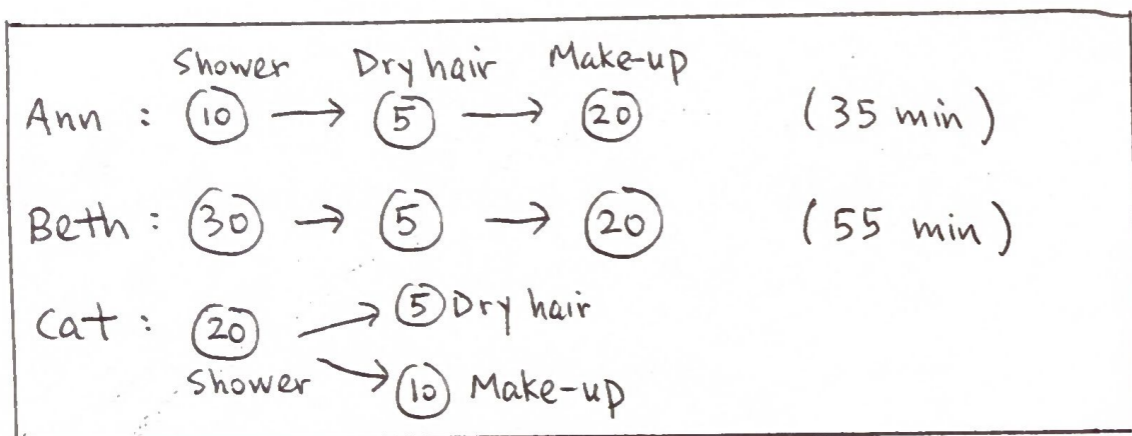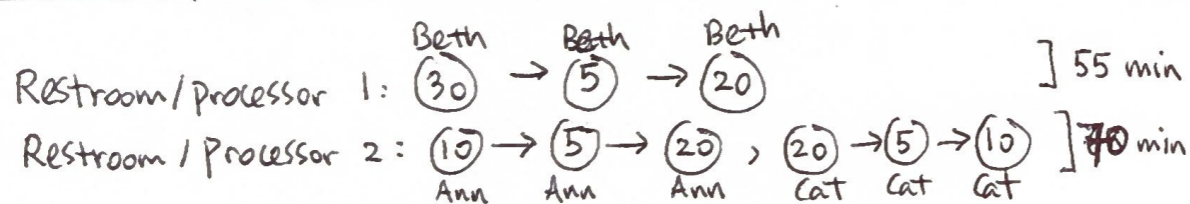# 10 Scheduling

Ann, Beth, Cat share two bathrooms (the processors). Their order-requirement digraph

(≡ directed graph of which tasks must go first)

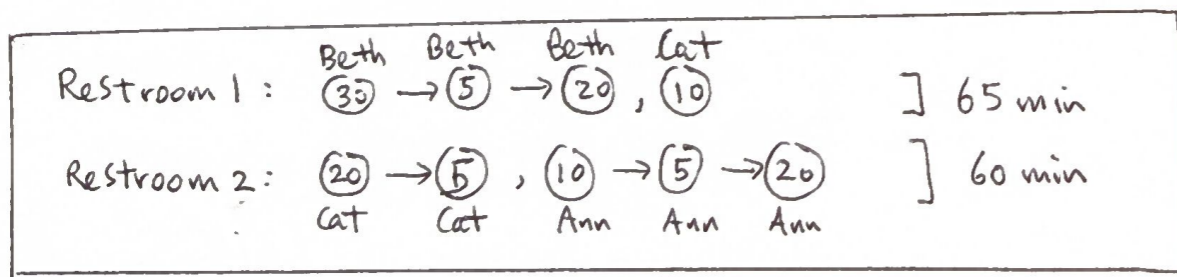is given below in minutes:

```
           Shower   Dry hair   Make-up
Ann :       (10) →    (5)  →    (20)          (35 min)

Beth :      (30) →    (5)  →    (20)          (55 min)

Cat :       (20) →  (5) Dry hair
            Shower  ↘ (10) Make-up
```

