R-Based Statistical Distribution Analysis Toolkit

Overall Summary for Sampling Distributions

The normality of the sampling distribution of a sample statistic depends on three key factors:

1. **Nature of the parent distribution** (e.g., symmetric like Normal, or skewed like Exponential).
2. **Type of statistic** (e.g., mean, median, IQR, standard deviation, etc.).
3. **Sample size (n)**, with larger sample sizes generally aiding convergence to normality.

General Findings:

- **Statistics with Quick Normality:**
  - **Mean:** Achieves normality the fastest across most distributions, starting at small sample sizes ($n \geq 10$ to $n \geq 100$).
  - **Standard Deviation (SD):** Achieves normality relatively quickly ($n \geq 50$ to $n \geq 500$) for many distributions, but lags behind the mean.
  - **Median:** Achieves normality for symmetric distributions like Normal, but requires larger sample sizes ($n \geq 50$ to $n \geq 1000$) for skewed distributions like Exponential and Beta.
- **Statistics Requiring Larger Samples:**
  - **IQR:** Requires $n \geq 500$ or more to approximate normality due to sensitivity to variability in distributions like Exponential and Beta.
- **Statistics That Rarely Achieve Normality:**
  - **Minimum and Maximum:** Consistently fail to achieve normality for most distributions due to their dependence on extreme values, especially in heavy-tailed distributions like Cauchy.
- **Parent Distribution Impact:**
  - Symmetric distributions (e.g., Normal, Beta with $\alpha=\beta$) show faster convergence to normality for most statistics.
  - Skewed or heavy-tailed distributions (e.g., Exponential, Cauchy) delay or prevent normality for statistics like median, IQR, and extreme values.

Notable Situations:

1. **Cauchy Distribution:**
   - Mean and SD never achieve normality which is opposite of others. Only the median and IQR converge at larger sample sizes.
2. **Beta Distributions**
   - Symmetric Beta distributions ($\alpha=\beta$) show faster convergence to normality compared to highly skewed Beta distributions ($\alpha\neq\beta$). For example:
     - $\alpha=2$, $\beta=2$: Mean achieves normality at $n\geq10$.
     - $\alpha=0.2$, $\beta=5$: Mean require $n\geq1000$.
3. **Poisson Distribution:**
   - Higher values of λ (e.g., $\lambda=25$) lead to faster convergence to normality for the mean and SD ($n\geq10$ to $n\geq50$), while smaller λ (e.g., $\lambda=0.001$) delays convergence significantly ($n\geq10{,}000$)
4. **Gamma Distribution:**
   - Minimum achieved normality for $n \geq 10$ but lose normality again after $n = 1000$, showing a non-monotonic pattern in convergence.

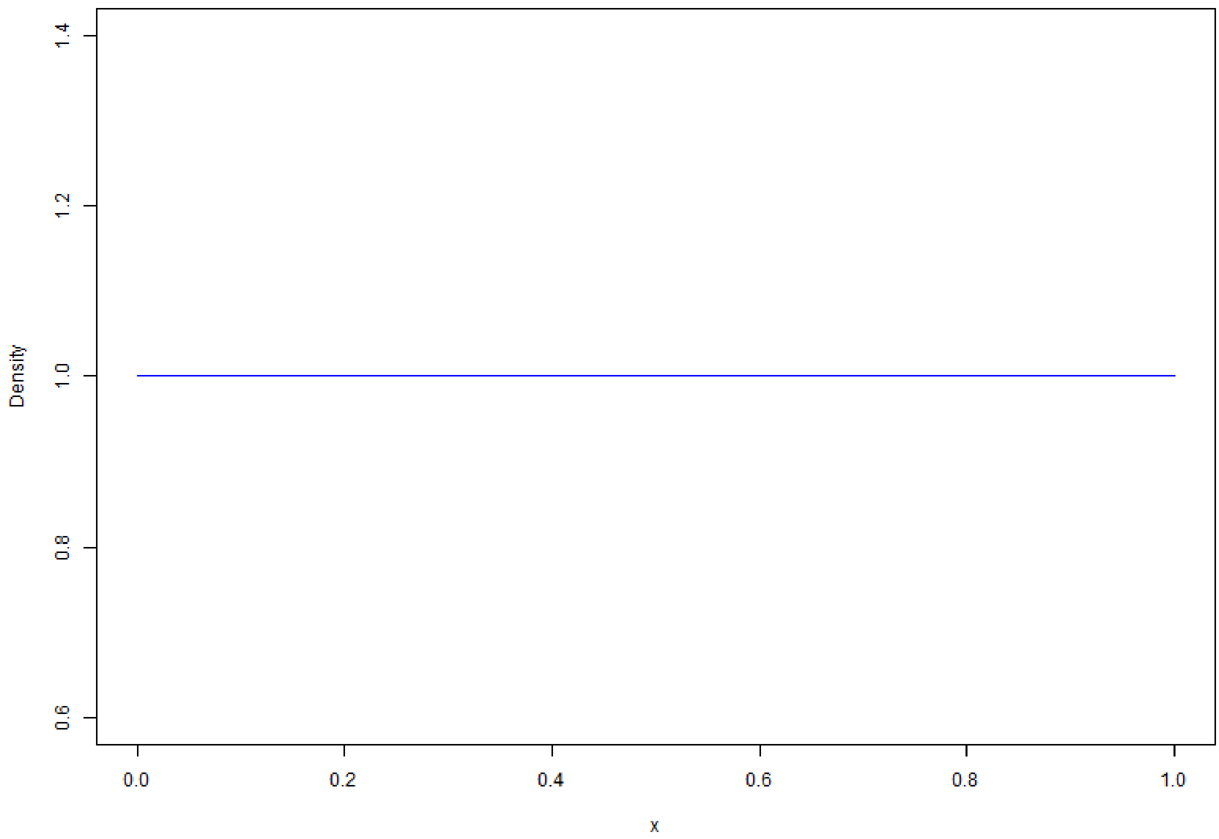
Summary of Key Results:

- The mean is the most reliable statistic for achieving normality, converging quickly for symmetric and moderately skewed distributions.
- Extreme-value statistics like minimum and maximum almost never achieve normality.
- Distributions with higher symmetry (e.g., Normal, Beta with $\alpha=\beta$) lead to faster and more consistent normality.
- Heavy-tailed distributions (e.g., Cauchy) or skewed distributions (e.g., Exponential, Beta with $\alpha\ll\beta$) require much larger sample sizes for most statistics to approximate normality.

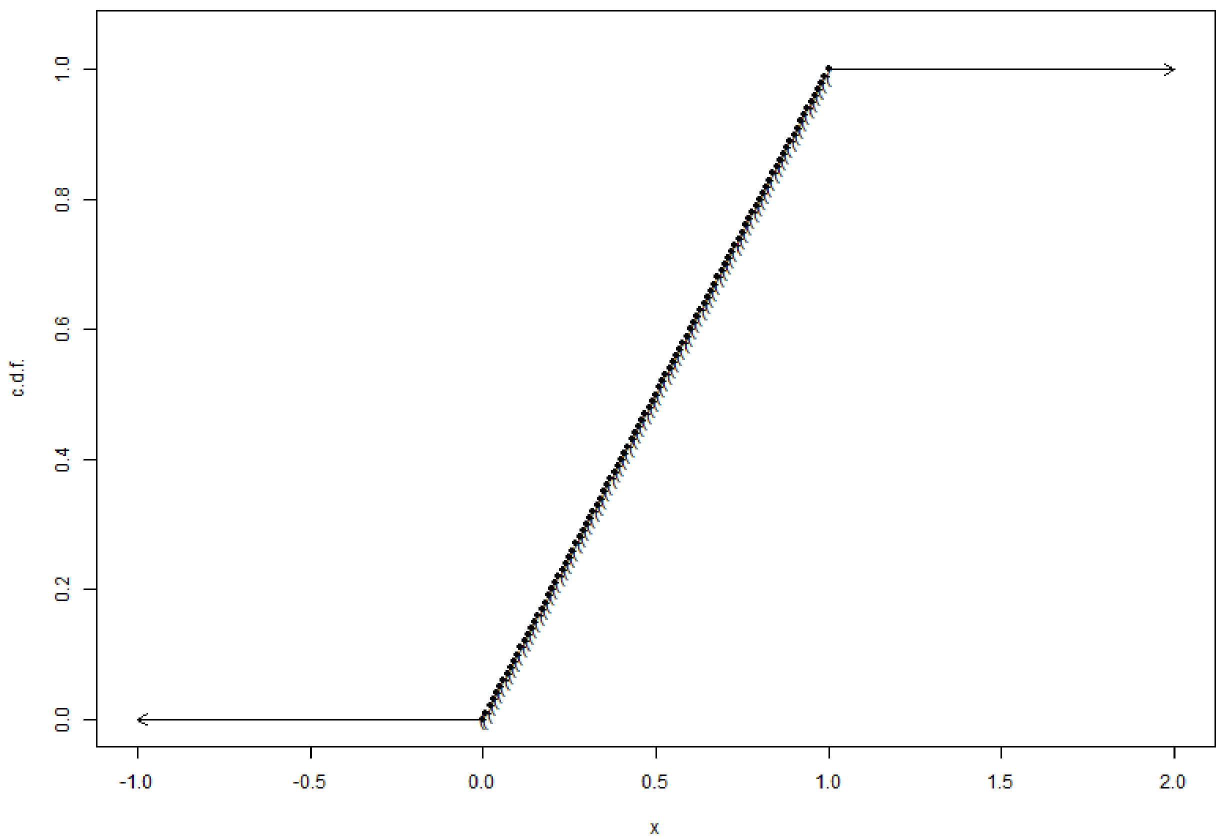
The detailed report and plots of distributions are on the following pages.

UNIFORM DISTRIBUTION (0,1)

PDF of Uniform(0,1)



CDF of Uniform(0,1)



UNIFORM DISTRIBUTION

	Values of n to achieve normality (nn=1000, a=0, b=1)							
Statistic	n=10	n=50	n=100	n=500	n=1000	n=5000	Any n	Min(n)
Mean	Yes	Yes	Yes	Yes	Yes	Yes	Yes	10
Median	Yes	Yes	Yes	Yes	Yes	Yes	Yes	10
Std Dev	Yes	Yes	Yes	Yes	Yes	Yes	Yes	10
Min	No	No	No	No	No	No	No	NA
Max	No	No	No	No	No	No	No	NA
IQR	Yes	Yes	Yes	Yes	Yes	Yes	Yes	10

Conclusion for Exponential Distribution

Normality Achieved:

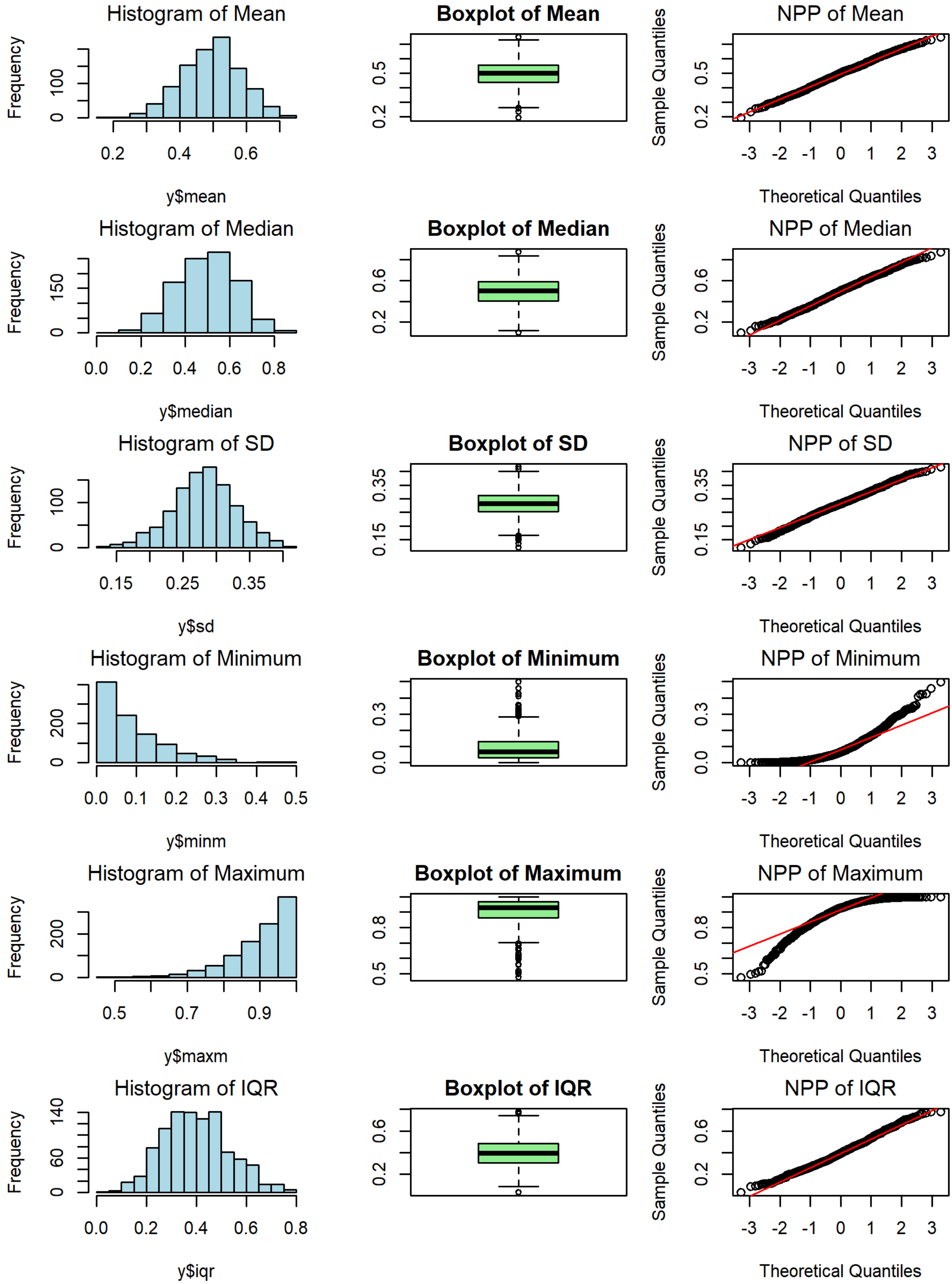
- **Mean**: Achieves normality for n≥10, converging very quickly due to the Central Limit Theorem (CLT).
- **Median**: Achieves normality for n≥10, reflecting rapid convergence even in the presence of skewness.
- **Standard Deviation (SD)**: Achieves normality for n≥10, indicating strong consistency across sample sizes.
- **IQR**: Achieves normality for n≥10, showing fast convergence despite the Uniform distribution's variability.

Normality Not Achieved:

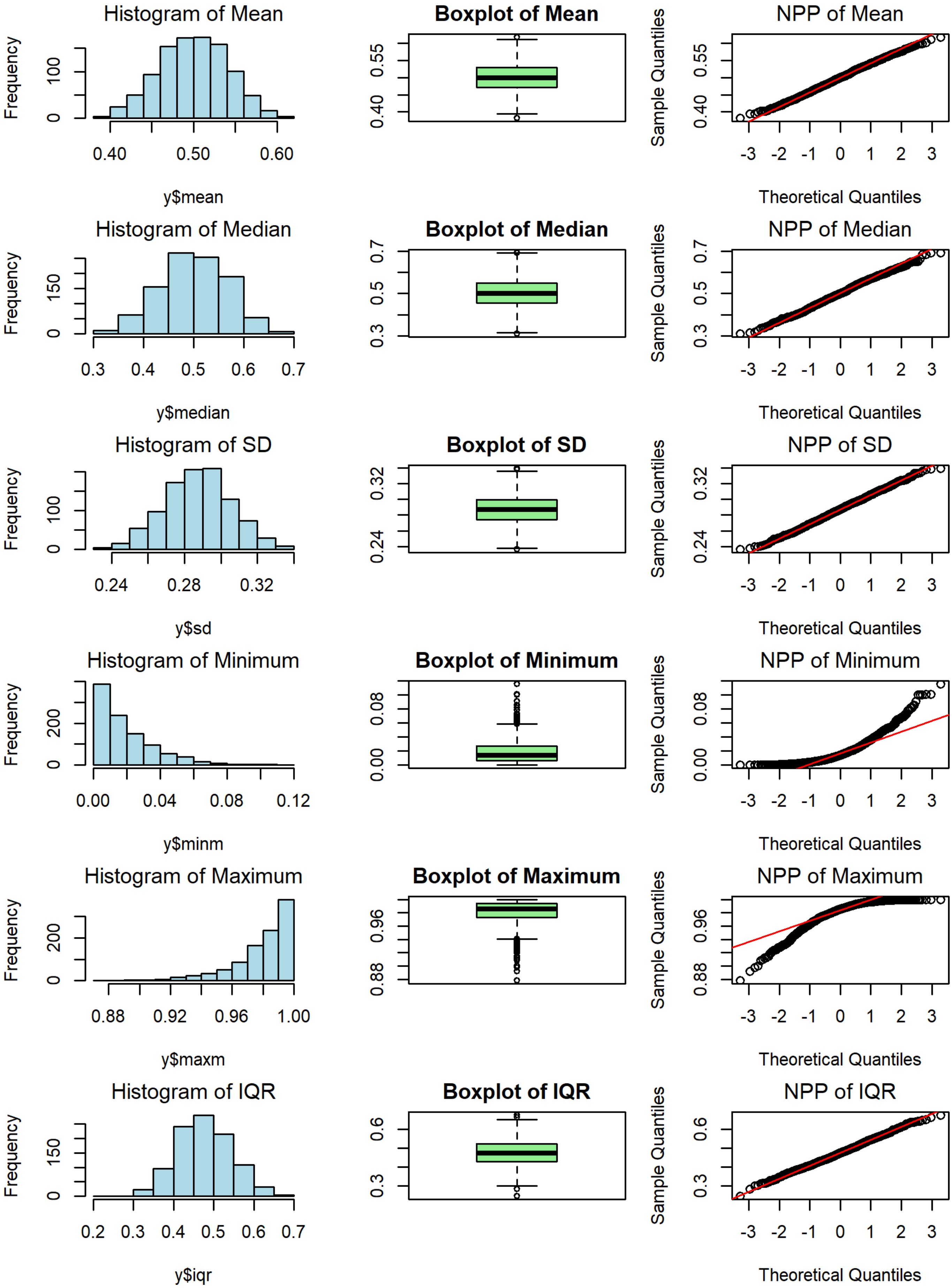
- **Minimum and Maximum**: Do not achieve normality for any n, as they are highly influenced by extreme values and the Uniform distribution's inherent asymmetry.

Overall:
The mean, median, standard deviation, and IQR converge to normality extremely quickly, requiring only small sample sizes (n≥10). However, the minimum and maximum fail to achieve normality due to their sensitivity to extreme values and the skewed nature of the Uniform distribution.

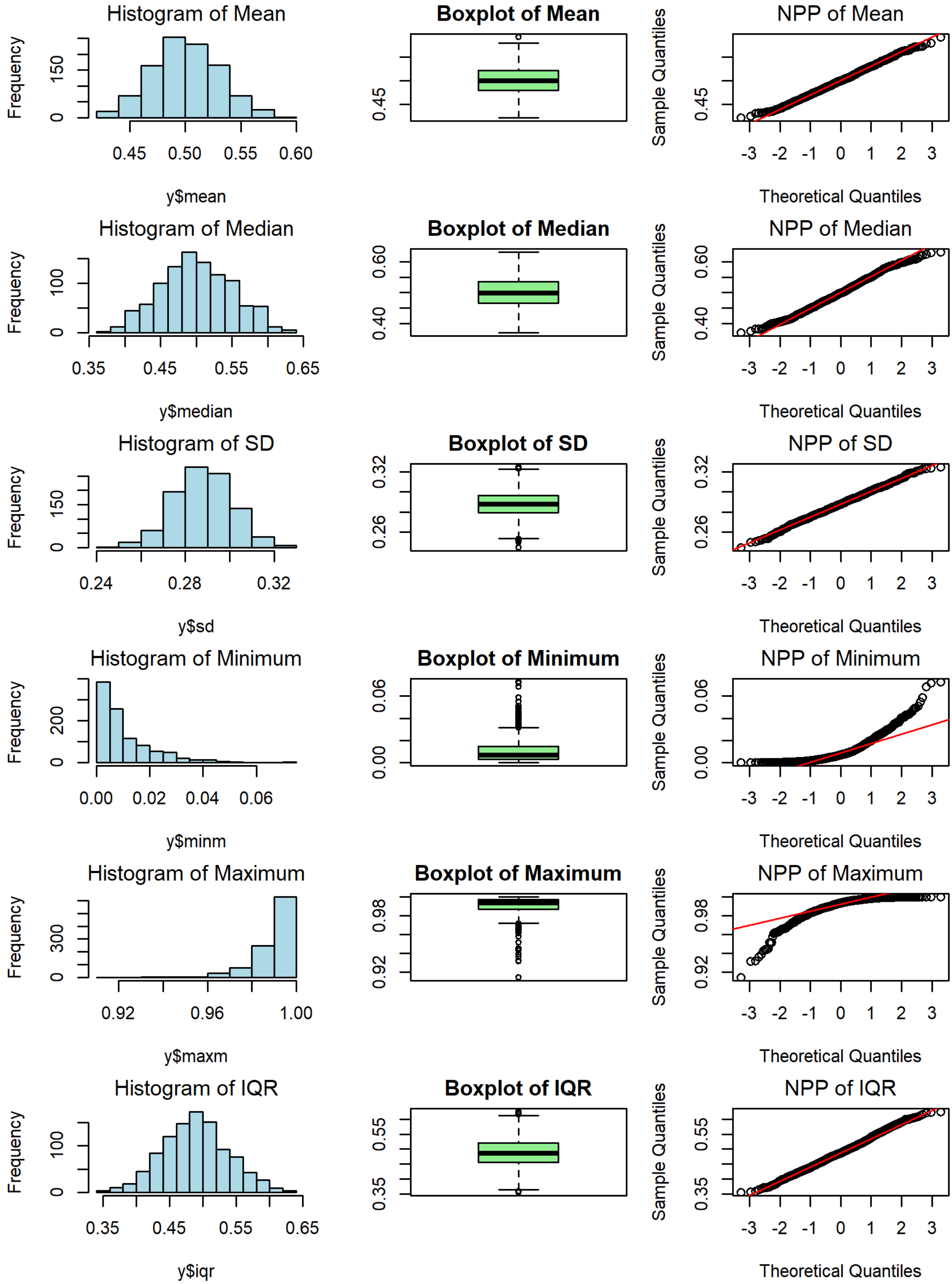
UNIFORM DISTRIBUTION PLOT
(n=10, nn=1000, a=0, b=1)



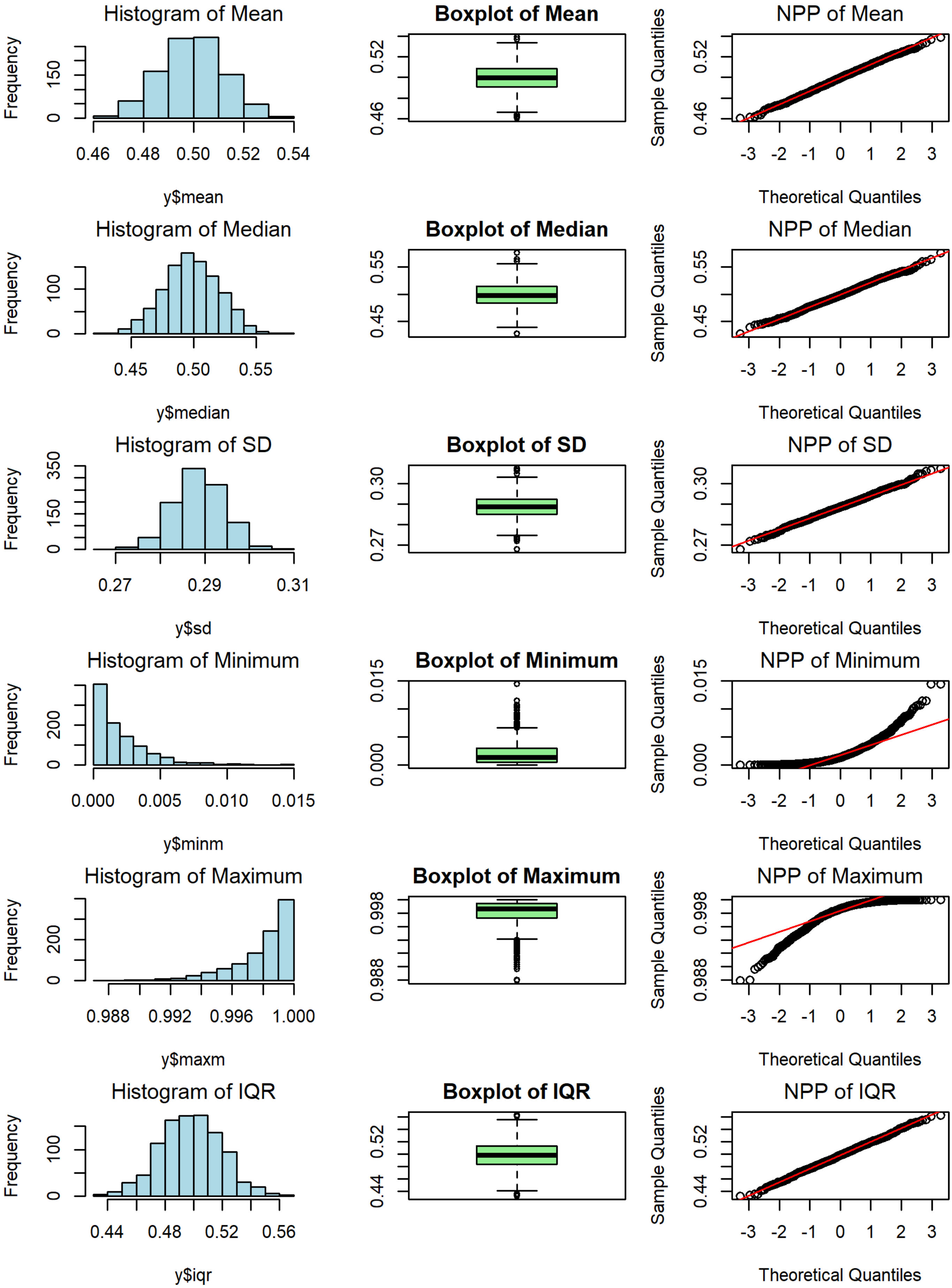
UNIFORM DISTRIBUTION PLOT
(n=50, nn=1000, a=0, b=1)



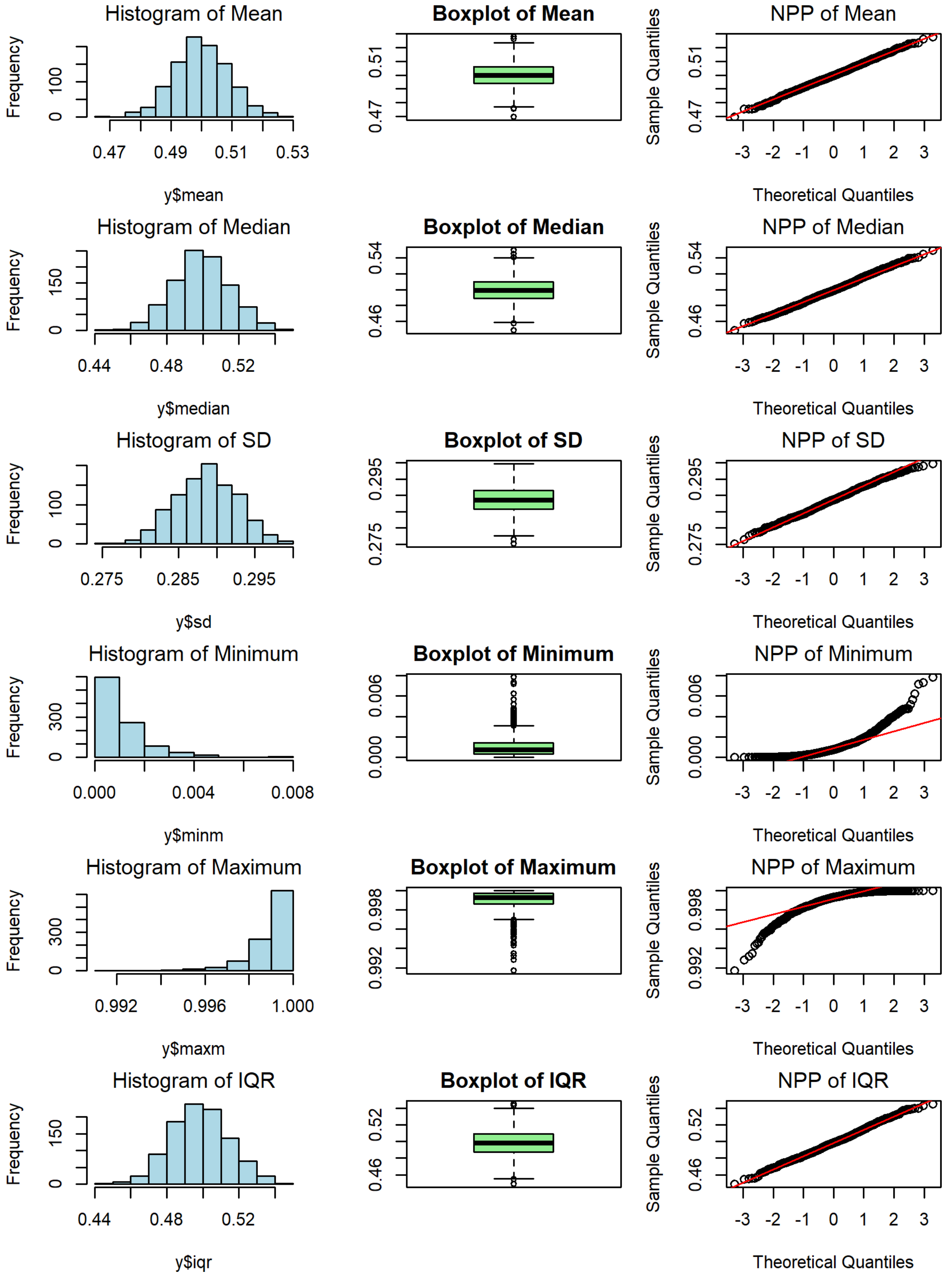
UNIFORM DISTRIBUTION PLOT
(n=100, nn=1000, a=0, b=1)

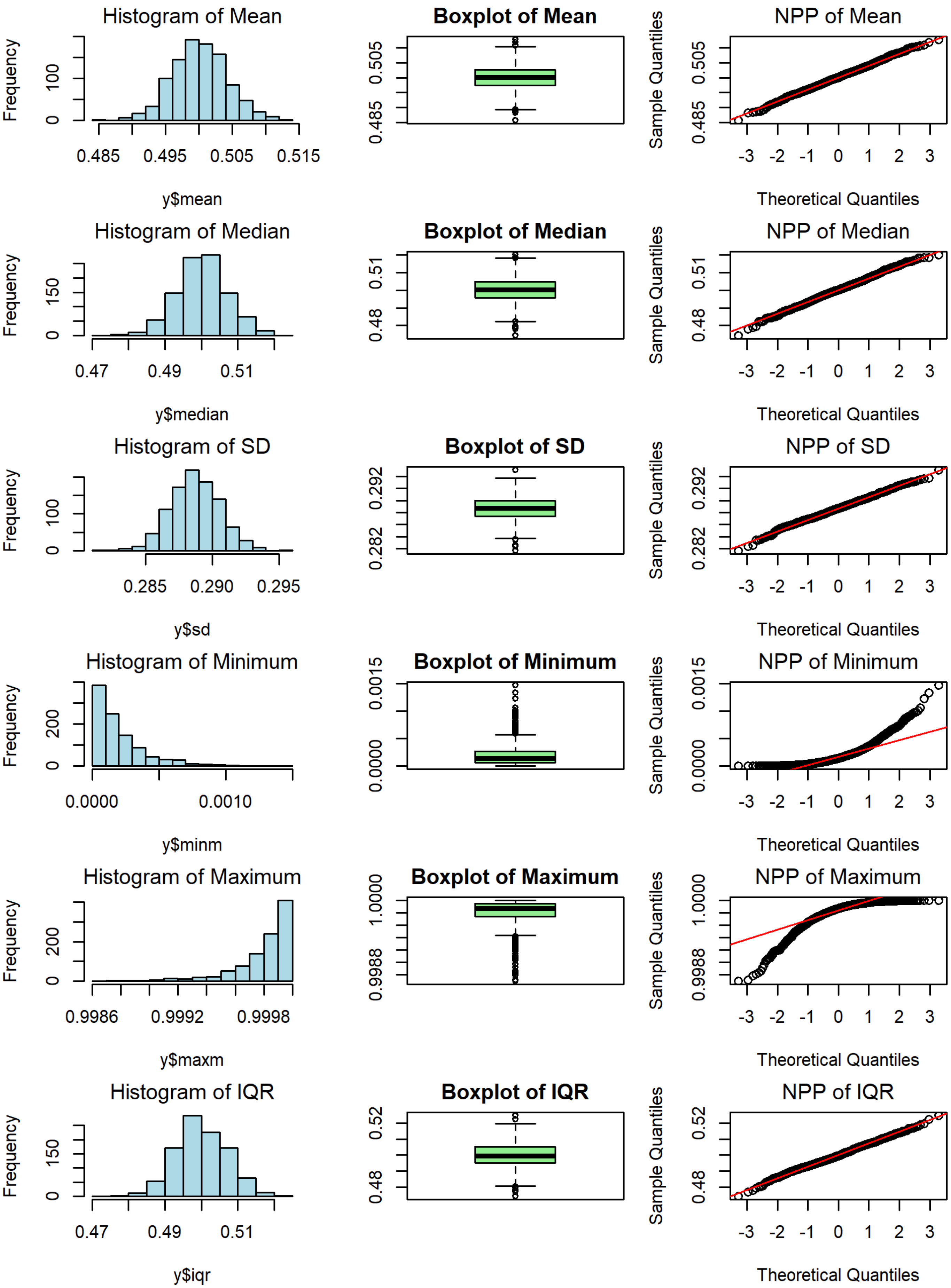


UNIFORM DISTRIBUTION PLOT
(n=500, nn=1000, a=0, b=1)



UNIFORM DISTRIBUTION PLOT
(n=1000, nn=1000, a=0, b=1)




Histogram of Mean
Frequency
0
100
0.485
0.495
0.505
0.515
y$mean
Boxplot of Mean
0.485
0.505
NPP of Mean
Sample Quantiles
0.485
0.505
-3
-2
-1
0
1
2
3
Theoretical Quantiles
Histogram of Median
Frequency
0
150
0.47
0.49
0.51
y$median
Boxplot of Median
0.48
0.51
NPP of Median
Sample Quantiles
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0.51
-3
-2
-1
0
1
2
3
Theoretical Quantiles
Histogram of SD
Frequency
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100
0.285
0.290
0.295
y$sd
Boxplot of SD
0.282
0.292
NPP of SD
Sample Quantiles
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0.292
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-2
-1
0
1
2
3
Theoretical Quantiles
Histogram of Minimum
Frequency
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200
0.0000
0.0010
y$minm
Boxplot of Minimum
0.0000
0.0015
NPP of Minimum
Sample Quantiles
0.0000
0.0015
-3
-2
-1
0
1
2
3
Theoretical Quantiles
Histogram of Maximum
Frequency
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200
0.9986
0.9992
0.9998
y$maxm
Boxplot of Maximum
0.9988
1.0000
NPP of Maximum
Sample Quantiles
0.9988
1.0000
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-2
-1
0
1
2
3
Theoretical Quantiles
Histogram of IQR
Frequency
0
150
0.47
0.49
0.51
y$iqr
Boxplot of IQR
0.48
0.52
NPP of IQR
Sample Quantiles
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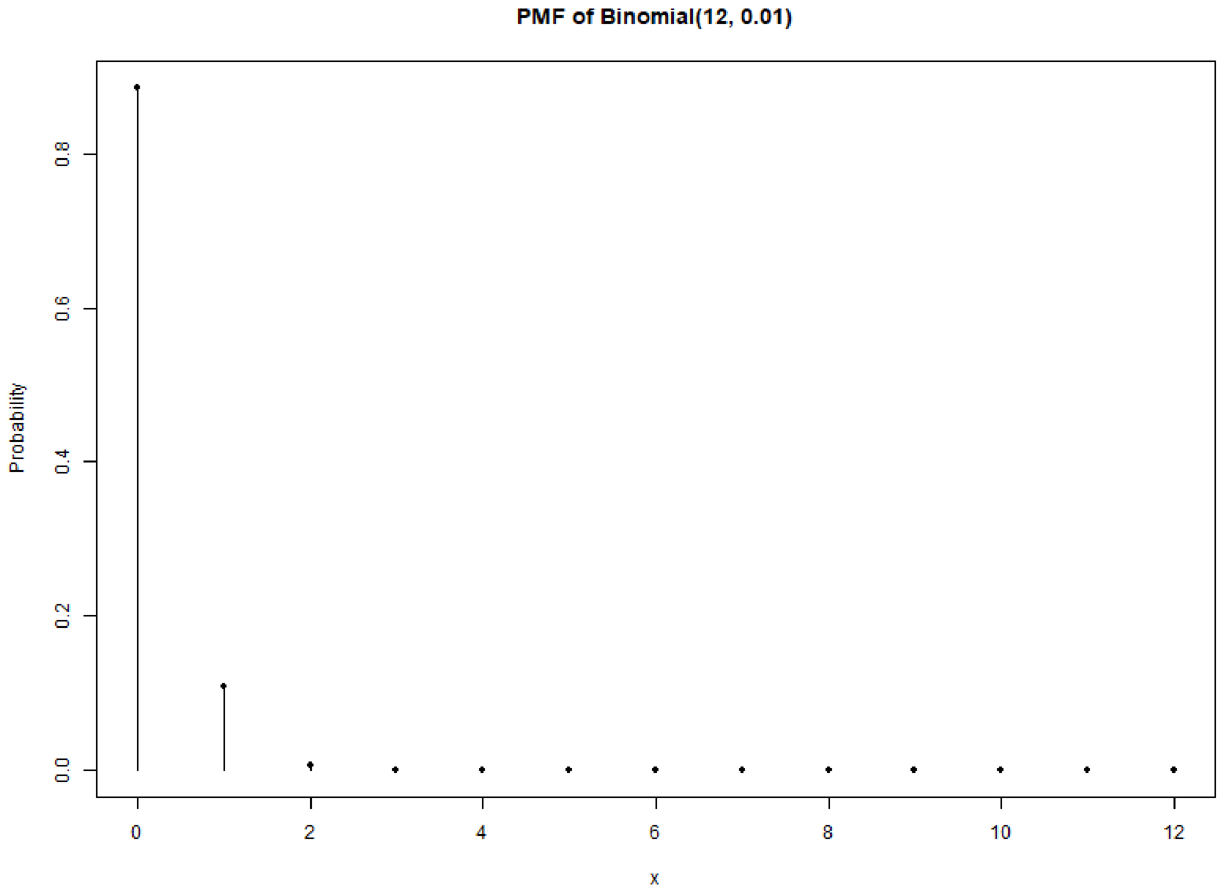
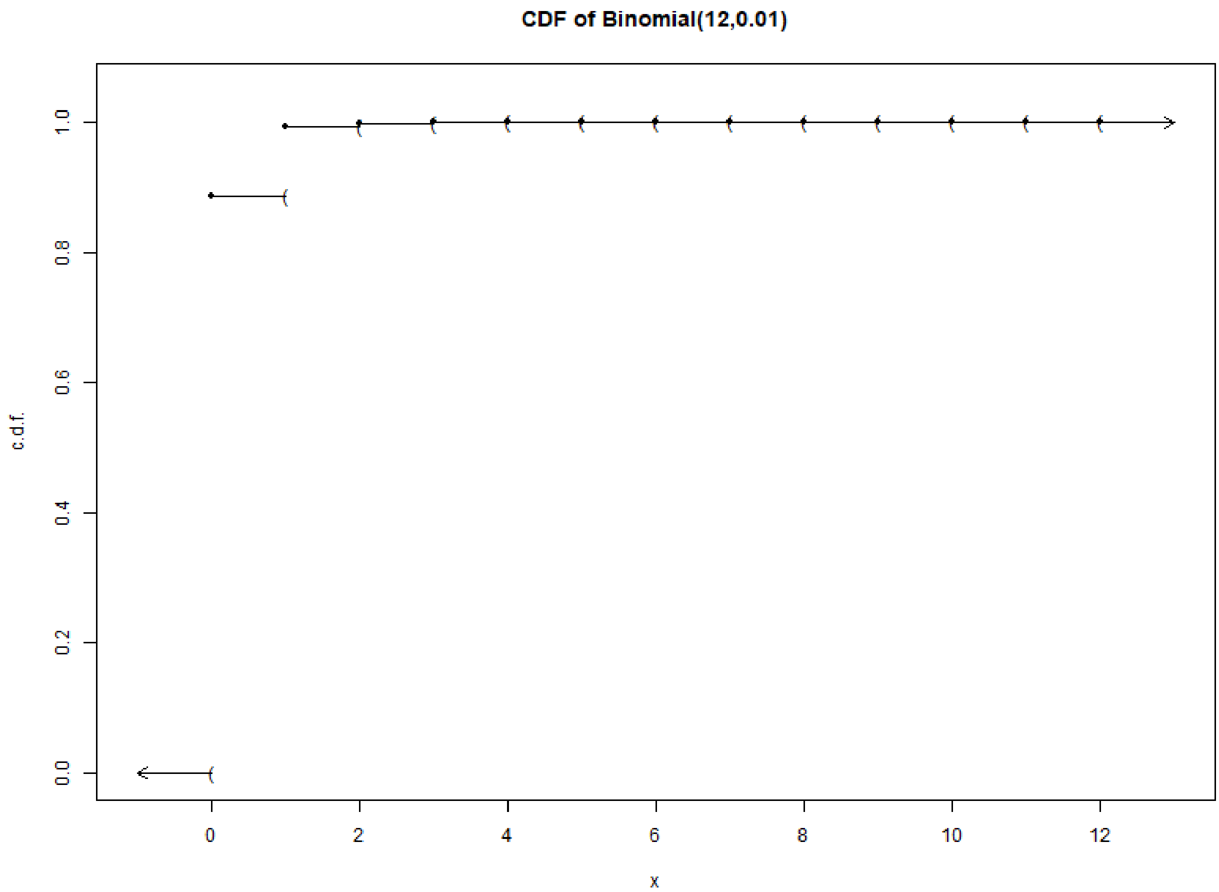
-1
0
1
2
3
Theoretical Quantiles

BINOMIAL DISTRIBUTION $(12,0.01)$





BINOMIAL DISTRIBUTION

	Values of n to achieve normality (nn=1000, m=12, p=0.01)							
Statistic	n=10	n=50	n=100	n=500	n=1000	n=5000	Any n	Min(n)
Mean	No	No	No	Yes	Yes	Yes	Yes	500
Median	No	No	No	No	No	No	No	NA
Std Dev	No	No	Yes	Yes	Yes	Yes	Yes	100
Min	No	No	No	No	No	No	No	NA
Max	No	No	No	No	No	No	No	NA
IQR	No	No	No	No	No	No	No	NA

Conclusion for Binomial Distribution (m = 12, p = 0.01)

Normality Achieved:

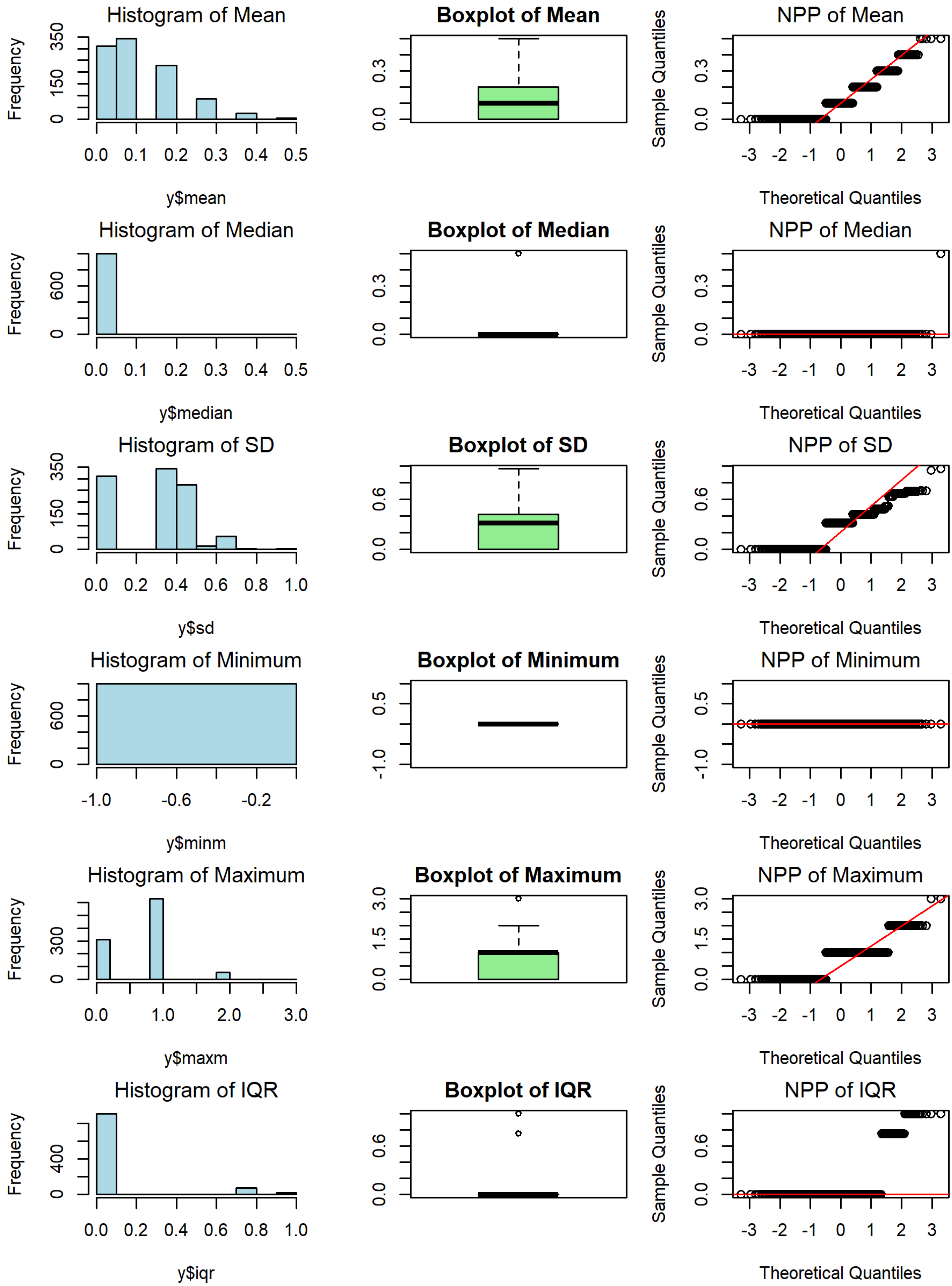
- **Mean:** Achieves normality for n ≥ 500, indicating that a moderately large sample size is necessary for the mean to approximate a normal distribution.
- **Standard Deviation (SD):** Achieves normality for n ≥ 100, converging faster compared to the mean.

Normality Not Achieved:

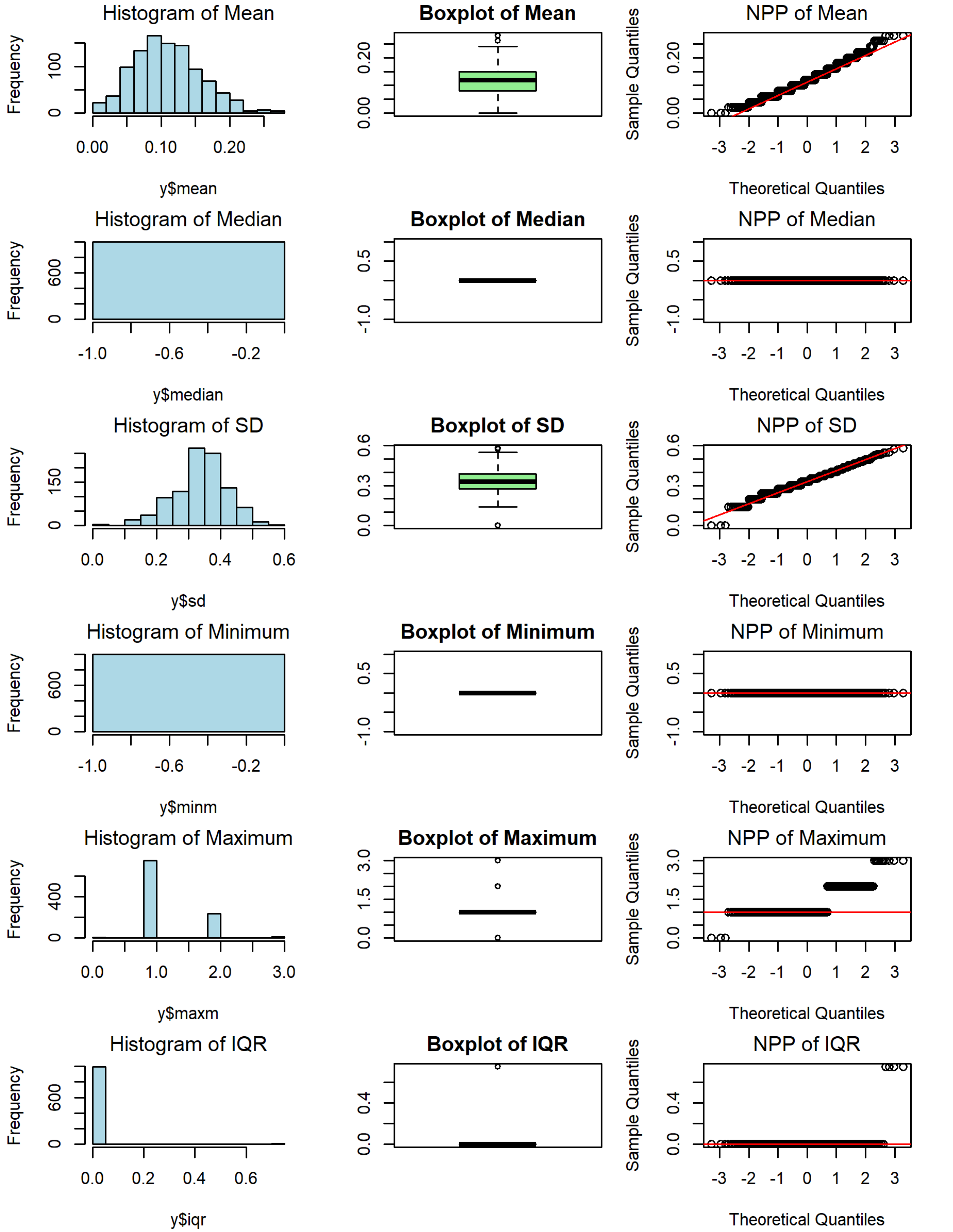
- **Median:** Does not achieve normality for any sample size, as it remains influenced by the discrete and skewed nature of the Binomial distribution.
- **Minimum and Maximum:** Do not achieve normality for any sample size due to their sensitivity to the extremes of the distribution.
- **IQR:** Does not achieve normality for any sample size, reflecting its dependency on the underlying discrete structure.

Overall: For the Binomial distribution with m = 12 and p = 0.01, the mean and standard deviation are the only statistics that achieve normality. The standard deviation converges to normality faster (n ≥ 100), while the mean requires a larger sample size (n ≥ 500). The median, minimum, maximum, and IQR remain non-normal regardless of the sample size due to the distribution's discrete and highly skewed characteristics.

BINOMIAL DISTRIBUTION PLOT
(n=10, nn=1000, m=12, p=0.01)

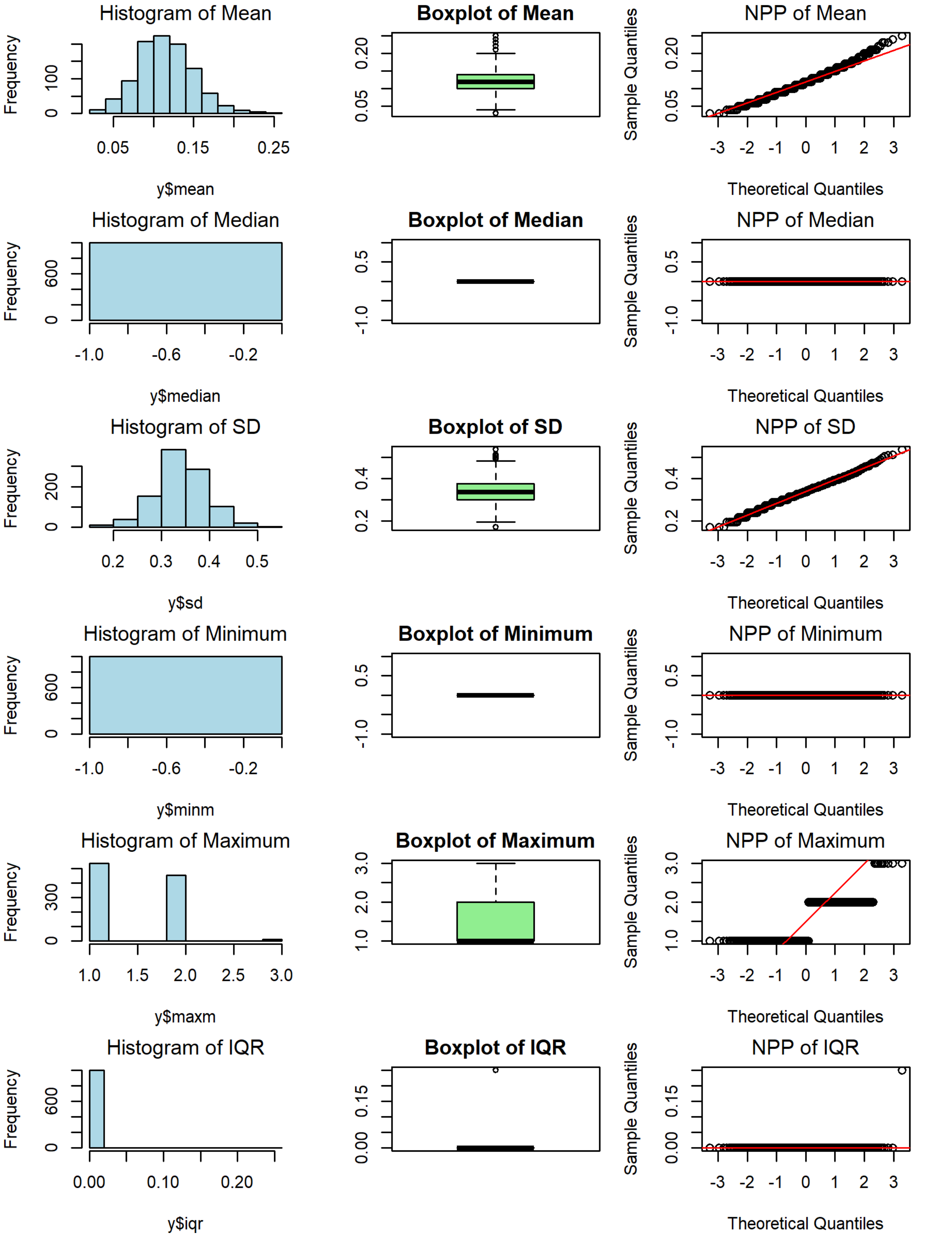


BINOMIAL DISTRIBUTION PLOT
(n=50, nn=1000, m=12, p=0.01)

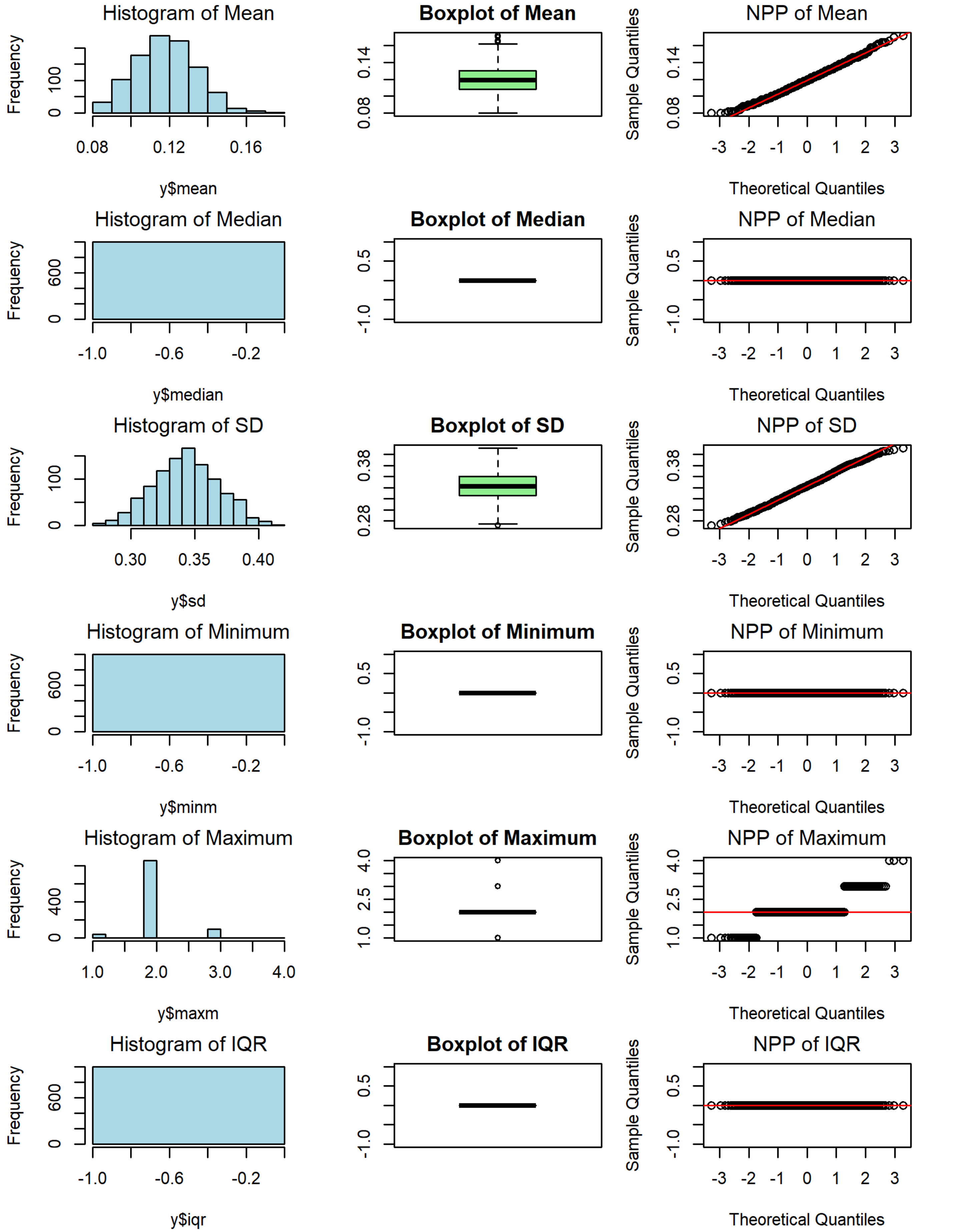


BINOMIAL DISTRIBUTION PLOT

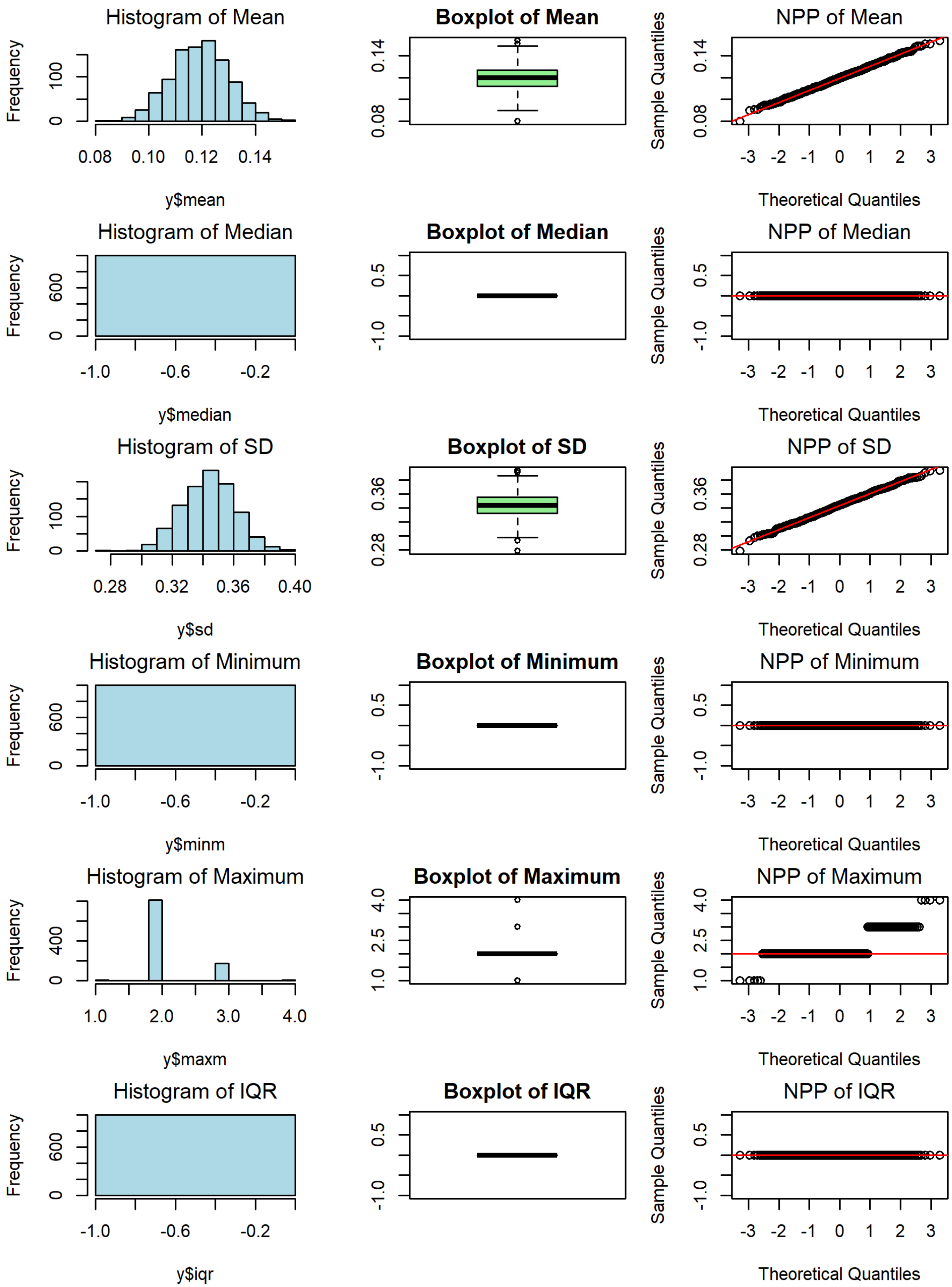
(n=100, nn=1000, m=12, p=0.01)

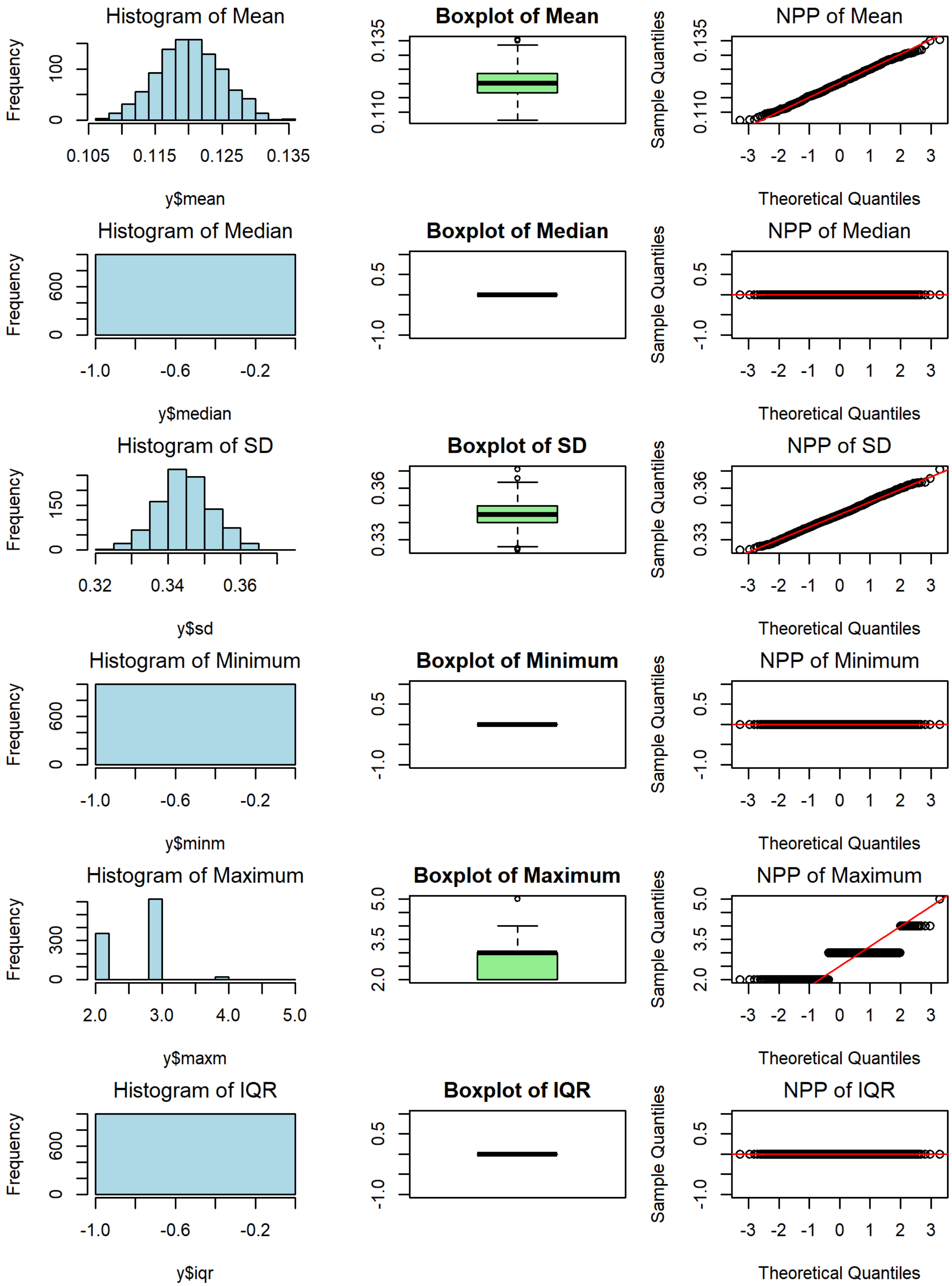


BINOMIAL DISTRIBUTION PLOT
(n=500, nn=1000, m=12, p=0.01)



BINOMIAL DISTRIBUTION PLOT
(n=1000, nn=1000, m=12, p=0.01)




Histogram of Mean
Frequency
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100
0.105
0.115
0.125
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y$mean
Boxplot of Mean
0.110
0.135
NPP of Mean
Sample Quantiles
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0.135
-3
-2
-1
0
1
2
3
Theoretical Quantiles
Histogram of Median
Frequency
0
600
-1.0
-0.6
-0.2
y$median
Boxplot of Median
-1.0
0.5
NPP of Median
Sample Quantiles
-1.0
0.5
-3
-2
-1
0
1
2
3
Theoretical Quantiles
Histogram of SD
Frequency
0
150
0.32
0.34
0.36
y$sd
Boxplot of SD
0.33
0.36
NPP of SD
Sample Quantiles
0.33
0.36
-3
-2
-1
0
1
2
3
Theoretical Quantiles
Histogram of Minimum
Frequency
0
600
-1.0
-0.6
-0.2
y$minm
Boxplot of Minimum
-1.0
0.5
NPP of Minimum
Sample Quantiles
-1.0
0.5
-3
-2
-1
0
1
2
3
Theoretical Quantiles
Histogram of Maximum
Frequency
0
300
2.0
3.0
4.0
5.0
y$maxm
Boxplot of Maximum
2.0
3.5
5.0
NPP of Maximum
Sample Quantiles
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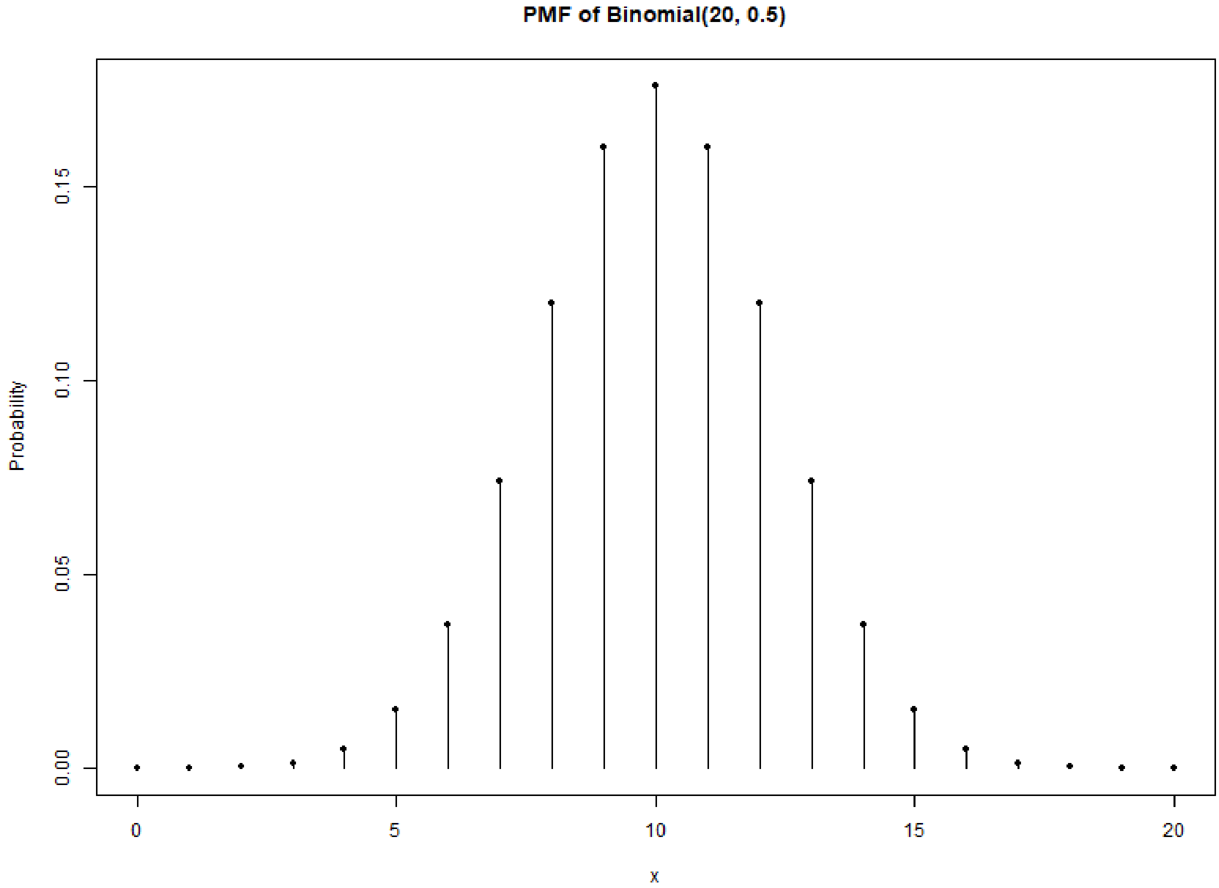
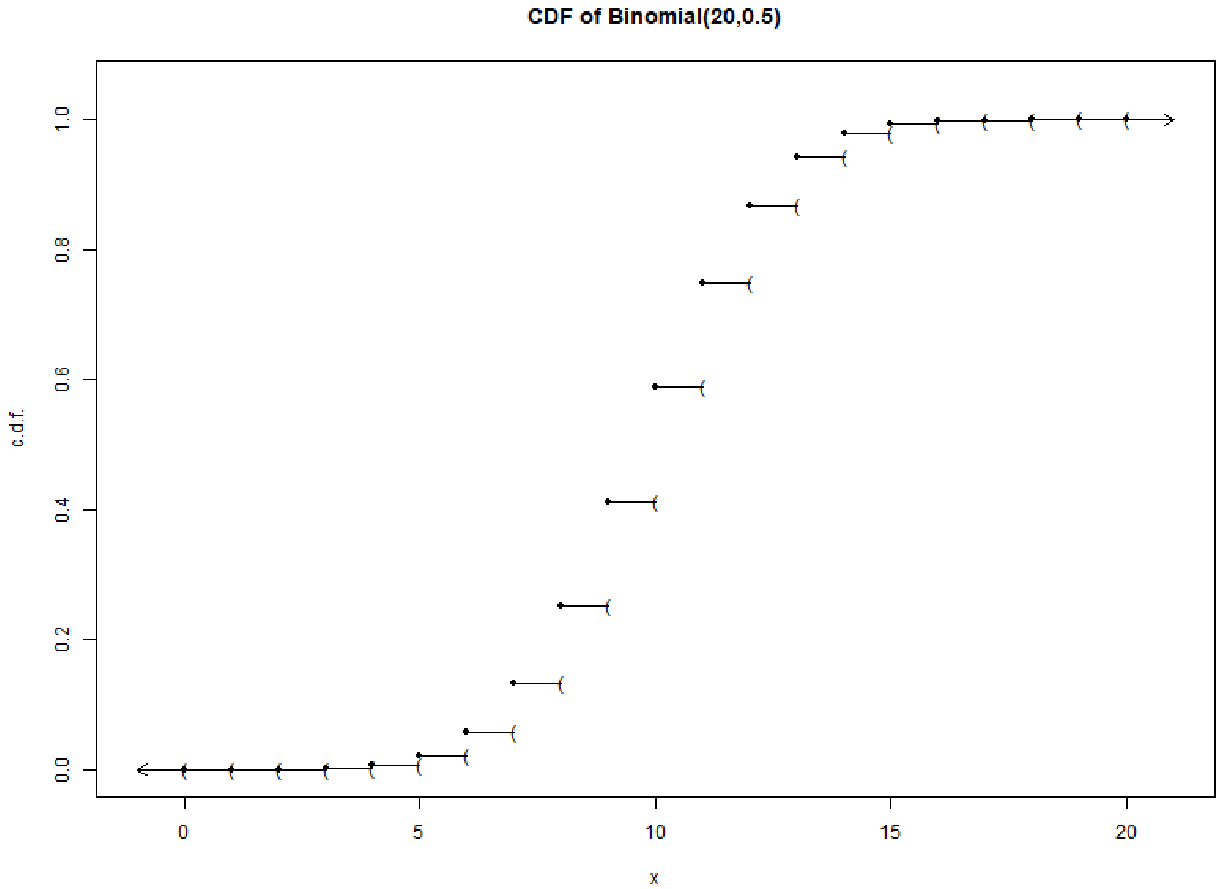
3.5
5.0
-3
-2
-1
0
1
2
3
Theoretical Quantiles
Histogram of IQR
Frequency
0
600
-1.0
-0.6
-0.2
y$iqr
Boxplot of IQR
-1.0
0.5
NPP of IQR
Sample Quantiles
-1.0
0.5
-3
-2
-1
0
1
2
3
Theoretical Quantiles

BINOMIAL DISTRIBUTION $(20,0.5)$





BINOMIAL DISTRIBUTION

	Values of n to achieve normality (nn=1000, m=20, p=0.5)							
Statistic	n=10	n=50	n=100	n=500	n=1000	n=5000	Any n	Min(n)
Mean	No	Yes	Yes	Yes	Yes	Yes	Yes	50
Median	No	No	No	No	No	No	No	NA
Std Dev	No	Yes	Yes	Yes	Yes	Yes	Yes	50
Min	No	No	No	No	No	No	No	NA
Max	No	No	No	No	No	No	No	NA
IQR	No	No	No	No	No	No	No	NA

Conclusion for Binomial Distribution (m = 20, p = 0.5)

Normality Achieved:

- **Mean:** Achieves normality for n ≥ 50, indicating that the mean converges to normality relatively quickly due to the symmetric nature of the Binomial distribution with p = 0.5.
- **Standard Deviation (SD):** Achieves normality for n ≥ 50, showing a similar rate of convergence to the mean.

