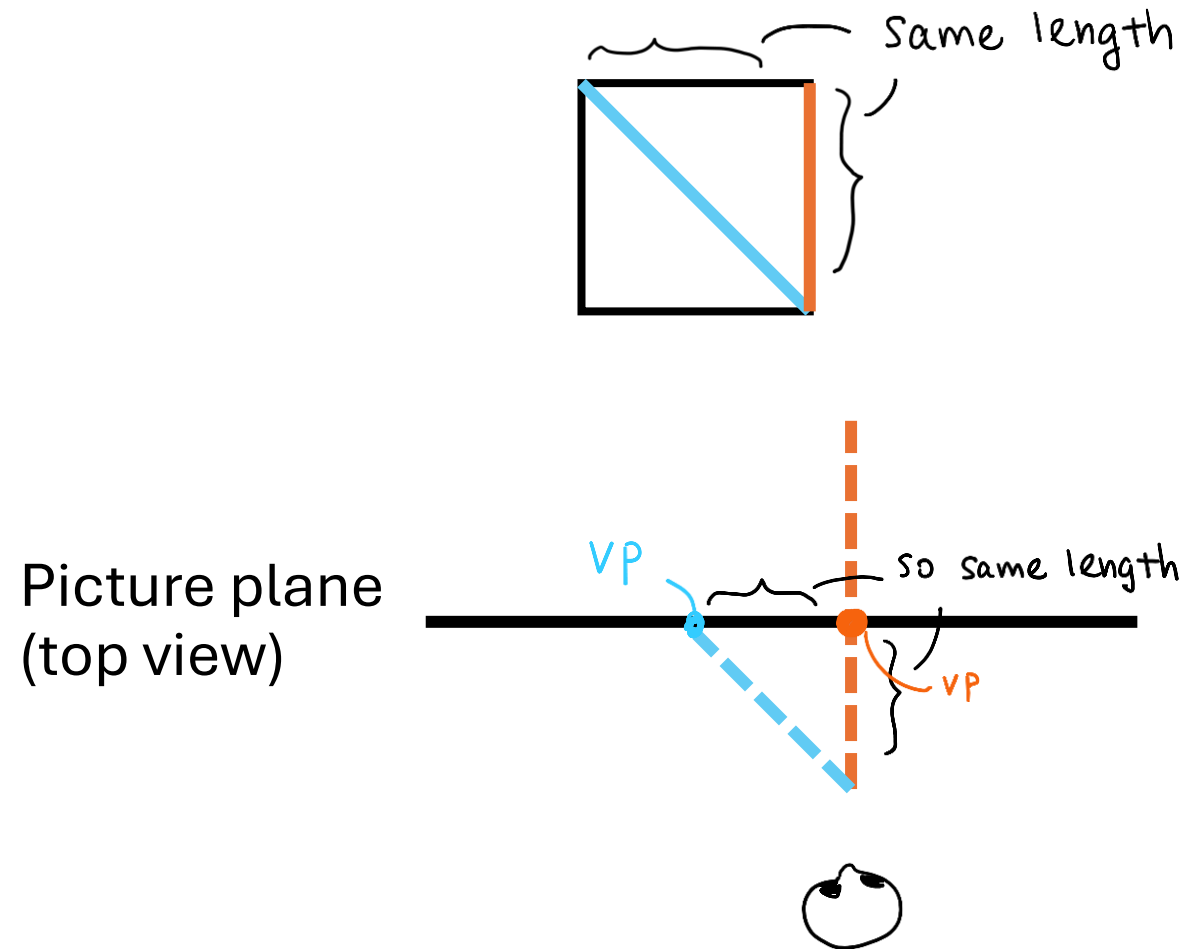
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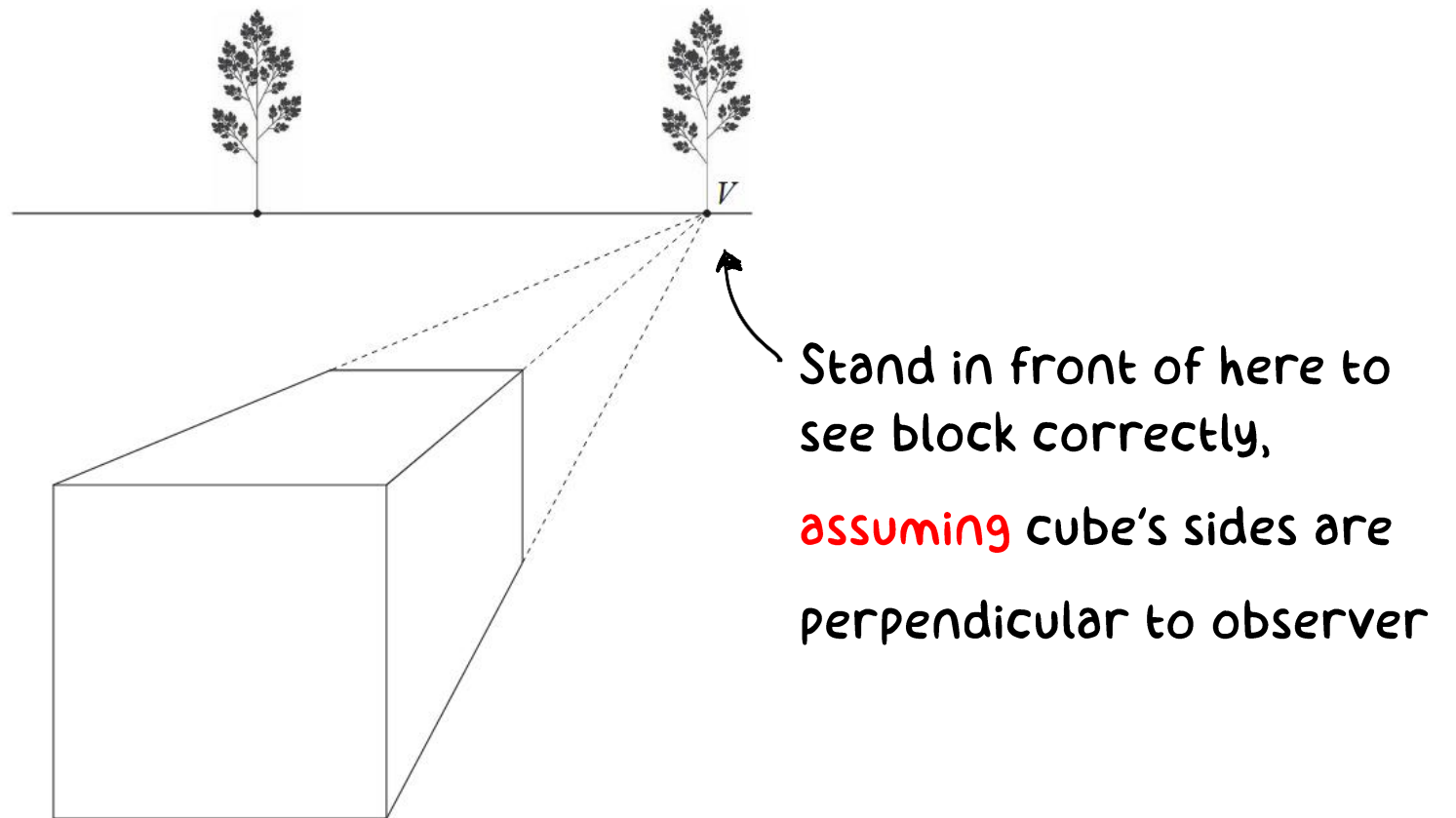
