

RAMAKRISHNA MISSION VIVEKANANDA
CENTENARY COLLEGE

KOLKATA-700118

MATHEMATICAL PHYSICS PRACTICAL NOTEBOOK

Core Course : 5

SUBMITTED BY –

ANIK MANDAL

(B.Sc. 3rd semester student of Department Of Physics)

REGISTRATION NO. : A01-1152-111-042-2018

ADMIT NO. :

YEAR : 2019

CONTENT

NO.	TOPIC	PAGE NO.
<u>1</u>	Determination of the value of nCr and nPr for a n=7 and r=[0,7]	
<u>2</u>	Determination of the value of sin(x) for x = 30° with error approximation	
<u>3</u>	Determination of the value of cos(x) for x= 60° with error approximation	
<u>4</u>	Determination of the value of e^x for x = 5 with error approximation	
<u>5</u>	Determination of the value of log_ex for x = 10 with error approximation	
<u>6</u>	Determination of the value of (1+x)ⁿ using Taylor expansion for x = 0.01 and n = 3 with error approximation	
<u>7</u>	Determination of the value of the value of π through Ramanujan's Pi formula with error approximation	
<u>8</u>	Matrix or Array operations	
<u>9</u>	Determination of the value of square root of a number(x=3) using Bisection method with error approximation	
<u>10</u>	Determination of the value of square root of a number(x=10) using Newton-Raphson method with error approximation	
<u>11</u>	Determination of the value of integration of a function using Trapezoidal method with error approximation	
<u>12</u>	Determination of the value of integration of a function using Simpson's 1/3 method with error approximation	
<u>13</u>	Determination of the value of variables of linear equation using Gaussian Elimination method	
<u>14</u>	Sorting of the elements of a list in Ascending or Descending order using Insertion sort method	
<u>15</u>	Sorting of the elements of a list in Ascending or Descending order using Bubble sort method	
<u>16</u>	Determination of the value of the bar graph of number of Heads and Tails for a number of trials	
<u>17</u>	Determination of the value of the bar graph of number of points in each separate window through generating random points	
<u>18</u>	Determination of the value of the integration of a function $[f(x) = \exp(-x^2)]$ using Monte Carlo Method	
<u>19</u>	Determination of the value of the mean graph of a data set(linearly dependent) using Least Square Fit method	
<u>20</u>	Determination of the value of differentiation of a function $[f(x) = x^3 - 3x]$ at a given point (x=5)	
<u>21</u>	Determination of the value of the function from its 1 st order differential equation using Euler's method	
<u>22</u>	Determination of the value of the equation of harmonic oscillator from its 2 nd order differential equation and determination of the value of the equation of <ol style="list-style-type: none"> 1. Damped harmonic oscillator 2. Critically damped harmonic oscillator 3. Over-damped harmonic oscillator 4. Forced harmonic oscillator(undamped) for $f(x) = 100\sin(x)$ 	
<u>23</u>	Determination of the value of the Fourier coefficients and Fourier function of a given function $[f(x)=x^3]$ in a given range $[-\pi,\pi]$ And Determination of the value of the same with Integration module for different function	
<u>24</u>	Determination of the value of the Maximum, Minimum, Average, RMS value and Standard Deviation of a function $[f(x)=x^2\exp(-x^2)]$ in a given range $[0,\pi]$	

1. Write a program to determine of the value of nCr and nPr for a n=7 and r =[0,7]:

PROGRAM:

```
# DETERMINATION OF THE VALUE OF nCr AND nPr :

print("\n\tDETERMINATION OF THE VALUE OF nCr AND nPr :\n")
n = 7
r = 0

while r <= n:
    (fn, fr, fnr) = (1, 1, 1)

    for i in range(1, n+1, 1):
        fn = fn*i

    for j in range(1, r+1, 1):
        fr = fr*j

    for k in range(1, n-r+1, 1):
        fnr = fnr*k

    nCr = fn/(fr*fnr)
    nPr = fn/fnr

    print("nCr for n:", n, "r :", r, " equals to :", nCr)
    print("nPr for n:", n, "r :", r, " equals to :", nPr)
    r = r+1
```

OUTPUT:

DETERMINATION OF THE VALUE OF nCr AND nPr :

```
nCr for n: 7 r : 0  equals to : 1.0
nPr for n: 7 r : 0  equals to : 1.0
nCr for n: 7 r : 1  equals to : 7.0
nPr for n: 7 r : 1  equals to : 7.0
nCr for n: 7 r : 2  equals to : 21.0
nPr for n: 7 r : 2  equals to : 42.0
nCr for n: 7 r : 3  equals to : 35.0
```

```

nPr for n: 7 r : 3 equals to : 210.0
nCr for n: 7 r : 4 equals to : 35.0
nPr for n: 7 r : 4 equals to : 840.0
nCr for n: 7 r : 5 equals to : 21.0
nPr for n: 7 r : 5 equals to : 2520.0
nCr for n: 7 r : 6 equals to : 7.0
nPr for n: 7 r : 6 equals to : 5040.0
nCr for n: 7 r : 7 equals to : 1.0
nPr for n: 7 r : 7 equals to : 5040.0

```

2. Write a program to determine the value of sin(x) with error for x= 30°:

Taylor Series of sin(x):

$$\sin(x) = \sum_{i=0}^{\infty} (-1)^i \frac{x^{(2i+1)}}{(2i+1)!}$$

PROGRAM:

```

# DETERMINATION OF THE VALUE OF sin(30) WITH ERROR APPROXIMATION :

import math
print("\n\tDETERMINATION OF THE VALUE OF sin(30) WITH ERROR APPROXIMATION
:\n")

a = 30.0
x = a*math.pi/180.0
eps = 0.00001

(i, fact, s, err, z, tm) = (0.0, 1.0, 0.0, 1.0, 1.0, 0.0)

while err > eps:
    i = i + 1.0
    fact = fact * i

    if i % 2.0 == 1.0:
        z = z+1.0
        t = (x ** i)*((-1.0)**z) / fact
        s = s + t

```

```

err = abs(t)
tm = tm+1.0

print("The value of the sin(", a, ") with maximum error approximation",
eps, " :\t", s)
print("The total number of terms required to reach the accuracy :", tm)

```

OUTPUT:

DETERMINATION OF THE VALUE OF sin(30) WITH ERROR APPROXIMATION :

The value of the sin(30.0) with maximum error approximation 1e-05 :
0.4999999918690232

The total number of terms required to reach the accuracy : 4.0

3. Write a program to determine the value of cos(x) with error for x= 60°:

Taylor Series of cos(x):

$$\cos(x) = \sum_{i=0}^{\infty} (-1)^i \frac{x^{2i}}{(2i)!}$$

PROGRAM:

DETERMINATION OF THE VALUE OF cos(60) WITH ERROR APPROXIMATION :

```

import math
print("\n\tDETERMINATION OF THE VALUE OF cos (60) WITH ERROR APPROXIMATION
:\n")

a = 60.0
x = a*math.pi/180
eps = 0.00001

(i, fact, s, err, z, tm) = (0.0, 1.0, 1.0, 1.0, 0.0, 0.0)

while err >= eps:
    i = i + 1.0
    fact = fact * i

```

```

if i % 2.0 == 0.0:
    z = z+1
    t = (x ** i)*((-1.0)**z) / fact
    s = s + t

err = abs(t)
tm = tm+1.0

print("The value of the cos(", a, ") with maximum error approximation",
eps, " :\t", s)
print("The total number of terms required to reach the accuracy :", tm)

```

OUTPUT:

DETERMINATION OF THE VALUE OF cos (60) WITH ERROR APPROXIMATION :

The value of the cos(60.0) with maximum error approximation 1e-05 :
0.4999999963909432

The total number of terms required to reach the accuracy : 5.0

4. Write a program to determine the value of exp(x) with error for x= 5:

Taylor Series of e^x:

$$e^x = \sum_{i=0}^{\infty} \frac{x^i}{i!}$$

PROGRAM:

DETERMINATION OF THE VALUE OF exp(5) WITH ERROR APPROXIMATION :

```

print("\n\tDETERMINATION OF THE VALUE OF exp(5) WITH ERROR APPROXIMATION
:\n")

```

```

x = 5.0

```

```

eps = 0.000001

```

```

(i, fact, s, err, tm) = (0.0, 1.0, 1.0, 1.0, 0.0)

```

```

while err > eps:
    i = i + 1.0
    fact = fact*i
    t = (x**i)/fact
    s = s+t

    err = abs(t)
    tm = tm+1.0

print("The value of the exp(", x, ") with maximum approximation", eps, "
:\t", s)
print("The total number of terms required to reach the accuracy :", tm)

```

OUTPUT:

DETERMINATION OF THE VALUE OF exp(5) WITH ERROR APPROXIMATION :

The value of the exp(5.0) with maximum approximation 1e-06 :
148.41315898276957

The total number of terms required to reach the accuracy : 23.0

5. Write a program to determine the value of $\log_e(x)$ with error for $x=10$:

Taylor Series of $\log_e(x)$:

$$\log_e(x) = - \sum_{i=1}^{\infty} \frac{(1-x)^i}{i} \quad \text{for } x < 1$$

PROGRAM:

DETERMINATION OF THE VALUE OF $\ln(10)$ WITH ERROR APPROXIMATION :

```

print("\n\tDETERMINATION OF THE VALUE OF  $\ln(10)$  WITH ERROR APPROXIMATION
:\n")

```

```

a = 10.0

```

```

x = 1 - (1/a)

```

```

eps = 0.00001
(err, s, i, tm) = (1.0, 0.0, 1.0, 0.0)

while err > eps:
    t = (x**i)/i
    s = s+t
    i = i+1.0

    err = abs(t)
    tm = tm+1.0

print("The value of the ln(", a, ") : ", s)
print("The total number of terms required to reach the accuracy :", tm)

```

OUTPUT:

DETERMINATION OF THE VALUE OF $\ln(10)$ WITH ERROR APPROXIMATION :

The value of the $\ln(10.0)$: 2.3025137650431016

The total number of terms required to reach the accuracy : 70.0

6. Write a program to determine the value of $(1+x)^n$ Taylor expansion with error for $x=0.01 \ll 1$ and $n = 3$:

Taylor Series of $(1+x)^n$:

$$(1+x)^n = \sum_{i=0}^{\infty} \binom{n}{i} x^i \quad \text{for } x \ll 1$$

PROGRAM:

```

# DETERMINATION OF THE VALUE OF THE  $(1+x)^n$  TAYLOR EXPANSION WITH ERROR
APPROXIMATION WHEN  $x \ll 1$ 

```

```

print("\n\tDETERMINATION OF THE VALUE OF THE  $(1+x)^n$  TAYLOR EXPANSION WITH
ERROR APPROXIMATION WHEN  $x \ll 1$ \n")

