

# Logic

We first finish logic with a final example to help solve Problem 4 from Lesson 20:

## Problem.

"Jo eats pizza or drinks tea (possibly both)" and "If Jo eats pizza, then Jo drinks tea" are true propositions. Describe the truth value of "If Jo does not drink tea, then Jo eats pizza". And more.

## Answer:

Let  $p$  = "Jo eats pizza." and  $q$  = "Jo drinks tea." Then

- "Jo eats pizza or drinks tea (possibly both)" becomes  $p \vee q$  which we assumed to be true.
- "If Jo eats pizza, then Jo drinks tea" becomes  $p \Rightarrow q$  which we also assumed true.
- "If Jo does not drink tea, then Jo eats pizza" becomes  $\neg q \Rightarrow p$ , whose truth value we wish to find.

Recalling the truth tables from the previous lesson, we can write a combined truth table that records  $p \vee q$ ,  $p \Rightarrow q$ , and  $\neg q \Rightarrow p$  simultaneously:

$p$	$q$	$p \vee q$	$p \Rightarrow q$	$\neg q$	$\neg q \Rightarrow p$
T	T	T	T	F	T
T	F	T	F	T	T
F	T	T	T	F	T
F	F	F	T	T	F

Because  $p \vee q$  is assumed true, we can eliminate Row 4.

Because  $p \Rightarrow q$  is assumed true, we can also eliminate Row 2.

Therefore we are left with Rows 1 and 3:

$p$	$q$	$p \vee q$	$p \Rightarrow q$	$\neg q$	$\neg q \Rightarrow p$
T	T	T	T	F	T
T	F	T	F	T	T
F	T	T	T	F	T
F	F	F	T	T	F

In both these cases,  $\neg q \Rightarrow p$  is true. In other words, "If Jo does not drink tea, then Jo eats pizza" is a true statement if we assume that the two statements given to us are true.

## Federal income taxes

In the US, income tax is levied at the following levels:

- federal (the entire country, USA),
- state (except a few states)
- some major metropolitan cities like San Francisco, Philadelphia, etc.

We focus on two types of federal income taxes: federal income tax and FICA. Note that state income tax is mostly structured similarly to federal income tax.

## Terms.

- **Gross income:** all income you received in the year, say, 2025.
- **Taxable income aka adjusted gross income (AGI):** Gross income minus deductions.
- **Itemized deduction:** charitable donations, home mortgage interest, high medical expense, etc. for which you have kept the receipt ("itemized") of such transactions.
- **Standard deduction:** \$15,750 for single filers in 2025, if you do not want to do itemized deduction or if your itemized deductions do not exceed the amount \$15,750. The standard deduction is there to incentivize you to use this option so that the IRS does not need to sort through all your receipts for small deductions.

## Income tax

**Federal income tax.** See table below. US federal income tax is **progressive** in that the first \$11,925 of your taxable income is taxed at a lower rate than the next higher portion of your income (\$11,925–\$48,475), which in turn is taxed at a lower rate than the next higher portion of your income (\$48,475–\$103,350), and so on.

### 2025 federal tax brackets

These brackets and rates apply to taxable income earned during 2025. The deadline for filing 2025 taxes will be April 15, 2026, or October 15, 2026, with an extension.

Tax rate	Single filer	Married filing jointly (or surviving spouse)	Head of household	Married filing separately
10%	\$0 to \$11,925	\$0 to \$23,850	\$0 to \$17,000	\$0 to \$11,925
12%	\$11,926 to \$48,475	\$23,851 to \$96,950	\$17,001 to \$64,850	\$11,926 to \$48,475
22%	\$48,476 to \$103,350	\$96,951 to \$206,700	\$64,851 to \$103,350	\$48,476 to \$103,350
24%	\$103,351 to \$197,300	\$206,701 to \$394,600	\$103,351 to \$197,300	\$103,351 to \$197,300

Screenshot from <https://www.nerdwallet.com/taxes/learn/federal-income-tax-brackets>

