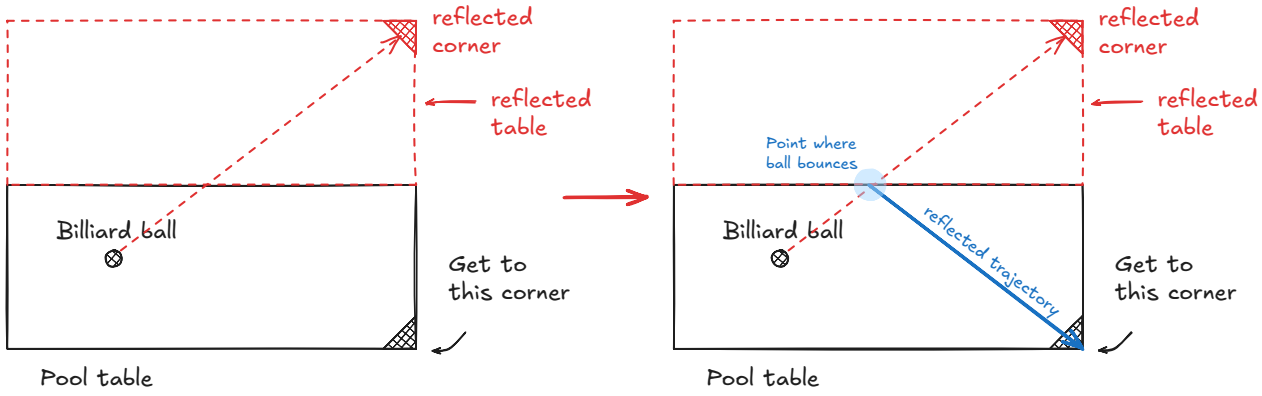
Lesson 2 — Reflection principle (filled)

Mirror

1. (a) Draw on the mirror an outline of your head. Then compare the size of this outline to your actual head.
   The size of the outline of the head on the mirror is half the size of my actual head.
   (b) How is (a) affected by your distance to the mirror?
   (a) is not affected by my distance to the mirror.
   (c) Explain the phenomenon in (a). Hint: draw a diagram of you and the mirror and how light bounces from what you see to your eye.
   Hint: see accompanying slides for the explanation.
   (d) How tall of a mirror do you need to buy to see your whole person? How high off the ground should this mirror be placed?
   I'd need to buy a mirror that is half my height to see my whole person.

Billiards

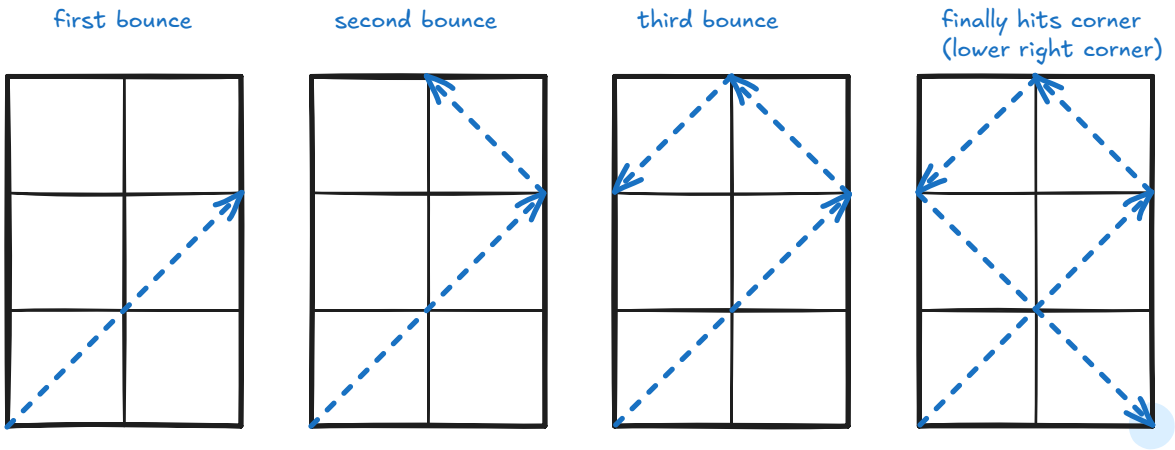
Decide where to hit the billiard ball against one wall of the rectangular pool table to get it to the corner shown.



