$$\nu(\tau) = \cosh^{-1}\left[\frac{c^2\tau^2-(r_S^2+r_R^2+(z_R-z_S)^2)}{2r_Sr_R}\right]$$

$$\beta(\tau) = \beta_{++}(\tau) + \beta_{+-}(\tau) + \beta_{-+}(\tau) + \beta_{--}(\tau)$$

$$\beta_{\pm\pm}(\tau) = \frac{\sin[\nu(\pi\pm\theta_S\pm\theta_R)]}{\cosh[\nu\eta(\tau)]-\cos[\nu(\pi\pm\theta_S\pm\theta_R)]}$$

which expands to:

$$\beta{+}{+}(\tau) = \frac{\sin[\nu(\pi+\theta_S+\theta_R)]}{\cosh[\nu\eta(\tau)]-\cosh[\nu(\pi+\theta_S+\theta_R)]}$$

$$\beta{+}{-}(\tau) = \frac{\sin[\nu(\pi+\theta_S-\theta_R)]}{\cosh[\nu\eta(\tau)]-\cosh[\nu(\pi+\theta_S-\theta_R)]}$$

$$\beta{-}{-}(\tau) = \frac{\sin[\nu(\pi-\theta_S-\theta_R)]}{\cosh[\nu\eta(\tau)]-\cosh[\nu(\pi-\theta_S-\theta_R)]}$$

$$\beta{-}{+}(\tau) = \frac{\sin[\nu(\pi-\theta_S+\theta_R)]}{\cosh[\nu\eta(\tau)]-\cosh[\nu(\pi-\theta_S+\theta_R)]}$$

$$h_d(\tau) = -\frac{c\nu}{2\pi}\frac{\beta(\tau)}{r_Sr_R}\sinh\eta(\tau)H(\tau-\tau_0)$$