```

```

x = 0.01

```

```

n = 3

```

```

eps = 0.00001

```



```

(a, s, err, i, f, fn, tm) = (1.0, 1.0, 1.0, 1, 1.0, 1.0, 1)

while err > eps:
    f = f*i
    fn = fn*(n-i+1)

    t = (fn/f)*(x**i)
    s = s+t

    i = i+1

    err = abs(t)
    tm = tm+1

print("The value of (" , a, "+", x, ")^", n, " is equals to : ", s)
print("The number of terms to reach the accuracy is :", tm)

```

OUTPUT:

DETERMINATION OF THE VALUE OF THE $(1+x)^n$ TAYLOR EXPANSION WITH ERROR APPROXIMATION WHEN $x \ll 1$

The value of $(1.0 + 0.01)^3$ is equals to : 1.030301

The number of terms to reach the accuracy is : 4

7. Write a program to determine the value of the value of π through Ramanujan's Pi formula with error approximation:

Ramanujan's Pi Formula:

$$\frac{1}{\pi} = \frac{\sqrt{8}}{9801} \sum_{k=0}^{\infty} \frac{(4k)! (1103 + 26390k)}{(k!)^4 (396)^{4k}}$$

PROGRAM:

FACTORIAL MODULE:

```

def factorial(n, i, step):
    fact = 1
    while i <= n:

```

```

    fact = fact*i
    i = i+step
return fact

```

```

# DETERMINATION OF THE VALUE OF  $\pi$  USING RAMANUJAN'S  $\pi$  SERIES

```

```

from factorial import *

```

```

print("\tDETERMINATION OF THE VALUE OF  $\pi$  USING RAMANUJAN'S  $\pi$  SERIES")

```

```

s0 = (8 ** 0.5) / 9801 # constant

```

```

(f1, f2, j, k, err, eps, t, c) = (1, 1, 0, 0, 1, 10 ** (-14), 0, 10)

```

```

t0 = (1 * (1103 + 26390 * 0)) / (1 * 1) # 1st term of the summation

```

```

# as we know that,

```

```

# 0! = 1

```

```

# so,

```

```

# (4 * 0)! = 1

```

```

# pow(0!,4) = 1

```

```

# and, we also know that,

```

```

# pow(396,4*0)= 1

```

```

sum = t0

```

```

while err > eps:

```

```

    t = t + 1

```

```

    for i in range(0, t):

```

```

        i = i + 1

```

```

        p = 4 * i

```

```

        f1 = factorial(p, 1, 1)

```

```

        f2 = factorial(i, 1, 1)

```

```

        s = (f1 * (1103 + 26390 * i)) / ((f2 ** 4) * (396 ** p)) # every
term of the summation

```

```

        sum = sum + s # summation part

```

```

pi = 1 / (s0 * sum)

```

```

err = abs(pi - c)
c = pi

(f1, f2, sum) = (1, 1, t0)

print("\nUltimate Approximated value of  $\pi$  :", pi)
print("Number of terms required to reach that approximated value :", t +
1)

```

OUTPUT:

```

DETERMINATION OF THE VALUE OF  $\pi$  USING RAMANUJAN'S  $\pi$  SERIES
Ultimate Approximated value of  $\pi$  : 3.141592653589793
Number of terms required to reach that approximated value : 3

```

8. Array and matrix operation:

PROGRAM:

```

# ARRAY OR MATRIX OPERATIONS :

import numpy as np
print("\n\tARRAY OR MATRIX OPERATIONS :\n")

xx = [[1.0, 2.0, 3.0], [-5.0, 0.0, 5.0], [0.0, -1.0, 1.0]]
yy = [[0.0, 5.0, 10.0], [1.0, 2.0, 3.0], [-1.0, 0.0, 1.0]]

print("My matrices are \nxx :", xx, "\nny :", yy)

addxy = np.zeros((3, 3))
sbtxy = np.zeros((3, 3))
mltxy = np.zeros((3,3))
trsx = np.zeros((3, 3))

trx = 0.0

for i in range(3):
    for j in range(3):

```

```

addxy[i][j] = xx[i][j]+yy[i][j]
sbtxy[i][j] = xx[i][j]-yy[i][j]

for k in range(3):
    mltxy[i][j] += xx[i][k]*yy[k][j]

trsx[j][i] = xx[i][j]

trx += xx[i][i]

print("The addition of xx and yy matrix is : \n", addxy)
print("The subtraction of xx from yy matrix is : \n", sbtxy)
print("The multiplication of xx to yy matrix is : \n", mltxy)
print("The transpose of xx matrix is : \n", trsx)
print("The tr(xx) : ", trx)

```

OUTPUT:

ARRAY OR MATRIX OPERATIONS :

My matrices are

xx : [[1.0, 2.0, 3.0], [-5.0, 0.0, 5.0], [0.0, -1.0, 1.0]]

yy : [[0.0, 5.0, 10.0], [1.0, 2.0, 3.0], [-1.0, 0.0, 1.0]]

The addition of xx and yy matrix is :

[[1. 7. 13.]

[-4. 2. 8.]

[-1. -1. 2.]]

The subtraction of xx from yy matrix is :

[[1. -3. -7.]

[-6. -2. 2.]

[1. -1. 0.]]

The multiplication of xx to yy matrix is :

[[-1. 9. 19.]

[-5. -25. -45.]

[-2. -2. -2.]]

The transpose of xx matrix is :

[[1. -5. 0.]

[2. 0. -1.]

[3. 5. 1.]]

The tr(xx) : 2.0

❖ Write a program to determine the square root of a number(x= 3) using Bisection method with error:

PROGRAM:

```
# DETERMINATION OF THE SQUARE ROOT OF A GIVEN NUMBER THROUGH BISECTION :
```

```
print("\n\tDETERMINATION OF THE SQUARE ROOT OF A GIVEN NUMBER THROUGH  
BISECTION : \n")
```

```
x = 3.0
```

```
a = 1.0
```

```
b = 20.0
```

```
eps = 0.00001
```

```
err = 1.0
```

```
tm = 0.0
```

```
while err > eps:
```

```
    c = (a+b)/2.0
```

```
    if c**2.0 < x:
```

```
        a = c
```

```
    elif c**2.0 == x:
```

```
        err = eps
```

```
    else:
```

```
        b = c
```

```
    d = (a+b)/2.0
```

```
    err = abs(c-d)
```

```
    tm = tm+1.0
```

```
print("The value of the square root of", x, "is equals to :", c)
```

```
print("And the number of terms to reach the accuracy is :", tm)
```

OUTPUT:

```
DETERMINATION OF THE SQUARE ROOT OF A GIVEN NUMBER THROUGH BISECTION :
```

The value of the square root of 3.0 is equals to : 1.7320585250854492

And the number of terms to reach the accuracy is : 20.0

❖ Write a program to determine the square root of a given number (x = 10) using Newton-Raphson method with error:

PROGRAM:

```
# DETERMINATION OF THE VALUE OF SQUARE ROOT OF A GIVEN NUMBER THROUGH
NEWTON-RAPHSON METHOD WITH ERROR APPROXIMATION :

n = 2.0

print("\n\tDETERMINATION OF THE VALUE OF SQUARE ROOT OF", n, " THROUGH
NEWTON-RAPHSON METHOD WITH ERROR APPROXIMATION :\n")

x = 10.0
eps = 0.00001
(err, i, tm) = (1.0, 1.0, 0.0)

while err > eps:
    x1 = x
    m = 2.0*x1
    y = (x1**2.0)-n
    x = x1-(y/m)
    z = (x**2.0)-n

    err = abs(y-z)
    tm = tm+1.0

print("The value of square root of", n, "with given accuracy", eps, " is :
", x)
print("The number of terms to reach the accuracy is : ", tm)
```

OUTPUT:

DETERMINATION OF THE VALUE OF SQUARE ROOT OF 2.0 THROUGH NEWTON-RAPHSON METHOD WITH ERROR APPROXIMATION :

The value of square root of 2.0 with given accuracy 1e-05 is :
1.4142135623730954

The number of terms to reach the accuracy is : 7.0

❖ Write a program to determine the value of integration of a given function through trapezoidal method with error:

PROGRAM:

```
# DETERMINATION OF THE VALUE OF INTEGRATION OF A FUNCTION THROUGH
TRAPEZOIDAL METHOD :

print("\n\tDETERMINATION OF THE VALUE OF INTEGRATION OF A FUNCTION THROUGH
TRAPEZOIDAL METHOD :\n")

xi = 0.0
xf = 10.0

n = 10000
h = (xf-xi)/(n-1)

# Suppose,
# My function is : yi=f(xi)=xi^2

yi = xi**2
yf = xf**2

(i, err, s) = (1, 1.0, 0.0)

while i < (n-1):
    c = xi+i*h
    fc = c**2
    s = s+fc
    i = i+1

s = (yi+yf+2*s)*h/2

print("The value of the integration of my function from", xi, "to", xf, "
is : ", s)
```

OUTPUT:

```
DETERMINATION OF THE VALUE OF INTEGRATION OF A FUNCTION THROUGH
TRAPEZOIDAL METHOD :
```

```
The value of the integration of my function from 0.0 to 10.0 is :
333.3333350003328
```

❖ Write a program to determine the value of integration of a given function through Simpson's 1/3 method of integration with error:

PROGRAM:

```
# DETERMINATION OF THE VALUE OF INTEGRATION OF A FUNCTION THROUGH SIMPSON
1/3 METHOD :

import numpy as np

print("\n\tDETERMINATION OF THE VALUE OF INTEGRATION OF A FUNCTION THROUGH
SIMPSON 1/3 METHOD : \n")

xi = 0.0
xf = 1.0

# Suppose,
# my function is : yi=sin(x)*exp(x^3/3)

yi = np.sin(xi) * np.exp((xi ** 3) / 3)
yf = np.sin(xf) * np.exp((xf ** 3) / 3)

(n, err, eps, sn, t) = (2, 1, 0.0005, 10, 0)
while err > eps:
    h = (xf-xi)/(n-1.0)

    (i, s) = (1, 0.0)

    while i < (n-1):
        c = xi+i*h
        yc = np.sin(c)*np.exp((c**3)/3)

        if i % 2.0 == 1.0:
            s = s+4.0*yc
        else:
            s = s+2.0*yc
        i = i+1

    n = n+1
    s = (yi+yf+s)*h/3.0

    err = abs(sn-s)
```



```

sn = s
t += 1

s = sn
print("The value of the integration of my function from", xi, "to", xf,
      "is equals to : ", s)
print("Number of terms to reach the accuracy : ", t)

