Lesson 17: Interest rates

What to do with your money?

Some basic ways to store money and assets:

- Bank account for short-term money.
- Retirement account for low tax or tax-free investing.
- Other (real estate, etc.)

Bank account:

- **Checking accounts** for transactions. You may want to consider opting out of overdraft coverage.
- **Savings account** to keep perhaps 3 to 6 months of emergency funds.
    - Regular savings accounts provide low interest. Some also limit the number of monthly withdrawals.
    - High-yield savings account (HYSA, mine is now 3.5%; sometimes matches inflation)
- **Certificate of deposit** (CD ladder) for higher yield returns. Basically you are loaning money to the bank and can't get your money back until the contract ends.

You should open a **Roth IRA** (individual retirement account) where you contribute (post-tax) money:

- The gains you make in this account (by dividend, by buying and selling, etc.) are not taxed.
- You can withdraw your principal at any time.
- The gains you make in this account cannot be withdrawn until retirement age; early withdrawal of gains is penalized.

Traditional investment advice:

- **In youth, put most in index funds** (higher risk, but higher reward), and re-invest the dividends. This gives you a 6-7% gain per year (adjusted for inflation).
- **Near retirement, shift more into U.S. treasury bonds** (lower risk, but lower reward) in case of stock market crash. **However**, this strategy makes the assumption that U.S. treasury bond is regarded as one of the safest asset in the world since the 1980s. Not all government bonds are safe: notably, Russia defaulted on its debt (bond) in 1998, Argentina in 2001, Greece in 2012-2015, and Lebanon in 2020.
- **Dollar cost average**: contribute some money into the index fund at roughly regular intervals even if the market seems to be falling or rising. Such lows and highs are miniscule to the steady gains of the U.S. stock market in the timespan of 30-40 years. **However**, this strategy makes some assumption of the fact that the U.S. stock market has been rising since the Crash of 1929.

Interest rates

Definitions.

- The **principal** is how much money you put in if you invest, or how much you owe if you want to buy something.
- The interest rate is often given as the **annual percentage yield** or **rate** (APY, APR).

Simple interest

Simple interest is when interest rate is a percent of the original loan. This is usually for:

- Small purchase loans like furniture, cars, etc.
- Treasury bonds (government IOU's)
- Student loans (with some exceptions, e.g. if you don't make enough payment.)

Example 1.

Furniture costs $2000. Interest rate is 5% APY. You pay it over two years. Then the total interest is the product of all three quantities:

$$\$2000 \times \frac{0.05}{\text{year}} \times 2 \text{ year} = \$200$$

The monthly payment is the furniture cost and its interest spread over the 24 months:

$$\frac{\$2000 + \$200}{24 \text{ months}} \approx \$91.67 \text{ per month}$$

Example 2.

You buy a car for $20,000.

- Option A: pay over 4 years at APR of 7%
- Option B: pay over 6 years at APR of 7.5%

(a) What is the total interest of each option?

(b) Which loan has a lower monthly payment?

Answer:

(a)

$$\text{Option A: } \$20,000 \times \frac{0.07}{\text{year}} \times 4 \text{ year} = \$5,600$$

$$\text{Option B: } \$20,000 \times \frac{0.075}{\text{year}} \times 6 \text{ year} = \$10,500$$

(b)

$$\text{Option A monthly payment: } \frac{\$20,000 + \$5,600}{48 \text{ months}} \approx \$533.33 \text{ per month}$$

$$\text{Option B monthly payment: } \frac{\$30,000 + \$10,500}{72 \text{ months}} \approx \$423.61 \text{ per month}$$

Compound interest

Compound interest is when interest added to the old principal from the previous time period becomes the new principal for the new time period.

Instances where this is used:

- Savings account
- Credit card debt, etc.
- Inflation (on average)
- Stocks (on average)
- Fixed rate mortgage

APR vs EAR

Example 1. Credit card charges each month by:

- whether you make the minimum payment to avoid penalty, and
- the interest on the debt you have remaining, this monthly **periodic rate** of say 1.65%

The annual percentage rate is the monthly rate multiplied by 12:

$$1.65 \times 12 = 19.80\%$$

More accurate is the **effective annual rate** (EAR) which notes that your debt is being compounded:

$$1.0165^{12} = 1.217 = +21.7\%$$

(This ignores minimum payment penalties.) In Europe credit cards must report the EAR where we see the APR.

One-time contribution

Example 2. (One-time investment)
You invest $1 into the stock market which has an average return of 6% yearly (when adjusted for inflation). After how many years does your (inflation-adjusted) investment double in value?

Answer: Let us try some values:

Year 0: $1
Year 1: $1.06^1 = \$1.06$
Year 2: $1.06^2 = \$1.1236$
...
Year 6: $1.06^5 = \$1.33$
...
Year 10: $1.06^{10} = \$1.7908476965$
Year 12: $1.06^{12} = \$2.0121964718$

So it takes 12 years to double in value.

Regular contributions

Example 3. (Regular contributions)
Suppose you contribute $1000 annually to your retirement account
(We assume $1000 is what you contribute yearly when adjusted to inflation; so then we can use the 6% computation.)

Year 0: 1000
Year 1: $1000 + 1000 \cdot 1.06$
Year 2: $1000 + 1000 \cdot 1.06 + 1000 \cdot 1.06^2$
Year 20: $1000 + 1000 \cdot 1.06 + 1000 \cdot 1.06^2 + \cdots + 1000 \cdot 1.06^{20} = 1000 \cdot \dfrac{1.06^{21} - 1}{1.06 - 1}$

$$= 1000 \cdot \frac{2.399}{0.06} \approx 1000 \cdot 39.9927 \approx 39992.72$$

Here we used the geometric series formula

$$a + ar + ar^2 + ar^3 + \cdots + ar^n = a \cdot \frac{r^{n+1} - 1}{r - 1}$$

with

- $a = 1$
- $r = 1.06$
- $n = 20$