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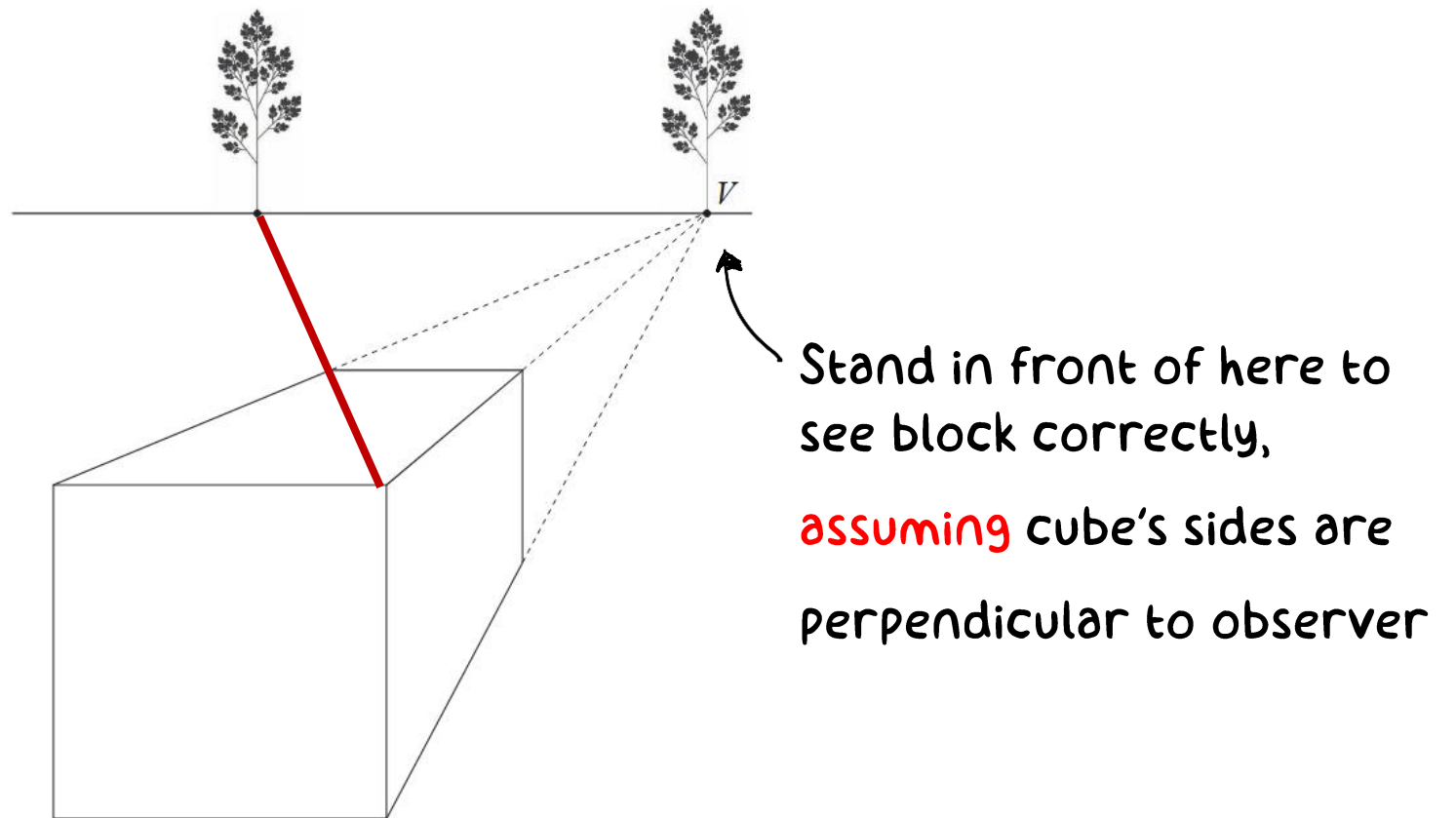
For a square or cube in 1-point perspective:

The viewing distance is the distance from a side's VP to diagonal's VP.

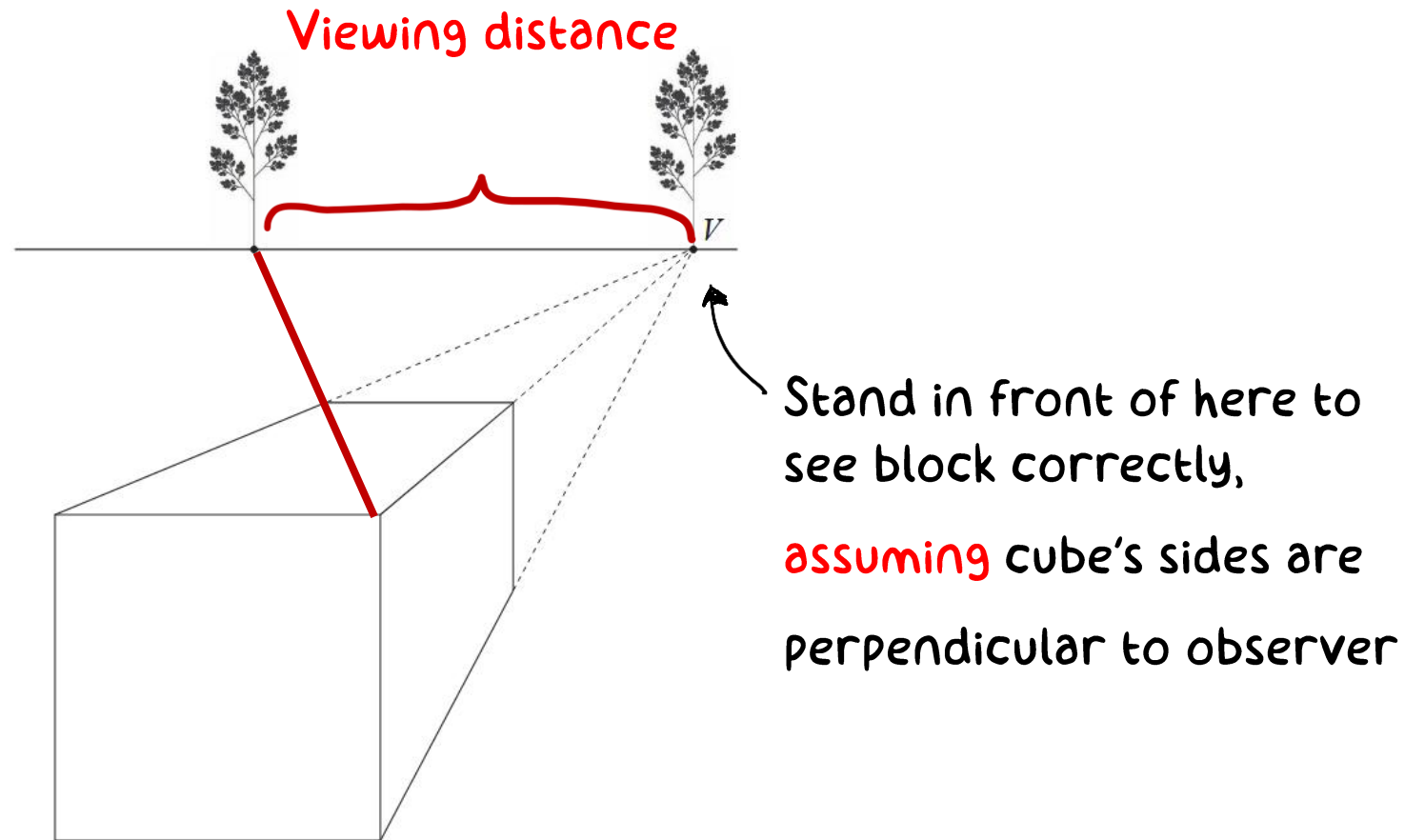
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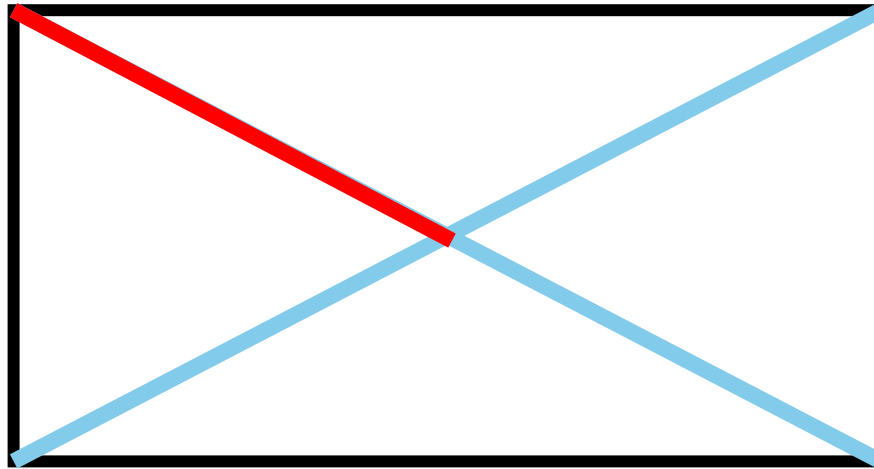
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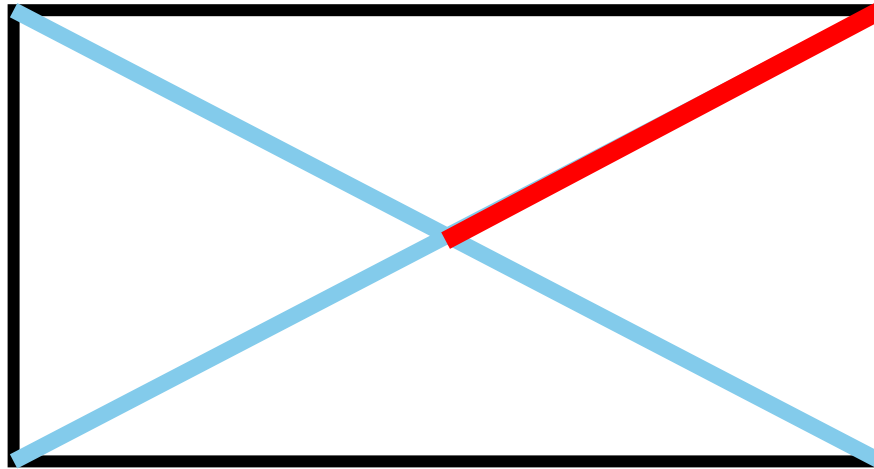
Viewing distance in 2PP

Where to stand to make a shape look like a **rectangular prism**.

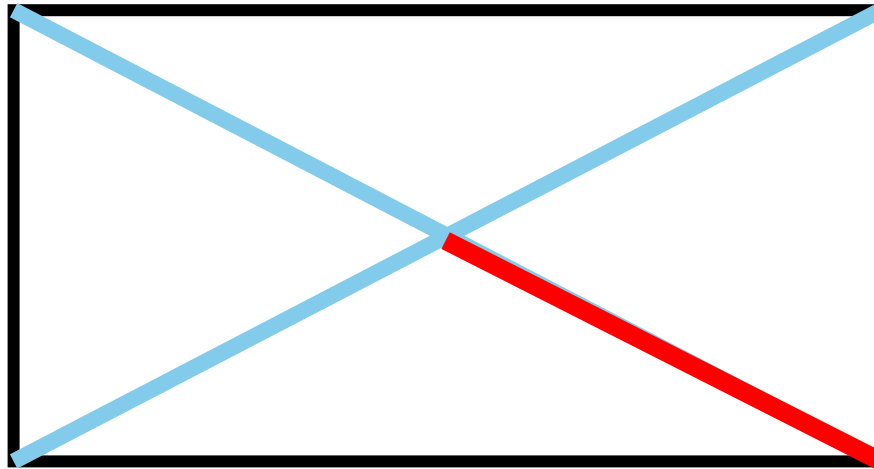
Rectangle Theorem. The diagonal half-lines in a rectangle are the same length.