```

OUTPUT:

DETERMINATION OF THE VALUE OF INTEGRATION OF A FUNCTION THROUGH SIMPSON
1/3 METHON :

The value of the integration of my function from 0.0 to 1.0 is equals to :
0.5248062730696056

Number of terms to reach the accuracy : 783

- **Simpson's 1/3 with array operation:**

PROGRAM:

```

# DETERMINATION OF THE VALUE OF INTEGRATION OF A FUNCTION THROUGH SIMPSON
1/3 METHOD USING ARRAY:

import numpy as np

print("\n\tDETERMINATION OF THE VALUE OF INTEGRATION OF A FUNCTION THROUGH
SIMPSON 1/3 METHOD :")

xi = 0.0
xf = 10.0

n = 10000
h = (xf-xi)/(n-1)

xx = np.linspace(xi, xf, n)

# Suppose,
# my function is : yi=f(xi)=xi^2

yy = xx**2.0

```

```
(i, s) = (1, 0.0)
```

```
while i < n-1.0:
```

```
    k = i % 2.0
```

```
    if k == 1.0:
```

```
        s = s+4.0*yy[i]
```

```
    else:
```

```
        s = s+2.0*yy[i]
```

```
    i = i+1
```

```
s = (yy[0]+yy[n-1]+s)*h/3.0
```

```
print("\nThe value of the integration of my function from", xi, "to", xf,  
      "is equals to : ", s)
```

OUTPUT:

DETERMINATION OF THE VALUE OF INTEGRATION OF A FUNCTION THROUGH SIMPSON 1/3
METHOD :

The value of the integration of my function from 0.0 to 10.0 is equals to :
333.30000000033306

❖ Write a program to determine the value of variables from three linear equation through Gaussian Elimination method :

PROGRAM:

```
# DETERMINATION OF THE VALUE OF VARIABLES FROM THREE LINEAR EQUATION  
THROUGH GAUSSIAN ELIMINATION METHOD :
```

```
print("\n\tDETERMINATION OF THE VALUE OF VARIABLES FROM THREE LINEAR  
EQUATION THROUGH GAUSSIAN ELIMINATION METHOD : \n")
```

```
x = 1.0
```

```
y = 2.0
```

```
z = 3.0
```

```
eps = 0.00001
```

```
erx = ery = erz = 1.0
```

```
# My equations are:
```

```
#      12x+3y-5z=1          (1)
```

```
#      x+5y+3z=28          (2)
```

```
#      3x+7y+13z=76        (3)
```

```
while erx and ery and erz > eps:
```

```
    x1 = (1.0-3.0*y+5.0*z)/12.0
```

```
    y1 = (28.0-x1-3.0*z)/5.0
```

```
    z1 = (76.0-3.0*x1-7.0*y1)/13.0
```

```
    erx = abs(x-x1)
```

```
    ery = abs(y-y1)
```

```
    erz = abs(z-z1)
```

```
    x = x1
```

```
    y = y1
```

```
    z = z1
```

```
print("So, the general solution of my equations are x =", x, "y =", y, "z  
=", z)
```

OUTPUT:

```
DETERMINATION OF THE VALUE OF VARIABLES FROM THREE LINEAR EQUATION THROUGH  
GAUSSIAN ELIMINATION METHOD :
```

```
So, the general solution of my equations are x = 1.0000035820556061 y =  
2.9999977236707336 z = 4.000000399087542
```

❖ Write a program to sort a numerical list in ascending order or descending order through Insertion sort method:

PROGRAM:

```
# SORTING A NUMERICAL LIST IN ASCENDING ORDER OR DESCENDING ORDER THROUGH  
INSERTION SORT METHOD :  
  
print("\n\tSORTING A NUMERICAL LIST IN ASCENDING ORDER OR DESCENDING ORDER  
THROUGH INSERTION SORT METHOD :\n")  
  
xx = [0, -1, -10, 5, 7, 2, 11, 8, 9, -6]  
print("My list :", xx)  
  
for i in range(0, len(xx)):  
    for j in range(i+1, len(xx)):  
        if xx[i] > xx[j]:                # ASCENDING ORDER  
            (xx[i], xx[j]) = (xx[j], xx[i])  
  
print("My list after sorting in ascending order : ", xx)
```

OUTPUT:

```
SORTING A NUMERICAL LIST IN ASCENDING ORDER OR DESCENDING ORDER THROUGH  
INSERTION SORT METHOD :
```

```
My list : [0, -1, -10, 5, 7, 2, 11, 8, 9, -6]
```

```
My list after sorting in ascending order :  [-10, -6, -1, 0, 2, 5, 7, 8, 9, 11]
```

❖ Write a program to sort a numerical list in ascending order or descending order through Bubble sort method:

PROGRAM:

```
# SORTING A NUMERICAL LIST IN ASCENDING ORDER OR DESCENDING ORDER THROUGH
BUBBLE SORT METHOD :

print("\n\tSORTING A NUMERICAL LIST IN ASCENDING ORDER OR DESCENDING ORDER
THROUGH BUBBLE SORT METHOD :\n")

xx = [0, -1, -10, 5, 7, 2, 11, 8, 9, -6]
print("My list :", xx)

for k in range(100):
    for i in range(0, len(xx)-1):
        if xx[i] < xx[i+1]:                # DESCENDING ORDER
            (xx[i], xx[i+1]) = (xx[i+1], xx[i])

print("My list after sorting in descending order : ", xx)
```

OUTPUT:

```
        SORTING A NUMERICAL LIST IN ASCENDING ORDER OR DESCENDING ORDER THROUGH
BUBBLE SORT METHOD :
```

```
My list : [0, -1, -10, 5, 7, 2, 11, 8, 9, -6]
```

```
My list after sorting in descending order :  [11, 9, 8, 7, 5, 2, 0, -1, -6, -10]
```

16. Write a program to determine the bar graph of the number of heads and tails through random number generation:

PROGRAM:

```
# DETERMINATION OF THE BAR GRAPH OF THE NUMBER OF HEADS AND TAILS THROUGH  
RANDOM NUMBER GENERATION :
```

```
import random as rd  
import matplotlib.pyplot as plt
```

```
i = 1  
n = 10000
```

```
ctH = 0.0  
ctT = 0.0
```

```
while i <= n:  
    x = rd.random()  
  
    if x < 0.5:  
        ctH += 1.0  
    else:  
        ctT += 1.0  
  
    i += 1
```

```
xx = [0.35, 0.65]  
yy = [ctH, ctT]  
zz = ["Head", "Tail"]
```

```
plt.bar(xx, yy, width=0.3, color=('r', 'g'))
```

```
plt.suptitle("DETERMINATION OF THE BAR GRAPH OF THE NUMBER OF HEADS AND  
TAILS THROUGH RANDOM NUMBER GENERATION")
```

```
plt.title("No. of Heads : "+str(ctH)+"    No. of Tails : "+str(ctT))
```

```
plt.xticks(xx, zz)
```

```
plt.xlim(-0.1, 1.1)
```

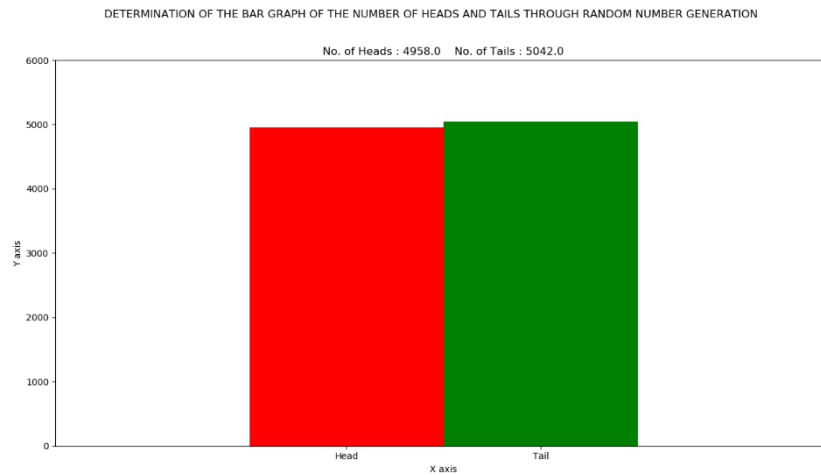
```
plt.ylim(0, n*0.6)
```

```
plt.ylabel("Y axis")
```

```
plt.xlabel("X axis")
```

```
plt.show()
```

OUTPUT:



17. Write a program to determine the bar graph of the number of the number of points in each windows between 0 to 1 through random number generation:

PROGRAM:

```
# DETERMINATION OF THE BAR GRAPH OF THE NUMBER OF POINTS IN EACH WINDOWS  
THROUGH RANDOM NUMBER GENERATION :
```

```
import random as rd  
import matplotlib.pyplot as plt
```

```
n = 1000000
```

```
a = 0  
xx = []  
cc = []  
aa = []  
ss = []  
for i in range(n):  
    x = rd.random()  
    xx.append(x)
```

```

while a < 0.9:
    c = 0
    for i in range(n):
        if (xx[i] > a) & (xx[i] < (a+0.1)):
            c += 1
    cc.append(c)
    s = "$Points No.:" + str(c)
    ss.append(s)

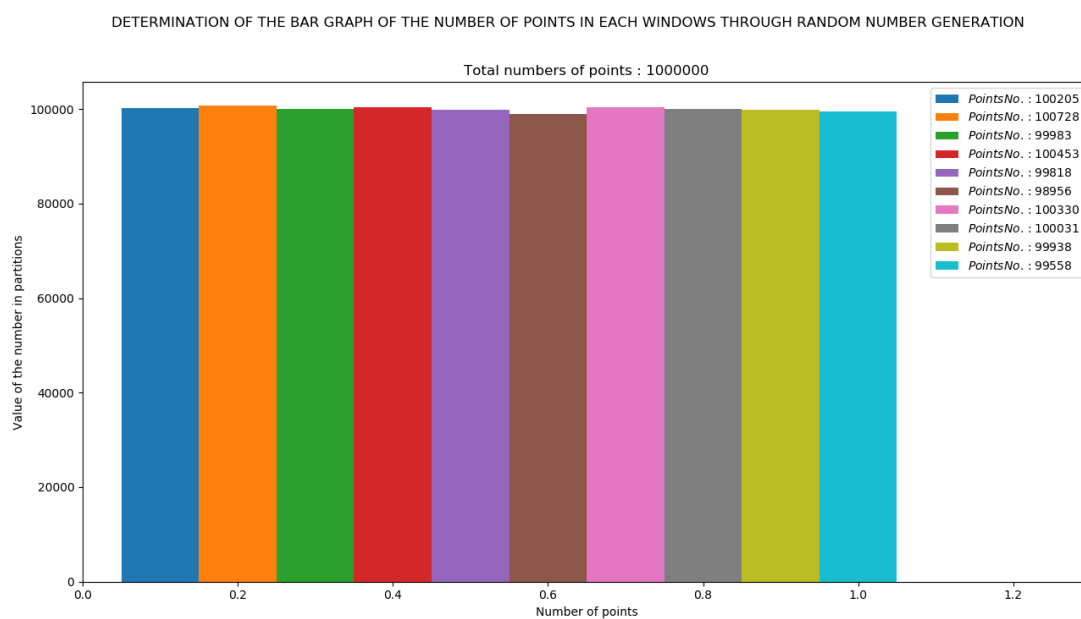
    a += 0.1
    aa.append(a)

for i in range(len(aa)):
    plt.bar(aa[i], cc[i], width=0.1)

plt.suptitle("DETERMINATION OF THE BAR GRAPH OF THE NUMBER OF POINTS IN
EACH WINDOWS THROUGH RANDOM NUMBER GENERATION ")
plt.title("Total numbers of points : "+str(n))
plt.legend(ss)
plt.ylabel("Value of the number in partitions")
plt.xlabel("Number of points")
plt.xlim(0, 1.3)
plt.show()

