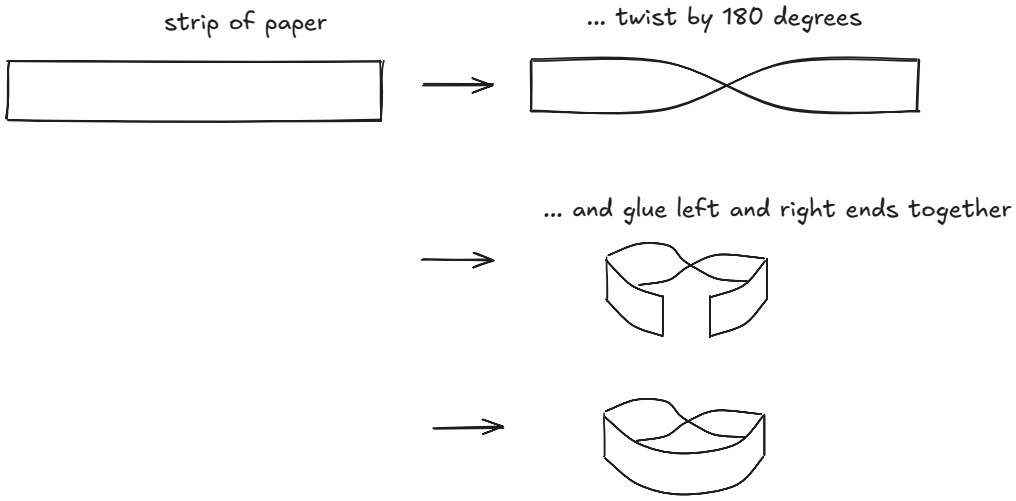
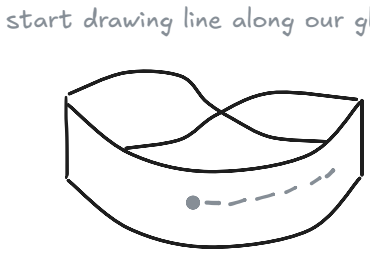
Lesson 14 — Surfaces

Problem 1

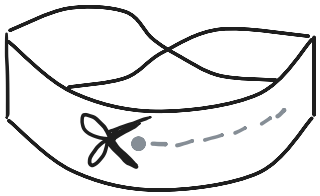
Take a strip of paper, twist one end by 180 degrees, and glue it to the other end. This creates



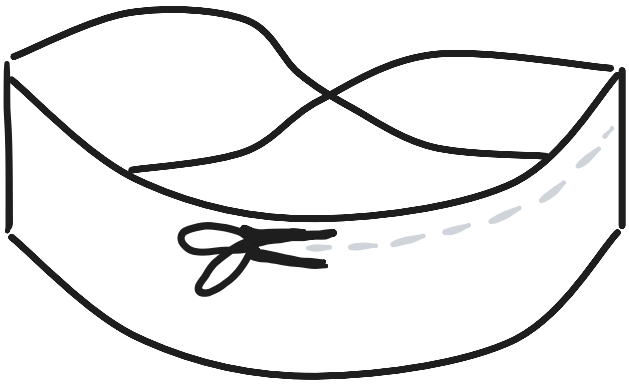
(a) If you use a pencil and draw a line starting from the outside of this paper as shown, will you eventually reach the inside of this paper?



(b) Cut the strip along the center line of the strip, all the way around until you meet back up. How many pieces of paper do you get? Explain why this is so.



(c) Cut along the strip 1/3 of the way to the edge of the strip, all the way around until you meet back up. How many pieces of paper do you get? Explain why this is so.

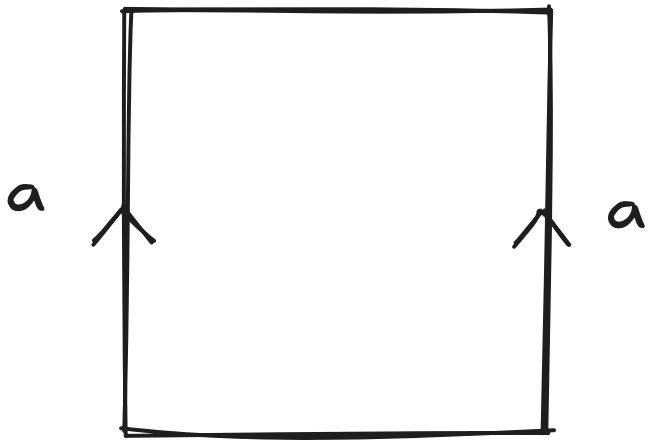


Problem 2

Repeat parts (a), (b), (c) of Problem 1 where the piece of paper in Problem 1 is twisted by 360 degrees instead of 180 degrees.