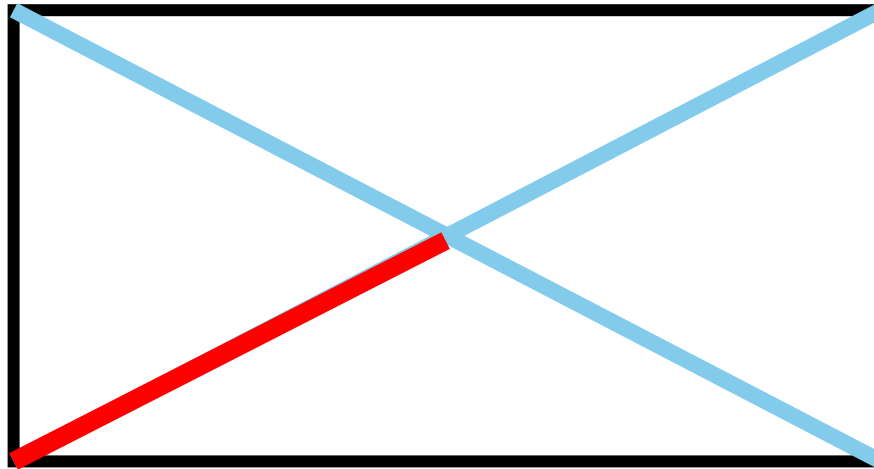
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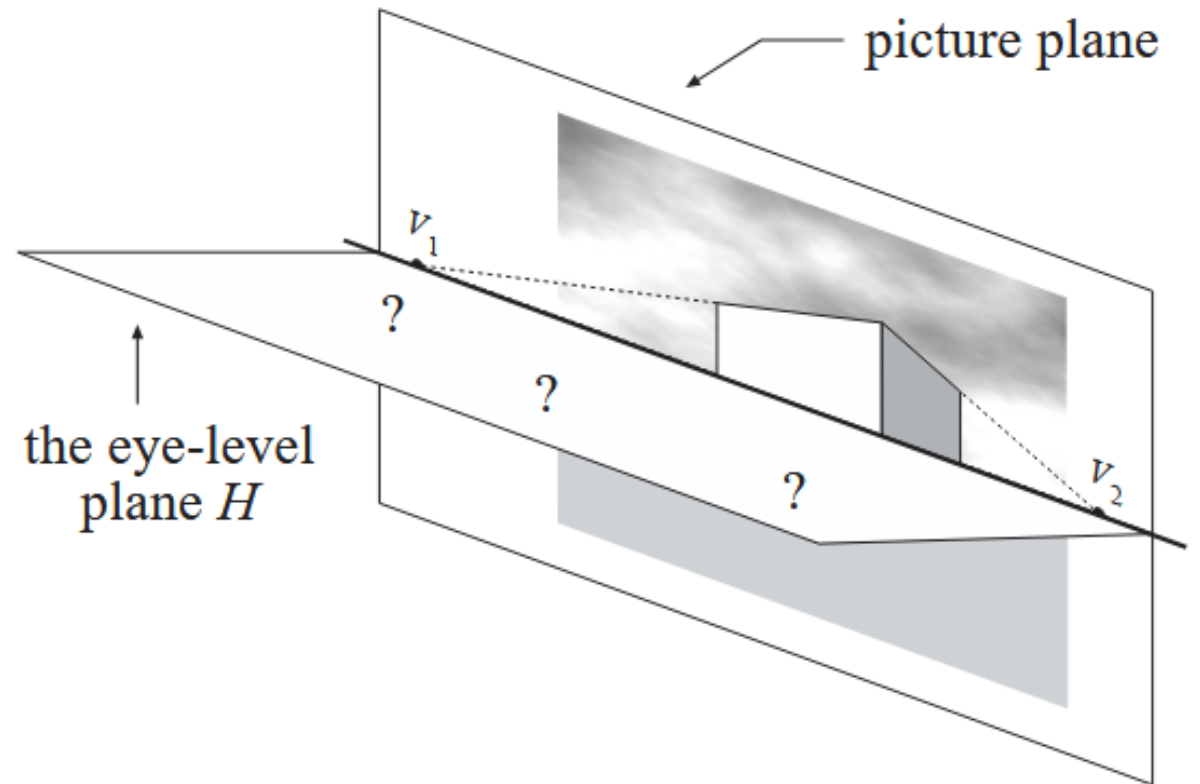


Rectangle Theorem. The diagonal half-lines in a rectangle are the same length.



Assume observer is looking level with ground plane

Figure 5.2. The eye-level plane H . The correct viewpoint for the painting is somewhere in this plane, but where?



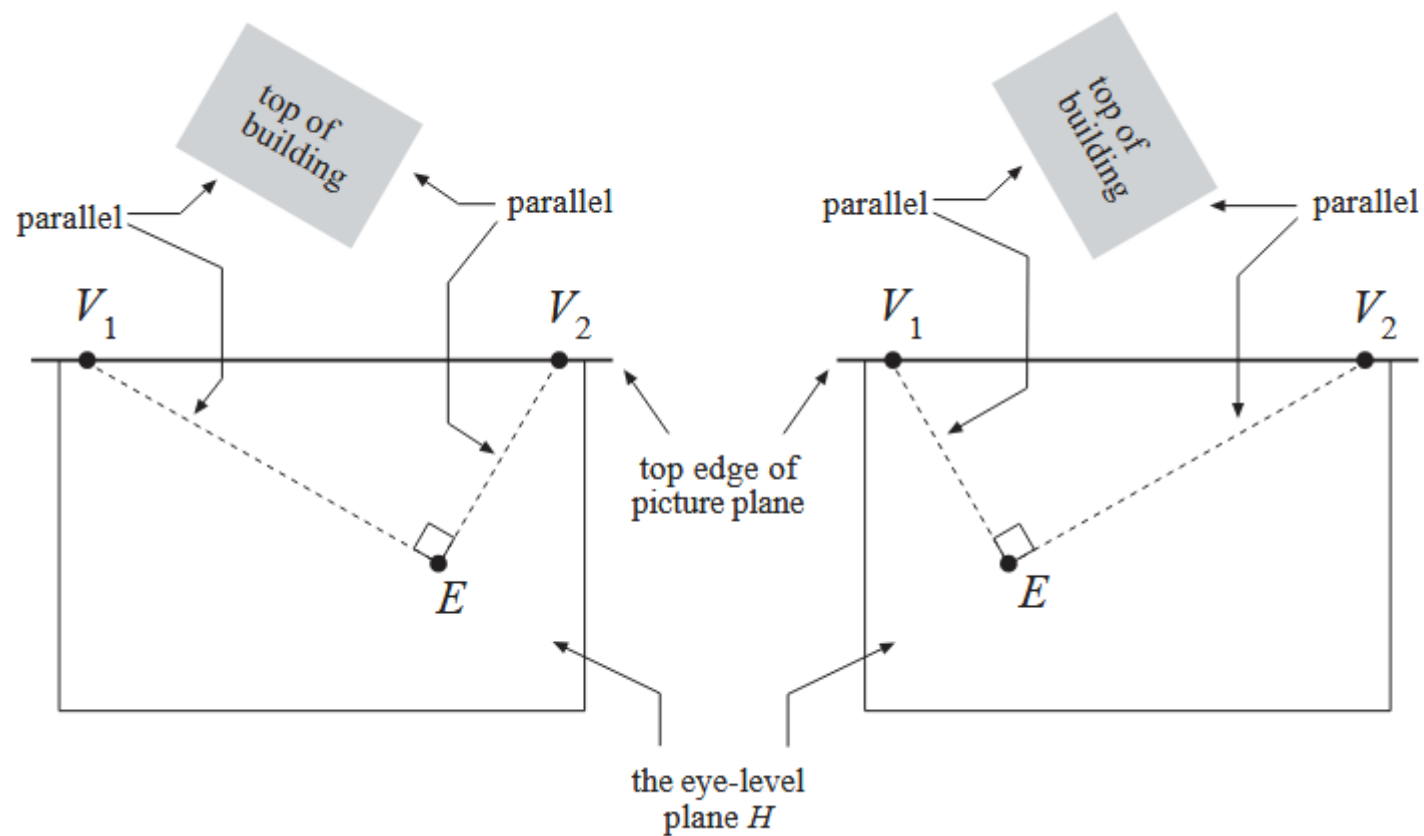


Figure 5.3. Two of many possible locations for the viewpoint E . Because the edges of the building form a right angle, the lines of sight to the vanishing points must form a right angle at the point E .