```

OUTPUT:



❖ Write a program to determine the integration of a function in a given range using Monte Carlo method:

PROGRAM:

```
# DETERMINATION OF THE INTEGRATION OF A FUNCTION THROUGH MONTE CARLO  
RANDOM NUMBER GENERATION METHOD :
```

```
import random as rd  
import matplotlib.pyplot as plt  
import numpy as np
```

```
n = 1000000
```

```
ct = 0.0
```

```
xx = []
```

```
yy = []
```

```
ax = []
```

```
ay = []
```

```
ux = []
```

```
uy = []
```

```
# Suppose,
```

```
# my function is :  $y_i = \exp(-x^2)$ 
```

```
for i in range(n):
```

```
    x = rd.uniform(-2.0, 2.0)
```

```
    y = rd.uniform(0, 1.0)
```

```
    xx.append(x)
```

```
    yy.append(y)
```

```
for i in range(n):
```

```
    if np.exp(-(xx[i])**2) >= yy[i]:
```

```
        ux.append(xx[i])
```

```
        uy.append(yy[i])
```

```
        ct += 1.0
```

```
    else:
```

```
        ax.append(xx[i])
```

```
        ay.append(yy[i])
```

```
a = 4.0*1.0  
s = (ct/n)*a
```

```
plt.plot(ux, uy, '.k', ax, ay, '.r')
```

```
plt.suptitle("DETERMINATION OF THE INTEGRATION OF A FUNCTION THROUGH MONTE  
CARLO RANDOM NUMBER GENERATION METHOD ")
```

```
plt.title("The value of the integration : "+str(s))
```

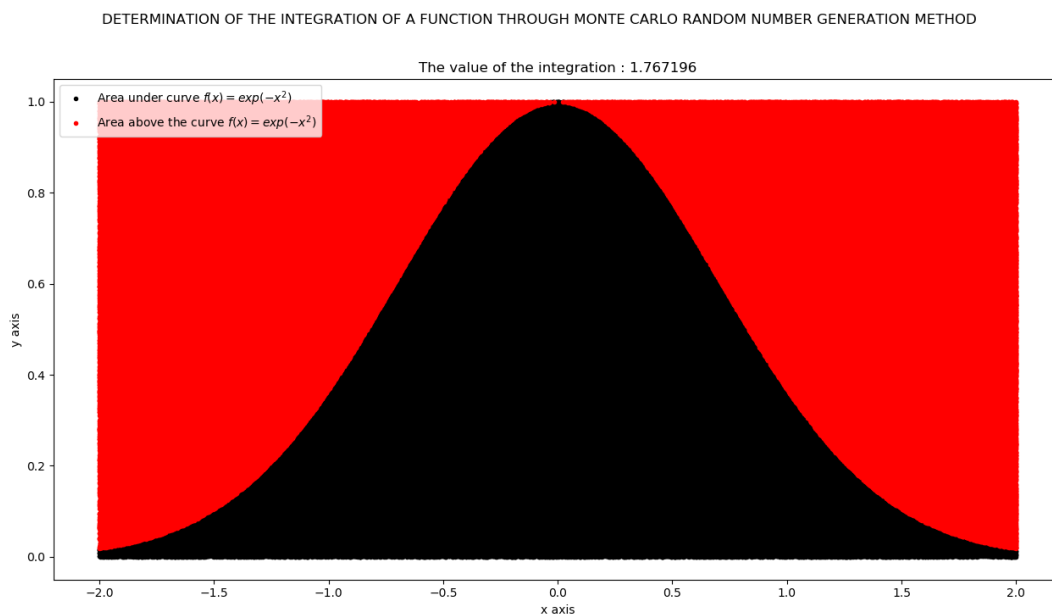
```
plt.xlabel("x axis")
```

```
plt.ylabel("y axis")
```

```
plt.legend(["Area under curve  $f(x)=\exp(-x^2)$ ", "Area above the curve  
 $f(x)=\exp(-x^2)$ "], loc="upper left")
```

```
plt.show()
```

OUTPUT:



❖ Write a program to determine the mean graph of some experimental data (with linear equation) through least square method:

PROGRAM:

```
# DETERMINATION OF THE MEAN GRAPH USING LEAST SQUARE METHOD :

import matplotlib.pyplot as plt
import numpy as np

xx = np.array([-5.0, -4.0, -3.0, -2.0, -1.0, 0.0, 1.0, 2.0, 3.0, 4.0,
5.0])
yy = np.array([-10.0, -7.0, -3.0, 2.0, 6.0, 8.0, 11.0, 13.0, 17.0, 22.0,
25.0])

(sumx, sumy, p1, p2) = (0.0, 0.0, 0.0, 0.0)

for k in range(len(xx)):
    sumx = sumx+xx[k]
    sumy = sumy+yy[k]

xb = sumx/len(xx)
yb = sumy/len(yy)

for l in range(len(xx)):
    p1 = p1+(xx[l]-xb)*(yy[l]-yb)
    p2 = p2+(xx[l]-xb)**2

m = p1/p2
c = yb-m*xb

xx1 = xx
yy1 = m*xx1+c

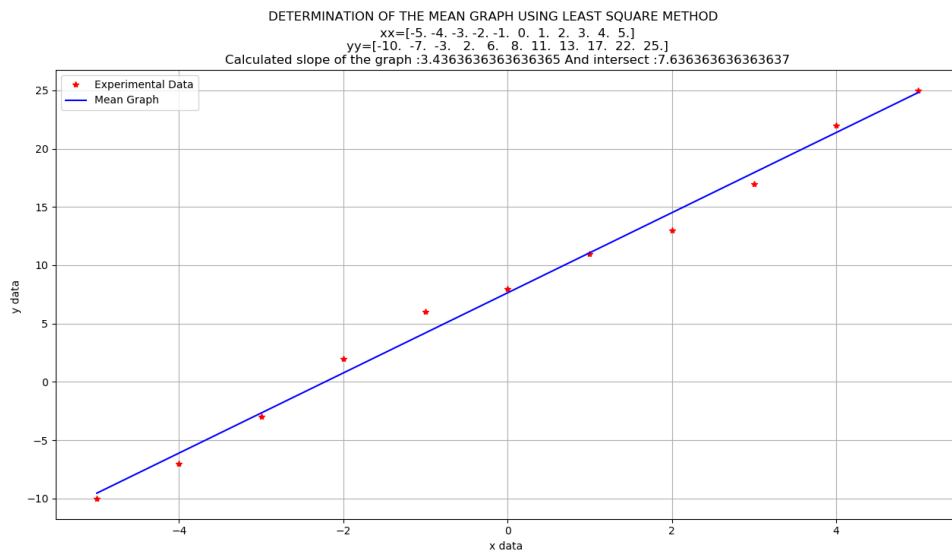
plt.plot(xx, yy, "*r", xx1, yy1, "-b")

plt.xlabel("x data ")
plt.ylabel("y data ")
plt.legend(["Experimental Data", "Mean Graph"])
```

```
plt.suptitle("DETERMINATION OF THE MEAN GRAPH USING LEAST SQUARE METHOD")
plt.title("xx="+str(xx)+"\n yy="+str(yy)+"\nCalculated slope of the graph\n:"+str(m)+" And intersect :"+str(c))
plt.grid()

plt.show()
```

OUTPUT:



❖ **Write a program to determine the value of differentiation of a function at a given point:**

PROGRAM:

```
# DETERMINATION OF THE DIFFERENTIATION OF A FUNCTION AT A GIVEN POINT :
```

```
print("\n\tDETERMINATION OF THE DIFFERENTIATION OF A FUNCTION AT A GIVEN  
POINT :")
```

```
def f(x):
```

```
    return x**3.0-3.0*x
```

```
a = 5.0
```

```
h = 0.00001
```

```
df = (f(a+h)-f(a))/h
```

```
print("So, the differentiated value of the function at the point", a, "is  
: ", df)
```

OUTPUT:

```
DETERMINATION OF THE DIFFERENTIATION OF A FUNCTION AT A GIVEN POINT :
```

```
So, the differentiated value of the function at the point 5.0 is :  
72.00014999710902
```

❖ Write a program to determine the function from it's 1st order differential equation using Euler's method:

PROGRAM:

```
# PLOTTING OF THE FUNCTION THROUGH IT'S 1ST ORDER DIFFERENTIAL EQUATION  
AND BOUNDARY CONDITIONS :
```

```
import matplotlib.pyplot as plt  
import numpy as np
```

```
# BOUNDARY CONDITIONS :
```

```
x = 0.0
```

```
y = 0.0
```

```
xx = []
```

```
yy = []
```

```
n = 500000
```

```
xs = 0.00001
```

```
i = 1
```

```
while i <= n:
```

```
    xx.append(x)
```

```
    yy.append(y)
```

```
    # SLOPE OF THE GRAPH AT BOUNDARY CONDITION:
```

```
    m = np.sin(x)-2*y
```

```
    c = y - m*x
```

```
    x = x + xs
```

```
    y = m*x + c
```

```
    i = i+1
```

```
xx = np.array(xx)
```