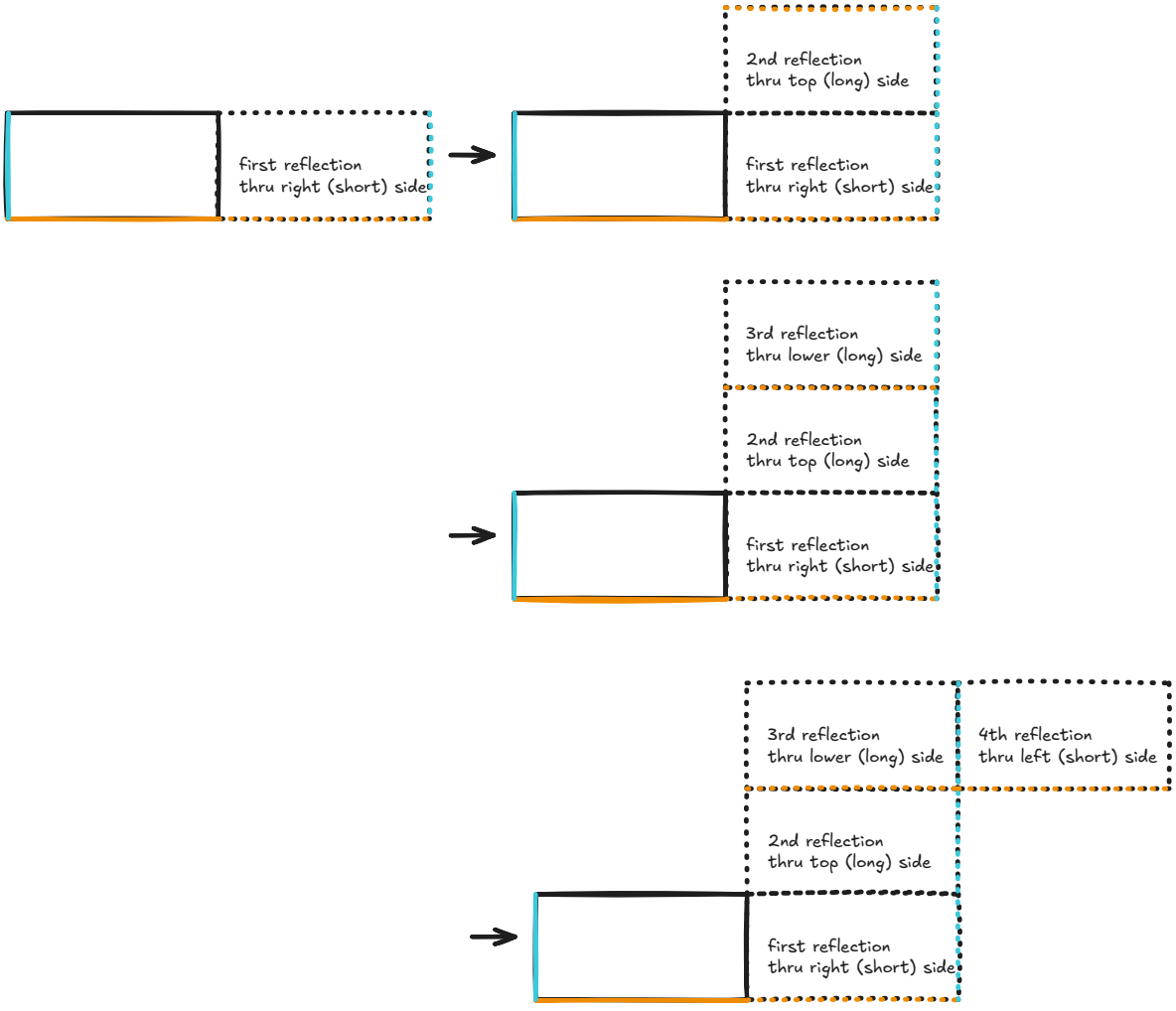
Group questions

Repeatedly use the reflection principle to help answer the following questions. Assume that the billiard ball follows a straight line trajectory without the effects of spin, friction, etc.

