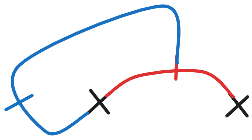
Lesson 13 — Planar graphs

Problem 1

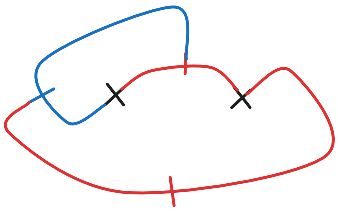
Two players take turns playing the game of **Brussel Sprouts** as follows. Two x's are drawn on paper:

The first player (red) connects two free ends of these x's with an edge and adds one tick mark on their edge.

The second player (blue) does the same, making sure their edge does not cross over any existing edge:



And then the first player goes again.



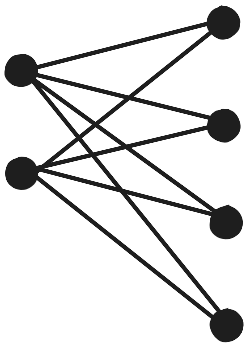
This goes on until some player cannot make a legal move, in which case that player loses.

Answer the following two questions.

(a) How many turns or moves were collectively taken by the two players?
(b) Which player loses, and why?

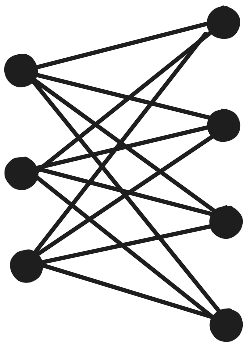
Problem 2

Show that the graph $K_{2,4}$ below is planar.



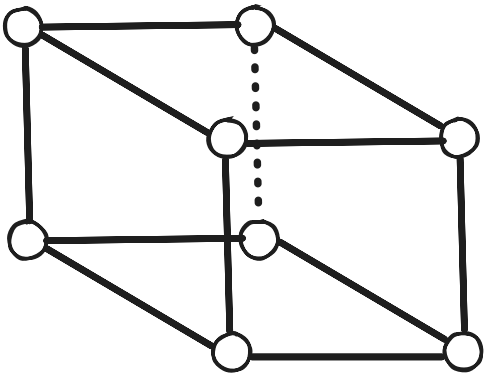
Problem 3

Argue that the graph $K_{3,4}$ below is **not** planar.



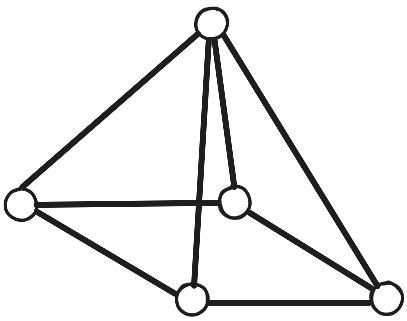
Problem 4

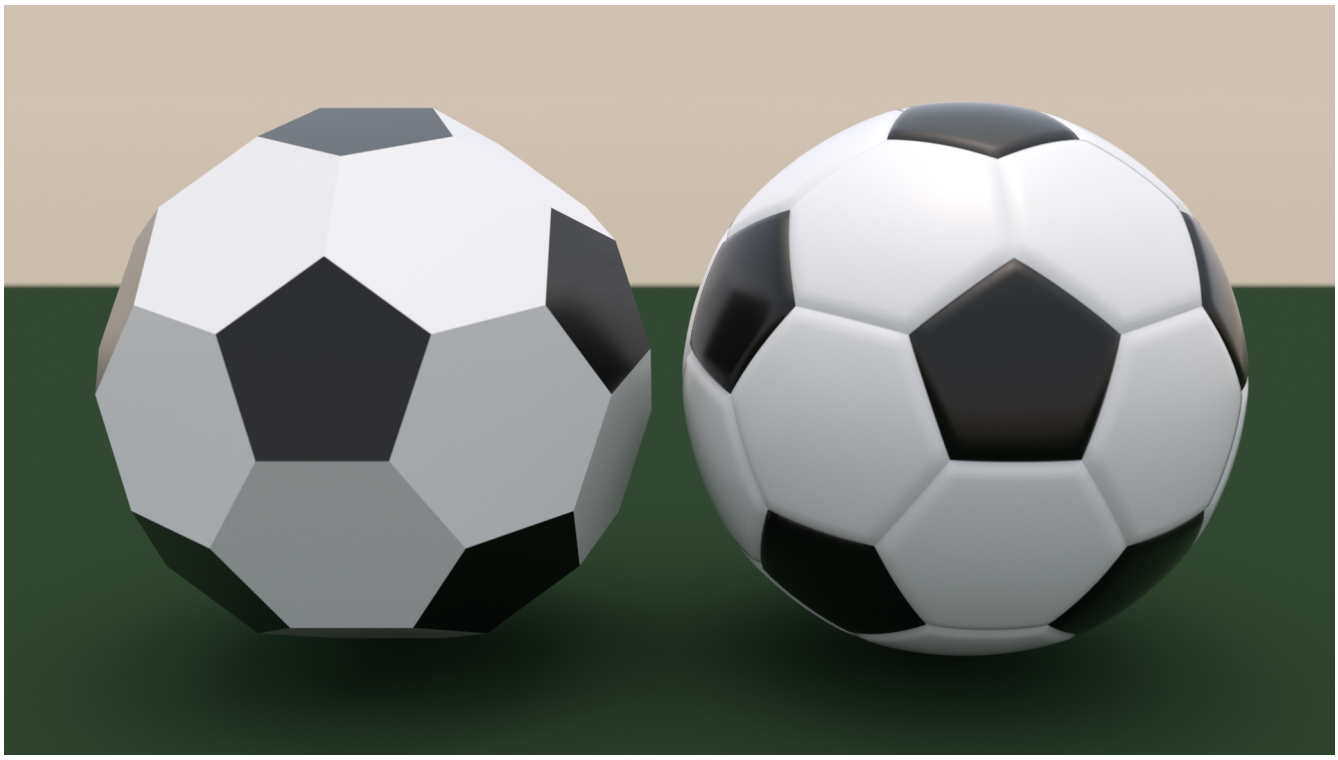
(a) Is the graph of a cube planar?
(b) Color the vertices of the cube with as few colors as possible, such that vertices along the same edge receive different colors.



Problem 5

(a) Is the graph of a square pyramid planar?
(b) Color the vertices of the square pyramid with as few colors as possible, such that vertices along the same edge receive different colors.



Problem 6

A soccer ball has 32 faces and 90 edges. Explain how this information alone helps you deduce the number of its vertices on the soccer ball.