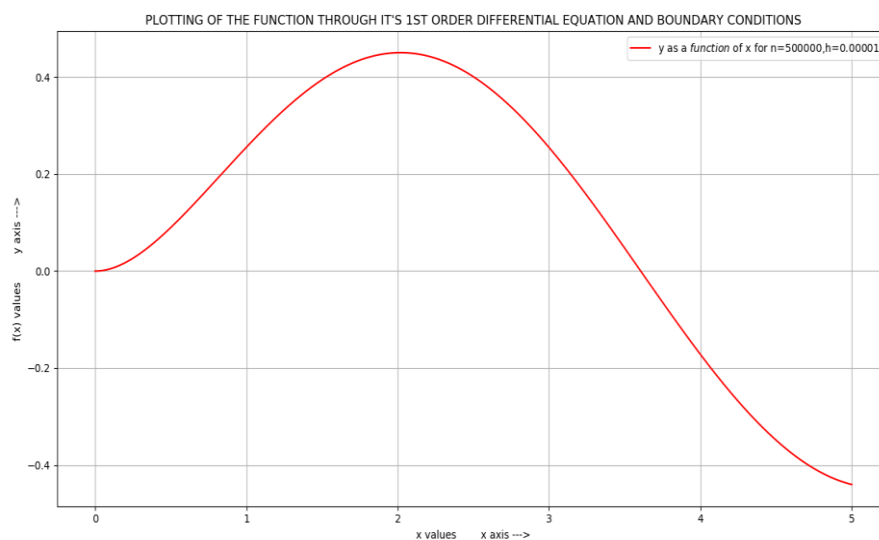
```
yy = np.array(yy)
```

```
plt.plot(xx, yy, 'r')

plt.title("PLOTING OF THE FUNCTION THROUGH IT'S 1ST ORDER DIFFERENTIAL
EQUATION AND BOUNDARY CONDITIONS")
plt.xlabel("x values      x axis ---->")
plt.ylabel("f(x) values      y axis ---->")
plt.legend(["y as a $function$ of x for n=500000,h=0.00001 "], loc="upper
right")
plt.grid()

plt.show()
```

OUTPUT:



- *Changes in the graph due to different values of approximated arc length(h):*

PROGRAM:

```
# PLOTING OF THE FUNCTION THROUGH IT'S 1ST ORDER DIFFERENTIAL EQUATION
AND BOUNDARY CONDITIONS :
```

```
import matplotlib.pyplot as plt
import numpy as np
```

```
# BOUNDARY CONDITIONS :
```

```
x = x1 = 0.0
```

```
y = y1 = 0.0
```

```

xx = []
yy = []

xx1 = []
yy1 = []

n = 1000000
xs = 0.00001

n2 = 50
xs1 = 0.2

i = 1
j = 1
while i <= n:
    xx.append(x)
    yy.append(y)

    # SLOPE OF THE GRAPH AT BOUNDARY CONDITION:
    m = np.sin(x)-2*y

    c = y - m*x
    x = x + xs
    y = m*x + c

    i = i+1

while j <= n2:
    xx1.append(x1)
    yy1.append(y1)

    # SLOPE OF THE GRAPH AT BOUNDARY CONDITION:
    m1 = np.sin(x1)-2*y1

    c1 = y1 - m1*x1
    x1 = x1 + xs1
    y1 = m1*x1 + c1

    j = j+1

```



```

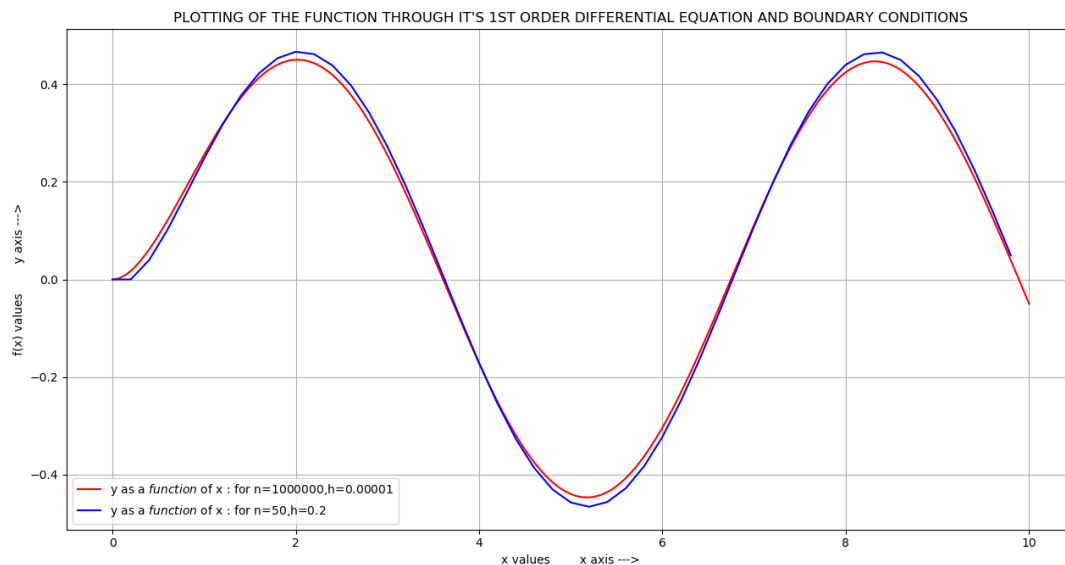
xx1 = np.array(xx1)
yy1 = np.array(yy1)

xx = np.array(xx)
yy = np.array(yy)

plt.plot(xx, yy, 'r')
plt.plot(xx1, yy1, 'b')
plt.title("PLOTING OF THE FUNCTION THROUGH IT'S 1ST ORDER DIFFERENTIAL
EQUATION AND BOUNDARY CONDITIONS")
plt.xlabel("x values          x axis --->")
plt.ylabel("f(x) values          y axis --->")
plt.legend(["y as a $function$ of x : for n=1000000,h=0.00001", "y as a
$function$ of x : for n=50,h=0.2"], loc="lower left")
plt.grid()
plt.show()

```

OUTPUT:



- **Determination of the function of radio-active decay from it's differential equation:**

PROGRAM:

```
# PLOTING OF THE FUNCTION OF RADIO-ACTIVE DECAY :
```

```
import matplotlib.pyplot as plt
```

```

import numpy as np

# BOUNDARY CONDITIONS :
x0 = x = 0.0
y0 = y = 10**5
l = 0.01

thf = 2.303*np.log10(2)/l

xx = []
yy = []

n = 70000
xs = 0.01
i = 1

while i <= n:
    xx.append(x)
    yy.append(y)

    # SLOPE OF THE GRAPH AT BOUNDARY CONDITION:
    m = -l*y

    c = y - m*x
    x = x + xs
    y = m*x + c

    i = i+1

xx = np.array(xx)
yy = np.array(yy)

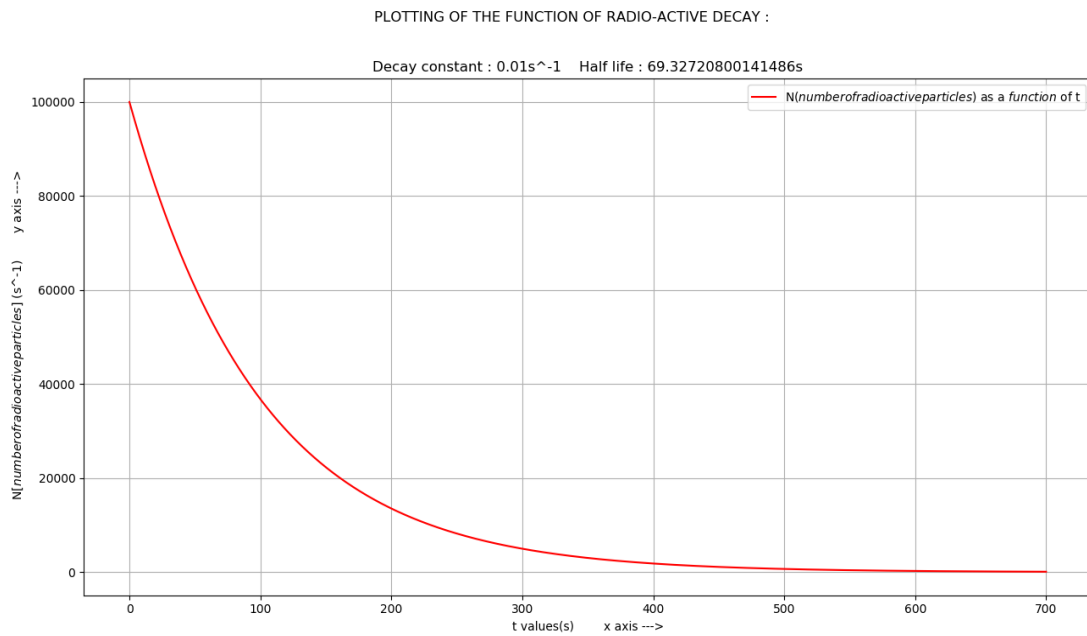
plt.plot(xx, yy, 'r')

plt.suptitle("PLOTING OF THE FUNCTION OF RADIO-ACTIVE DECAY :")
plt.title("Decay constant : "+str(l)+"s^-1    Half life : "+str(thf)+"s")
plt.xlabel("t values(s)          x axis --->")
plt.ylabel(" N[$number of radioactive particles$] (s^-1)          y axis --->")

```

```
plt.legend(["N($number of radioactive particles$) as a $function$ of t"])
plt.grid()
plt.show()
```

OUTPUT:



❖ Write a program to determine the function of harmonic oscillator (second order equation):