Normality Not Achieved:

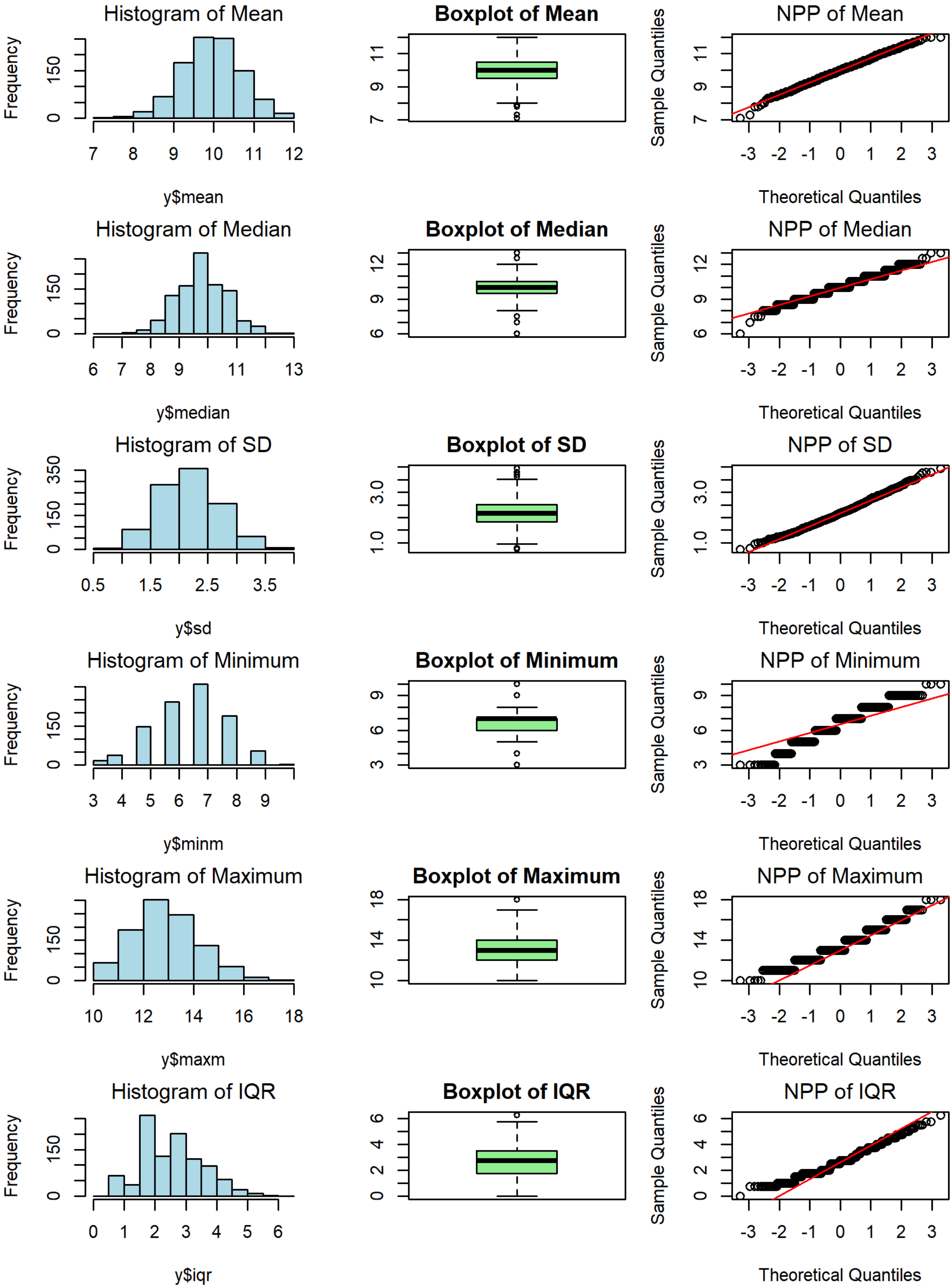
- **Median:** Does not achieve normality for any sample size, remaining non-normal due to the discrete characteristics of the Binomial distribution.
- **Minimum and Maximum:** Do not achieve normality for any sample size, as they are sensitive to the extremes of the distribution.
- **IQR:** Does not achieve normality for any sample size, reflecting its dependency on the underlying discrete structure.

Comparison with Binomial Distribution (m = 12, p = 0.01):

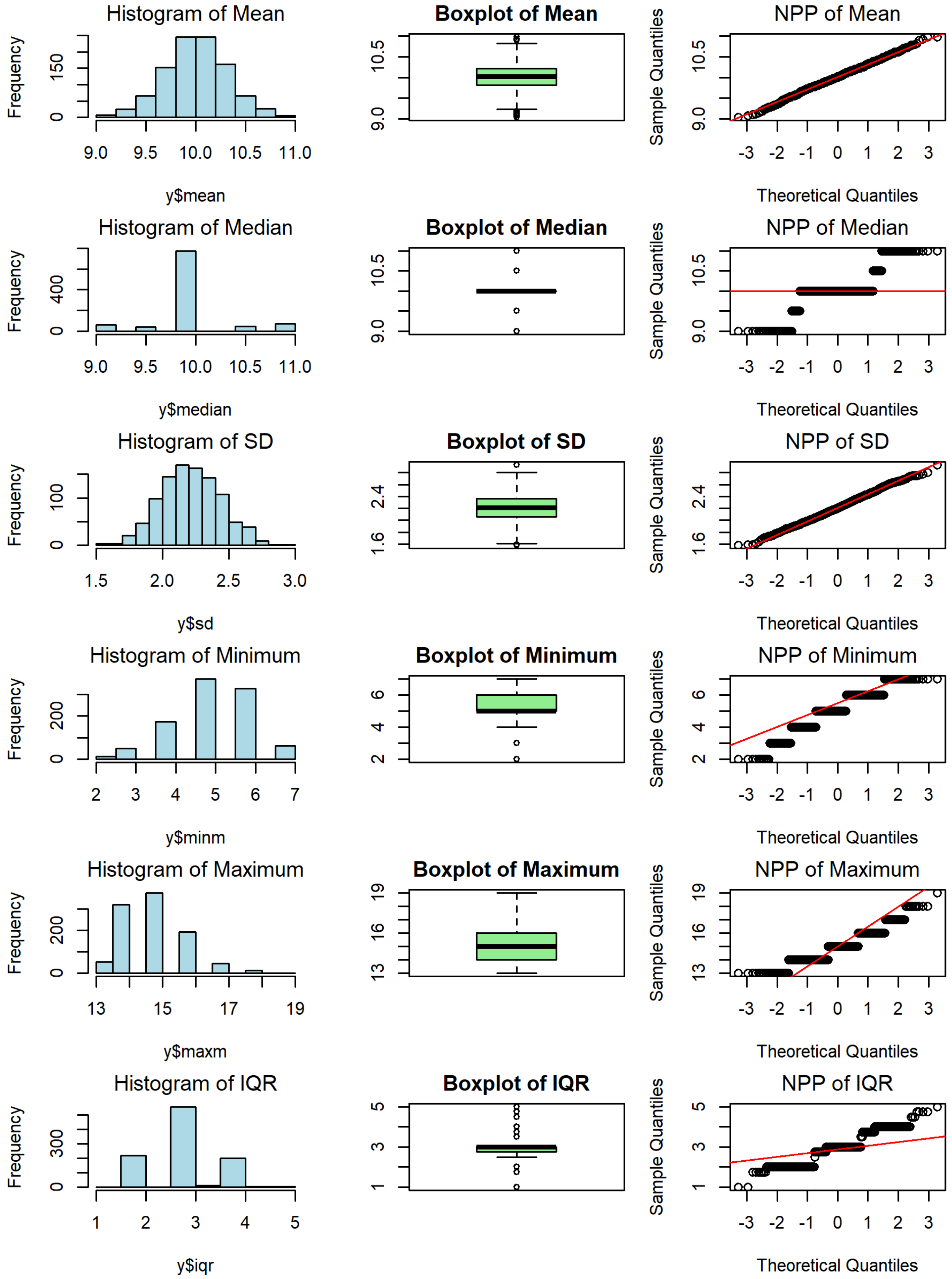
- For m = 20 and p = 0.5, normality is achieved at smaller sample sizes (n ≥ 50) for both the mean and standard deviation compared to m = 12 and p = 0.01. This faster convergence is due to the more symmetric and less skewed nature of the distribution when p = 0.5.

Overall: The mean and standard deviation achieve normality at n ≥ 50, significantly faster than in the case of m = 12 and p = 0.01. The median, minimum, maximum, and IQR remain non-normal regardless of sample size, consistent with the discrete nature of the distribution.

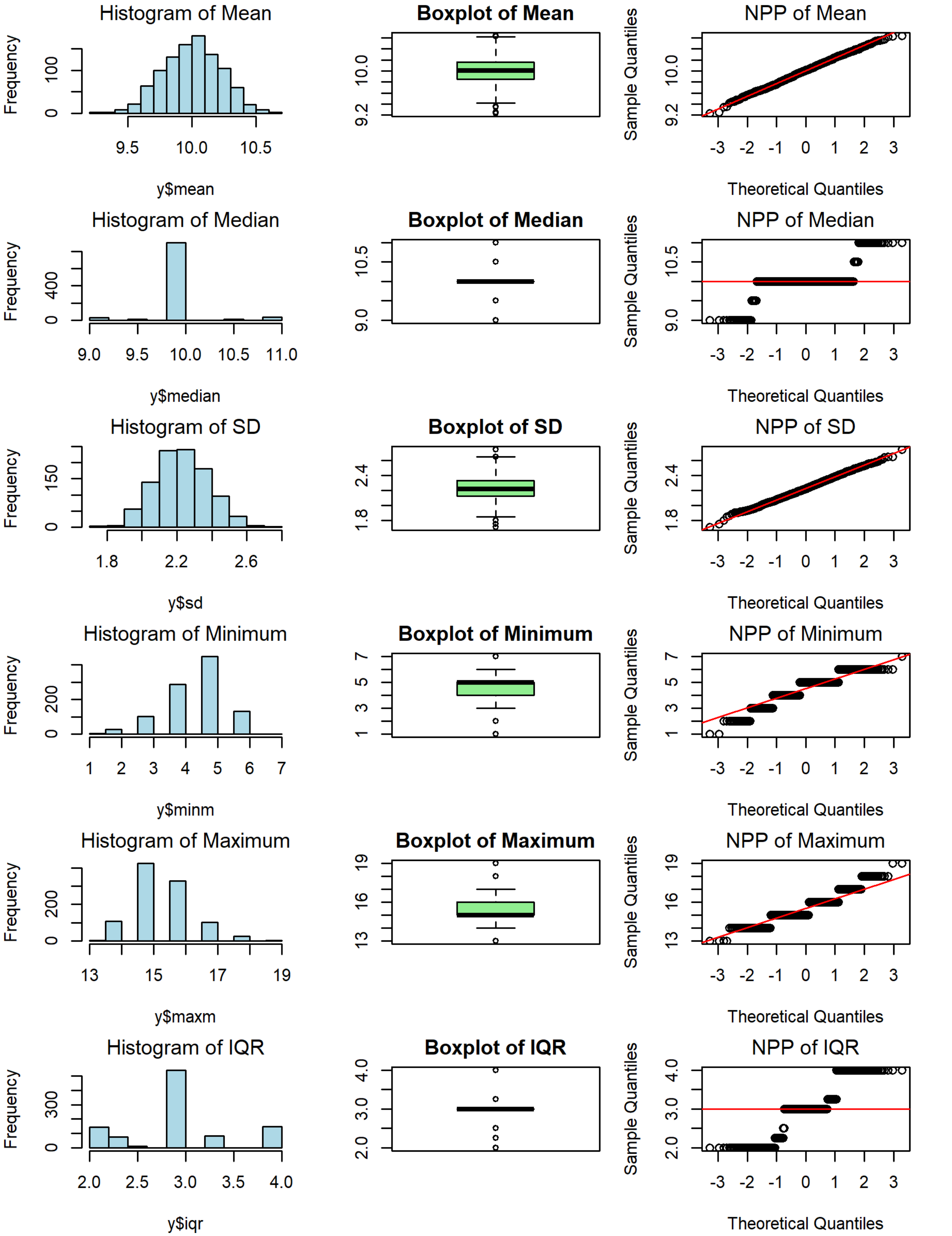
BINOMIAL DISTRIBUTION PLOT
(n=10, nn=1000, m=20, p=0.50)



BINOMIAL DISTRIBUTION PLOT
(n=50, nn=1000, m=20, p=0.50)

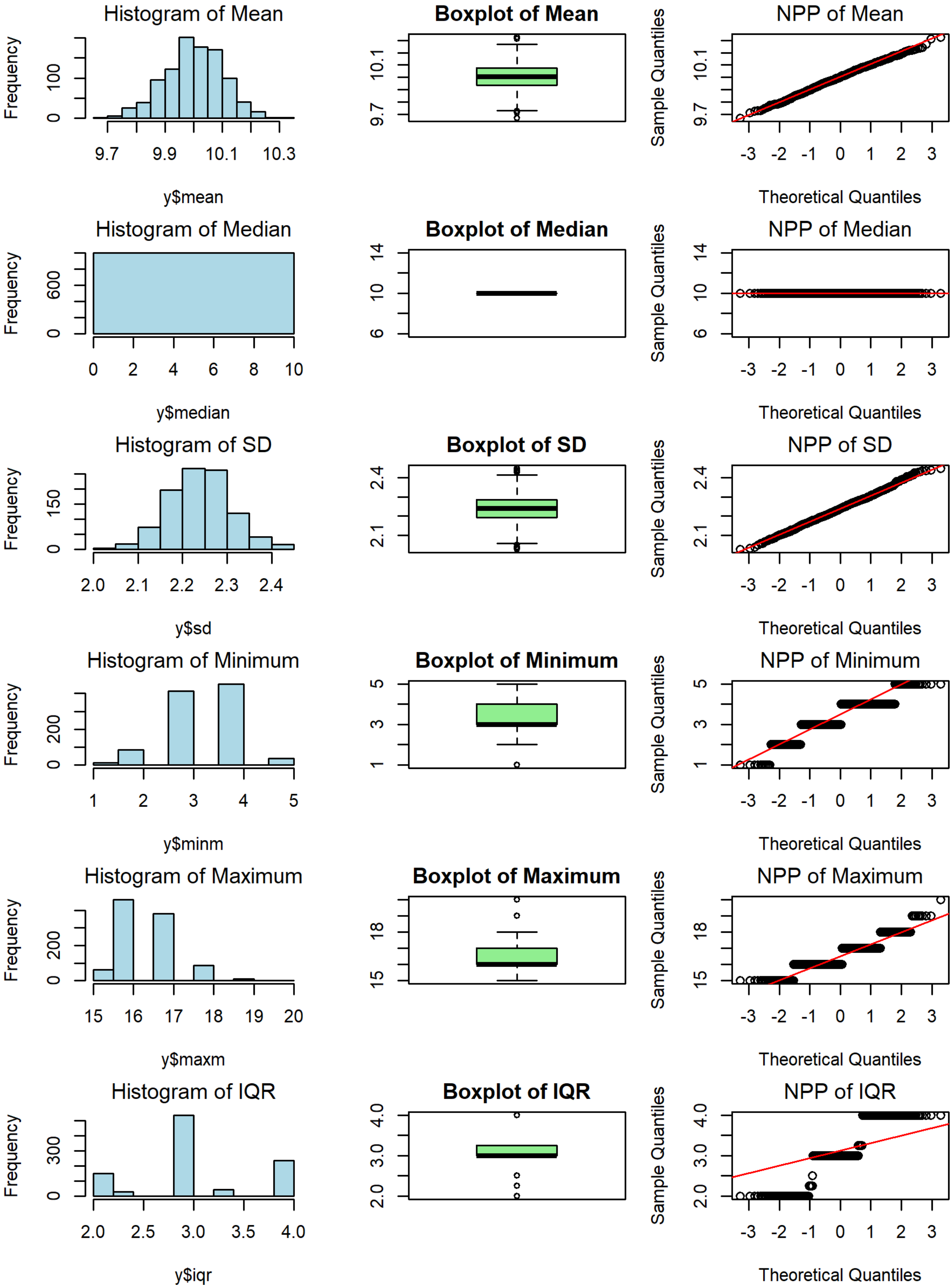


BINOMIAL DISTRIBUTION PLOT
(n=100, nn=1000, m=20, p=0.50)

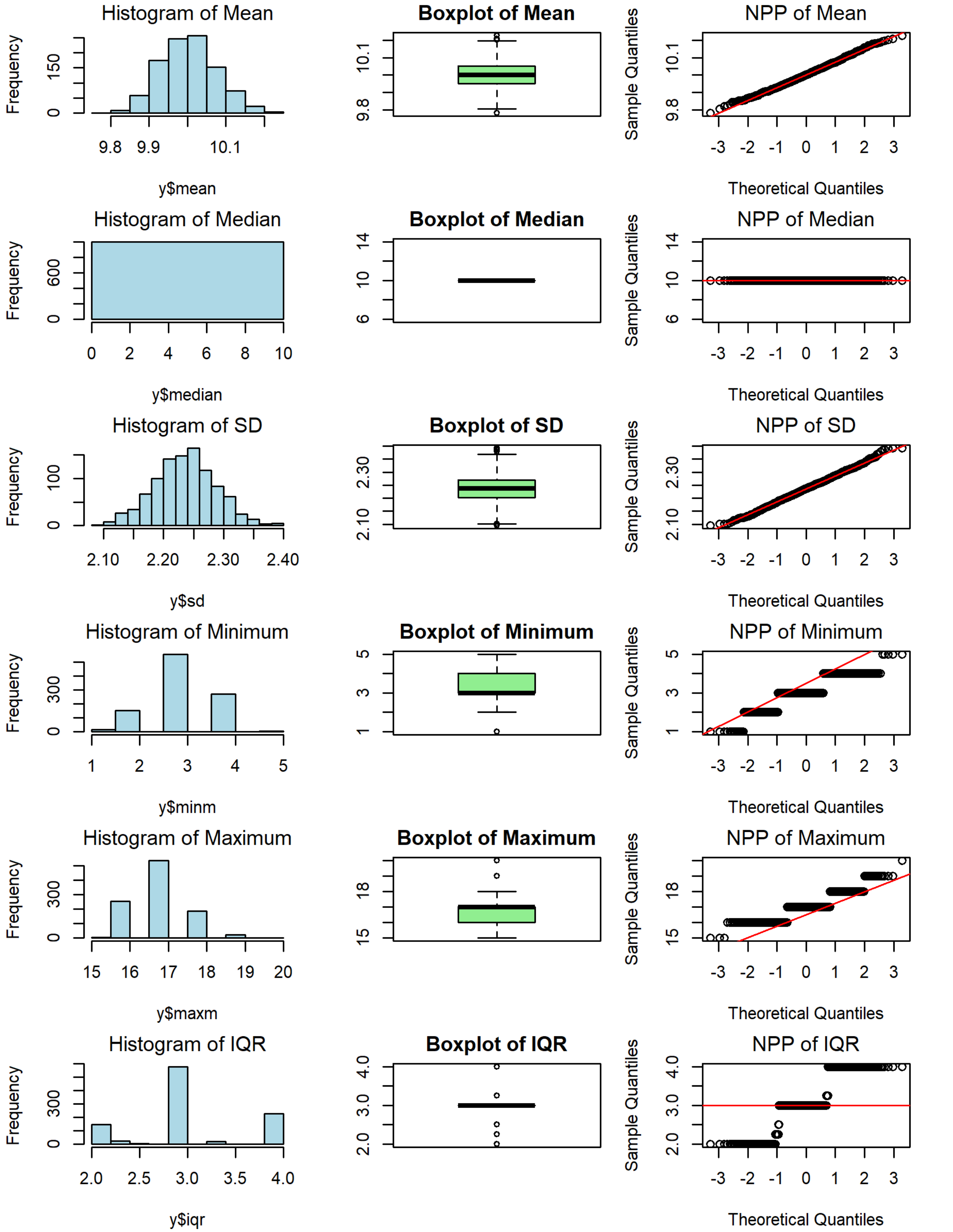


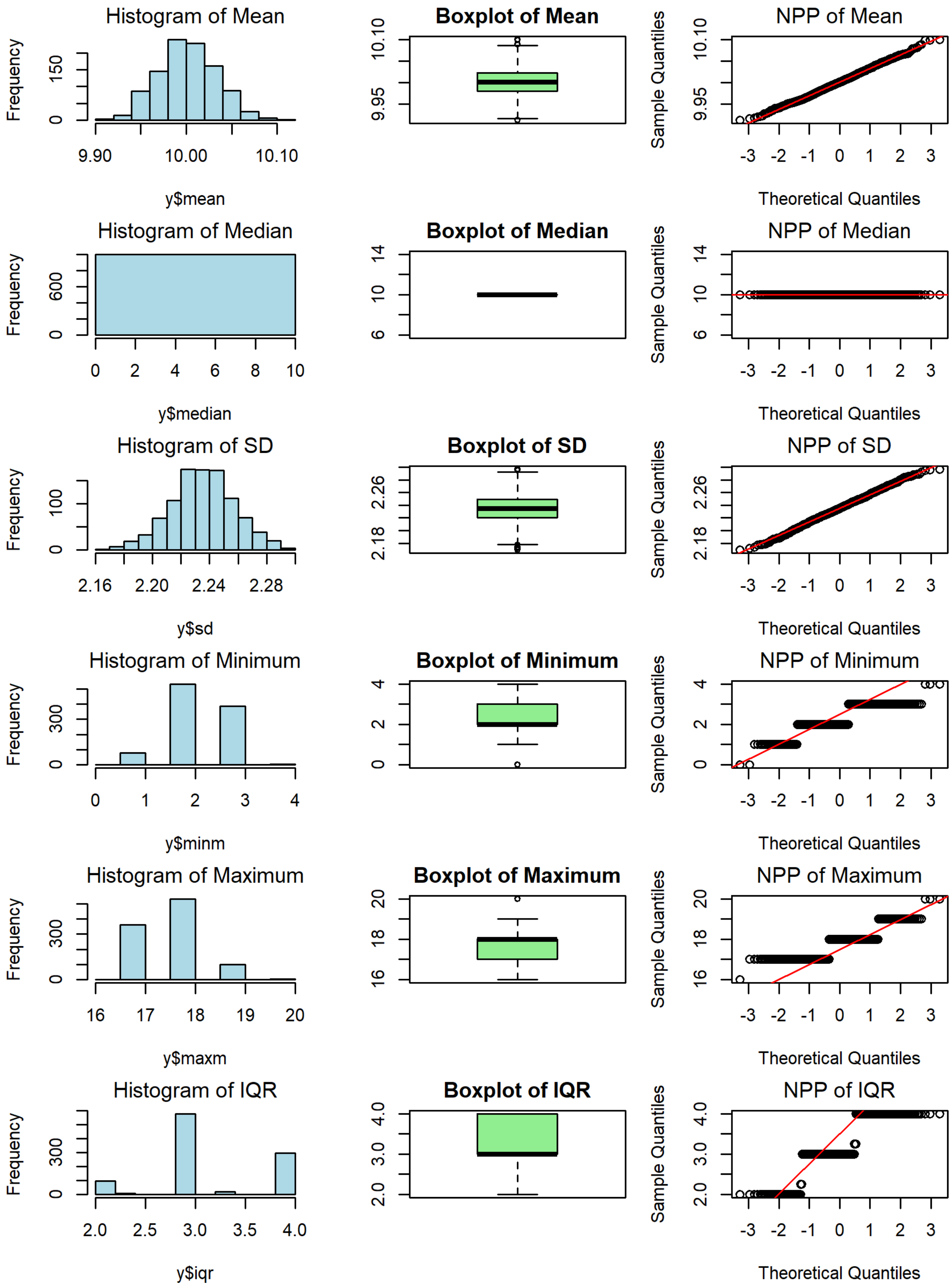
BINOMIAL DISTRIBUTION PLOT

(n=500, nn=1000, m=20, p=0.50)



BINOMIAL DISTRIBUTION PLOT
(n=1000, nn=1000, m=20, p=0.50)





Histogram of Mean
Frequency
0
150
9.90
10.00
10.10
y$mean
Boxplot of Mean
9.95
10.10
NPP of Mean
Sample Quantiles
9.95
10.10
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of Median
Frequency
0
600
0 2 4 6 8 10
y$median
Boxplot of Median
6
10
14
NPP of Median
Sample Quantiles
6
10
14
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of SD
Frequency
0
100
2.16 2.20 2.24 2.28
y$sd
Boxplot of SD
2.18
2.26
NPP of SD
Sample Quantiles
2.18
2.26
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of Minimum
Frequency
0
300
0 1 2 3 4
y$minm
Boxplot of Minimum
0
2
4
NPP of Minimum
Sample Quantiles
0
2
4
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of Maximum
Frequency
0
300
16 17 18 19 20
y$maxm
Boxplot of Maximum
16
18
20
NPP of Maximum
Sample Quantiles
16
18
20
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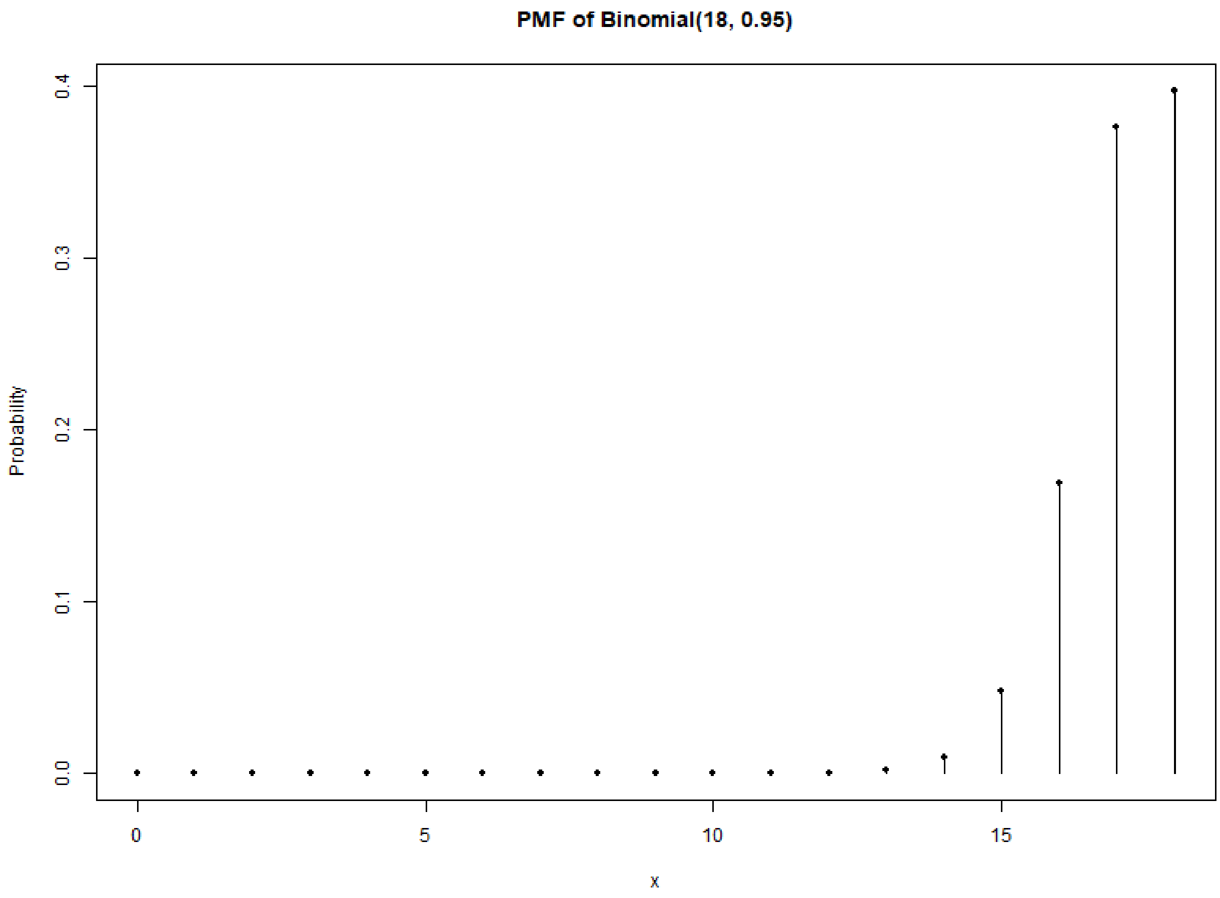
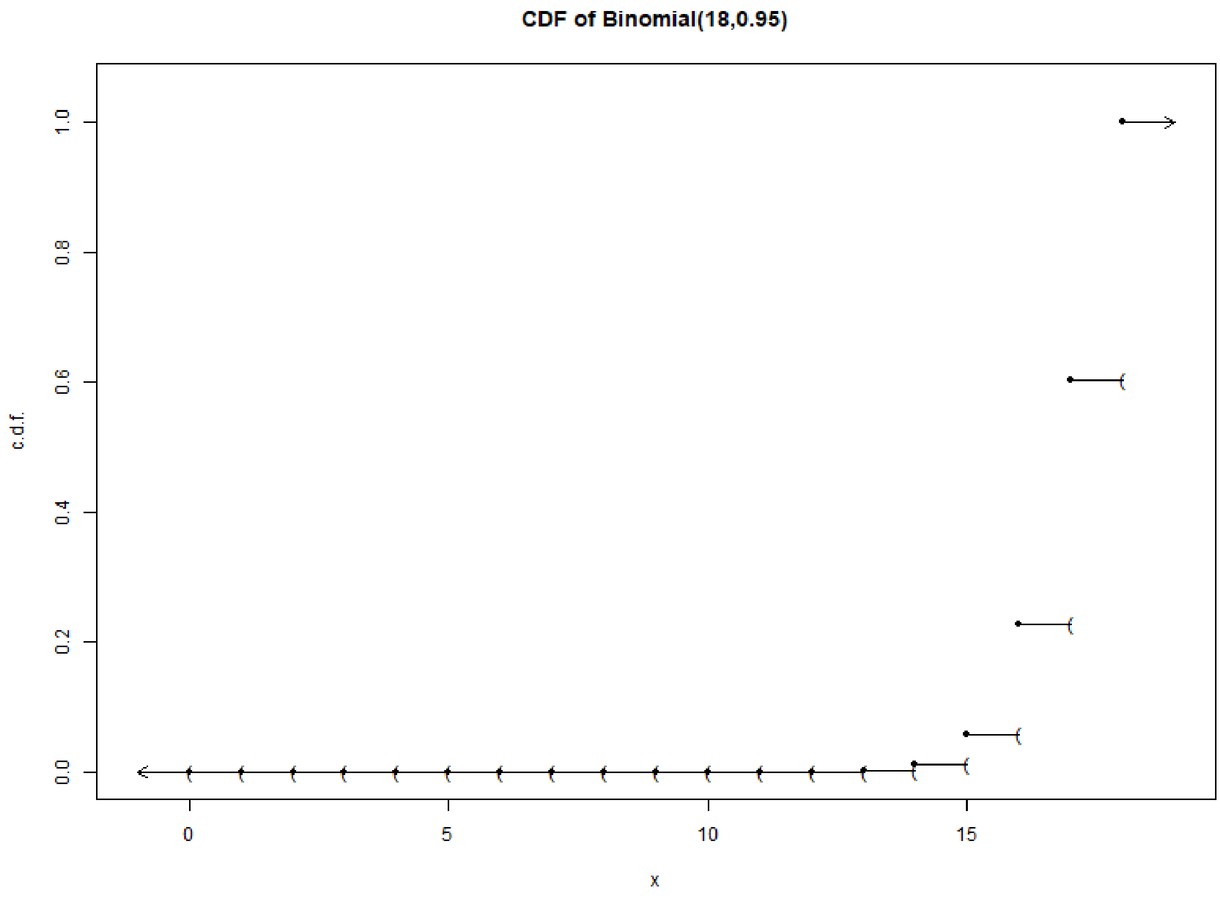
Theoretical Quantiles
Histogram of IQR
Frequency
0
300
2.0 2.5 3.0 3.5 4.0
y$iqr
Boxplot of IQR
2.0
3.0
4.0
NPP of IQR
Sample Quantiles
2.0
3.0
4.0
-3 -2 -1 0 1 2 3
Theoretical Quantiles

BINOMIAL DISTRIBUTION $(18,0.95)$





BINOMIAL DISTRIBUTION

	Values of n to achieve normality (nn=1000, m=18, p=0.95)							
Statistic	n=10	n=50	n=100	n=500	n=1000	n=5000	Any n	Min(n)
Mean	No	Yes	Yes	Yes	Yes	Yes	Yes	50
Median	No	No	No	No	No	No	No	NA
Std Dev	No	Yes	Yes	Yes	Yes	Yes	Yes	50
Min	No	No	No	No	No	No	No	NA
Max	No	No	No	No	No	No	No	NA
IQR	No	No	No	No	No	No	No	NA

Conclusion for Binomial Distribution (m = 18, p = 0.95)

Normality Achieved:

- **Mean:** Achieves normality for n ≥ 50, demonstrating relatively quick convergence due to the high probability of success (p = 0.95) creating a concentrated distribution.
- **Standard Deviation (SD):** Achieves normality for n ≥ 50, with a convergence rate similar to the mean.

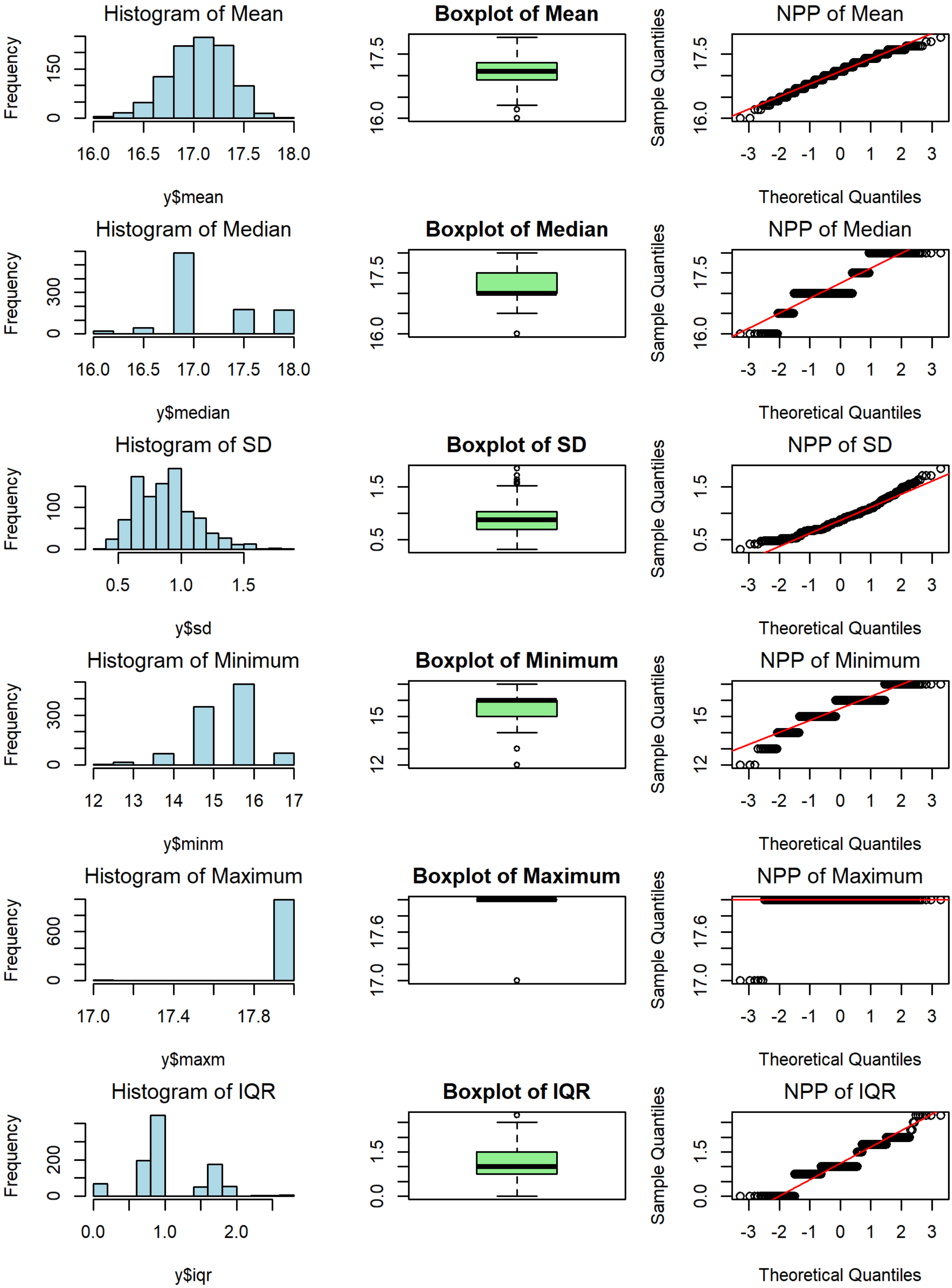
Normality Not Achieved:

- **Median:** Does not achieve normality for any sample size, consistent with the discrete nature of the Binomial distribution.
- **Minimum and Maximum:** Do not achieve normality for any sample size, as they are strongly influenced by the extremes of the distribution.
- **IQR:** Does not achieve normality for any sample size, reflecting the dependency on the underlying discrete structure and skewness.

Comparison with Other Parameters:

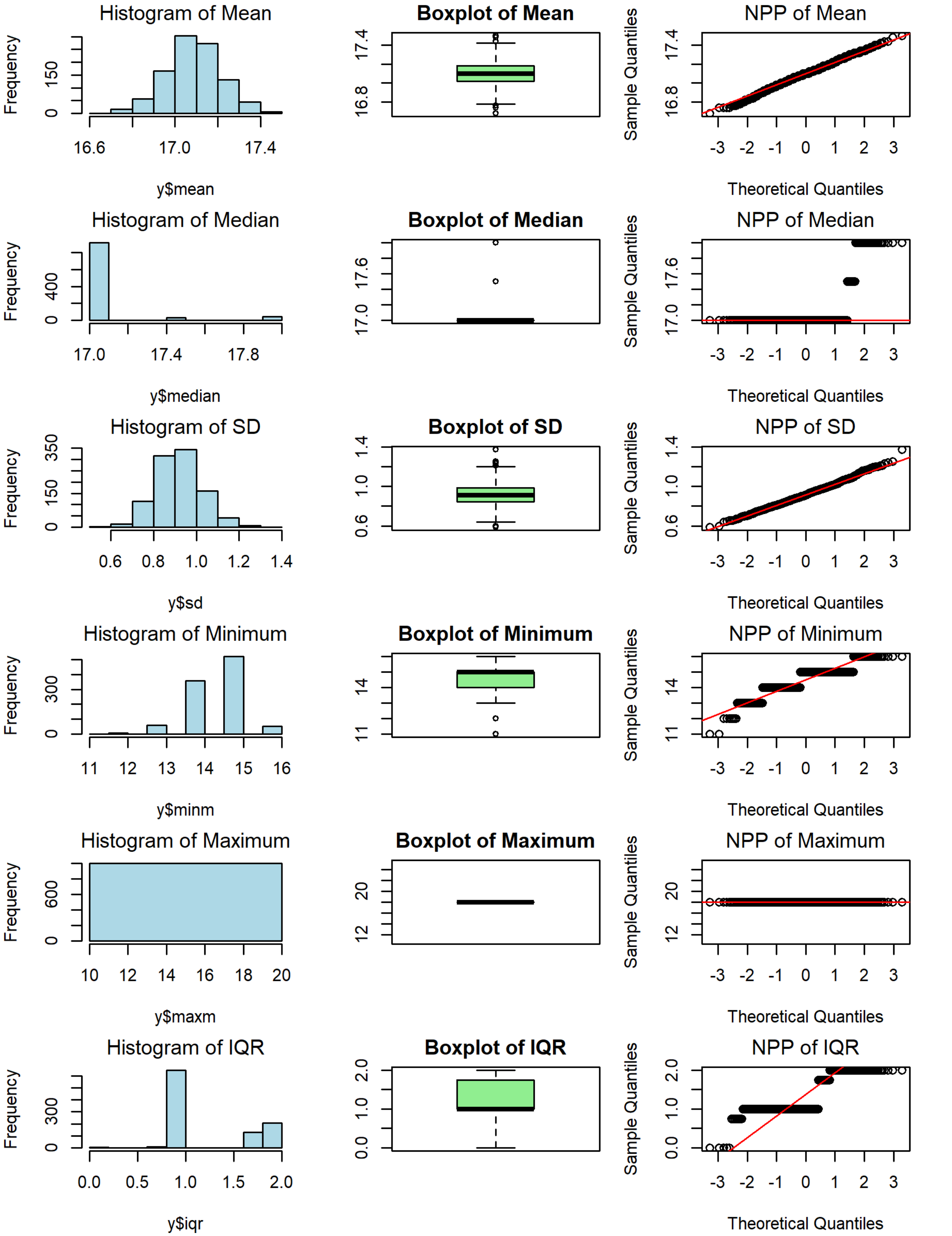
- Compared to **m = 20, p = 0.5**, the mean and standard deviation converge to normality at the same sample size (n ≥ 50). However, the distribution with **p = 0.95** is more skewed, making the normality less representative of the overall distribution characteristics.
- Compared to **m = 12, p = 0.01**, normality for the mean and standard deviation is achieved at much smaller sample sizes (n ≥ 50 vs. n ≥ 500 for the mean and n ≥ 100 for SD). This is due to the concentration of outcomes near the maximum for p = 0.95.

BINOMIAL DISTRIBUTION PLOT
(n=10, nn=1000, m=18, p=0.95)

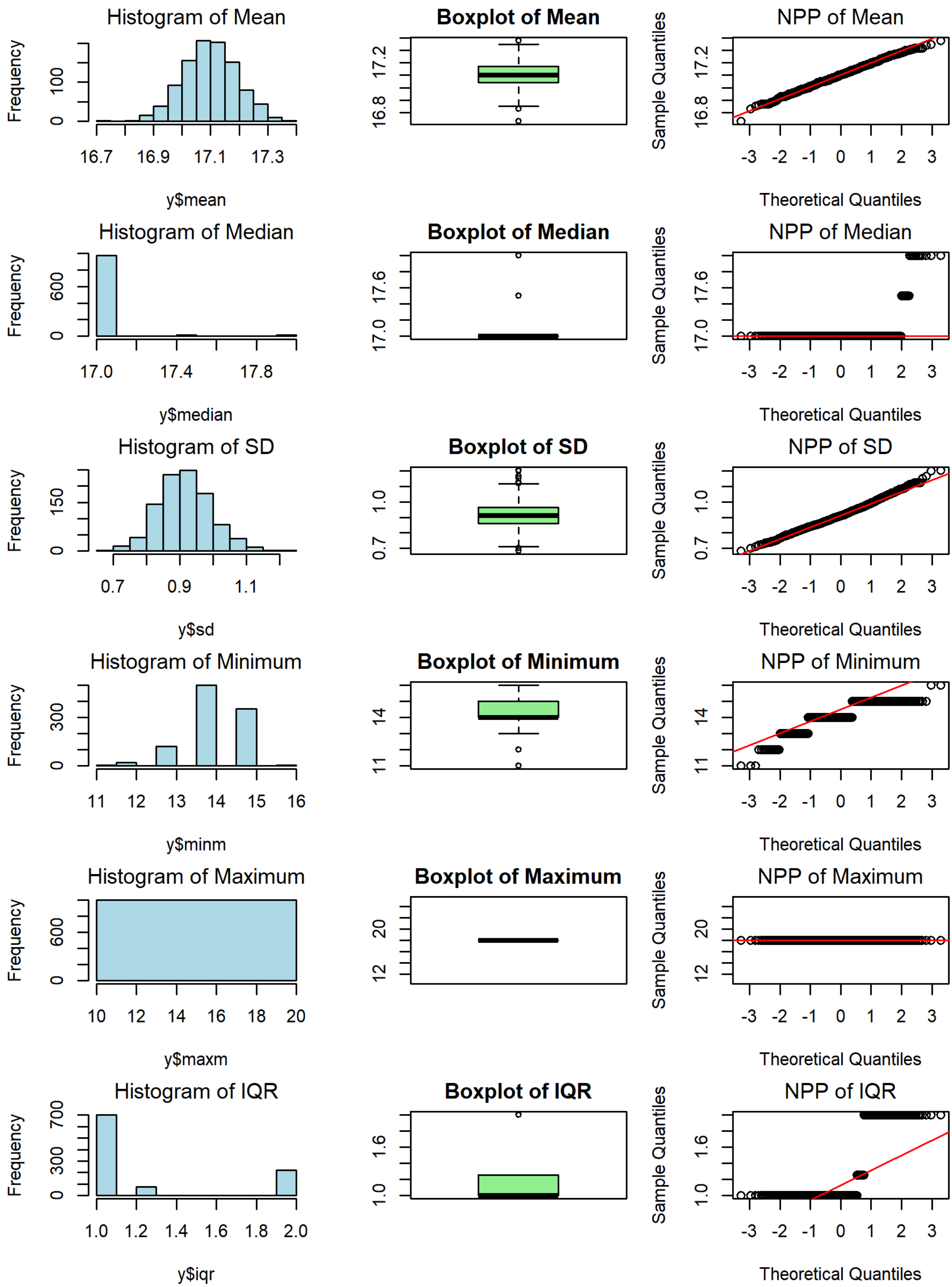


BINOMIAL DISTRIBUTION PLOT

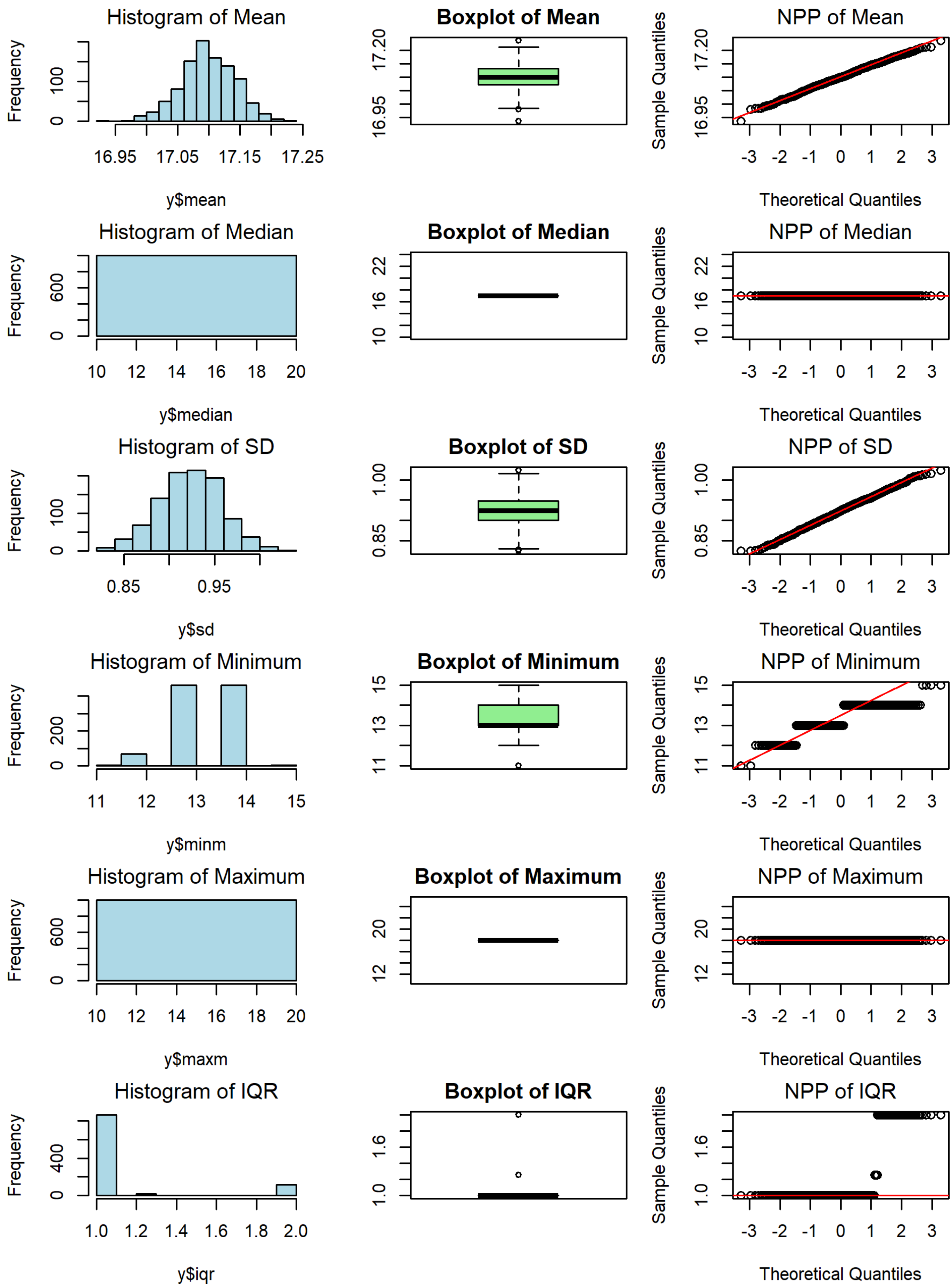
(n=50, nn=1000, m=18, p=0.95)



BINOMIAL DISTRIBUTION PLOT
(n=100, nn=1000, m=18, p=0.95)

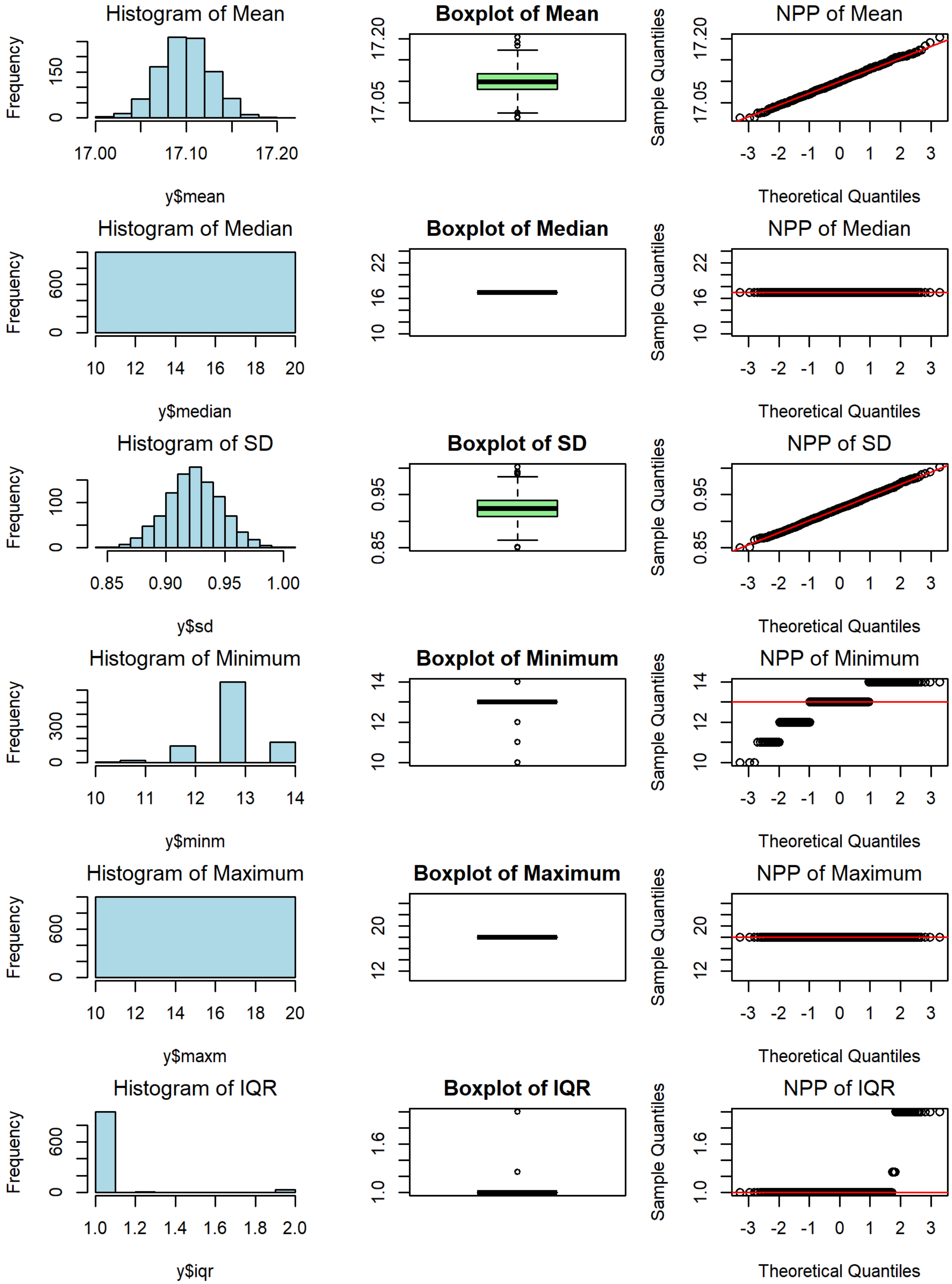


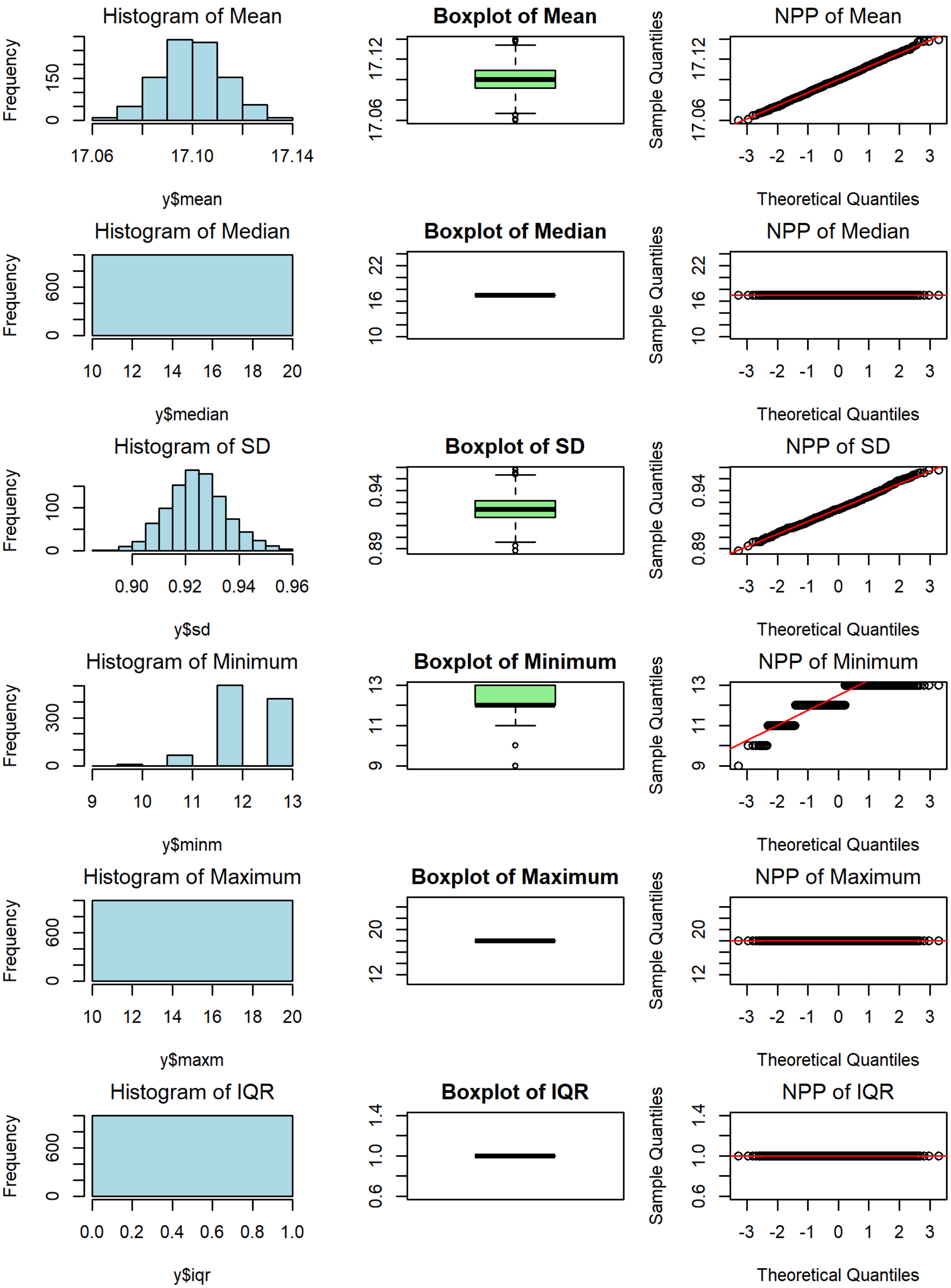
BINOMIAL DISTRIBUTION PLOT
(n=500, nn=1000, m=18, p=0.95)



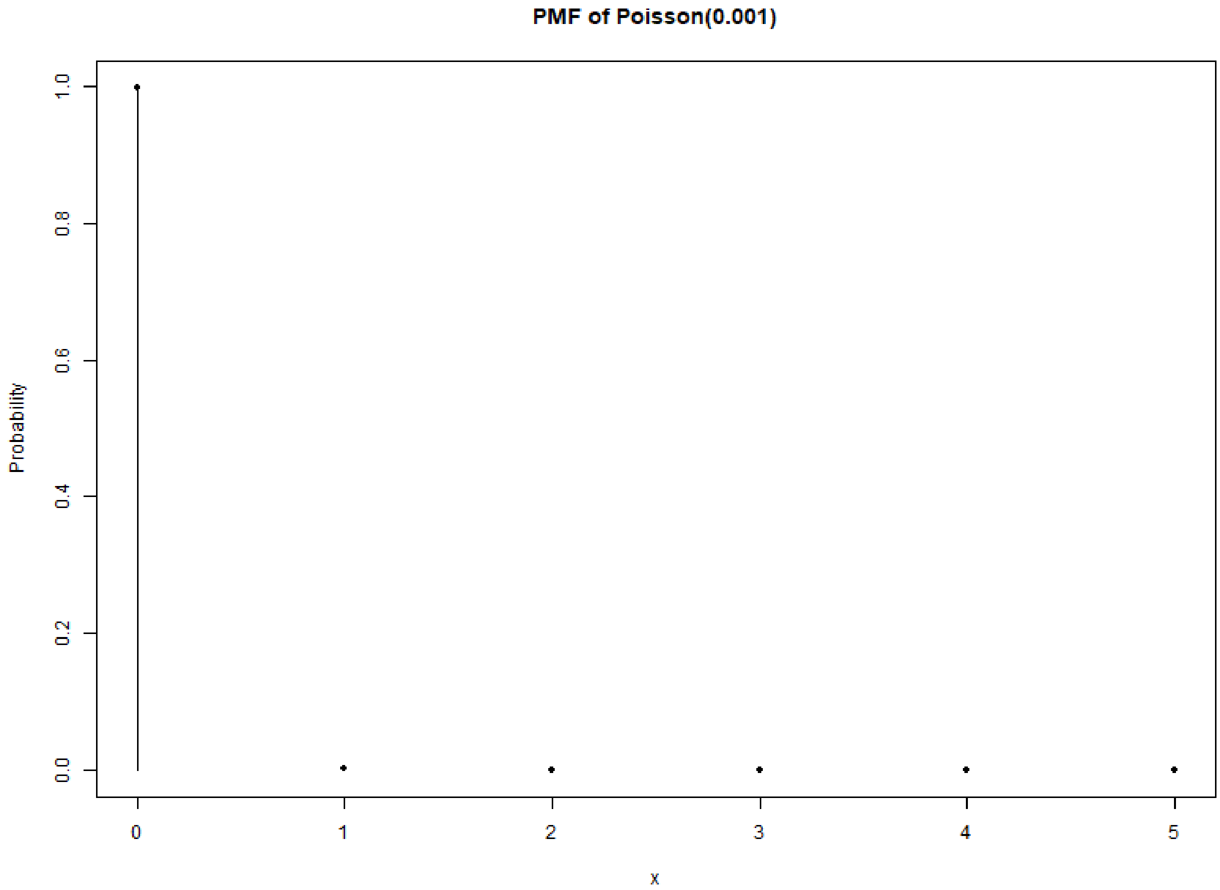
BINOMIAL DISTRIBUTION PLOT

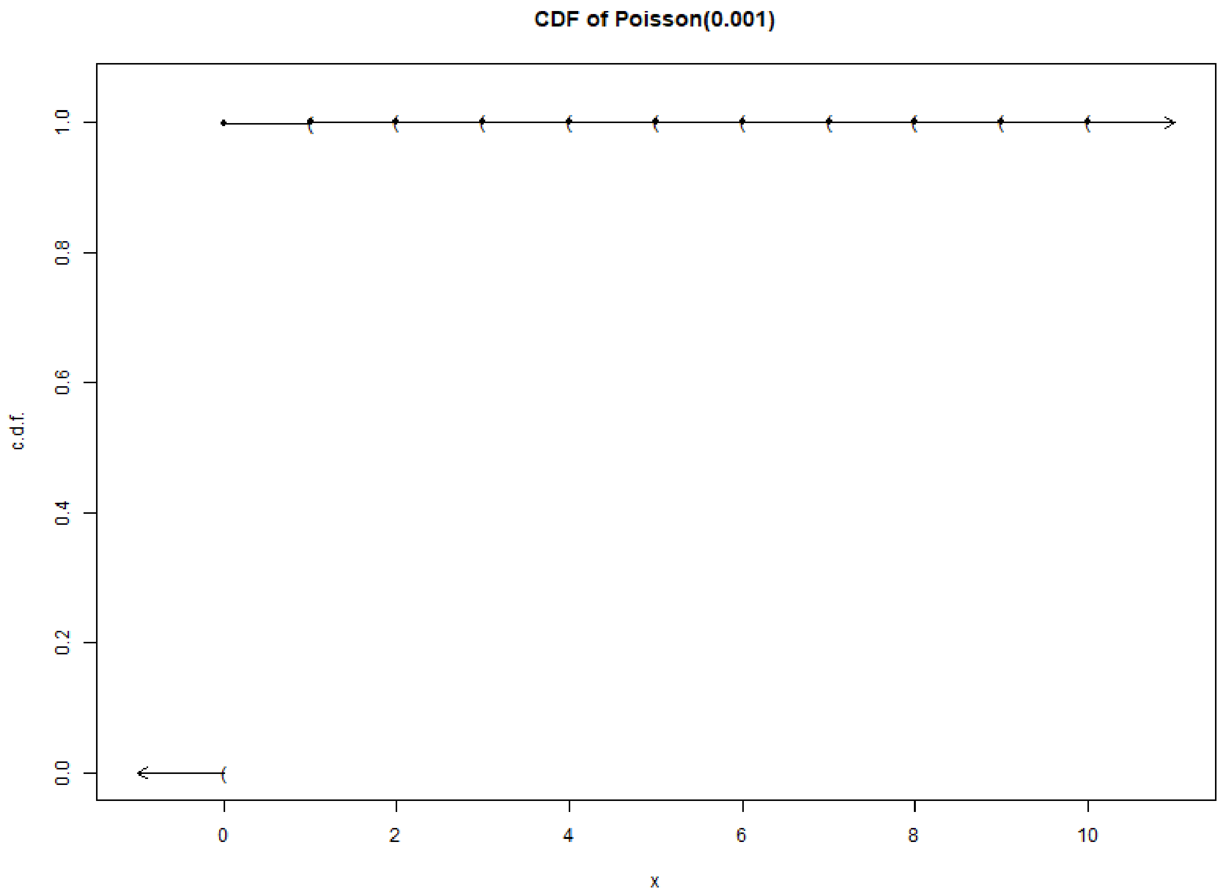
(n=1000, nn=1000, m=18, p=0.95)




Histogram of Mean
Frequency
y$mean
Boxplot of Mean
NPP of Mean
Sample Quantiles
Theoretical Quantiles
Histogram of Median
Frequency
y$median
Boxplot of Median
NPP of Median
Sample Quantiles
Theoretical Quantiles
Histogram of SD
Frequency
y$sd
Boxplot of SD
NPP of SD
Sample Quantiles
Theoretical Quantiles
Histogram of Minimum
Frequency
y$minm
Boxplot of Minimum
NPP of Minimum
Sample Quantiles
Theoretical Quantiles
Histogram of Maximum
Frequency
y$maxm
Boxplot of Maximum
NPP of Maximum
Sample Quantiles
Theoretical Quantiles
Histogram of IQR
Frequency
y$iqr
Boxplot of IQR
NPP of IQR
Sample Quantiles
Theoretical Quantiles

POISSON DISTRIBUTION (0.001)





POISSON DISTRIBUTION

	Values of n to achieve normality (nn=1000, λ=0.001)							
Statistic	n=10	n=50	n=100	n=500	n=1000	n=5000	Any n	Min(n)
Mean	No	No	No	No	No	No	Yes	10000
Median	No	No	No	No	No	No	No	NA
Std Dev	No	No	No	No	No	No	Yes	15000
Min	No	No	No	No	No	No	No	NA
Max	No	No	No	No	No	No	No	NA
IQR	No	No	No	No	No	No	No	NA

Conclusion for Poisson Distribution (λ = 0.001)

Normality Achieved:

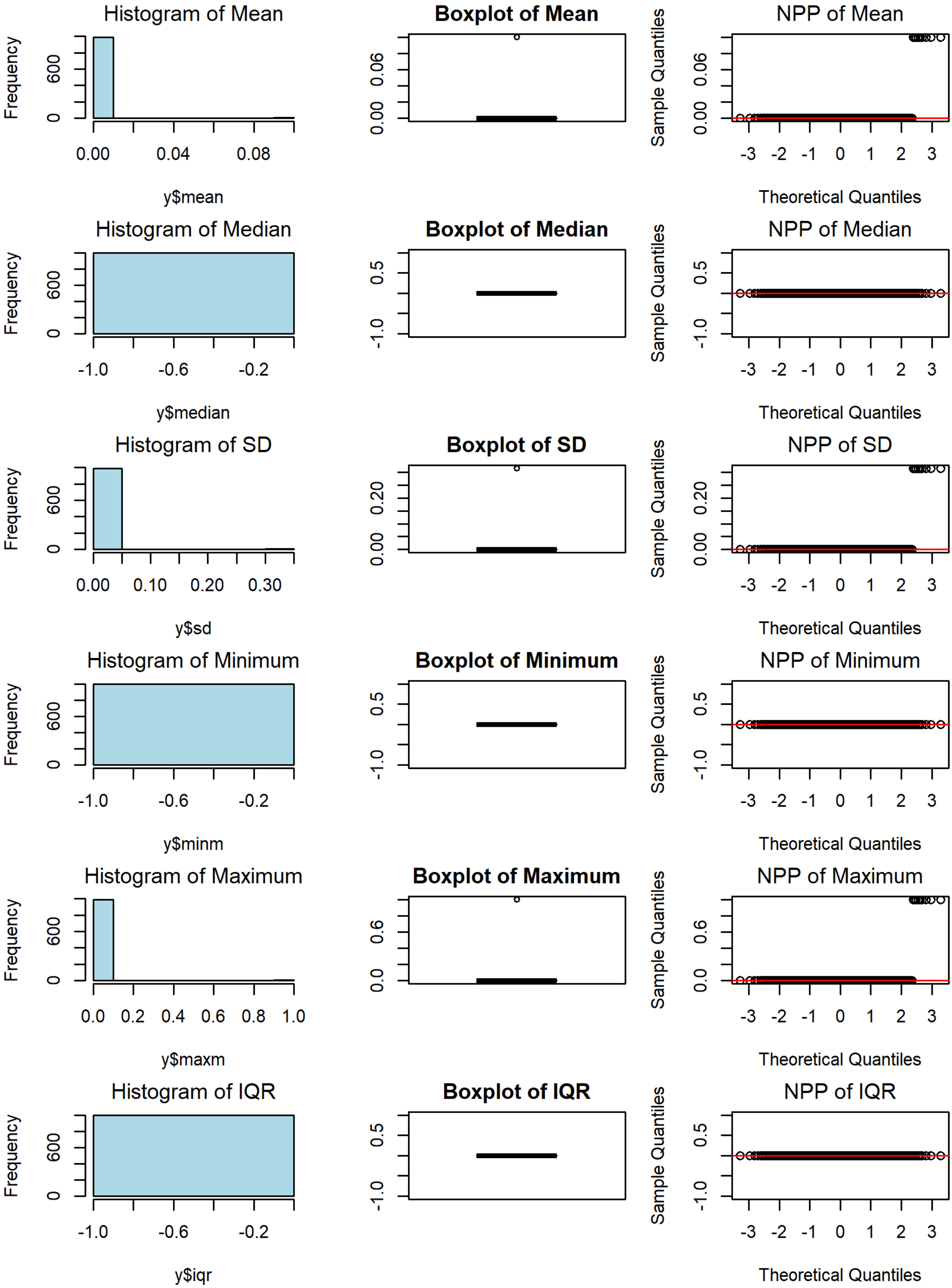
- **Mean:** Achieves normality for n ≥ 10,000, requiring a very large sample size due to the extremely low λ, which results in a highly skewed distribution.
- **Standard Deviation (SD):** Achieves normality for n ≥ 15,000, reflecting even slower convergence compared to the mean.

Normality Not Achieved:

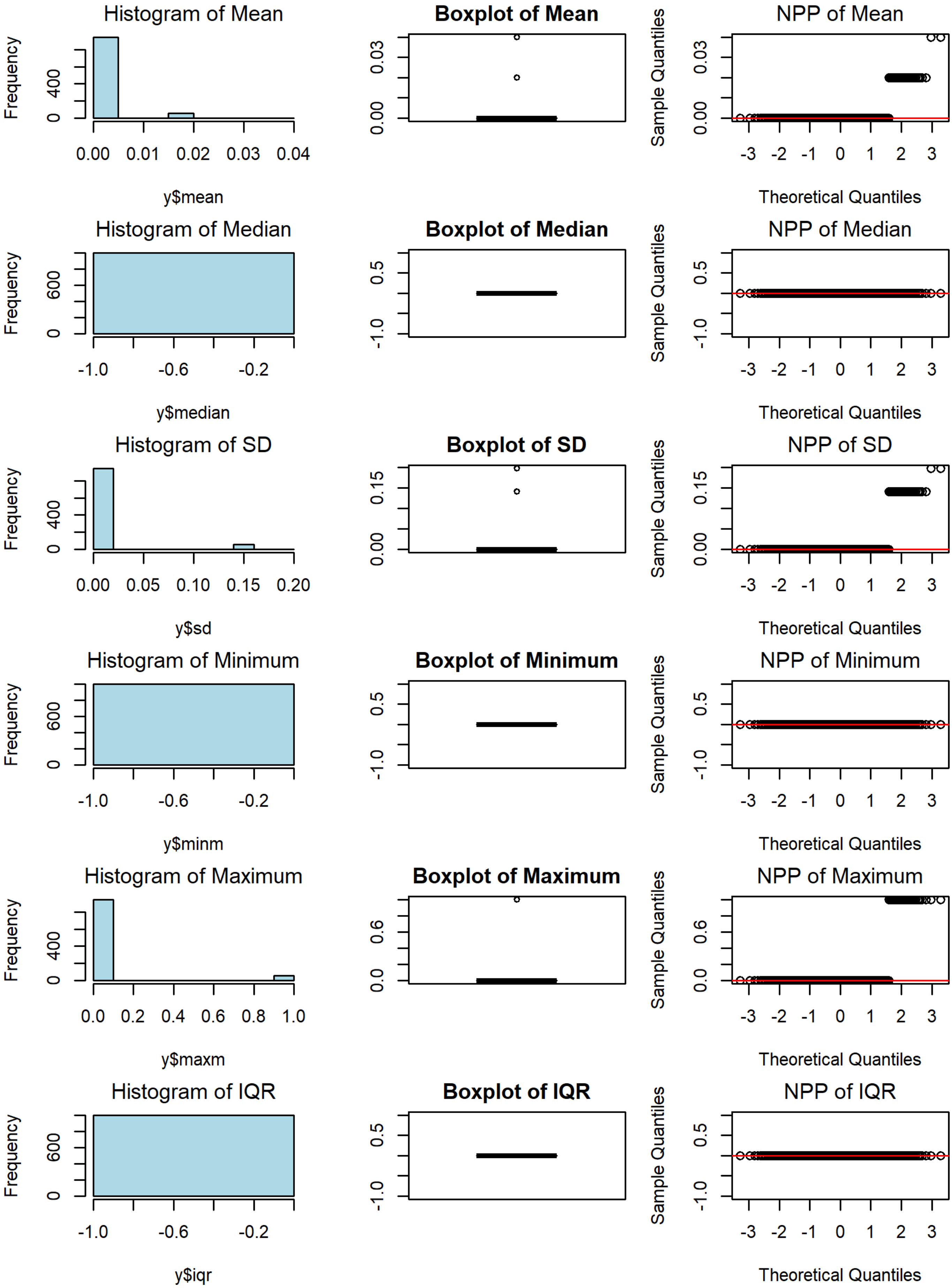
- **Median:** Does not achieve normality for any sample size due to the discrete nature of the Poisson distribution and the extreme skewness at λ = 0.001.
- **Minimum and Maximum:** Do not achieve normality for any sample size, as they are heavily influenced by the rarity of events in a distribution with such a low λ.
- **IQR:** Does not achieve normality for any sample size, as it remains tied to the structure of the underlying Poisson distribution.

Overall: For the Poisson distribution with λ = 0.001, achieving normality for the mean and standard deviation requires very large sample sizes (n ≥ 10,000 for the mean and n ≥ 15,000 for SD). This is significantly slower compared to Poisson distributions with larger λ values, where normality is achieved more easily due to reduced skewness. The median, minimum, maximum, and IQR remain non-normal, consistent with the highly discrete and sparse nature of the distribution at such a low λ.

