Oscillations, Wave and Optics

(SPRING 2025)

ASSIGNMENT-1

Topics: SHO, Damped and Driven Harmonic Oscillation — Total : 40

Date: 3th Feb, 2025 — **Due: 16th Feb, 2025** (EoD)

Part:A [20]

(i) A small body rests on a horizontal diaphragm of a loudspeaker that is supplied with an alternating current of constant amplitude but variable frequency. If the diaphragm executes simple harmonic oscillation in the vertical direction of amplitude $10\mu m$, at all frequencies, find the greatest frequency (in hertz) for which the small body stays in contact with the diaphragm. **2**

(ii) A U-tube of constant cross-sectional area A consists of a horizontal section connected at either end to two vertical sections. Suppose that the tube is filled with an incompressible liquid of mass density ρ. Let the total length of the liquid column be l. (Where l exceeds the length of the horizontal section.) Suppose that the surface of the liquid in one of the vertical sections is initially displaced (vertically) a small distance h from its equilibrium position. Show that the surface displacement subsequently executes simple harmonic oscillation at the angular frequency $\omega = \sqrt{2\,g/l}$, where g is the acceleration due to gravity. **3**

(iii) A particle of mass m slides in a frictionless semi-circular depression in the ground of radius R. Find the angular frequency of small amplitude oscillations about the particle's equilibrium position, assuming that the oscillations are essentially one-dimensional, so that the particle passes through the lowest point of the depression during each oscillation cycle. **3**

(iv) Show that the average speed of a particle executing simple harmonic oscillation is $2/\pi$ times the maximum speed. **3**

(v) A compound pendulum consists of a uniform bar of length l that pivots about one of its ends. Show that the pendulum has the same period of oscillation as a simple pendulum of length $(2/3)l$. **3**

(vi) Show that the ratio of two successive maxima in the displacement of a damped harmonic oscillator is constant. **2**

(vii) An LC circuit is such that at $t = 0$ the capacitor is uncharged and a current I_0 flows through the inductor. Find an expression for the charge Q stored on the positive plate of the capacitor as a function of time. [*Hint : You have to find Q(t) from I(t) or you can solve ODE for Q(t) and find I(t) then apply BCs*] **4**

Part:B [20]

(i) A particle of mass m executes one-dimensional simple harmonic oscillation such that its instantaneous x coordinate is

$$x(t) = a\cos(\omega t - \phi)$$

(a) Find the average values of x, x^2, $\dot{x}$, and $\dot{x}^2$ over a single cycle of the oscillation.
(b) Find the average values of the kinetic and potential energies of the particle over a single cycle of the oscillation. **2+2**

(ii) A particle executing simple harmonic oscillation in one dimension has speeds u and v at displacements a and b, respectively, from its equilibrium position. Show that the period of the motion can be written

$$T = 2\pi\left(\frac{b^2 - a^2}{u^2 - v^2}\right)^{1/2}$$

Show that the amplitude of the motion can be written

$$A = \left(\frac{u^2b^2 - v^2a^2}{u^2 - v^2}\right)^{1/2}$$

2+2

(iii) Imagine a straight tunnel passing through the center of the Earth, which is regarded as a sphere of radius R and uniform mass density. A particle is dropped into the tunnel from the surface. Show that the particle undergoes simple harmonic oscillation at the angular frequency $\omega = \sqrt{g/R}$, where g is the gravitational acceleration at Earth's surface. (Hint: The gravitational acceleration at a point inside a spherically symmetric mass distribution is the same as if all of the mass interior to the point were concentrated at the center, and all of the mass exterior to the point were neglected.) Estimate how long it takes the particle to reach the other end of the tunnel.

Assuming that the tunnel is smooth (i.e., ignoring friction), show that motion is simple harmonic even if the tunnel does not pass through the center of the Earth, and that the travel time from one end of the tunnel to the other is the same as before. **2+2**

(iv) According to classical electromagnetic theory, an accelerated electron radiates energy at the rate Ke^2a^2/c^3, where $K = 6\times10^9\mathrm{Nm}^2/\mathrm{C}^2, \mathrm{e}$ is the charge on an electron, a the instantaneous acceleration, and c the velocity of light in vacuum.

(a)If an electron were oscillating in a straight line with displacement $x(t) = A\sin(2\pi ft)$, how much energy would it radiate away during a single cycle?

(b)What is the effective Q_f of this oscillator?

(c)How many periods of oscillation would elapse before the energy of the oscillation was reduced to half of its initial value?

(d)Substituting a typical optical frequency (e.g., for green light) for f, give numerical estimates for the Q_f and half-life of the radiating system. **8**