PROGRAM:

```
# PLOTTING OF THE DIFFERENTIAL EQUATION OF A HARMONIC OSCILLATOR WITH  
BOUNDARY CONDITIONS :
```

```
import matplotlib.pyplot as plt  
import numpy as np
```

```
# BOUNDARY CONDITIONS :
```

```
x = 0.0
```

```
y = 0.0
```

```
r = 1
```

```
g = np.pi
```

```
w = 10*np.pi
```

```
xx = []
```

```
yy = []
```

```
n = 500000
```

```
xs = 0.00001
```

```
i = 1
```

```
while i <= n:
```

```
    xx.append(x)
```

```
    yy.append(y)
```

```
# SLOPE OF THE GRAPH AT BOUNDARY CONDITION:
```

```
m = r*w*(np.cos(w*x+g))
```

```
c = y - m*x
```

```
x = x + xs
```

```
y = m*x + c
```

```
i = i+1
```

```
xx = np.array(xx)
```

```

yy = np.array(yy)

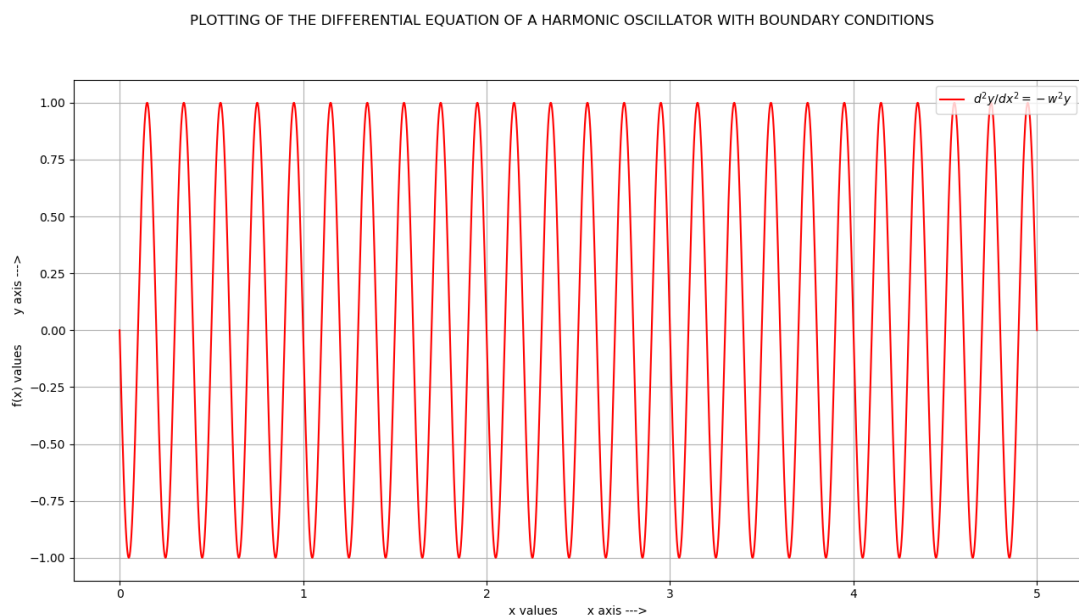
plt.plot(xx, yy, 'r')

plt.suptitle("PLOTING OF THE DIFFERENTIAL EQUATION OF A HARMONIC
OSCILLATOR WITH BOUNDARY CONDITIONS")
plt.xlabel("x values      x axis --->")
plt.ylabel("f(x) values      y axis --->")
plt.legend([" $d^2y/dx^2 = -w^2y$ "], loc="upper right")
plt.grid()

plt.show()

```

OUTPUT:



- **Determination of the function of damped harmonic oscillator:**

PROGRAM:

PLOTING OF THE DIFFERENTIAL EQUATION OF A DAMPED HARMONIC OSCILLATOR WITH
BOUNDARY CONDITIONS :

```

import matplotlib.pyplot as plt
import numpy as np

```

BOUNDARY CONDITIONS :

x = 0

y = 1

```

r = 1
e = 0

w = 100*np.pi
g = 2
w1 = (w-(g**2)/4)**0.5

xx = []
yy = []

n = 500000
xs = 0.00001
i = 1

while i <= n:
    xx.append(x)
    yy.append(y)

    # SLOPE OF THE GRAPH AT BOUNDARY CONDITION:
    m = r*np.exp(-0.5*g*x)*(-0.5*g*np.cos(w1*x+e)-w1*np.sin(w1*x+e))

    c = y - m*x
    x = x + xs
    y = m*x + c

    i = i+1

xx = np.array(xx)
yy = np.array(yy)

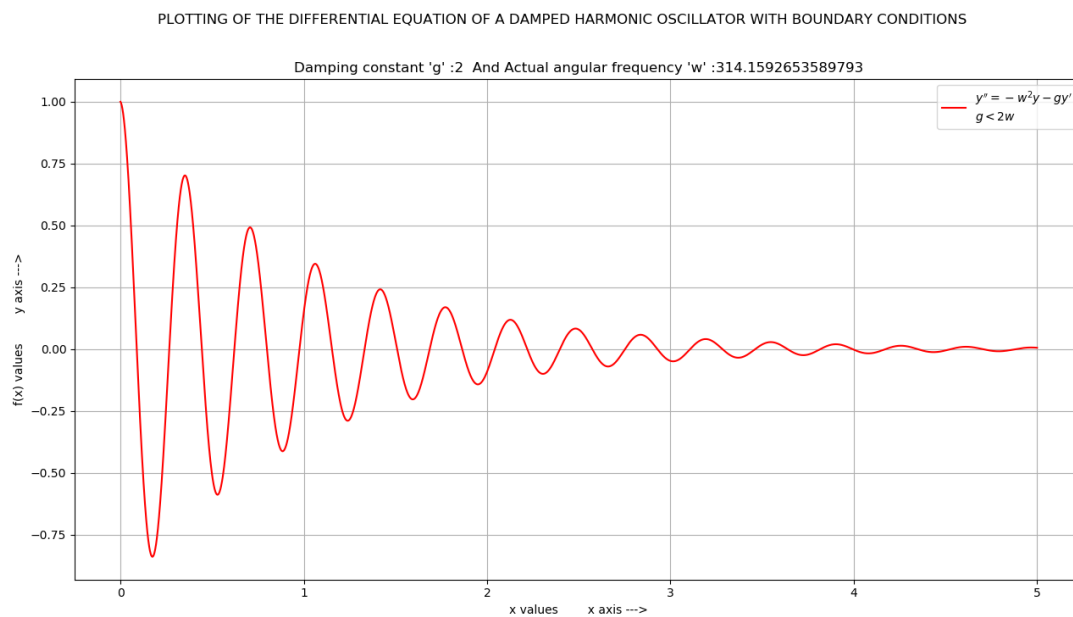
plt.plot(xx, yy, 'r')

plt.suptitle("PLOTING OF THE DIFFERENTIAL EQUATION OF A DAMPED HARMONIC
OSCILLATOR WITH BOUNDARY CONDITIONS")
plt.title("Damping constant 'g' :"+str(g)+" And Actual angular frequency 'w'
:"+str(w))
plt.xlabel("x values          x axis --->")
plt.ylabel("f(x) values          y axis --->")
plt.legend(["$y'' = -w^2y -gy'$\n$g<2w$"], loc="upper right")
plt.grid()

```

plt.show()

OUTPUT:



- **Determination of the function of critically damped harmonic oscillator:**

PROGRAM:

PLOTTING OF THE DIFFERENTIAL EQUATION OF A CRITICALLY DAMPED HARMONIC OSCILLATOR WITH BOUNDARY CONDITIONS :

```
import matplotlib.pyplot as plt
import numpy as np
```

```
# BOUNDARY CONDITIONS :
```

```
x = 0
```

```
y = 1
```

```
a = 1
```

```
b = -10
```

```
w = np.pi/2
```

```
g = 2*w
```

```
xx = []
```

```
yy = []
```

```

n = 500000
xs = 0.00001
i = 1

while i <= n:
    xx.append(x)
    yy.append(y)

    # SLOPE OF THE GRAPH AT BOUNDARY CONDITION:
    m = np.exp(-0.5*g*x)*(-0.5*g*(a+x*b)+b)

    c = y - m*x
    x = x + xs
    y = m*x + c

    i = i+1

xx = np.array(xx)
yy = np.array(yy)

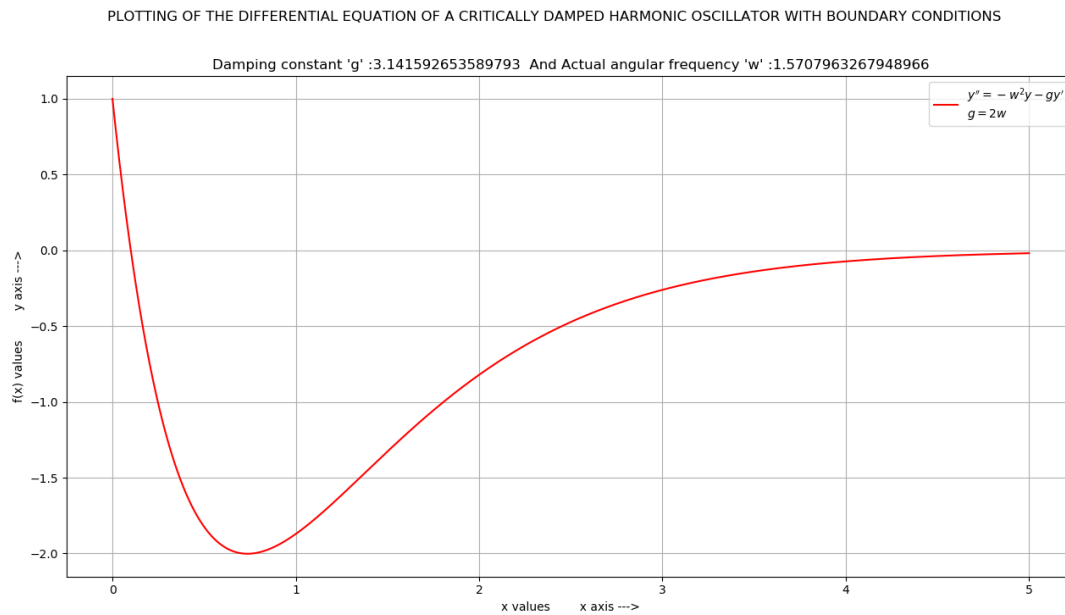
plt.plot(xx, yy, 'r')

plt.suptitle("PLOTING OF THE DIFFERENTIAL EQUATION OF A CRITICALLY DAMPED
HARMONIC OSCILLATOR WITH BOUNDARY CONDITIONS")
plt.title("Damping constant 'g' :"+str(g)+" And Actual angular frequency
'w' :"+str(w))
plt.xlabel("x values          x axis --->")
plt.ylabel("f(x) values          y axis --->")
plt.legend(["$y'' = -w^2y -gy'$\n$g =2w$"], loc="upper right")
plt.grid()

plt.show()

```

OUTPUT:



