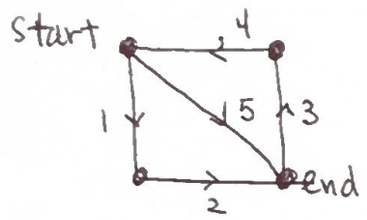
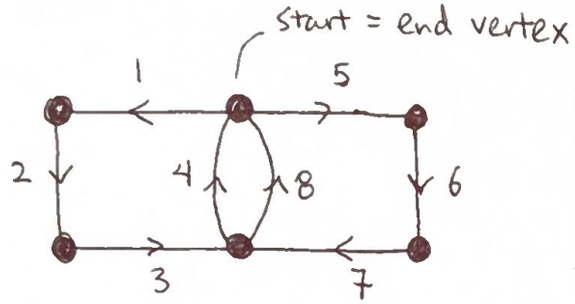
8 Euler paths and circuits

In a connected graph, an Euler path is a connected sequence of edges that uses each edge exactly once.

An Euler circuit is an Euler path that ends at the starting vertex.



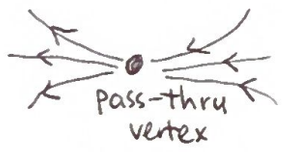
Euler path



Euler circuit

Insight Euler path uses each edge once.

So at a pass-thru vertex, each in-edge has a corresponding out-edge; so degree of this vertex is even.

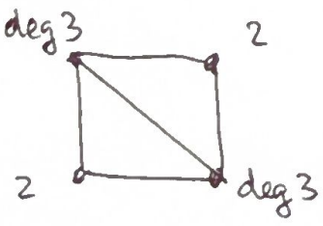


(Start & end vertex ~~need~~ don't need even degree.)

Euler path theorem

- Graph has Euler path ⇔ at most two vertex degrees are odd.
- Graph has Euler circuit ⇔ no degree is odd.

Ex



has no Euler circuit but has Euler path.

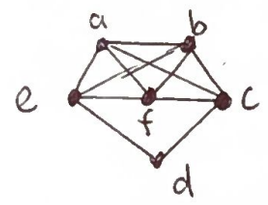
[Do exercises 1-3]

Aside Efficient paths save money for mail delivery, Ohio county snow removal (30% less trucks), reading electricity meters (Israel, 40% less time).

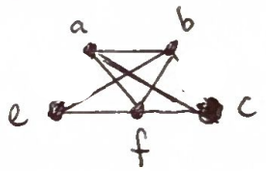
In 1970s NYC, suggestions for efficient trash pickup routes not implemented due to politics, bureaucracy, potential job loss, etc.

How to find Euler paths? (Hierholzer's algorithm)

Ex (All even degrees)



Form cycle, say, abcdea



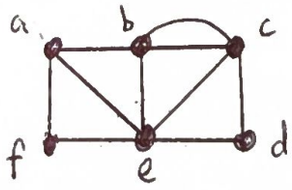
We subtracted off a cycle so leftover degrees still even, so form cycle, say befb & insert into main cycle at b:

a befb cdea

New cycle is afca, insert say at 1st a:

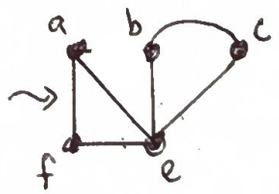
afca befb cdea.

Ex (Two odd degrees)



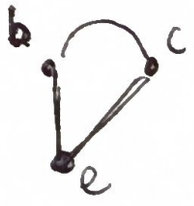
Start & end at odd degree vertices (a,e):

abcde.



form cycle aefa insert@a:

aefa bcde



Form cycle bceb, insert at b:

aefa bceb cde