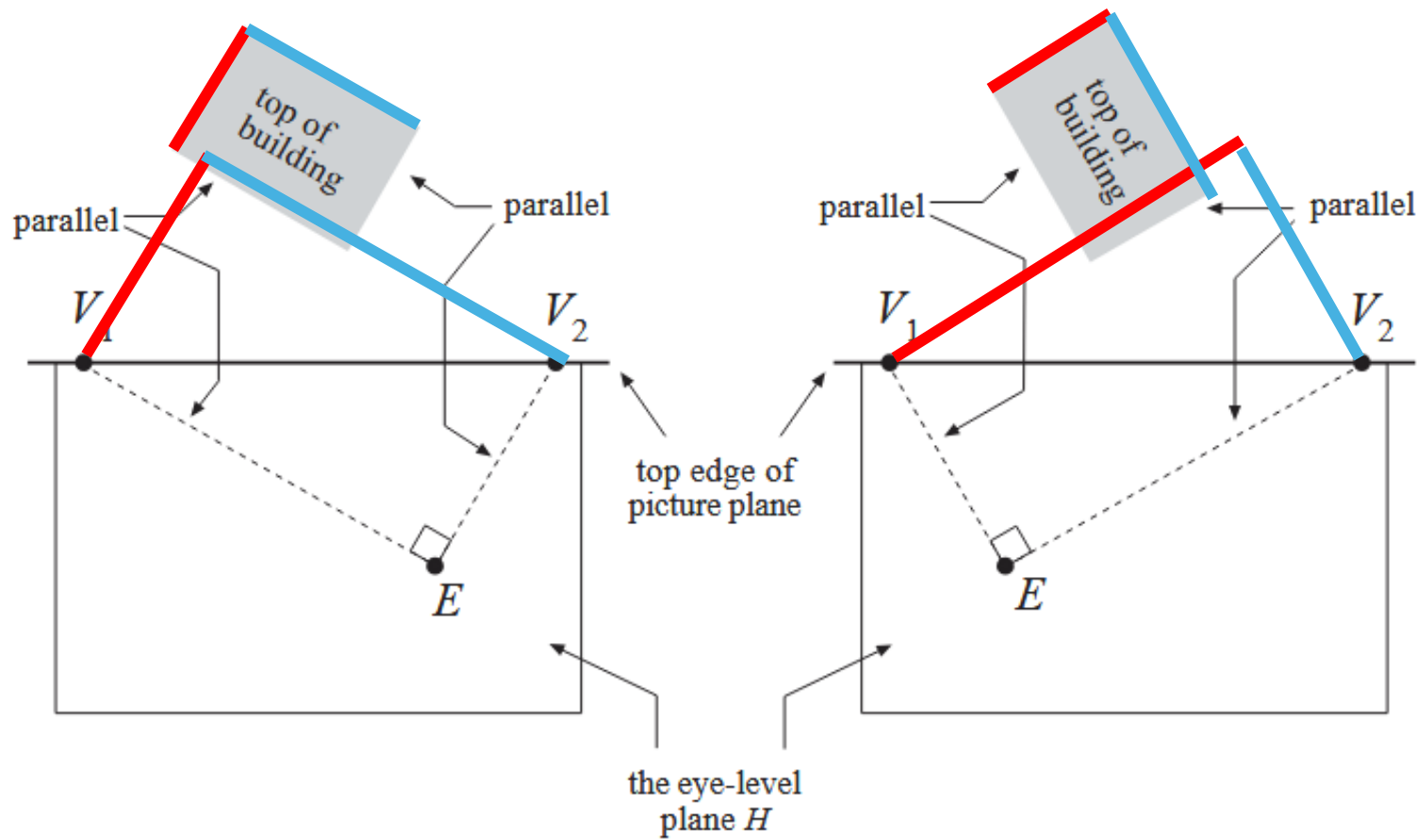


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