- **Determination of the function of over-damped harmonic oscillator:**

PROGRAM:

```
# PLOTTING OF THE DIFFERENTIAL EQUATION OF A OVER-DAMPED HARMONIC
OSCILLATOR WITH BOUNDARY CONDITIONS :
```

```
import matplotlib.pyplot as plt
import numpy as np
```

```
# BOUNDARY CONDITIONS :
```

```
x = 0
```

```
y = 1
```

```
a = -1
```

```
b = 2
```

```
w = np.pi
```

```
g = 2.5*w
```

```
k1 = -0.5*g-(0.25*g**2-w**2)**0.5
```

```
k2 = -0.5*g+(0.25*g**2-w**2)**0.5
```

```
xx = []
```

```
yy = []
```

```

n = 500000
xs = 0.00001
i = 1

while i <= n:
    xx.append(x)
    yy.append(y)

    # SLOPE OF THE GRAPH AT BOUNDARY CONDITION:
    m = k1*a*np.exp(k1*x)+k2*b*np.exp(k2*x)

    c = y - m*x
    x = x + xs
    y = m*x + c

    i = i+1

xx = np.array(xx)
yy = np.array(yy)

plt.plot(xx, yy, 'r')

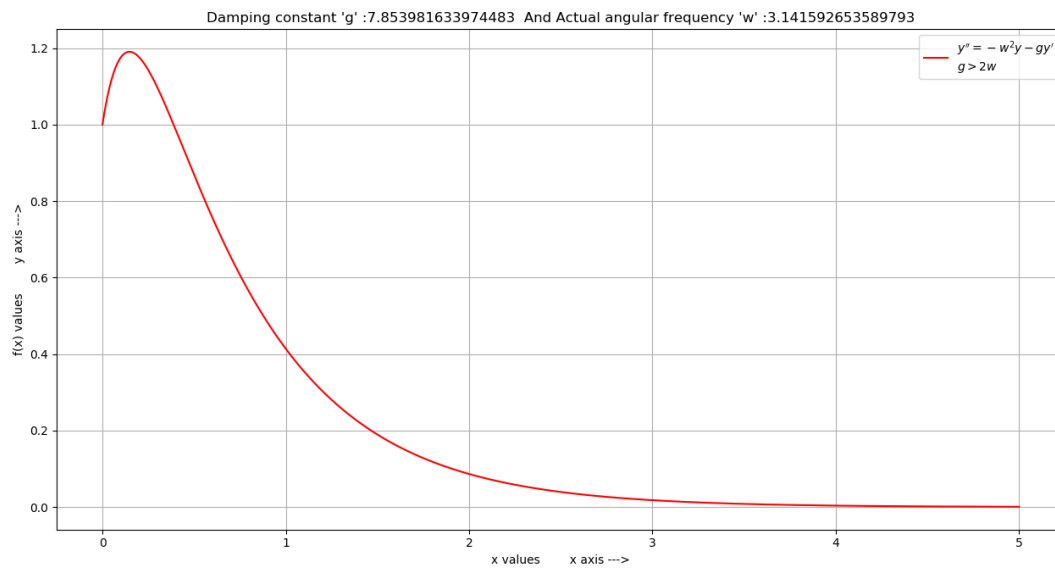
plt.suptitle("PLOTING OF THE DIFFERENTIAL EQUATION OF A OVER-DAMPED
HARMONIC OSCILLATOR WITH BOUNDARY CONDITIONS")
plt.title("Damping constant 'g' :"+str(g)+" And Actual angular frequency
'w' :"+str(w))
plt.xlabel("x values          x axis --->")
plt.ylabel("f(x) values          y axis --->")
plt.legend(["$y'' = -w^2y -gy'$"], loc="upper right")
plt.grid()

plt.show()

```

OUTPUT:

PLOTTING OF THE DIFFERENTIAL EQUATION OF A OVER-DAMPED HARMONIC OSCILLATOR WITH BOUNDARY CONDITIONS



- **Determination of the function of forced harmonic oscillator for a given function $f(x) = 100\sin(x)$:**

PROGRAM:

PLOTTING OF THE DIFFERENTIAL EQUATION OF A FORCED HARMONIC OSCILLATOR WITH BOUNDARY CONDITIONS :

```
import matplotlib.pyplot as plt
import numpy as np
```

BOUNDARY CONDITIONS :

x = 0

y = 0

a = 1

b = 1

r = 1

e = np.pi

w = 5*np.pi

xx = []

yy = []

n = 1000000

```

xs = 0.00001
i = 1
while i <= n:
    xx.append(x)
    yy.append(y)

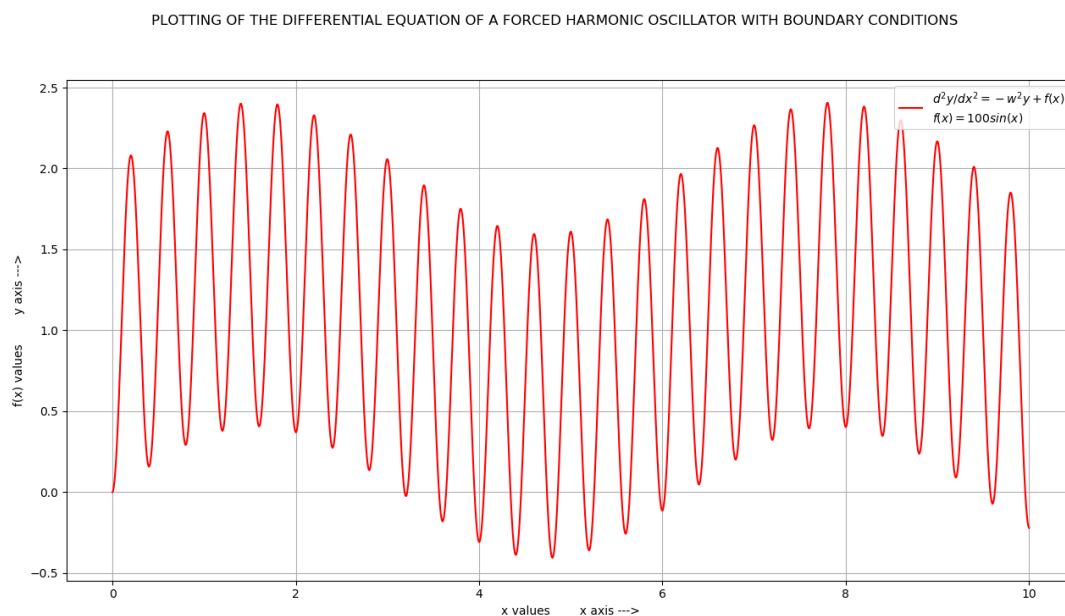
    # SLOPE OF THE GRAPH AT BOUNDARY CONDITION:
    m = -a*r*w*(np.sin(w*x-e)) + b*100*np.cos(x)/(w**2-1)

    c = y - m*x
    x = x + xs
    y = m*x + c
    i = i+1
xx = np.array(xx)
yy = np.array(yy)

plt.plot(xx, yy, 'r')
plt.suptitle("PLOTING OF THE DIFFERENTIAL EQUATION OF A FORCED HARMONIC
OSCILLATOR WITH BOUNDARY CONDITIONS")
plt.xlabel("x values          x axis --->")
plt.ylabel("f(x) values          y axis --->")
plt.legend([" $d^2y/dx^2 = -w^2y + f(x)$ \n $f(x)=100\sin(x)$ "], loc="upper
right")
plt.grid()
plt.show()

```

OUTPUT:



❖ Write a program to determine the Fourier coefficients of a function and the Fourier function :

PROGRAM:

```
# DETERMINATION OF THE FOURIER SERIES COEFFICIENTS FOR GIVAN SITUATION AND
BOUNDARY CONDITIONS :

# AND PLOTTING OF THE EXACT FOURIER FUNCTION FOR DIFFERENT VALUES OF x :

import matplotlib.pyplot as plt
import numpy as np

print("\n\tDETERMINATION OF THE FOURIER SERIES COEFFICIENTS FOR GIVAN
SITUATION AND BOUNDARY CONDITIONS :\n")

xi = -np.pi
xf = np.pi

n = 1000
h = (xf-xi)/(n-1)
nc = 25

xx = np.linspace(xi, xf, n)
# y=xx^3
cc = []
cs = []
yy = []

for i in range(nc):
    yc = (xx**3)*np.cos(i*xx)
    ys = (xx**3)*np.sin(i*xx)

    j = 1
    (sc, ss) = (0, 0)

    while j < (n-1):
        k = j % 2

        if k == 1:
            sc = sc+4*yc[j]
            ss = ss+4*ys[j]
```

```

        else:
            sc = sc+2*yc[j]
            ss = ss+2*ys[j]

        j = j+1

    sc = (yc[0]+yc[n-1]+sc)*h/(3*np.pi)
    ss = (ys[0]+ys[n-1]+ss)*h/(3*np.pi)

    cc.append(sc)
    cs.append(ss)

cc[0] = cc[0]/2
yy = []
zz = xx**3

print("Coefficients of cosine function : ", cc, "\nCoefficients of sine
function : ", cs)

for i in range(n):
    s = 0

    for j in range(nc):
        s = s + cc[j]*np.cos(j*xx[i]) + cs[j]*np.sin(j*xx[i])

    yy.append(s)

plt.plot(xx, yy, '-r', xx, zz, '-.b')

plt.title("FOURIER ANALYSIS OF A FUNCTION"+"Number of points :"+str(n)+"
Number of coefficients :"+str(nc))
plt.xlabel("x Data --->")
plt.ylabel("y Data --->")
plt.legend(["Fourier function", "Actual function"])
plt.grid()

plt.show()

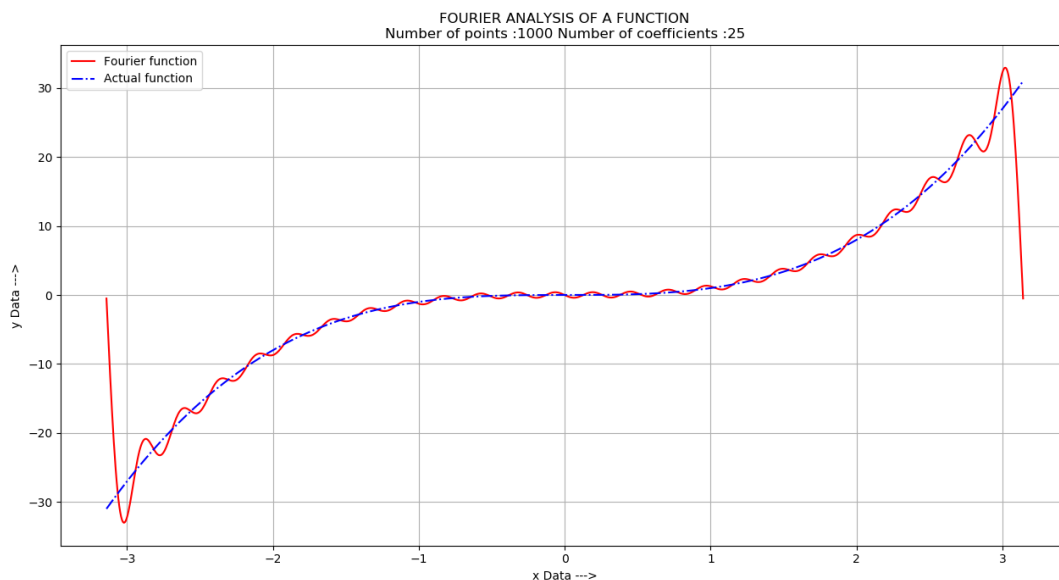
```