So let's put Beth in her own bathroom:

```
                 Beth      Beth      Beth
Restroom/processor 1:  (30) →  (5) →  (20)        ] 55 min

Restroom/processor 2:  (10)→ (5)→ (20) , (20)→(5)→(10)  ] 70 min
                        Ann   Ann   Ann    Cat  Cat  Cat
```

But Cat is flexible, so let's put part of her schedule in Processor 1:

```
              Beth  Beth  Beth  Cat
Restroom 1 :  (30) → (5) → (20) , (10)         ] 65 min

Restroom 2 :  (20) → (5) , (10) → (5) → (20)   ] 60 min
              Cat   Cat   Ann   Ann   Ann
```

So now both bathrooms are done being used by 65 mins.

Definition Critical path of an order-requirement digraph is its longest path; in above example: Beth's.

Note overall tasks completion time ≥ overall time of critical path.
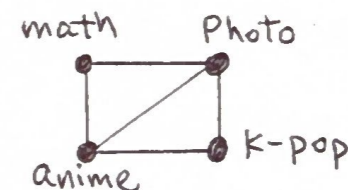

Scheduling Conflicts

Problem Table below shows with "X" which pairs of clubs share members:

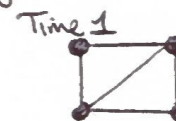|	Math	Anime	Photo	K-Pop
Math		X	X	
Anime	X		X	X
Photo	X	X		X
K-Pop		X	X	

(a) Graph the above info:

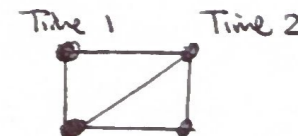vertex = club.
edge = common members (scheduling conflicts)

(b) At least How many different meeting times are needed for all club members to attend all club meetings?
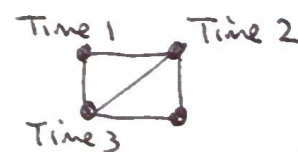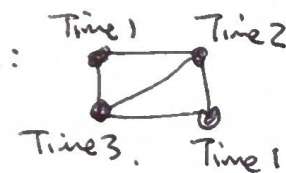
First ~~make~~ put Math at Time 1 :

But now Photo can't be at same time :

Now Anime can't be at Times 1 & 2 :

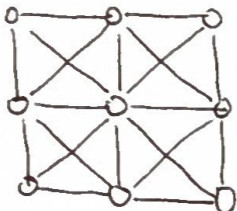
And K-pop can't be at Times 2 & 3) So Time 1 :

Schedule :

Time 1	Math, K-Pop
Time 2	~~Anime~~ Photo, ~~Photo~~
Time 3	Anime

We can think of such scheduling conflicts as a
vertex coloring problem, where we try to color endpoints of
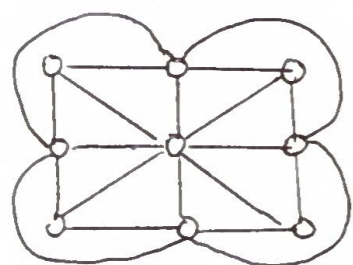the same edge by different colors, so

Black ○——————○ white
                              is one such coloring of the vertices.
Red ●   ● Black

useful fact: Four Color Theorem (proved in 1995):
Any graph whose edges don't overlap can be colored
by ≤4 colors.

Ex   The graph   seems to require
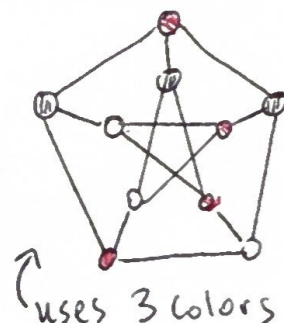
many colors to   color its vertices.

But bending some of its edges out, like so:

, shows that the graph can also be
drawn in a way where its edges don't
cross, so we can color the vertices of
this graph with ≤4 colors.

More examples   Use as few colors to color the vertices of
the graphs below, so that endpoints of the same edge
receive different colors.

(a)   (b)

↑ uses 3 colors   ↑ uses 2 colors