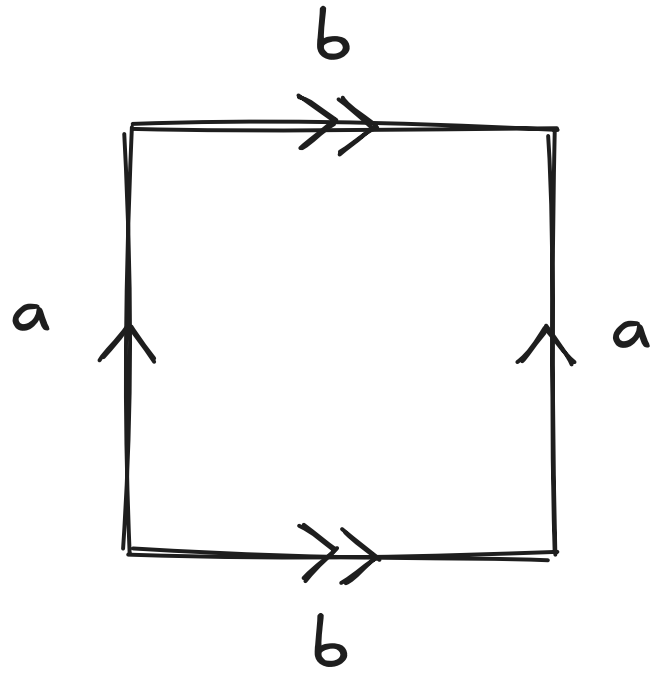
Problem 3

Suppose you glue the following square piece of very stretchy paper along the edges indicated: more precisely, edge a is glued to the other edge a so that their arrows point in the same direction. What surface do you get as a result? (Draw the process of gluing the sides together, including the surface that results.)



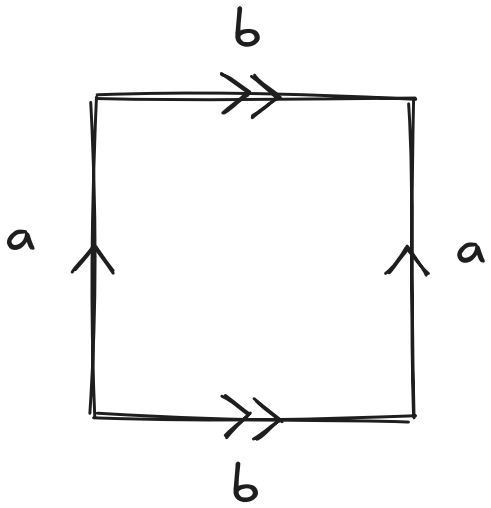
Problem 4

Suppose you glue the following square piece of very stretchy paper along the edges indicated: more precisely, edge a is glued to the other edge a so that their arrows point in the same direction, and edge b is glued to the other edge b so that their arrows point in the same direction. What surface do you get as a result? (Draw the process of gluing the sides together, including the surface that results.)



Problem 5

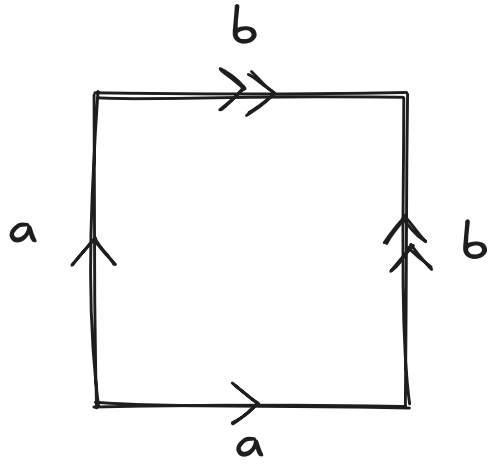
Suppose you glue the following square piece of very stretchy paper along the edges indicated: more precisely, edge a is glued to the other edge a so that their arrows point in the same direction, and edge b is glued to the other edge b so that their arrows point in the same direction. Is the resulting surface orientable (meaning it has two sides) or not orientable?



Problem 6

Suppose you glue the following square piece of very stretchy paper along the edges indicated: more precisely, edge a is glued to the other edge a so that their arrows point in the same direction, and edge b is glued to the other edge b so that their arrows point in the same direction.

(a) Is the resulting surface orientable (meaning it has two sides) or not orientable?
(b) Up to stretching, what surface results? (Draw the process of gluing the sides together, including the surface that results.)



Problem 7

Glue the corresponding sides a, b, c of the two pieces of very stretchy, triangular papers together until you get one surface.

(a) Is the resulting surface orientable (meaning it has two sides) or not orientable?
(b) Up to stretching and bending, what surface results? (Draw the process of gluing the sides together, including the surface that results.)