OUTPUT:

DETERMINATION OF THE FOURIER SERIES COEFFICIENTS FOR GIVEN SITUATION AND BOUNDARY CONDITIONS :

Coefficients of cosine function : $[-0.010314702969813013, 0.020629405325126794, -0.020629403481579073, 0.020629400408861288, -0.020629396106711818, 0.02062939057478826, -0.020629383812656523, 0.02062937581978188, -0.020629366595531017, 0.020629356139174406, -0.020629344449882958, 0.02062933152673188, -0.020629317368695162, 0.020629301974651567, -0.020629285343379735, 0.020629267473561943, -0.02062924836377787, 0.020629228012511092, -0.02062920641814757, 0.020629183578967373, -0.020629159493153484, 0.020629134158790568, -0.020629107573866382, 0.020629079736260627, -0.02062905064375045]$

Coefficients of sine function : $[0.0, 7.739143732746151, -8.369474261966861, 6.135096613621219, -4.747041920240474, 3.851516408124934, -3.233922152536394, 2.784446048556541, -2.4434430231892748, 2.1761988613315393, -1.9612701434178348, 1.7847419294680844, -1.6372087155875916, 1.5120926871323432, -1.4046592188342801, 1.3114155122617714, -1.2297295766430356, 1.1575810396020911, -1.0933935987834598, 1.0359195875093448, -0.9841587089905687, 0.9372997128754436, -0.8946778079595052, 0.8557430771308967, -0.8200367191523152]$



- **Determination of Fourier coefficients with integration module:**

PROGRAM:

```
# INTEGRATION MODULE (SIMPSON'S 1-THIRD METHOD):
```

```
def integrate(yy, xi, xf, n):
```

```
    h = (xf-xi)/(n-1)
```

```
    (i, s) = (1, 0)
```

```
    while i < (n-1):
```

```

        k = i % 2

        if k == 1:
            s = s+4*yy[i]
        else:
            s = s+2*yy[i]

        i = i+1

s = (yy[0]+yy[n-1]+s)*h/3

return s

# DETERMINATION OF THE FOURIER SERIES COEFFICIENTS FOR GIVAN SITUATION AND
BOUNDARY CONDITIONS :

# AND PLOTTING OF THE EXACT FOURIER FUNCTION FOR DIFFERENT VALUES OF x :

import matplotlib.pyplot as plt
import numpy as np
from integration import *

print("\n\tDETERMINATION OF THE FOURIER SERIES COEFFICIENTS FOR GIVAN
SITUATION AND BOUNDARY CONDITIONS :\n")

xi = -1
xf = 1

xm = -1/2

n = 1000
n1 = int(n*abs((xm-xi)/(xf-xi)))
n2 = int(n*abs((xf-xm)/(xf-xi)))
nc = 25

xx = np.linspace(xi, xf, n)
xx1 = np.linspace(xi, xm, n1)
xx2 = np.linspace(xm, xf, n2)

cc = []

```



```

cs = []
yy = []

for i in range(nc):
    yc1 = -np.cos(np.pi*i*xx1)
    yc2 = np.cos(np.pi*i*xx2)

    ys1 = -np.sin(np.pi*i*xx1)
    ys2 = np.sin(np.pi*i*xx2)

    sc = integrate(yc1, xi, xm, n1) + integrate(yc2, xm, xf, n2)
    ss = integrate(ys1, xi, xm, n1) + integrate(ys2, xm, xf, n2)

    cc.append(sc)
    cs.append(ss)

cc[0] = cc[0]/2
yy = []
zz = np.zeros(n)

print("Coefficients of cosine function : ", cc, "\nCoefficients of sine
function : ", cs)

for i in range(n):
    s = 0

    for j in range(nc):
        s = s + cc[j]*np.cos(np.pi*j*xx[i]) + cs[j]*np.sin(np.pi*j*xx[i])

    yy.append(s)

    if xx[i] <= -1/2:
        zz[i] = -1
    else:
        zz[i] = 1

plt.plot(xx, yy, '-r', xx, zz, '-.b')

```

```
plt.title("FOURIER ANALYSIS OF A FUNCTION"+"\\nNumber of points :"+str(n)+"
Number of coefficients :"+str(nc))

plt.xlabel("x Data --->")
plt.ylabel("y Data --->")
plt.legend(["Fourier function", "Actual function"])
plt.grid()

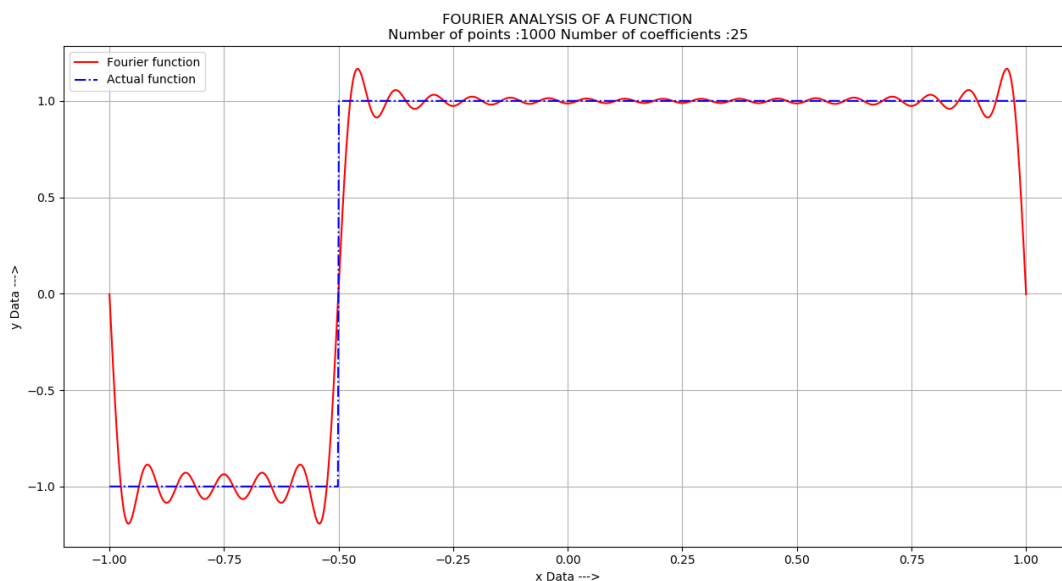
plt.show()
```

OUTPUT:

DETERMINATION OF THE FOURIER SERIES COEFFICIENTS FOR GIVAN SITUATION AND BOUNDARY CONDITIONS :

Coefficients of cosine function : [0.5000008936502575, 0.6372852178571413, -0.0013369007851609278, -0.21153270025425822, 1.7873005150495329e-06, 0.12798095478040422, -0.0013369007851609582, -0.09026334577458535, 1.787300515065579e-06, 0.0713840847007406, -0.0013369007851610035, -0.057183742349962915, 1.7873005150146216e-06, 0.04961085905302777, -0.0013369007851610124, -0.041742087817690655, 1.787300515083902e-06, 0.03807988034526334, -0.0013369007851609907, -0.032798623478153086, 1.787300515209019e-06, 0.030938434469283935, -0.0013369007851609868, -0.026962987604537837, 1.7873005150837937e-06]

Coefficients of sine function : [0.0, 0.6359483283320411, -0.6366113498599372, 0.21286963482139154, -4.504471568258329e-08, 0.1266441103044902, -0.2121813227313752, 0.09160032540521143, -9.011804719941371e-08, 0.07004728530978864, -0.12728183926070374, 0.058520767094192044, -1.3524866468287407e-07, 0.04827410481150616, -0.09088671672375252, 0.04307915775438066, -1.804653481714296e-07, 0.03674317134645662, -0.07065971164332047, 0.03413573871505965, -2.2579704168480092e-07, 0.029601770835470366, -0.05778184774541635, 0.02830014827852027, -2.7127290947448344e-07]



❖ Write a program to determine the maximum, minimum, average, rms value and standard deviation of a function $f(x) = x^2 \exp(-x^2)$ in a given range $[0, \pi]$:

PROGRAM:

```
# DETERMINATION OF THE VALUE OF MAXIMUM, AVERAGE, RMS AND STANDARD  
DEVIATION OF A GIVEN FUNCTION:
```

```
import numpy as np  
from integration import *  
from summation import *
```

```
print("\n\tDETERMINATION OF THE VALUE OF MAXIMUM, AVERAGE, RMS AND  
STANDARD DEVIATION OF A GIVEN FUNCTION:\n")
```

```
xi = 0  
xf = 5
```

```
n = 10000
```

```
xx = np.linspace(xi, xf, n)  
# Suppose,  
# my function is : yi=f(xi)= (x^2)*exp(-x^2)
```

```
yy = (xx**2)*np.exp(-xx**2)  
zz = np.ones(len(xx))
```

```
# MAXIMUM VALUE OF THE FUNCTION:
```

```
max = yy[0]  
for i in range(len(yy)):  
    if max < yy[i]:  
        max = yy[i]
```

```
print("Maximum value of the function in range ", xi, "to ", xf, " :", max)
```

```
# MINIMUM VALUE OF THE FUNCTION:
```

```
min = yy[0]  
for i in range(len(yy)):
```

```

        if min > yy[i]:
            min = yy[i]

print("Minimum value of the function in range ", xi, "to ", xf, " :", min)

# DETERMINATION OF AVG. VALUE:
s = integrate(yy, xi, xf, n)
z = integrate(zz, xi, xf, n)

avg = s/z
print("Average value of the function in range ", xi, "to ", xf, " :", avg)

# DETERMINATION OF THE R.M.S VALUE:
sum = integrate(yy**2, xi, xf, n)
rms = (sum/z)**(1/2)

print("RMS value of the function in range ", xi, "to ", xf, " :", rms)

# DETERMINATION OF THE STANDARD DEVIATION:
v = (yy - avg)**2
add = summation(v,0,n-1,1)

sd = (add/(n-1))**0.5

print("Standard deviation of the function in range ", xi, "to ", xf, " :",
sd)

```

OUTPUT:

DETERMINATION OF THE VALUE OF MAXIMUM, AVERAGE, RMS AND STANDARD DEVIATION OF A GIVEN FUNCTION:

```

Maximum value of the function in range 0 to 5 : 0.36787943381262717
Minimum value of the function in range 0 to 5 : 0.0
Average value of the function in range 0 to 5 : 0.08862564702186711
RMS value of the function in range 0 to 5 : 0.15329847846752445
Standard deviation of the function in range 0 to 5 : 0.1250857465091682

```