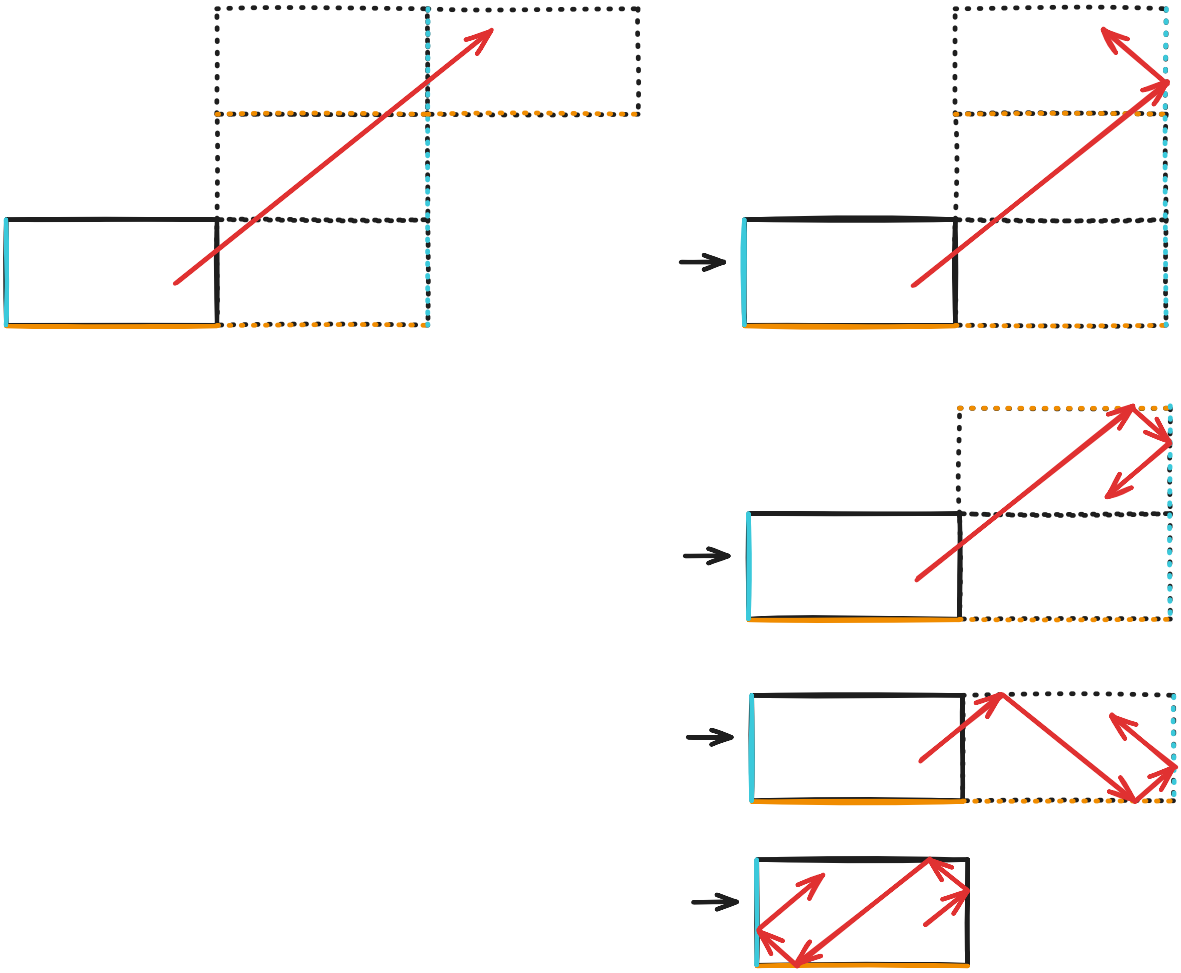
2. Shoot a billiard ball at a 45° angle from the bottom left corner of a 3×2 pool table. After how many bounces will it hit a corner again? Which corner will be hit?
   Solution: First we grid up the 3×2 pool table. Next, notice that a ball that shoots out at a 45° trajectory from horizontal must cut through each grid square diagonally each time. Therefore, the trajectory looks as follow, with the ball hitting the lower right corner after three bounces.



