Oscillations, Wave and Optics

(SPRING 2025)

ASSIGNMENT-0

Topics: Basic Math (Calculus, Taylor Series, Fourier Series, ODE) Total Marks: 50

Date: 23nd Jan, 2025 **Due: 23rd Feb, 2025** (EoD)

Problem-1 [10]

(a) Solve $4y'' + 4y' + 37y = 0$ and find y(x) for the given boundary conditions:
(i) $y(x = 0) = 0$,(ii) $y(x = \frac{\pi}{6}) = exp(-\frac{\pi^2}{12})$.
Crosscheck the solution; check whether your solution satisfies the ODE. **3+2**

(b) Make a hand-drawn plot of the solution in the x-y plane using the reference informations:
(i) $exp(-\frac{1}{2}) \approx 0.6$, (ii) $exp(-\frac{5}{4}) \approx 0.3$, (iii) $exp(-\frac{9}{4}) \approx 0.1$
Also briefly mention how you are using the provided informations for plotting. **2+1**

(c) Taylor expand the solution about x=0. (*Error/deviation of the order $x^5$is acceptable*).
Also, estimate the leading order error term in your truncated Taylor series at x=1. **2**

Problem-2 [10]

(a) (i) $x = sint$, (ii) $y = cos2t$
plot these two equations in the t-x and t-y planes, respectively. (*In range* $t = [0, 2\pi]$). **2+2**

(b) For a constant t, you will get the x-value and y-value using those two equations. Use a set of t-values to get a set of x-values and y-values. Use those x-values and y-values to find the trajectory of the particle in the x-y plane. **3**

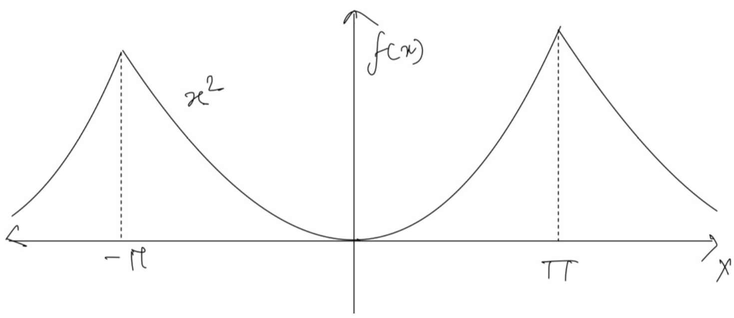
(c) You can also use the trigonometric identities to solve those two equations for t to get y as a function of x. Plot that function in the respective limits of x and y. And check whether the plot is equivalent to the plot in section-(b). **3**

Problem-3 [10]

(a) Integrate the function $f(x) = x^2$ for the range of $x = [-\pi, \pi]$. **1**

(b) considering $f(x)$ to be periodic, that means $f(x + 2\pi) = f(x)$.
Find the Fourier series of the function. Determine whether it is necessary to evaluate the sine integral as part of the process, and justify your answer with proper reasoning. **3+1**

(c) Determine the number of terms n in the Fourier series expansion such that the leading-order error is less than 0.1% of the value of the truncated Taylor series at $x = \pi$. **3**



(d) Integrate the truncated Fourier series in the same limit of x and determine the deviation with respect to the integration result at part-(a). **2**

[You can use some advanced calculator or write a few lines of code to perform the term-wise summation. Just mention how you are doing the calculations.]

Problem-4: Calculate Integrals **[10]**

1. $\int (2cos2x - sin2x)\; e^{-x}\; dx$
2. $\int\; sinx\; sin5x\; cos2x\; dx$
3. $\int_0^\infty sinh3x\; e^{-2x}\; dx$
4. $\int_0^\pi cos2x\; dx$
5. $\int_0^\pi cos^2 2x\; dx$

Problem-5: Solve the equations and find the roots **[6]**

1. $\int f(x)\; dx = cos4x + sin^2 2x - 1$ find $f(x)$.
2. $\int f(x)\; dx = cos2x\; e^{-x} + t^5$ *[t is independent of x]* find $f(x)$.
3. $cos4x + sin^2 2x - 1 = 0$ find roots/ general solution.

Problem-6: **[4]**

Prove the relation:

$$cos\omega t + cos(\omega t - \phi) + cos(\omega t - 2\phi) + ... + cos(\omega t - (n-1)\phi) = \frac{sin(\frac{n\phi}{2})}{sin(\frac{\phi}{2})} cos(\omega t - \frac{1}{2}(n-1)\phi)$$

Hint: Try to use complex definition of $cos\theta$, rearrange the terms and use geometric series formula.