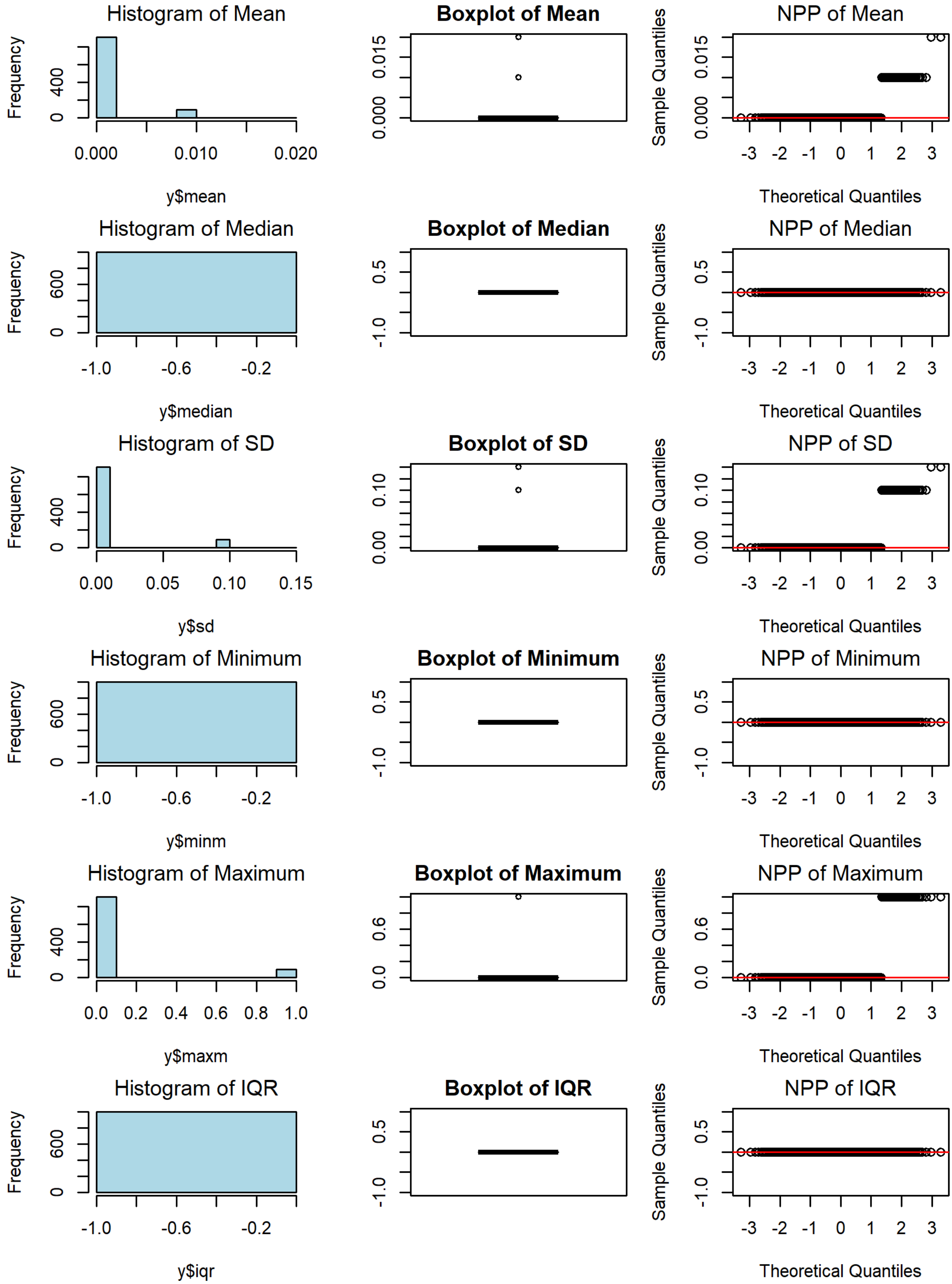
POISSON DISTRIBUTION PLOT
(n=10, nn=1000, λ =0.001)



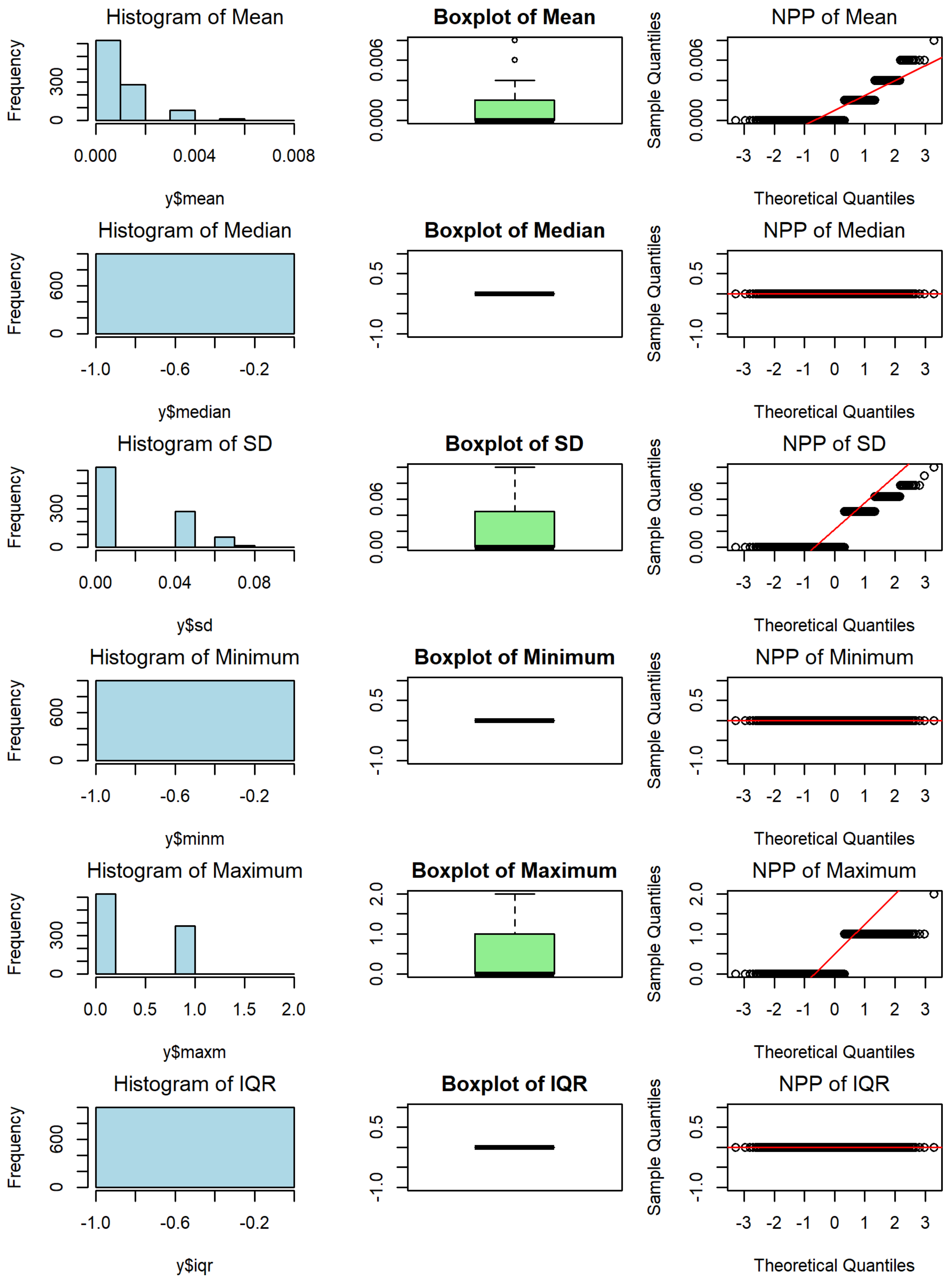
POISSON DISTRIBUTION PLOT
(n=50, nn=1000, λ =0.001)



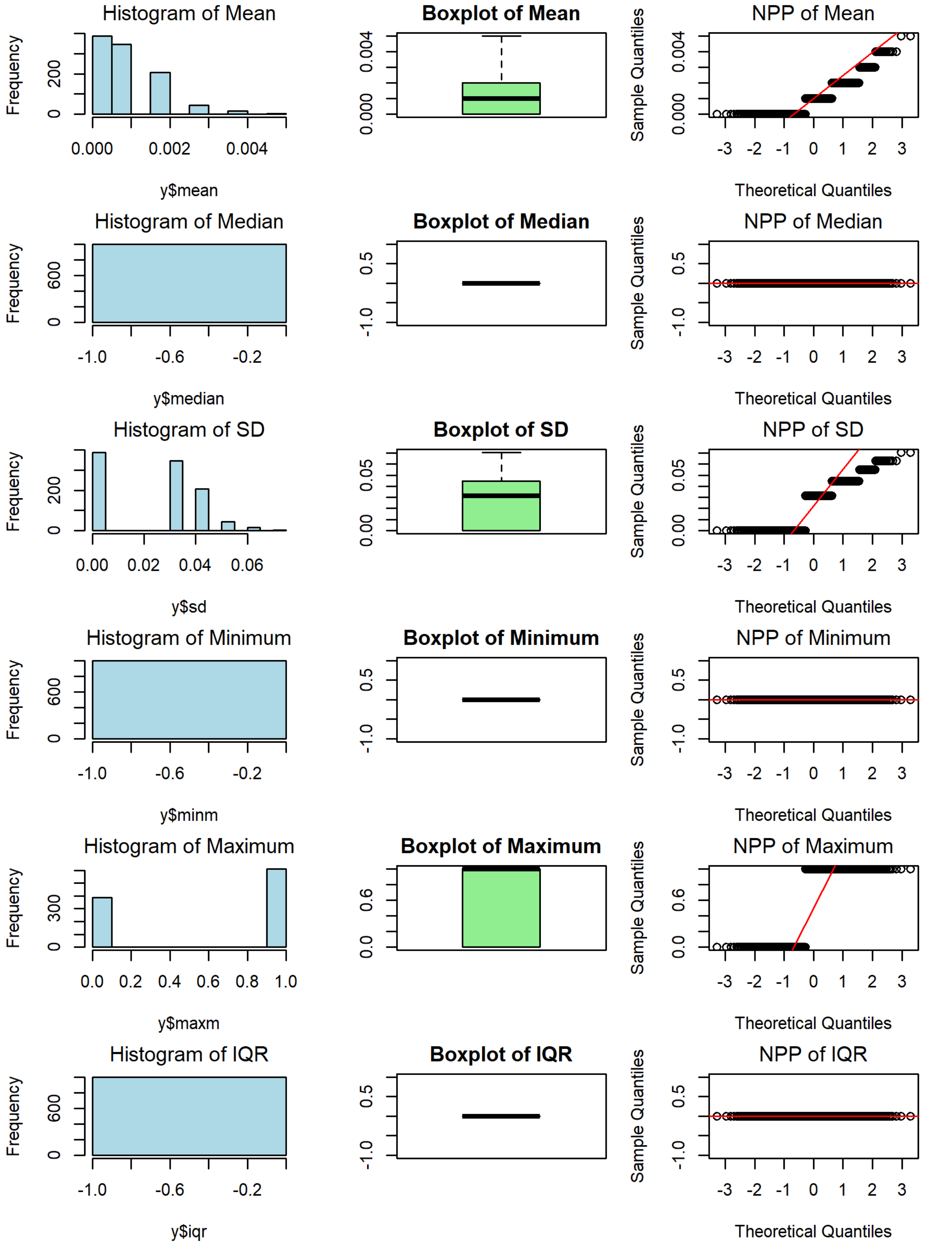
POISSON DISTRIBUTION PLOT
(n=100, nn=1000, λ =0.001)

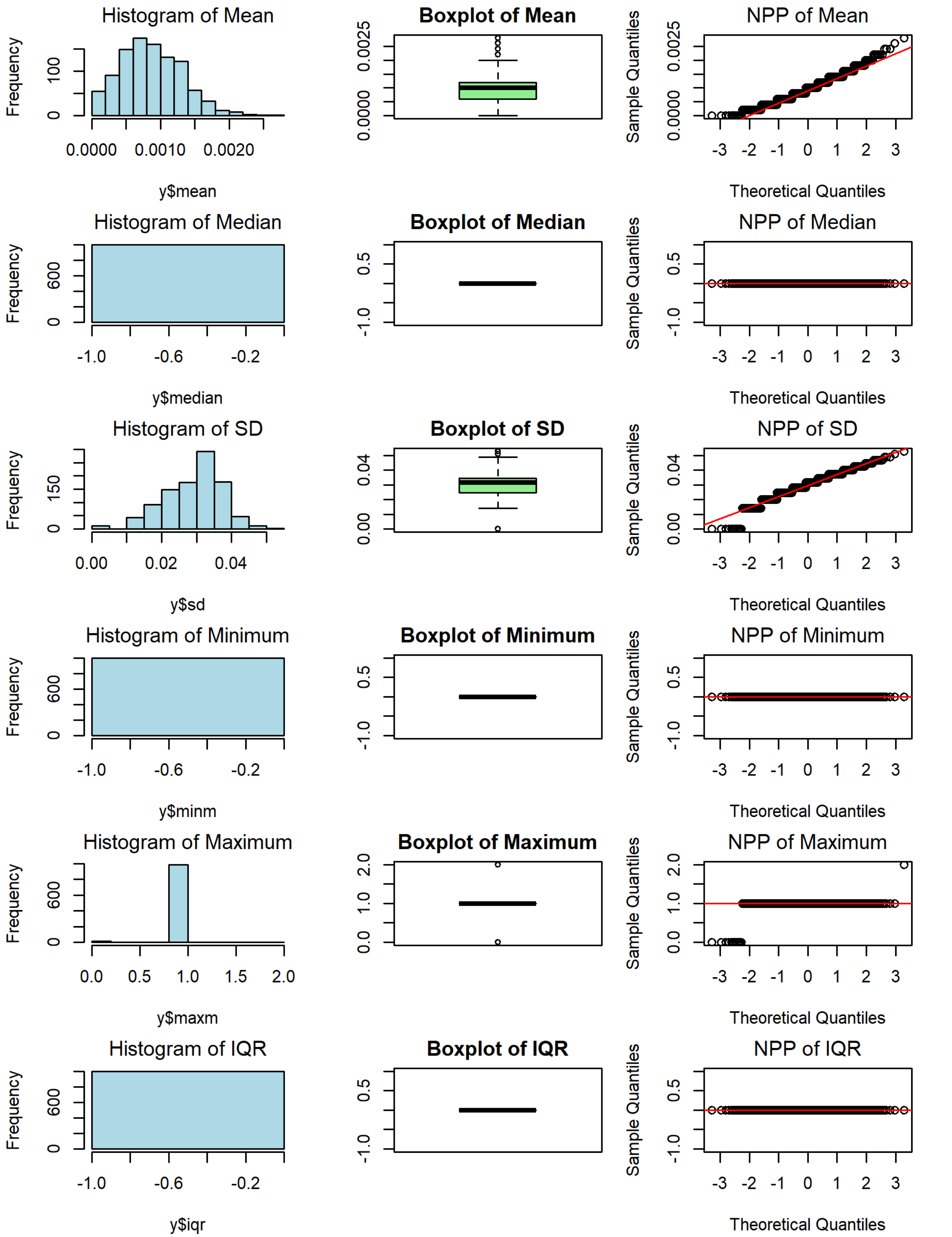


POISSON DISTRIBUTION PLOT
(n=500, nn=1000, λ =0.001)



POISSON DISTRIBUTION PLOT
(n=1000, nn=1000, λ =0.001)




Histogram of Mean
Frequency
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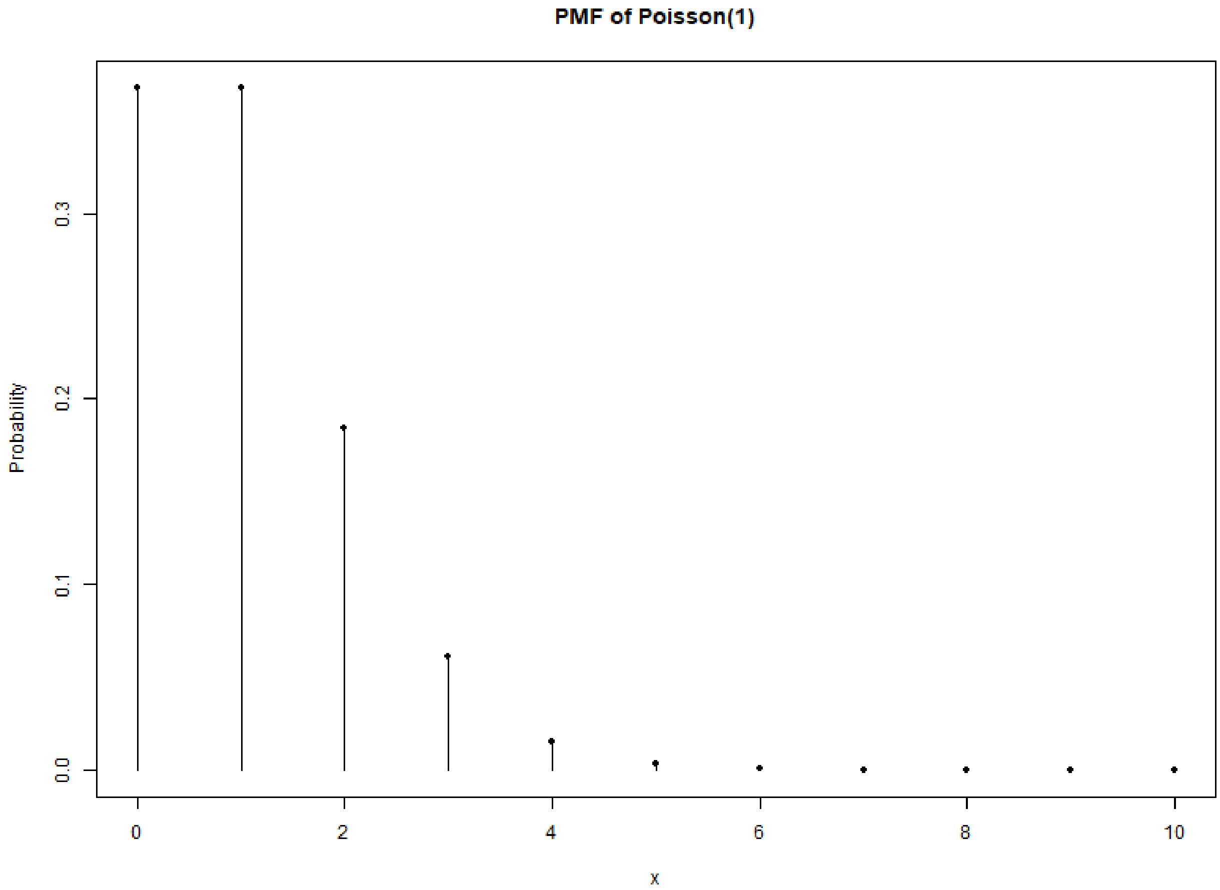
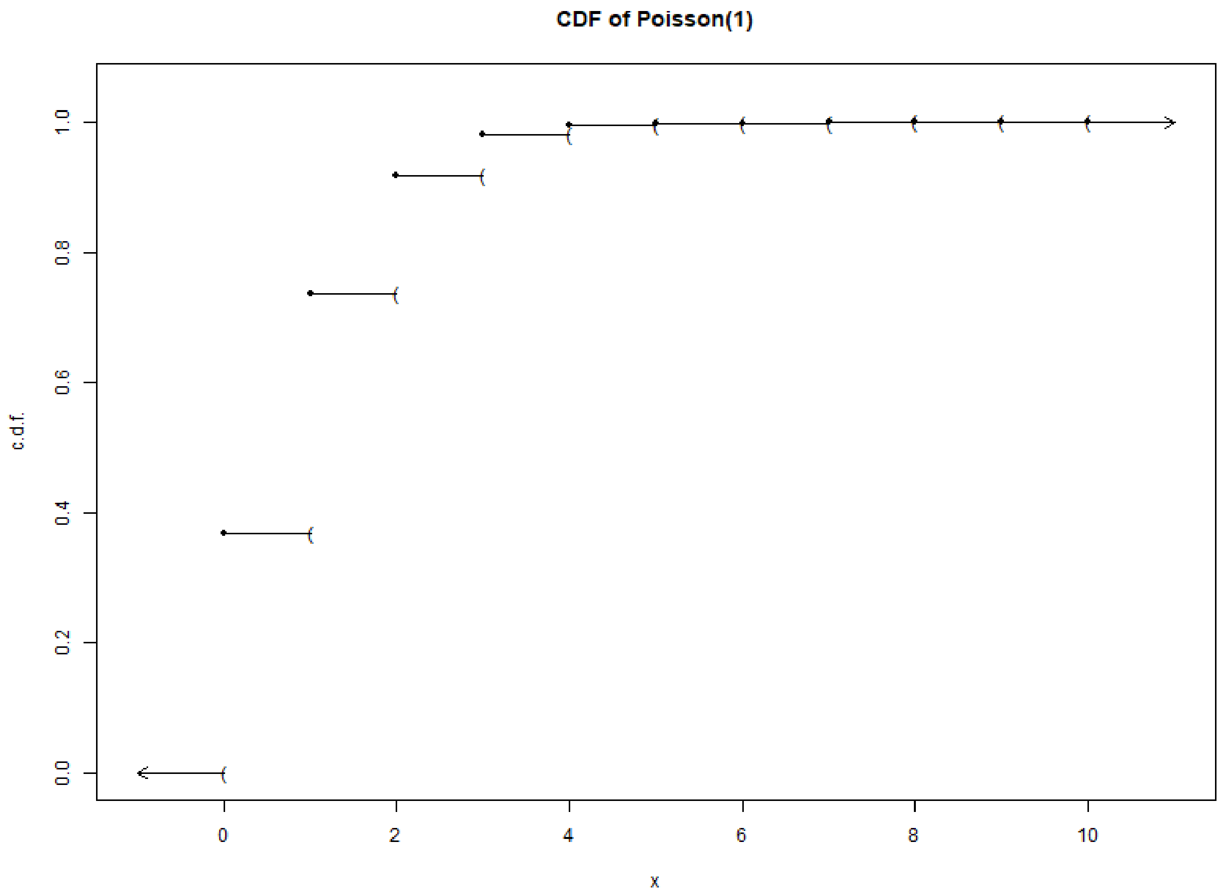
100
0.0000
0.0010
0.0020
y$mean
Boxplot of Mean
0.0000
0.0025
NPP of Mean
Sample Quantiles
0.0000
0.0025
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of Median
Frequency
0
600
-1.0
-0.6
-0.2
y$median
Boxplot of Median
-1.0
0.5
NPP of Median
Sample Quantiles
-1.0
0.5
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of SD
Frequency
0
150
0.00
0.02
0.04
y$sd
Boxplot of SD
0.00
0.04
NPP of SD
Sample Quantiles
0.00
0.04
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of Minimum
Frequency
0
600
-1.0
-0.6
-0.2
y$minm
Boxplot of Minimum
-1.0
0.5
NPP of Minimum
Sample Quantiles
-1.0
0.5
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of Maximum
Frequency
0
600
0.0
0.5
1.0
1.5
2.0
y$maxm
Boxplot of Maximum
0.0
1.0
2.0
NPP of Maximum
Sample Quantiles
0.0
1.0
2.0
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of IQR
Frequency
0
600
-1.0
-0.6
-0.2
y$iqr
Boxplot of IQR
-1.0
0.5
NPP of IQR
Sample Quantiles
-1.0
0.5
-3 -2 -1 0 1 2 3
Theoretical Quantiles

POISSON DISTRIBUTION (1)





POISSON DISTRIBUTION

	Values of n to achieve normality (nn=1000, λ =1)						
Statistic	n=10	n=50	n=100	n=500	n=1000	n=5000	Any n	Min(n)
Mean	No	Yes	Yes	Yes	Yes	Yes	Yes	50
Median	No	No	No	No	No	No	No	NA
Std Dev	No	Yes	Yes	Yes	Yes	Yes	Yes	50
Min	No	No	No	No	No	No	No	NA
Max	No	No	No	No	No	No	No	NA
IQR	No	No	No	No	No	No	No	NA

Conclusion for Poisson Distribution (λ = 1)

Normality Achieved:

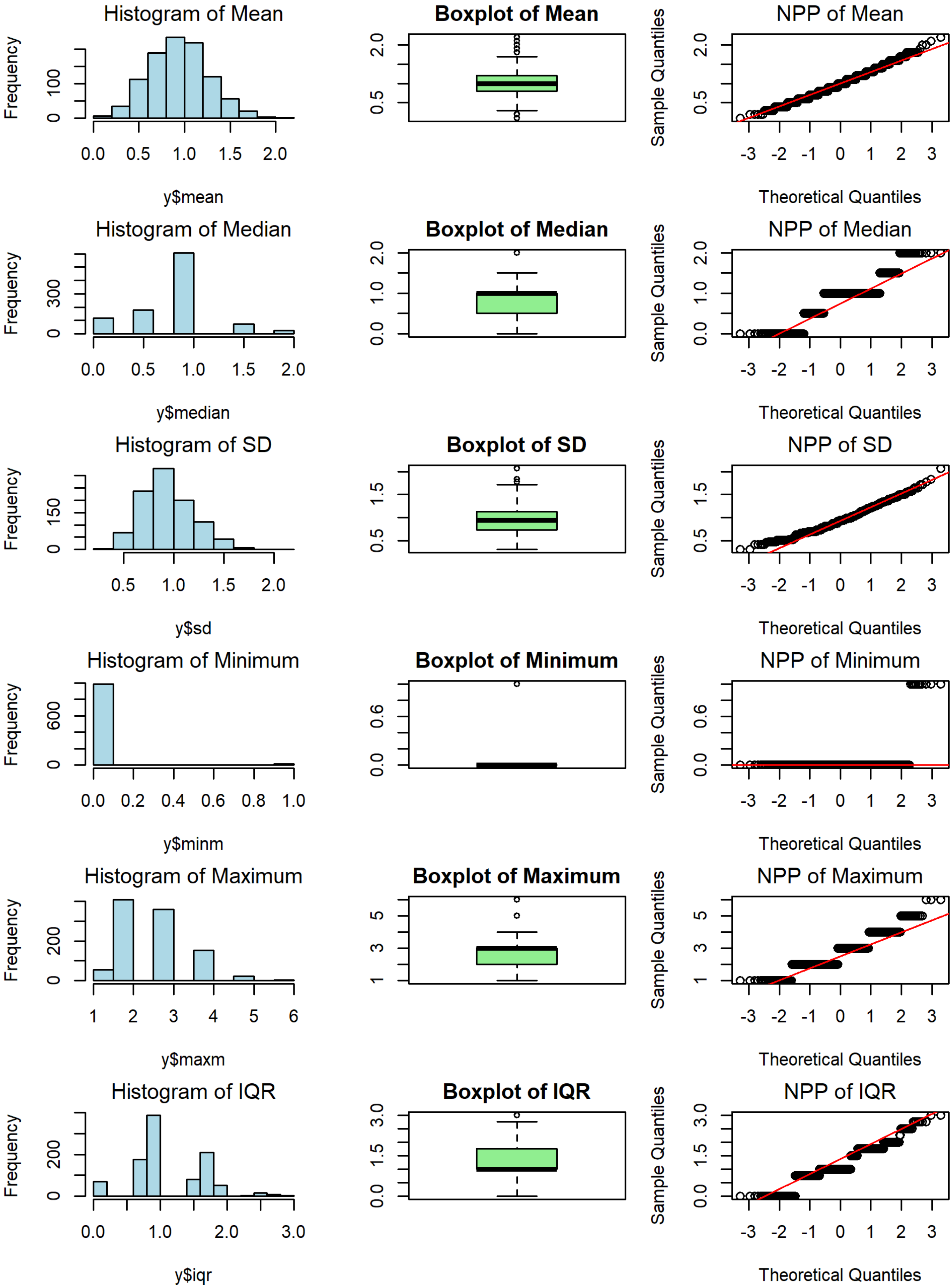
- **Mean:** Achieves normality for n ≥ 50, converging relatively quickly due to the moderate λ value, which reduces the distribution's skewness.
- **Standard Deviation (SD):** Achieves normality for n ≥ 50, matching the convergence rate of the mean.

Normality Not Achieved:

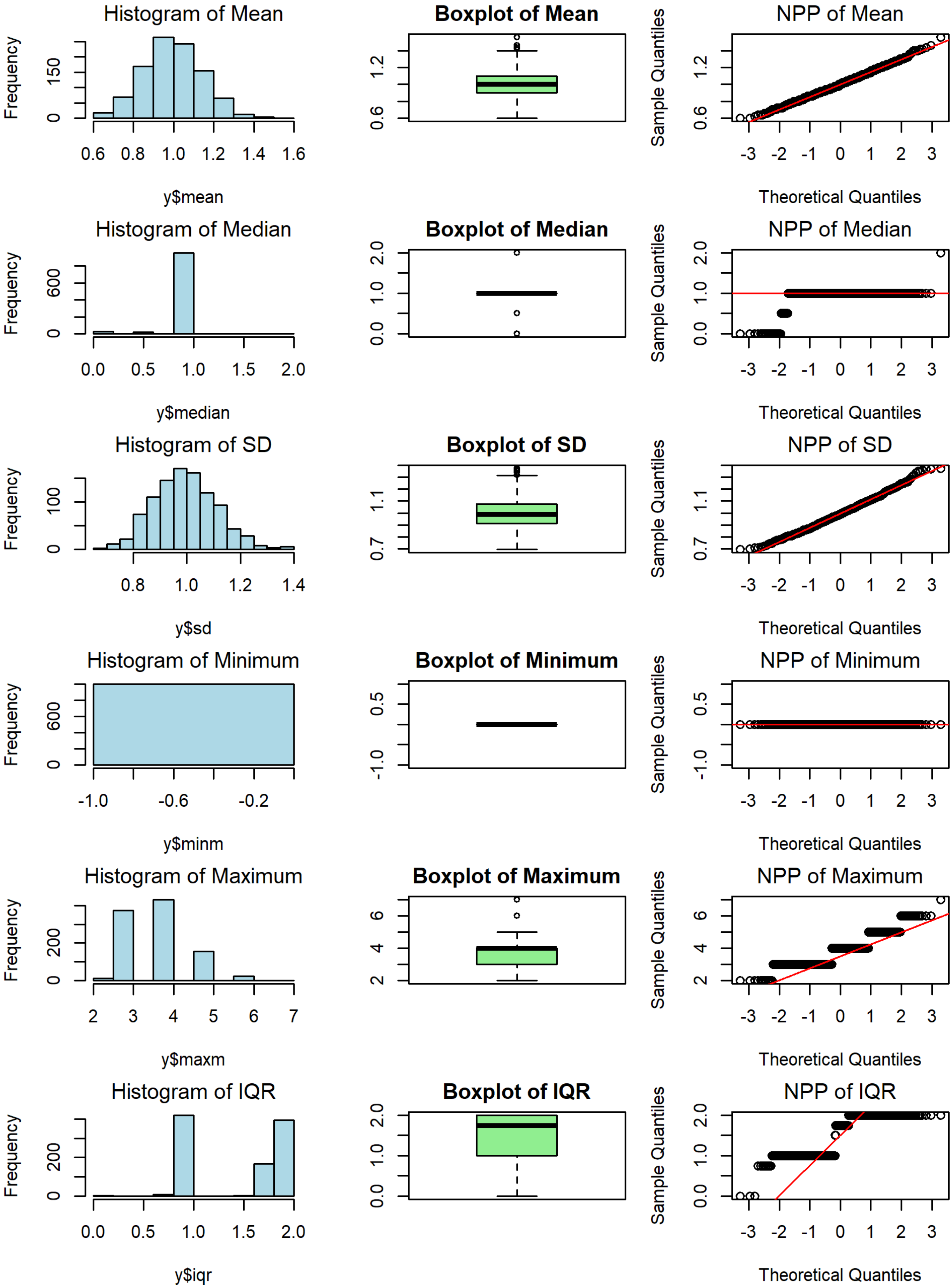
- **Median:** Does not achieve normality for any sample size, as the Poisson distribution remains discrete regardless of λ.
- **Minimum and Maximum:** Do not achieve normality for any sample size due to sensitivity to the extremes of the Poisson distribution.
- **IQR:** Does not achieve normality for any sample size, as it reflects the discrete and skewed nature of the distribution.

Overall: For the Poisson distribution with λ = 1, the mean and standard deviation achieve normality at n ≥ 50, indicating a faster convergence compared to λ = 0.001. However, the median, minimum, maximum, and IQR remain non-normal, consistent with the characteristics of the Poisson distribution. This highlights that as λ increases, the convergence to normality for key statistics improves significantly.

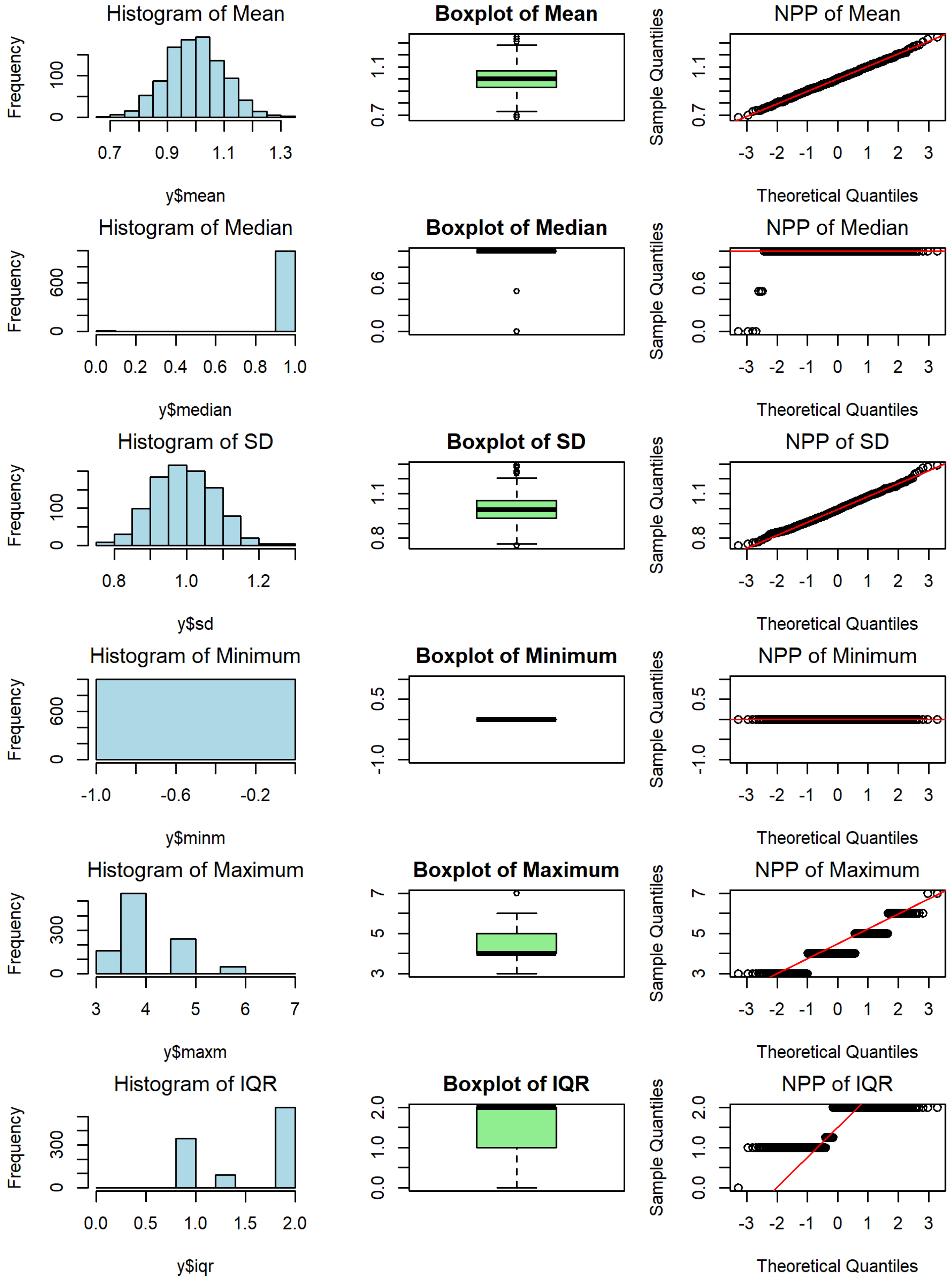
POISSON DISTRIBUTION PLOT
(n=10, nn=1000, λ =1)



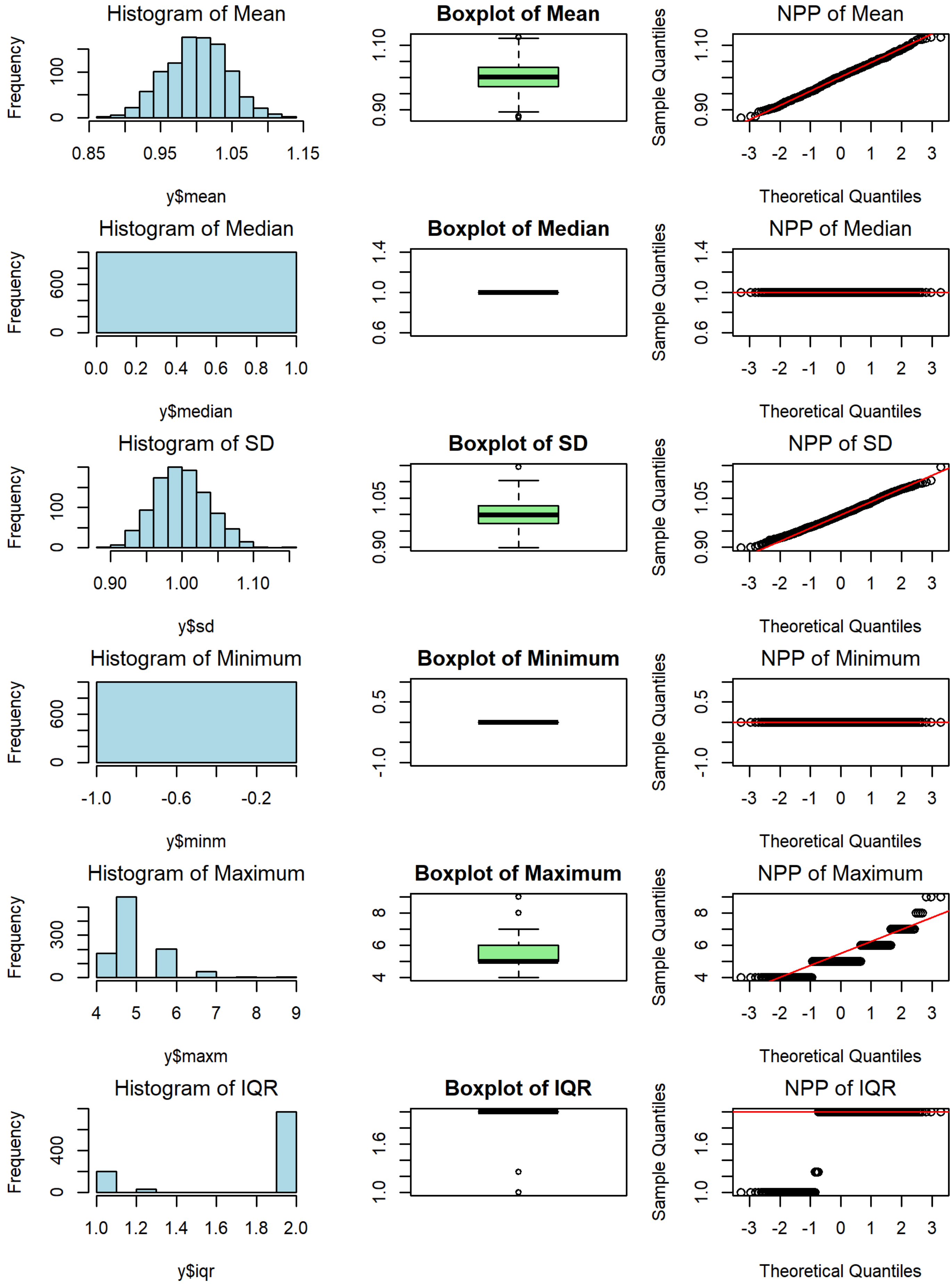
POISSON DISTRIBUTION PLOT
(n=50, nn=1000, λ =1)

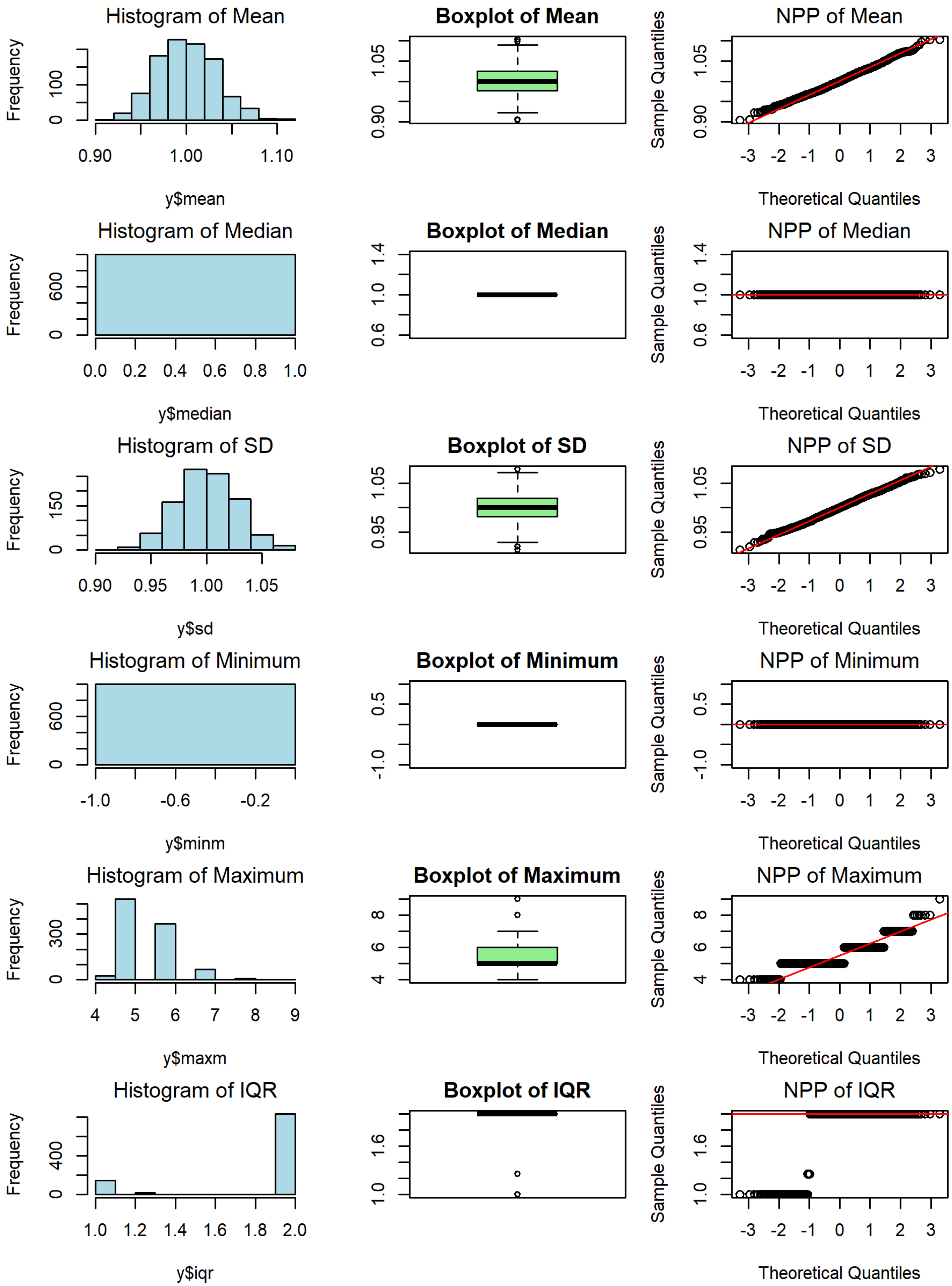


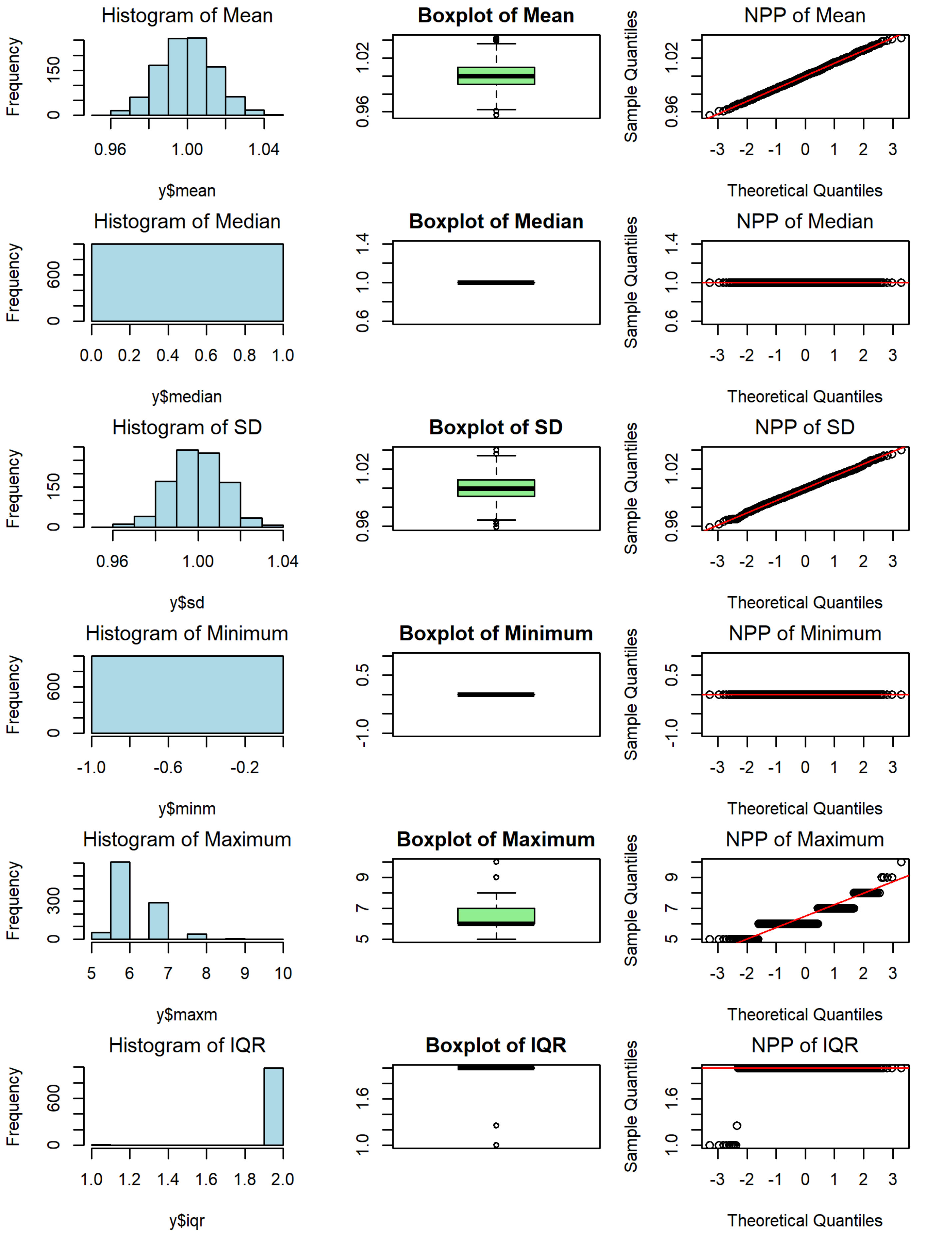
POISSON DISTRIBUTION PLOT
(n=100, nn=1000, λ =1)



POISSON DISTRIBUTION PLOT
(n=500, nn=1000, λ =1)




Histogram of Mean
Frequency
y$mean
Boxplot of Mean
NPP of Mean
Sample Quantiles
Theoretical Quantiles
Histogram of Median
y$median
Boxplot of Median
NPP of Median
Histogram of SD
y$sd
Boxplot of SD
NPP of SD
Histogram of Minimum
y$minm
Boxplot of Minimum
NPP of Minimum
Histogram of Maximum
y$maxm
Boxplot of Maximum
NPP of Maximum
Histogram of IQR
y$iqr
Boxplot of IQR
NPP of IQR


Histogram of Mean
Frequency
y$mean
Boxplot of Mean
NPP of Mean
Sample Quantiles
Theoretical Quantiles
Histogram of Median
Frequency
y$median
Boxplot of Median
NPP of Median
Sample Quantiles
Theoretical Quantiles
Histogram of SD
Frequency
y$sd
Boxplot of SD
NPP of SD
Sample Quantiles
Theoretical Quantiles
Histogram of Minimum
Frequency
y$minm
Boxplot of Minimum
NPP of Minimum
Sample Quantiles
Theoretical Quantiles
Histogram of Maximum
Frequency
y$maxm
Boxplot of Maximum
NPP of Maximum
Sample Quantiles
Theoretical Quantiles
Histogram of IQR
Frequency
y$iqr
Boxplot of IQR
NPP of IQR
Sample Quantiles
Theoretical Quantiles

POISSON DISTRIBUTION

	Values of n to achieve normality (nn=1000, λ = 25)							
Statistic	n=10	n=50	n=100	n=500	n=1000	n=5000	Any n	Min(n)
Mean	Yes	Yes	Yes	Yes	Yes	Yes	Yes	10
Median	No	No	No	No	No	No	No	NA
Std Dev	No	Yes	Yes	Yes	Yes	Yes	Yes	50
Min	No	No	No	No	No	No	No	NA
Max	No	No	No	No	No	No	No	NA
IQR	No	No	No	No	No	No	No	NA

Conclusion for Poisson Distribution (λ = 25)

Normality Achieved:

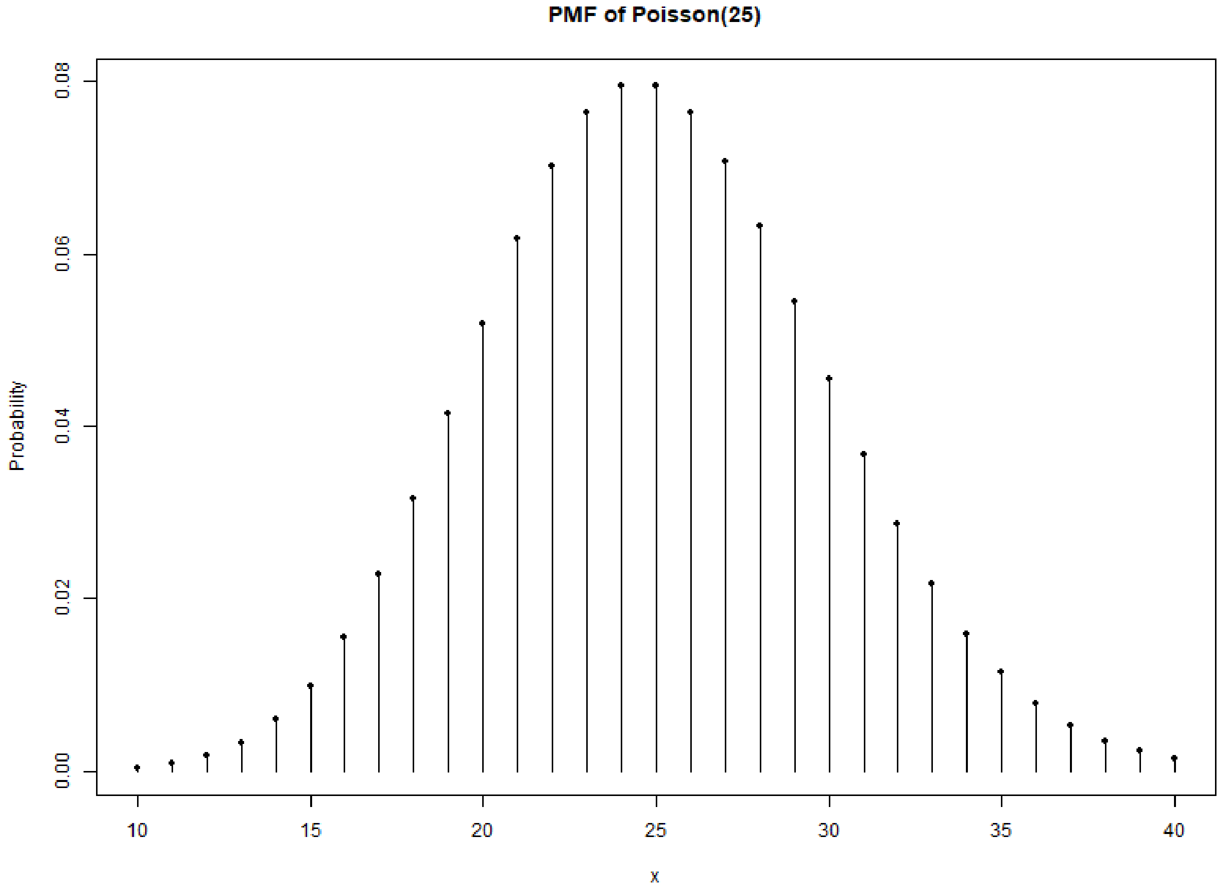
- **Mean:** Achieves normality for all sample sizes (n ≥ 10), as the higher λ value makes the distribution more symmetric and closer to the normal distribution even with smaller sample sizes.
- **Standard Deviation (SD):** Achieves normality for n ≥ 50, reflecting the rapid convergence due to reduced skewness at this higher λ.

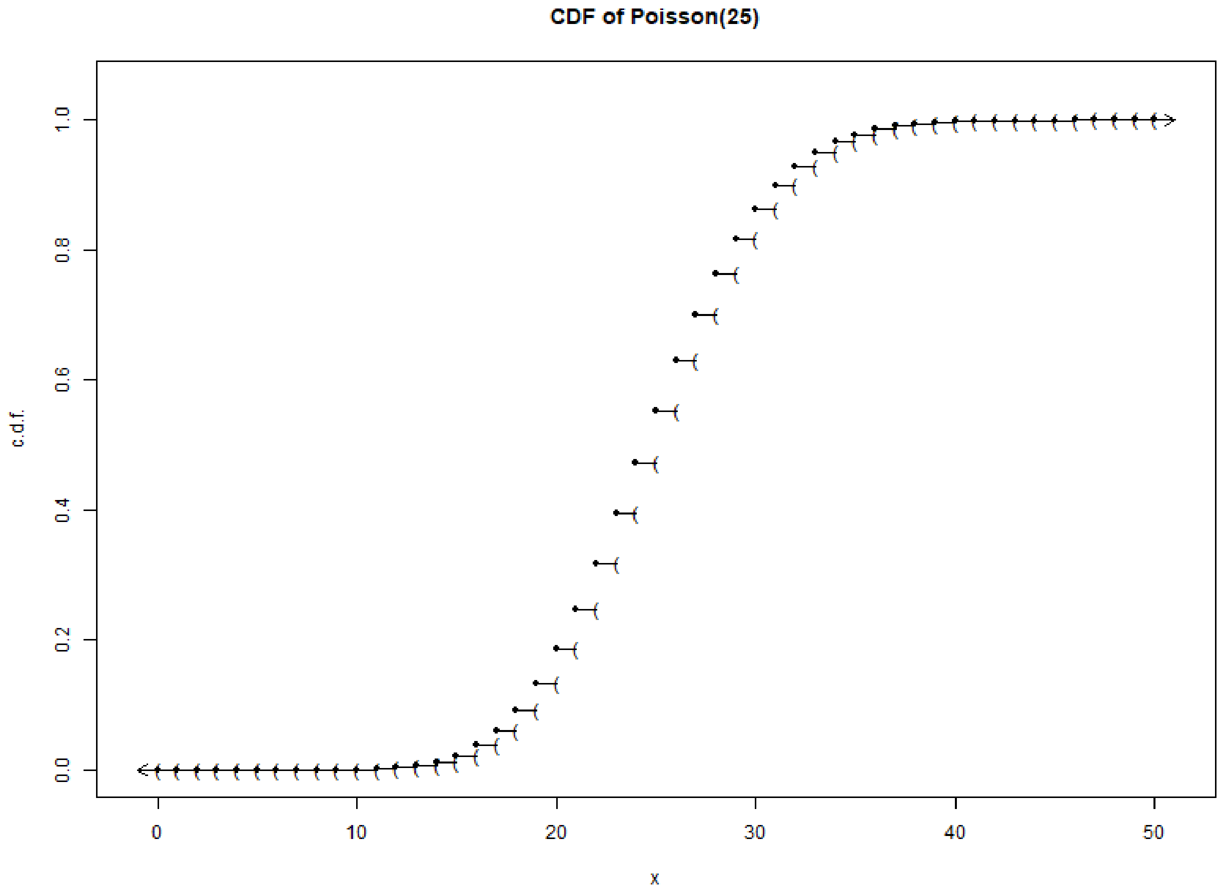
Normality Not Achieved:

- **Median:** Does not achieve normality for any sample size, as the Poisson distribution remains discrete despite the increased λ value.
- **Minimum and Maximum:** Do not achieve normality for any sample size, as they are influenced by the extreme tails of the distribution.
- **IQR:** Does not achieve normality for any sample size, indicating that interquartile properties are less affected by λ compared to the mean and SD.

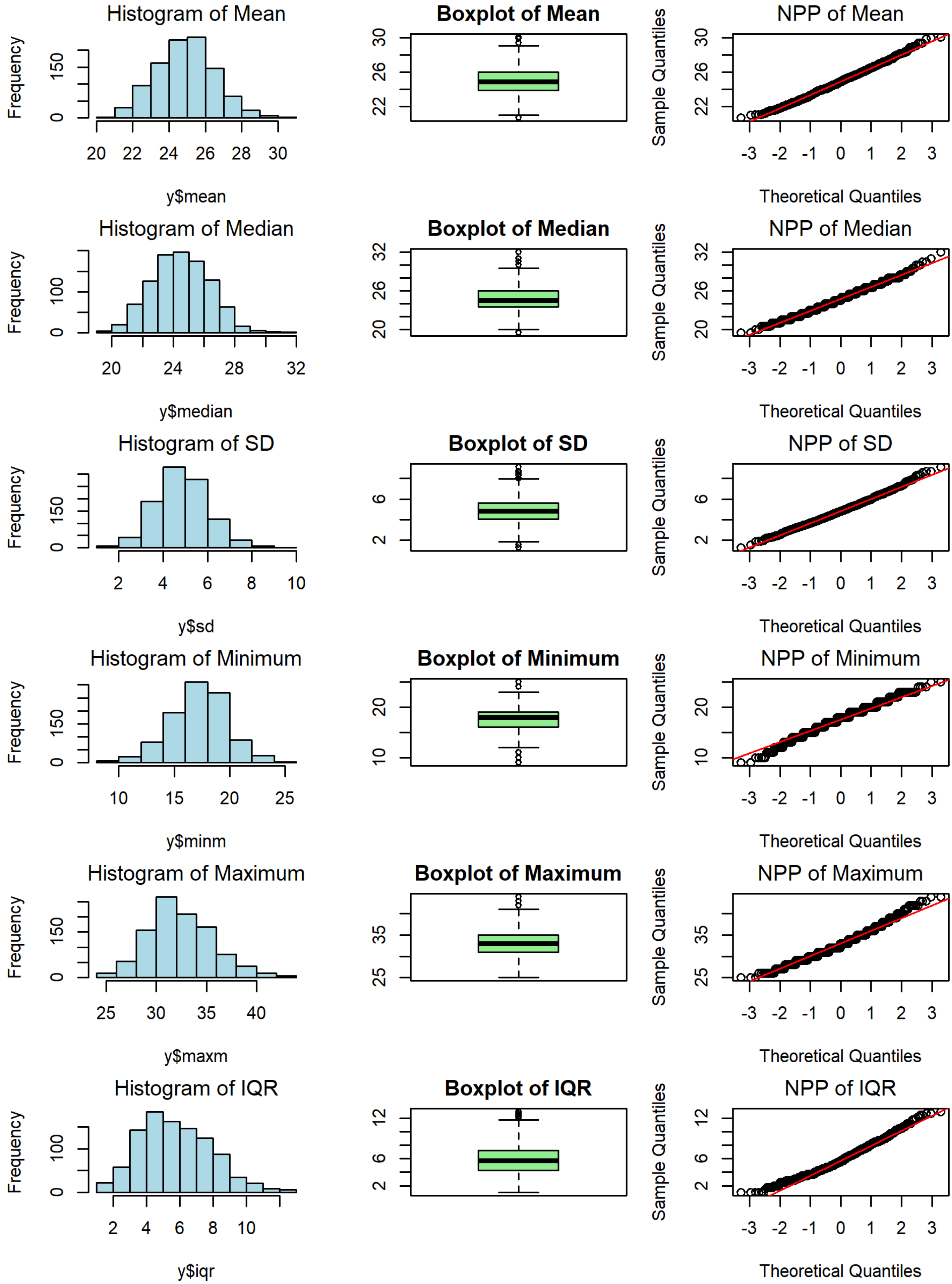
Overall: For the Poisson distribution with λ = 25, the mean achieves normality for all sample sizes, and the standard deviation follows closely at n ≥ 50. This demonstrates the rapid convergence facilitated by the higher λ value. However, the median, minimum, maximum, and IQR remain non-normal, highlighting their resistance to convergence regardless of λ. The results underscore how larger λ values significantly enhance the normality of key statistics in the Poisson distribution.

POISSON DISTRIBUTION (25)

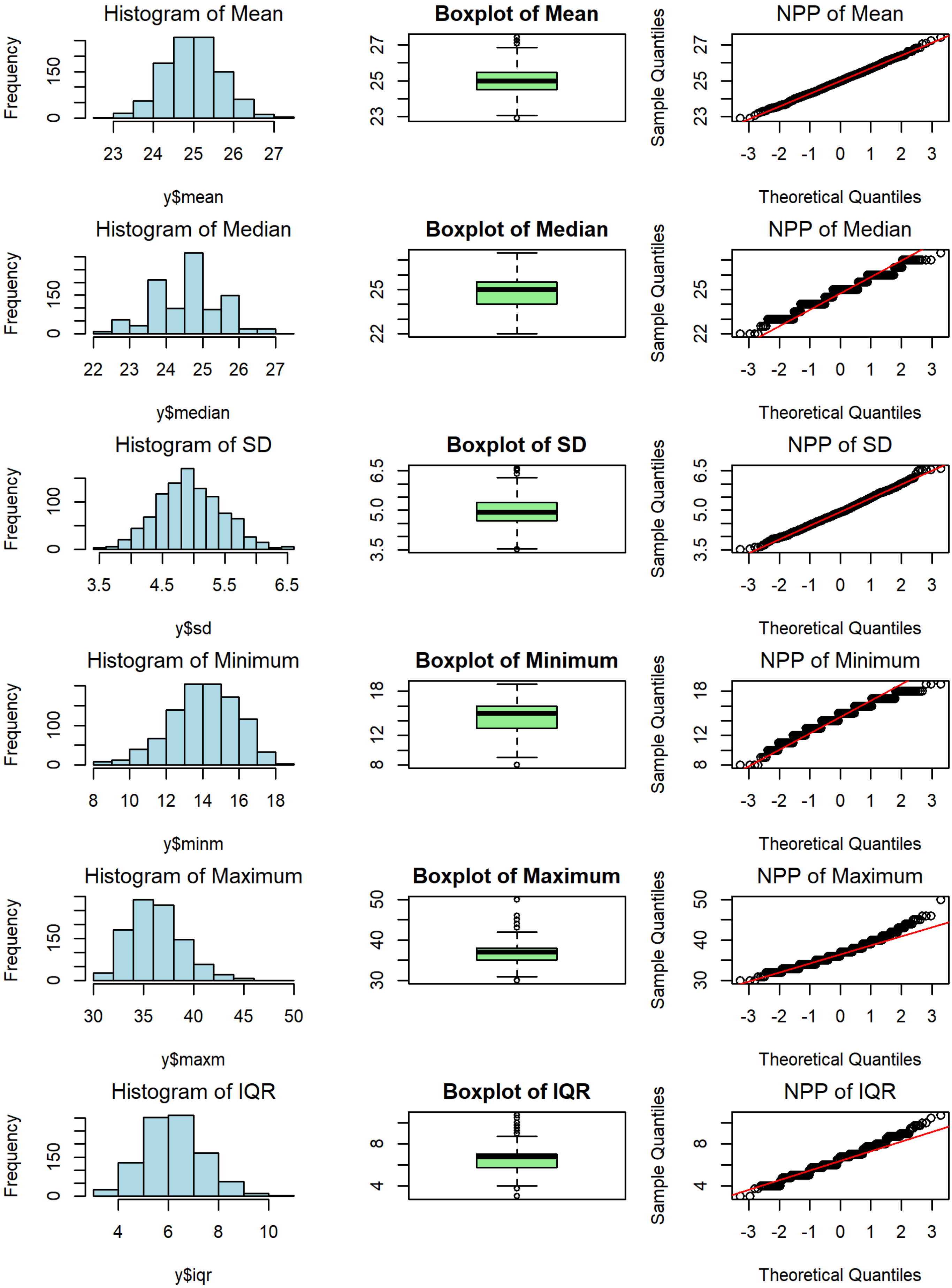




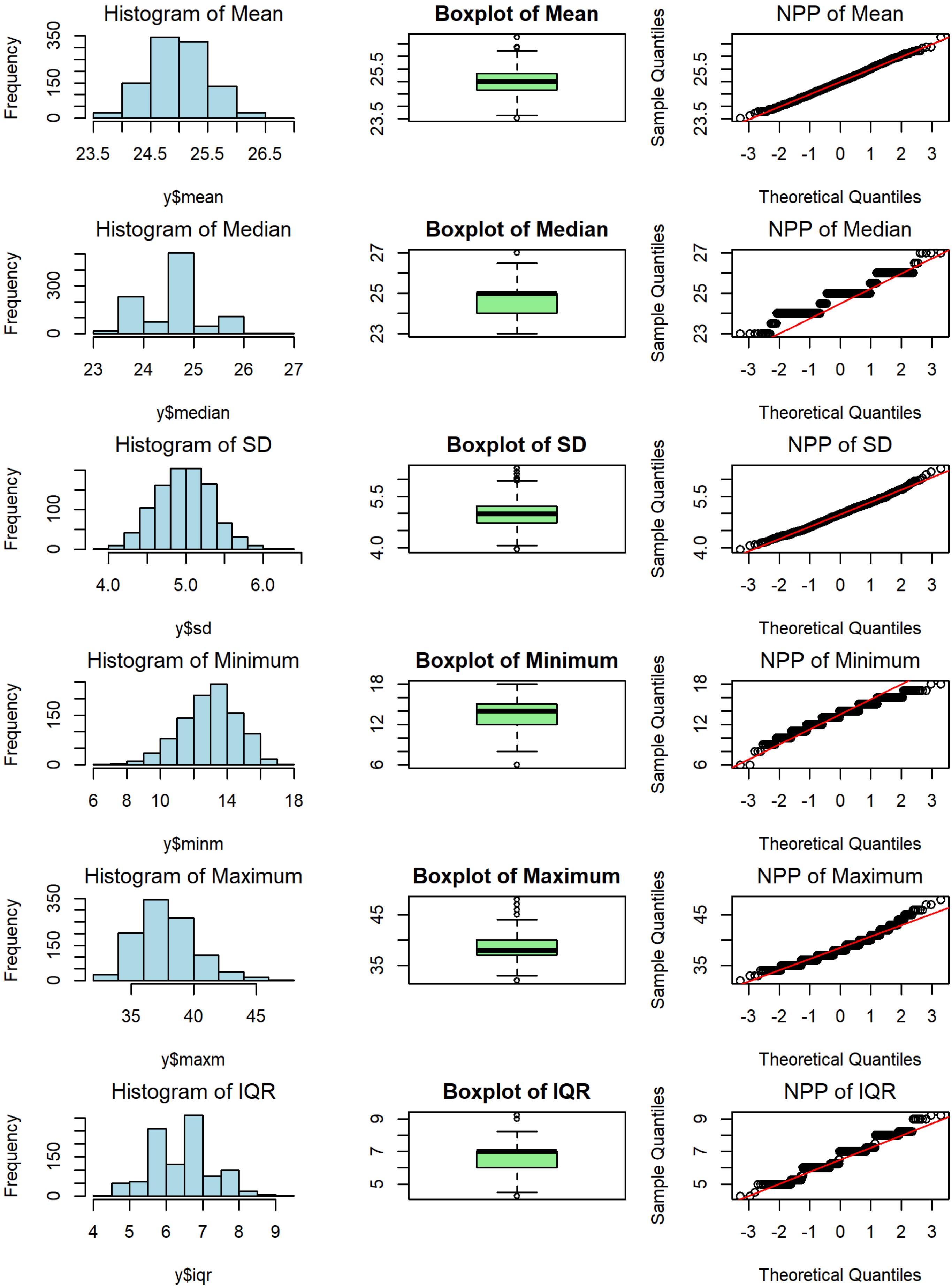
POISSON DISTRIBUTION PLOT
(n=10, nn=1000, λ =25)



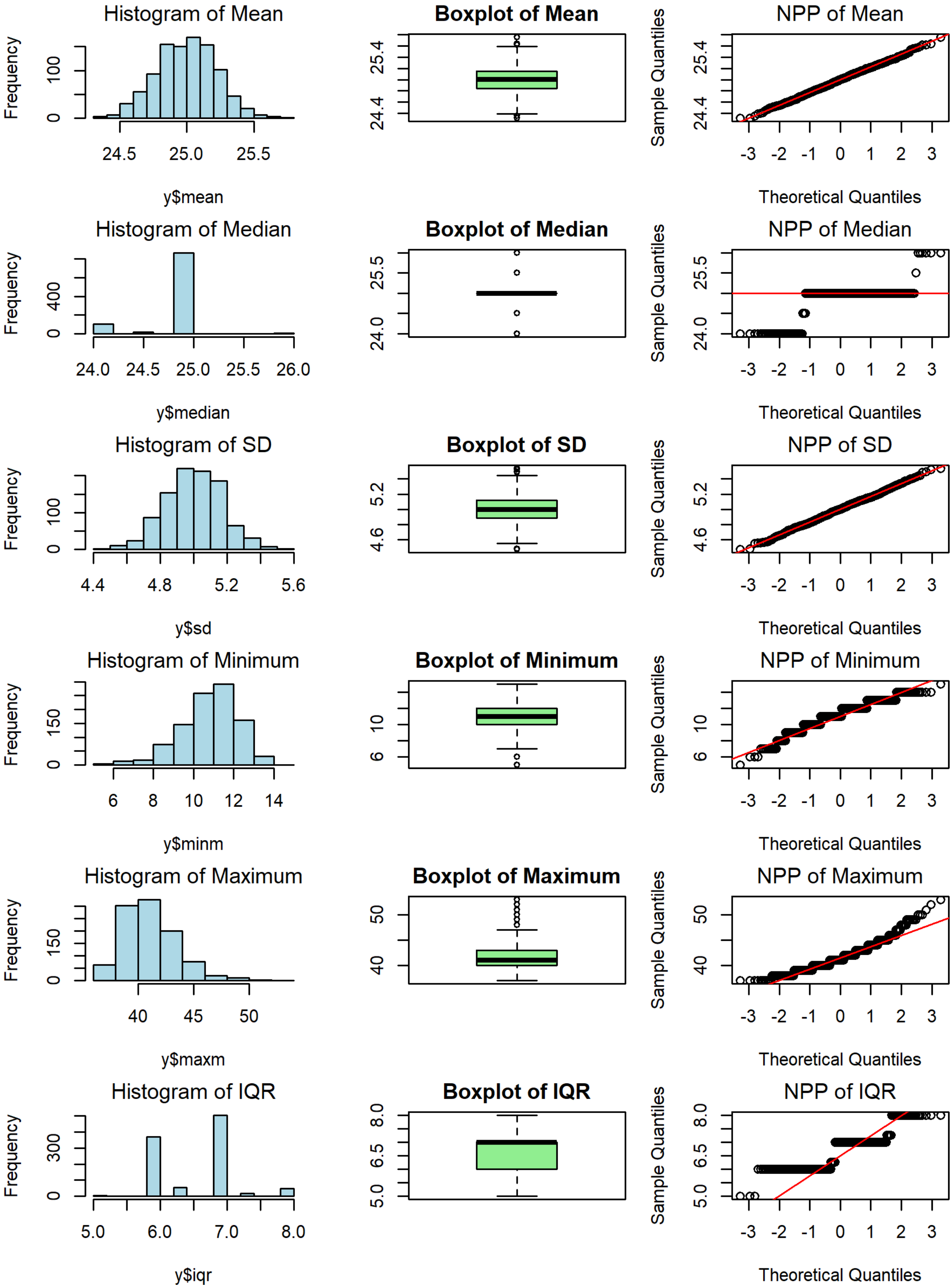
POISSON DISTRIBUTION PLOT
(n=50, nn=1000, λ =25)

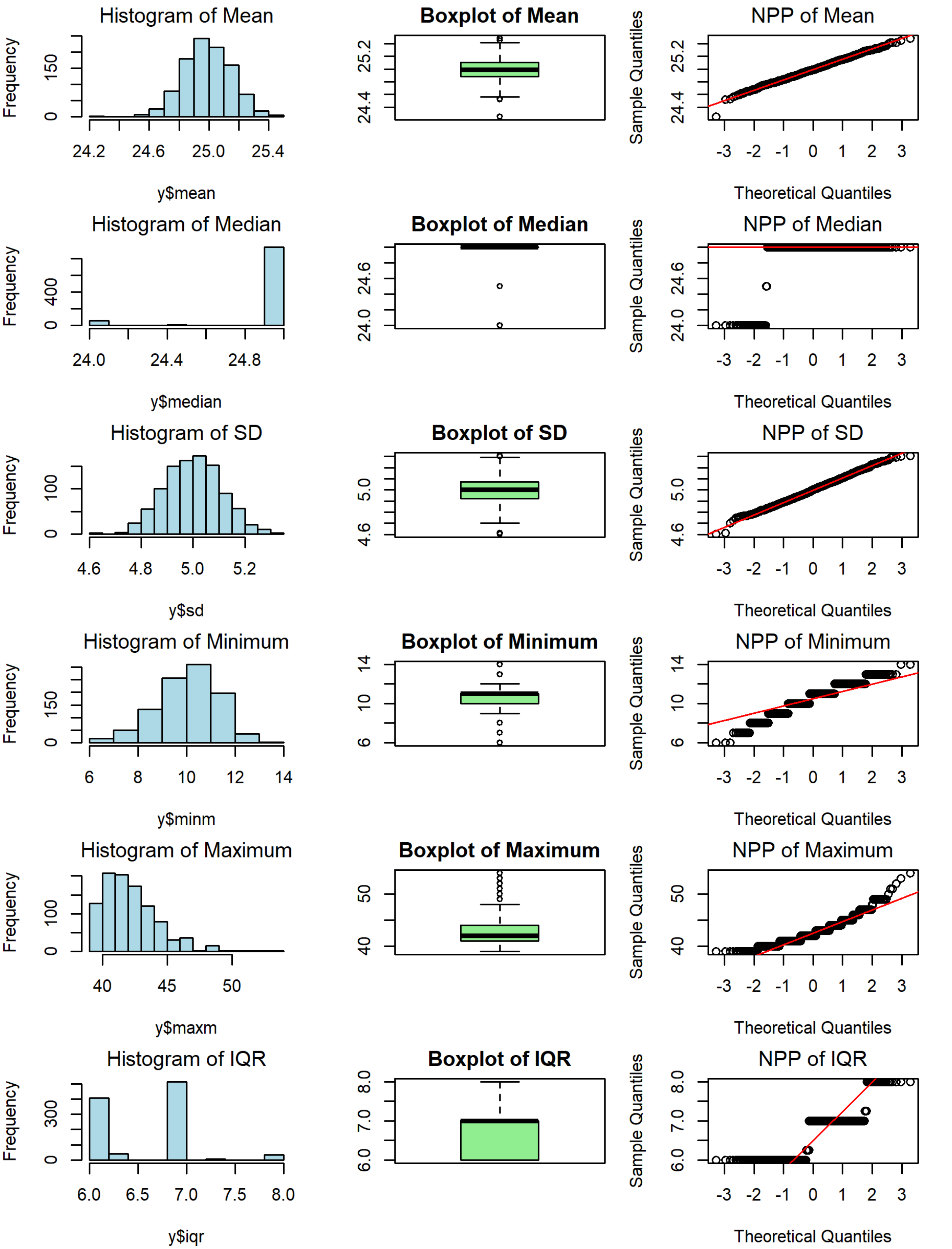


POISSON DISTRIBUTION PLOT
(n=100, nn=1000, λ =25)



POISSON DISTRIBUTION PLOT
(n=500, nn=1000, λ =25)




Histogram of Mean
Boxplot of Mean
NPP of Mean
Frequency
Sample Quantiles
y$mean
Theoretical Quantiles
Histogram of Median
Boxplot of Median
NPP of Median
y$median
Histogram of SD
Boxplot of SD
NPP of SD
y$sd
Histogram of Minimum
Boxplot of Minimum
NPP of Minimum
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Boxplot of Maximum
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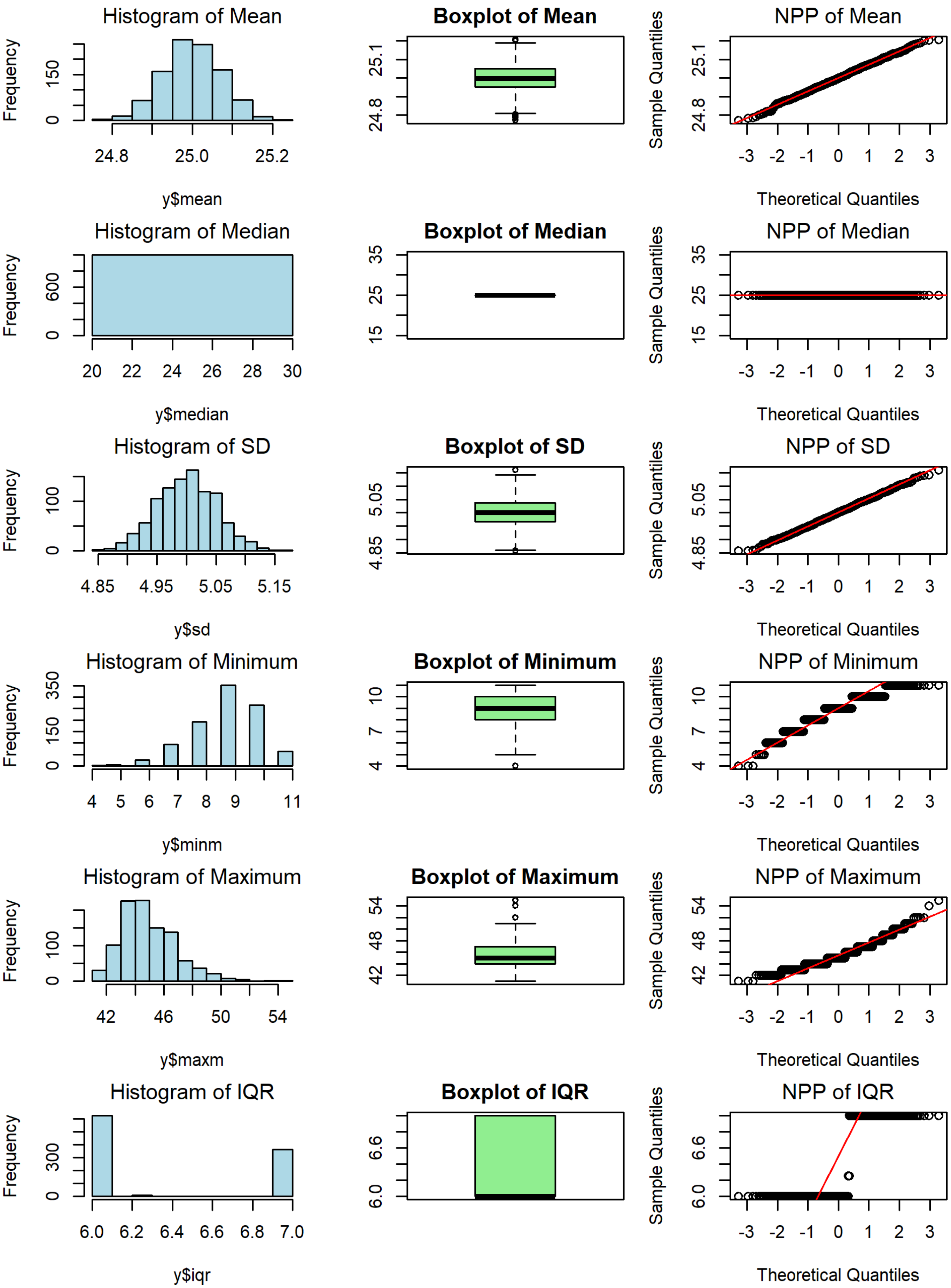
y$maxm
Histogram of IQR
Boxplot of IQR
NPP of IQR
y$iqr


Histogram of Mean
Frequency
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25.0
25.2
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Boxplot of Mean
24.8
25.1
NPP of Mean
Sample Quantiles
24.8
25.1
-3
-2
-1
0
1
2
3
Theoretical Quantiles
Histogram of Median
Frequency
0
600
20
22
24
26
28
30
y$median
Boxplot of Median
15
25
35
NPP of Median
Sample Quantiles
15
25
35
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-2
-1
0
1
2
3
Theoretical Quantiles
Histogram of SD
Frequency
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5.05
5.15
y$sd
Boxplot of SD
4.85
5.05
NPP of SD
Sample Quantiles
4.85
5.05
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-1
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1
2
3
Theoretical Quantiles
Histogram of Minimum
Frequency
0
150
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4
5
6
7
8
9
11
y$minm
Boxplot of Minimum
4
7
10
NPP of Minimum
Sample Quantiles
4
7
10
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-2
-1
0
1
2
3
Theoretical Quantiles
Histogram of Maximum
Frequency
0
100
42
46
50
54
y$maxm
Boxplot of Maximum
42
48
54
NPP of Maximum
Sample Quantiles
42
48
54
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-1
0
1
2
3
Theoretical Quantiles
Histogram of IQR
Frequency
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300
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6.2
6.4
6.6
6.8
7.0
y$iqr
Boxplot of IQR
6.0
6.6
NPP of IQR
Sample Quantiles
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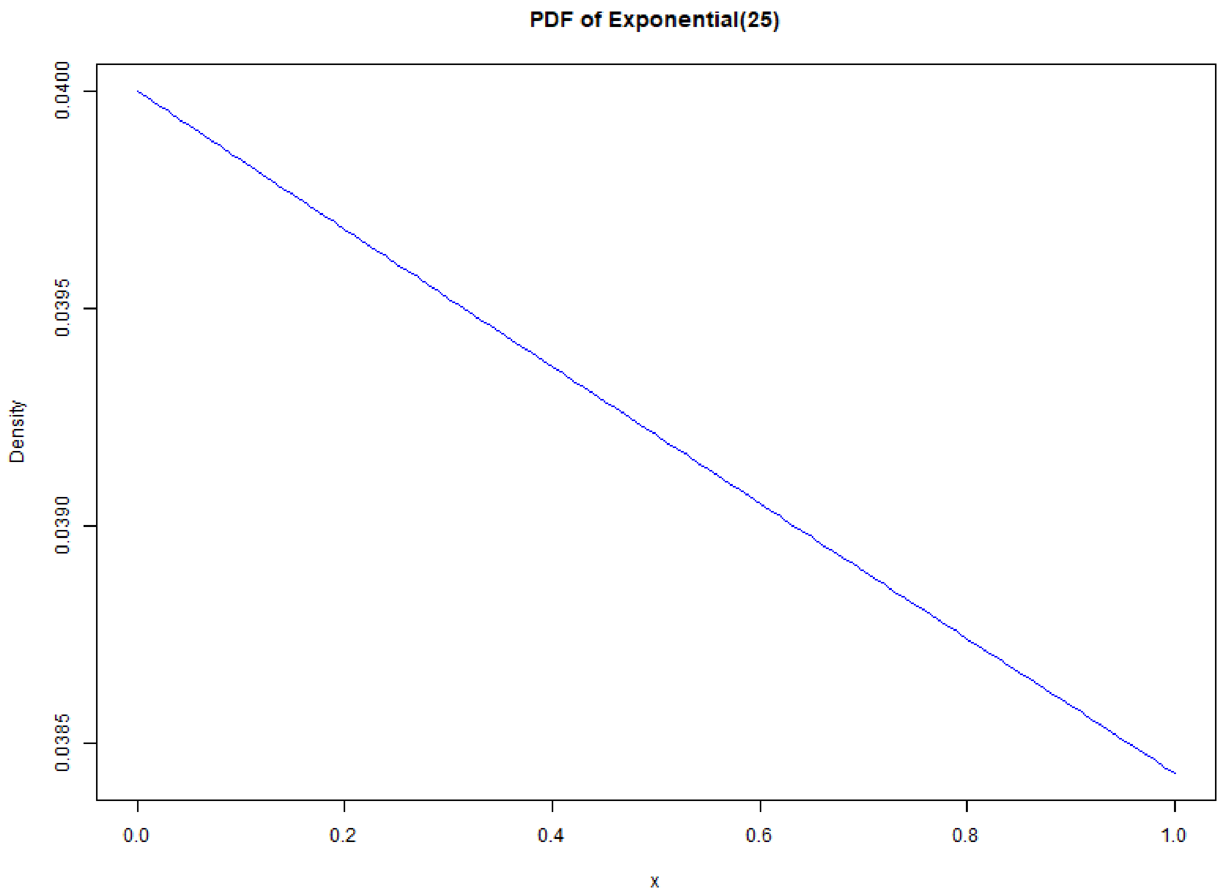
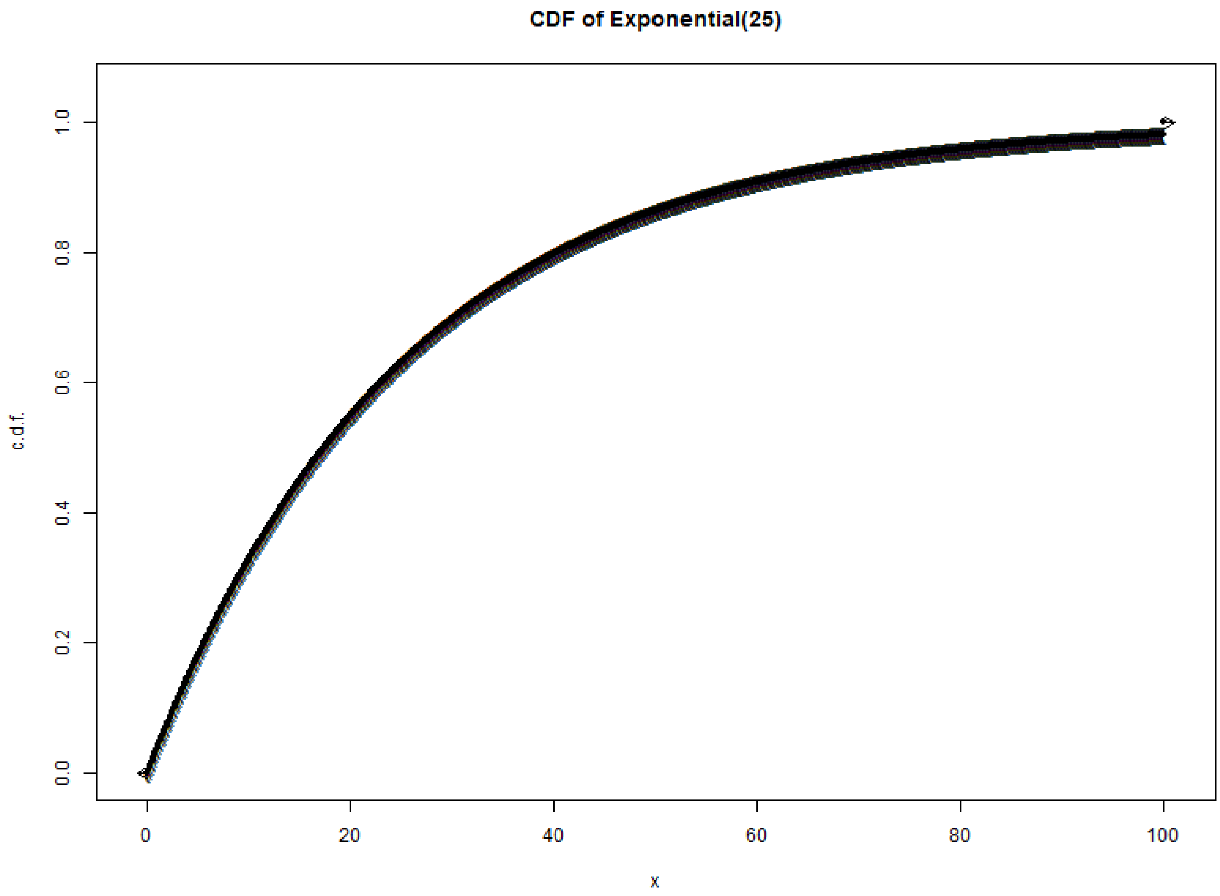
6.6
-3
-2
-1
0
1
2
3
Theoretical Quantiles

EXPONENTIAL DISTRIBUTION (25)





EXPONENTIAL DISTRIBUTION

	Values of n to achieve normality (nn=1000, λ=25)							
Statistic	n=10	n=50	n=100	n=500	n=1000	n=5000	Any n	Min(n)
Mean	No	Yes	Yes	Yes	Yes	Yes	Yes	50
Median	No	No	No	No	Yes	Yes	Yes	1000
Std Dev	No	No	No	Yes	Yes	Yes	Yes	500
Min	No	No	No	No	No	No	No	NA
Max	No	No	No	No	No	No	No	NA
IQR	No	No	No	Yes	Yes	Yes	Yes	500

Conclusion for Exponential Distribution (λ = 25)

Normality Achieved:

- **Mean:** Achieves normality for n ≥ 50, with a quicker convergence due to the relatively high λ value. This makes the distribution less skewed compared to lower λ values.
- **Median:** Achieves normality for n ≥ 1000, with larger sample sizes required due to the distribution's skewness that reduces as n increases.
- **Standard Deviation (SD):** Achieves normality for n ≥ 500, reflecting a slower convergence due to the distribution's variability at smaller sample sizes.
- **IQR:** Achieves normality for n ≥ 500, similar to the standard deviation, but with slower convergence than the mean.

