CS5691: Pattern Recognition and Machine Learning

Programming Assignment 1

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Motivation

To reconstruct the given image using SVD, EVD and compare the results obtained

Figure 1: Original image on which reconstruction is performed

1 Eigen Value decomposition (EVD)

Let A be an $n \times n$ square matrix, if $\vec{e}$ is an $n \times 1$ vector such that

$$A\vec{e} = \lambda\vec{e}$$

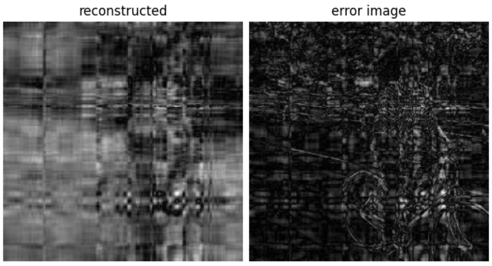
then $\vec{e}$ is an eigen vector of A and λ is the corresponding eigen value. We can write the following equation:

$$AX = X\Lambda$$

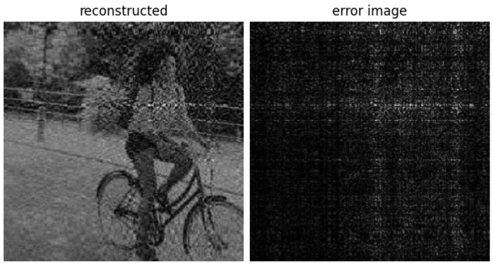
where columns of X are the eigen vectors of A and Λ is a diagonal matrix with corresponding eigen values. If eigen vectors are linearly independent, X^{-1} exists, and A can be decomposed as

$$A = X\Lambda X^{-1}$$

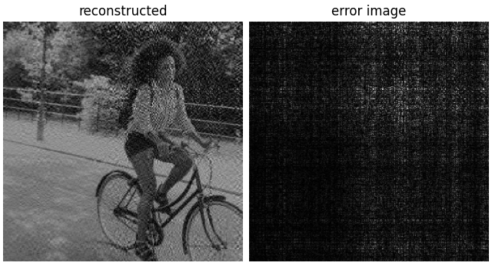
We are given a 256×256 pixels grayscale image, eigen values are sorted in descending order of magnitude and image is reconstructed from the first k eigen vectors, the following results are obtained for various values of k:



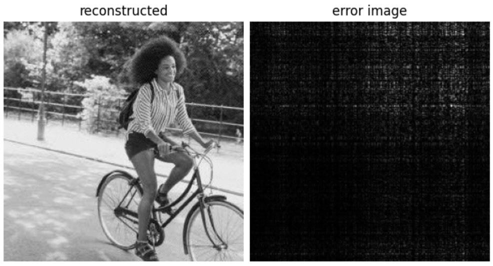
(a) k=10



(b) k=50



(c) k=100



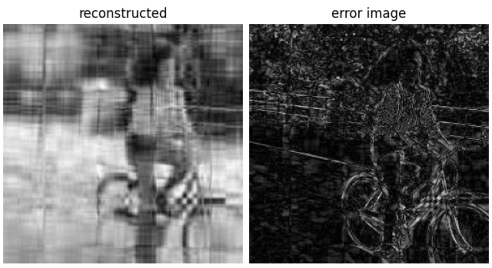
(d) k=200

2 Singular value decomposition (SVD)

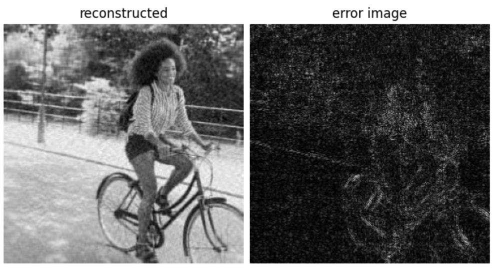
Let A be an $m \times n$ real valued matrix, then A can be decomposed as follows:

$$A = U\Sigma V^H$$

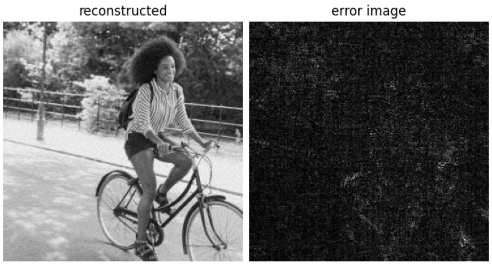
where U, V are unitary matrices of sizes $m \times m$ and $n \times n$ respectively and Σ is an $m \times n$ diagonal matrix with only non negative values which are called singular values. These singular values are sorted in descending order and the same image is reconstructed using top k singular values, the following results are obtained on various values of k:



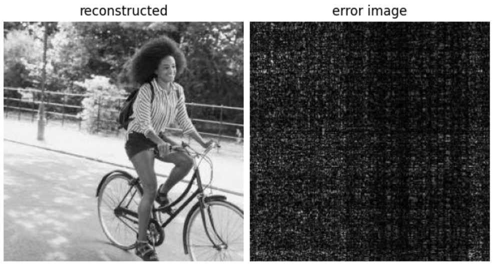
(e) k=10



(f) k=50



(g) k=100



(h) k=200

Frobenius norm is computed between the original image and reconstructed image for $0 \leq k \leq 255$ and plotted for both types of reconstruction as follows:

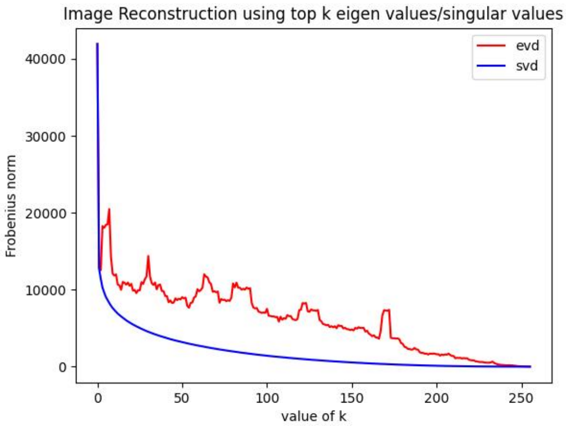


Figure 2: Error in reconstruction as a function of k

Inferences

- Reconstruction error decreases as the value of k is increased
- For a given value of k, better reconstruction is obtained in svd as compared to evd
- In case of svd, 95% of image is reconstructed just by the top $k = 100$ singular values.