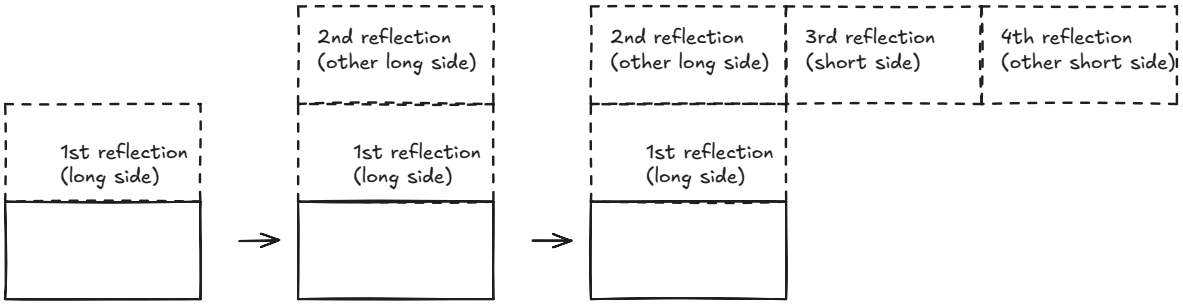
3. Shoot a billiard ball at a 45° angle from the bottom left corner of a 2×3 pool table. After how many bounces will it hit a corner again? Which corner will be hit?
   Hint: the same method in 2. works.
4. Shoot a billiard ball at a 45° angle from the bottom left corner of a 2×6 pool table. After how many bounces will it hit a corner again?
   Hint: the same method in 2. works.
5. A billiard ball is shot from the center of a square table. Describe how the ball can hit the north edge, east edge, south edge, and west edge of the table in that order.
   Hint: the same method used to solve Problem 6 below also works here.
6. Can a billiard ball shot on a 1×2 table first hit one shorter side, then one longer side, then the other longer side, and finally the other shorter side?
   This is possible: if we use the idea of reflecting the table through the short side, then long side, and so on, we see how to hit each side successively as shown in the following drawings:

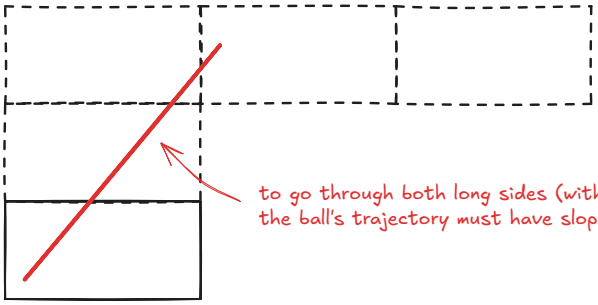


So now we just draw a straight line from the original pool table to the final pool table and fold back the straight line to obtain the desired ball's trajectory:

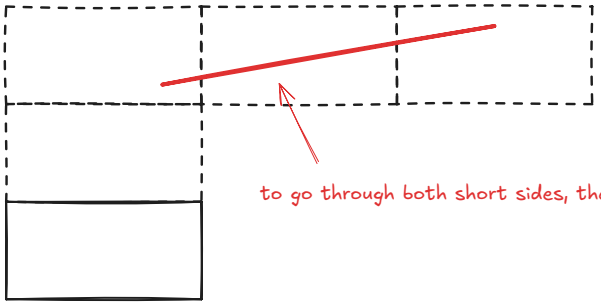


7. Can a billiard ball shot on a 1×2 table first hit one longer side, then the other longer side, then one shorter side, then the other shorter side? Hint: analyze the slope of the trajectory.
   Answer: It's impossible to hit long-short-other long side-other short side since unfolding the rectangle (as in the previous problem) gives us a trajectory that does not go through all the rectangles:





to go through both long sides (without hitting the pool table's corners), the ball's trajectory must have slope > 1/2



to go through both short sides, the ball's trajectory must have slope < 1/2

Clearly we can't have a slope that is both > 1/2 and < 1/2, so this trajectory is impossible.