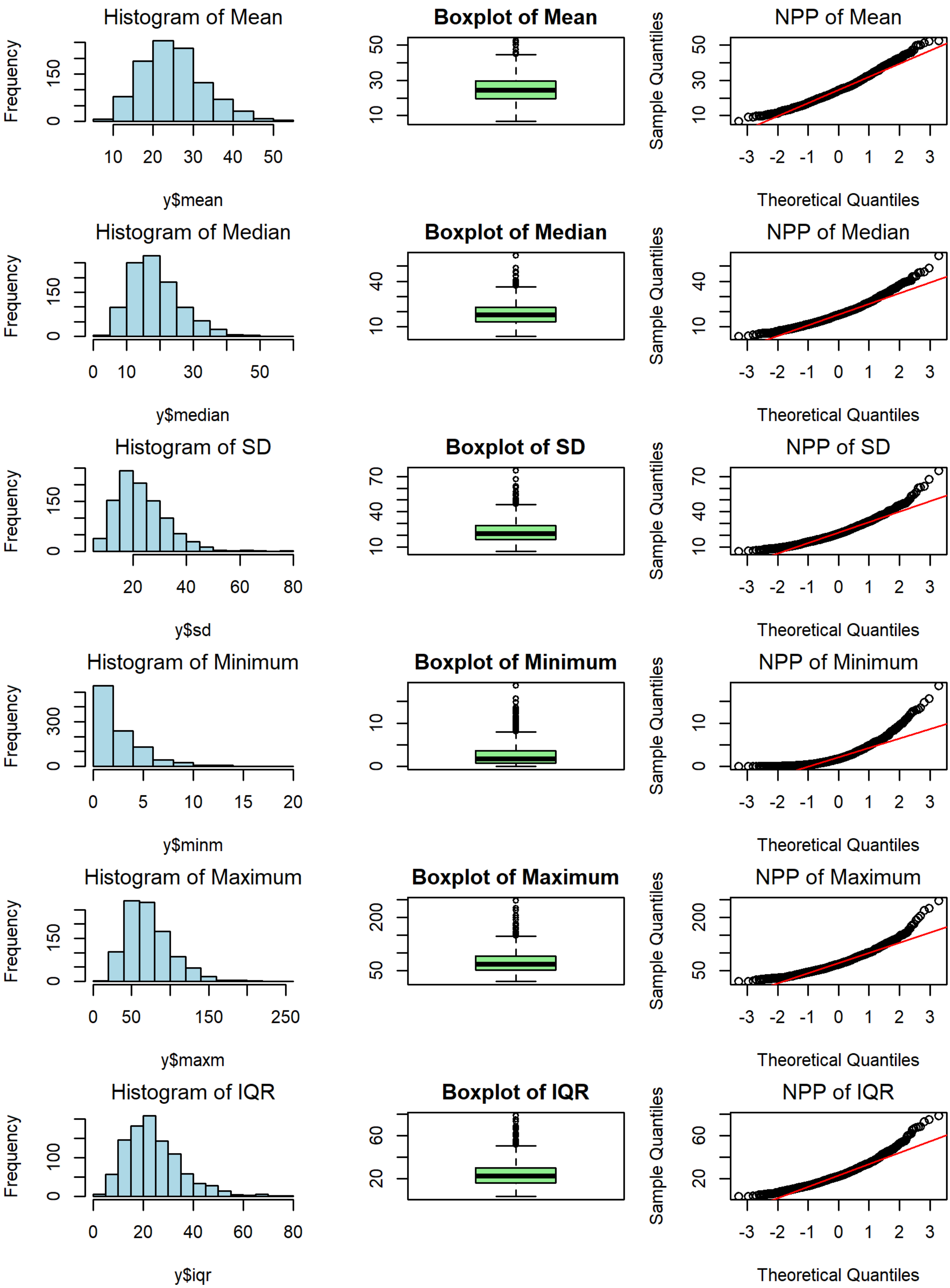
Normality Not Achieved:

- **Minimum and Maximum:** Do not achieve normality for any sample size, as they are highly sensitive to the extreme values in the Exponential distribution.

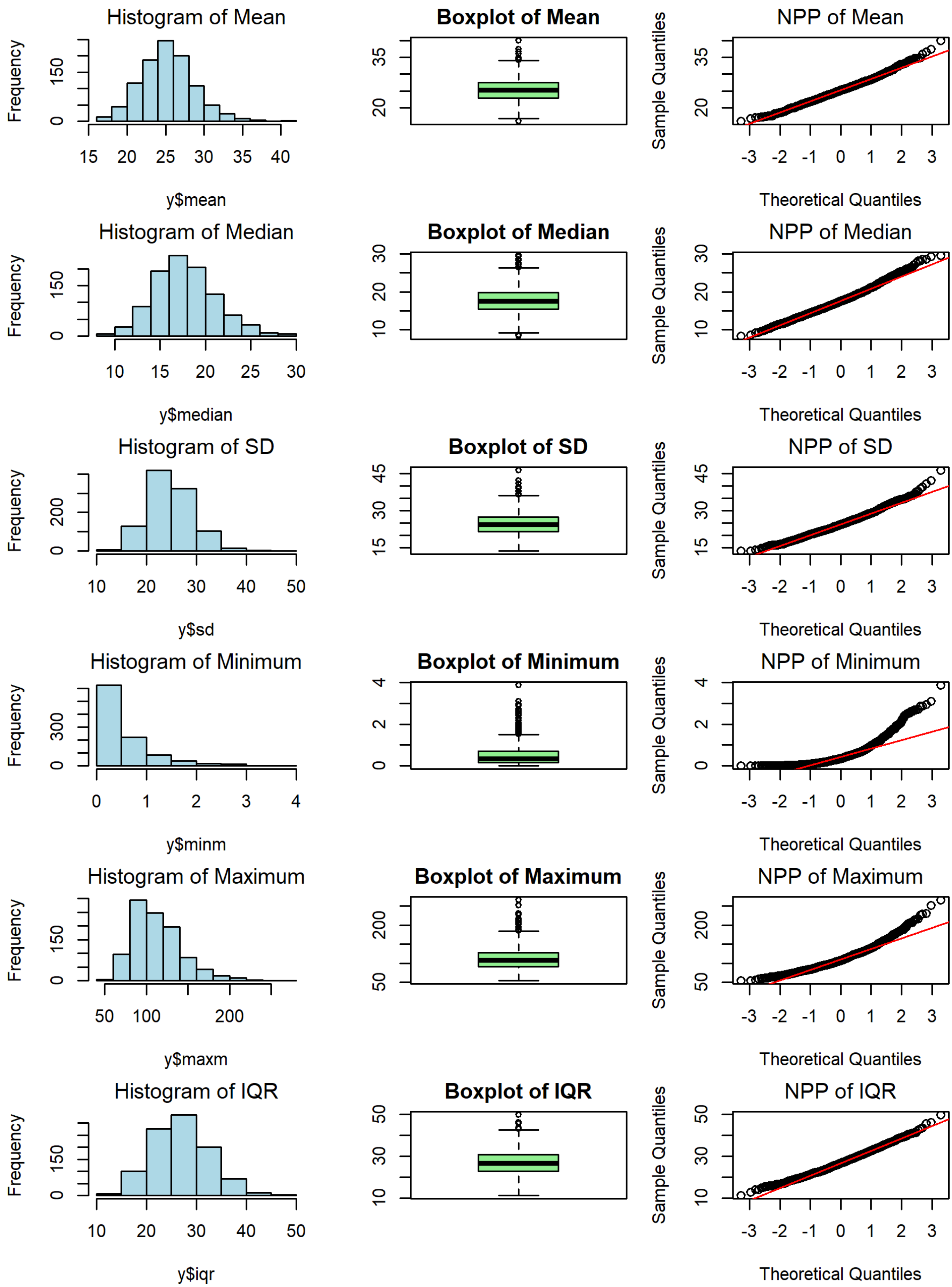
Overall: For the Exponential distribution with λ = 25, the mean achieves normality quickly at n ≥ 50, while the median, standard deviation, and IQR require larger sample sizes (n ≥ 500 to n ≥ 1000). The minimum and maximum remain non-normal due to the inherent skewness and extreme values in the distribution.

EXPONENTIAL DISTRIBUTION PLOT

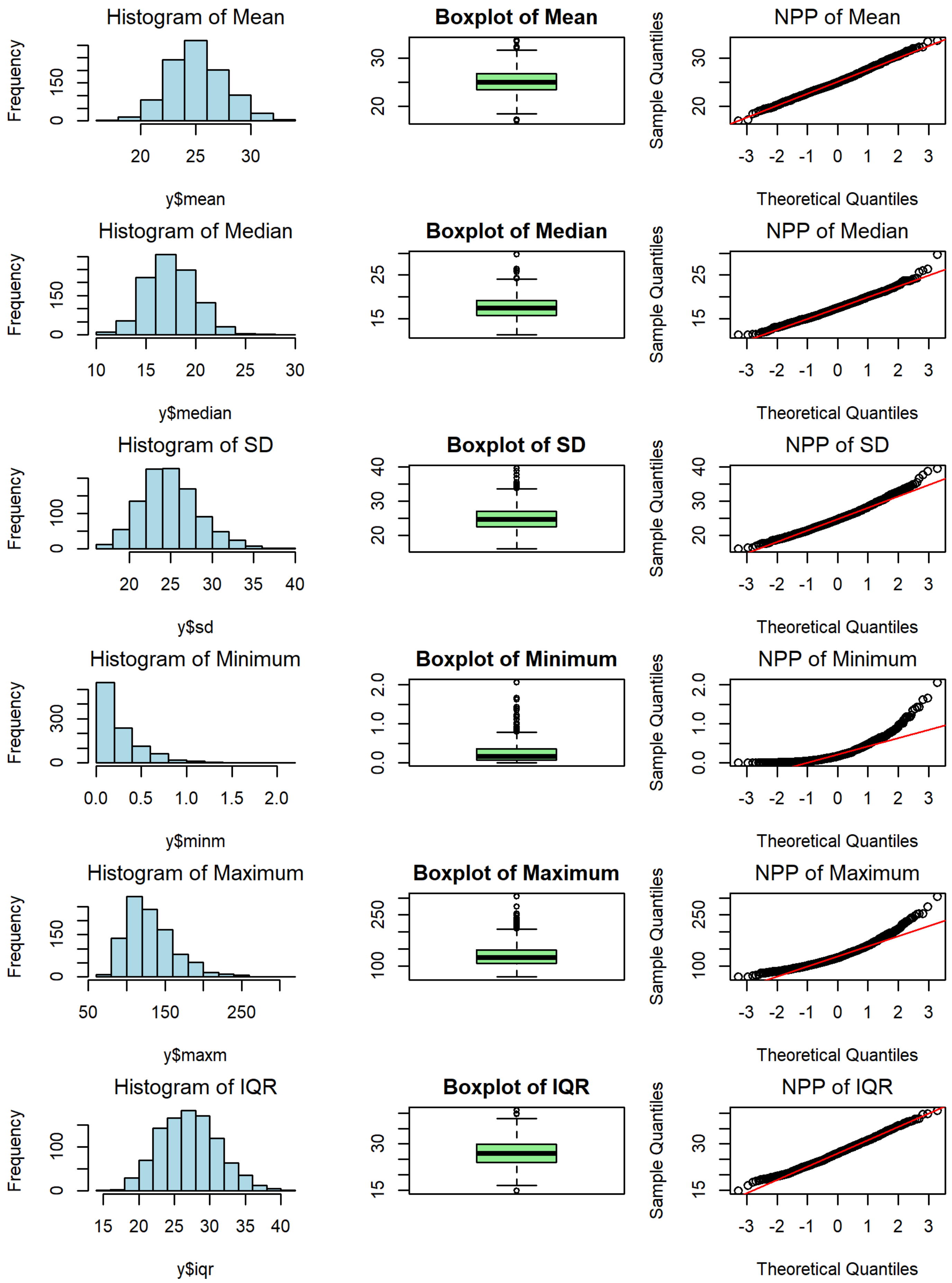
(n=10, nn=1000, λ=25)



EXPONENTIAL DISTRIBUTION PLOT
(n=50, nn=1000, λ=25)

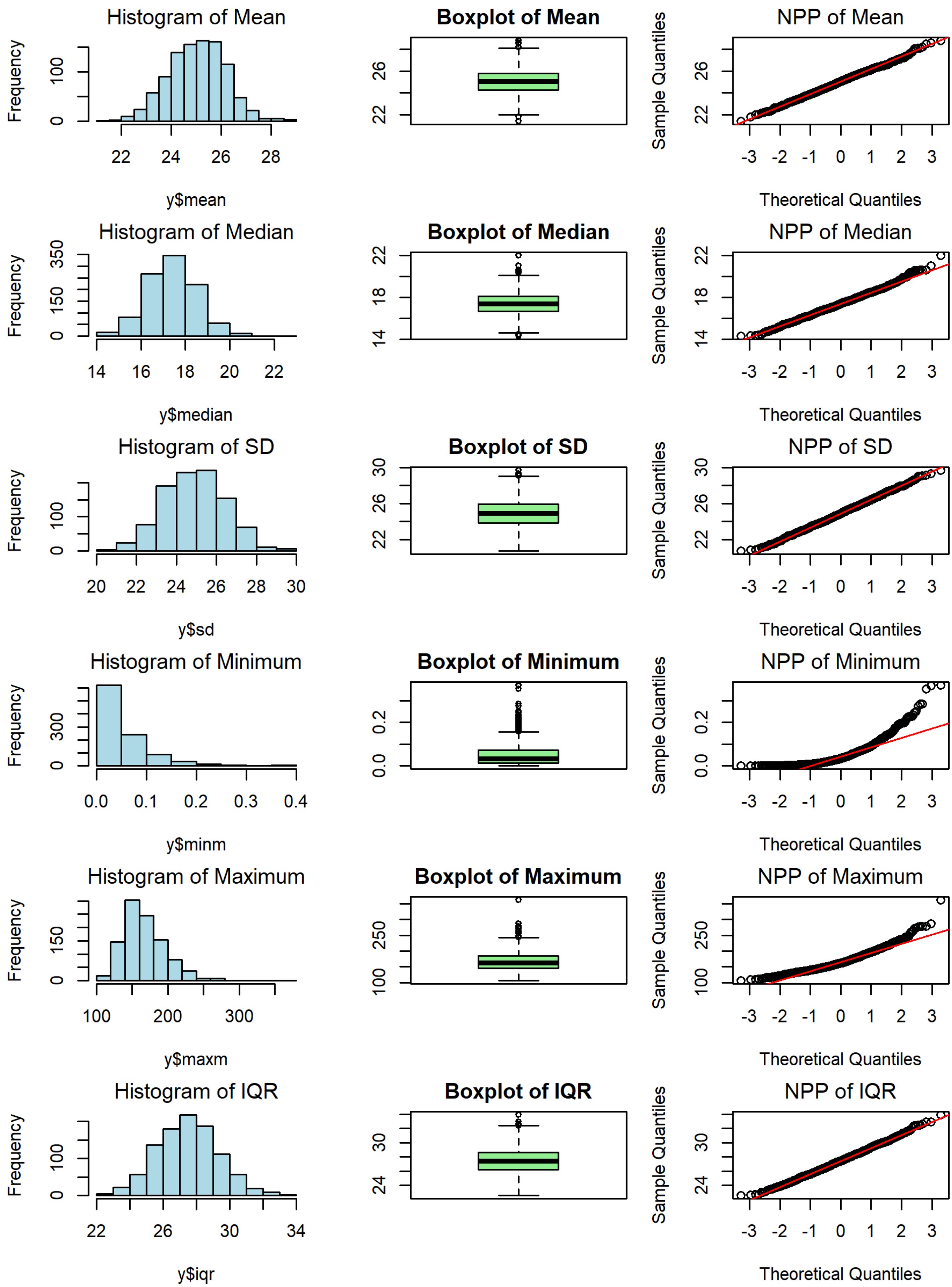


EXPONENTIAL DISTRIBUTION PLOT
(n=100, nn=1000, λ=25)

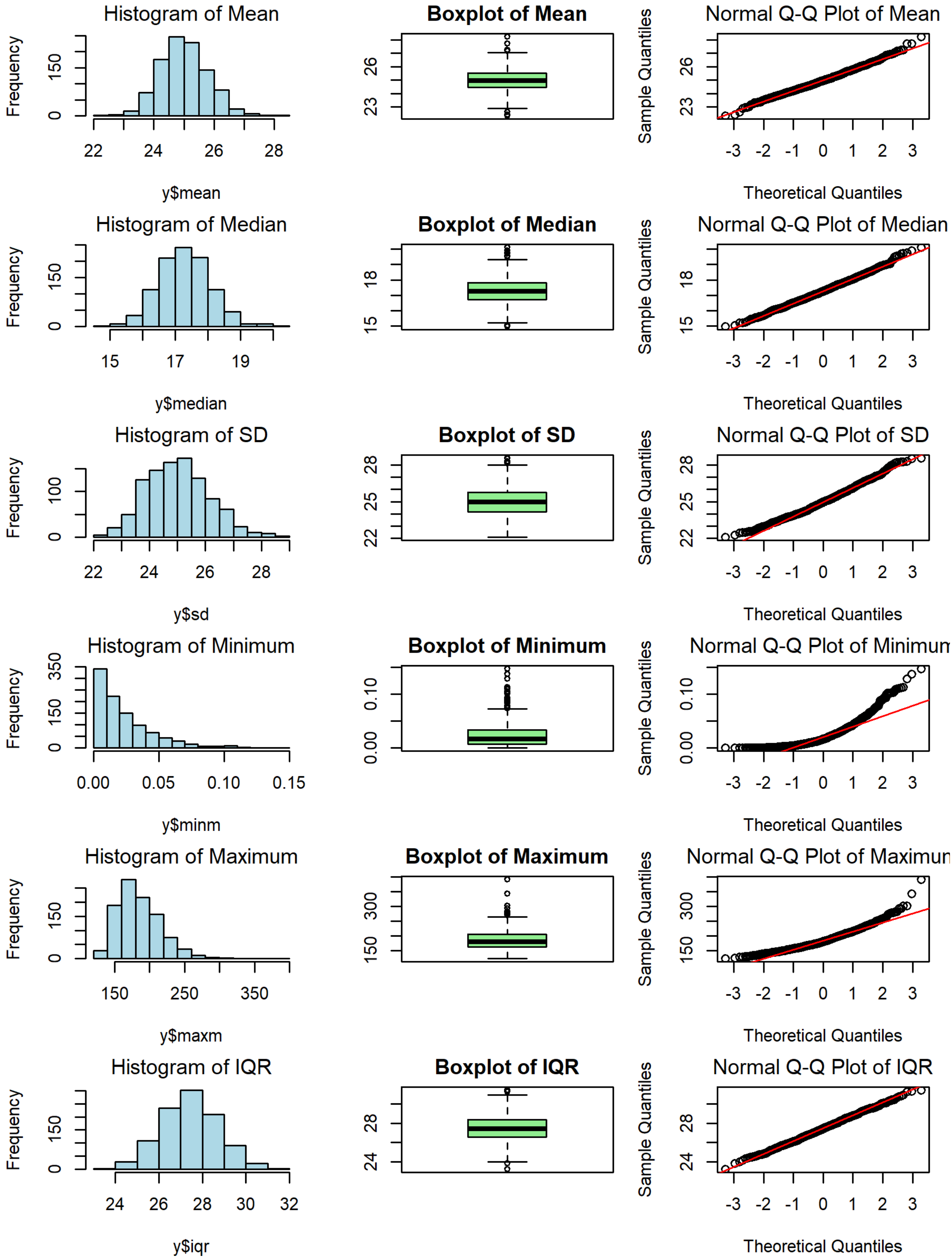


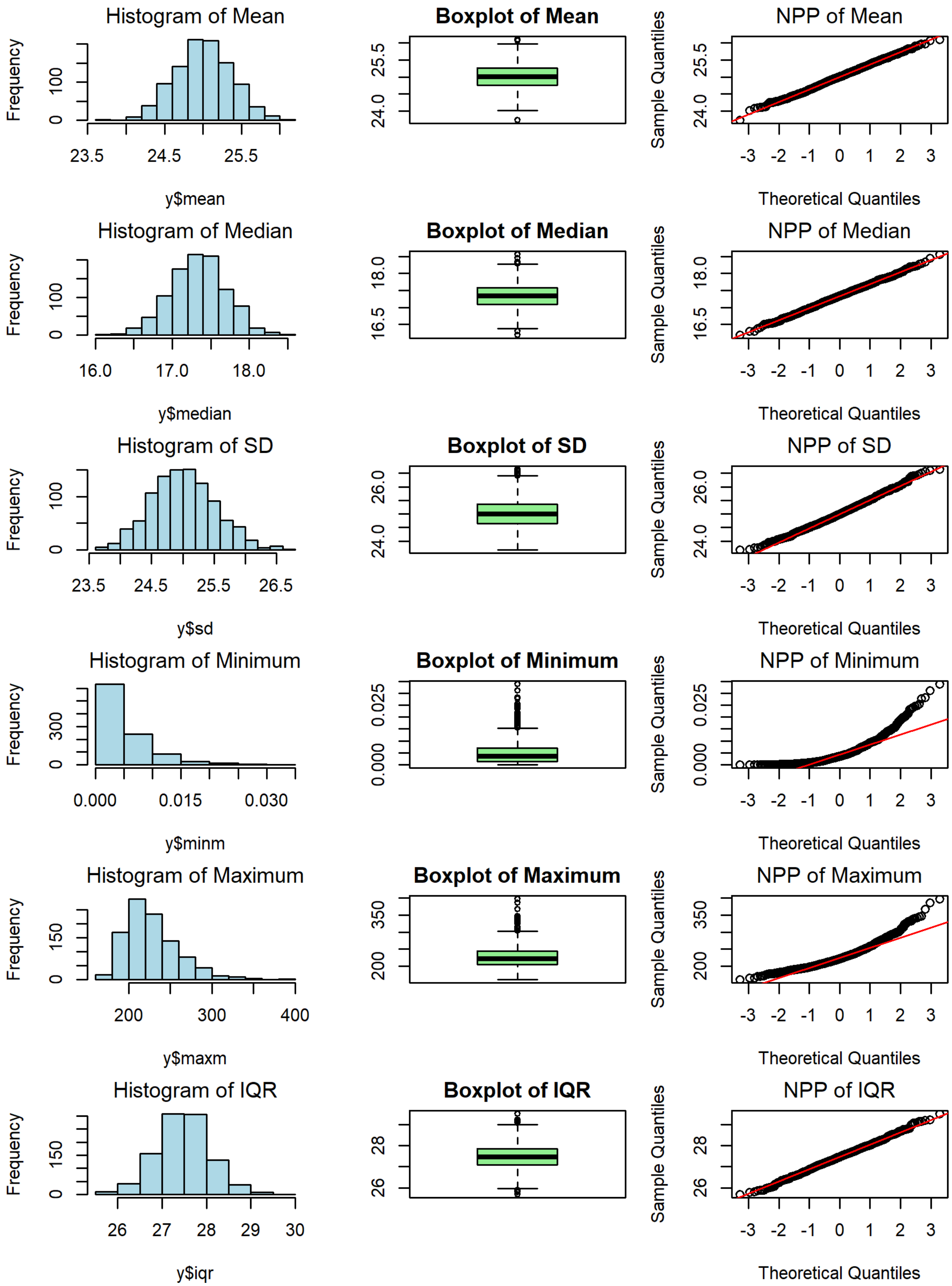
EXPONENTIAL DISTRIBUTION PLOT

(n=500, nn=1000, λ=25)

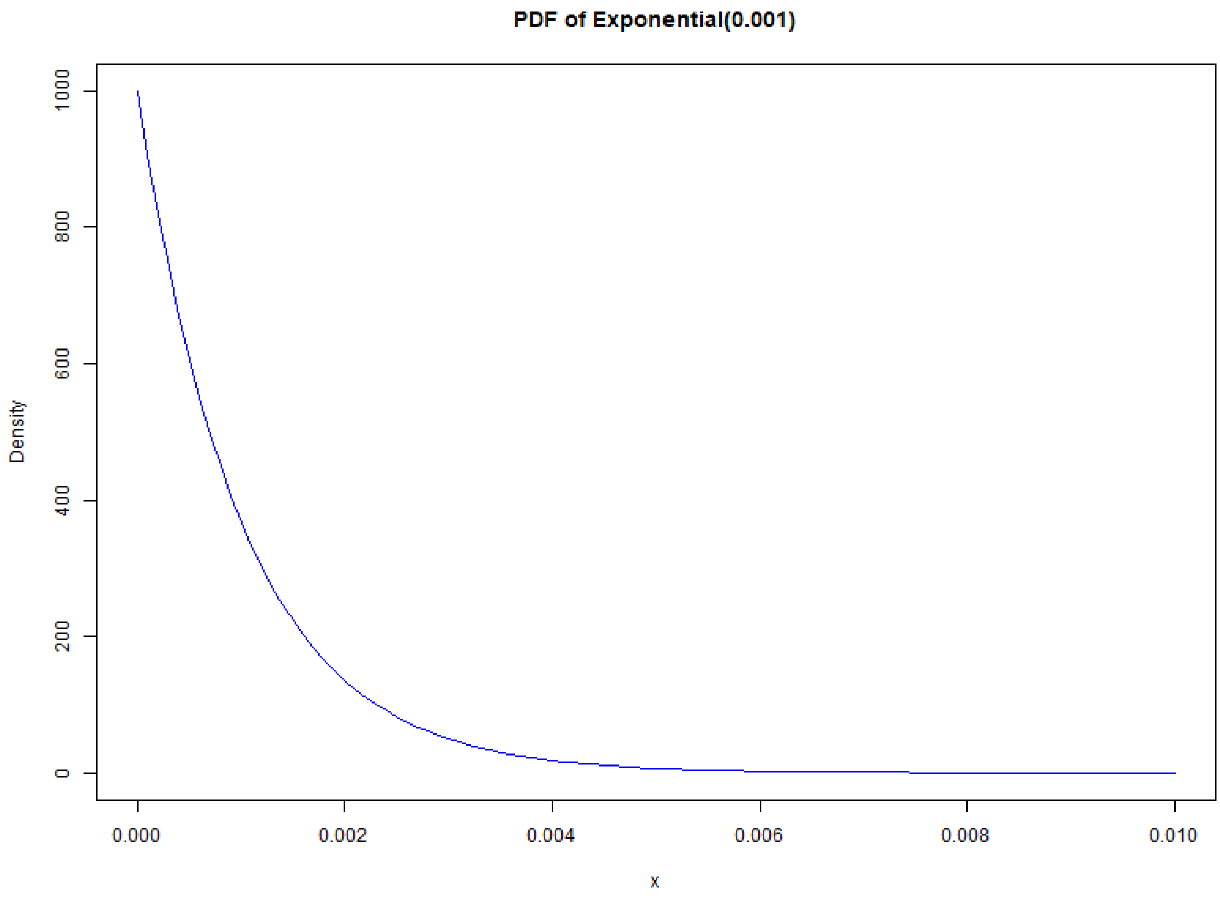


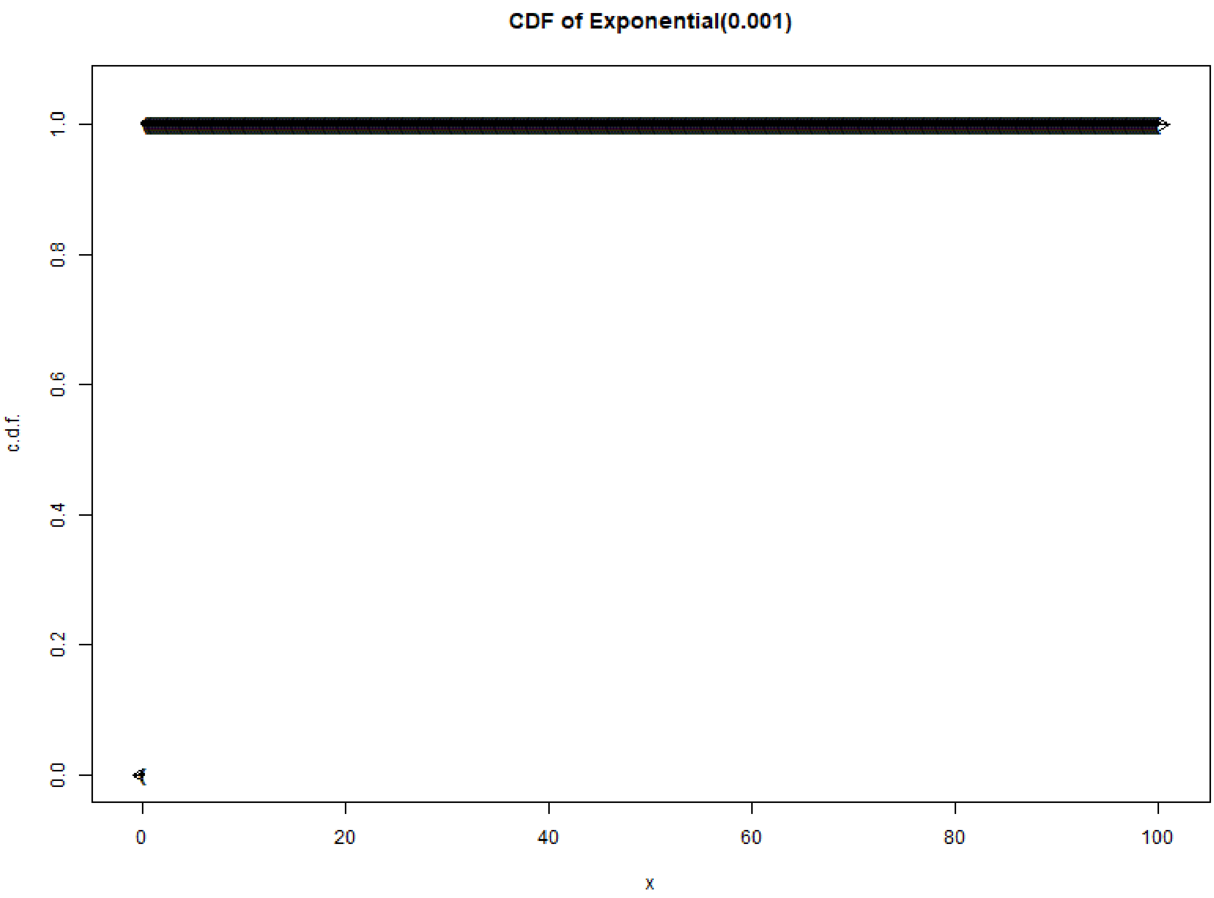
EXPONENTIAL DISTRIBUTION PLOT
(n=1000, nn=1000, λ=25)




Histogram of Mean
Frequency
0
100
23.5
24.5
25.5
y$mean
Boxplot of Mean
24.0
25.5
NPP of Mean
Sample Quantiles
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of Median
Frequency
0
100
16.0
17.0
18.0
y$median
Boxplot of Median
16.5
18.0
NPP of Median
Sample Quantiles
Theoretical Quantiles
Histogram of SD
Frequency
0
100
23.5
24.5
25.5
26.5
y$sd
Boxplot of SD
24.0
26.0
NPP of SD
Sample Quantiles
Theoretical Quantiles
Histogram of Minimum
Frequency
0
300
0.000
0.015
0.030
y$minm
Boxplot of Minimum
0.000
0.025
NPP of Minimum
Sample Quantiles
Theoretical Quantiles
Histogram of Maximum
Frequency
0
150
200
300
400
y$maxm
Boxplot of Maximum
200
350
NPP of Maximum
Sample Quantiles
Theoretical Quantiles
Histogram of IQR
Frequency
0
150
26
27
28
29
30
y$iqr
Boxplot of IQR
26
28
NPP of IQR
Sample Quantiles
Theoretical Quantiles

EXPONENTIAL DISTRIBUTION (0.001)





EXPONENTIAL DISTRIBUTION

	Values of n to achieve normality (nn=1000, λ=0.001)						
Statistic	n=10	n=50	n=100	n=500	n=1000	n=5000	Any n	Min(n)
Mean	No	No	Yes	Yes	Yes	Yes	Yes	100
Median	No	No	Yes	Yes	Yes	Yes	Yes	100
Std Dev	No	No	Yes	Yes	Yes	Yes	Yes	100
Min	No	No	No	No	No	No	No	NA
Max	No	No	No	No	No	No	No	NA
IQR	No	No	No	No	Yes	Yes	Yes	1000

Conclusion for Exponential Distribution

Normality Achieved:

- **Mean**: Achieves normality for n≥100, reflecting moderate convergence due to the Central Limit Theorem (CLT).
- **Median**: Achieves normality for n≥100, with convergence similar to the mean despite the parent distribution's skewness.
- **Standard Deviation (SD)**: Achieves normality for n≥100, requiring comparable sample sizes to the mean and median for convergence.
- **IQR**: Achieves normality for n≥1000, showing slower convergence due to variability and sensitivity to skewness.

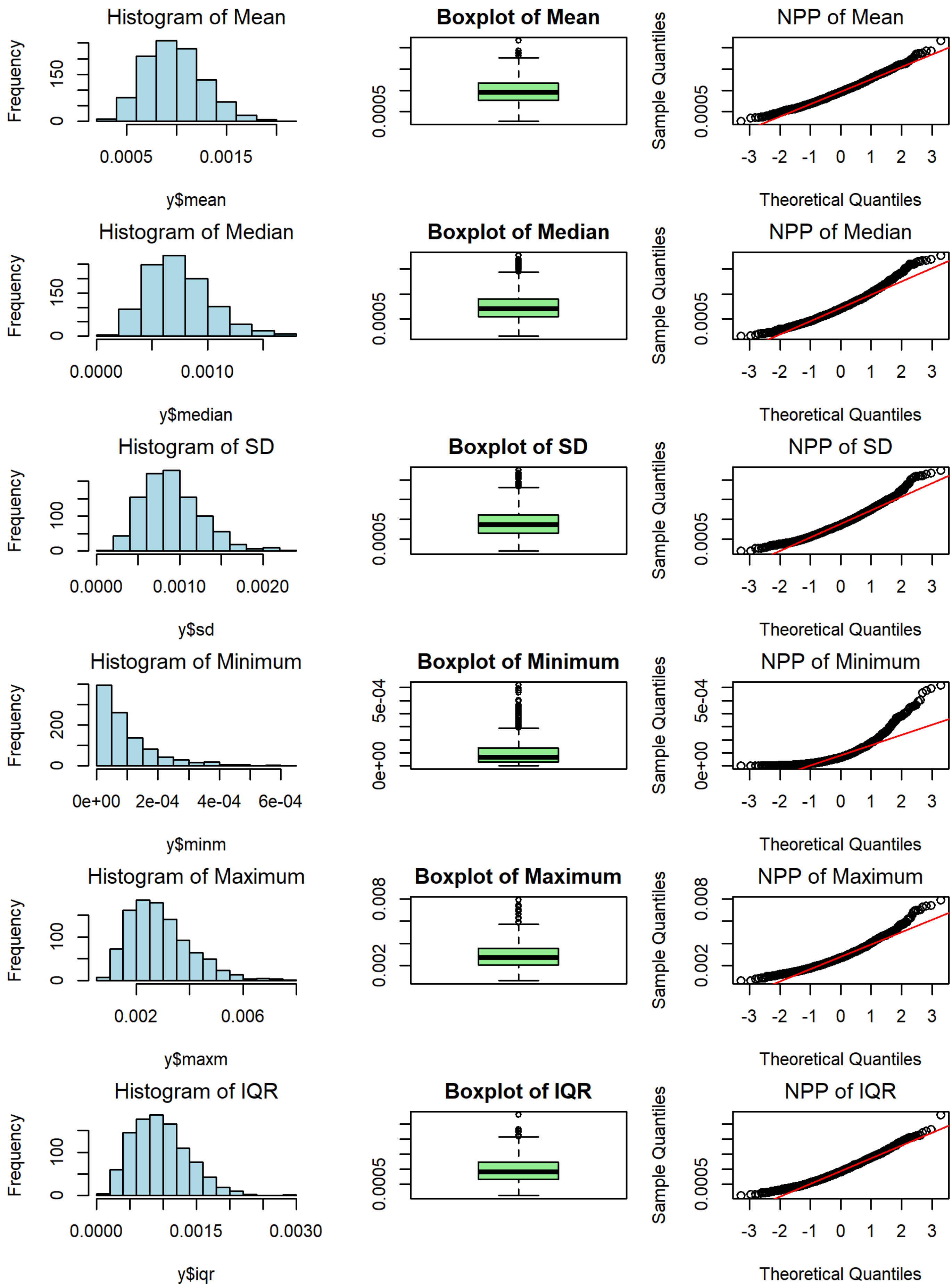
Normality Not Achieved:

- **Minimum**: Does not achieve normality for any n, as it is highly influenced by extreme values and the Exponential distribution's asymmetry.
- **Maximum**: Does not achieve normality for any n, reflecting its sensitivity to extreme values in the parent distribution.

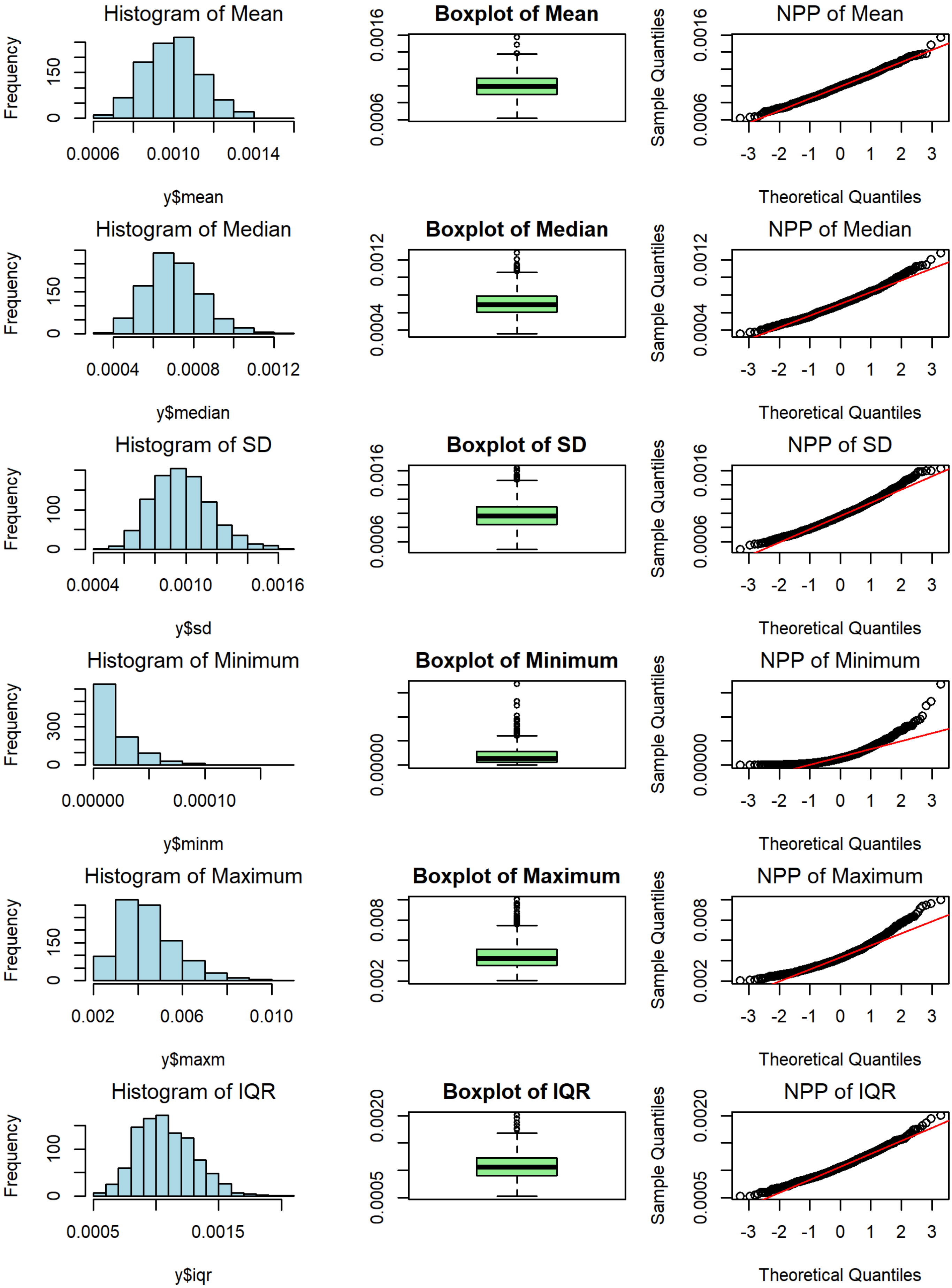
Overall:
The mean, median, and standard deviation converge to normality for moderate sample sizes (n≥100), making them reliable statistics. However, the IQR requires larger sample sizes (n≥1000) to approximate normality, while the minimum and maximum never achieve normality due to the skewness and heavy tail of the Exponential distribution.

EXPONENTIAL DISTRIBUTION PLOT

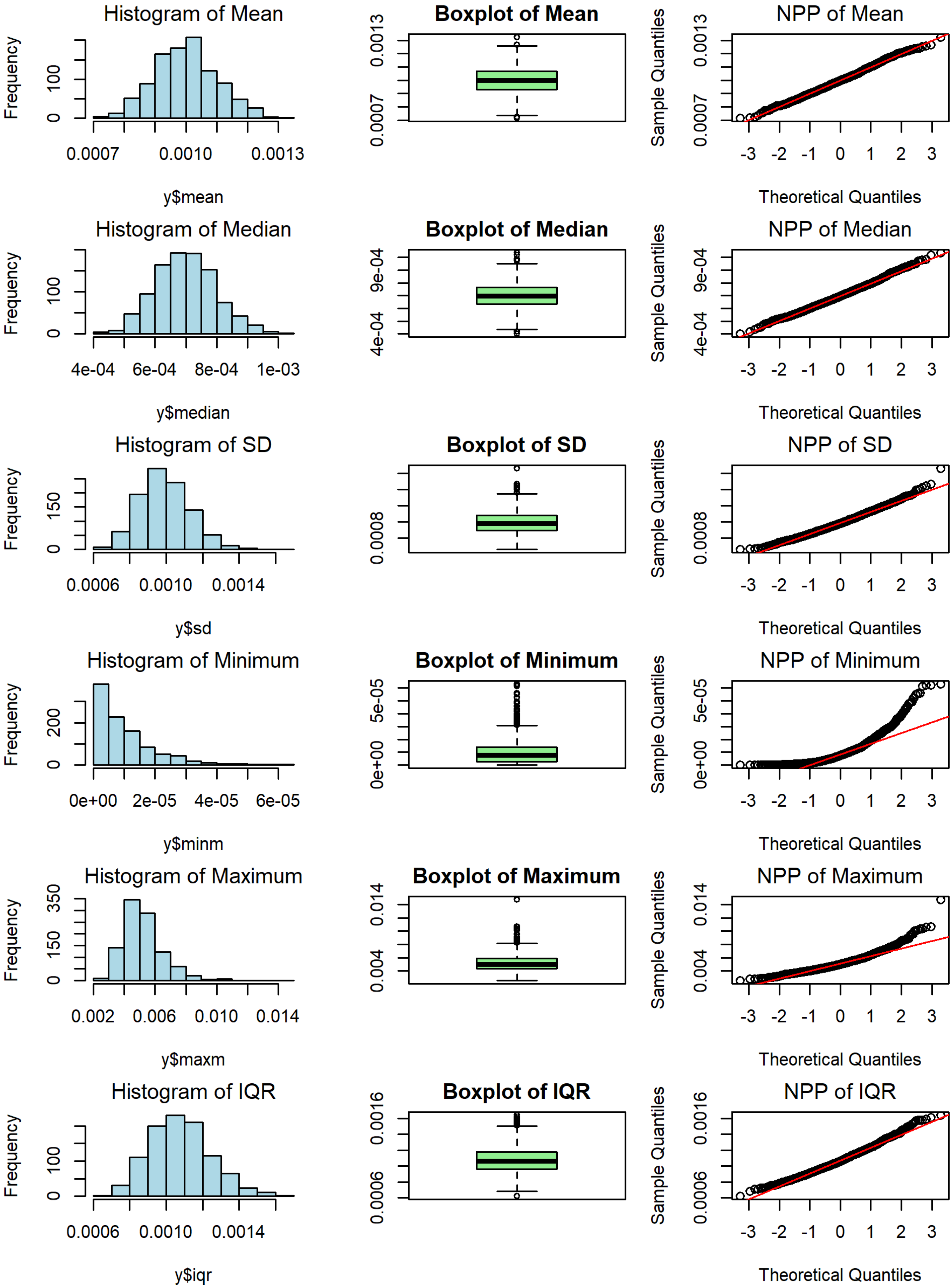
(n=10, nn=1000, λ=0.001)



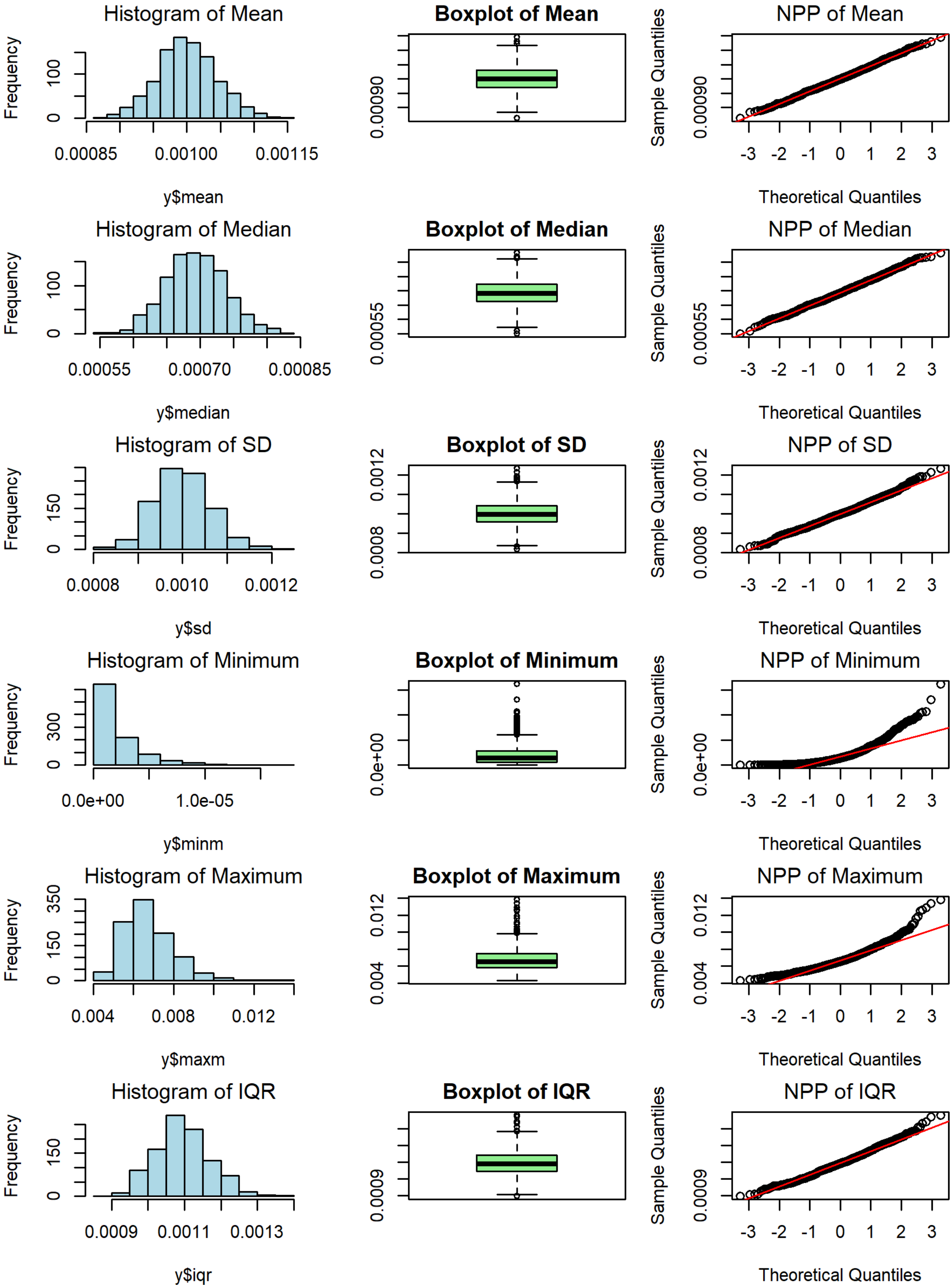
EXPONENTIAL DISTRIBUTION PLOT
(n=50, nn=1000, λ=0.001)



EXPONENTIAL DISTRIBUTION PLOT
(n=100, nn=1000, λ=0.001)

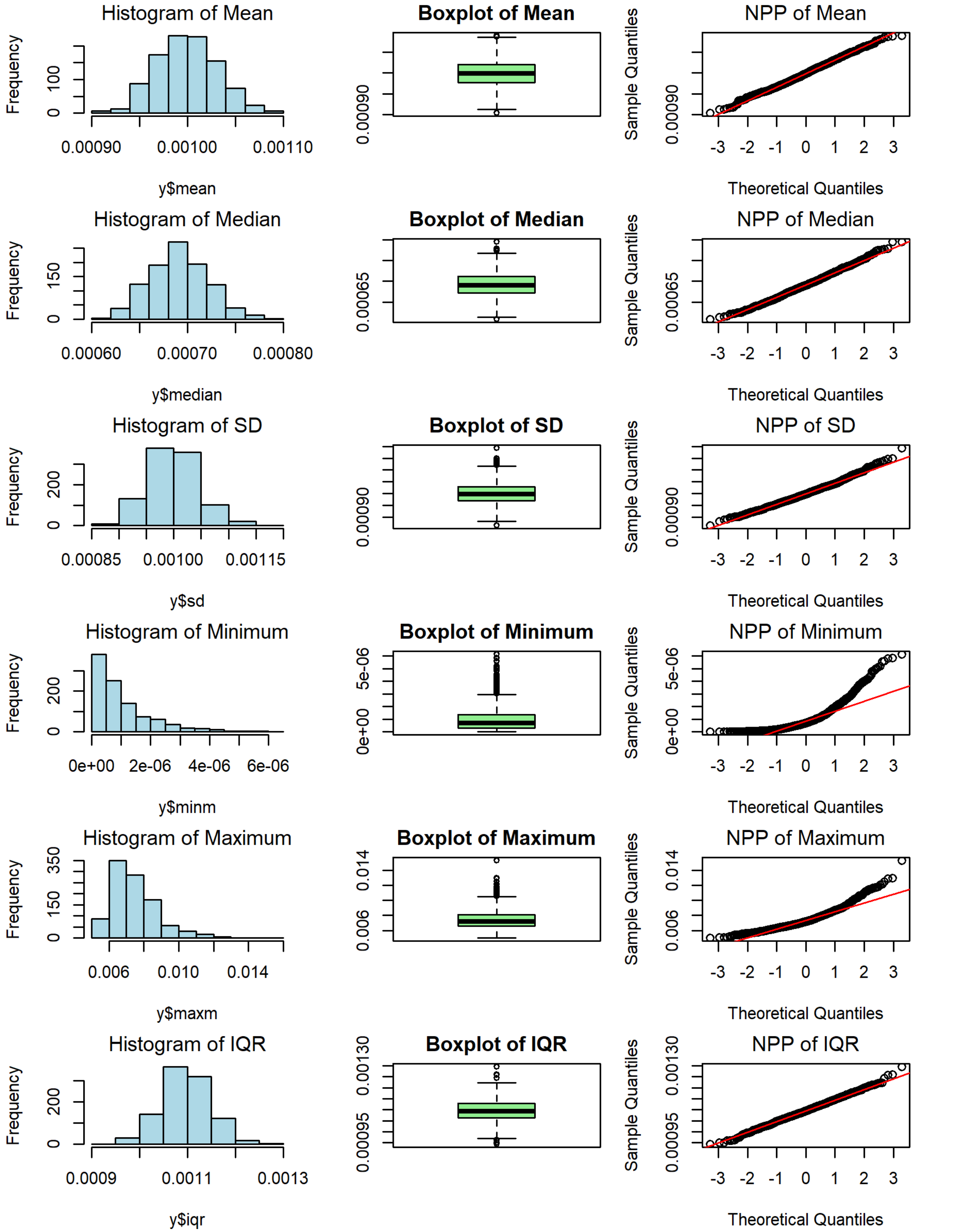


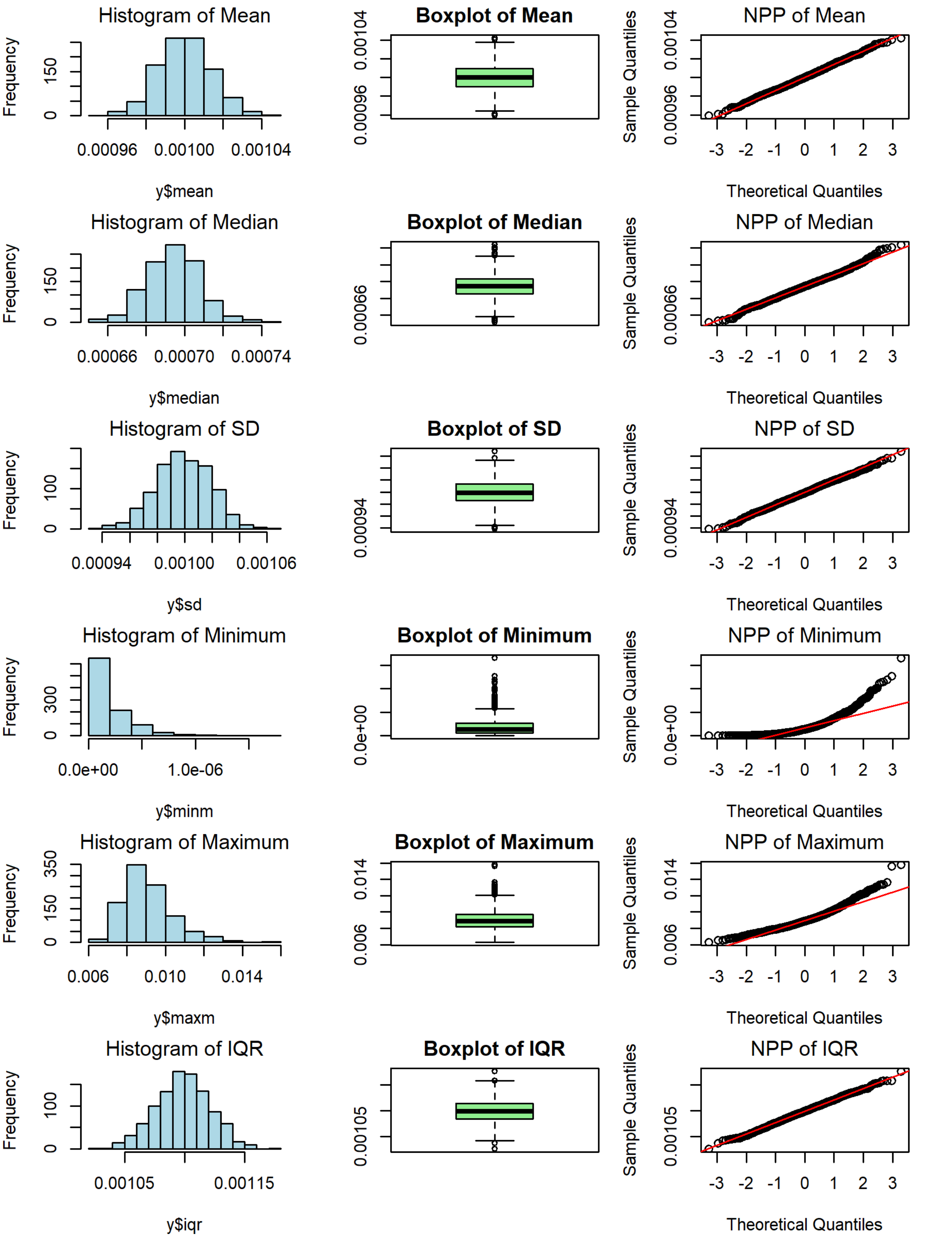
EXPONENTIAL DISTRIBUTION PLOT
(n=500, nn=1000, λ=0.001)



EXPONENTIAL DISTRIBUTION PLOT

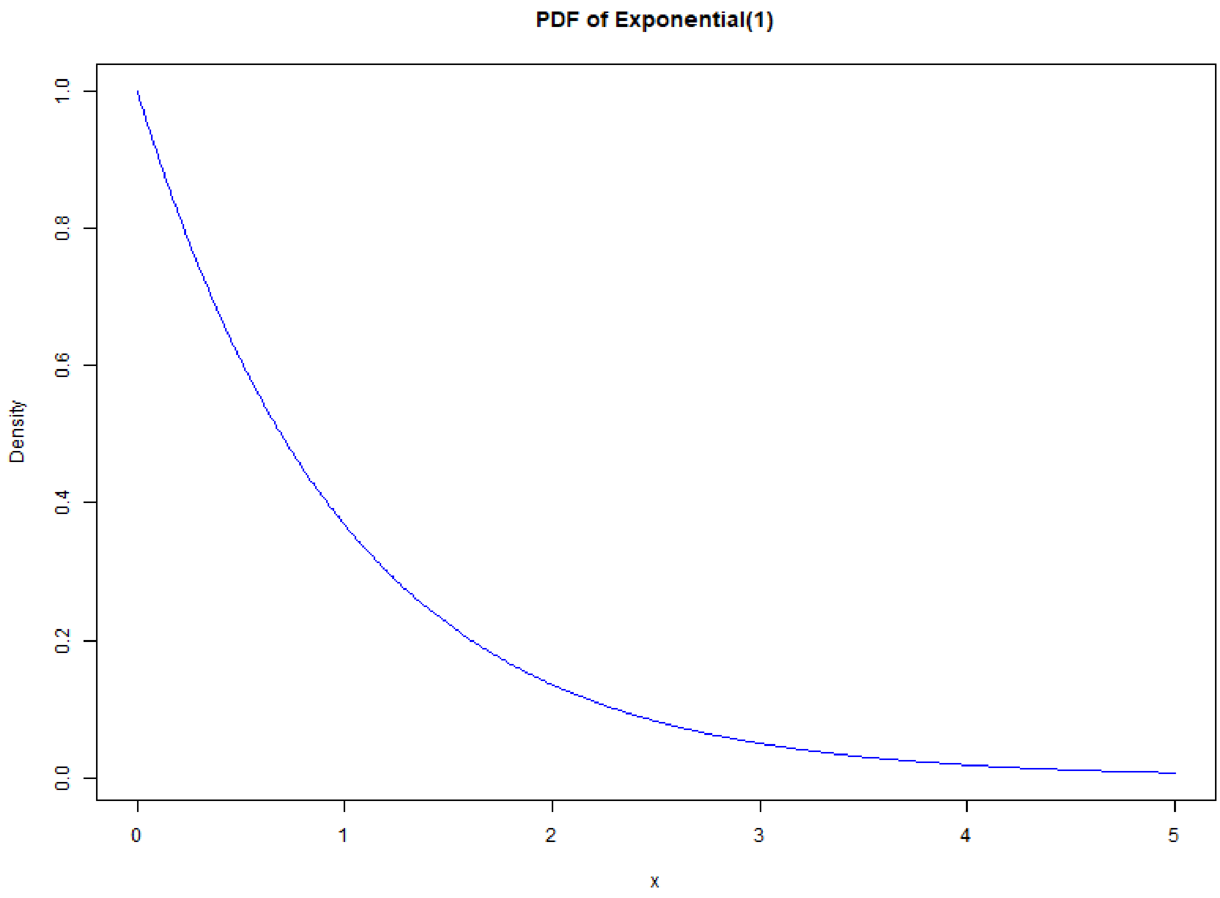
(n=1000, nn=1000, λ=0.001)

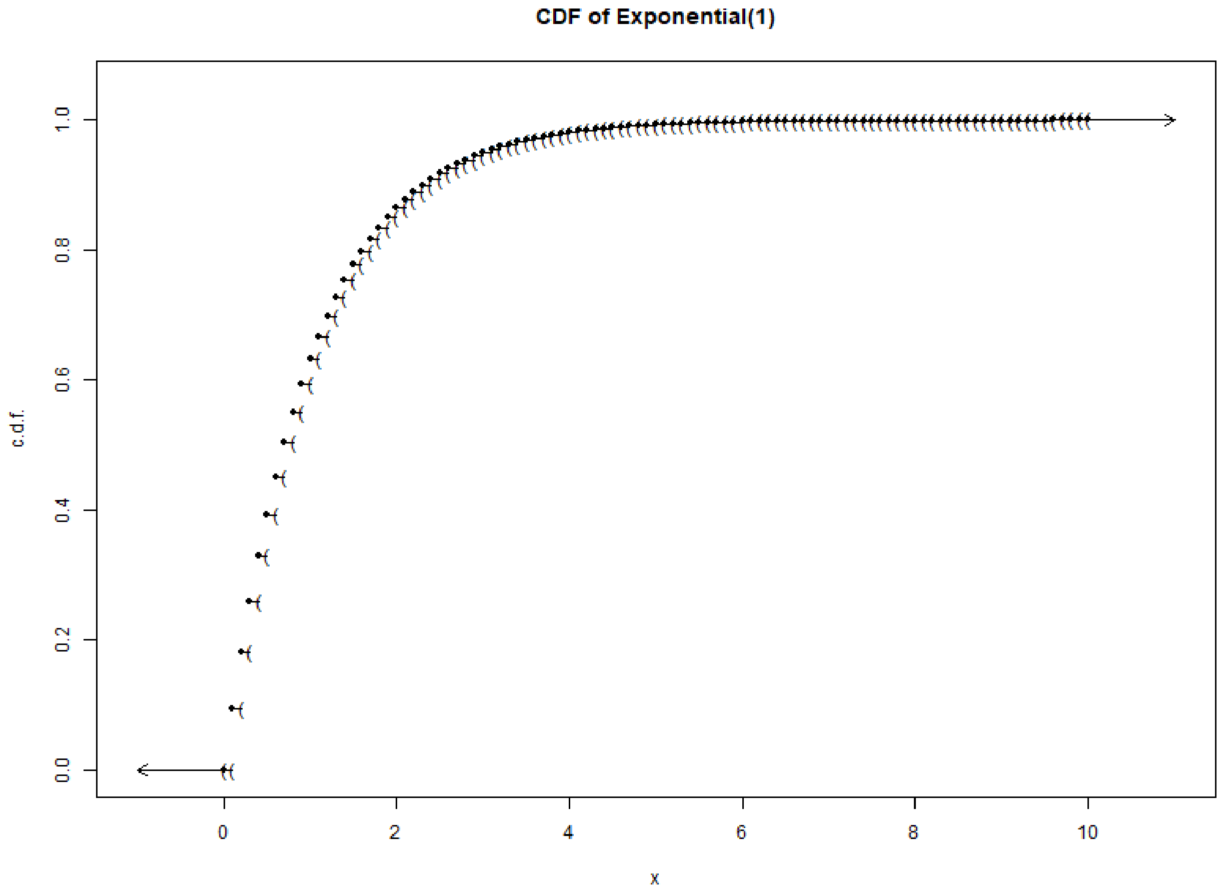




Histogram of Mean
Frequency
0
150
0.00096
0.00100
0.00104
y$mean
Boxplot of Mean
0.00096
0.00104
NPP of Mean
Sample Quantiles
Theoretical Quantiles
-3
-2
-1
0
1
2
3
Histogram of Median
0.00066
0.00070
0.00074
y$median
Boxplot of Median
0.00066
NPP of Median
Histogram of SD
100
0.00094
0.00100
0.00106
y$sd
Boxplot of SD
0.00094
NPP of SD
Histogram of Minimum
300
0.0e+00
1.0e-06
y$minm
Boxplot of Minimum
NPP of Minimum
Histogram of Maximum
350
0.006
0.010
0.014
y$maxm
Boxplot of Maximum
NPP of Maximum
Histogram of IQR
0.00105
0.00115
y$iqr
Boxplot of IQR
NPP of IQR

EXPONENTIAL DISTRIBUTION (1)





EXPONENTIAL DISTRIBUTION

	Values of n to achieve normality (nn=1000, λ=1)						
Statistic	n=10	n=50	n=100	n=500	n=1000	n=5000	Any n	Min(n)
Mean	No	Yes	Yes	Yes	Yes	Yes	Yes	50
Median	No	No	No	Yes	Yes	Yes	Yes	500
Std Dev	No	No	No	Yes	Yes	Yes	Yes	500
Min	No	No	No	No	No	No	No	NA
Max	No	No	No	No	No	No	No	NA
IQR	No	No	No	Yes	Yes	Yes	Yes	500

Conclusion for Exponential Distribution

Normality Achieved:

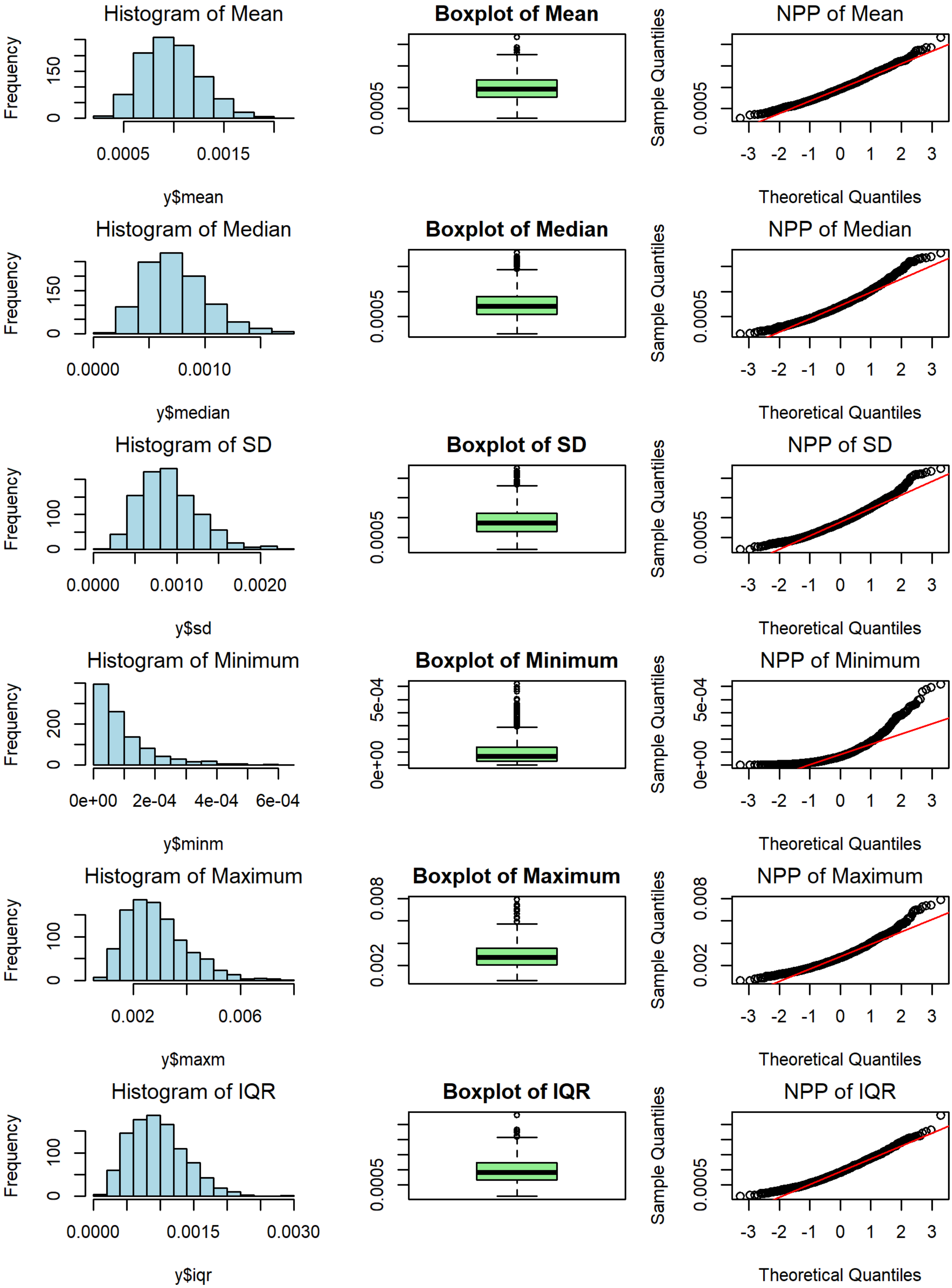
- **Mean**: Achieves normality for n≥50, converging relatively quickly due to the Central Limit Theorem (CLT).
- **Median**: Achieves normality for n≥500, requiring larger sample sizes due to sensitivity to the distribution’s skewness.
- **Standard Deviation (SD)**: Achieves normality for n≥500, reflecting a slower rate of convergence influenced by variability.
- **IQR**: Achieves normality for n≥500, indicating convergence similar to the standard deviation.

