

1.1

$P(\text{OR gate} | x=1, z=1)$

AND gate

x	y	z
1	1	1

OR gate

x	y	z
1	0	1
1	1	1

$$P(\text{OR gate}) = \frac{2}{3}$$

1.2

$$P(y=1 | x=1, z=1) = \frac{P(y=1, x=1, z=1)}{P(x=1, z=1)}$$

$$= \frac{2}{3} \text{ (from truth tables)}$$

1.3

when $x=1, z=1$

Abduction

$$P(\text{OR gate}) = \frac{2}{3} \text{ (choosing OR gate)}$$

$$P(\text{AND gate}) = \frac{1}{3} \text{ (choosing AND gate)}$$

Intervention: $x=0$

Prediction:

$$z=0 = \left(P(\text{OR}) * P(z=0 | x=0) + P(\text{AND}) * P(z=0 | x=0) \right)$$

$$= \left(\frac{2}{3} * \frac{1}{2} + \frac{1}{3} * 1 \right)$$

$$= \left(\frac{2}{3} \right)$$

$$z=1 = P(\text{OR}) * P(z=1 | x=0) + P(\text{AND}) * P(z=1 | x=0)$$

$$= \left(\frac{2}{3} * \frac{1}{2} + \frac{1}{3} * 0 \right) = \frac{1}{3}$$

2.2.a

$$\begin{aligned}PNS &= P(Y=1|X=1) - P(Y=1|X=0) \\&= 0.9198813 - 0.1992071 \\&= 0.720.\end{aligned}$$

$$PN = \frac{PNS}{P(Y=1|X=1)} = \frac{0.720}{0.919} = \underline{\underline{0.783}}$$

$$PS = \frac{PNS}{P(Y=0|X=0)}$$

$$P(Y=0|X=0) = \frac{P(Y=0, X=0)}{P(X=0)}$$

$$= \frac{0.4}{0.4995}$$

$$PS = \frac{0.720 \times 0.4995}{0.4} = \underline{\underline{0.8991}}$$