## Employee

**Example 1.** What is the federal income tax if you earn a \$130,000 salary? Assume you use the standard deduction.

**Answer.**

$$\text{Adjusted gross income} = 130,000 - 15,750 = 114,250$$

We now use the easier-to-use federal income tax table (Table 3) that calculates the tax burden on the portions of your income from the lower income brackets that "carry over"

TABLE 3 - Section 1(j)(2)(C) – Unmarried Individuals (other than Surviving Spouses and Heads of Households)

If Taxable Income Is:	The Tax Is:
Not over \$11,925	10% of the taxable income
Over \$11,925 but not over \$48,475	\$1,192.50 plus 12% of the excess over \$11,925
Over \$48,475 but not over \$103,350	\$5,578.50 plus 22% of the excess over \$48,475
Over \$103,350 but not over \$197,300	\$17,651 plus 24% of the excess over \$103,350
Over \$197,300 but not over \$250,525	\$40,199 plus 32% of the excess over \$197,300
Over \$250,525 but not over \$626,350	\$57,231 plus 35% of the excess over \$250,525
Over \$626,350	\$188,769.75 plus 37% of the excess over \$626,350

Screenshot from <https://www.irs.gov/pub/irs-drop/rp-24-40.pdf>

Since our adjusted gross income is 114,250, we fall under row 4 of this table. So:

$$\text{Federal income tax} = 17,651 + 0.24 \cdot (114,250 - 103,350) = \boxed{\$20,267}$$

## FICA (Federal Insurance Contributions Act)

For an employee, a 7.65% tax is applied to your **gross income**, of which 6.2% funds **social security** and 1.45% funds **Medicare**. Another identical 7.65% tax is paid by the employer, not taken from your stated salary.

**Example 2.** How much do you pay for FICA in Example 1?

$$0.0765 \times \text{Gross income} = 0.0765 \times 130,000 = \$9945.$$

## Part-time worker

Same tax situation as before. But if you work multiple jobs, then the withholding each employer makes on your behalf may assume a standard deduction being applied. Since you cannot double dip when using standard deduction, you may need to adjust your withholding with your employers. **Withholding** means tax that is automatically deducted from your paycheck and sent directly to the tax agencies (federal, state, etc.).

## Independent contractor

An independent contractor (aka self-employed worker) that earns enough money must pay taxes quarterly, and the federal income tax and FICA tax computation is given in Schedule SE (<https://www.irs.gov/pub/irs-pdf/f1040sse.pdf>).

- You can deduct more things like business expenses, contribution to retirement, etc. to lower your taxable income.
- You also pay the employer-half of the FICA contribution but this portion is used to adjust your taxable income: see Schedule SE line 13 for this deduction.
- FICA in the self-employment context is called "self-employment tax", and only applies to 92.35% of your gross income. (Here gross income means the income you get or "pay yourself" after you deduct business expenses. For instance, if you are self-employed and work from home, then your Wi-Fi bill or work laptop costs are deductible under business expenses.)
- As a general rule of thumb, contractors should charge 25%-40% higher than employed role for the same line of work to make up for costs and due to lack of typical employer-provided benefits.

**Example.**

I earn \$50,000 gross income as a contractor in the 2025 tax year and choose to use the standard deduction option. How much do I pay in federal income tax and FICA? Assume I have no deduction for business expenses, etc.

**Answer:**

FICA:

$$\begin{aligned} \text{Gross income} &= 50,000 \\ \Rightarrow \text{FICA} &= 0.153 \times 0.9235 \times \text{Gross income} = 0.153 \times 0.9235 \times 50,000 \approx 7064.78 \end{aligned}$$

Half of this FICA is allowed to lessen your taxable income:

$$\text{Adjusted gross income} = 50,000 - 15,750 - 7064.78/2 = 30717.61$$

Federal income tax is

$$0.10 \cdot 11,925 + 0.12 \cdot (30717.61 - 11,925) \approx 3447.61.$$

Total tax paid to IRS is:

$$\text{Federal income tax} + \text{FICA} = 7064.78 + 3447.61 = \$10512.39$$