Normality Not Achieved:

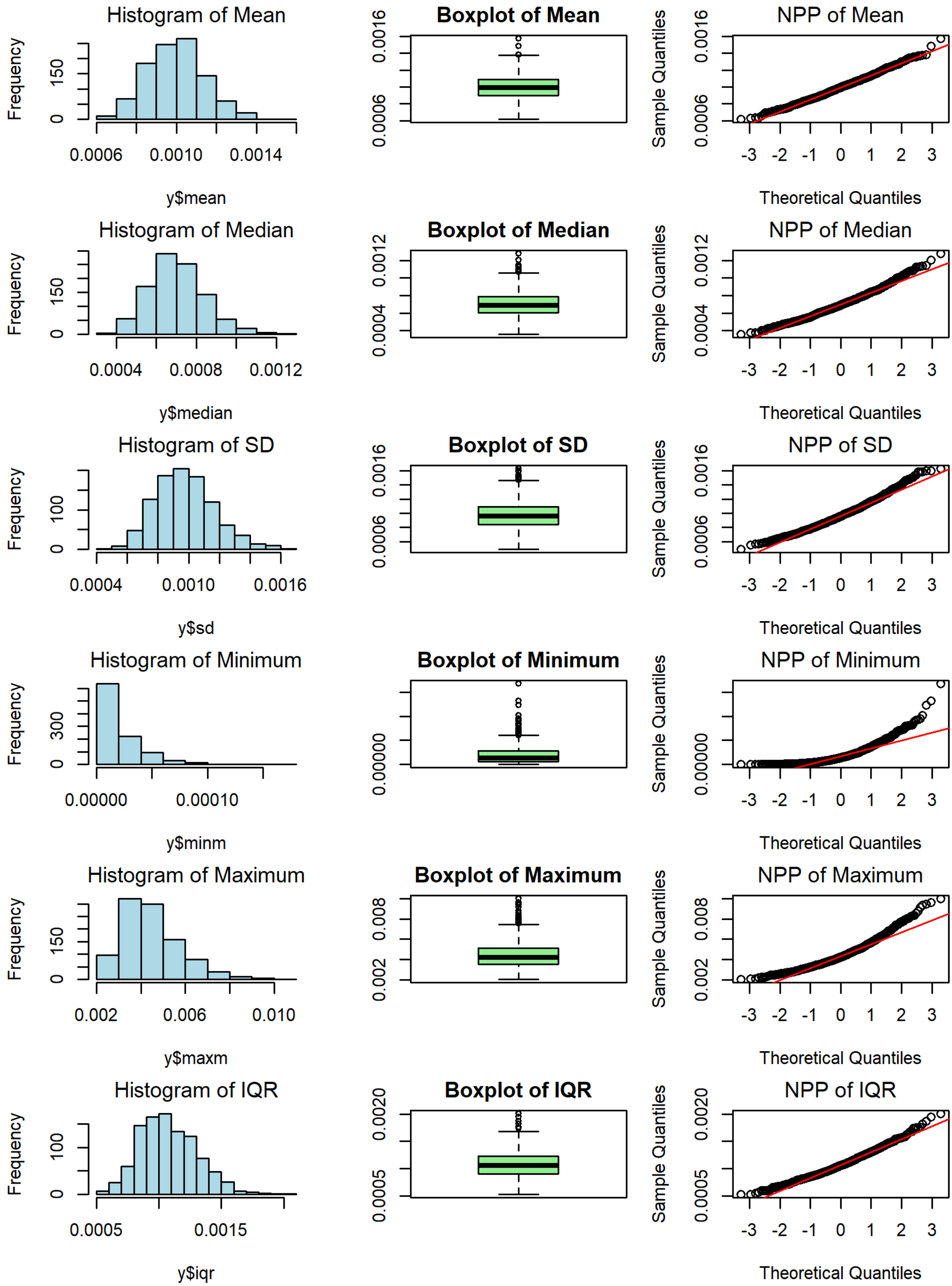
- **Minimum and Maximum**: Do not achieve normality for any n, as they are highly influenced by the Exponential distribution's extreme values and asymmetry.

Overall:
The mean converges to normality fastest (n≥50), making it the most reliable statistic for smaller sample sizes. The median, standard deviation, and IQR require larger sample sizes (n≥500) due to the distribution’s skewness and variability. The minimum and maximum remain non-normal regardless of the sample size.

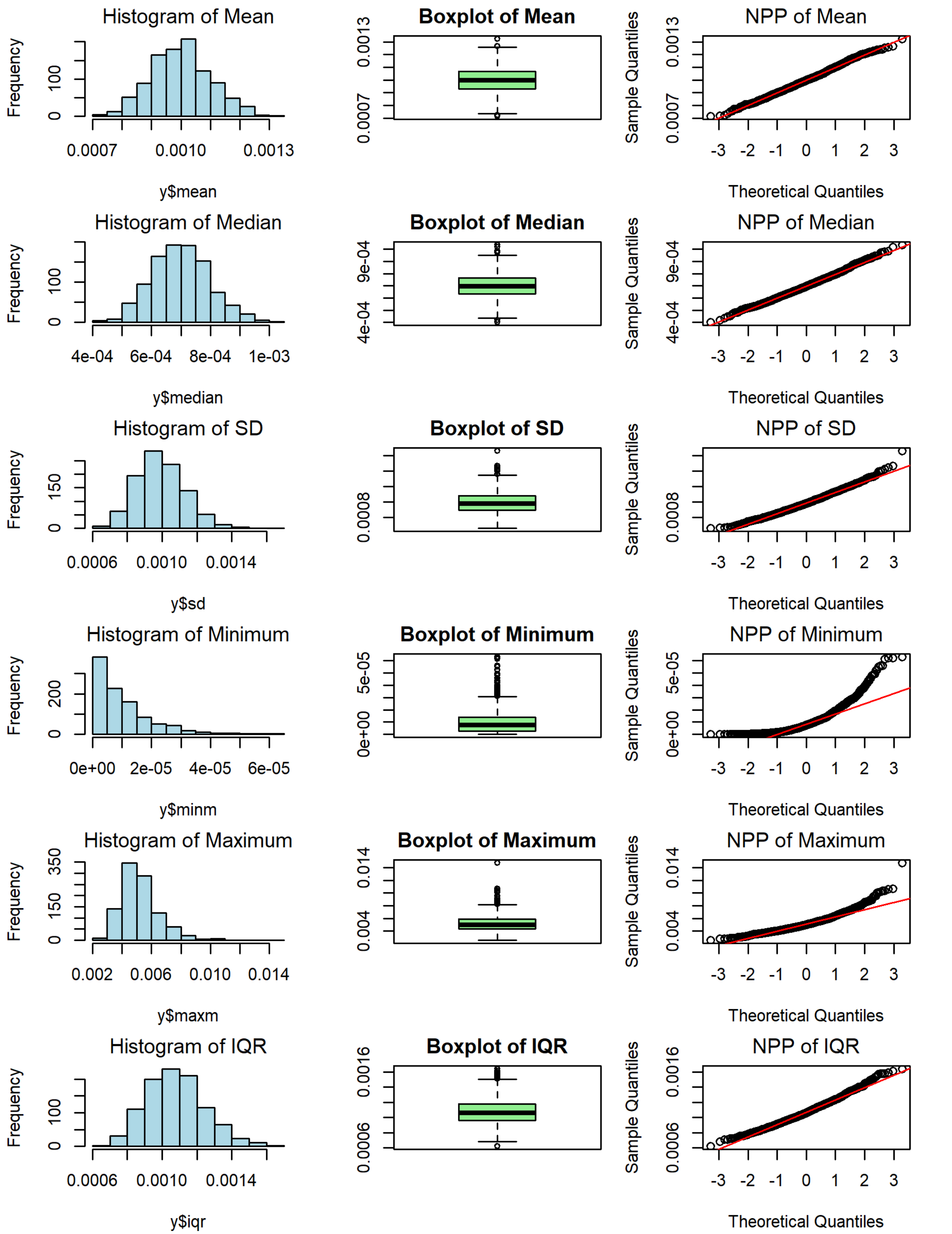
EXPONENTIAL DISTRIBUTION PLOT
(n=10, nn=1000, λ=1)



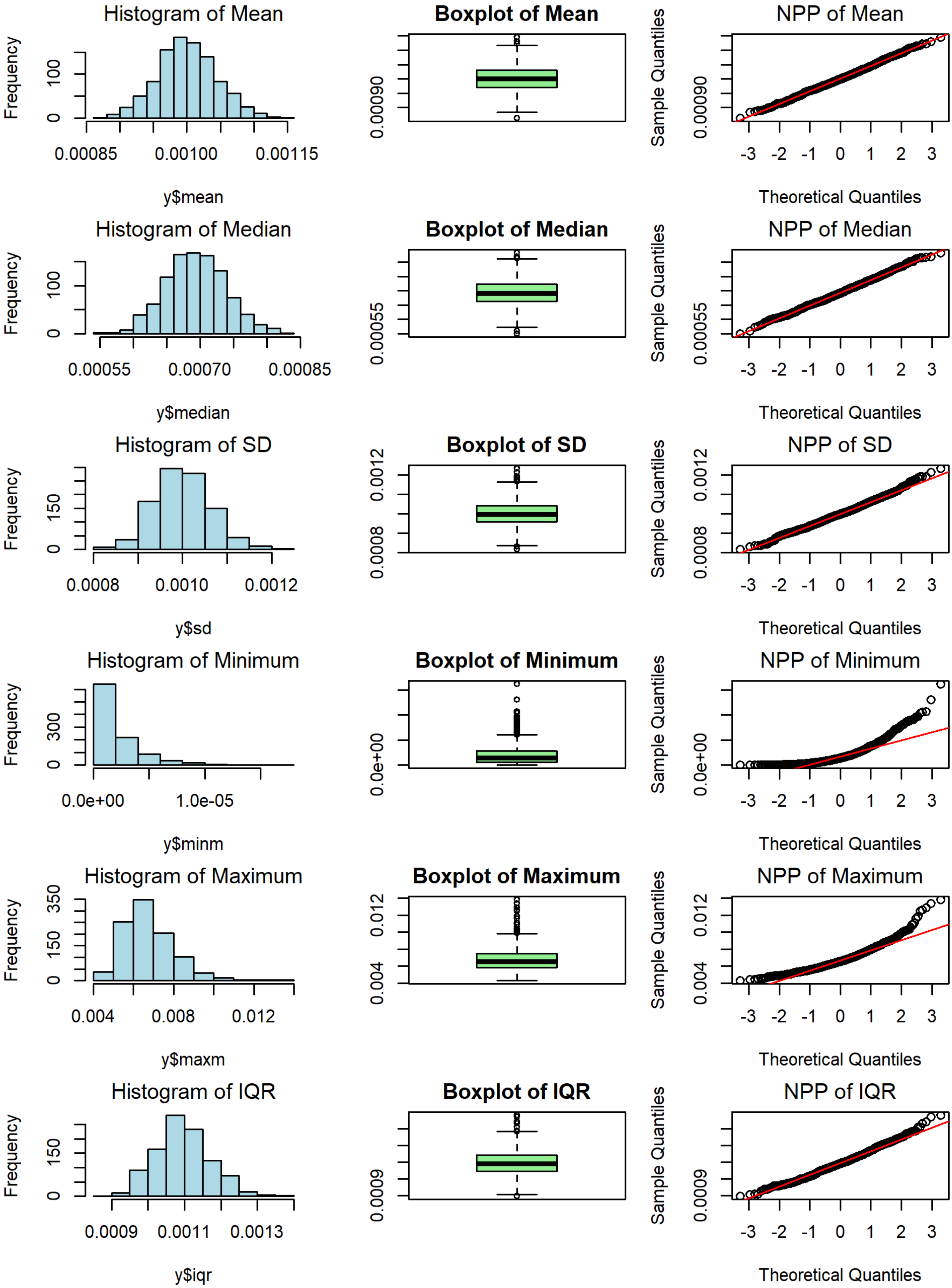
EXPONENTIAL DISTRIBUTION PLOT
(n=50, nn=1000, λ=1)



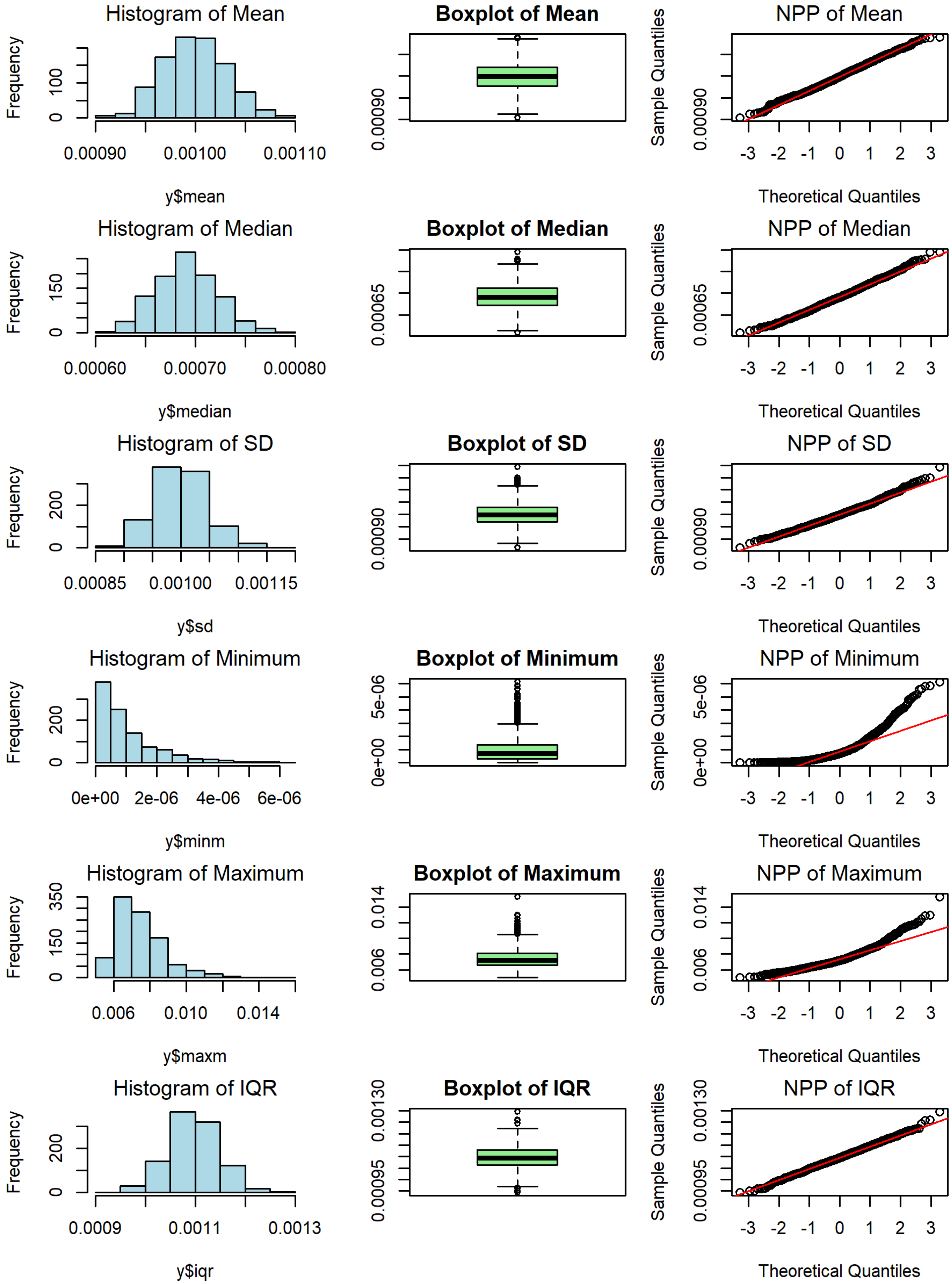
EXPONENTIAL DISTRIBUTION PLOT
(n=100, nn=1000, λ=1)

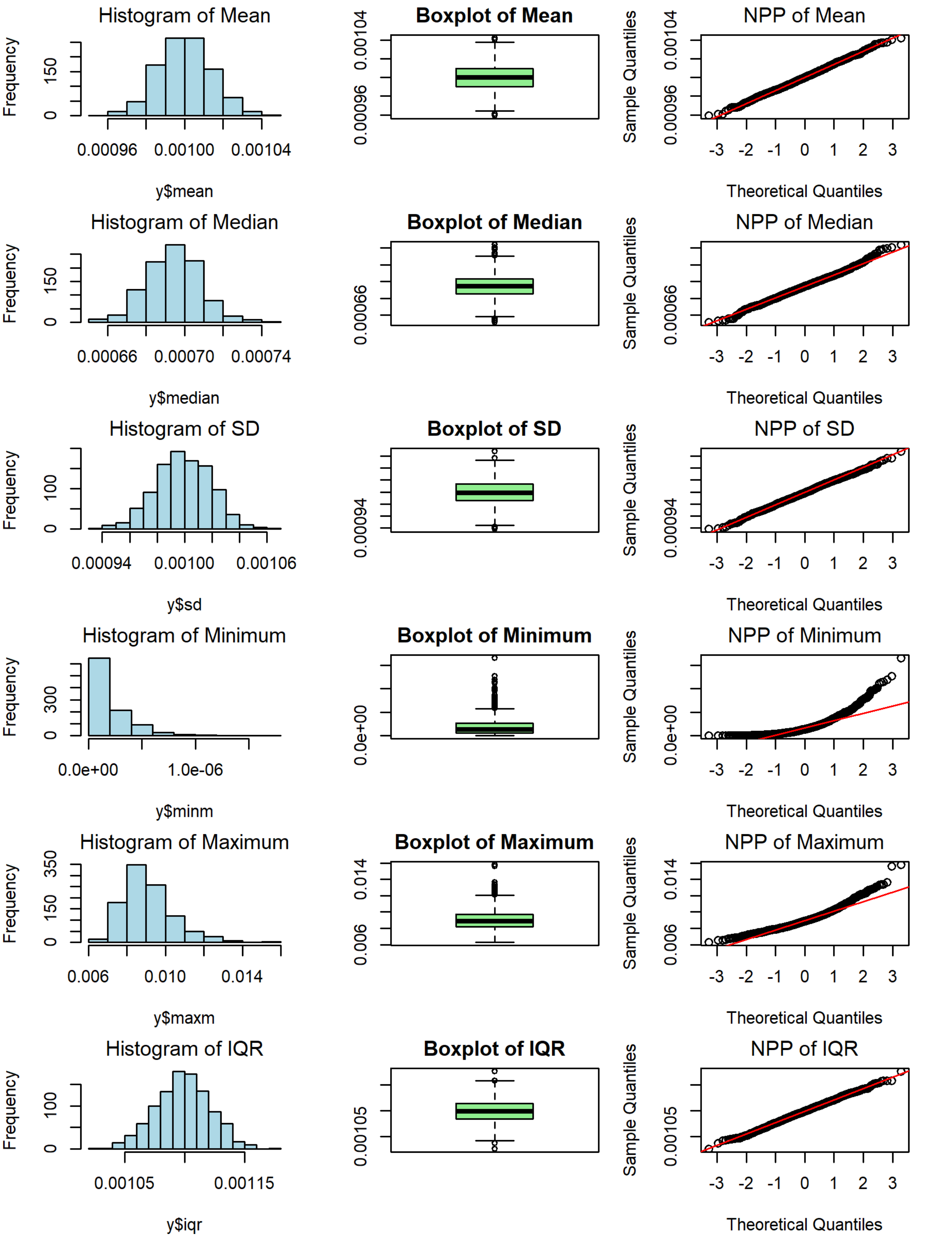


EXPONENTIAL DISTRIBUTION PLOT
(n=500, nn=1000, λ=1)

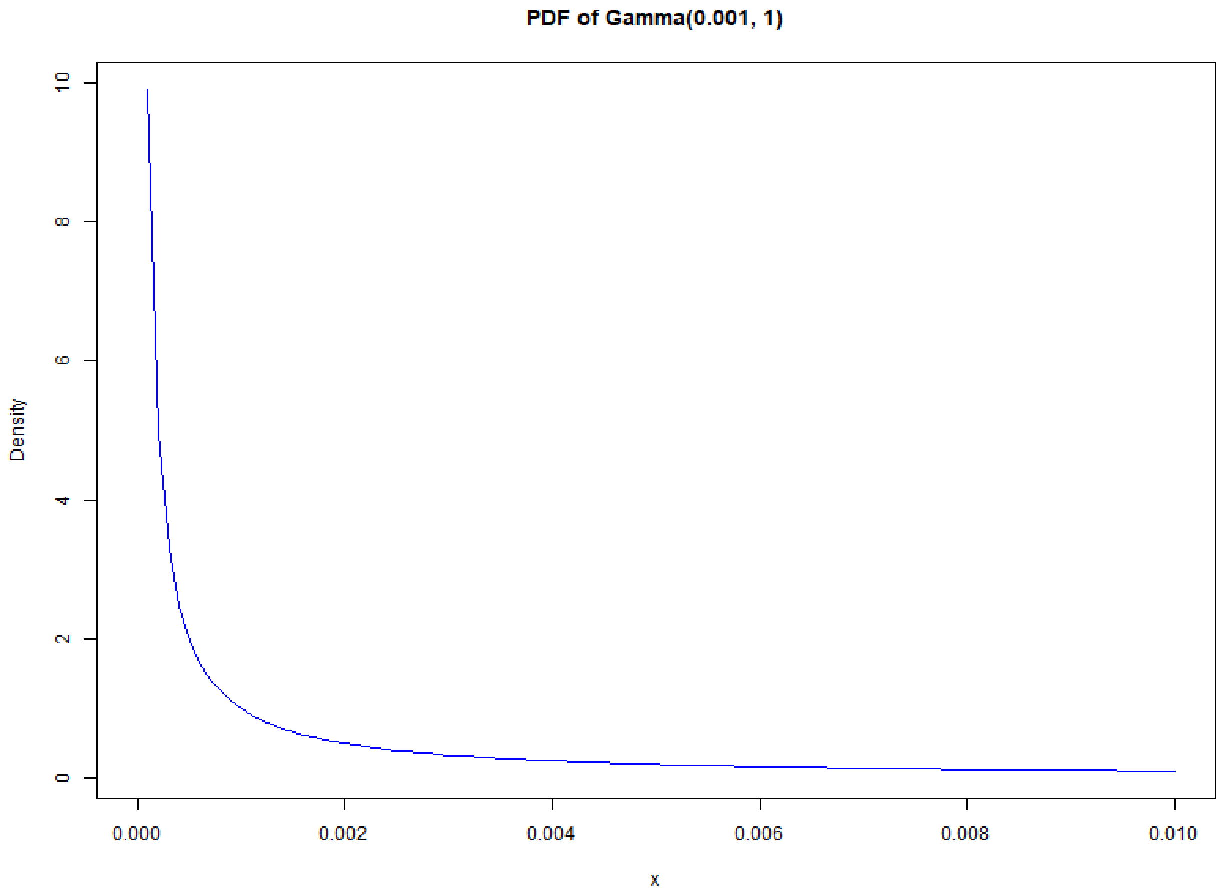


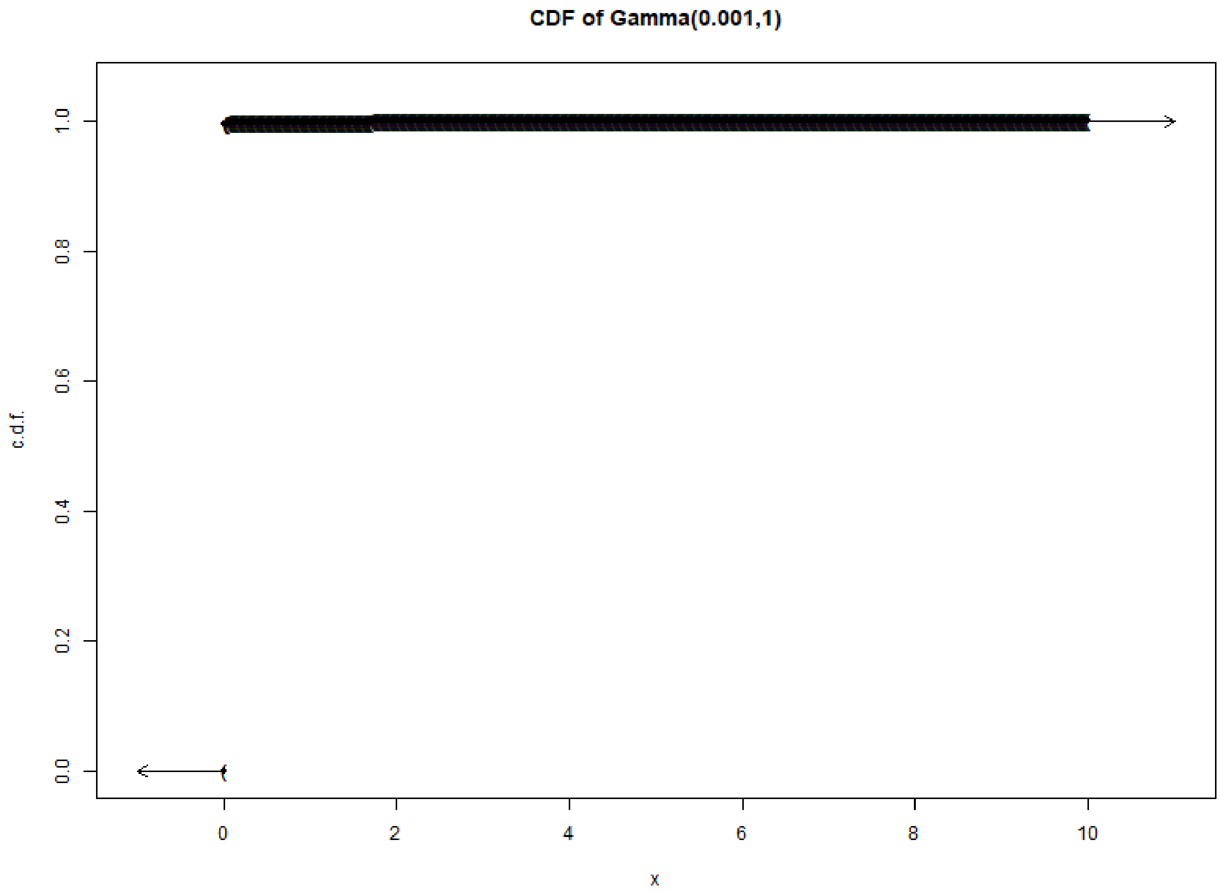
EXPONENTIAL DISTRIBUTION PLOT
(n=1000, nn=1000, λ=1)




Histogram of Mean
Frequency
0
150
0.00096
0.00100
0.00104
y$mean
Boxplot of Mean
0.00096
0.00104
NPP of Mean
Sample Quantiles
Theoretical Quantiles
-3
-2
-1
0
1
2
3
Histogram of Median
0.00066
0.00070
0.00074
y$median
Boxplot of Median
NPP of Median
Histogram of SD
100
0.00094
0.00100
0.00106
y$sd
Boxplot of SD
NPP of SD
Histogram of Minimum
300
0.0e+00
1.0e-06
y$minm
Boxplot of Minimum
NPP of Minimum
Histogram of Maximum
350
0.006
0.010
0.014
y$maxm
Boxplot of Maximum
NPP of Maximum
Histogram of IQR
0.00105
0.00115
y$iqr
Boxplot of IQR
NPP of IQR

GAMMA DISTRIBUTION (0.001,1)





GAMMA DISTRIBUTION

	Values of n to achieve normality (nn=1000, α=0.001, λ=1)							
Statistic	n=10	n=100	n=1000	n=5000	n=50e3	n=70e3	n=75e3	Min(n)
Mean	No	No	No	No	No	No	Yes	75000
Median	No	No	No	No	No	No	No	NA
Std.Dev	No	No	No	No	No	No	Yes	75000
Min	No	No	No	No	No	No	No	NA
Max	No	No	No	No	No	No	No	NA
IQR	No	No	No	No	No	No	No	NA

Conclusion for Gamma Distribution

Normality Achieved:

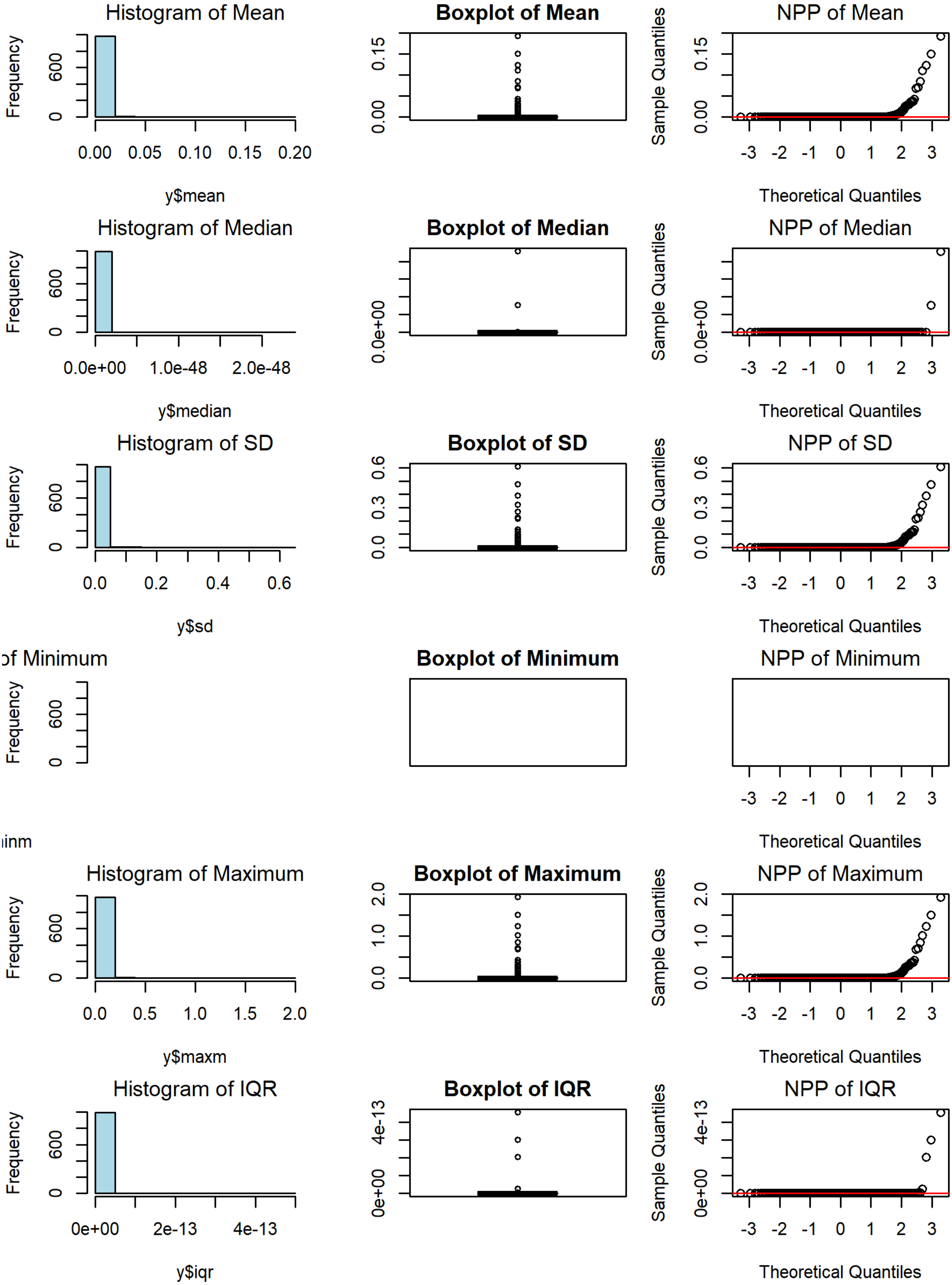
- **Mean:** Achieves normality for n ≥ 75,000, with convergence observed at larger sample sizes, indicating that higher sample sizes are necessary for the mean to approximate a normal distribution.
- **Standard Deviation (SD):** Achieves normality for n ≥ 75,000, showing a similar pattern of convergence as the mean with a high number of samples.

Normality Not Achieved:

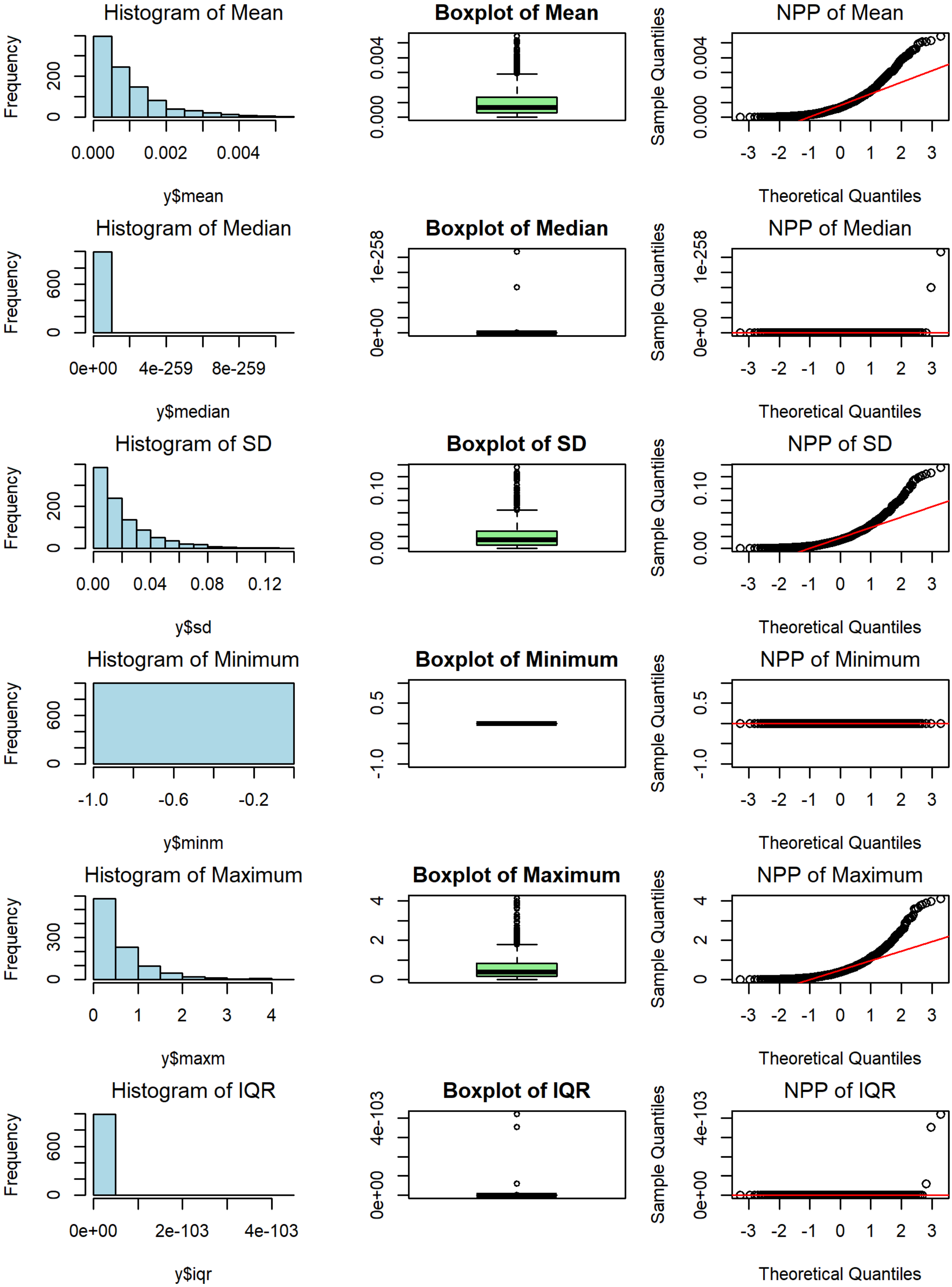
- **Median:** Does not achieve normality for any sample size, as it is still influenced by the shape of the Gamma distribution even with larger sample sizes.
- **Minimum and Maximum:** Do not achieve normality for any sample size, as these values are sensitive to the extreme values of the Gamma distribution.
- **IQR:** Does not achieve normality for any sample size, as it remains affected by the distribution's skewness and variability.

Overall: The mean and standard deviation are the most likely to achieve normality, but only for sample sizes n ≥ 75,000. Smaller sample sizes (n < 75,000) do not achieve normality for any of the statistics. The median, minimum, maximum, and IQR remain non-normal regardless of sample size due to the inherent characteristics of the Gamma distribution.

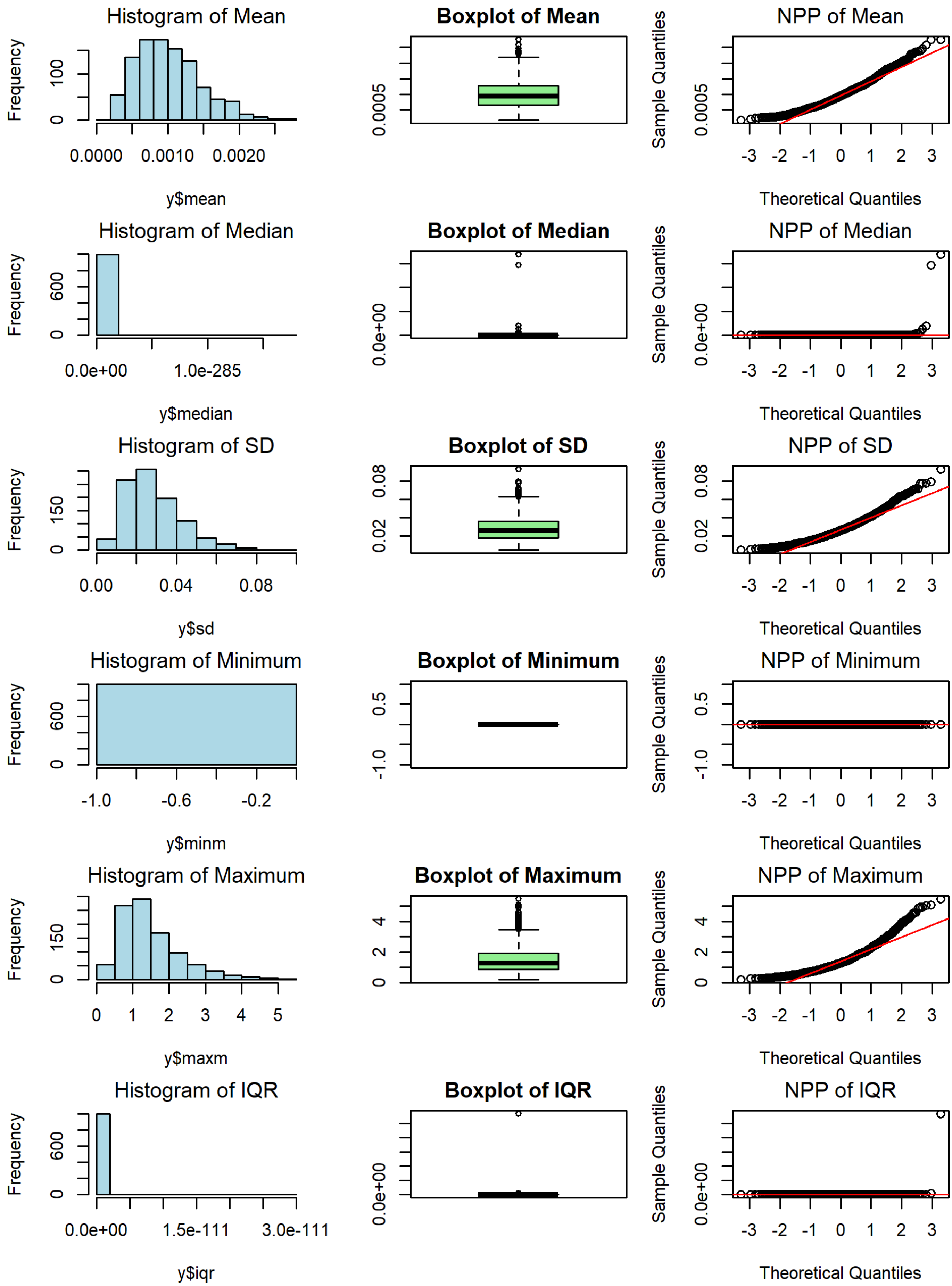
GAMMA DISTRIBUTION PLOT
(n=10, nn=1000, α=0.001, λ=1)



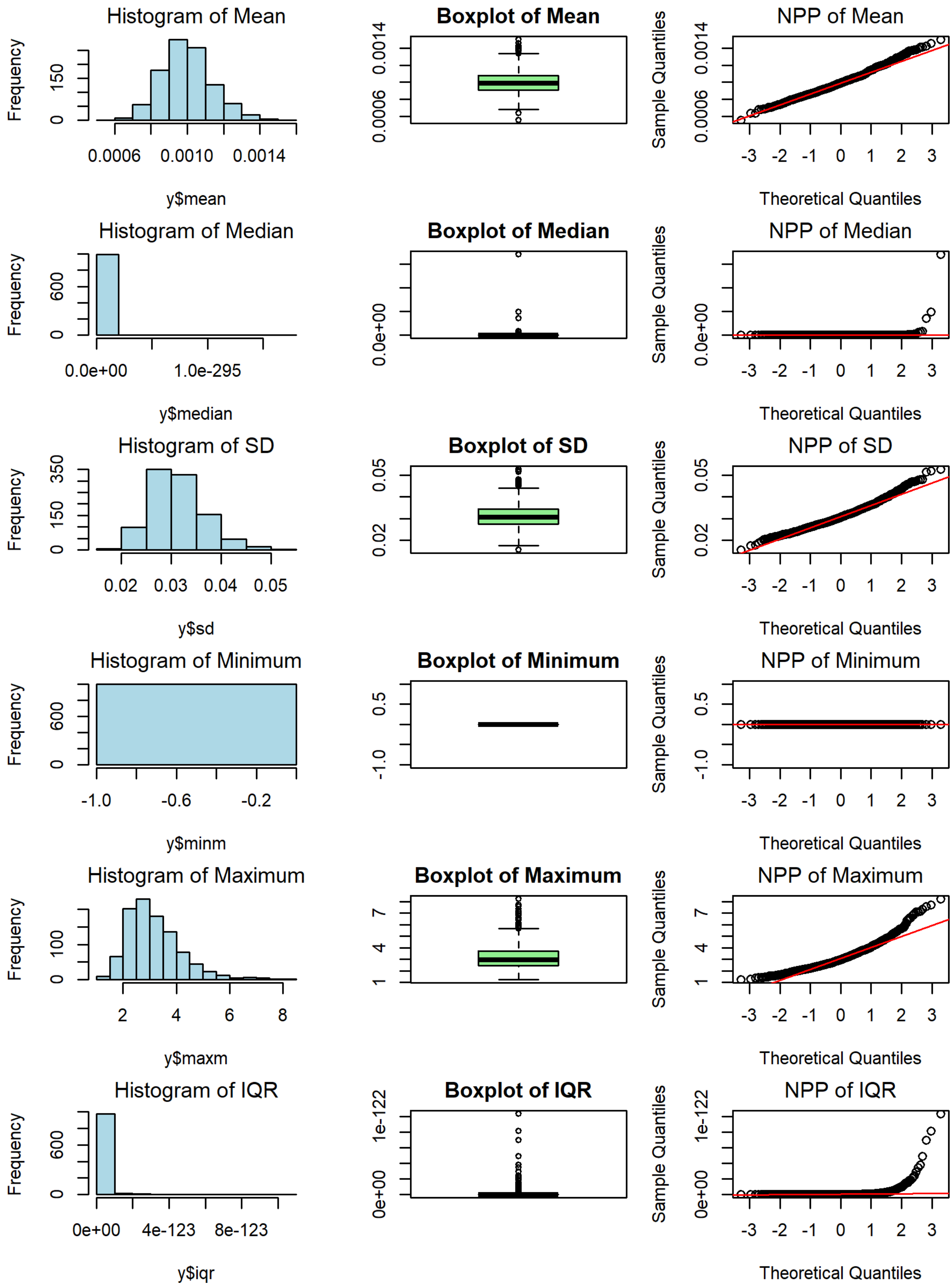
GAMMA DISTRIBUTION PLOT
(n=1000, nn=1000, α=0.001, λ=1)



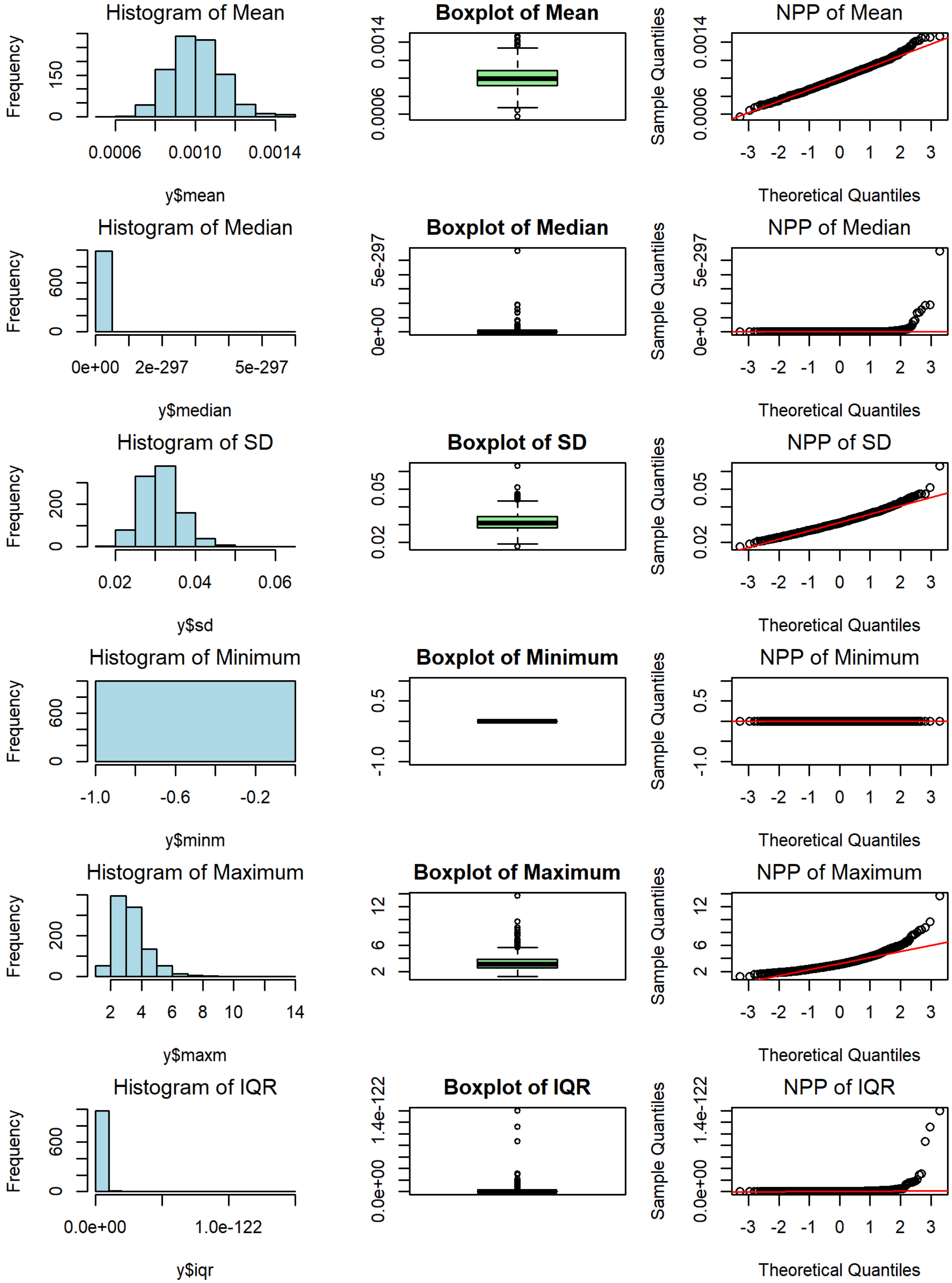
GAMMA DISTRIBUTION PLOT
(n=5000, nn=1000, α=0.001, λ=1)

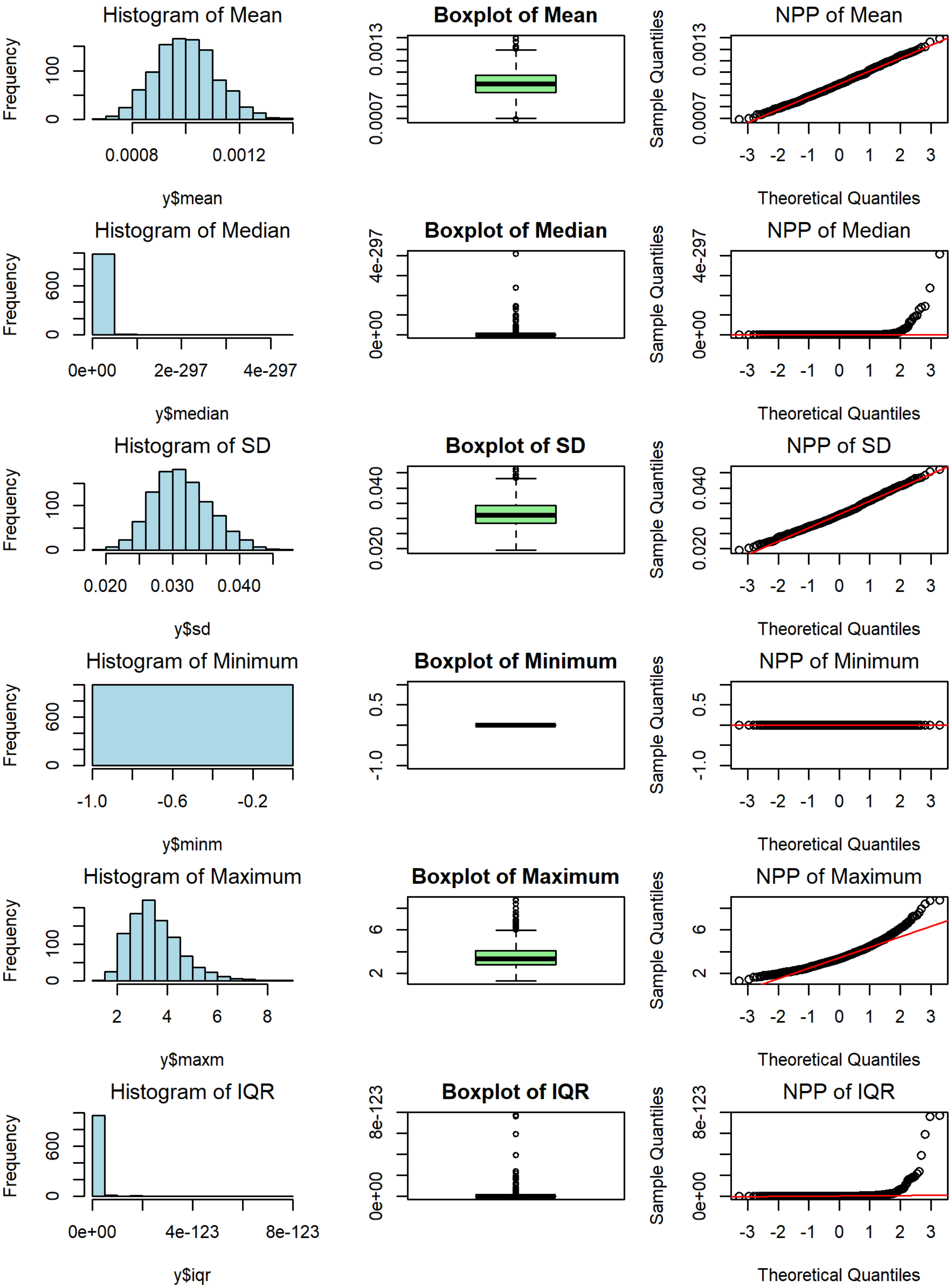


GAMMA DISTRIBUTION PLOT
(n=50000, nn=1000, α=0.001, λ=1)



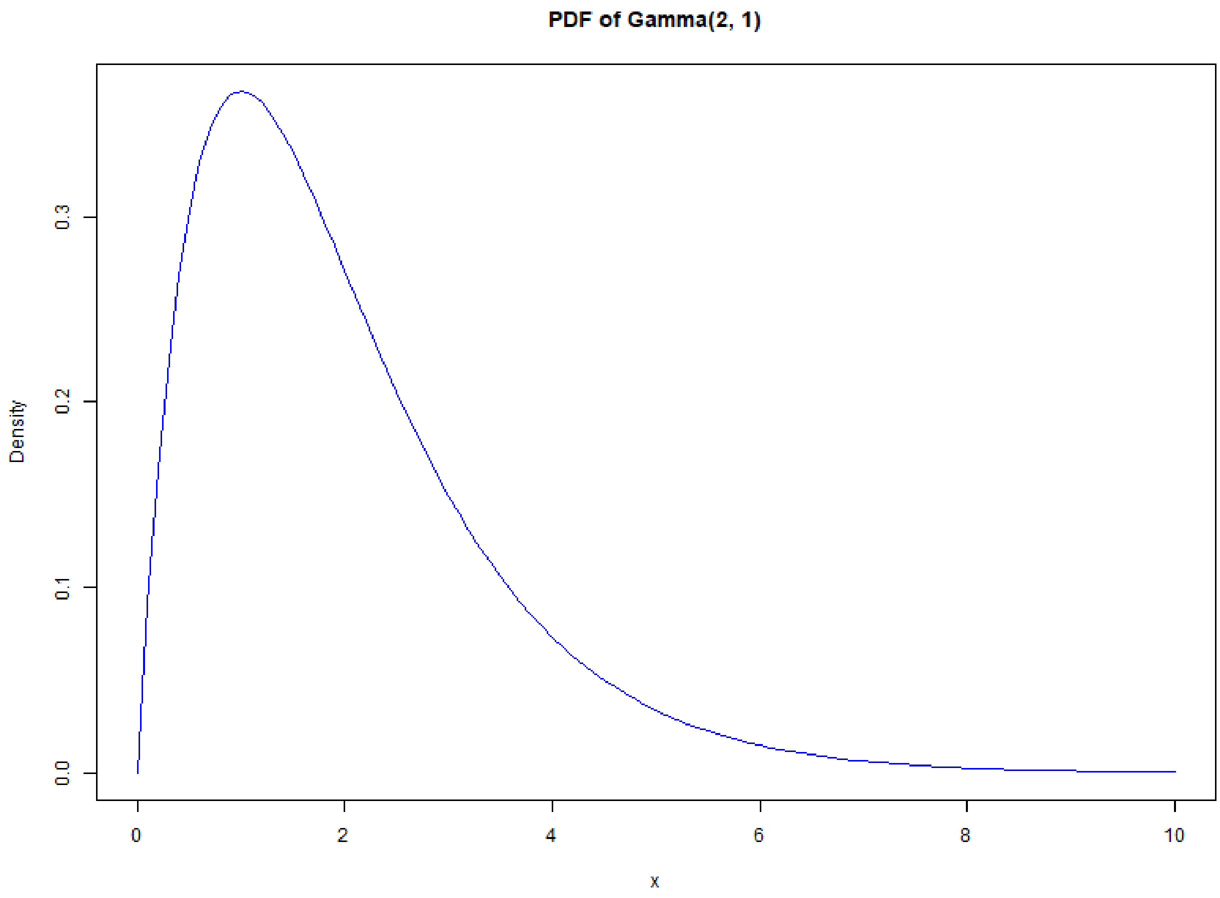
GAMMA DISTRIBUTION PLOT
(n=70000, nn=1000, α=0.001, λ=1)

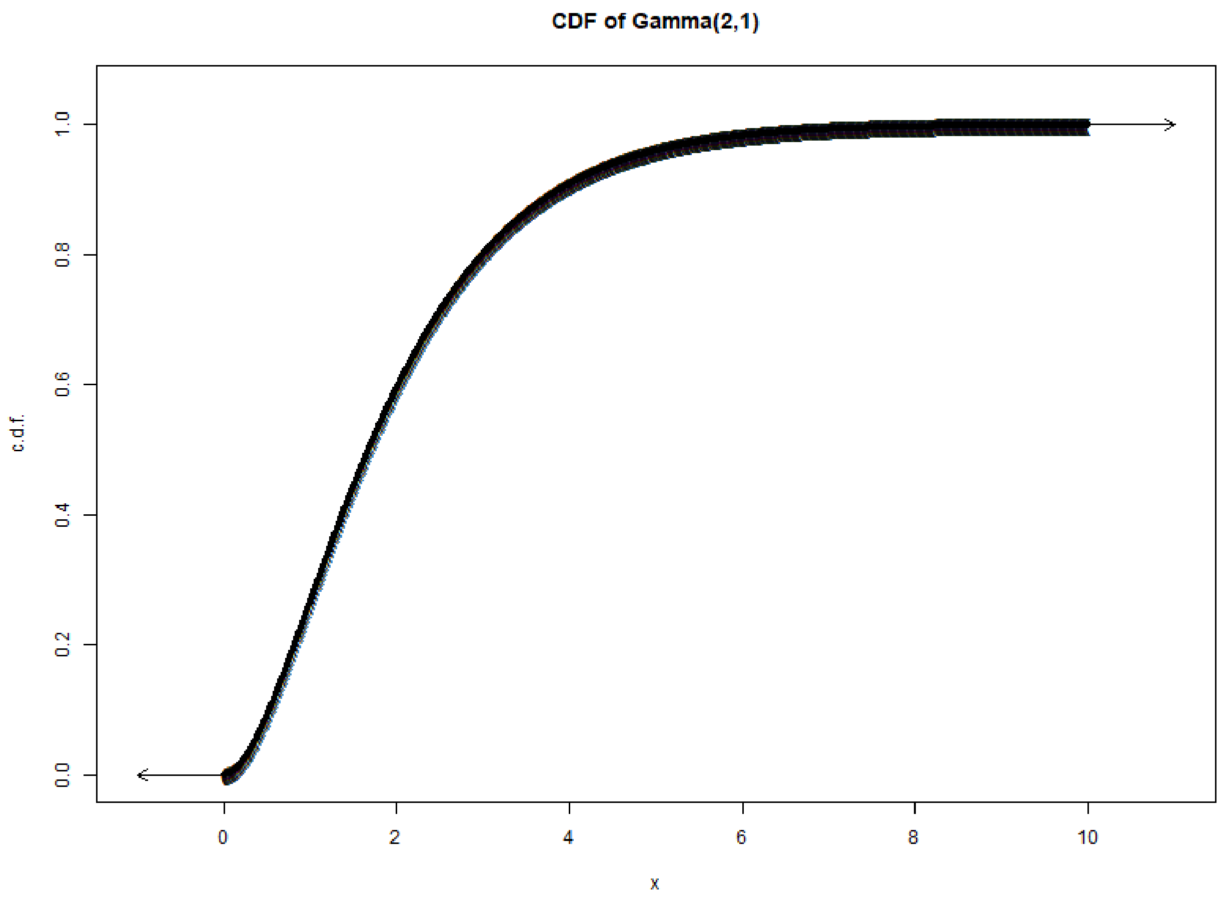




Histogram of Mean
Frequency
0
100
0.0008
0.0012
y$mean
Boxplot of Mean
0.0007
0.0013
NPP of Mean
Sample Quantiles
0.0007
0.0013
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of Median
Frequency
0
600
0e+00
2e-297
4e-297
y$median
Boxplot of Median
0e+00
4e-297
NPP of Median
Sample Quantiles
0e+00
4e-297
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of SD
Frequency
0
100
0.020
0.030
0.040
y$sd
Boxplot of SD
0.020
0.040
NPP of SD
Sample Quantiles
0.020
0.040
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of Minimum
Frequency
0
600
-1.0
-0.6
-0.2
y$minm
Boxplot of Minimum
-1.0
0.5
NPP of Minimum
Sample Quantiles
-1.0
0.5
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of Maximum
Frequency
0
100
2
4
6
8
y$maxm
Boxplot of Maximum
2
6
NPP of Maximum
Sample Quantiles
2
6
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of IQR
Frequency
0
600
0e+00
4e-123
8e-123
y$iqr
Boxplot of IQR
0e+00
8e-123
NPP of IQR
Sample Quantiles
0e+00
8e-123
-3 -2 -1 0 1 2 3
Theoretical Quantiles

GAMMA DISTRIBUTION $(2,1)$





GAMMA DISTRIBUTION

	Values of n to achieve normality (nn=1000, α=2, λ=1)						
Statistic	n=10	n=50	n=100	n=500	n=1000	n=5000	Any n	Min(n)
Mean	No	No	Yes	Yes	Yes	Yes	Yes	100
Median	No	No	No	Yes	Yes	Yes	Yes	500
Std Dev	No	No	Yes	Yes	Yes	Yes	Yes	100
Min	No	No	No	No	No	No	No	NA
Max	No	No	No	No	No	No	No	NA
IQR	No	No	Yes	Yes	Yes	Yes	Yes	100

Conclusion for Gamma Distribution (with α = 2)

Normality Achieved:

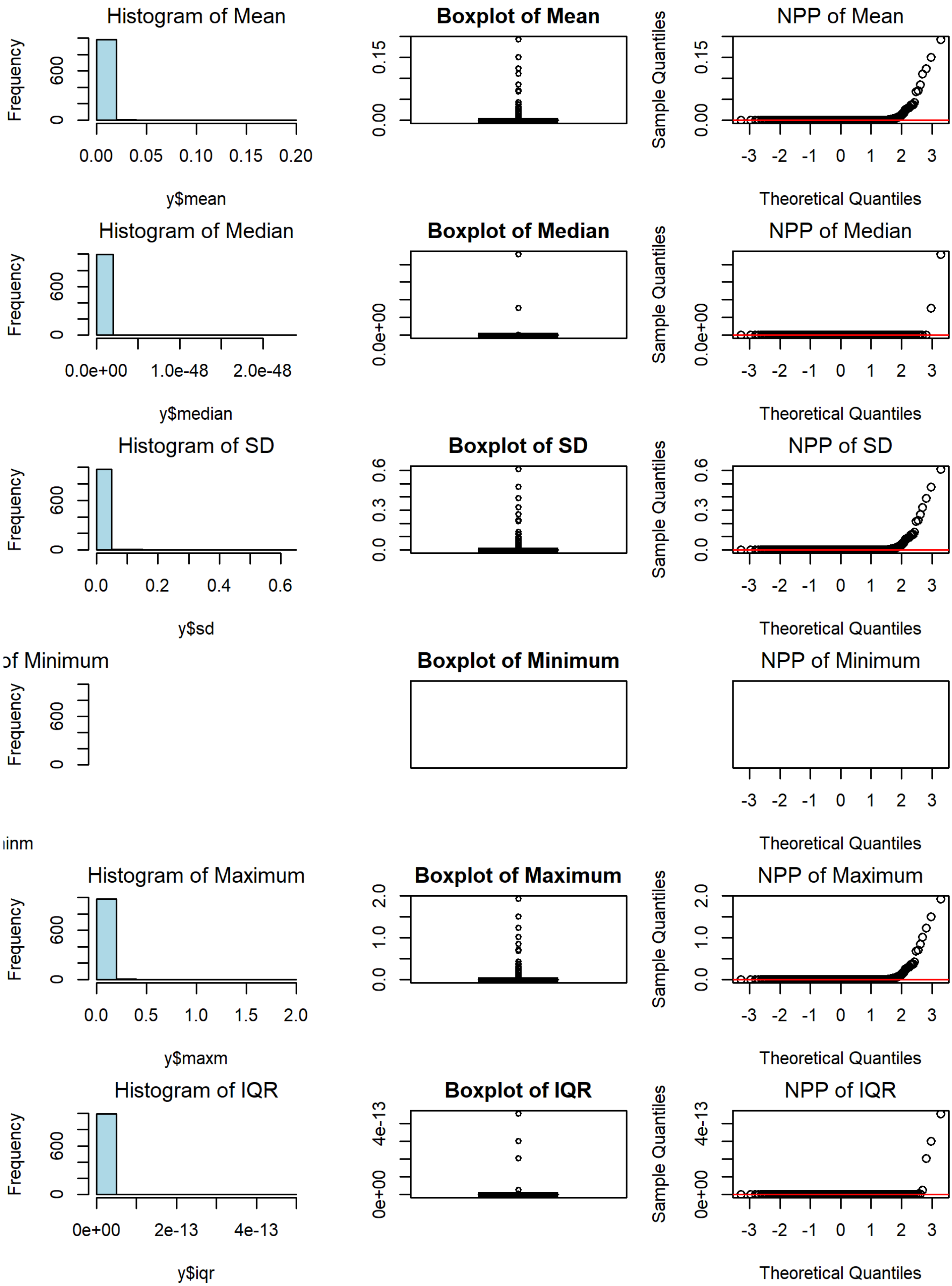
- **Mean:** Achieves normality for n ≥ 100, with convergence occurring relatively quickly as the sample size increases.
- **Median:** Achieves normality for n ≥ 500, indicating that slightly larger sample sizes are necessary for the median to approximate a normal distribution.
- **Standard Deviation (SD):** Achieves normality for n ≥ 100, showing that the standard deviation converges to normality at smaller sample sizes compared to the median.
- **IQR:** Achieves normality for n ≥ 100, with the IQR showing similar convergence patterns as the standard deviation.

Normality Not Achieved:

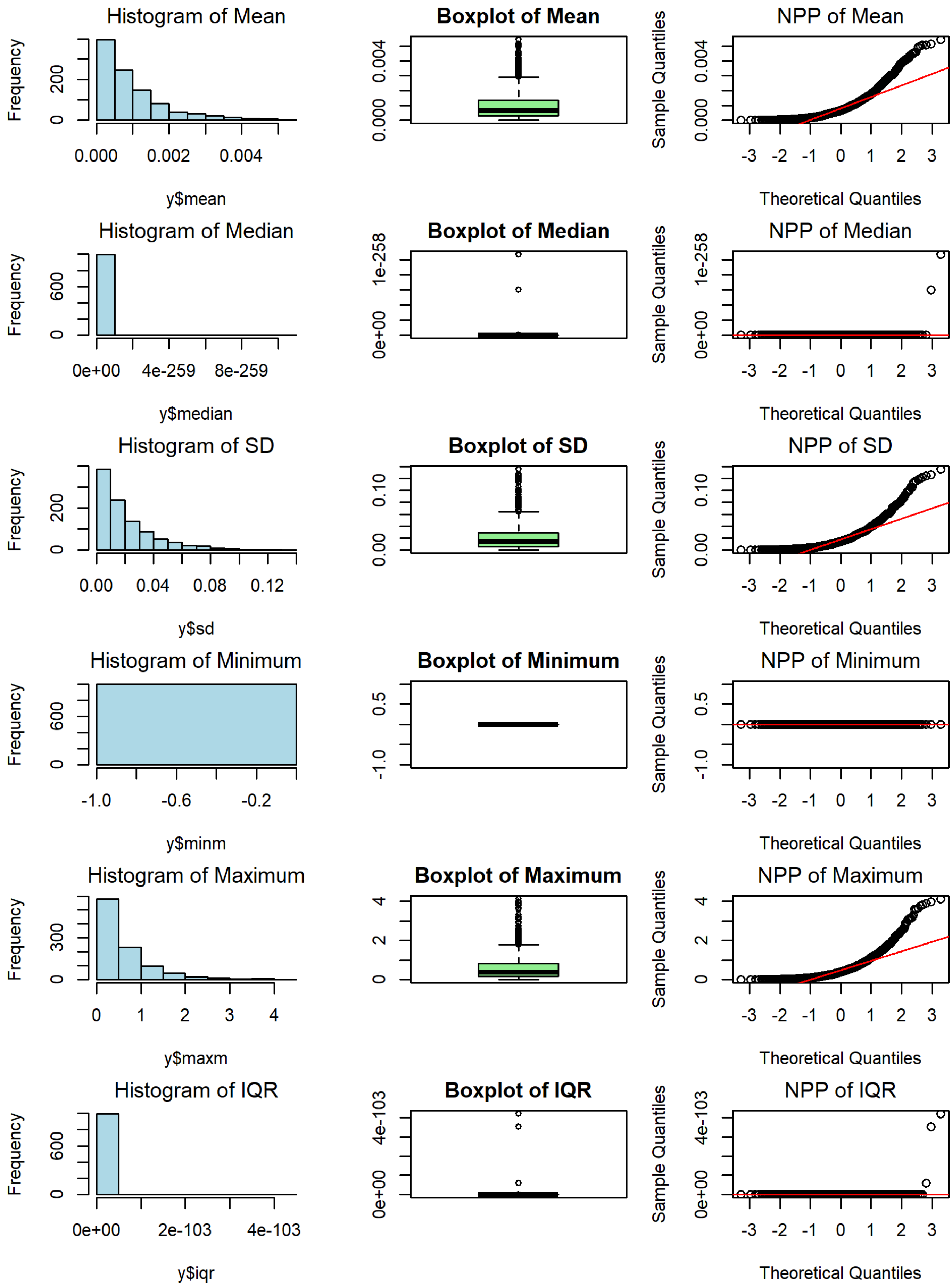
- **Minimum and Maximum:** Do not achieve normality for any sample size, as these values are still influenced by extreme values in the Gamma distribution.

Overall: For α = 2, the mean, standard deviation, and IQR achieve normality at n ≥ 100, which is much faster compared to α = 0.001. The extreme values (minimum and maximum) still require large sample sizes (n ≥ 500) to approximate normality, and they remain non-normal for smaller sample sizes.