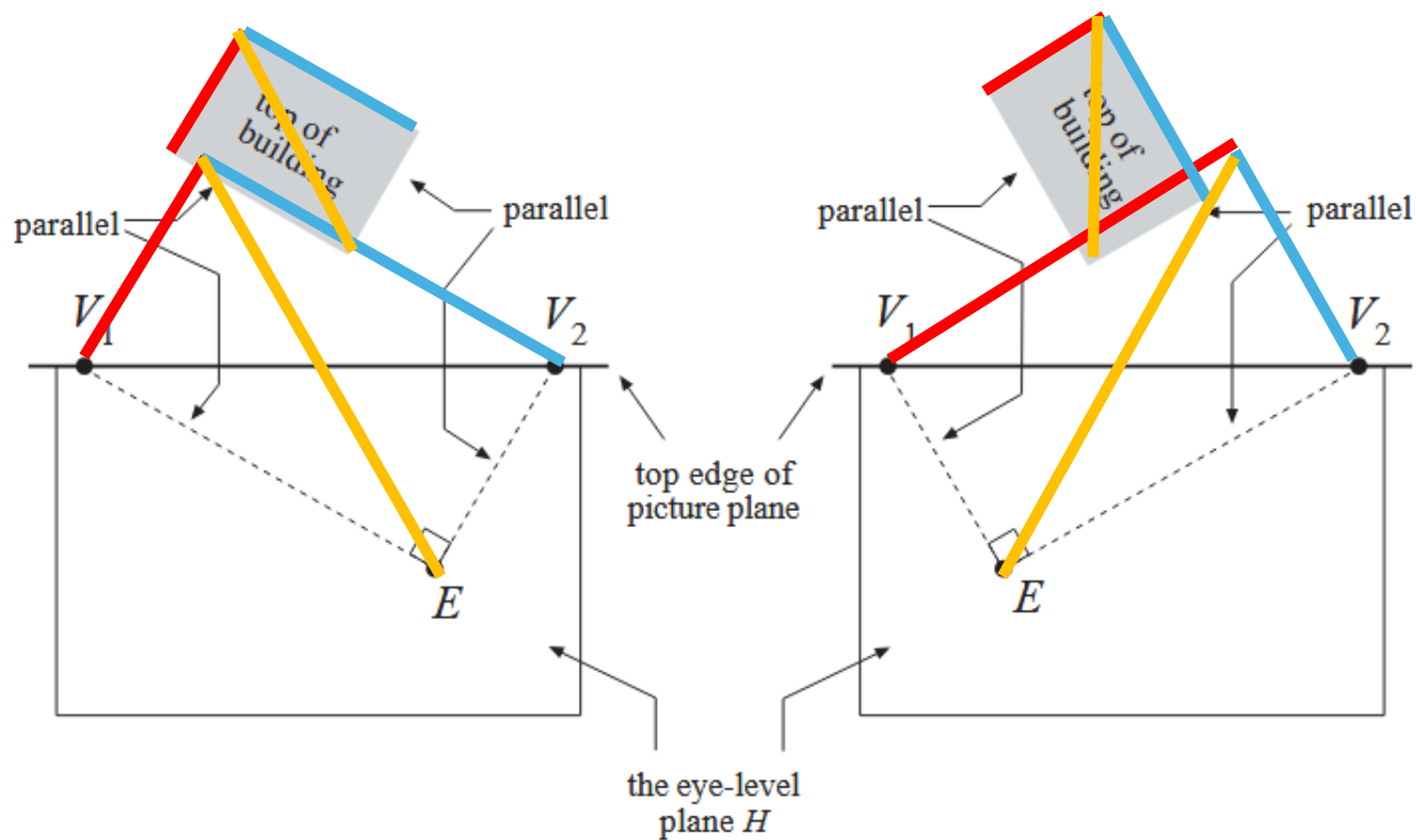
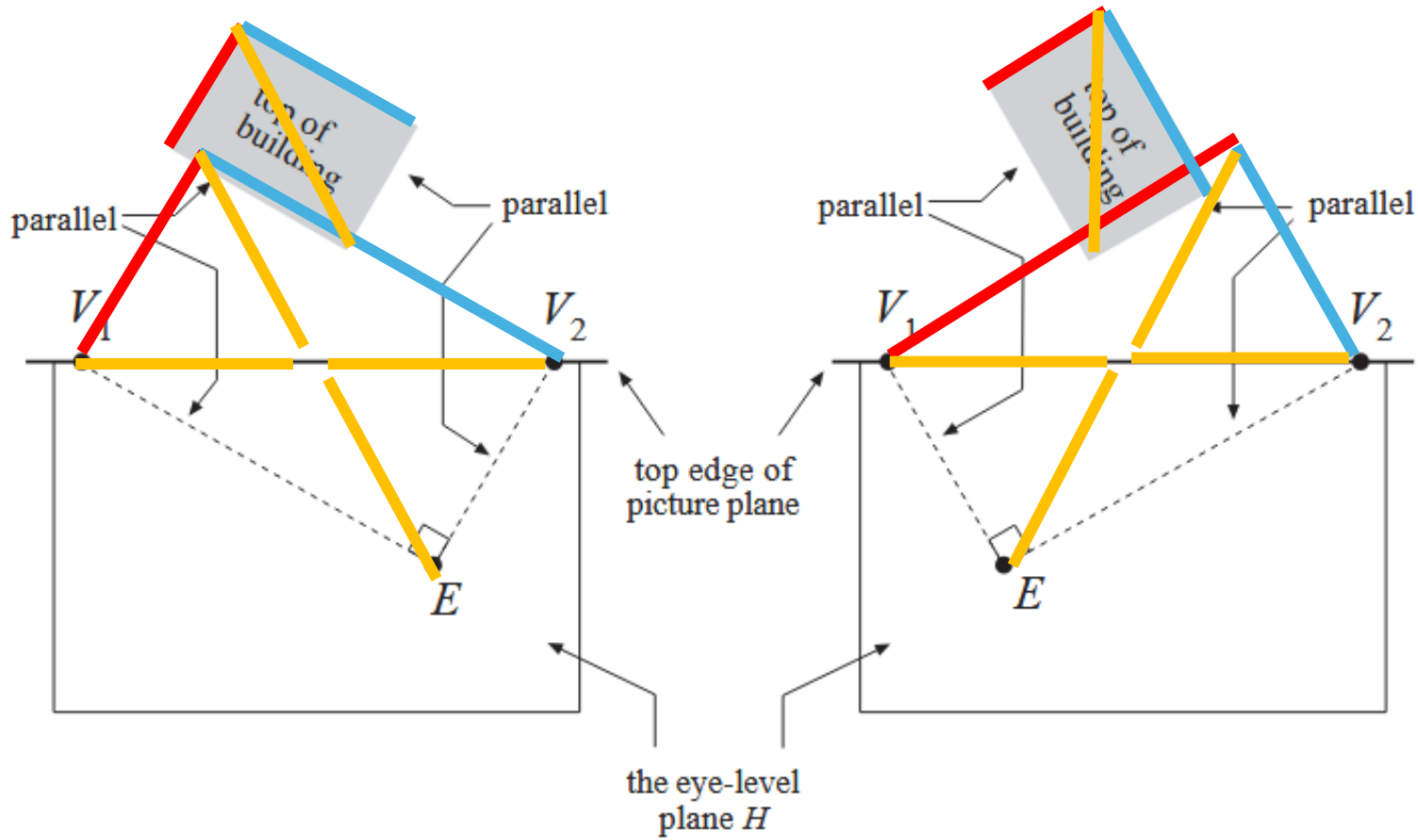
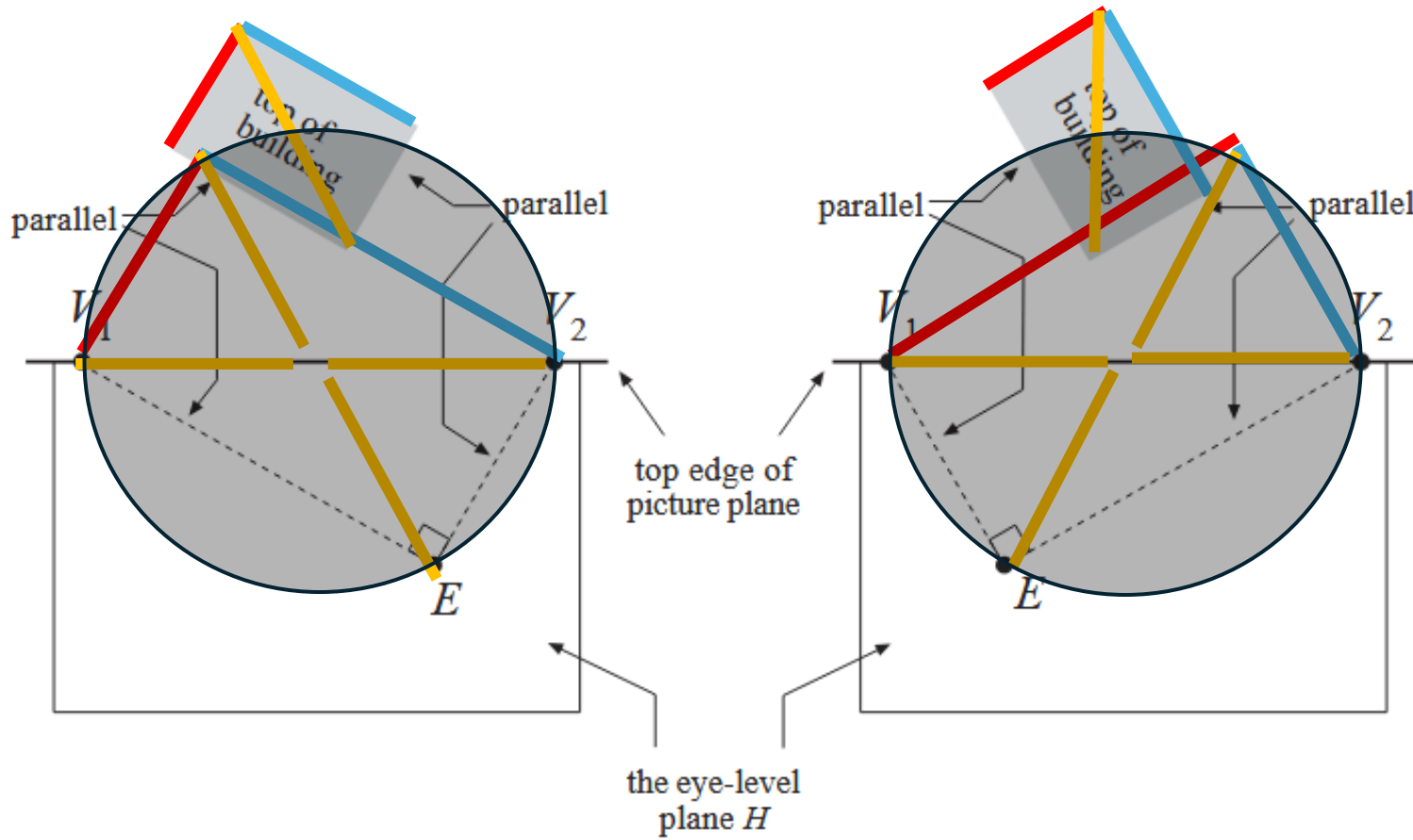


