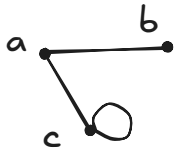
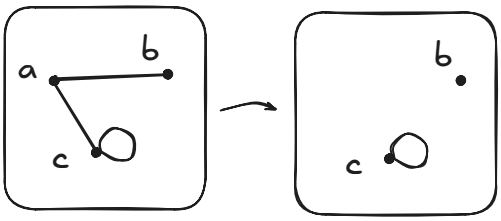
Lesson 7 — Graphs

A **graph** consists of **vertices** (dots) and **edges** (curved lines) connecting the vertices. A **loop** is an edge from a vertex to the same vertex. For instance, the graph with vertices a, b, c and edges $\overline{ab}, \overline{ac}, \overline{cc}$ can be draw as follows.



The **degree** of a vertex v counts the number of times v is listed as an endpoint of an edge. So a loop with endpoint v contributes $+2$ to the degree of v, while a non-loop edge incident to v contributes $+1$ to the degree of v. In the above graph, $\deg(a) = 2$, $\deg(b) = 1$, $\deg(a) = 3$.

A vertex v of a graph is a **cut-vertex** if the removal of v and its incident edges disconnects the graph into more separated parts. In the above graph, removal of the vertex a and its incident edges $\overline{ab}$ and $\overline{ac}$ produces the graph where c is no longer connected to b by traversing the edges.



Data set

The problems that follow in this assignment will refer to the following figures.

Figure 1: Marriages of prominent Florentine families in the early 15th century

Peruzzi — Castellani
Peruzzi — Strozzi
Peruzzi — Bischeri
Castellani — Strozzi
Castellani — Barbadori
Strozzi — Bischeri
Strozzi — Ridolfi
Bischeri — Guadagni
Barbadori — Ridolfi
Barbadori — Medici
Ridolfi — Tornabuoni
Guadagni — Lamberteschi
Guadagni — Albizzi
Albizzi — Medici
Albizzi — Ginori
Medici — Acciaiuoli
Medici — Salviati
Medici — Tornabuoni
Salviati — Pazzi

Figure 2: Trade network of the Southern Lau islands

In the graph below, each vertex represents an island, and each edge represents a trade route.