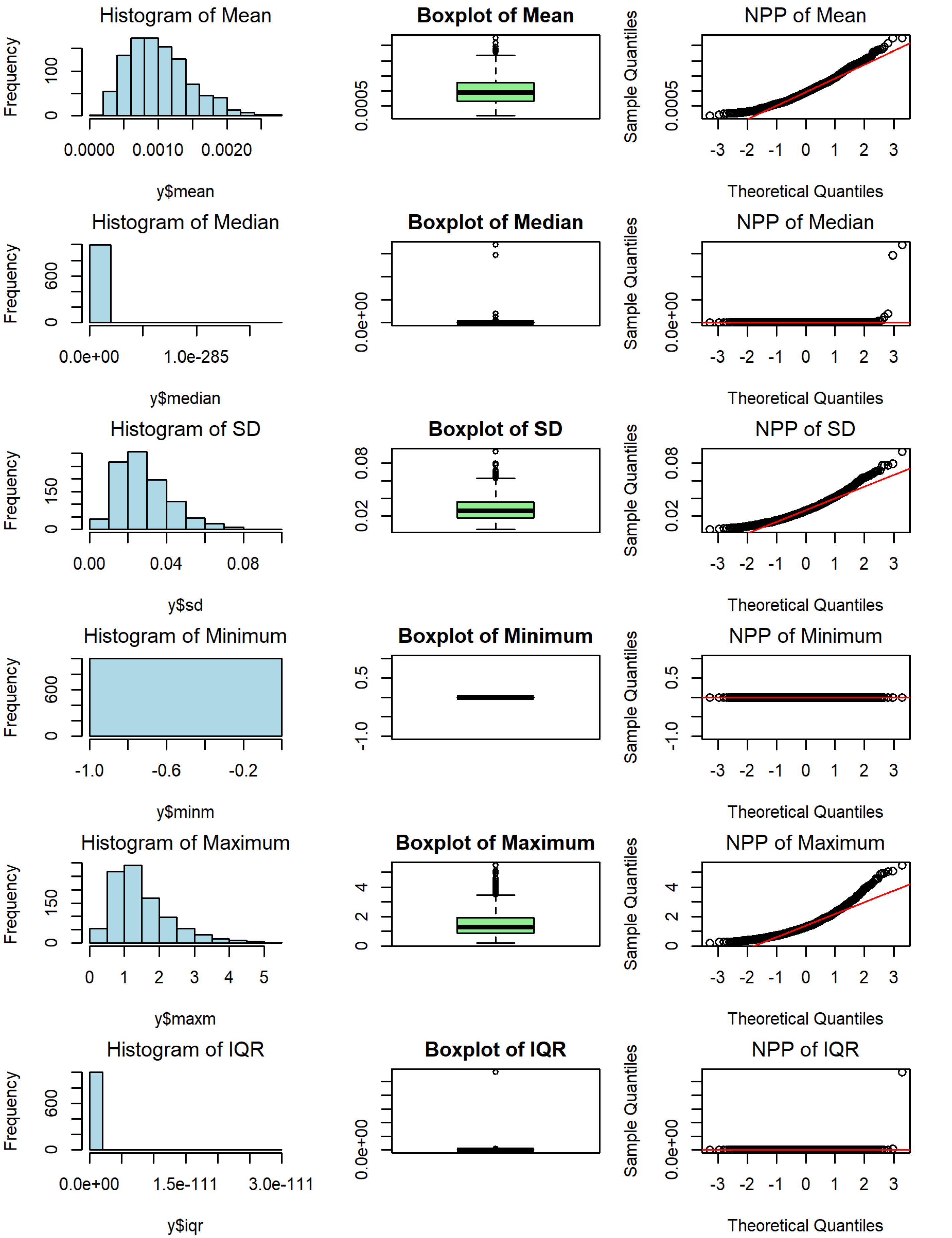
GAMMA DISTRIBUTION PLOT
(n=10, nn=1000, α=2, λ=1)



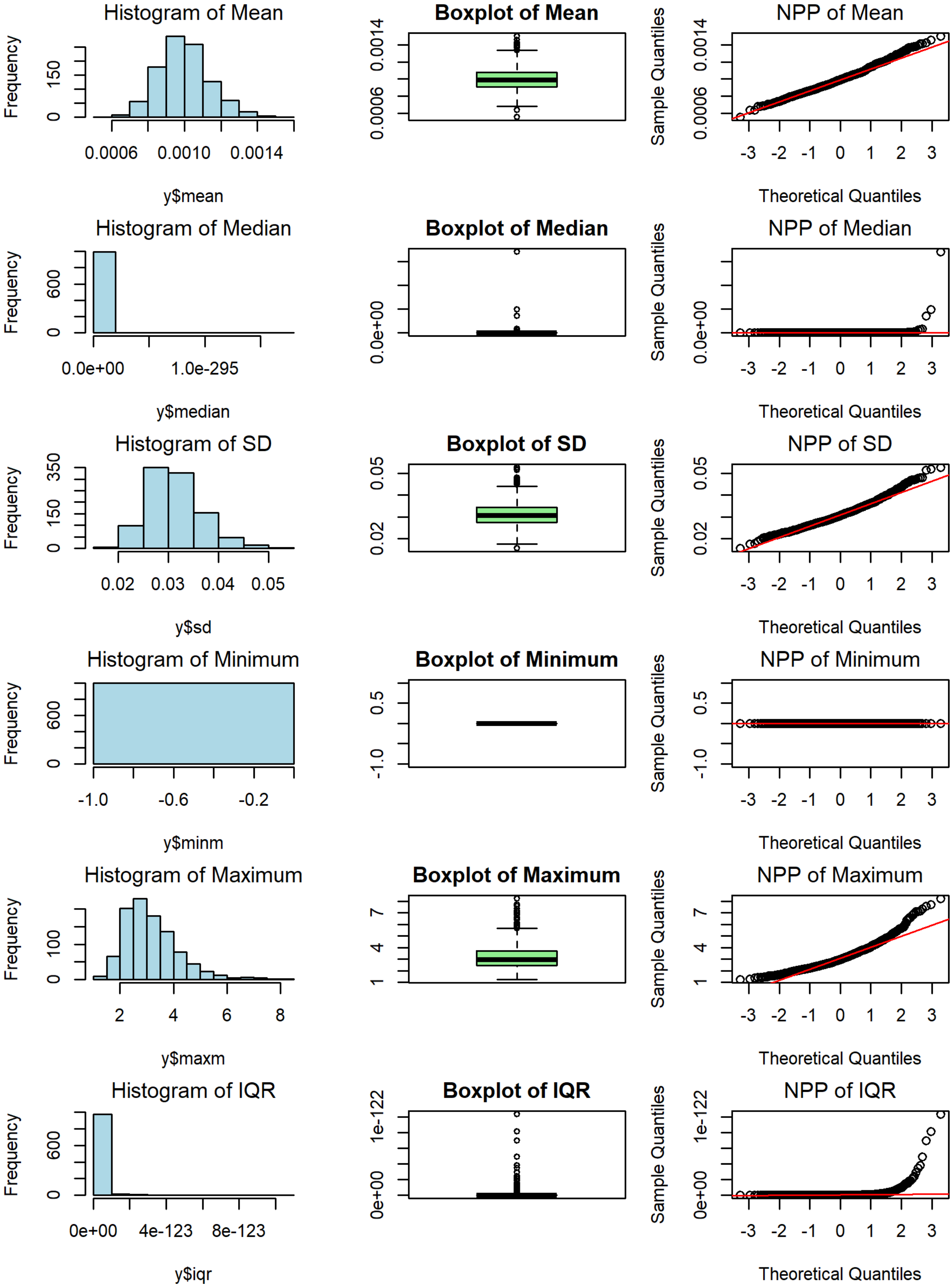
GAMMA DISTRIBUTION PLOT
(n=1000, nn=1000, α=2, λ=1)



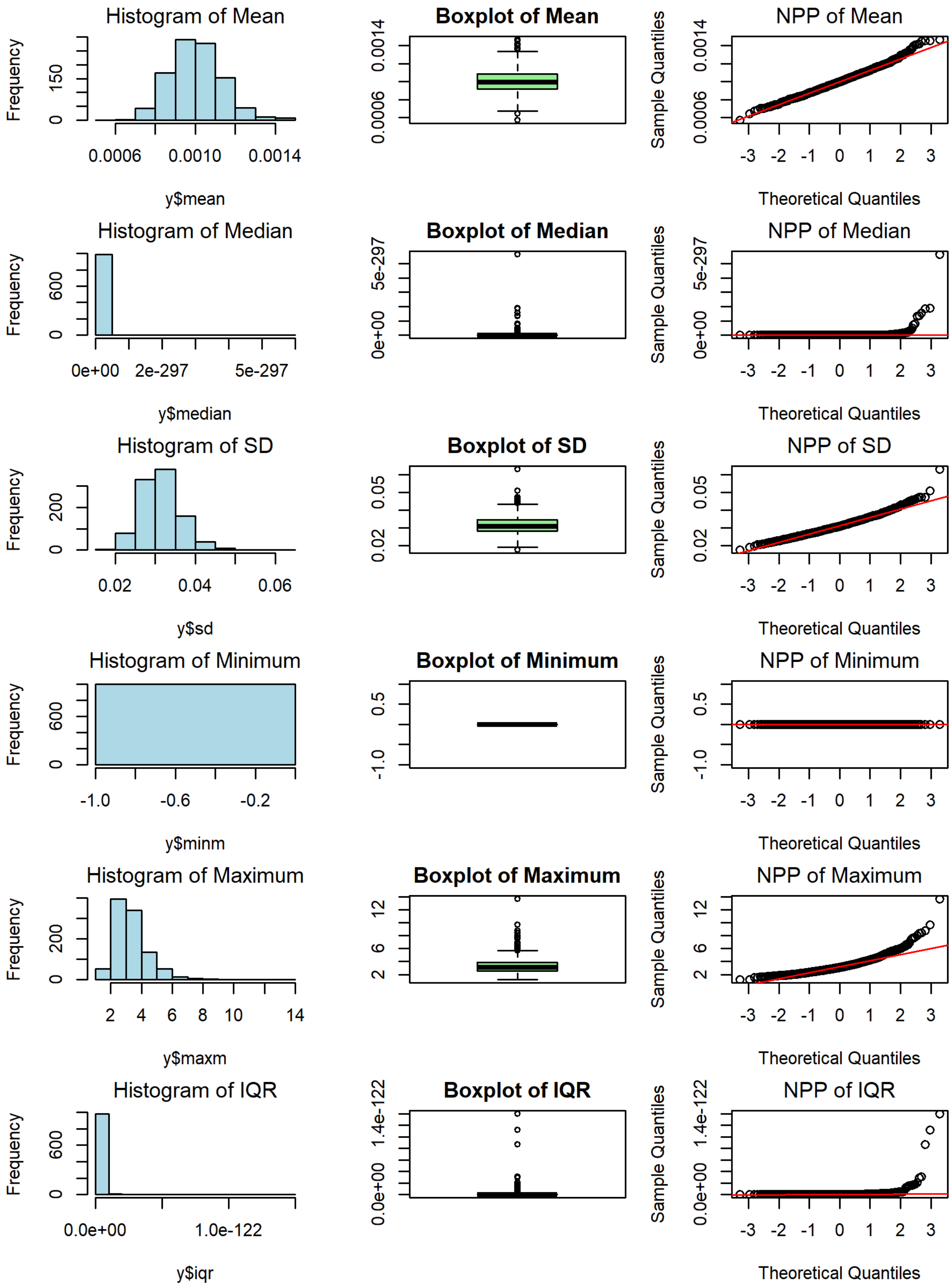
GAMMA DISTRIBUTION PLOT
(n=5000, nn=1000, α=2, λ=1)

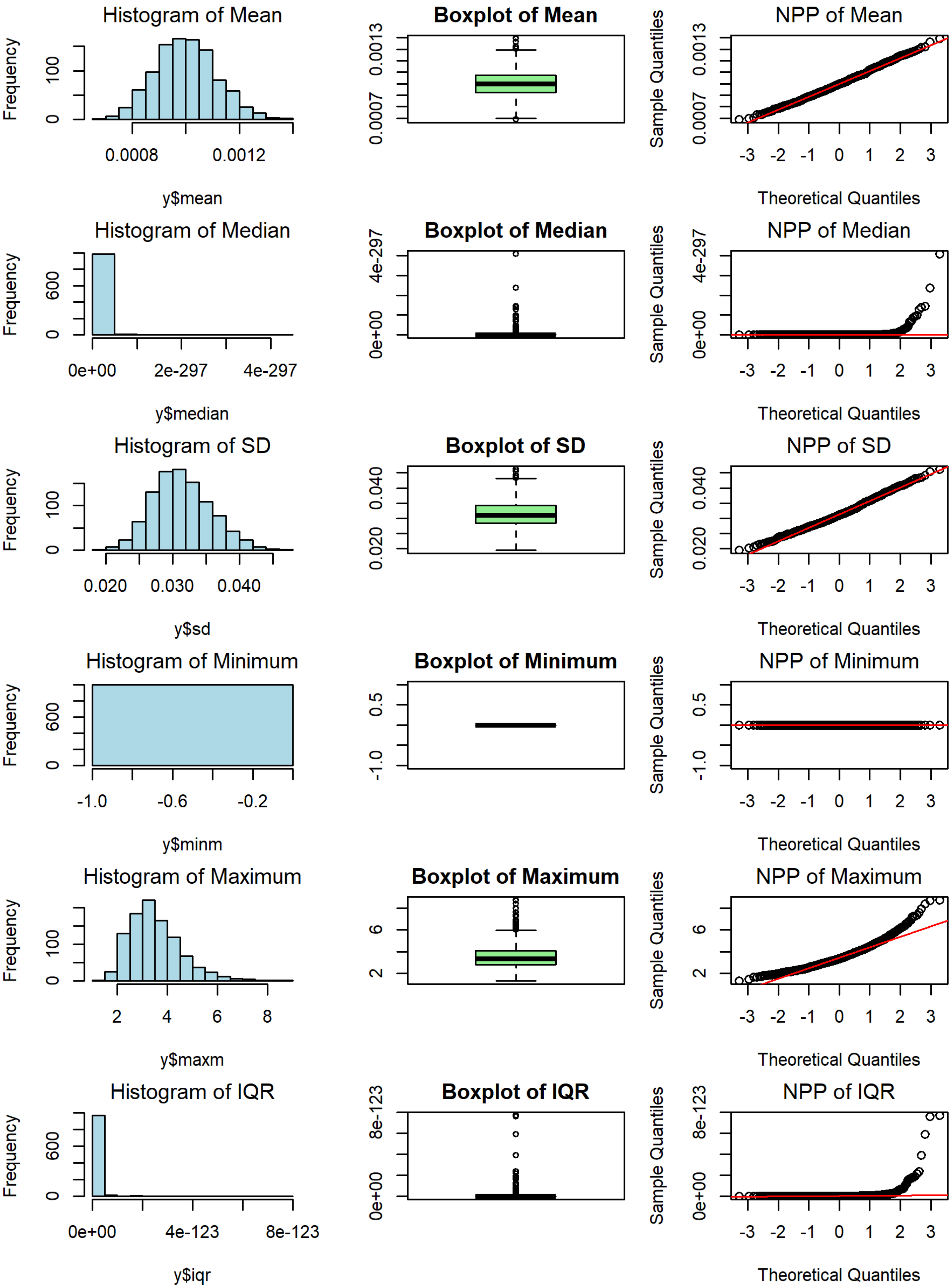


GAMMA DISTRIBUTION PLOT
(n=50000, nn=1000, α=2, λ=1)



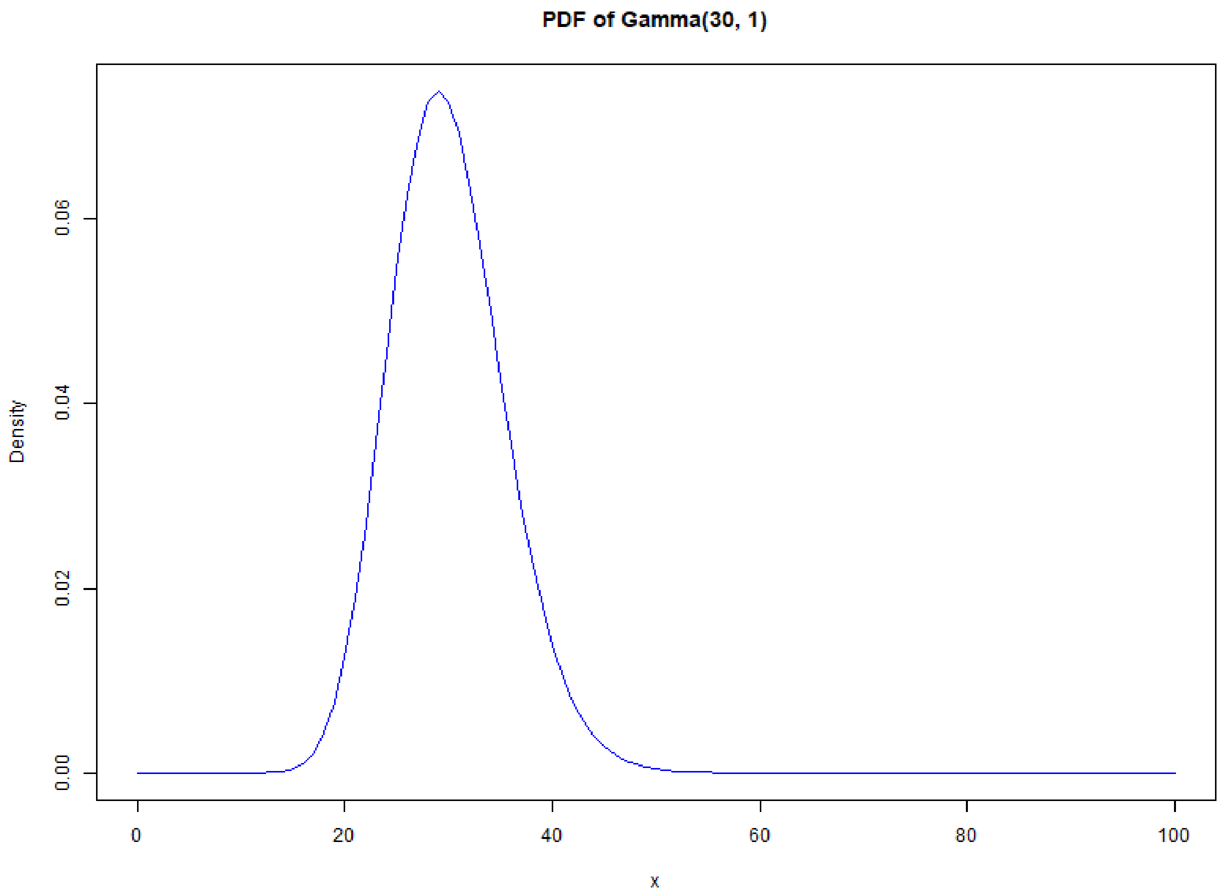
GAMMA DISTRIBUTION PLOT
(n=70000, nn=1000, α=2, λ=1)

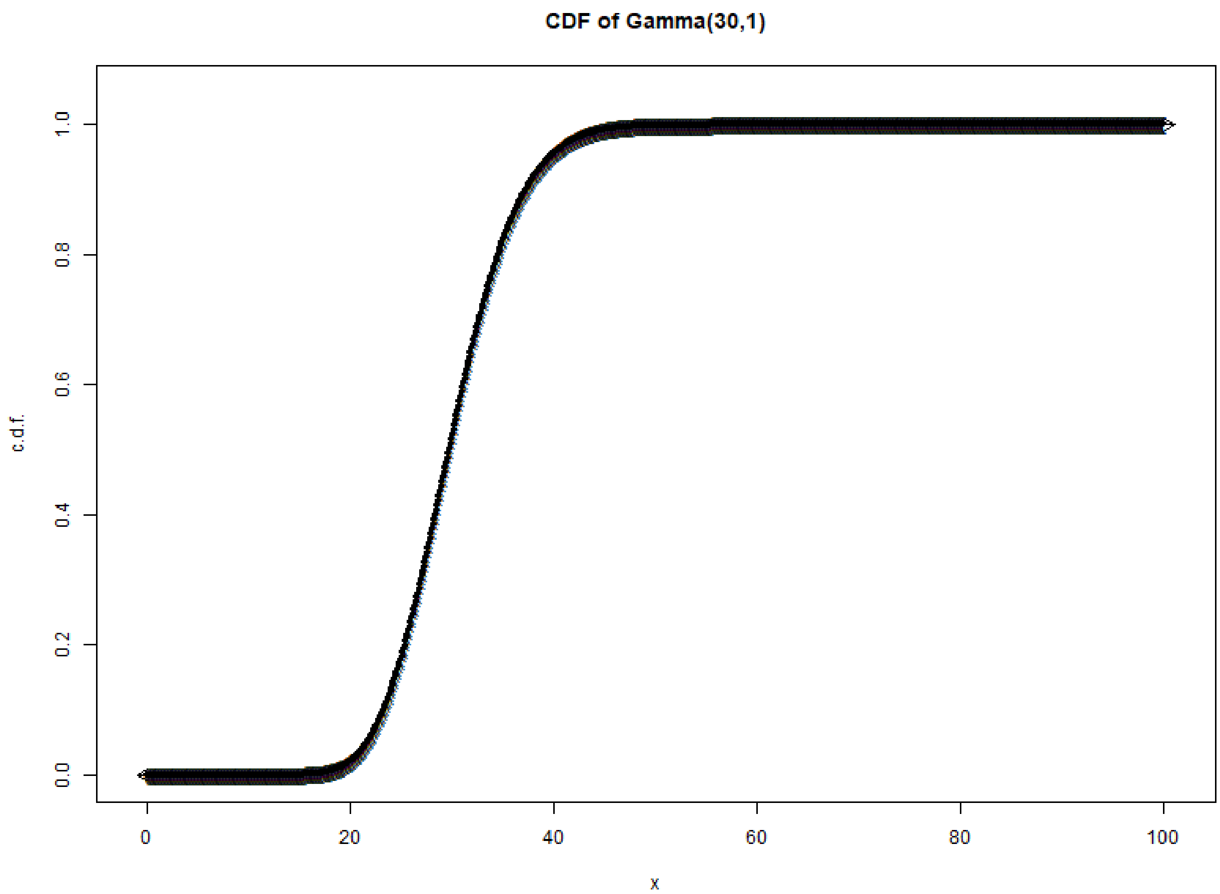




Histogram of Mean
Frequency
y$mean
Boxplot of Mean
NPP of Mean
Sample Quantiles
Theoretical Quantiles
Histogram of Median
y$median
Boxplot of Median
NPP of Median
Histogram of SD
y$sd
Boxplot of SD
NPP of SD
Histogram of Minimum
y$minm
Boxplot of Minimum
NPP of Minimum
Histogram of Maximum
y$maxm
Boxplot of Maximum
NPP of Maximum
Histogram of IQR
y$iqr
Boxplot of IQR
NPP of IQR

GAMMA DISTRIBUTION $(30,1)$





GAMMA DISTRIBUTION

	Values of n to achieve normality (nn=1000, α=30, λ=1)							
Statistic	n=10	n=50	n=100	n=500	n=1000	n=5000	Any n	Min(n)
Mean	Yes	Yes	Yes	Yes	Yes	Yes	Yes	10
Median	Yes	Yes	Yes	Yes	Yes	Yes	Yes	10
Std Dev	No	No	Yes	Yes	Yes	Yes	Yes	100
Min	Yes	Yes	Yes	Yes	No	No	Yes	10
Max	No	No	No	No	No	No	No	NA
IQR	No	No	Yes	Yes	Yes	Yes	Yes	100

Conclusion for Gamma Distribution (with α = 30)

Normality Achieved:

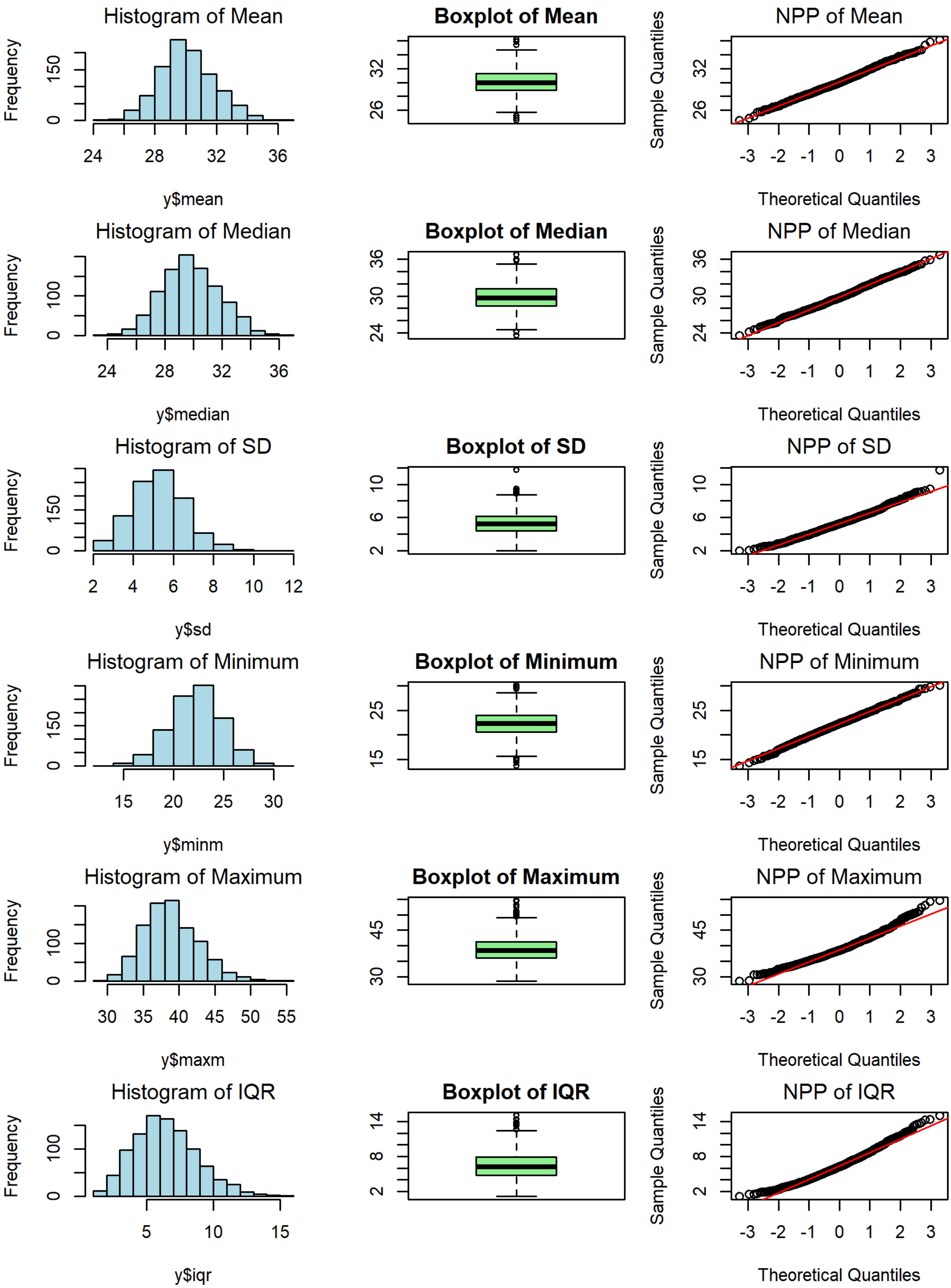
- **Mean:** Achieves normality for n ≥ 10, with quick convergence to normality as the sample size increases.
- **Median:** Achieves normality for n ≥ 10, indicating a very fast convergence compared to lower values of α.
- **Minimum:** Achieves normality for n ≥ 10 but loses normality again after n = 1000, showing a non-monotonic pattern in convergence.
- **Standard Deviation (SD):** Achieves normality for n ≥ 100, with convergence occurring at slightly larger sample sizes compared to the mean and median.
- **IQR:** Achieves normality for n ≥ 100, with convergence following a similar pattern to the standard deviation.

Normality Not Achieved:

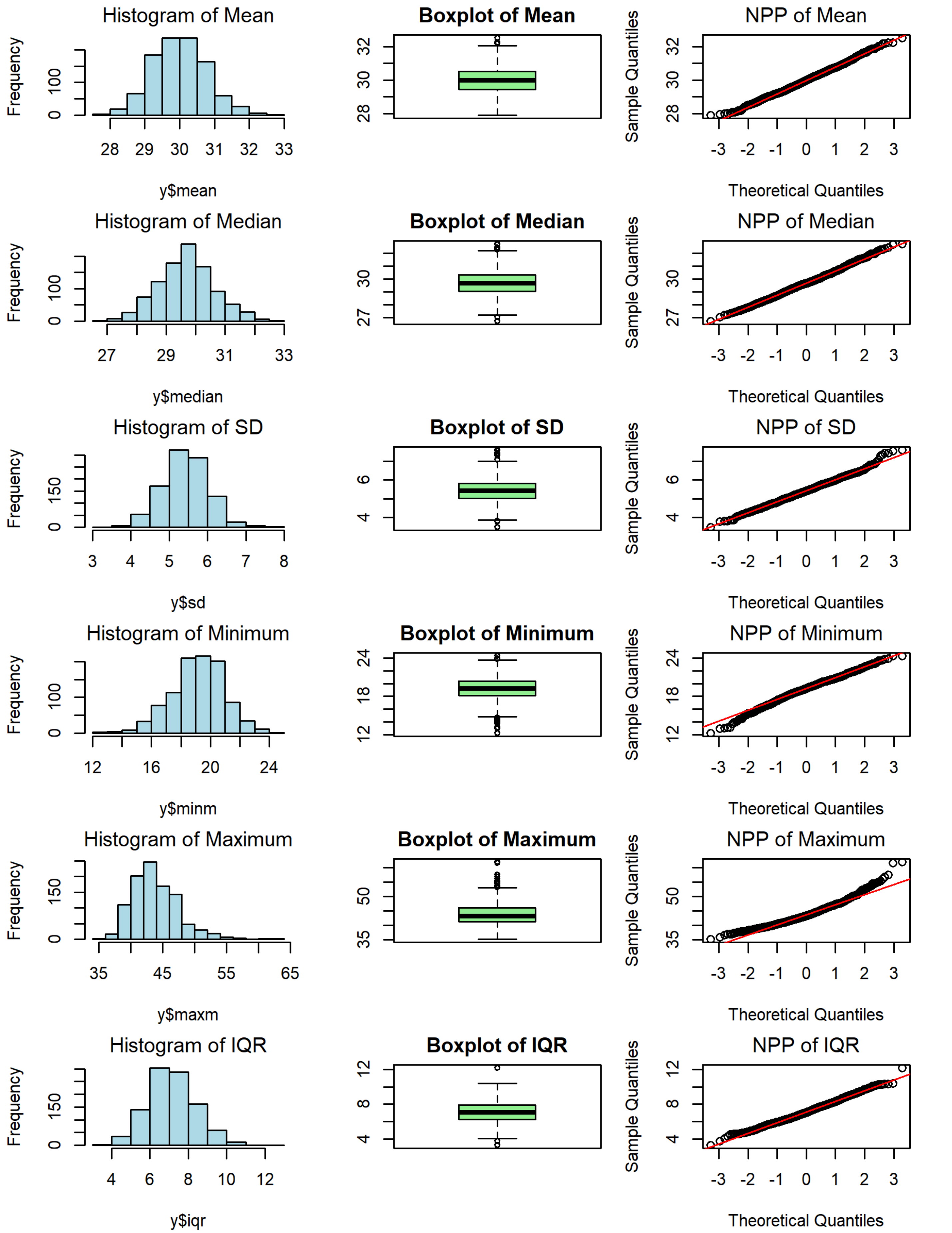
- **Maximum:** Does not achieve normality for any sample size, as it remains highly influenced by extreme values in the Gamma distribution.

Overall: For α = 30, normality is achieved at very small sample sizes, especially for the mean, median, and minimum, where convergence is observed from n ≥ 10. However, the minimum loses normality again after n = 1000. The standard deviation and IQR converge to normality at n ≥ 100, showing faster convergence compared to lower values of α (e.g., α = 0.001 or α = 2). The maximum remains non-normal regardless of the sample size.

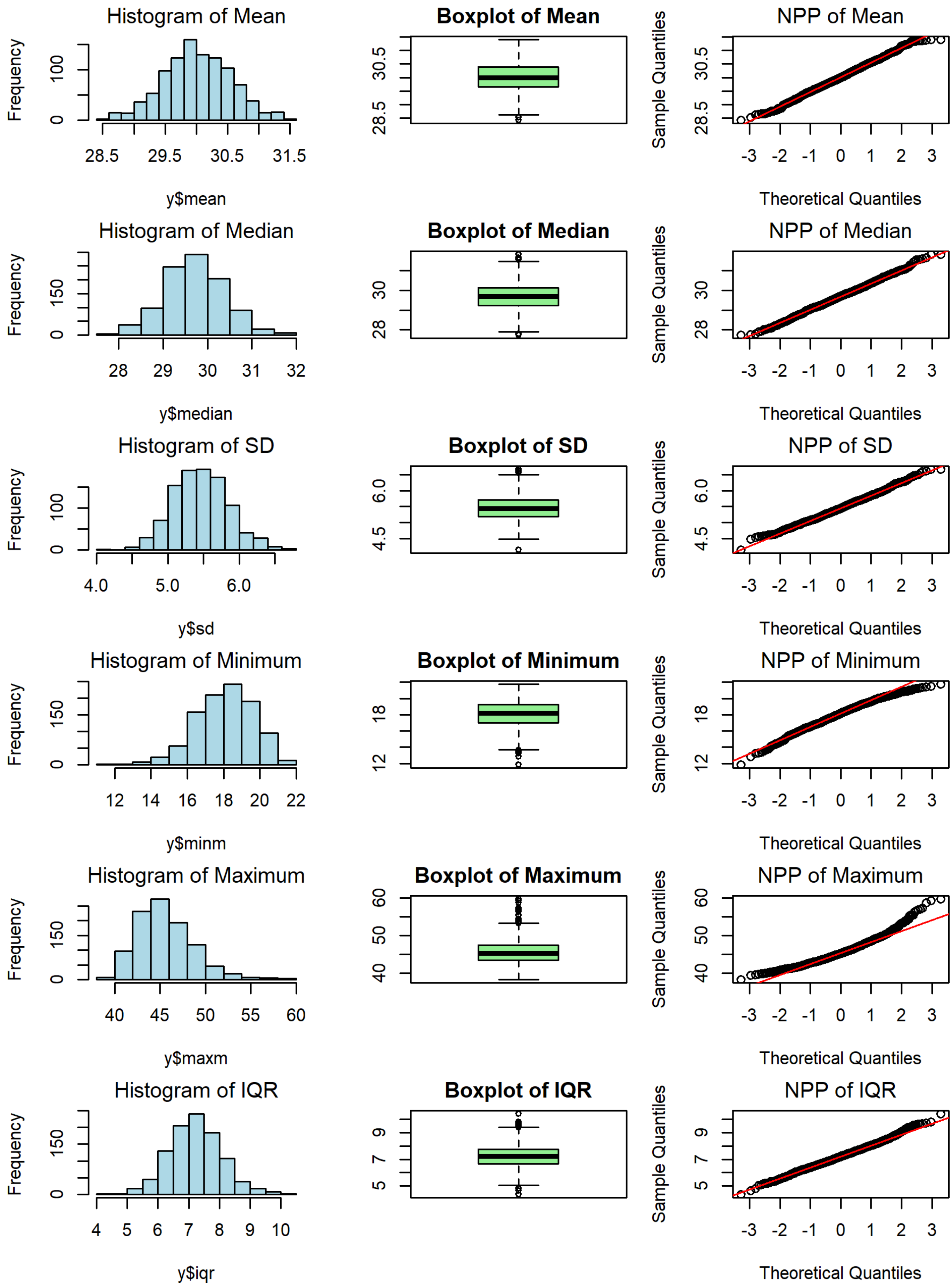
GAMMA DISTRIBUTION PLOT
(n=10, nn=1000, α=30, λ=1)



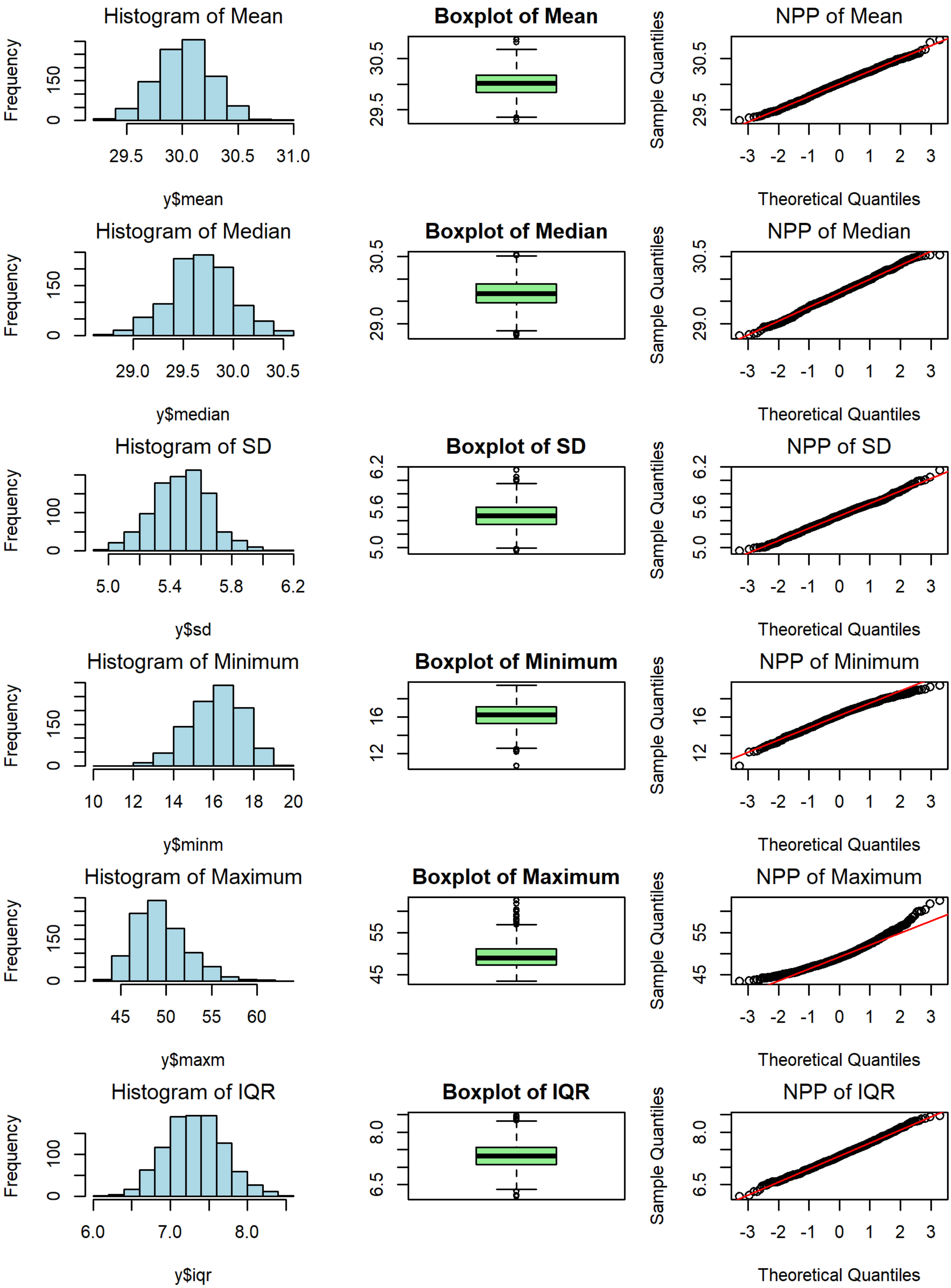
GAMMA DISTRIBUTION PLOT
(n=50, nn=1000, α=30, λ=1)



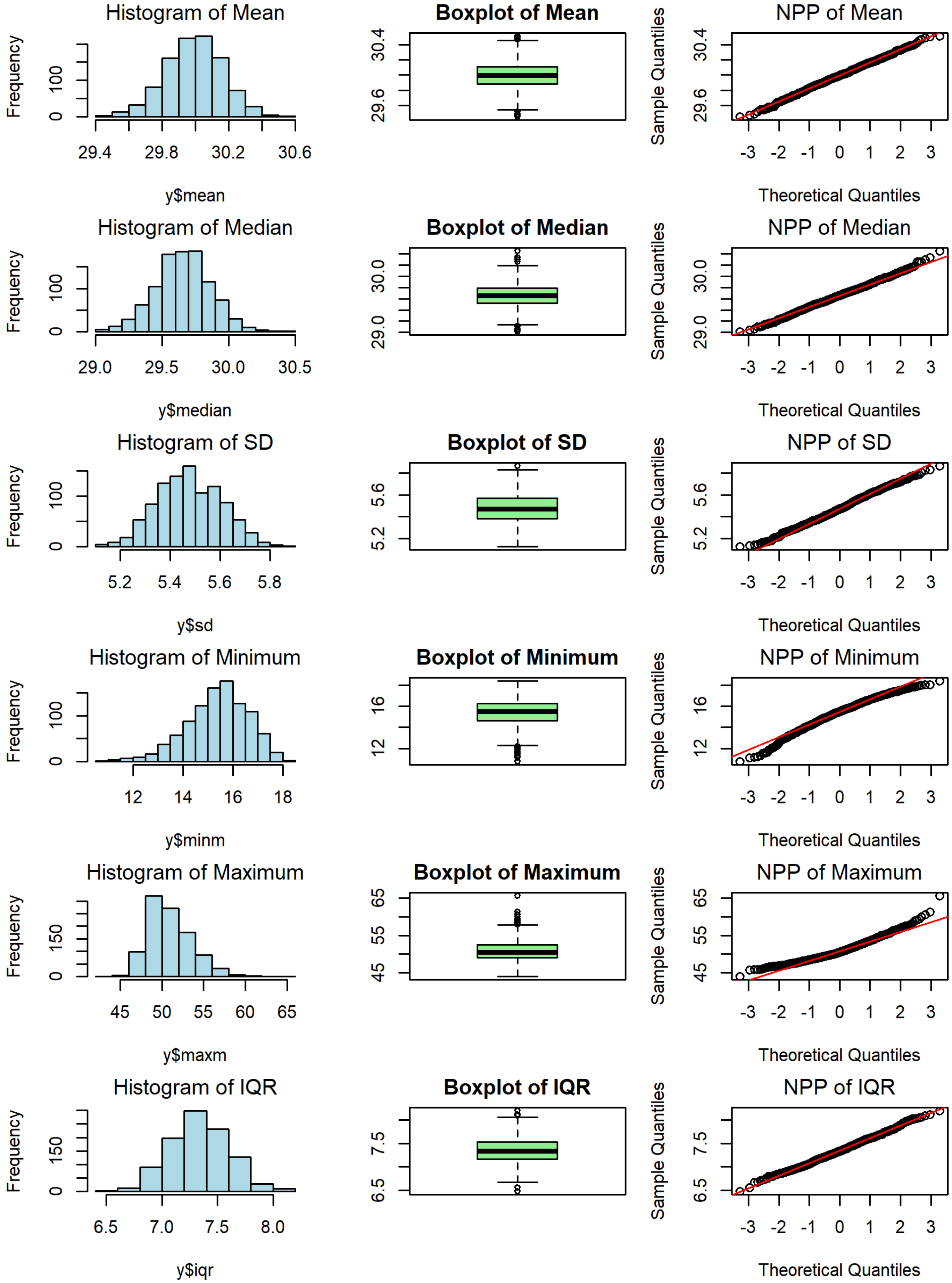
GAMMA DISTRIBUTION PLOT
(n=100, nn=1000, α=30, λ=1)

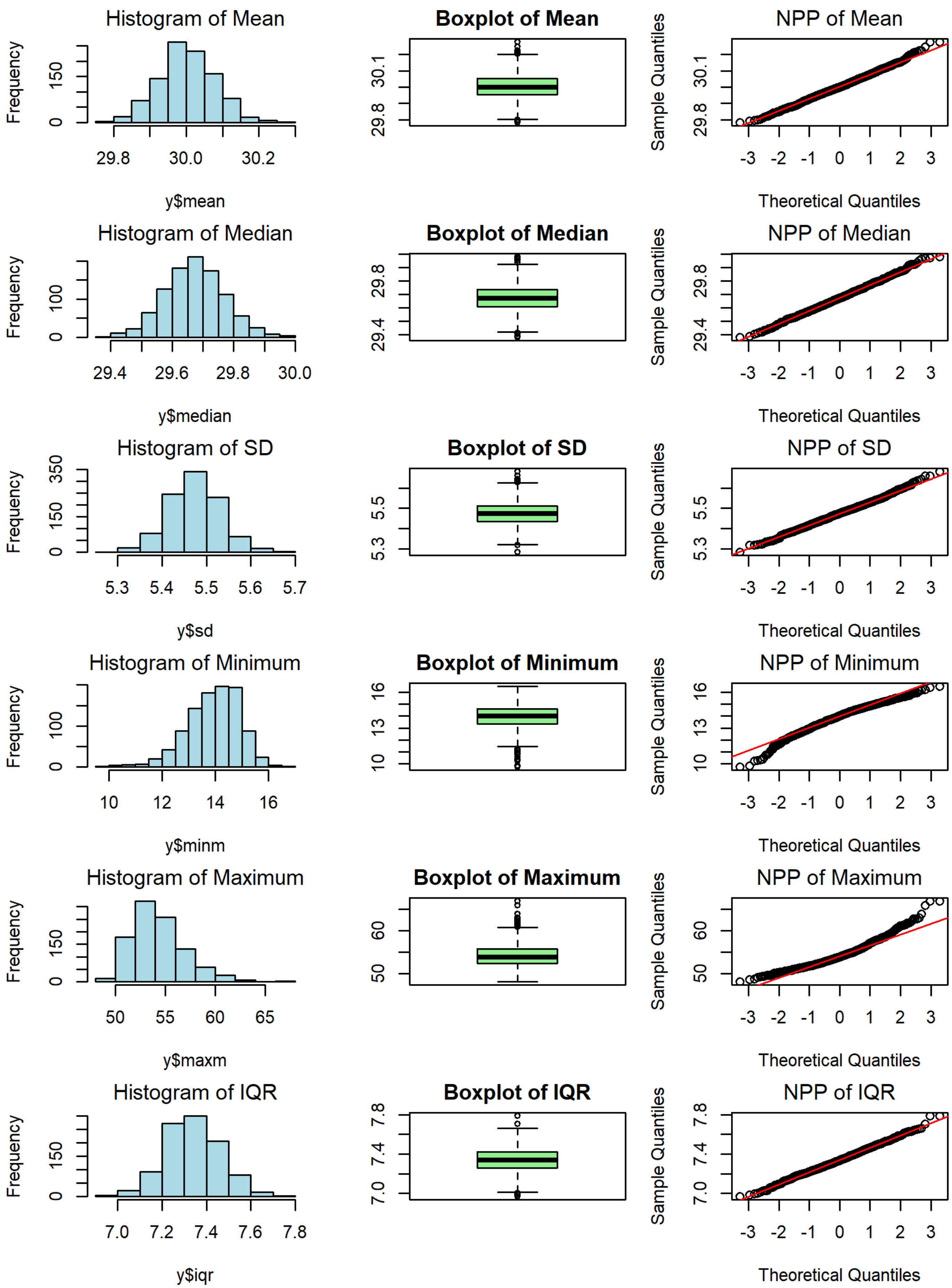


GAMMA DISTRIBUTION PLOT
(n=500, nn=1000, α=30, λ=1)

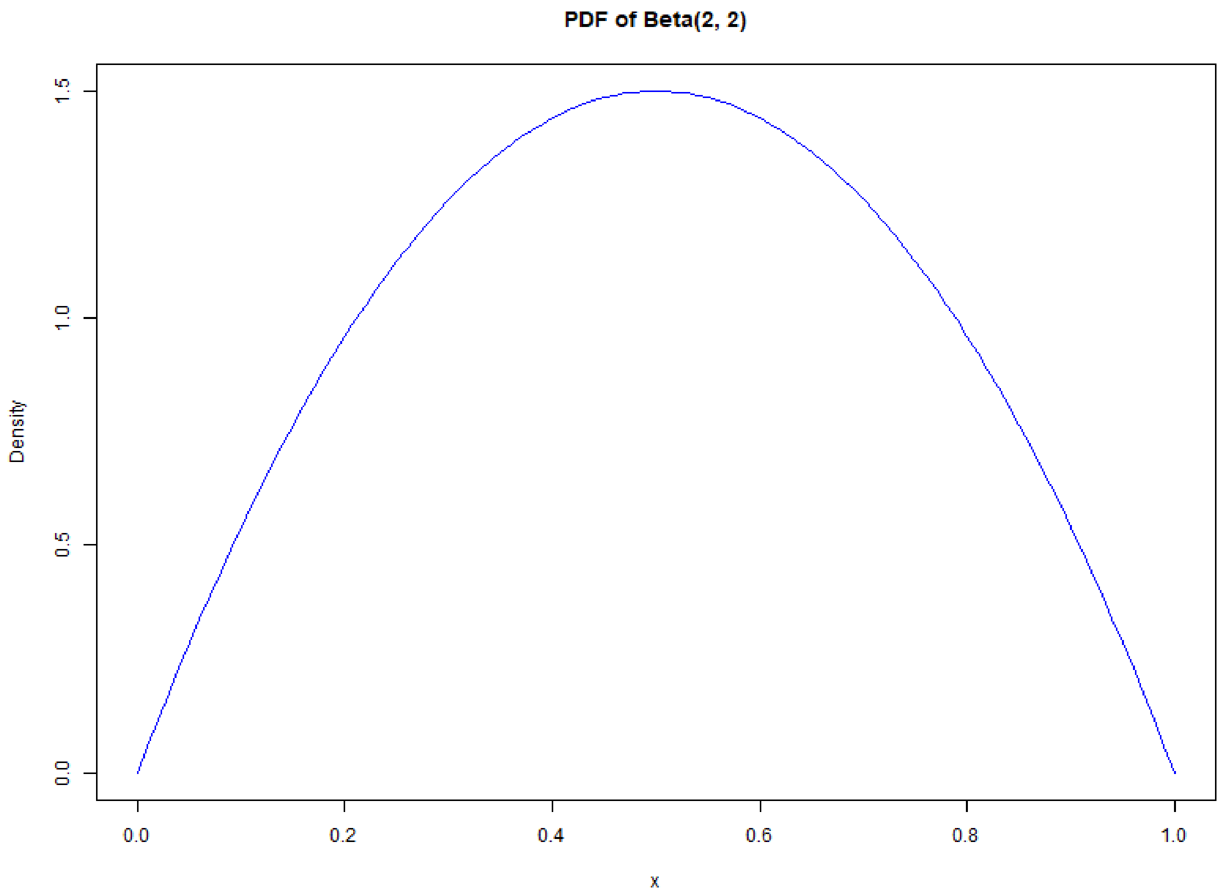


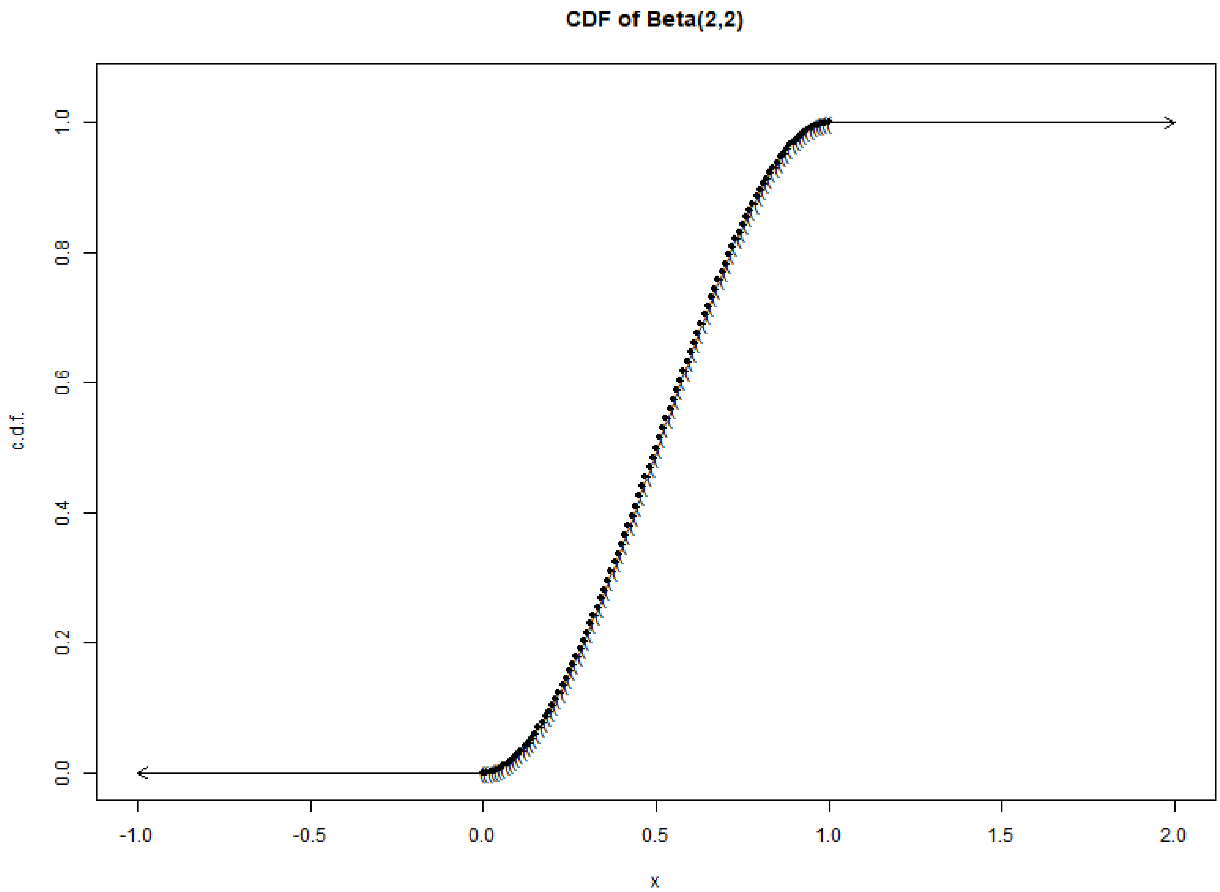
GAMMA DISTRIBUTION PLOT
(n=1000, nn=1000, α=30, λ=1)




Histogram of Mean
Boxplot of Mean
NPP of Mean
Histogram of Median
Boxplot of Median
NPP of Median
Histogram of SD
Boxplot of SD
NPP of SD
Histogram of Minimum
Boxplot of Minimum
NPP of Minimum
Histogram of Maximum
Boxplot of Maximum
NPP of Maximum
Histogram of IQR
Boxplot of IQR
NPP of IQR
Frequency
Sample Quantiles
Theoretical Quantiles
y$mean
y$median
y$sd
y$minm
y$maxm
y$iqr

BETA DISTRIBUTION (2,2)





BETA DISTRIBUTION

	Values of n to achieve normality (nn=1000, α=2, β=2)						
Statistic	n=10	n=50	n=100	n=500	n=1000	n=5000	Any n	Min(n)
Mean	Yes	Yes	Yes	Yes	Yes	Yes	Yes	10
Median	No	Yes	Yes	Yes	Yes	Yes	Yes	50
Std Dev	Yes	Yes	Yes	Yes	Yes	Yes	Yes	10
Min	No	No	No	No	No	No	No	NA
Max	No	No	No	No	No	No	No	NA
IQR	No	Yes	Yes	Yes	Yes	Yes	Yes	50

Conclusion for Beta Distribution (α = 2, β = 2)

Normality Achieved:

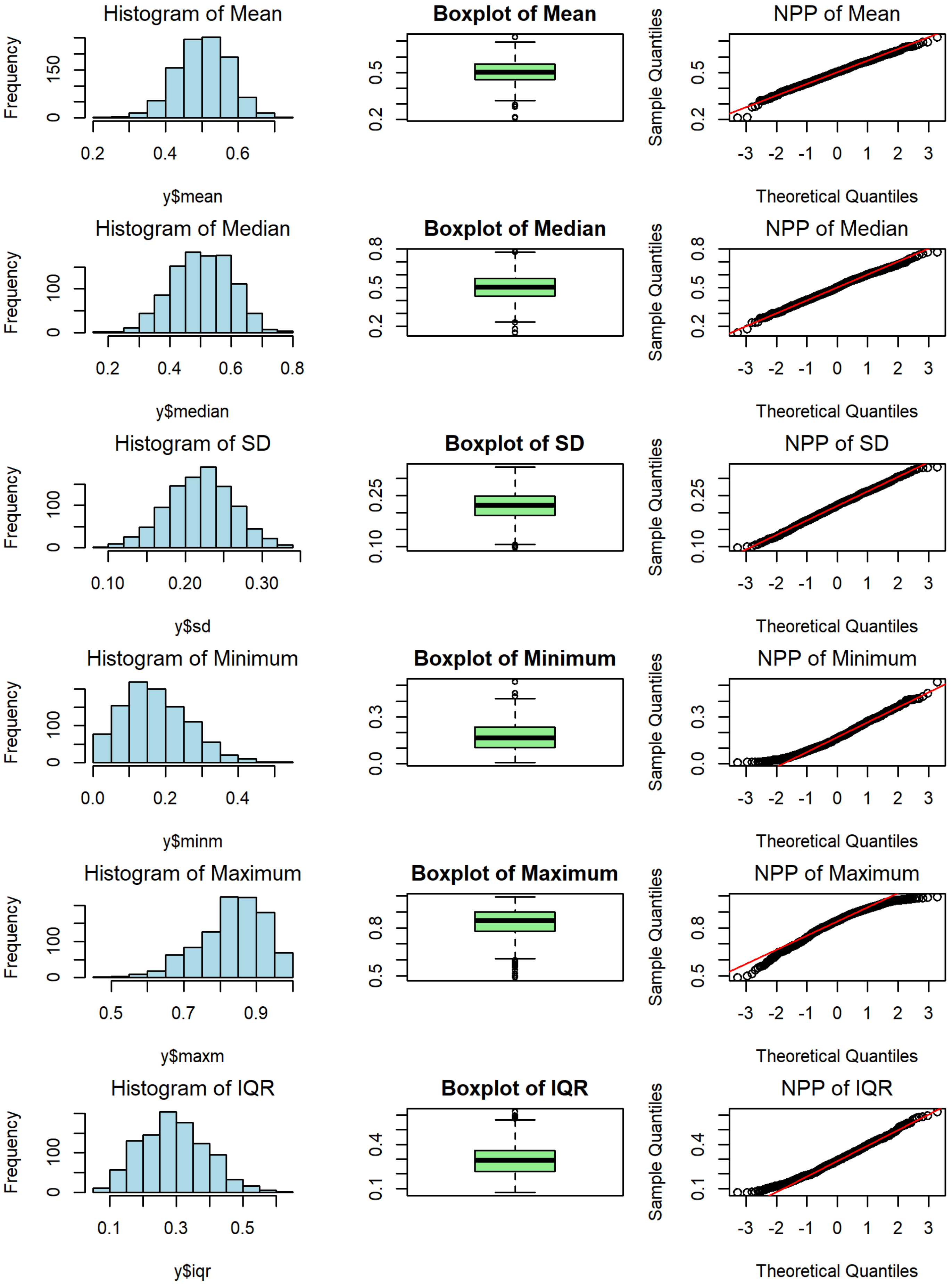
- **Mean:** Achieves normality across all sample sizes (n ≥ 10), with normality reached relatively quickly due to the balanced parameters (α = 2, β = 2) of the Beta distribution.
- **Standard Deviation (SD):** Achieves normality for all sample sizes (n ≥ 10), with convergence starting from very small sample sizes.

Normality Not Achieved:

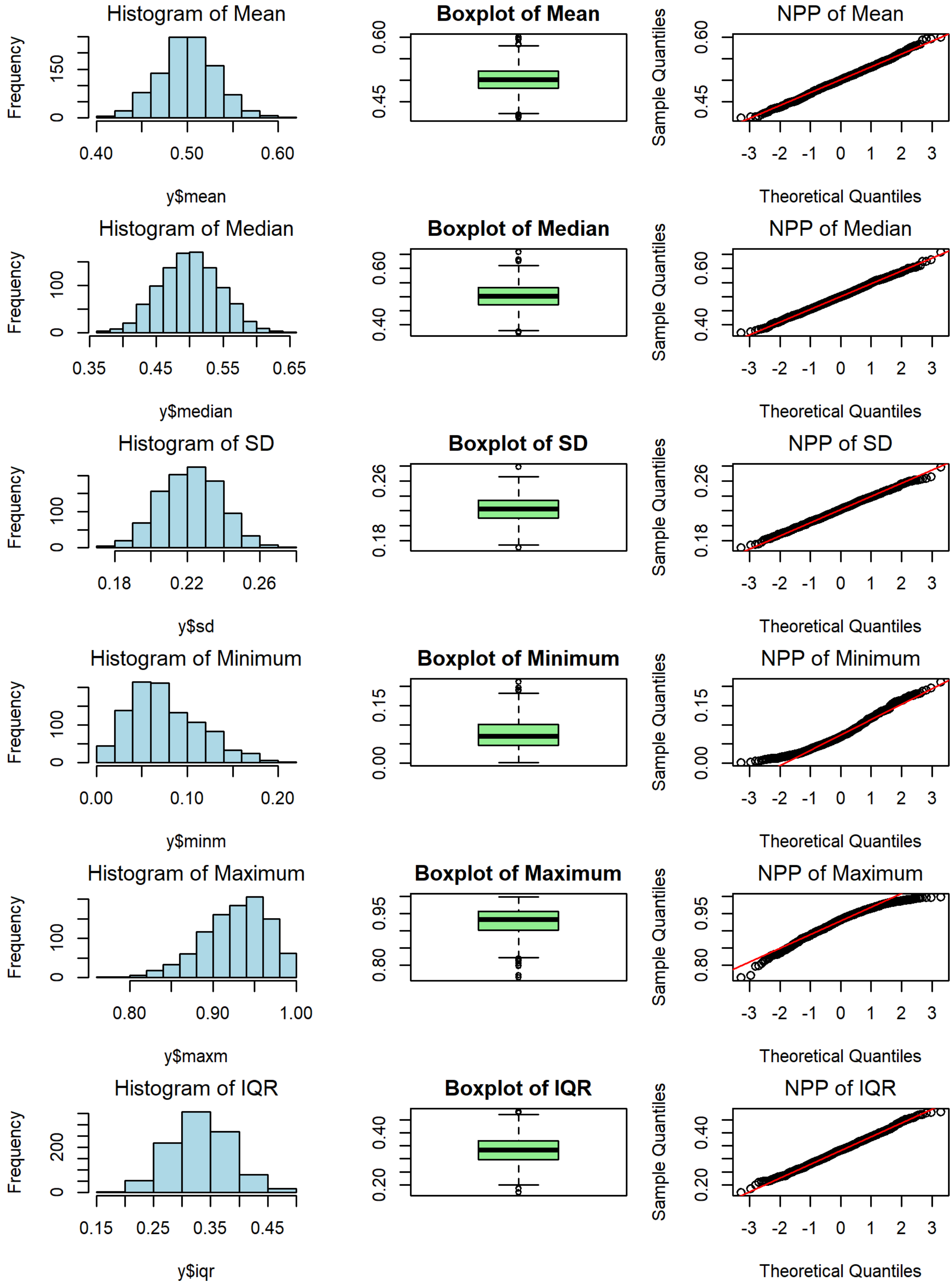
- **Median:** Achieves normality for n ≥ 50, with smaller sample sizes (n ≤ 10) showing a more skewed distribution. Larger sample sizes help reduce the skewness.
- **IQR:** Achieves normality for n ≥ 50, reflecting a similar trend to the median. The smaller the sample size, the more likely it is to deviate from normality.
- **Minimum and Maximum:** Do not achieve normality for any sample size due to their sensitivity to the extreme values in the Beta distribution.

Overall: The Beta distribution with α = 2 and β = 2 shows the fastest convergence to normality for the mean, standard deviation, and IQR, even at smaller sample sizes (n ≥ 10). The median and IQR require larger sample sizes (n ≥ 50) to show normality. The minimum and maximum values do not show normality due to their inherent sensitivity to extreme values.

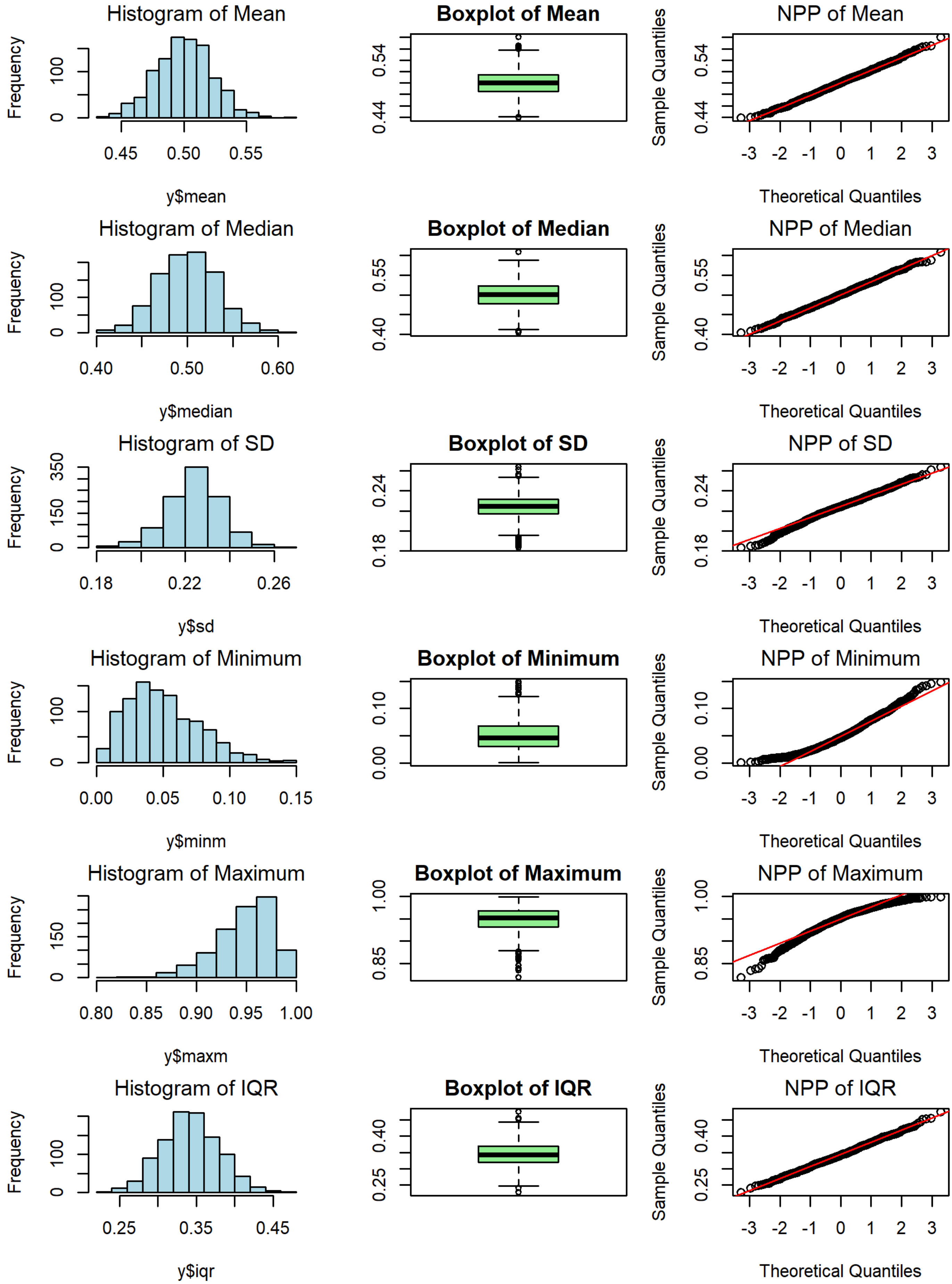
BETA DISTRIBUTION PLOT
(n=10, nn=1000, α=2, β=2)



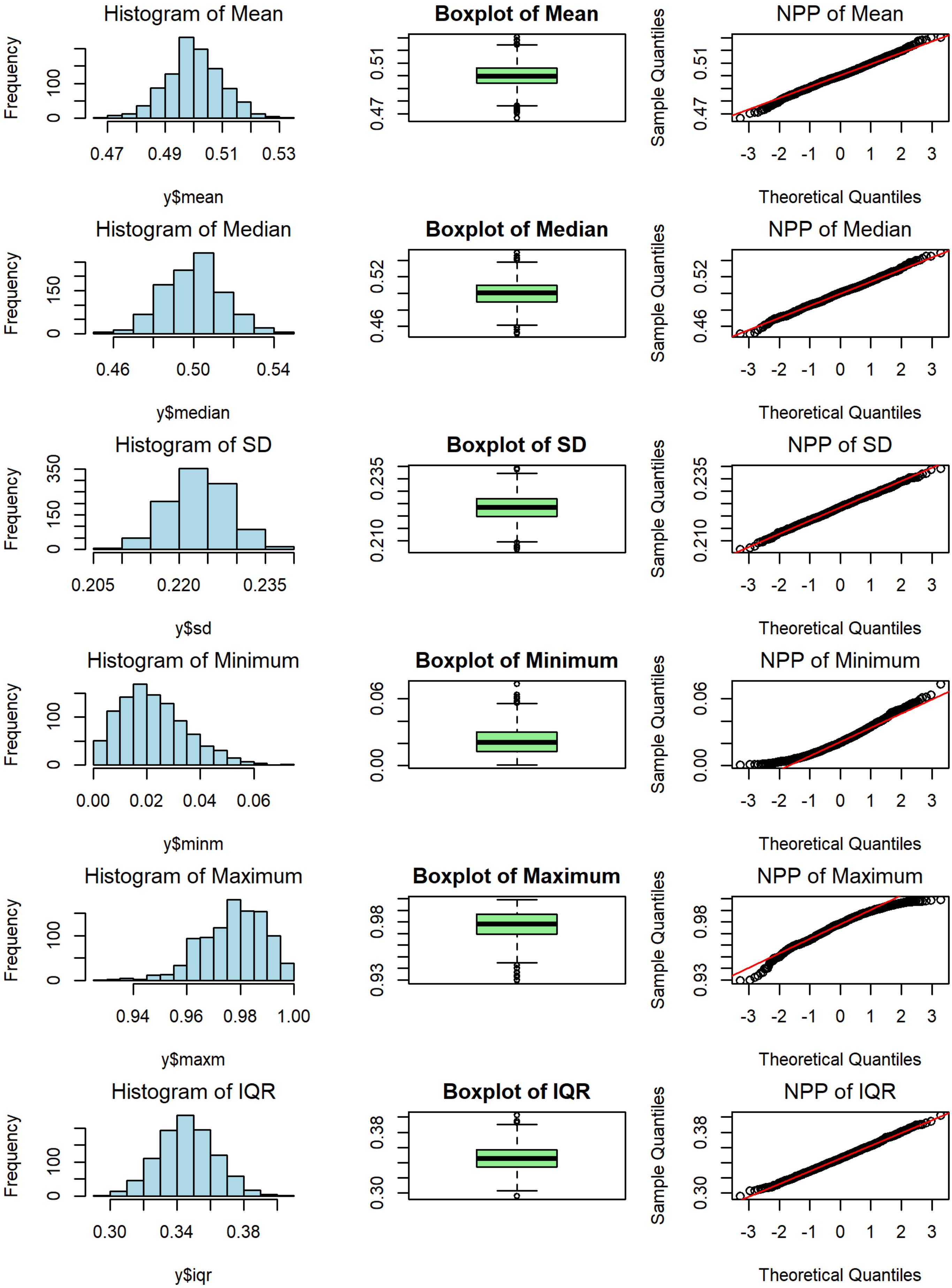
BETA DISTRIBUTION PLOT
(n=50, nn=1000, α=2, β=2)



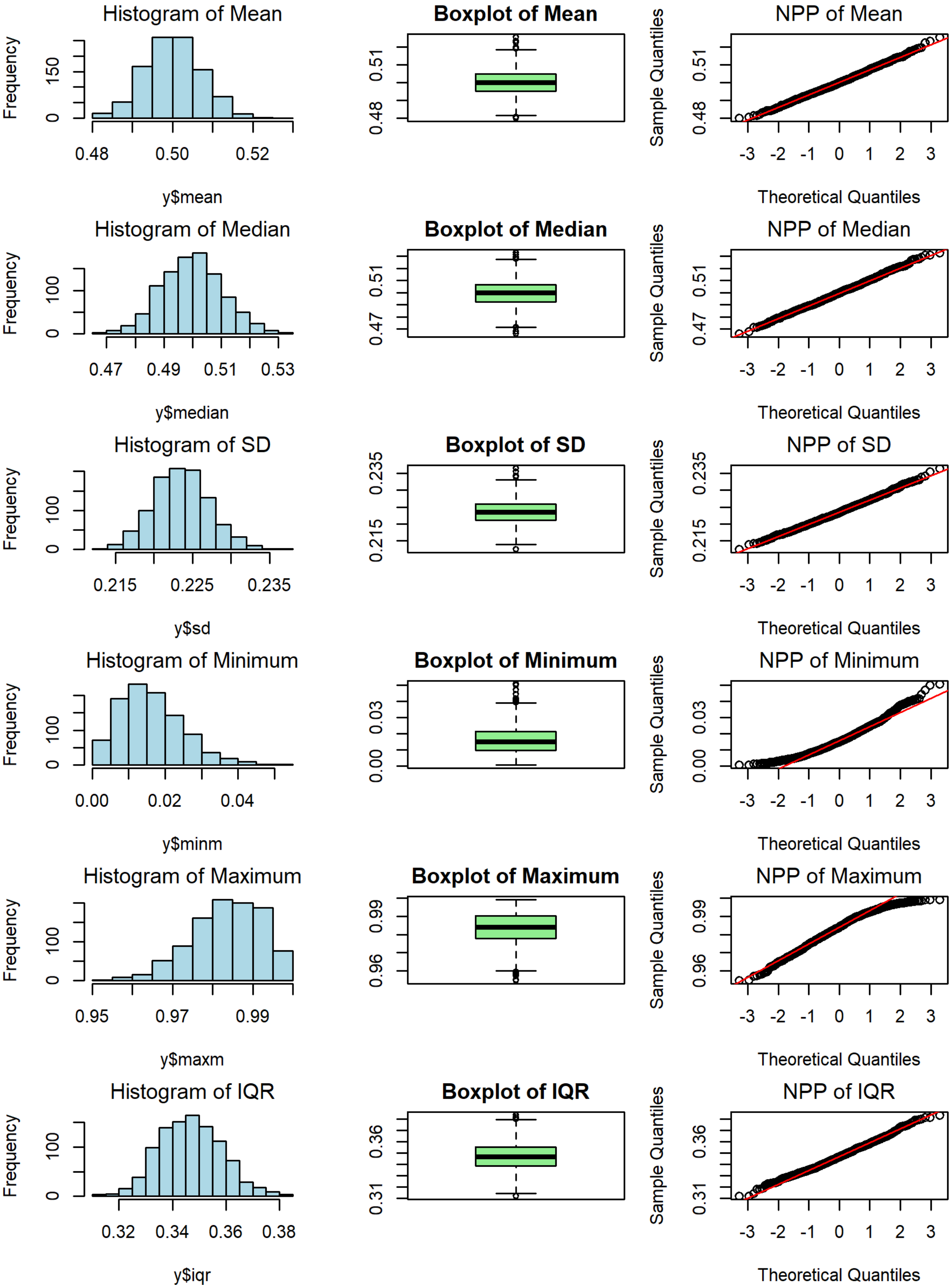
BETA DISTRIBUTION PLOT
(n=100, nn=1000, α=2, β=2)

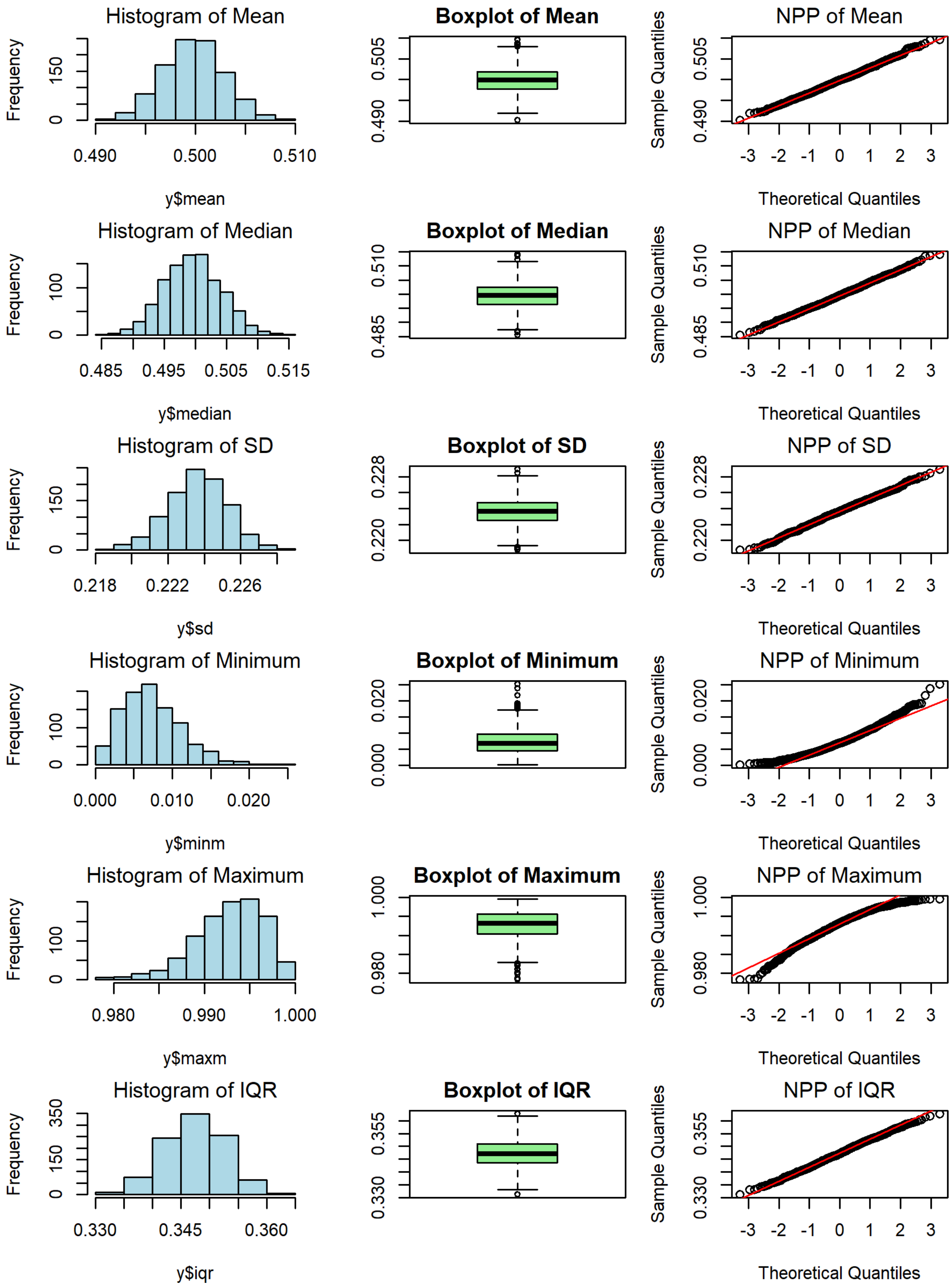


BETA DISTRIBUTION PLOT
(n=500, nn=1000, α=2, β=2)

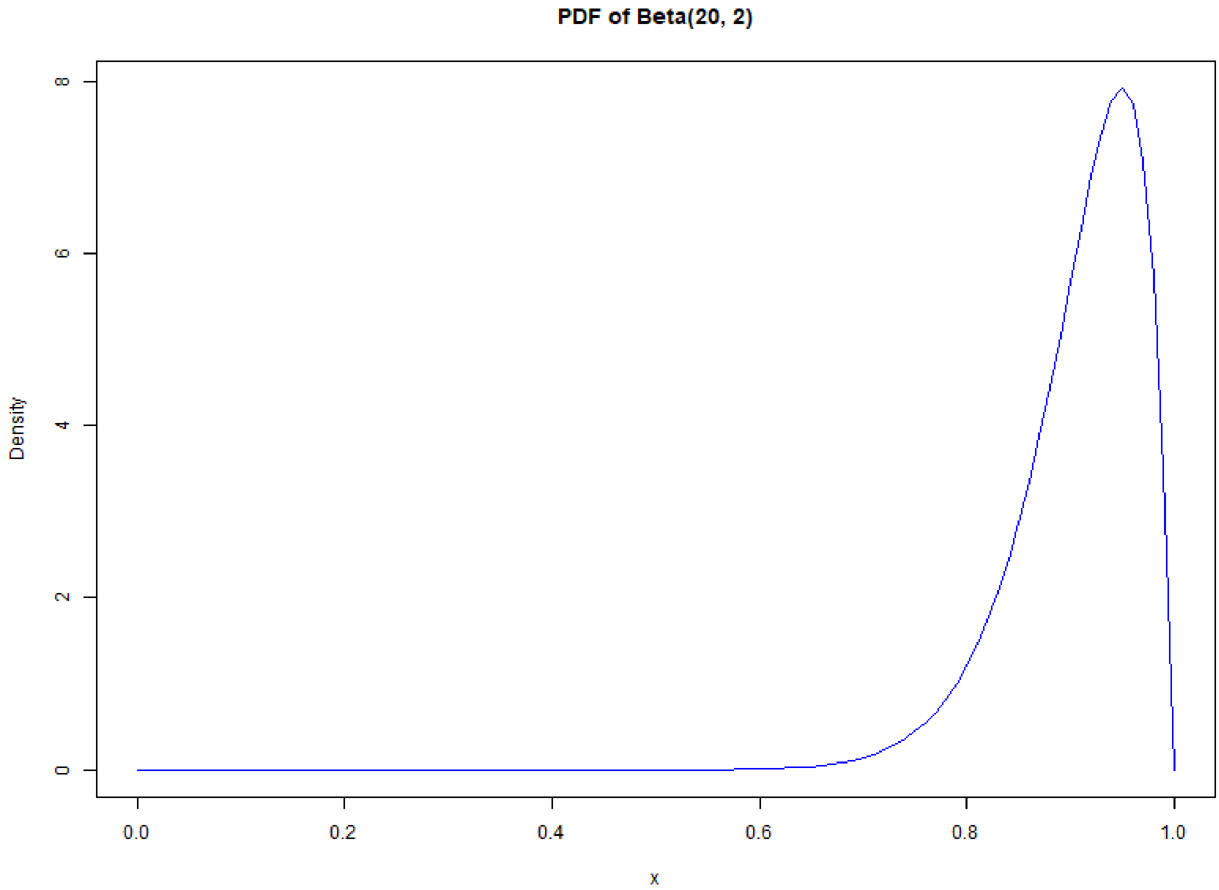


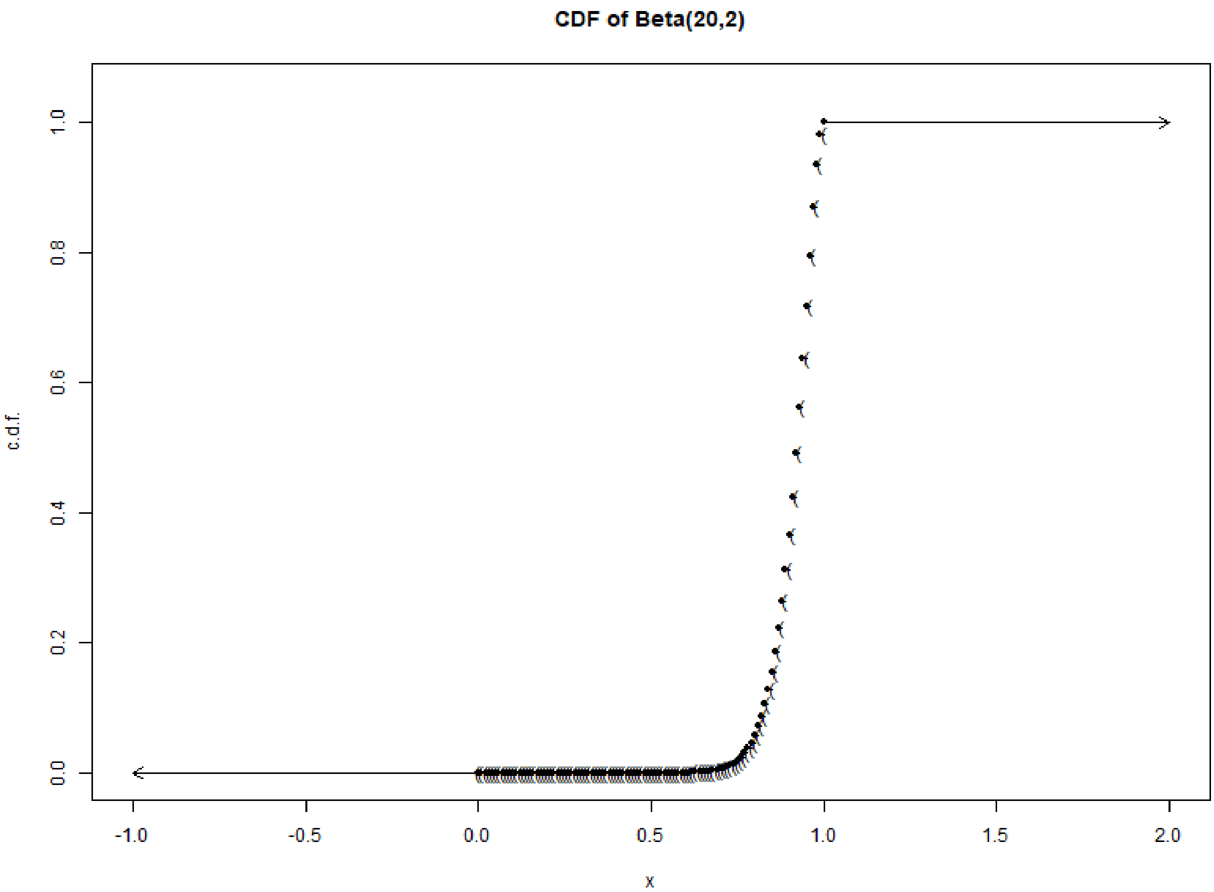
BETA DISTRIBUTION PLOT
(n=1000, nn=1000, α=2, β=2)




Histogram of Mean
Frequency
0
150
0.490
0.500
0.510
y$mean
Boxplot of Mean
0.490
0.505
NPP of Mean
Sample Quantiles
0.490
0.505
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of Median
Frequency
0
100
0.485
0.495
0.505
0.515
y$median
Boxplot of Median
0.485
0.510
NPP of Median
Sample Quantiles
0.485
0.510
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of SD
Frequency
0
150
0.218
0.222
0.226
y$sd
Boxplot of SD
0.220
0.228
NPP of SD
Sample Quantiles
0.220
0.228
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of Minimum
Frequency
0
100
0.000
0.010
0.020
y$minm
Boxplot of Minimum
0.000
0.020
NPP of Minimum
Sample Quantiles
0.000
0.020
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of Maximum
Frequency
0
100
0.980
0.990
1.000
y$maxm
Boxplot of Maximum
0.980
1.000
NPP of Maximum
Sample Quantiles
0.980
1.000
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of IQR
Frequency
0
150
350
0.330
0.345
0.360
y$iqr
Boxplot of IQR
0.330
0.355
NPP of IQR
Sample Quantiles
0.330
0.355
-3 -2 -1 0 1 2 3
Theoretical Quantiles

BETA DISTRIBUTION (20,2)





BETA DISTRIBUTION

	Values of n to achieve normality (nn=1000, α=20, β=2)						
Statistic	n=10	n=50	n=100	n=500	n=1000	n=5000	Any n	Min(n)
Mean	No	Yes	Yes	Yes	Yes	Yes	Yes	50
Median	No	No	Yes	Yes	Yes	Yes	Yes	100
Std Dev	No	No	Yes	Yes	Yes	Yes	Yes	100
Min	No	No	No	No	No	No	No	NA
Max	No	No	No	No	No	No	No	NA
IQR	No	Yes	Yes	Yes	Yes	Yes	Yes	50

Conclusion for Beta Distribution (α = 20, β = 2)

Normality Achieved:

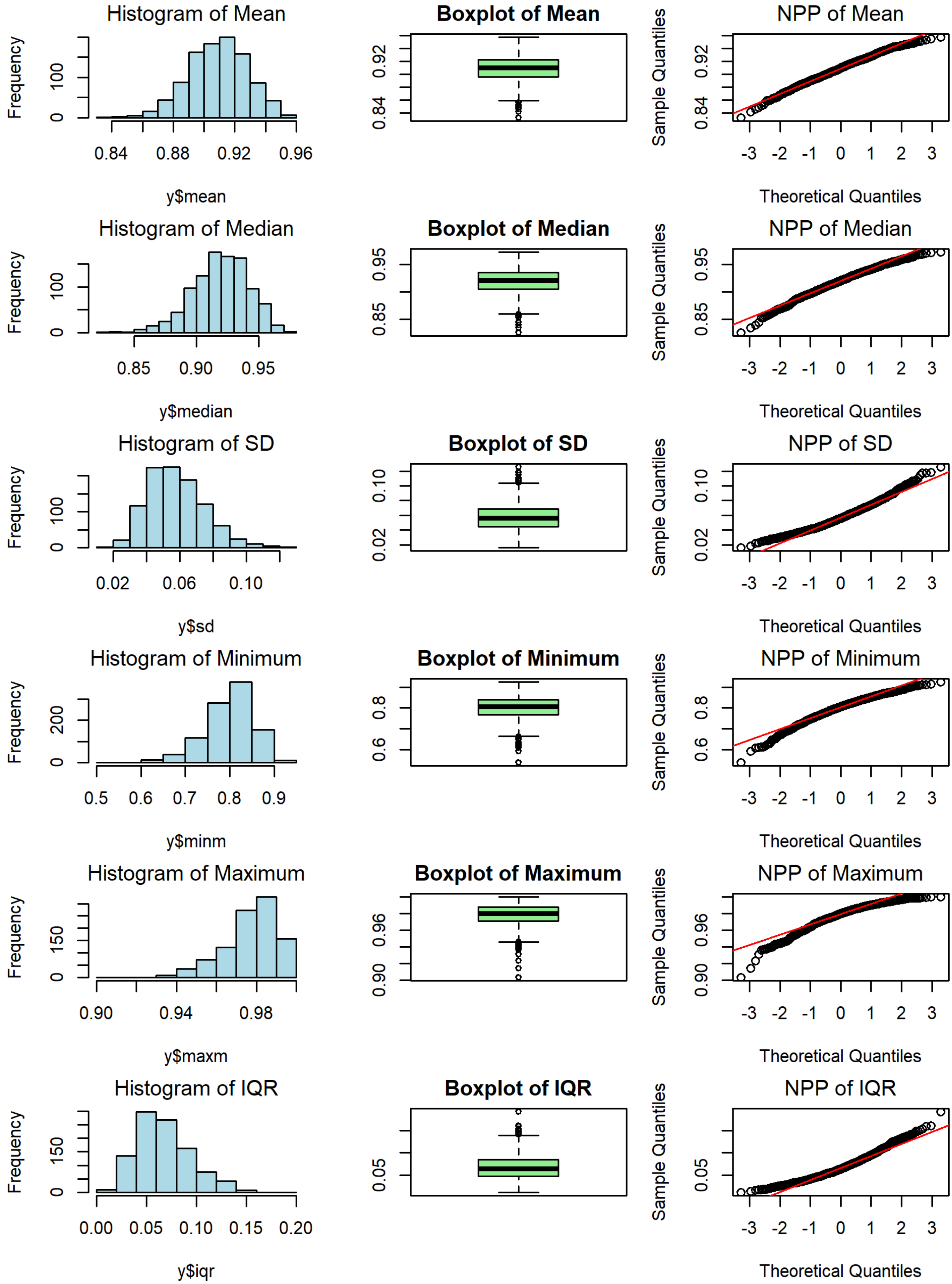
- **Mean:** Achieves normality for n ≥ 50, with rapid convergence as sample size increases.
- **Median:** Achieves normality for n ≥ 100, with smaller sample sizes (n ≤ 50) exhibiting more variability.
- **Standard Deviation (SD):** Achieves normality for n ≥ 100, showing increasing stability as sample size grows.
- **IQR:** Achieves normality for n ≥ 50, with larger sample sizes reflecting a smoother distribution.