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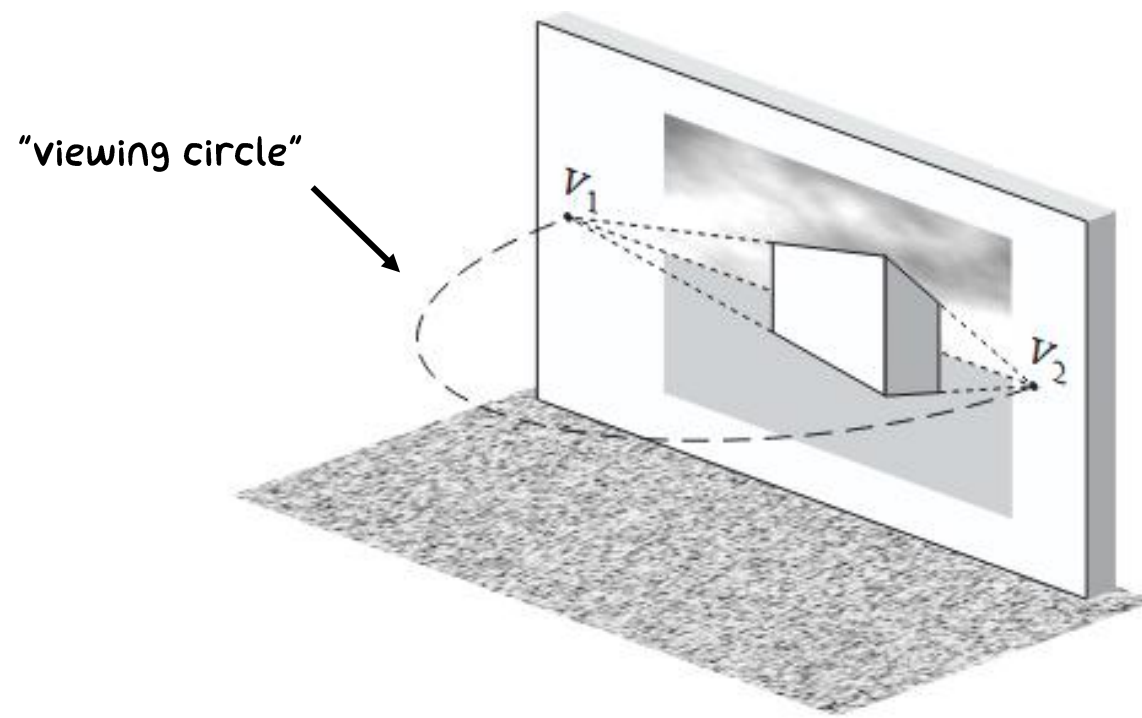
Yellow lines have same length (radius) by Rectangle Theorem.

So observer E is on the circle, the “viewing circle”.