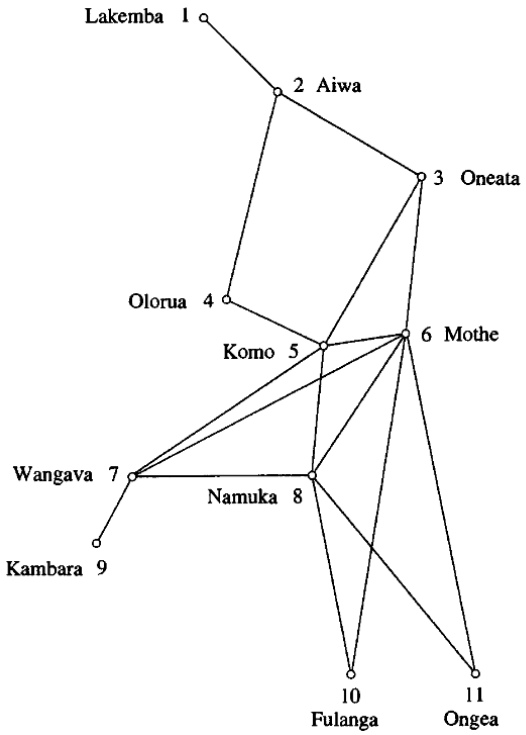


Figure 3: Greater Lauan Island trade routes

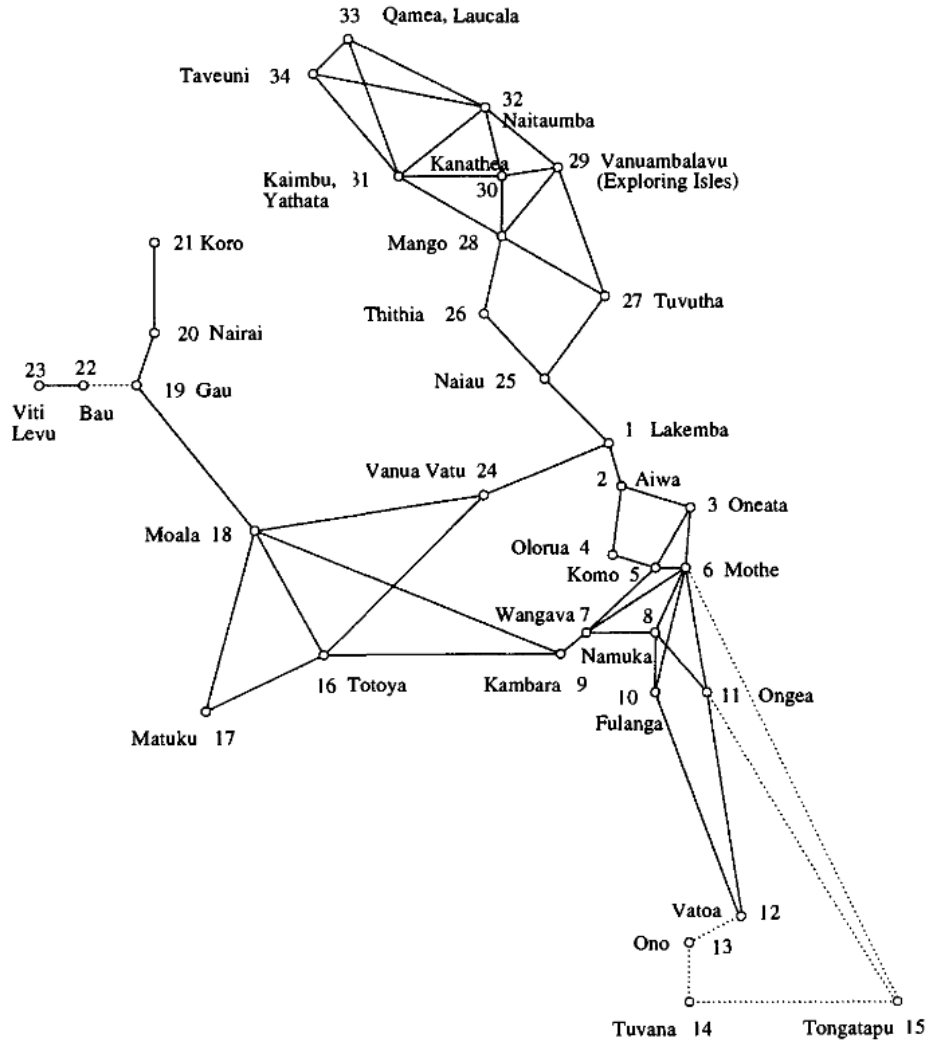


Figure 4: Torres Strait-Papua New Guinea canoe-purchasing trade routes

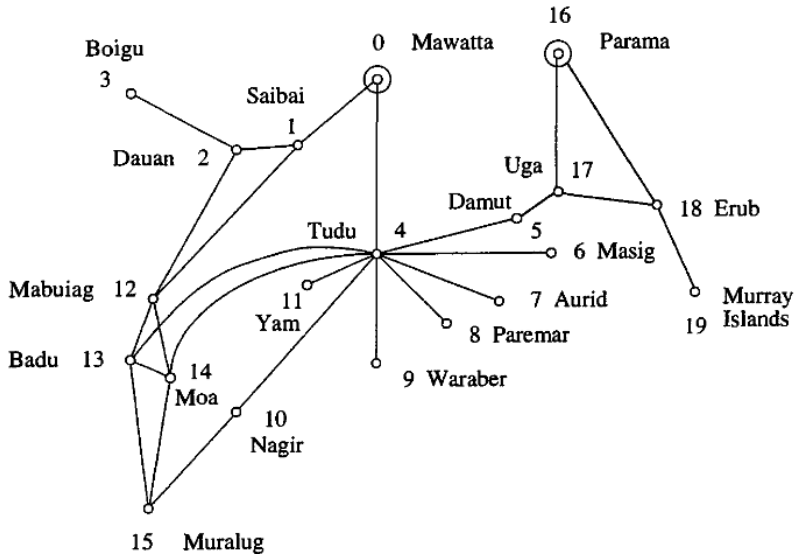
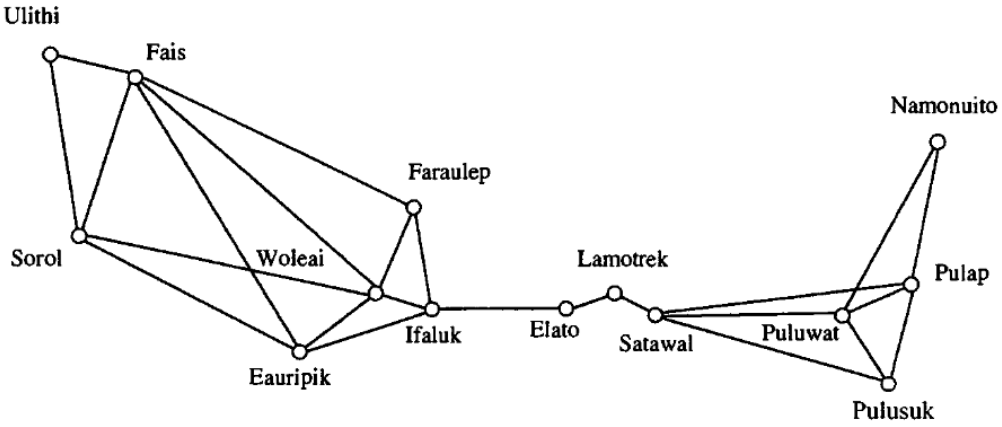


Figure 5: Western Carolines voyaging network



Problems

1. Draw a graph of the friendship network with the six vertices Ava, Ben, Cat, Dre, Eve, Finn and edges Ava-Ben, Ben-Cat, Cat-Ava, Cat-Dre, Dre-Eve, Eve-Finn, Finn-Dre. Find all the cut-vertices of this graph.
2. (a) Construct a graph from Figure 1 where each vertex is a family and each edge is a marriage between some two members of the two families.
   (b) Find the two vertices with the highest degrees in the resulting graph.
   (c) Describe the relationship between the families given by the two vertices of highest degrees.
3. Find the vertex of the highest degree in Figure 2. Is this island sensibly located?
4. Lakemba is not a vertex of a high degree in the trade network of the Southern Lau islands (Figure 2), but it is still very important in trades. Explain why using Figure 3 and the notion of a cut-vertex.
5. Specify at least two islands that are important in the trade network given in Figure 4, and discuss why these islands are important.
6. Analyze the power dynamics of the islands of the western Carolines as shown in Figure 5 using graph-theoretic concepts we have discussed so far.