Normality Not Achieved:

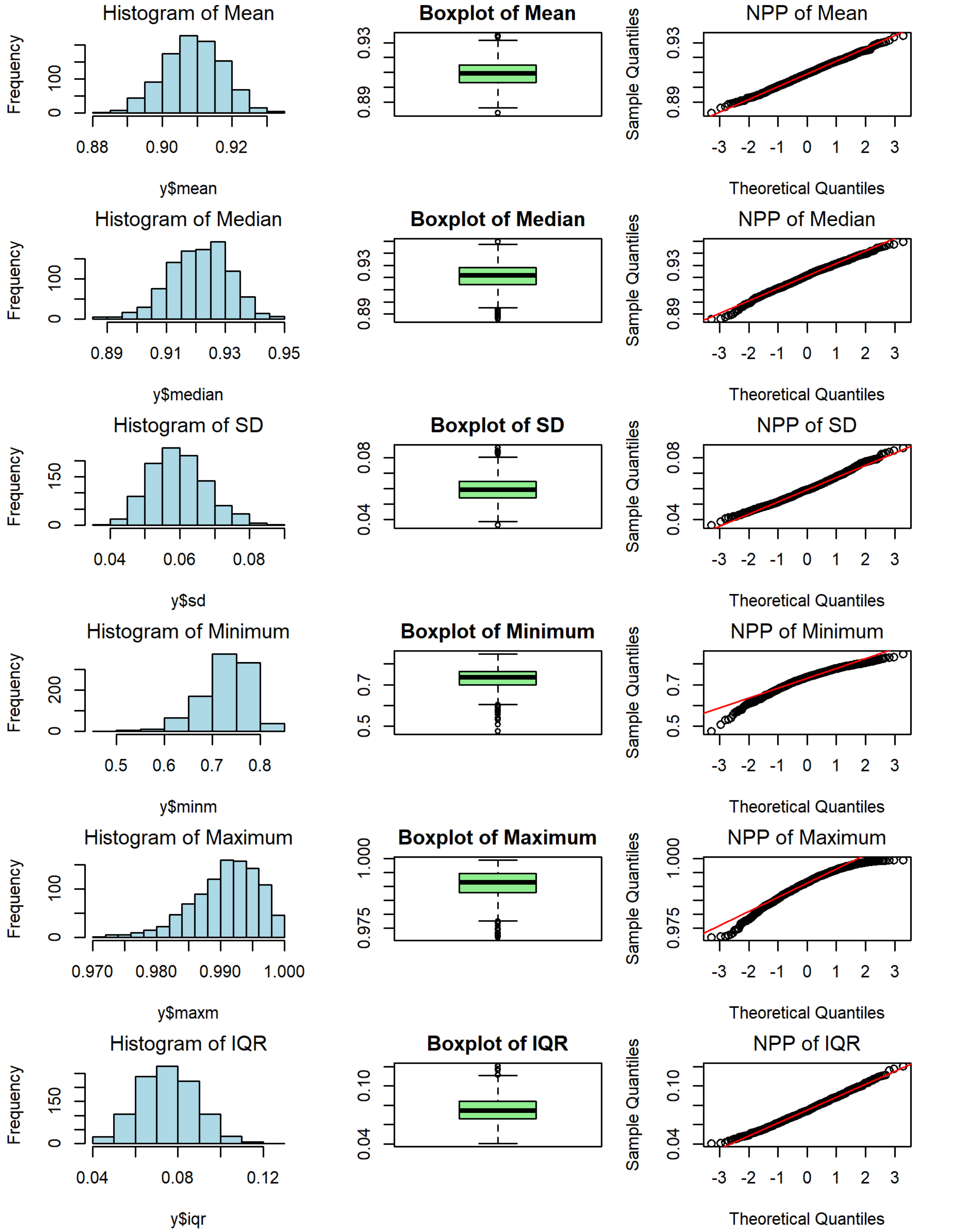
- **Minimum and Maximum:** Do not achieve normality for any sample size due to the distribution's skewness, especially at smaller sample sizes.

Overall: The Beta distribution with α = 20 and β = 2 demonstrates the fastest convergence to normality for the mean, median, standard deviation, and IQR as the sample size increases, with n ≥ 50 being sufficient for the mean and n ≥ 100 for other statistics. The minimum and maximum values remain non-normal across all sample sizes due to the skewed nature of the distribution.

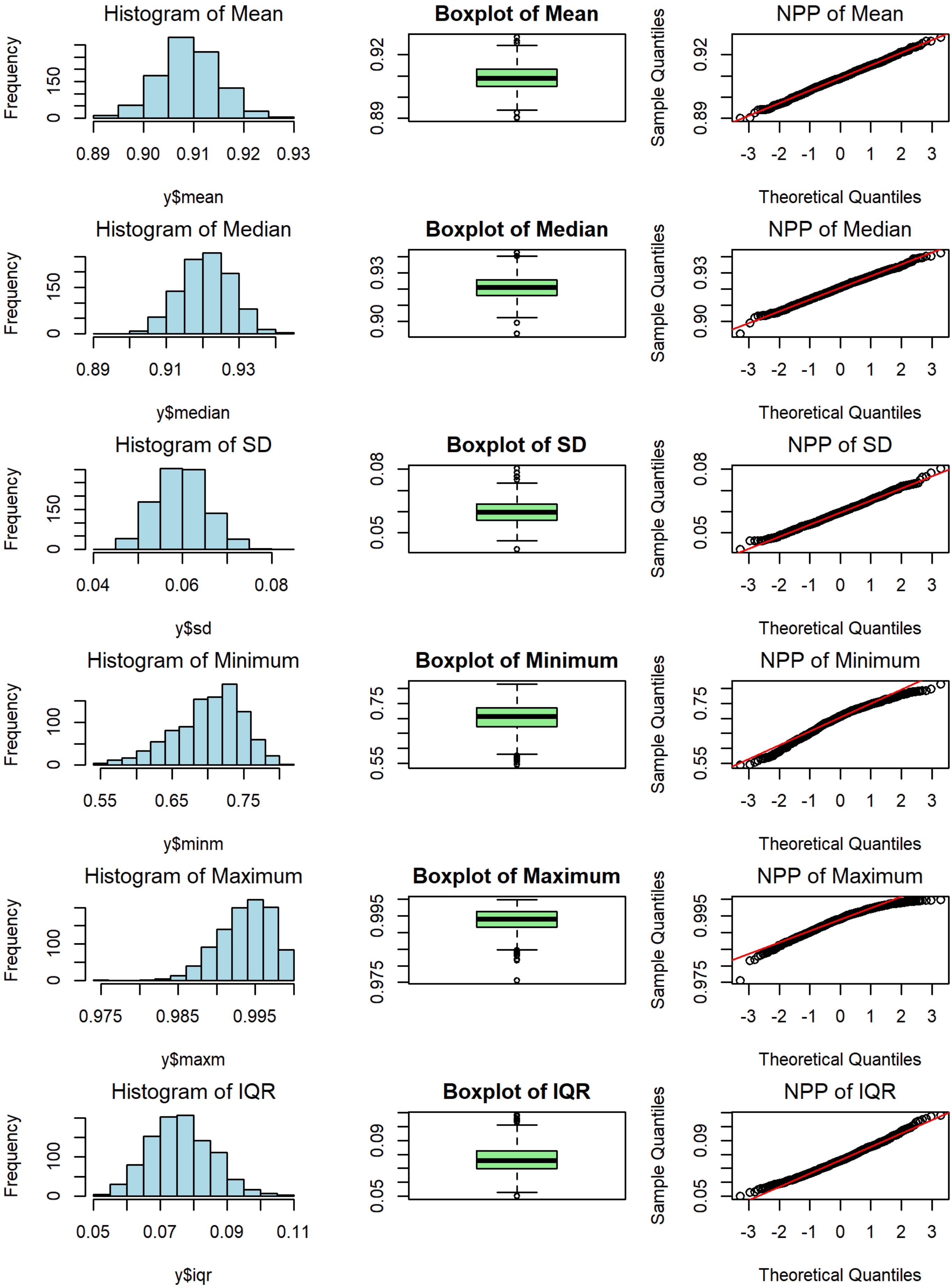
BETA DISTRIBUTION PLOT
$(n=10, nn=1000, \alpha=20, \beta=2)$



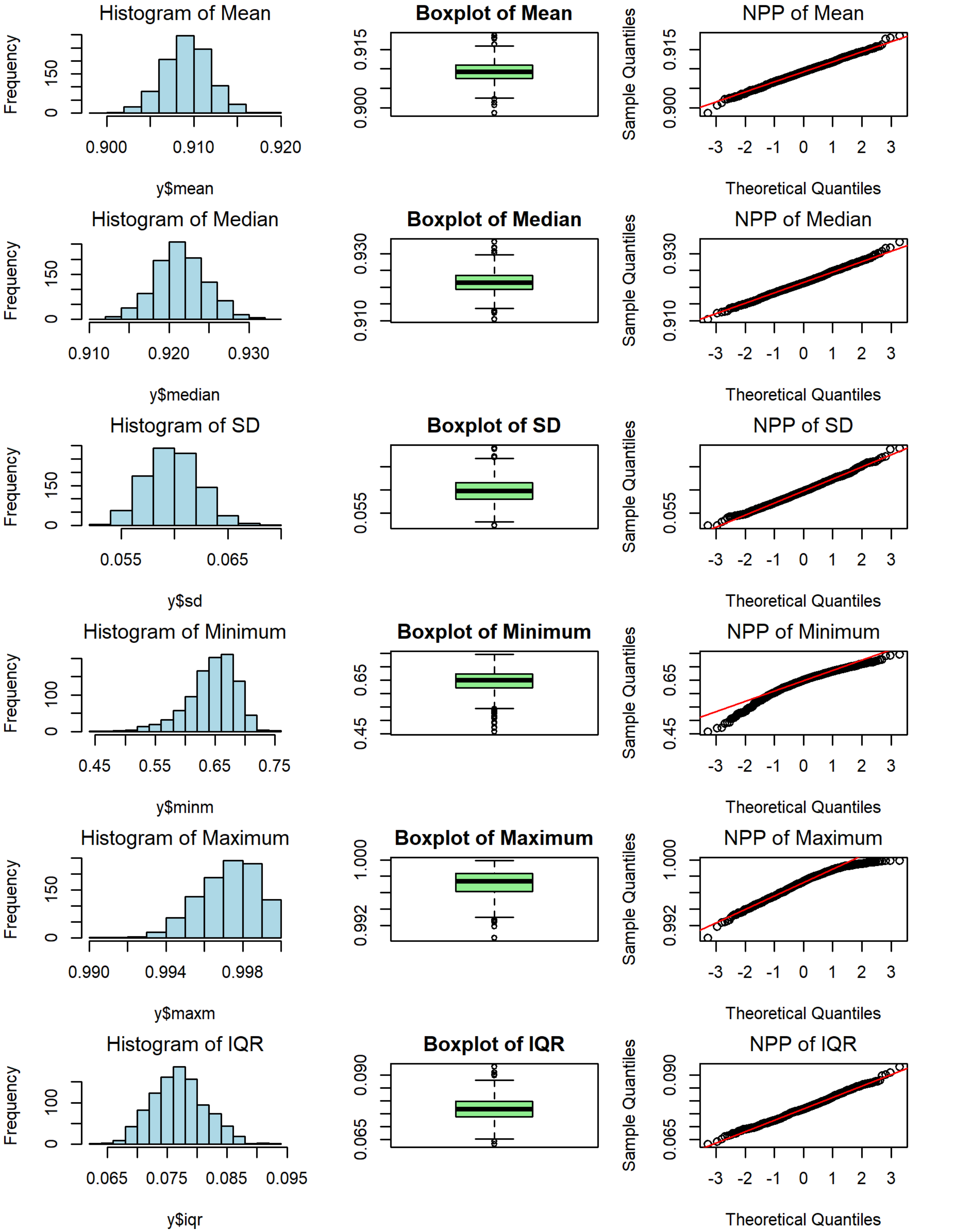
BETA DISTRIBUTION PLOT
(n=50, nn=1000, α=20, β=2)



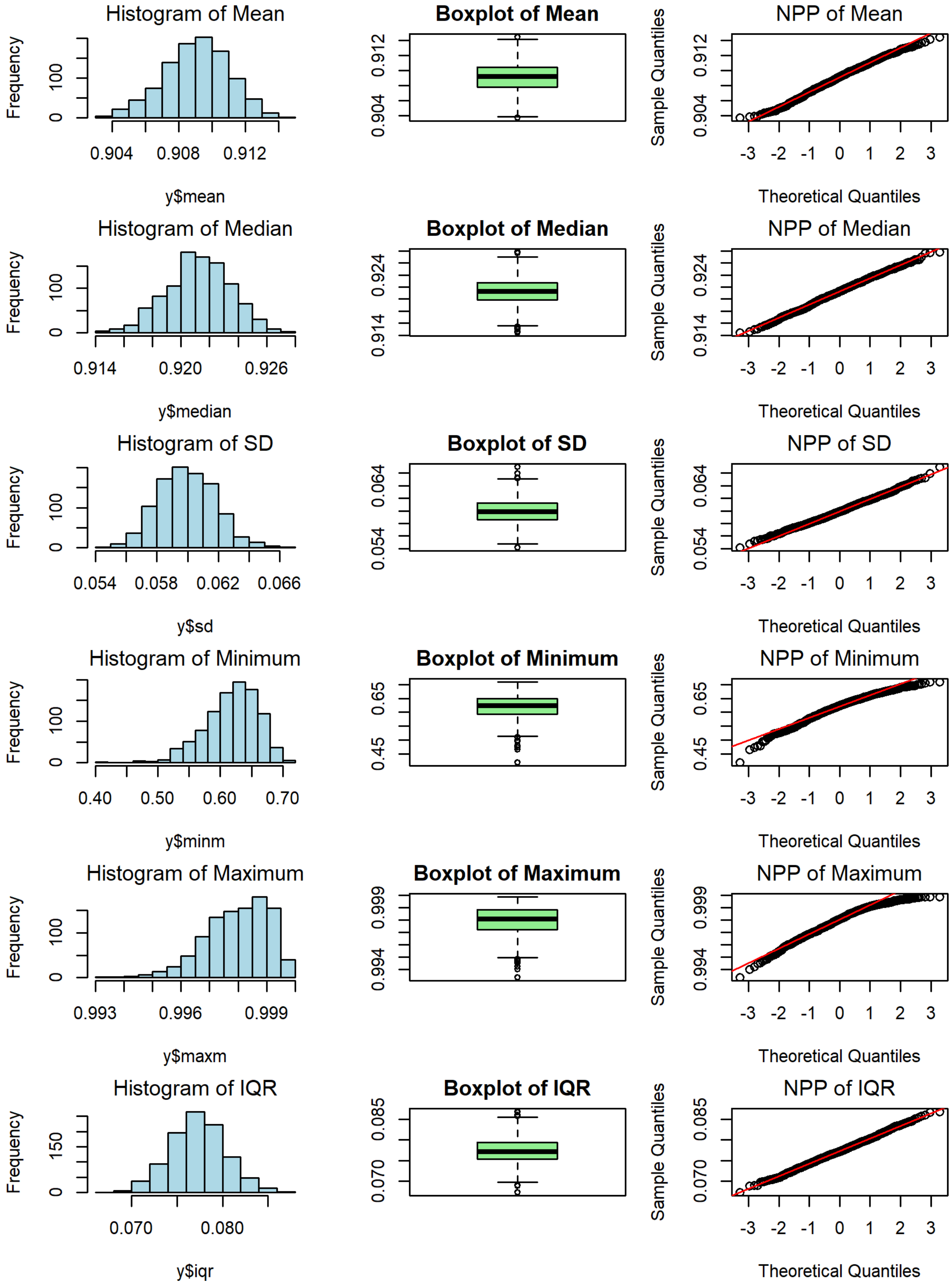
BETA DISTRIBUTION PLOT
(n=100, nn=1000, α=20, β=2)

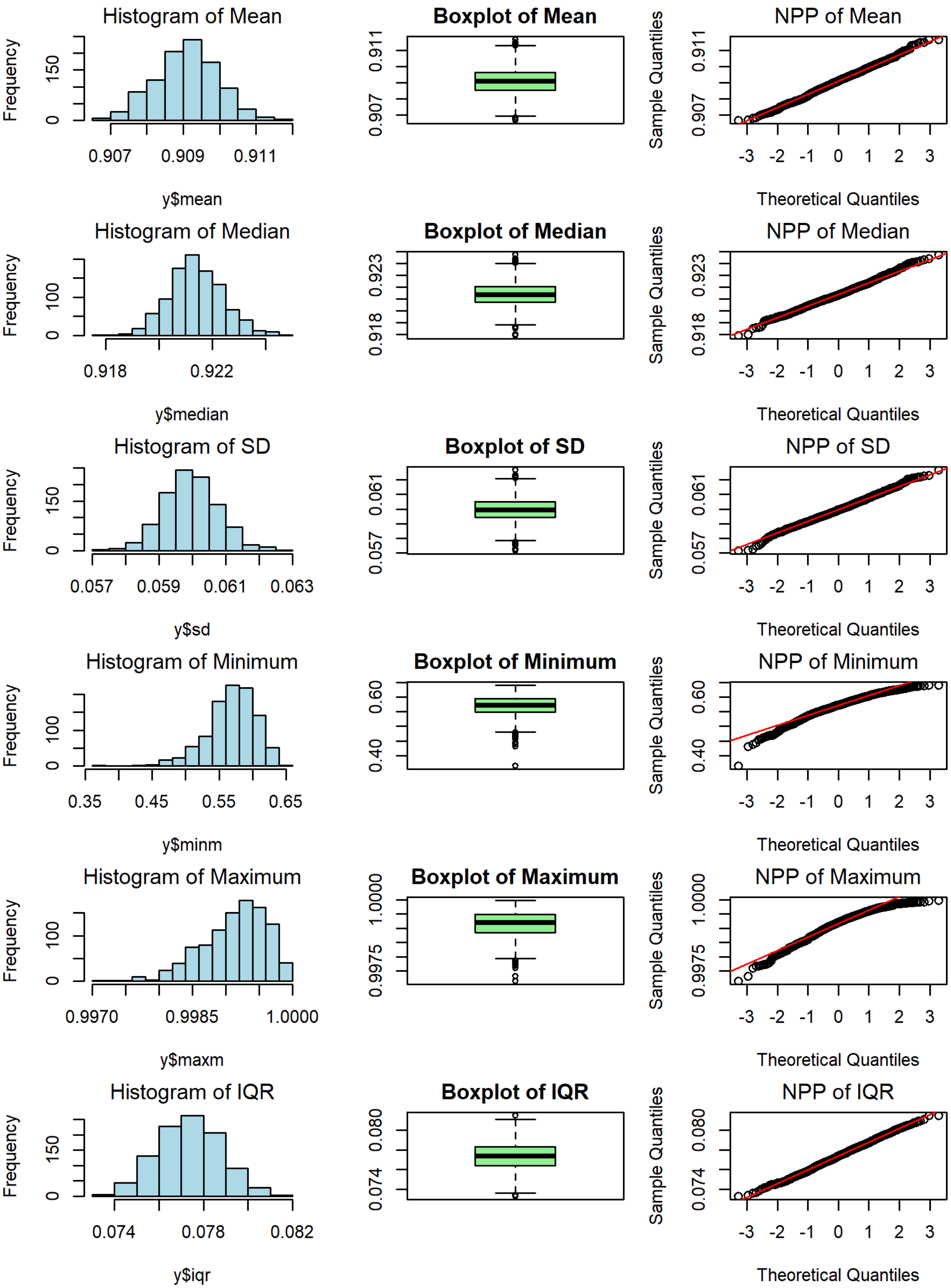


BETA DISTRIBUTION PLOT
(n=500, nn=1000, α=20, β=2)

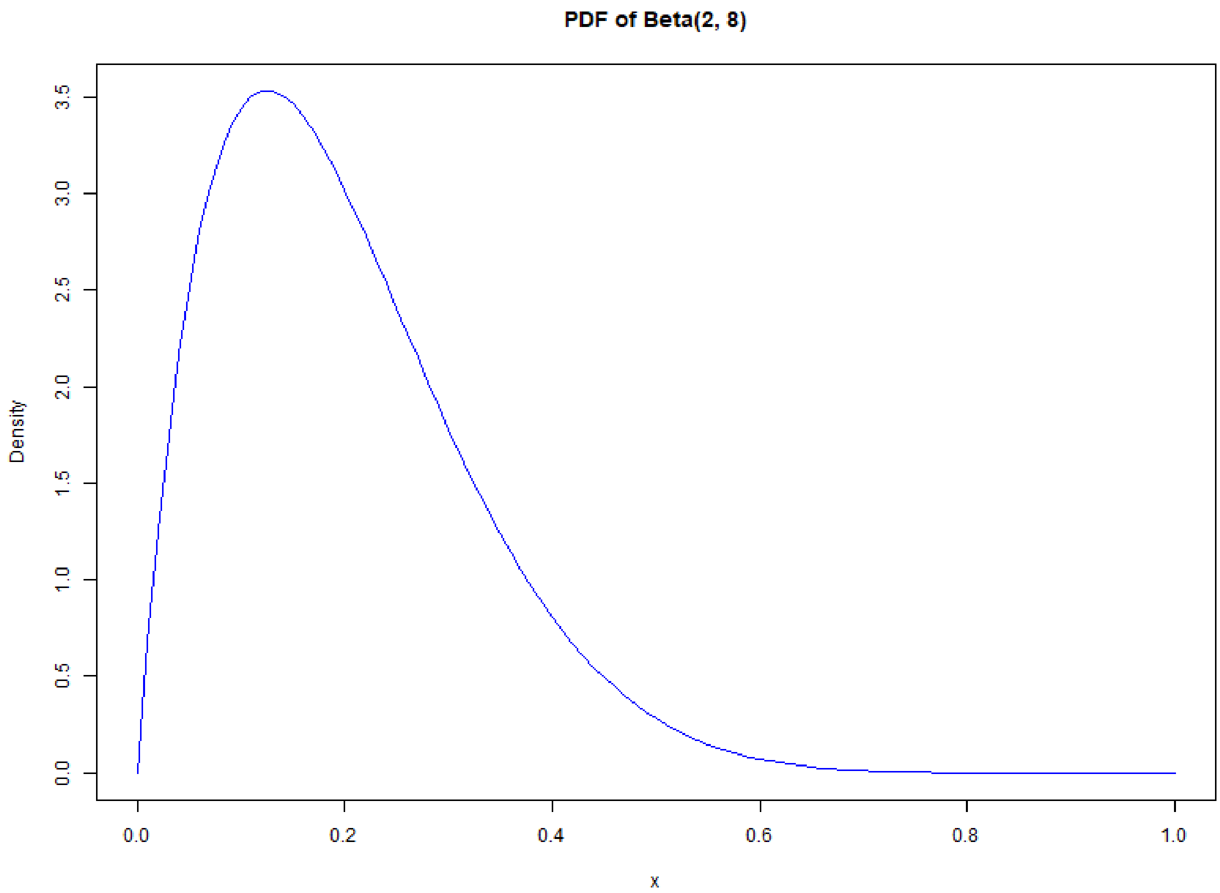


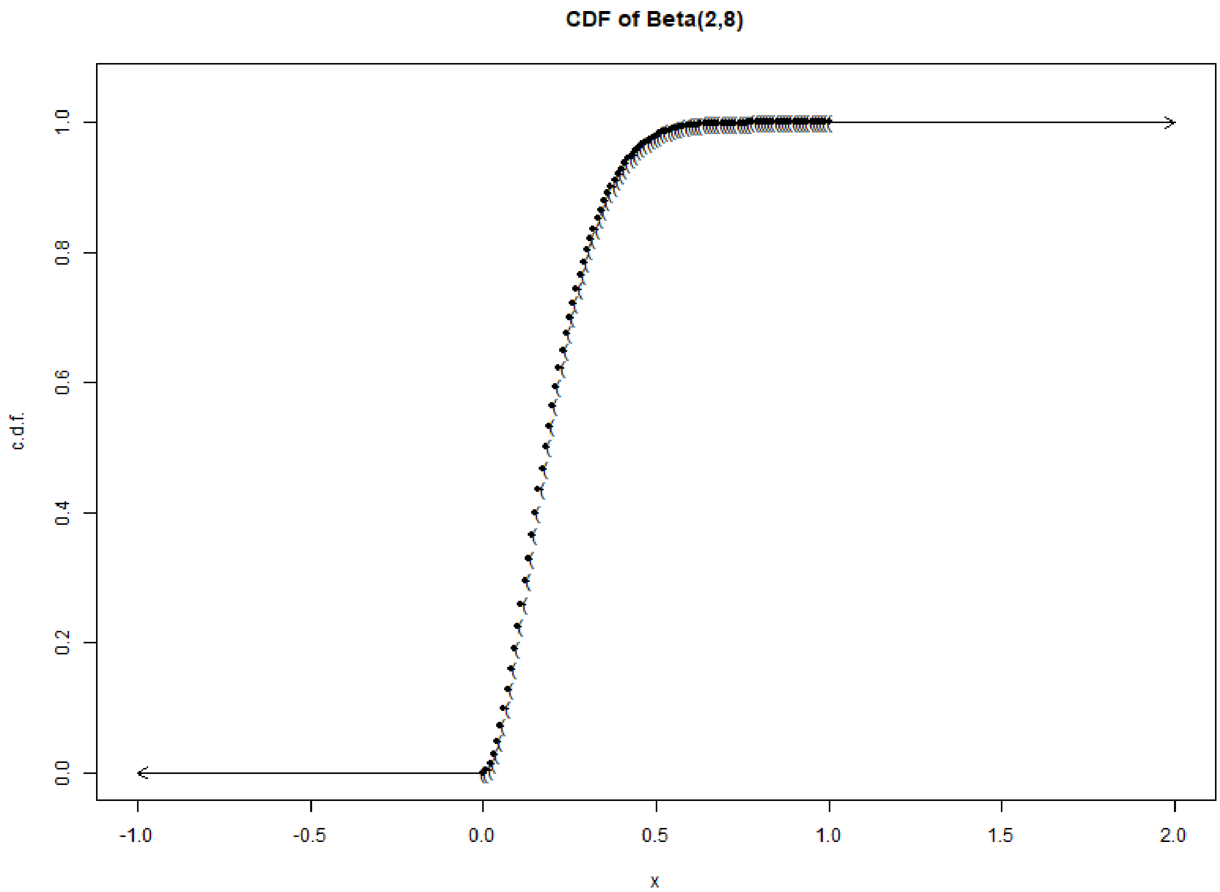
BETA DISTRIBUTION PLOT
(n=1000, nn=1000, α=20, β=2)




Histogram of Mean
Frequency
y$mean
Boxplot of Mean
NPP of Mean
Sample Quantiles
Theoretical Quantiles
Histogram of Median
y$median
Boxplot of Median
NPP of Median
Histogram of SD
y$sd
Boxplot of SD
NPP of SD
Histogram of Minimum
y$minm
Boxplot of Minimum
NPP of Minimum
Histogram of Maximum
y$maxm
Boxplot of Maximum
NPP of Maximum
Histogram of IQR
y$iqr
Boxplot of IQR
NPP of IQR

BETA DISTRIBUTION (2,8)





BETA DISTRIBUTION

	Values of n to achieve normality (nn=1000, α=2, β=8)						
Statistic	n=10	n=50	n=100	n=500	n=1000	n=5000	Any n	Min(n)
Mean	Yes	Yes	Yes	Yes	Yes	Yes	Yes	10
Median	No	Yes	Yes	Yes	Yes	Yes	Yes	50
Std Dev	No	No	Yes	Yes	Yes	Yes	Yes	100
Min	No	No	No	No	No	No	No	NA
Max	No	No	No	No	No	No	No	NA
IQR	No	No	Yes	Yes	Yes	Yes	Yes	100

Conclusion for Beta Distribution (α = 2, β = 8)

Normality Achieved:

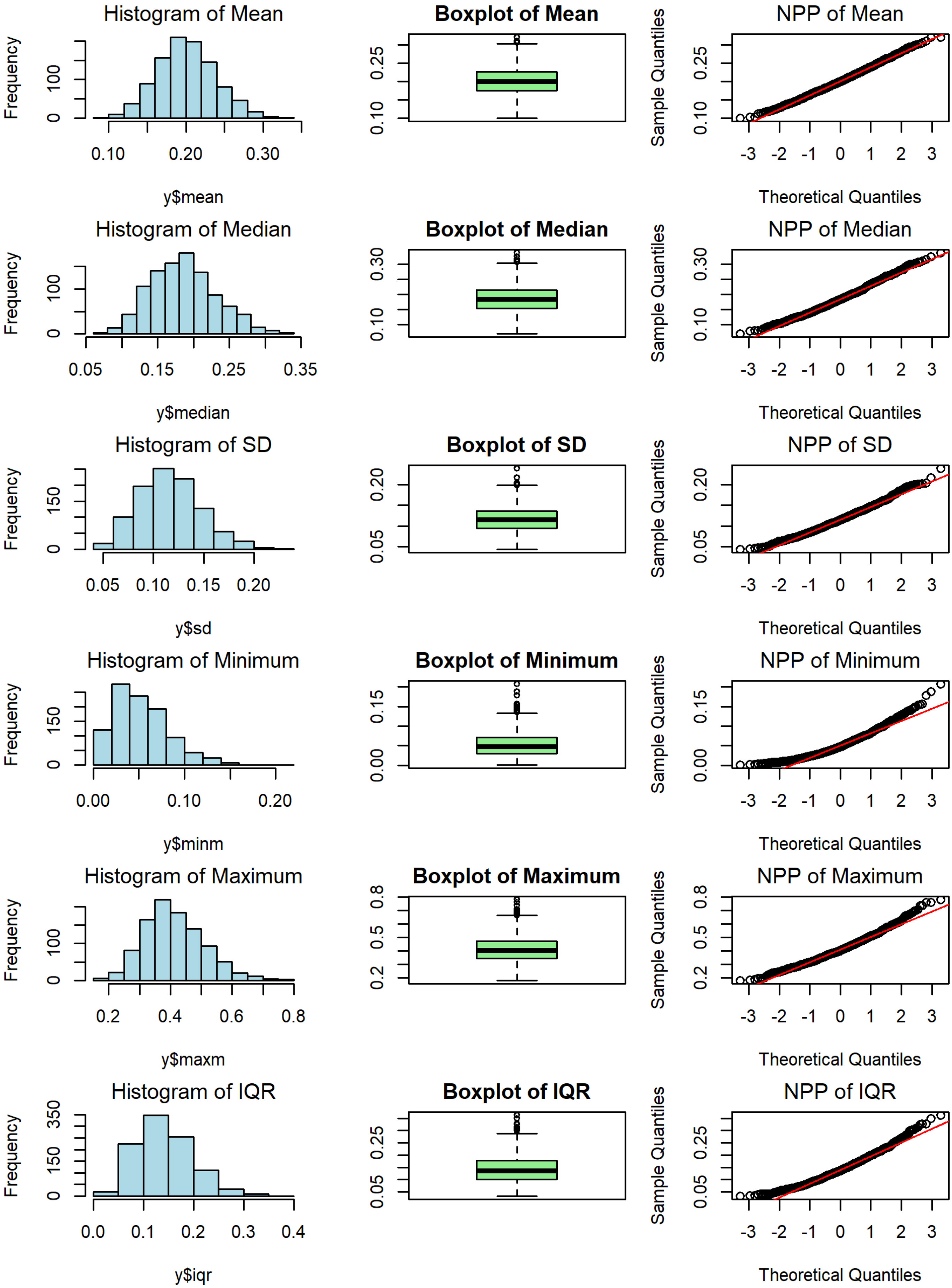
- **Mean:** Achieves normality for all sample sizes (n ≥ 10), with fast convergence, making it the most reliable statistic across sample sizes.
- **Median:** Achieves normality for n ≥ 50, with slower convergence compared to the mean.
- **Standard Deviation (SD):** Achieves normality for n ≥ 100, reflecting stabilization with larger sample sizes.
- **IQR:** Achieves normality for n ≥ 100, with a noticeable trend toward convergence as sample size increases.

Normality Not Achieved:

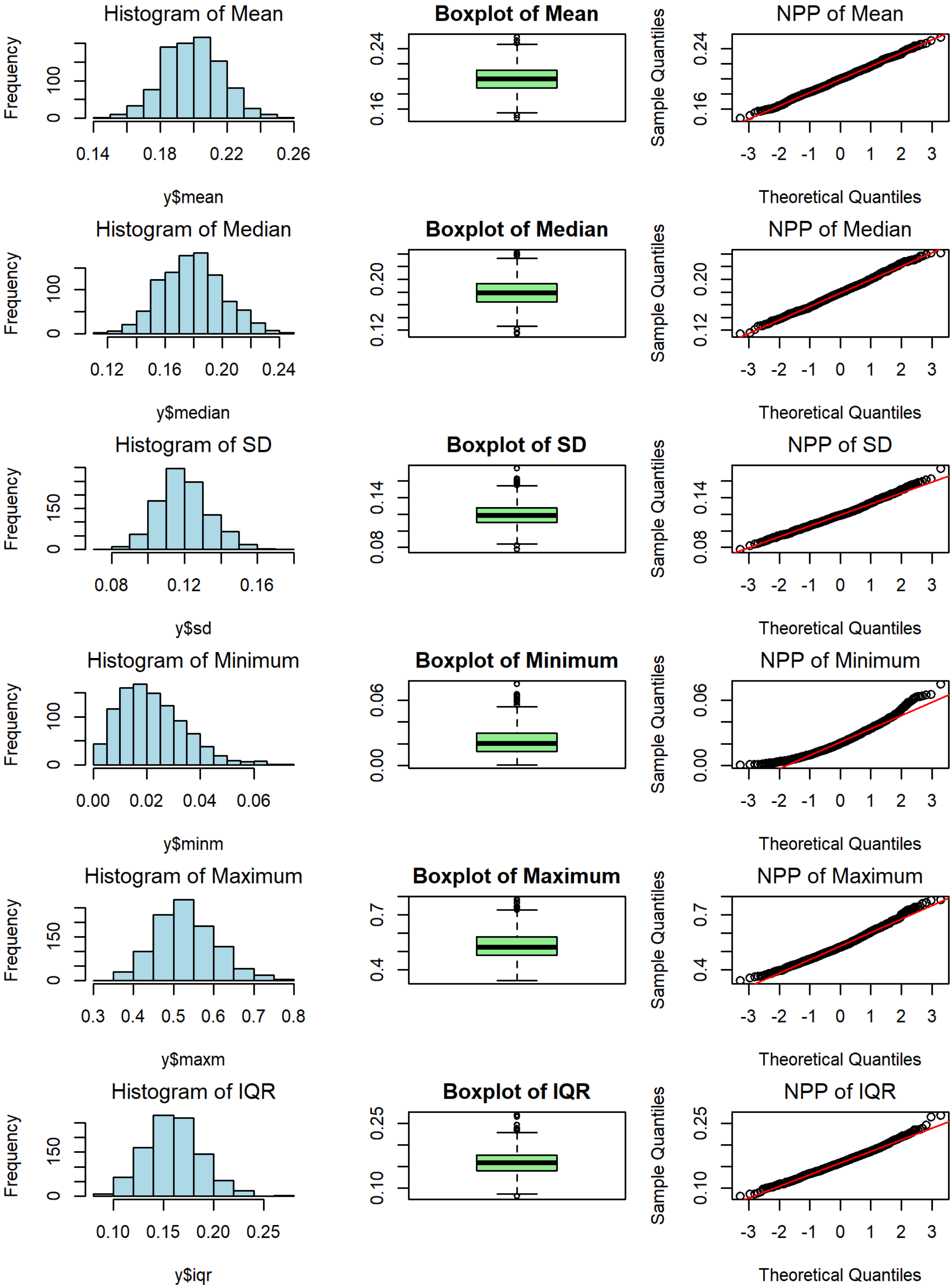
- **Minimum and Maximum:** Do not achieve normality for any sample size, as they are heavily influenced by the distribution's skewness, particularly at smaller sample sizes.

Overall: The Beta distribution with α = 2 and β = 8 achieves normality for the mean at n ≥ 10, with other statistics (median, standard deviation, and IQR) requiring larger sample sizes (n ≥ 50 for median, n ≥ 100 for SD and IQR) to stabilize. The minimum and maximum values remain non-normal across all sample sizes due to the skewed nature of the distribution.

BETA DISTRIBUTION PLOT
(n=10, nn=1000, α=2, β=8)

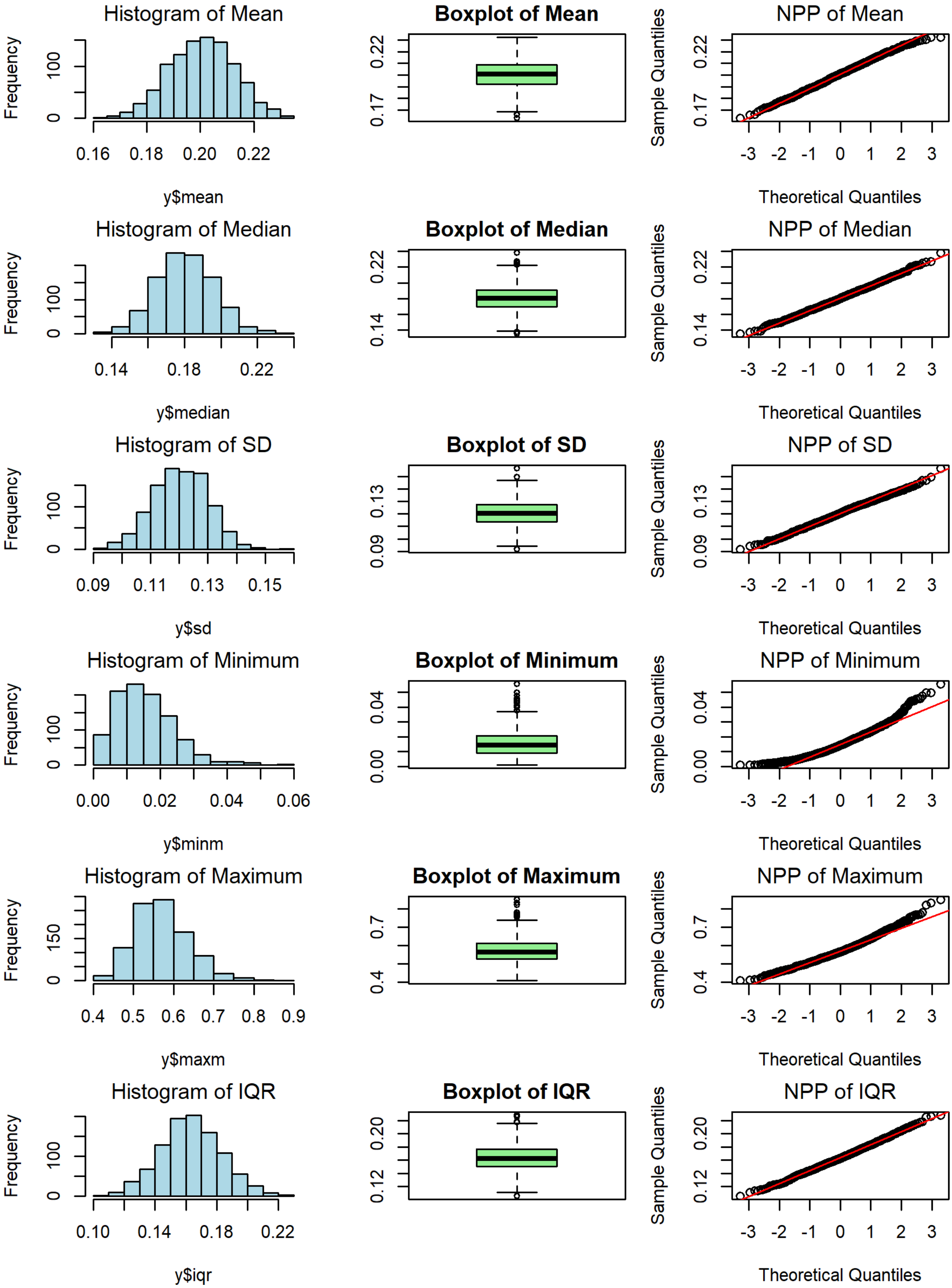


BETA DISTRIBUTION PLOT
(n=50, nn=1000, α=2, β=8)

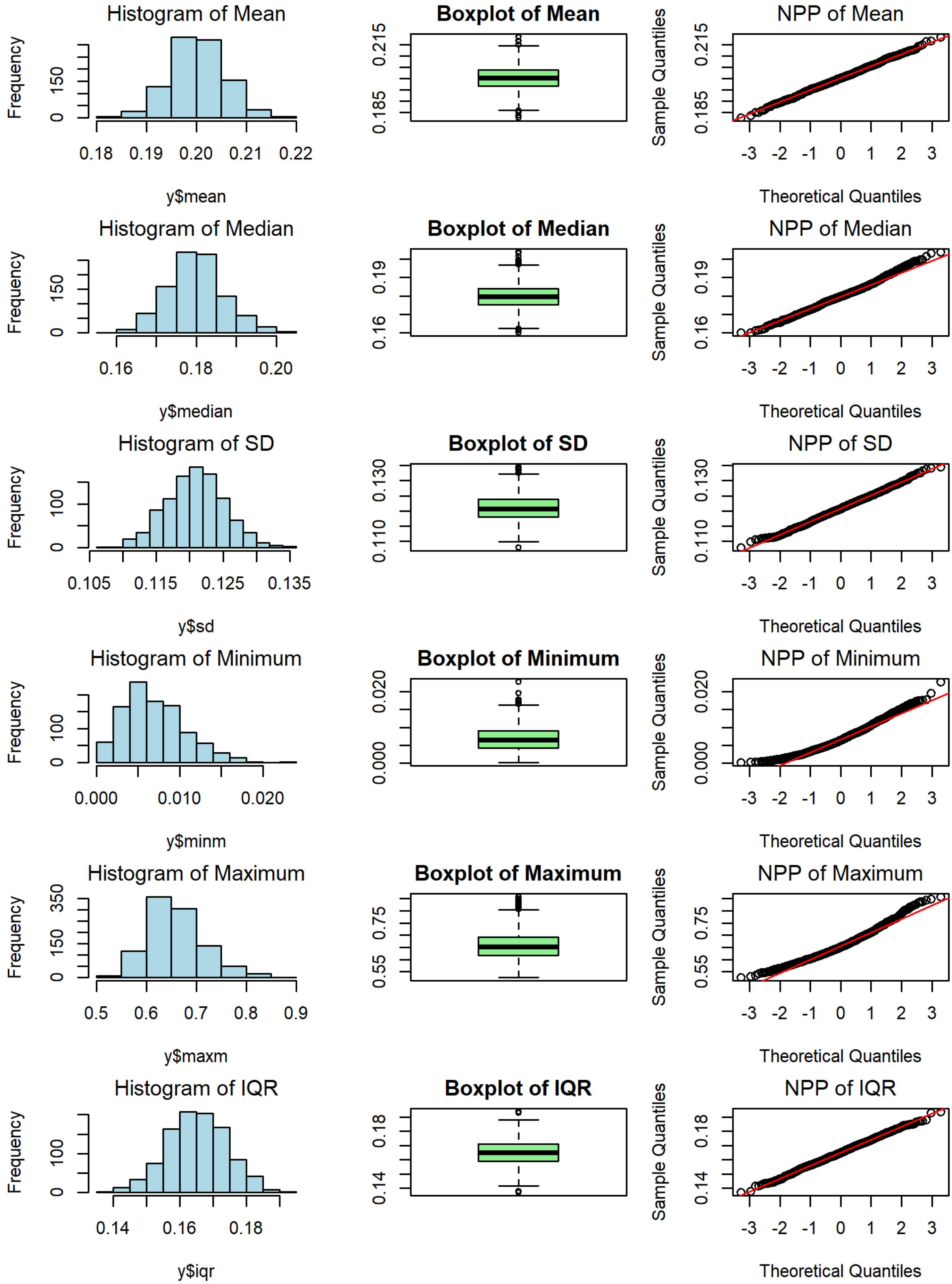


BETA DISTRIBUTION PLOT

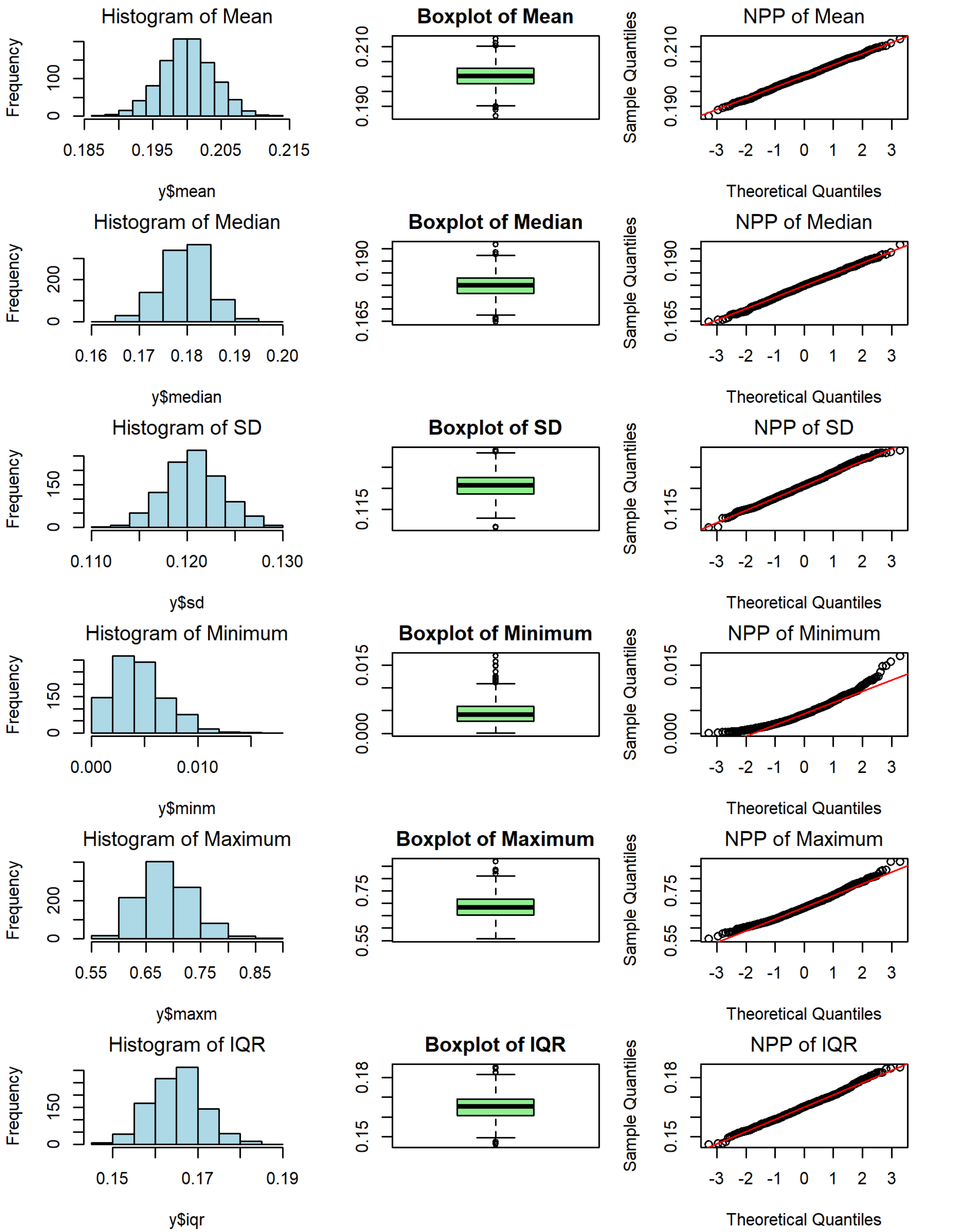
(n=100, nn=1000, α=2, β=8)

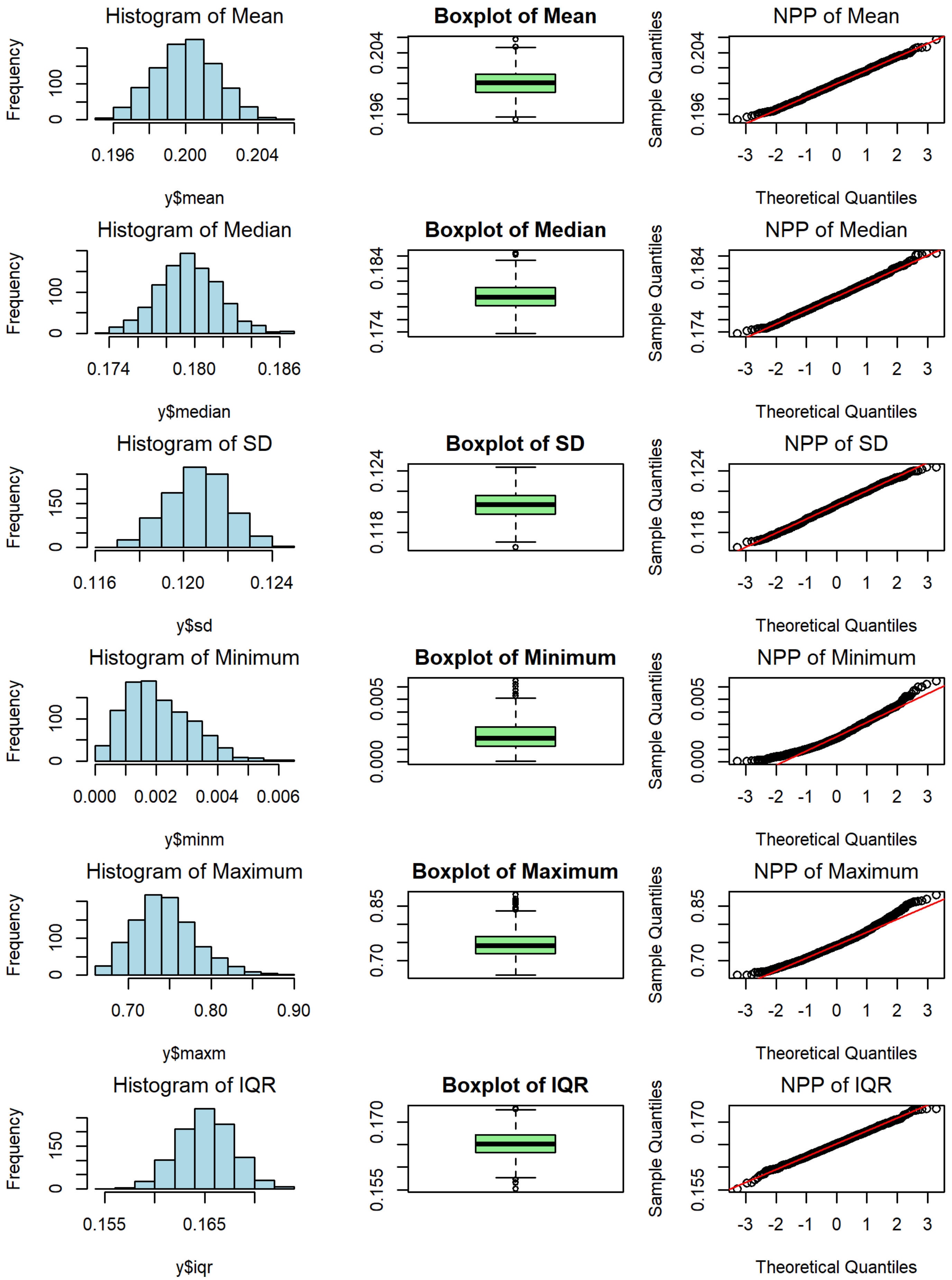


BETA DISTRIBUTION PLOT
(n=500, nn=1000, α=2, β=8)



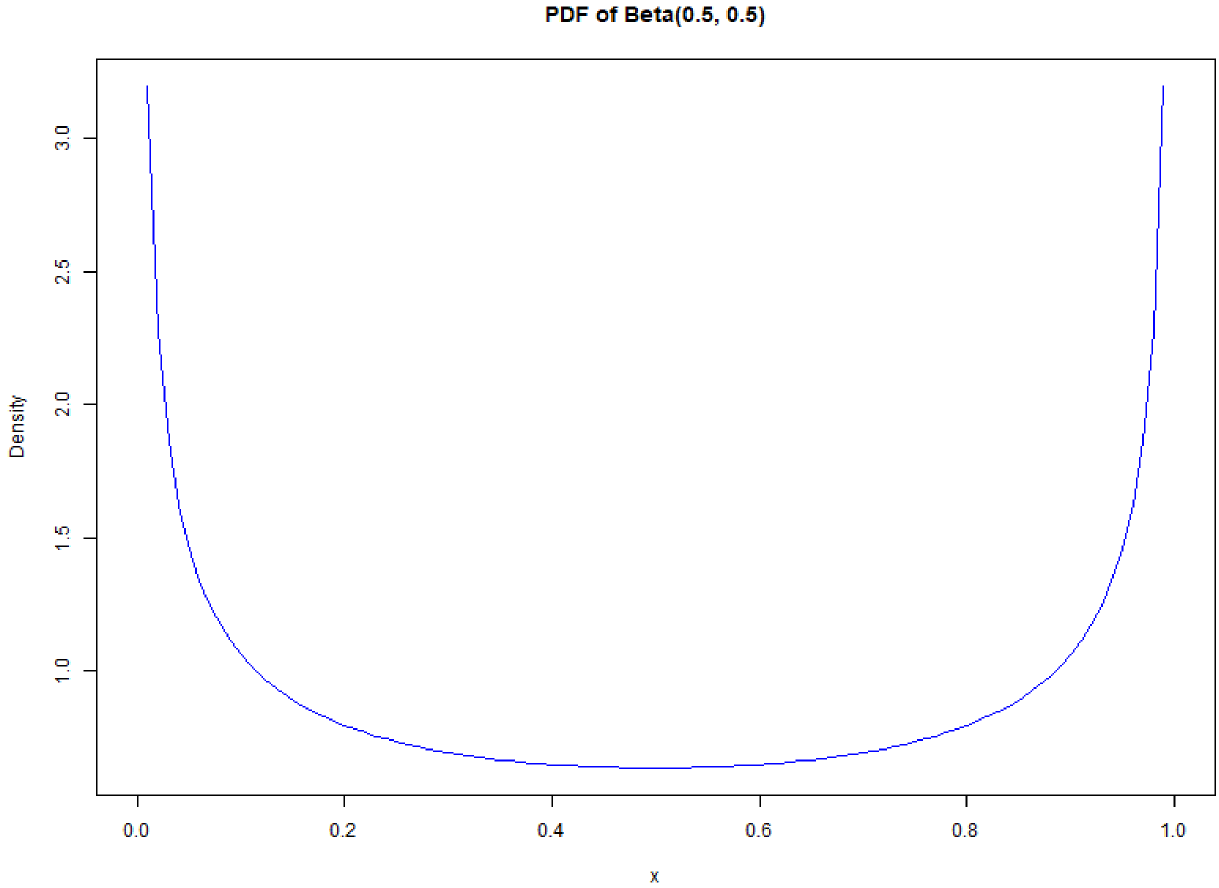
BETA DISTRIBUTION PLOT
(n=1000, nn=1000, α=2, β=8)

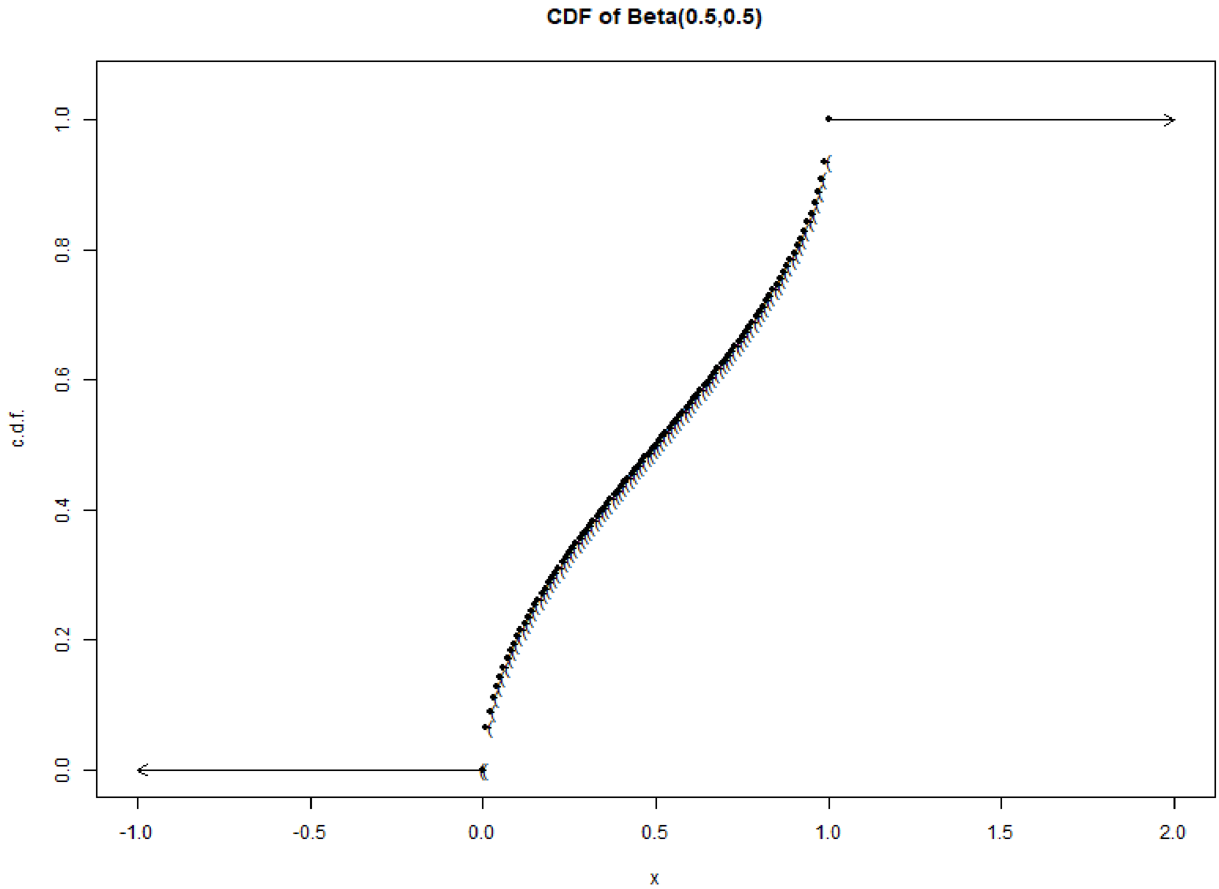




Histogram of Mean
Frequency
0
100
0.196
0.200
0.204
y$mean
Boxplot of Mean
0.196
0.204
NPP of Mean
Sample Quantiles
0.196
0.204
-3
-2
-1
0
1
2
3
Theoretical Quantiles
Histogram of Median
Frequency
0
100
0.174
0.180
0.186
y$median
Boxplot of Median
0.174
0.184
NPP of Median
Sample Quantiles
0.174
0.184
-3
-2
-1
0
1
2
3
Theoretical Quantiles
Histogram of SD
Frequency
0
150
0.116
0.120
0.124
y$sd
Boxplot of SD
0.118
0.124
NPP of SD
Sample Quantiles
0.118
0.124
-3
-2
-1
0
1
2
3
Theoretical Quantiles
Histogram of Minimum
Frequency
0
100
0.000
0.002
0.004
0.006
y$minm
Boxplot of Minimum
0.000
0.005
NPP of Minimum
Sample Quantiles
0.000
0.005
-3
-2
-1
0
1
2
3
Theoretical Quantiles
Histogram of Maximum
Frequency
0
100
0.70
0.80
0.90
y$maxm
Boxplot of Maximum
0.70
0.85
NPP of Maximum
Sample Quantiles
0.70
0.85
-3
-2
-1
0
1
2
3
Theoretical Quantiles
Histogram of IQR
Frequency
0
150
0.155
0.165
y$iqr
Boxplot of IQR
0.155
0.170
NPP of IQR
Sample Quantiles
0.155
0.170
-3
-2
-1
0
1
2
3
Theoretical Quantiles

BETA DISTRIBUTION $(0.5, 0.5)$





BETA DISTRIBUTION

	Values of n to achieve normality (nn=1000, α=0.5, β=0.5)						
Statistic	n=10	n=50	n=100	n=500	n=1000	n=5000	Any n	Min(n)
Mean	Yes	Yes	Yes	Yes	Yes	Yes	Yes	10
Median	No	Yes	Yes	Yes	Yes	Yes	Yes	50
Std Dev	No	Yes	Yes	Yes	Yes	Yes	Yes	50
Min	No	No	No	No	No	No	No	NA
Max	No	No	No	No	No	No	No	NA
IQR	No	Yes	Yes	Yes	Yes	Yes	Yes	50

Conclusion for Beta Distribution (α = 0.5, β = 0.5)

Normality Achieved:

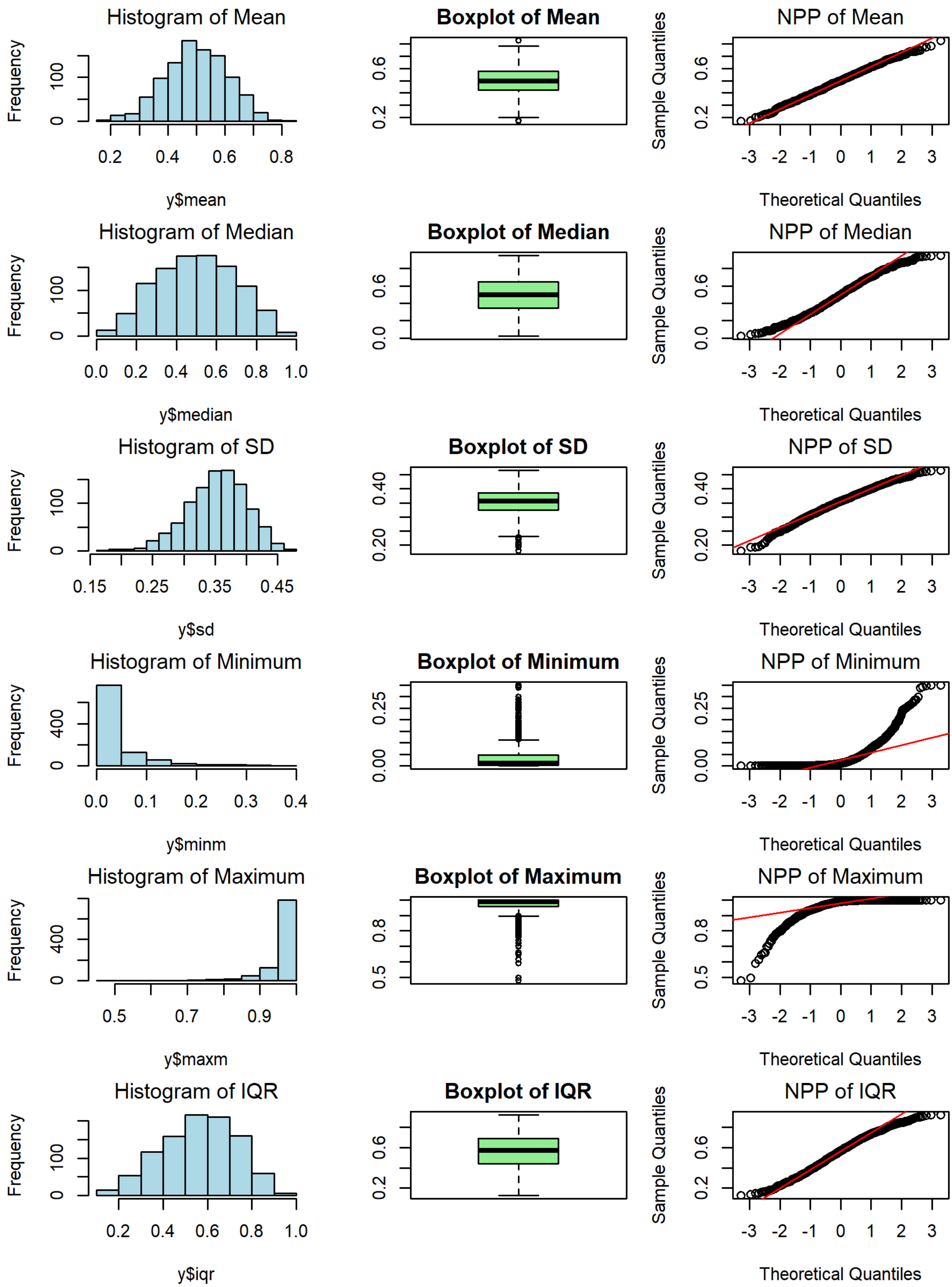
- **Mean:** Achieves normality for all sample sizes (n ≥ 10), showing quick convergence for this statistic.
- **Median:** Achieves normality starting from n ≥ 50, with gradual stabilization as the sample size increases.
- **Standard Deviation (SD):** Achieves normality from n ≥ 50, indicating convergence to a stable distribution with increasing sample sizes.
- **IQR:** Achieves normality from n ≥ 50, with more stable behavior as sample size increases.