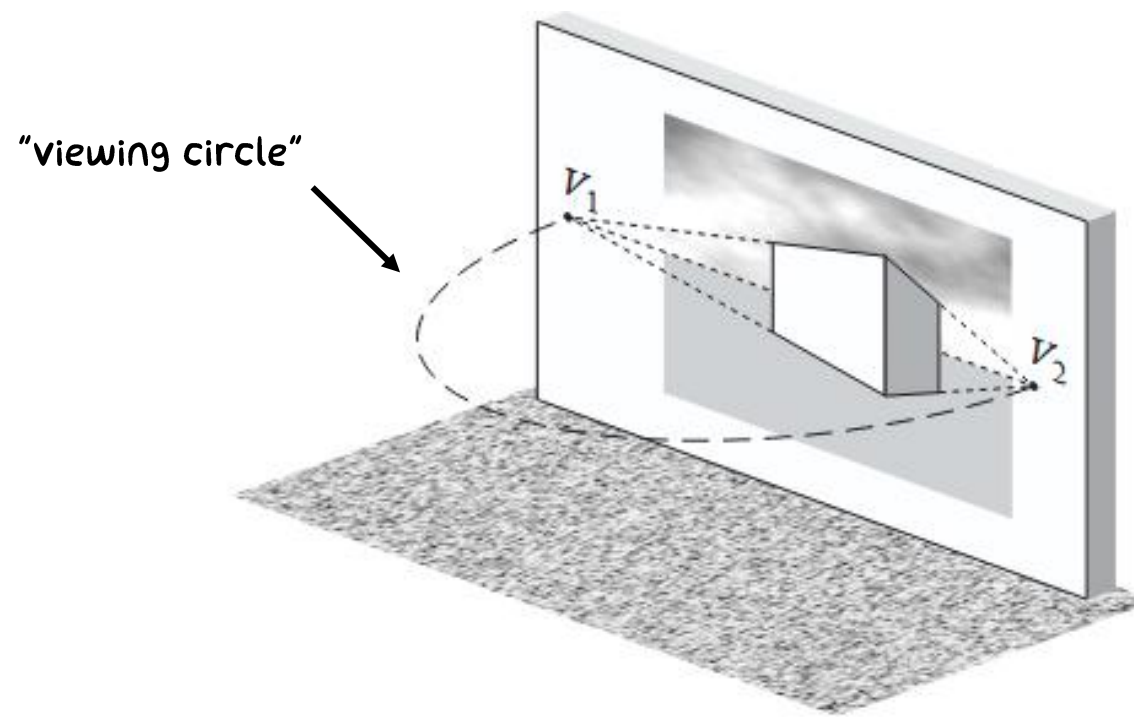


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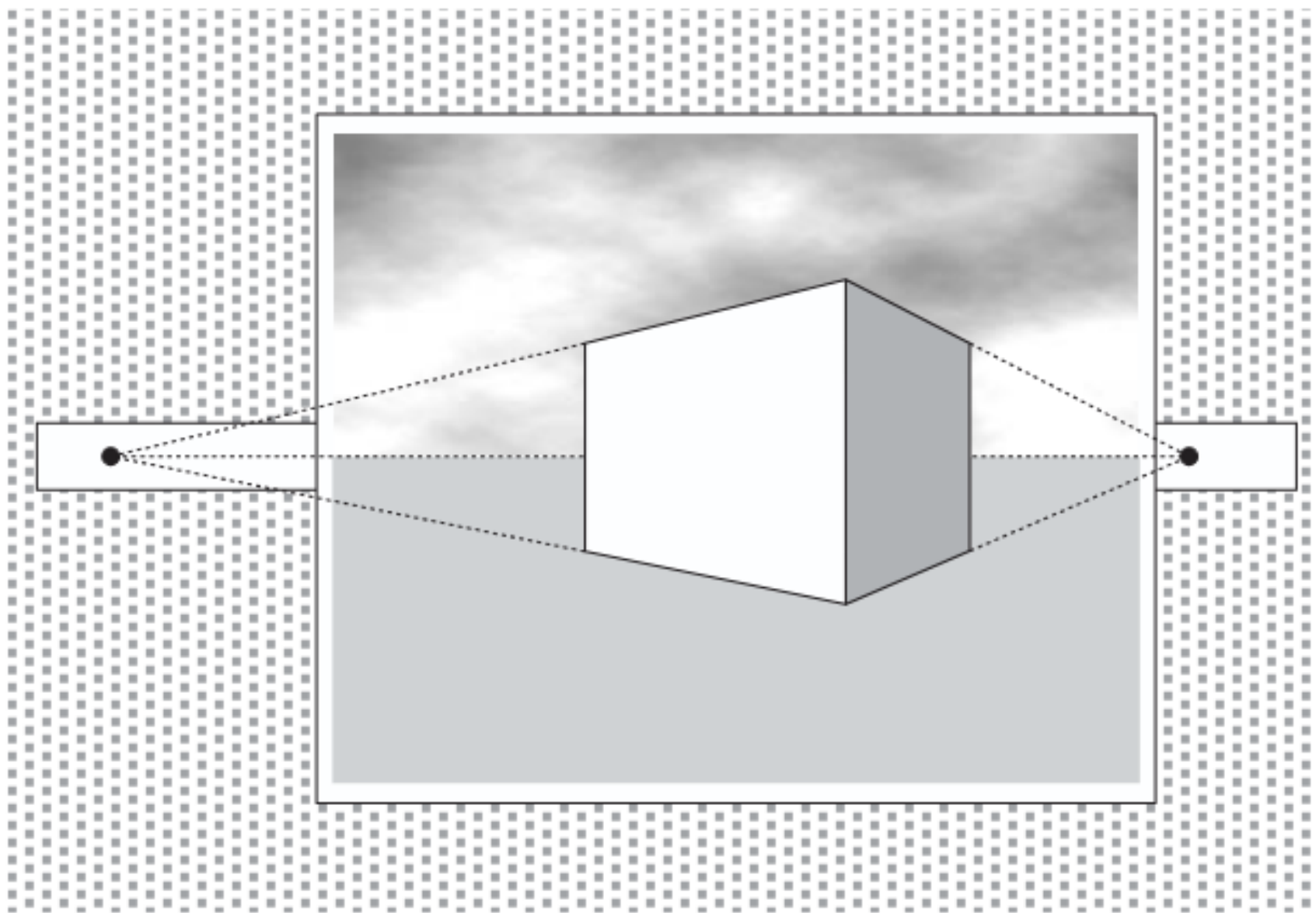
Theorem. The correct perspective viewpoints of a 2-point perspective scene is on the semicircle with endpoints V_1 and V_2 perpendicular to the picture plane.



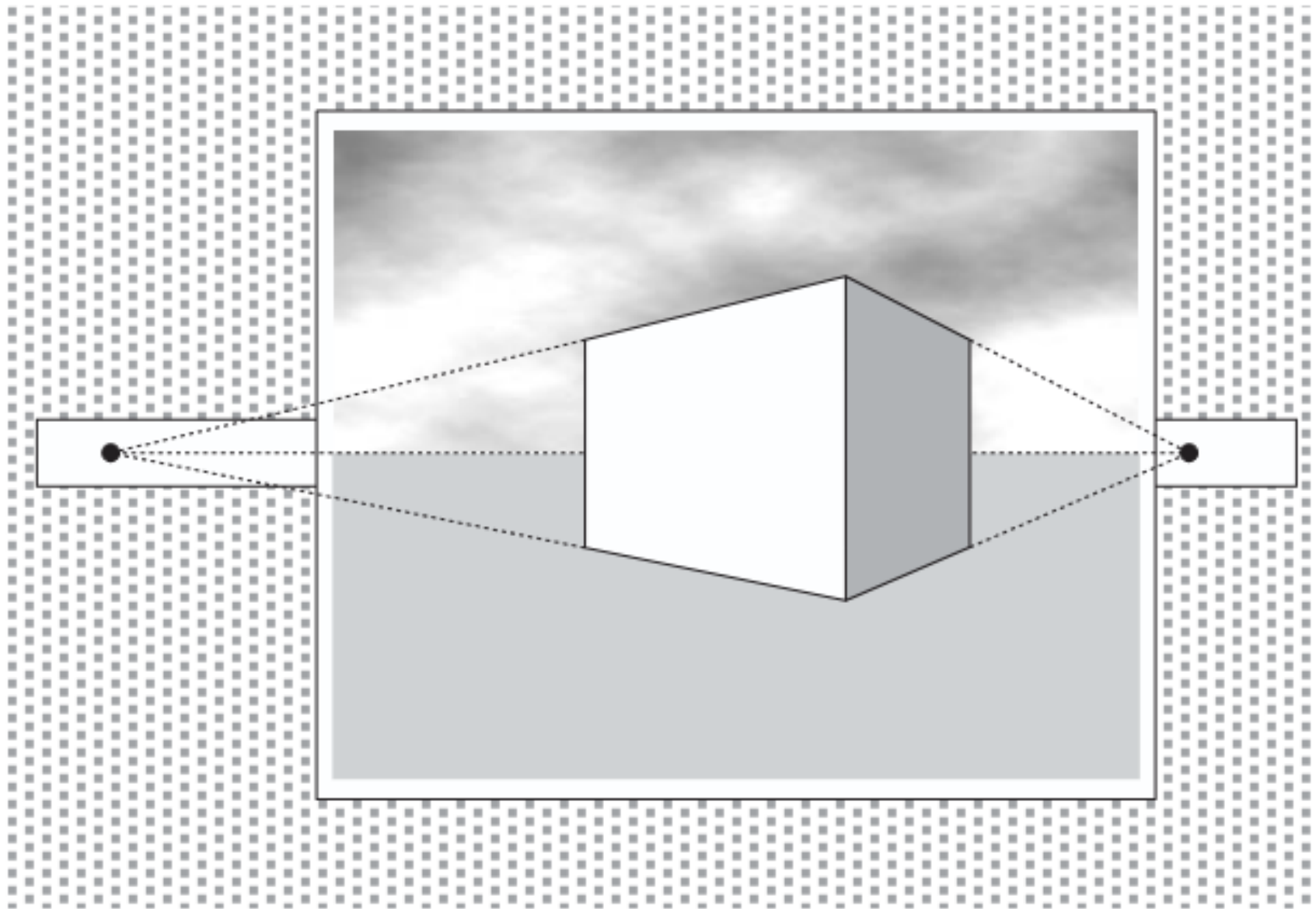
Theorem. The correct perspective viewpoints of a 2-point perspective scene is on the semicircle with endpoints V_1 and V_2 perpendicular to the picture plane.

Notes.

- Otherwise, the block above will not look like a rectangular prism, but instead look like a shape with non-right angle corners.
- There is a special point on the above "viewing circle" that makes the block look like a cube. See Chapter 5 of the referenced textbook at the end of these slides.



Spreading vanishing points far apart ensures a larger viewing circle, one that is more likely to be occupied by the casual viewer's eye.



We'll now find the viewing distance of this block by going to the assignment PDF...

Bibliography

Marc Frantz, Annalisa Crannell, *Viewpoints: mathematical perspective and fractal geometry in art*. Chapters 3–5.