Normality Not Achieved:

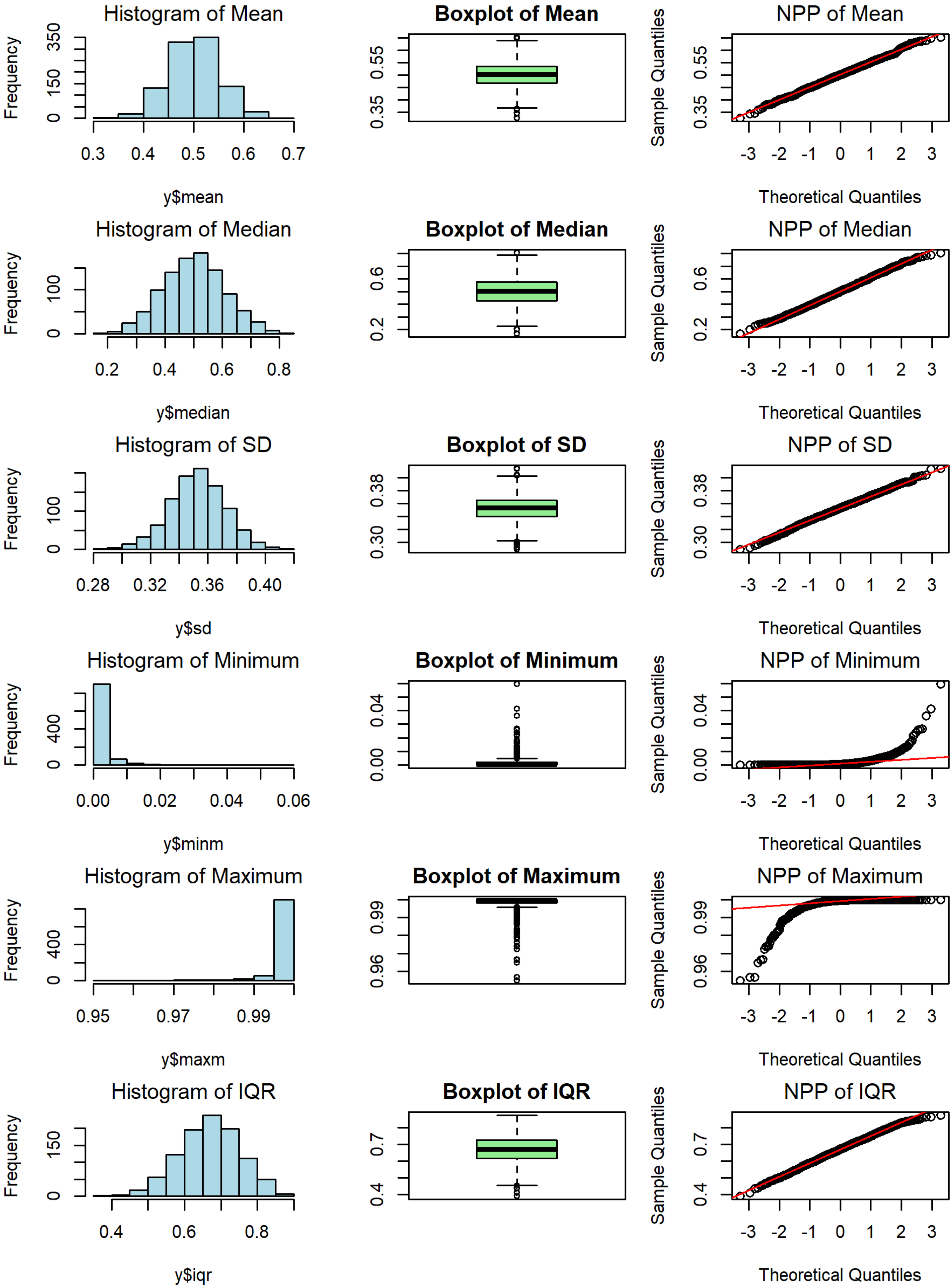
- **Minimum and Maximum:** Do not achieve normality for any sample size, as these statistics remain influenced by the highly skewed nature of the Beta distribution with α = 0.5 and β = 0.5.

Overall: The Beta distribution with α = 0.5 and β = 0.5 shows quick convergence for the mean across all sample sizes, but other statistics such as median, standard deviation, and IQR start stabilizing only after n ≥ 50. The minimum and maximum values remain non-normal at all sample sizes due to the pronounced skew of the distribution.

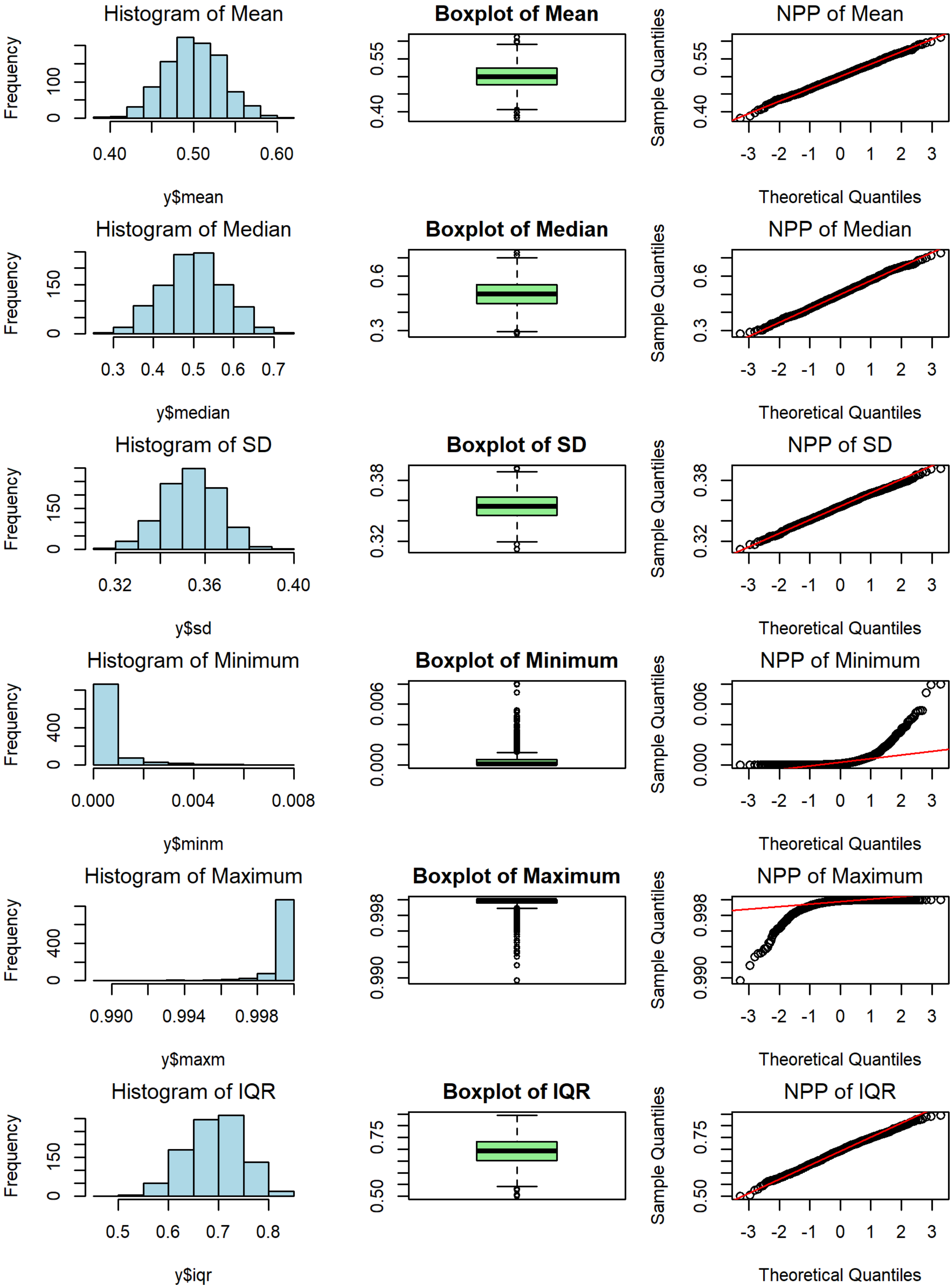
BETA DISTRIBUTION PLOT
(n=10, nn=1000, α=0.5, β=0.5)



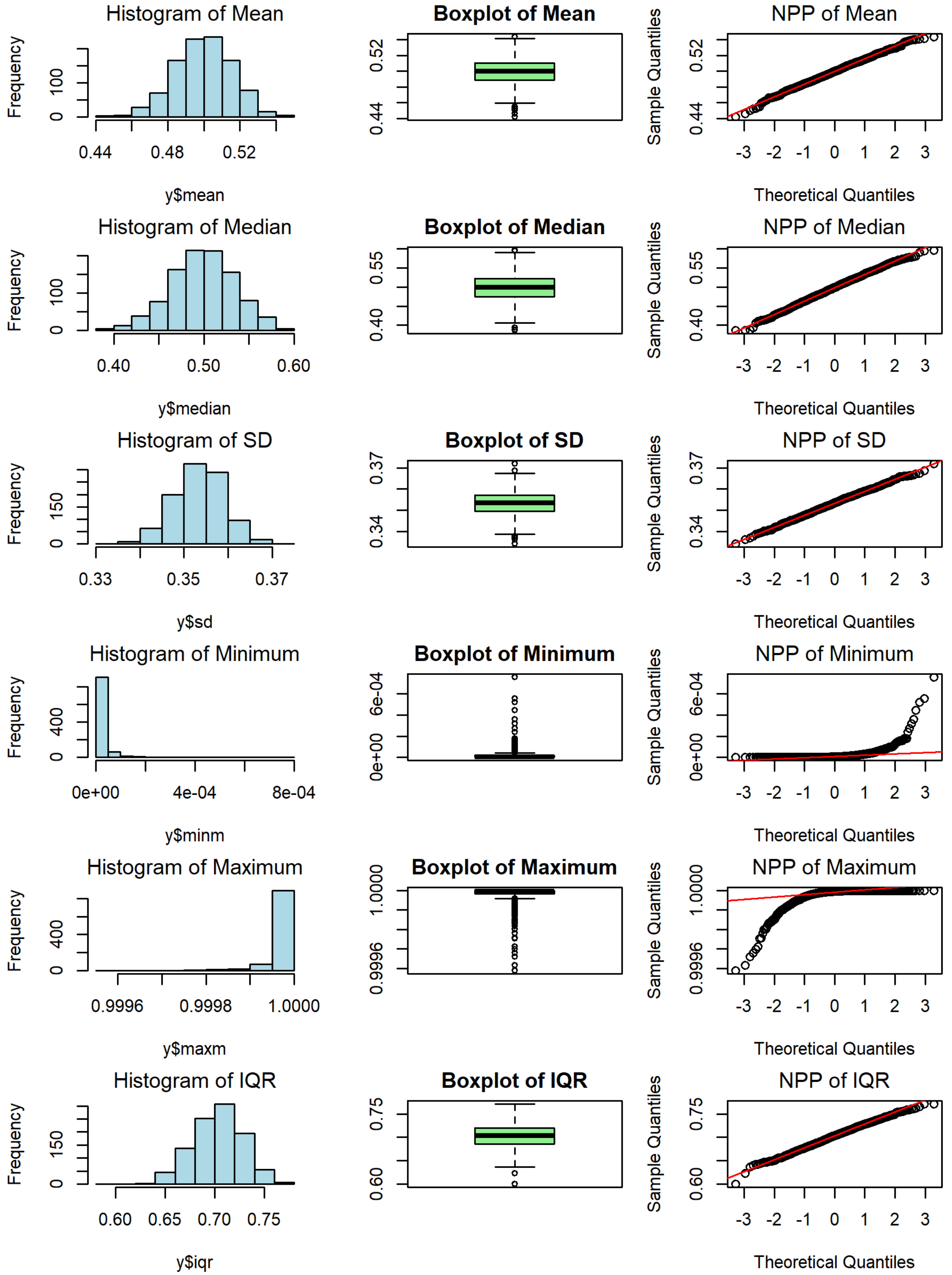
BETA DISTRIBUTION PLOT
(n=50, nn=1000, α=0.5, β=0.5)



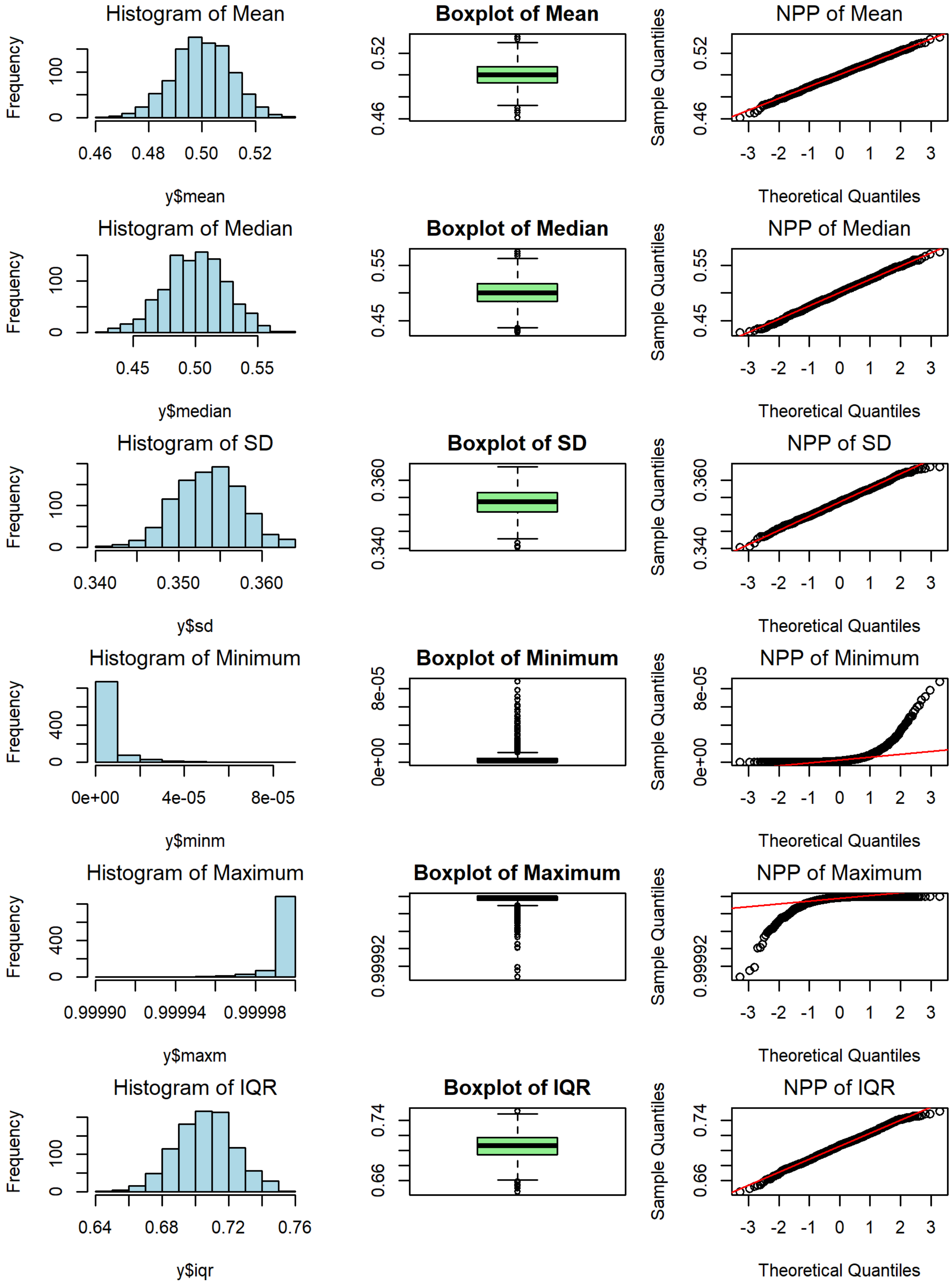
BETA DISTRIBUTION PLOT
(n=100, nn=1000, α=0.5, β=0.5)

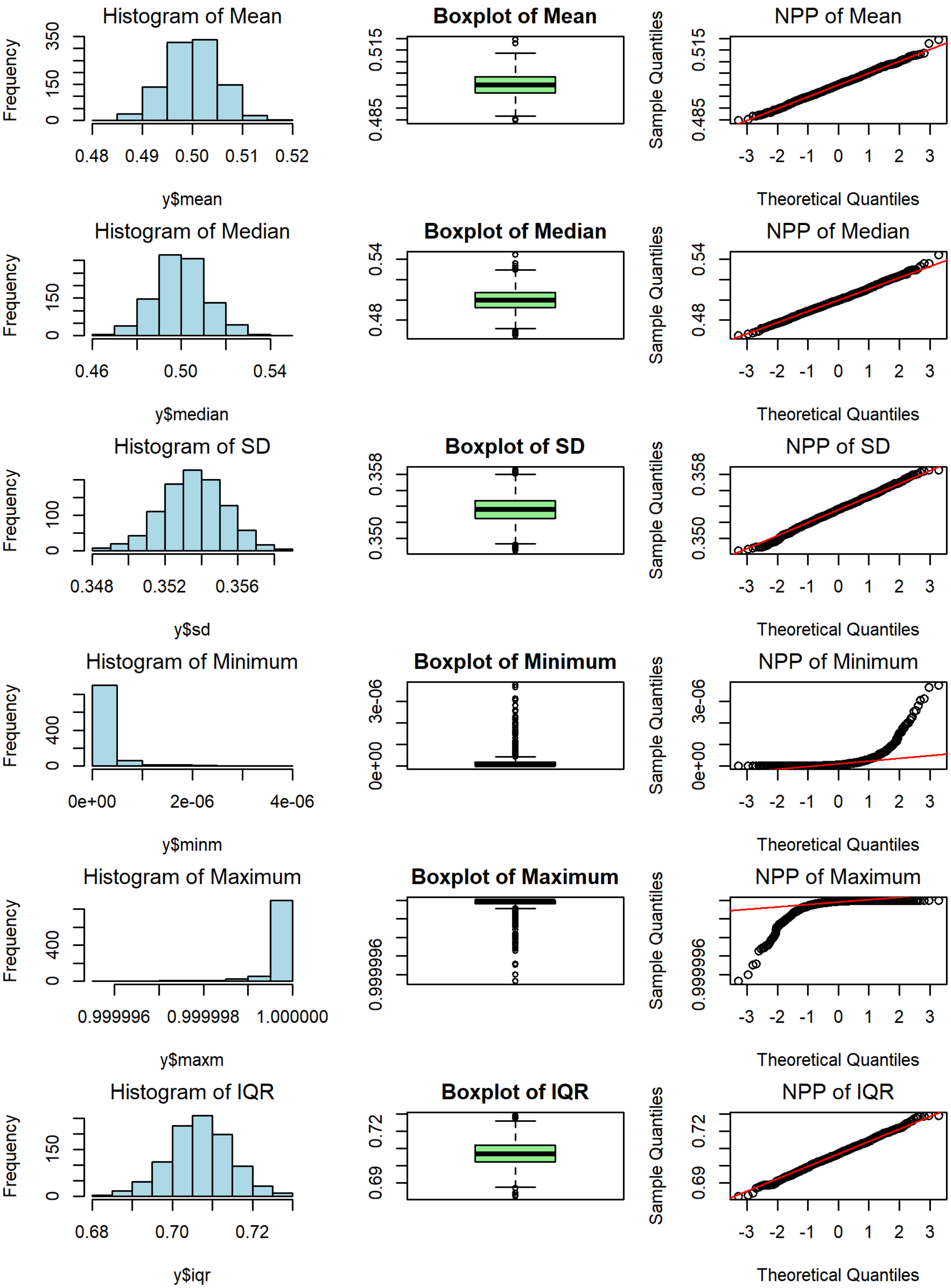


BETA DISTRIBUTION PLOT
(n=500, nn=1000, α=0.5, β=0.5)

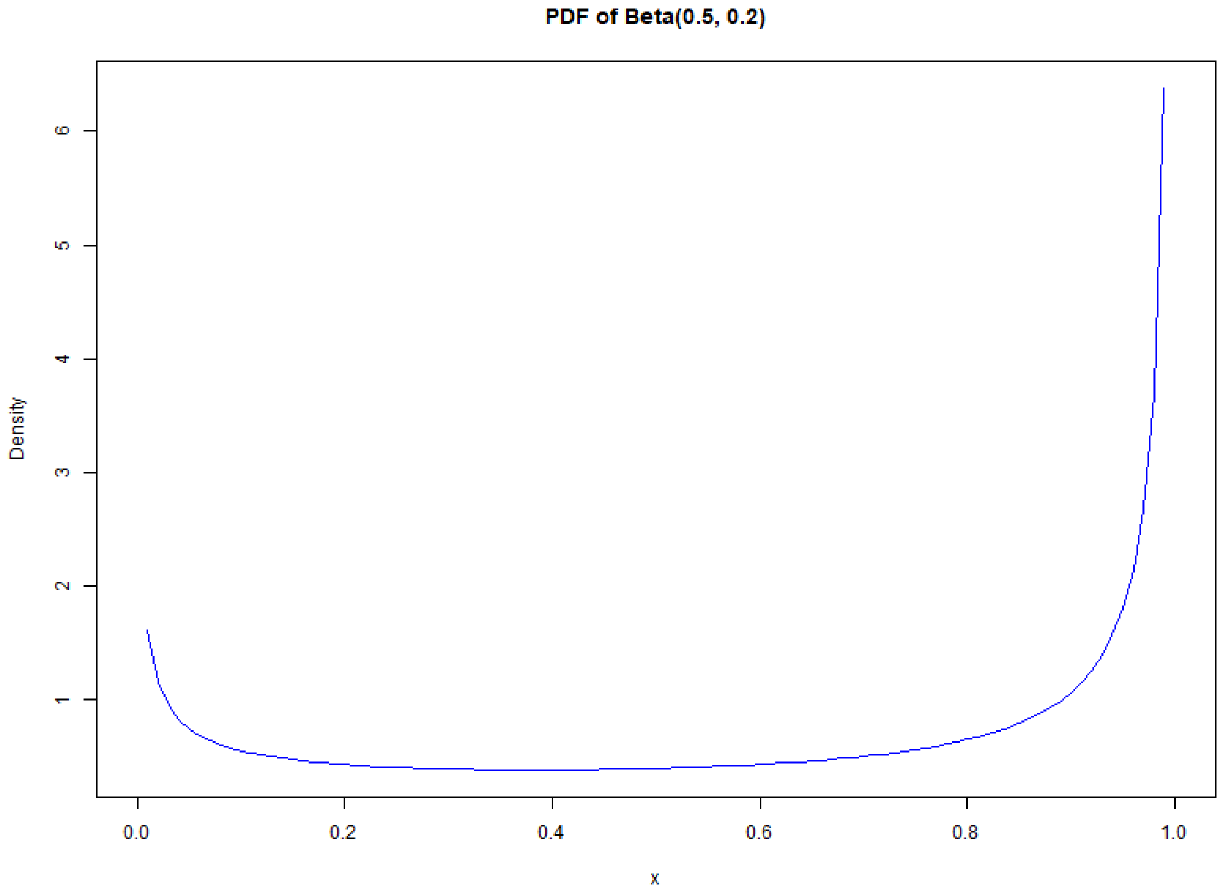


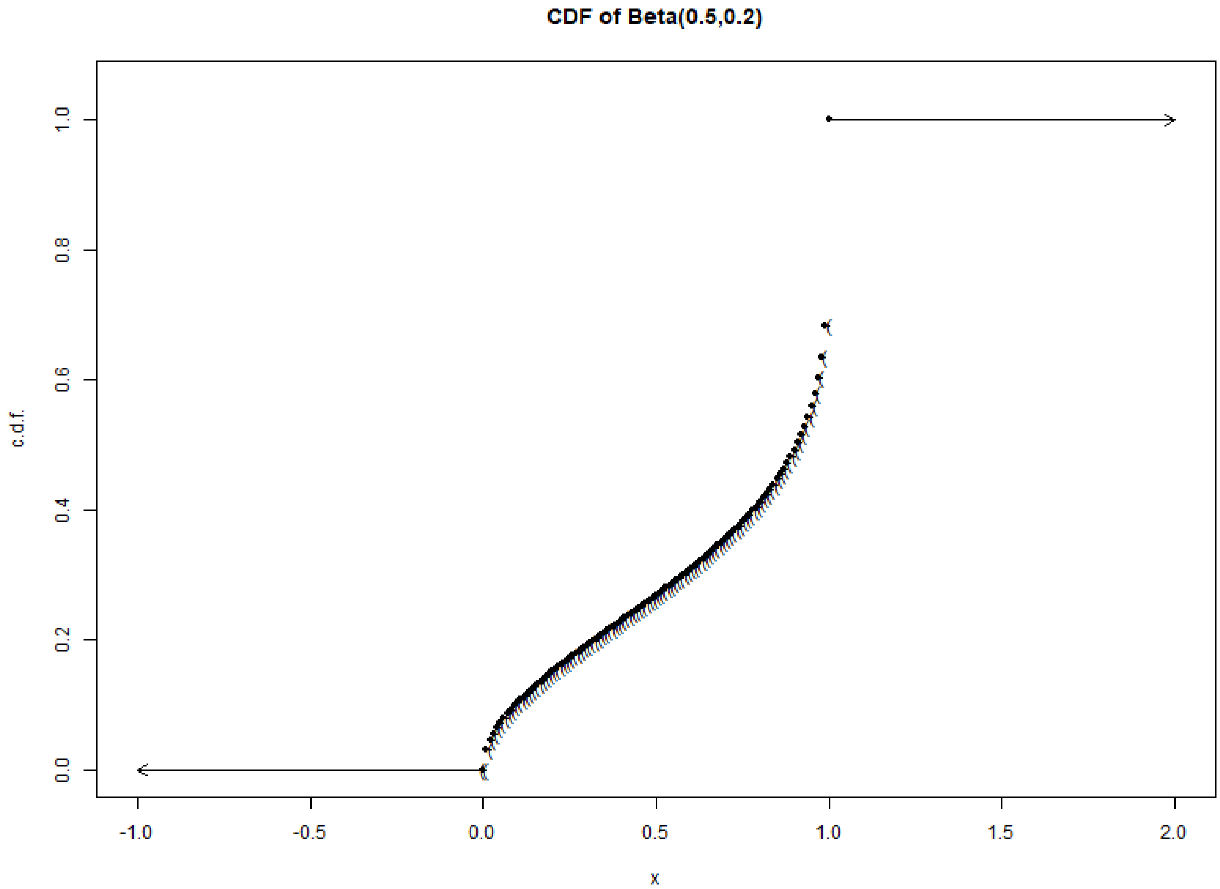
BETA DISTRIBUTION PLOT
(n=1000, nn=1000, α=0.5, β=0.5)




Histogram of Mean
Frequency
0
150
350
0.48
0.49
0.50
0.51
0.52
y$mean
Boxplot of Mean
0.485
0.515
NPP of Mean
Sample Quantiles
0.485
0.515
-3
-2
-1
0
1
2
3
Theoretical Quantiles
Histogram of Median
Frequency
0
150
0.46
0.50
0.54
y$median
Boxplot of Median
0.48
0.54
NPP of Median
Sample Quantiles
0.48
0.54
Theoretical Quantiles
Histogram of SD
Frequency
0
100
0.348
0.352
0.356
y$sd
Boxplot of SD
0.350
0.358
NPP of SD
Sample Quantiles
0.350
0.358
Theoretical Quantiles
Histogram of Minimum
Frequency
0
400
0e+00
2e-06
4e-06
y$minm
Boxplot of Minimum
0e+00
3e-06
NPP of Minimum
Sample Quantiles
0e+00
3e-06
Theoretical Quantiles
Histogram of Maximum
Frequency
0
400
0.999996
0.999998
1.000000
y$maxm
Boxplot of Maximum
0.999996
NPP of Maximum
Sample Quantiles
0.999996
Theoretical Quantiles
Histogram of IQR
Frequency
0
150
0.68
0.70
0.72
y$iqr
Boxplot of IQR
0.69
0.72
NPP of IQR
Sample Quantiles
0.69
0.72
Theoretical Quantiles

BETA DISTRIBUTION $(0.5,0.2)$





BETA DISTRIBUTION

	Values of n to achieve normality (nn=1000, α=0.5, β=0.2)						
Statistic	n=10	n=50	n=100	n=500	n=1000	n=5000	Any n	Min(n)
Mean	No	Yes	Yes	Yes	Yes	Yes	Yes	50
Median	No	No	No	No	No	Yes	Yes	5000
Std Dev	No	No	No	Yes	Yes	Yes	Yes	500
Min	No	No	No	No	No	No	No	NA
Max	No	No	No	No	No	No	No	NA
IQR	No	No	Yes	Yes	Yes	Yes	Yes	100

Conclusion for Beta Distribution (α = 0.5, β = 0.2)

Normality Achieved:

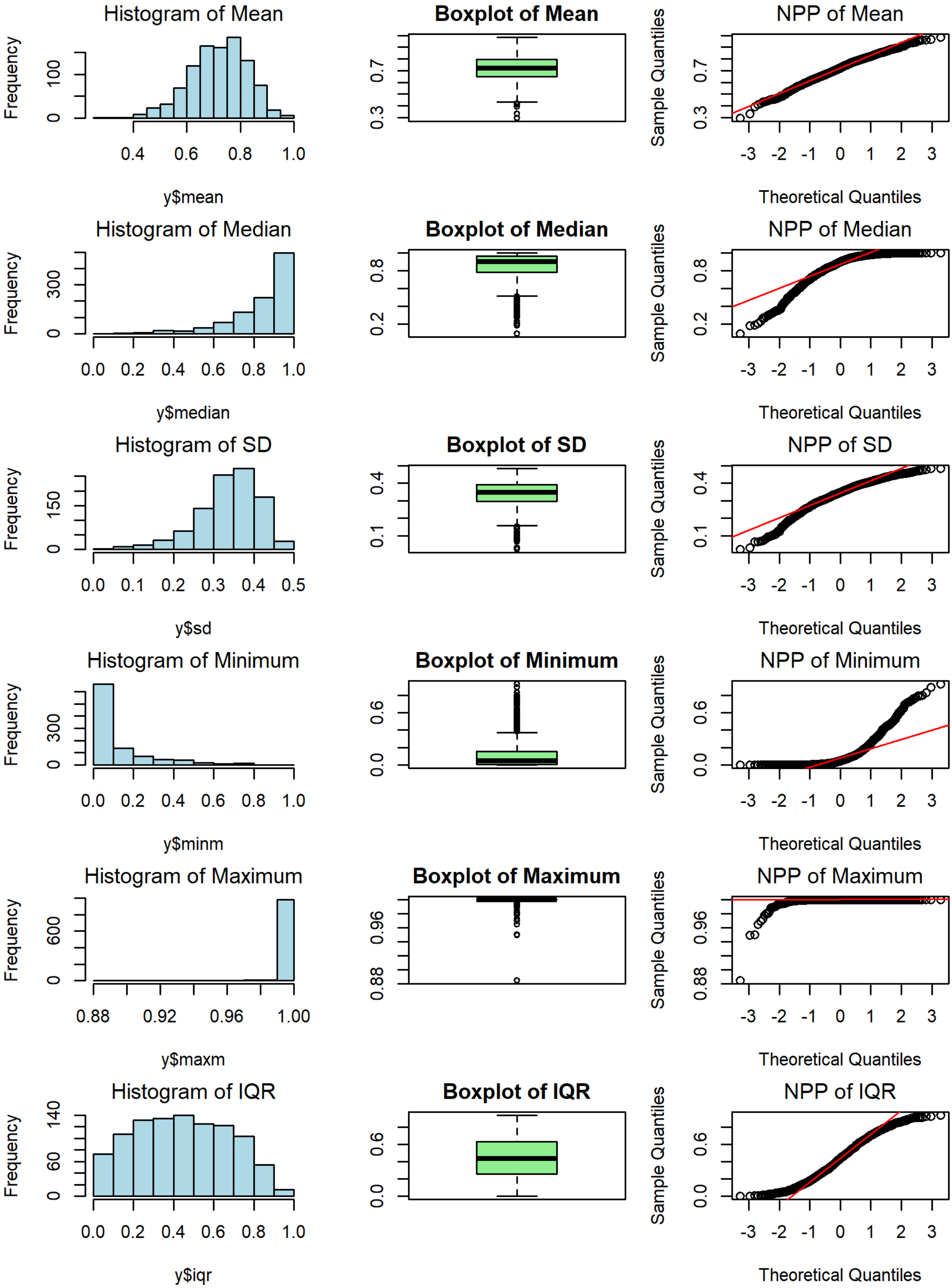
- **Mean:** Achieves normality starting from n ≥ 50, with gradual stabilization as sample size increases. The convergence is observed as the sample size grows.
- **Median:** Normality starts to appear only from n ≥ 5000. For smaller sample sizes, the median remains highly influenced by the distribution's skew.
- **Standard Deviation (SD):** Achieves normality from n ≥ 500, with more stable values as the sample size increases.
- **IQR:** Achieves normality from n ≥ 100, with stabilization observed at larger sample sizes.

Normality Not Achieved:

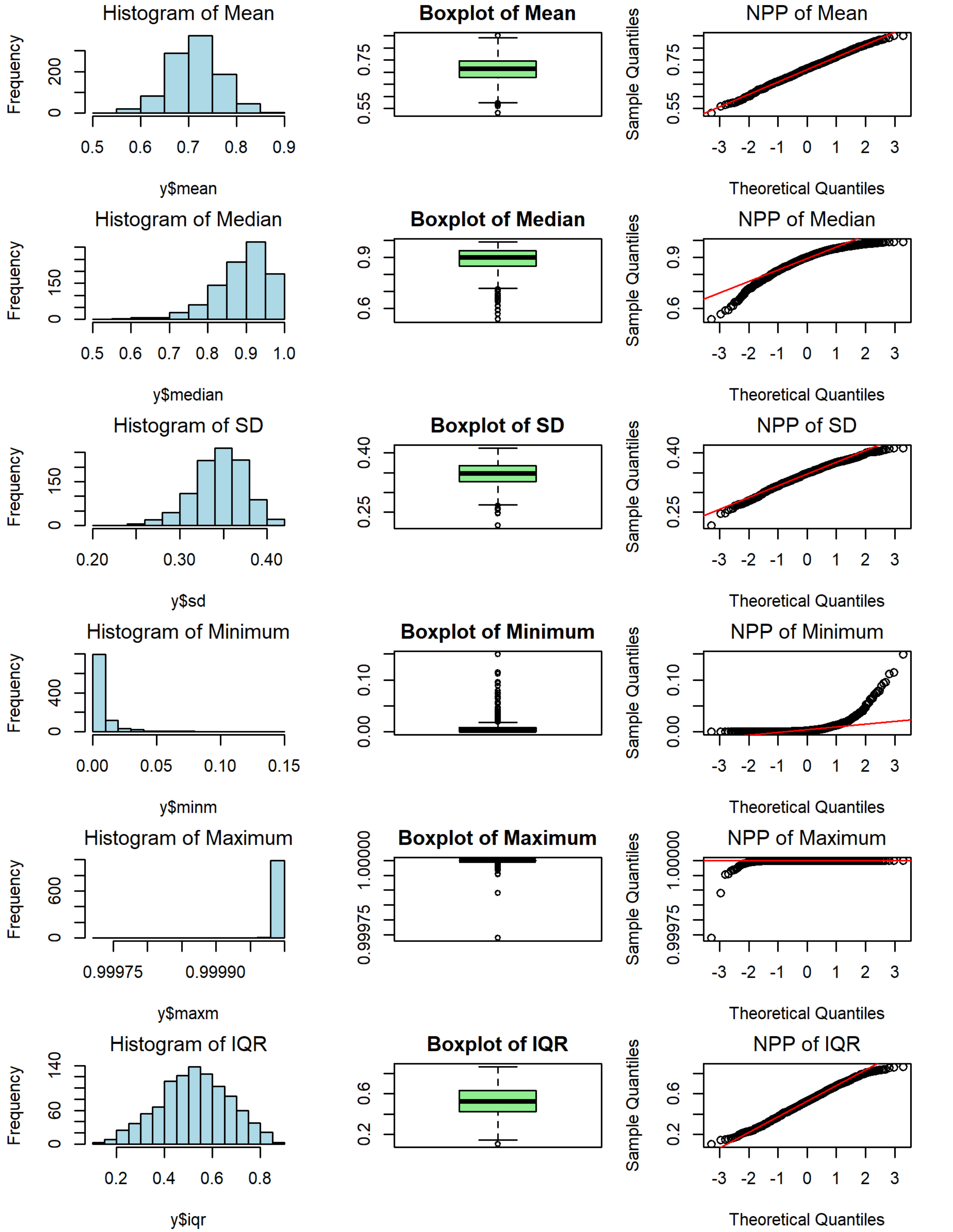
- **Minimum and Maximum:** These statistics do not achieve normality for any sample size due to the skewed nature of the Beta distribution with α = 0.5 and β = 0.2.

Overall: The Beta distribution with α = 0.5 and β = 0.2 shows delayed convergence for the median, which only achieves normality at n ≥ 5000. The mean, standard deviation, and IQR show earlier convergence to normality, starting from n ≥ 50, n ≥ 500, and n ≥ 100, respectively. However, the minimum and maximum values remain non-normal across all sample sizes due to the skewness of the distribution.

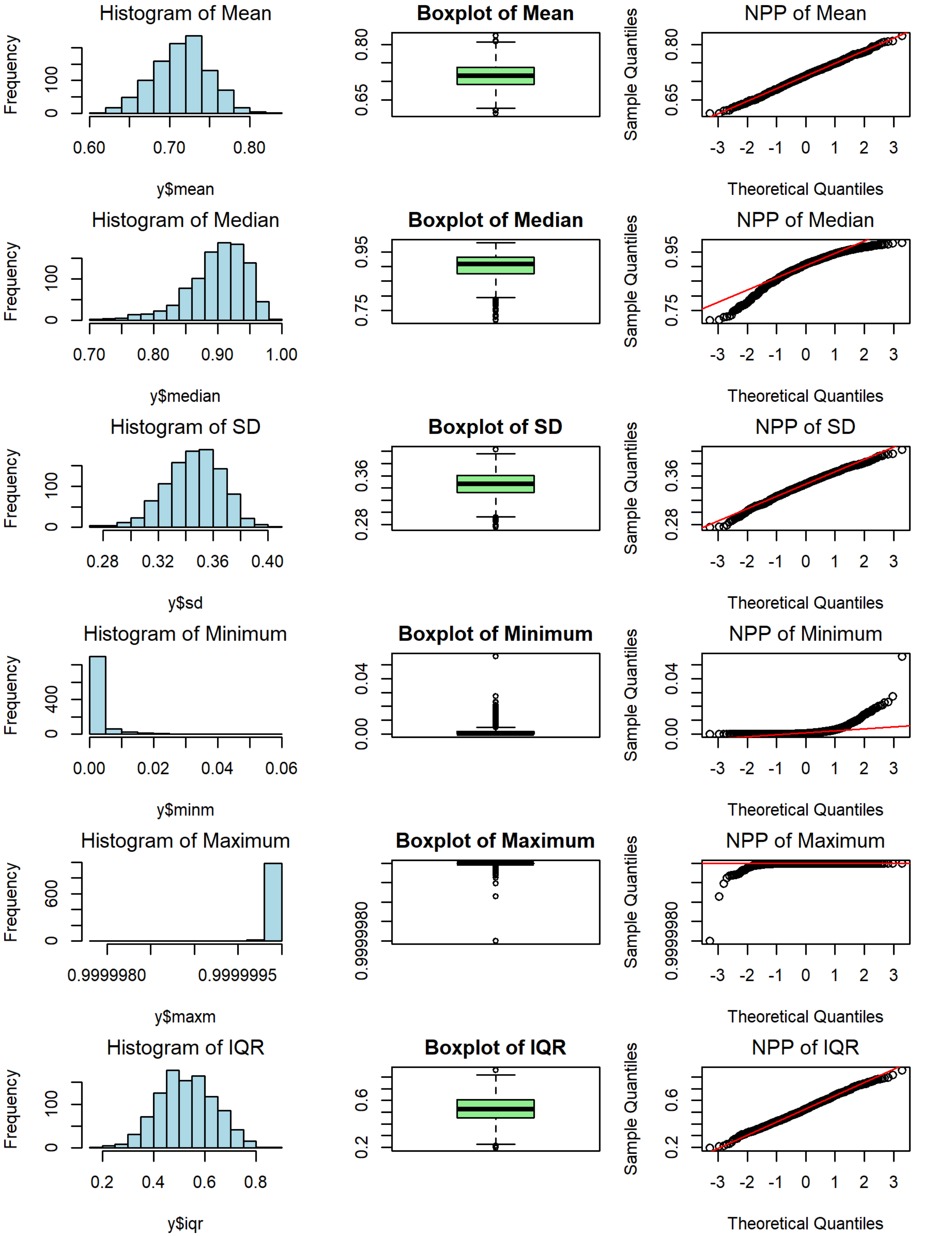
BETA DISTRIBUTION PLOT
(n=10, nn=1000, α=0.5, β=0.2)



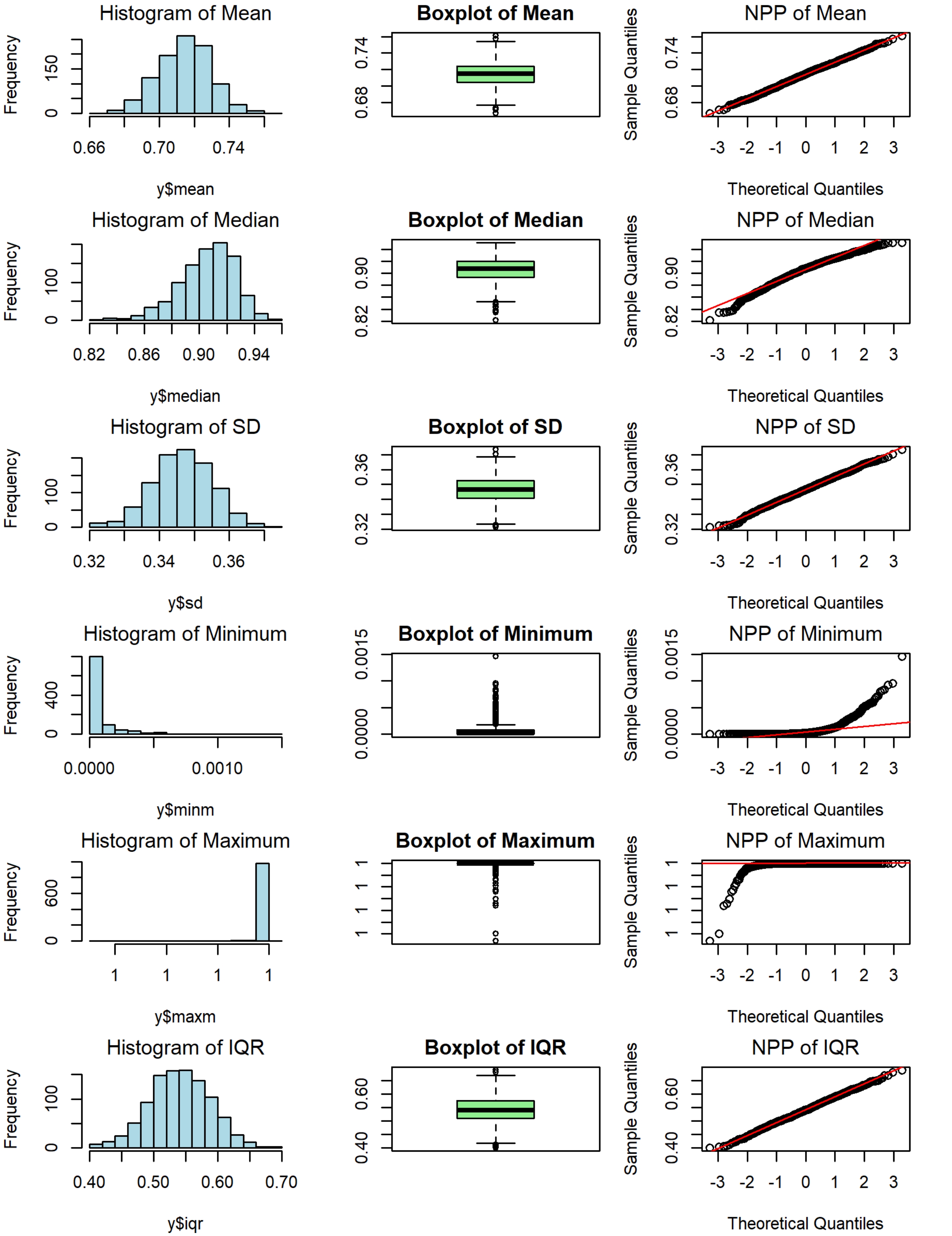
BETA DISTRIBUTION PLOT
(n=50, nn=1000, α=0.5, β=0.2)



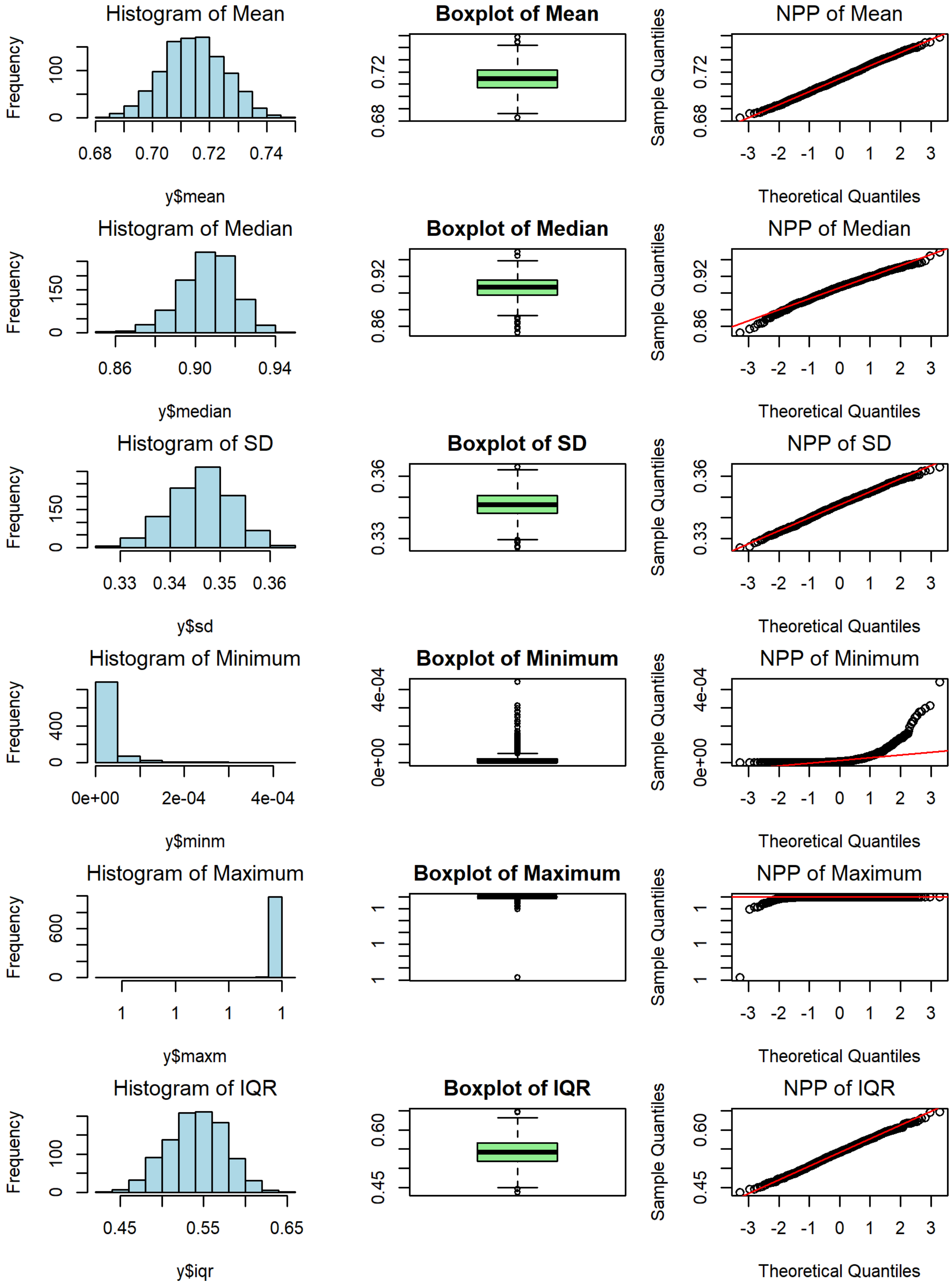
BETA DISTRIBUTION PLOT
(n=100, nn=1000, α=0.5, β=0.2)

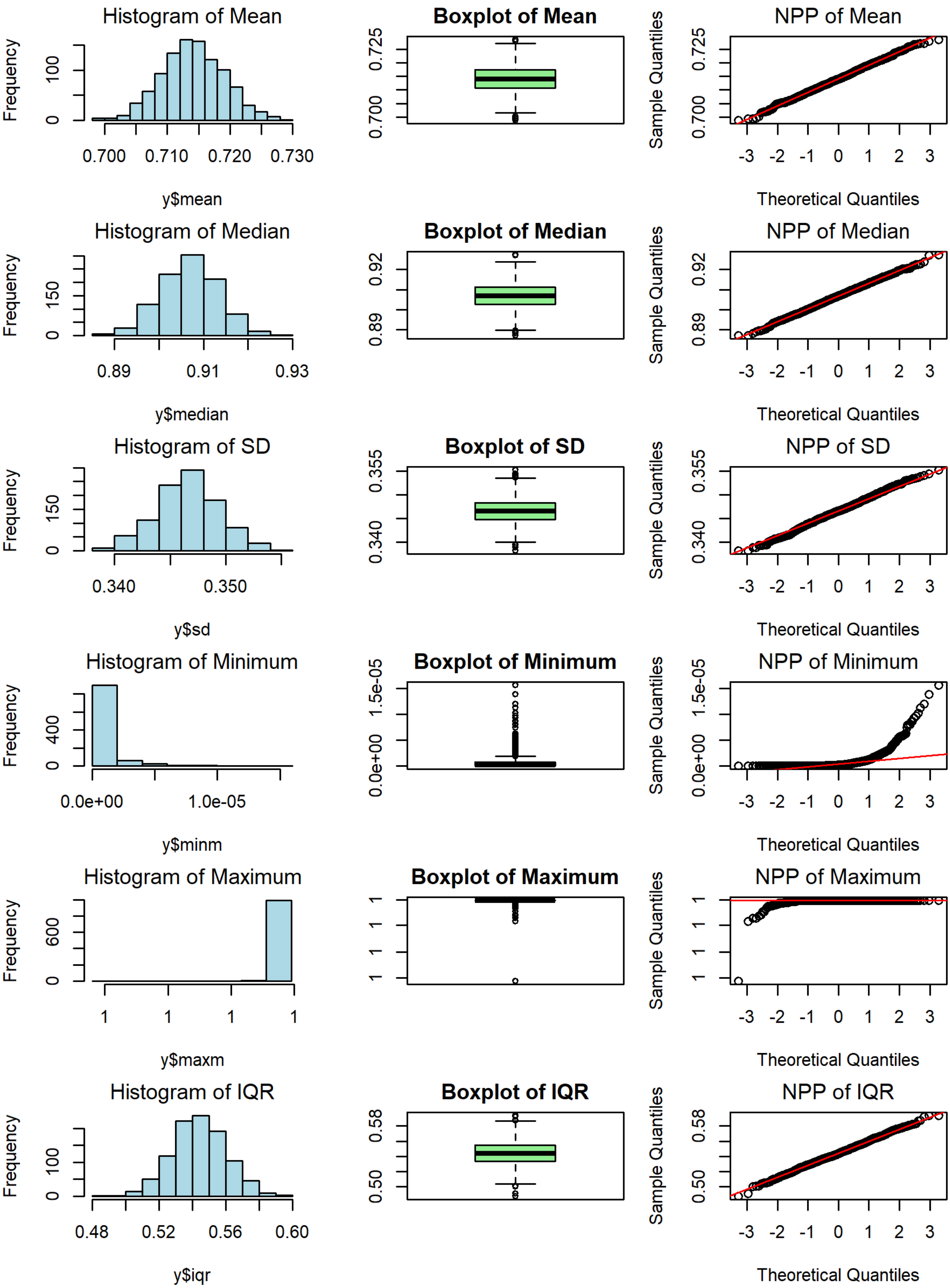


BETA DISTRIBUTION PLOT
(n=500, nn=1000, α=0.5, β=0.2)



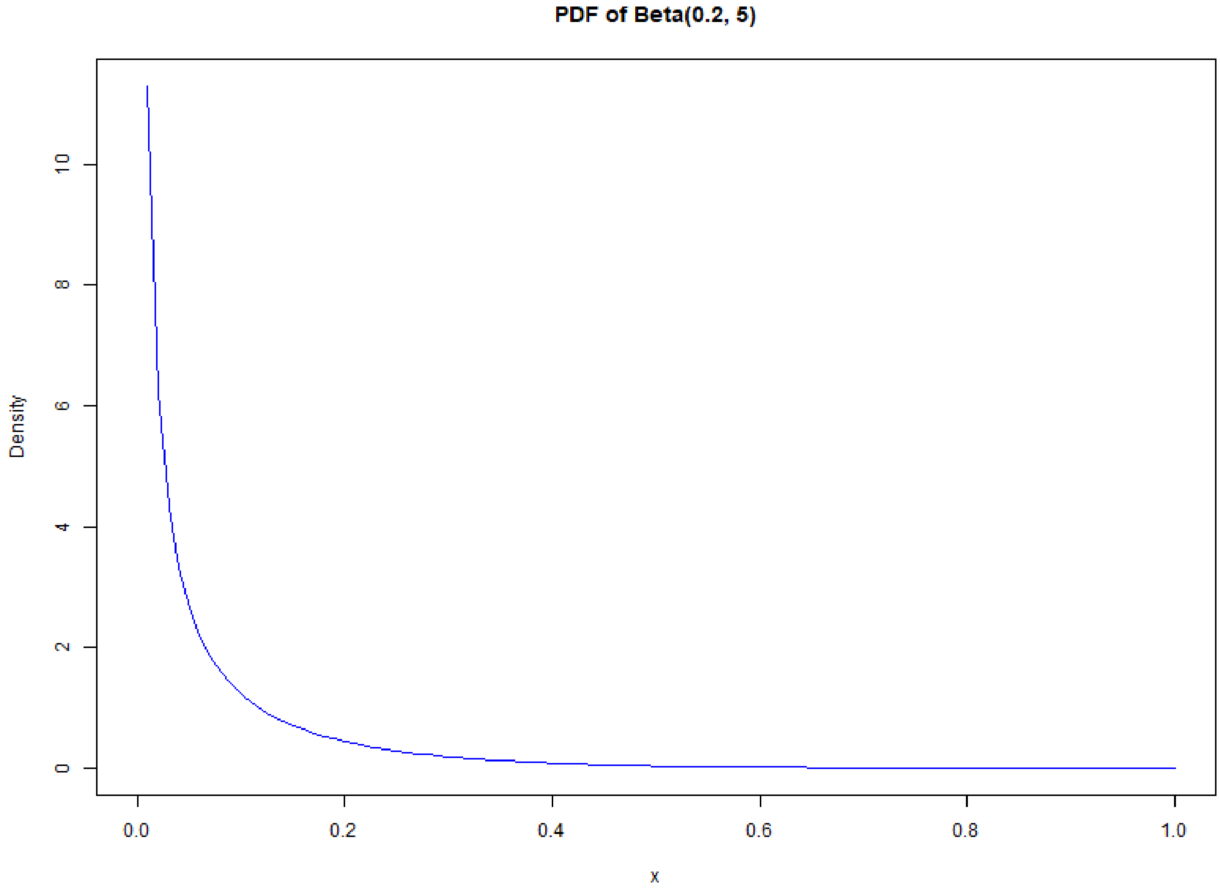
BETA DISTRIBUTION PLOT
(n=1000, nn=1000, α=0.5, β=0.2)

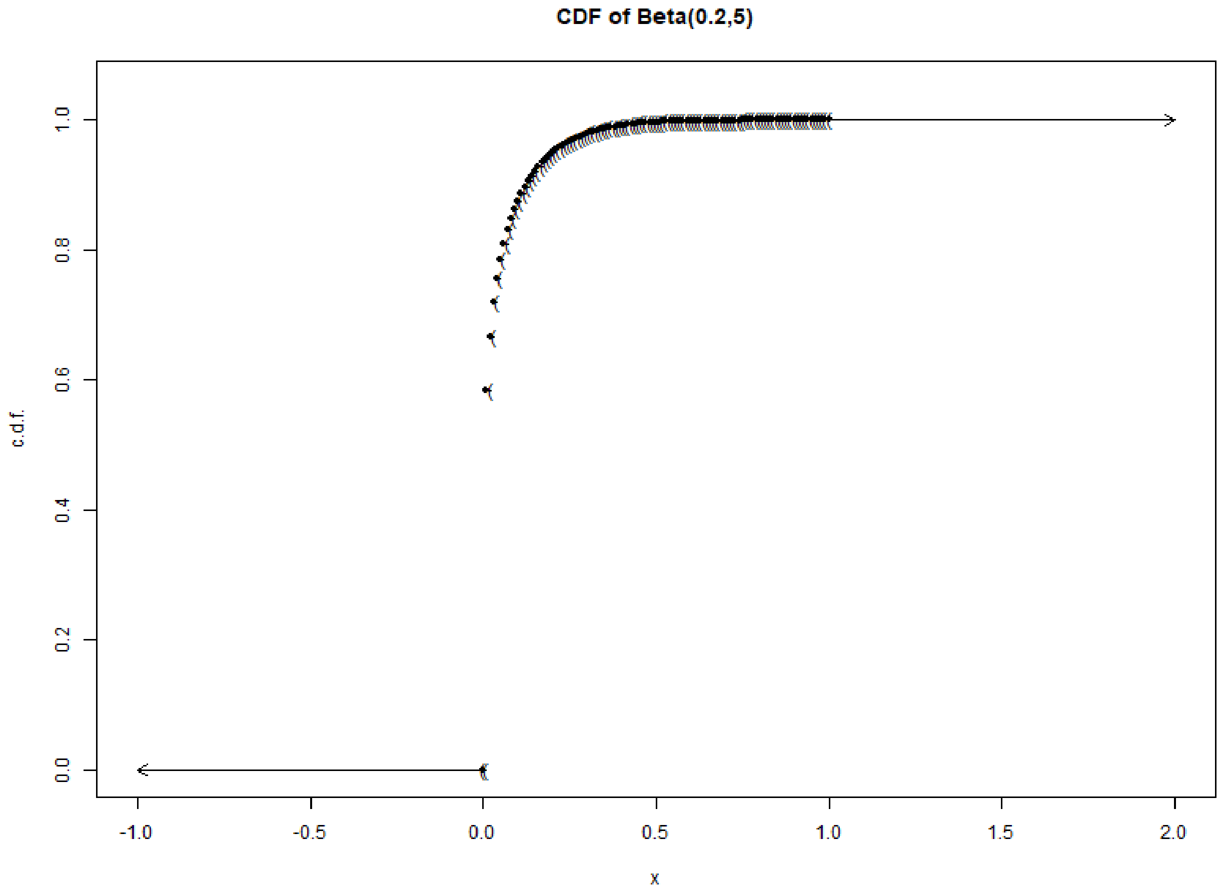




Histogram of Mean
Frequency
0
100
0.700
0.710
0.720
0.730
y$mean
Boxplot of Mean
0.700
0.725
NPP of Mean
Sample Quantiles
0.700
0.725
-3
-2
-1
0
1
2
3
Theoretical Quantiles
Histogram of Median
Frequency
0
150
0.89
0.91
0.93
y$median
Boxplot of Median
0.89
0.92
NPP of Median
Sample Quantiles
0.89
0.92
Theoretical Quantiles
Histogram of SD
Frequency
0
150
0.340
0.350
y$sd
Boxplot of SD
0.340
0.355
NPP of SD
Sample Quantiles
0.340
0.355
Theoretical Quantiles
Histogram of Minimum
Frequency
0
400
0.0e+00
1.0e-05
y$minm
Boxplot of Minimum
0.0e+00
1.5e-05
NPP of Minimum
Sample Quantiles
0.0e+00
1.5e-05
Theoretical Quantiles
Histogram of Maximum
Frequency
0
600
1
1
1
1
y$maxm
Boxplot of Maximum
1
1
1
1
NPP of Maximum
Sample Quantiles
1
1
1
1
Theoretical Quantiles
Histogram of IQR
Frequency
0
100
0.48
0.52
0.56
0.60
y$iqr
Boxplot of IQR
0.50
0.58
NPP of IQR
Sample Quantiles
0.50
0.58
Theoretical Quantiles

BETA DISTRIBUTION $(0.2, 5)$





BETA DISTRIBUTION

	Values of n to achieve normality (nn=1000, α=0.2, β=5)						
Statistic	n=10	n=50	n=100	n=500	n=1000	n=5000	Any n	Min(n)
Mean	No	No	No	No	Yes	Yes	Yes	1000
Median	No	No	No	No	No	Yes	Yes	5000
Std Dev	No	No	No	Yes	Yes	Yes	Yes	500
Min	No	No	No	No	No	No	No	NA
Max	No	No	No	No	No	No	No	NA
IQR	No	No	No	No	No	Yes	Yes	5000

Conclusion for Beta Distribution (α = 0.2, β = 5)

Normality Achieved:

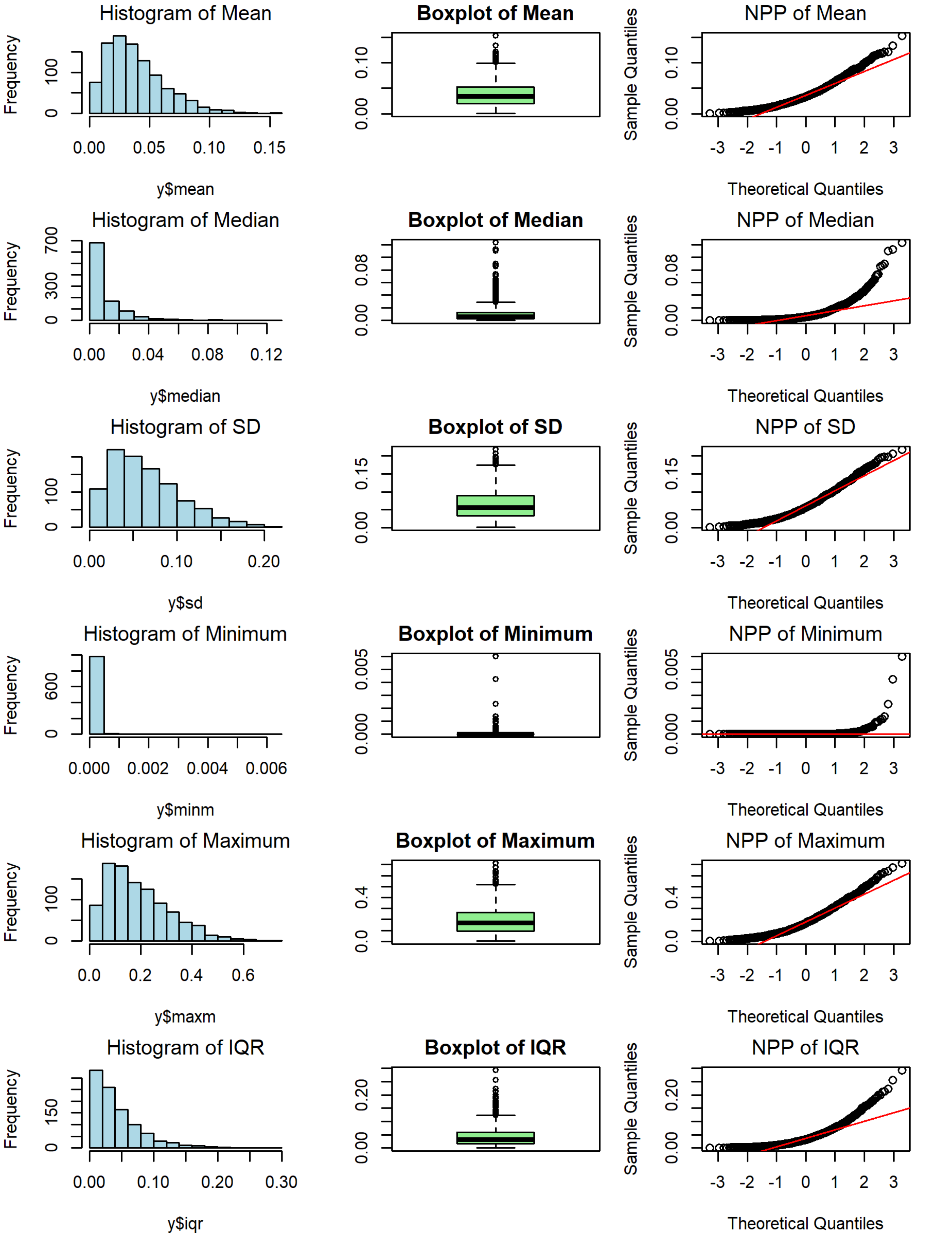
- **Mean:** Achieves normality starting from n ≥ 1000, with stabilization as the sample size increases.
- **Median:** Achieves normality at n ≥ 5000. Prior to that, it is still highly influenced by the skewed nature of the distribution.
- **Standard Deviation (SD):** Achieves normality starting from n ≥ 500, becoming more stable as sample size increases.
- **IQR:** Achieves normality from n ≥ 5000, with gradual stabilization for larger sample sizes.

Normality Not Achieved:

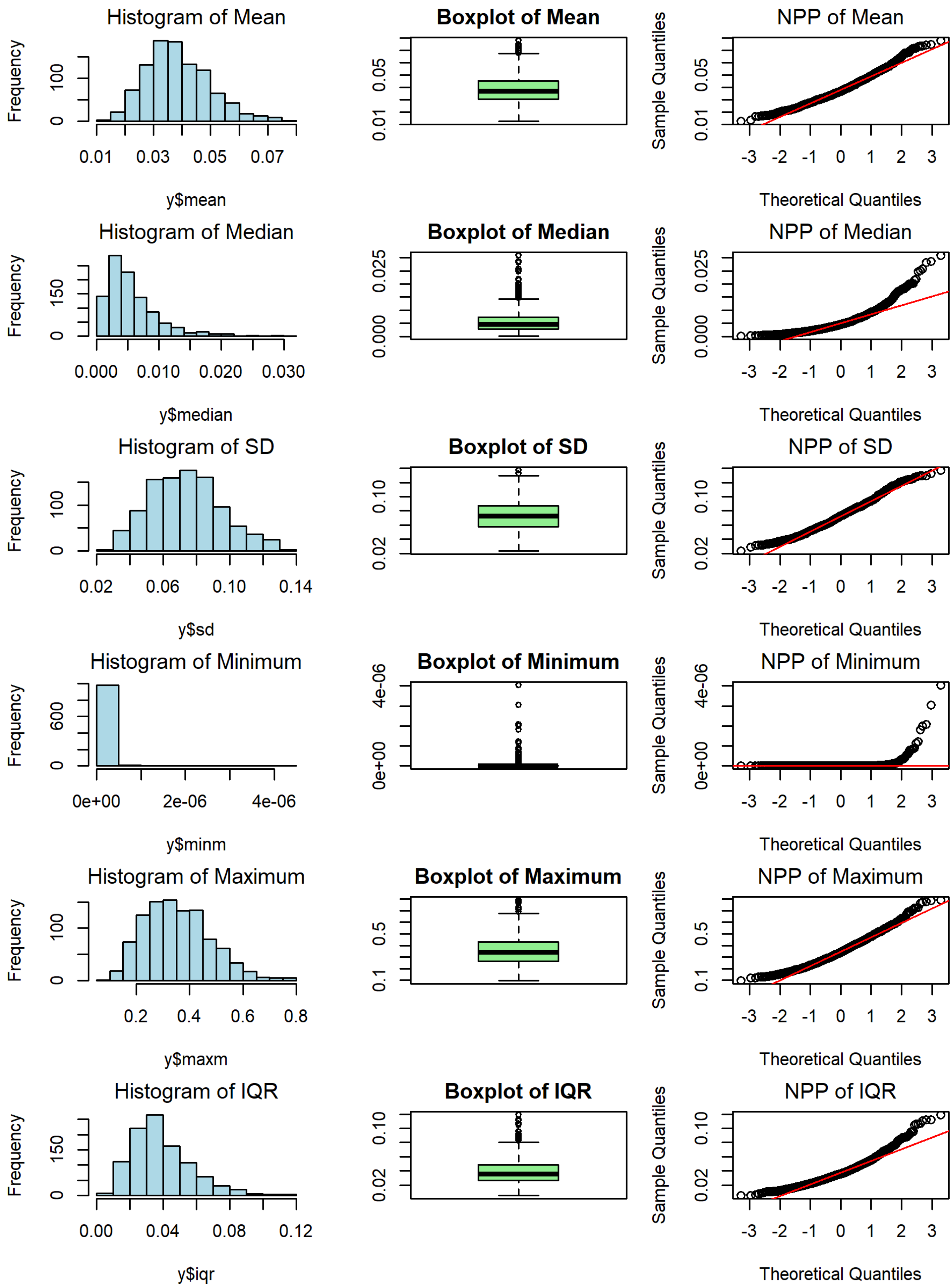
- **Minimum and Maximum:** These statistics do not achieve normality for any sample size due to the skewed shape of the Beta distribution with α = 0.2 and β = 5, where the distribution is highly skewed toward 0.

Overall: The Beta distribution with α = 0.2 and β = 5 shows a delayed convergence to normality. The mean stabilizes around n ≥ 1000, while the median and IQR only show normality at n ≥ 5000. The standard deviation begins to stabilize at n ≥ 500. As with other skewed Beta distributions, the minimum and maximum values do not achieve normality, reflecting the distribution's inherent skewness.

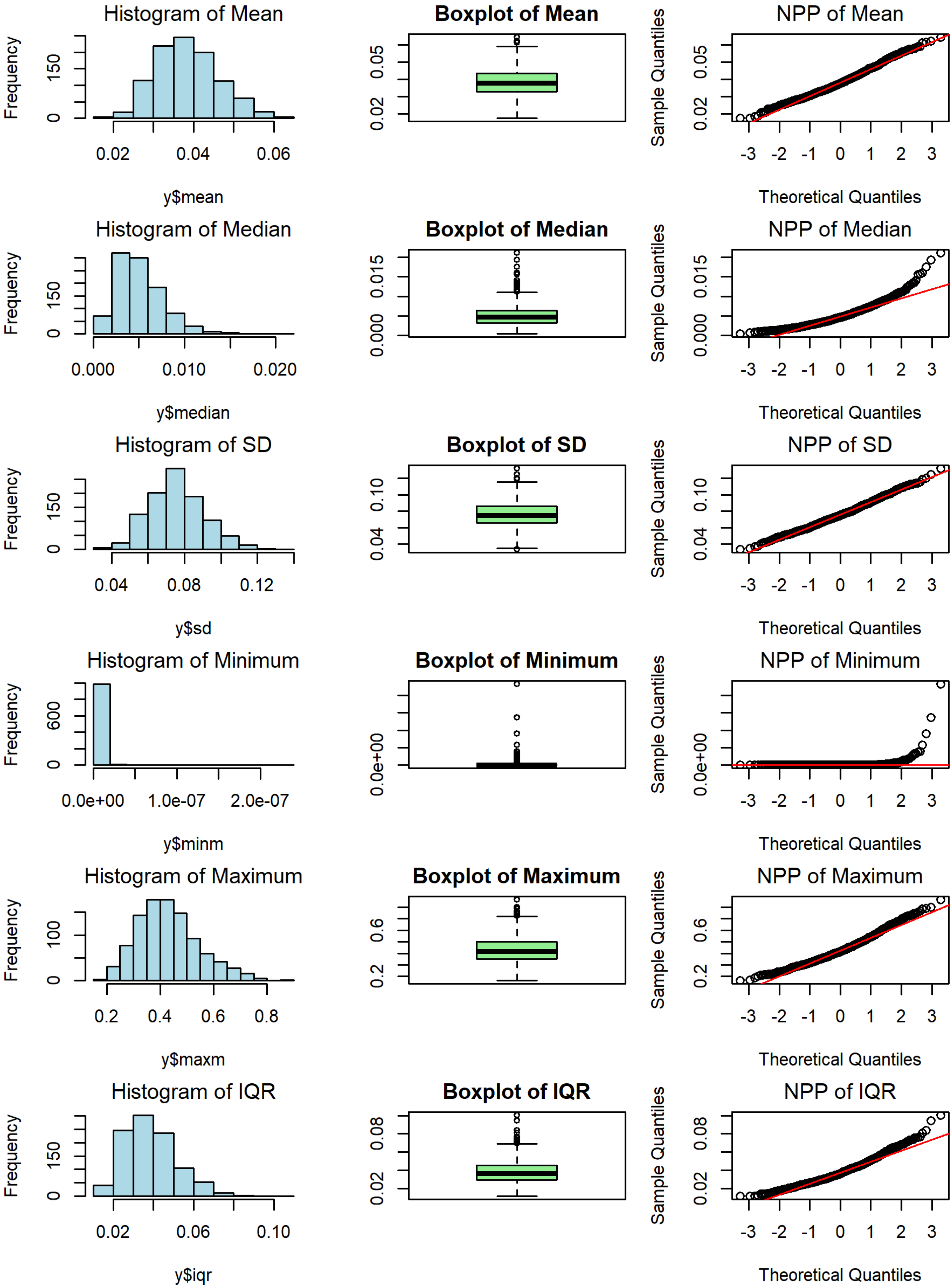
BETA DISTRIBUTION PLOT
(n=10, nn=1000, α=0.2, β=5)



BETA DISTRIBUTION PLOT
(n=50, nn=1000, α=0.2, β=5)

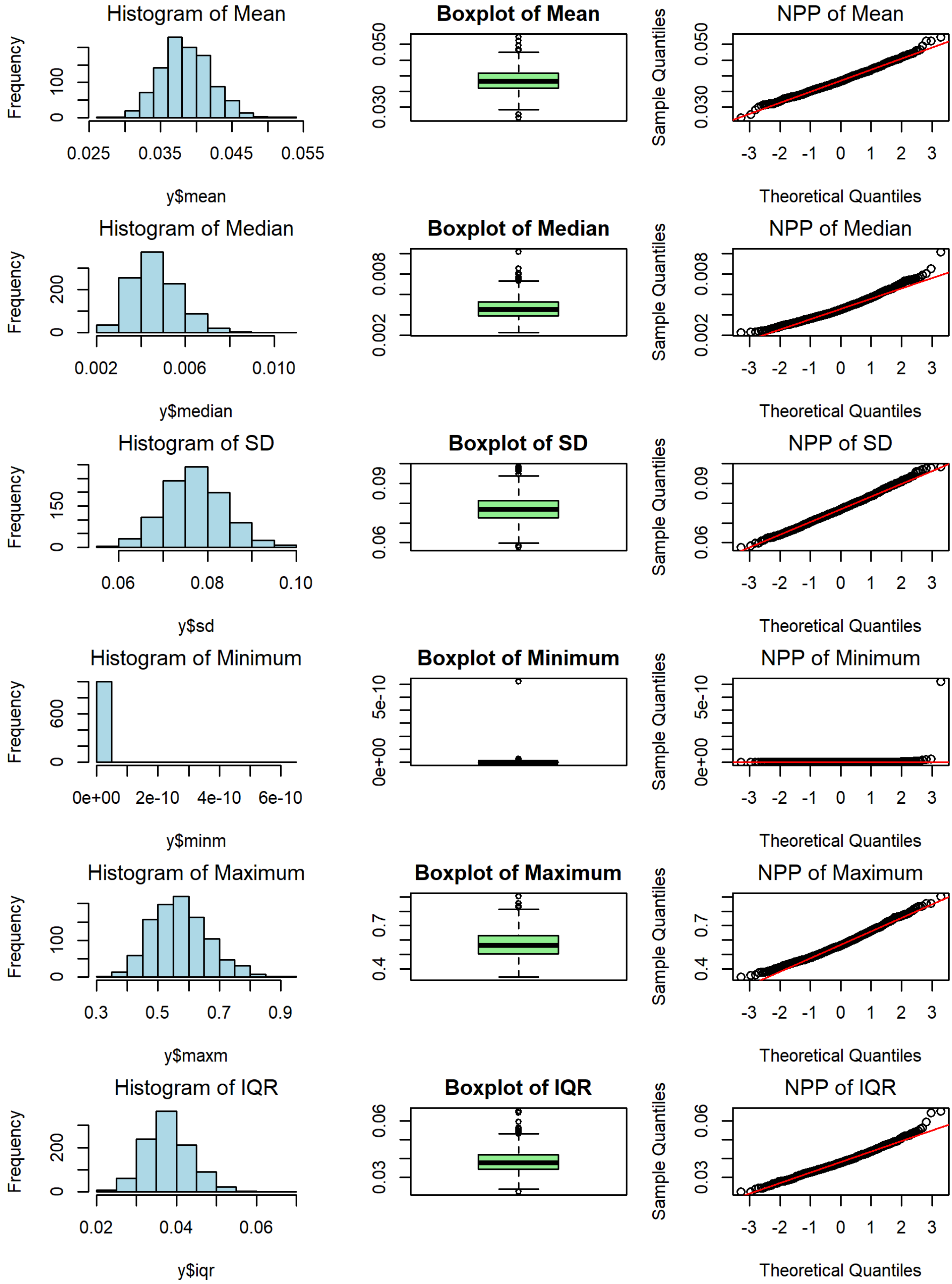


BETA DISTRIBUTION PLOT
(n=100, nn=1000, α=0.2, β=5)

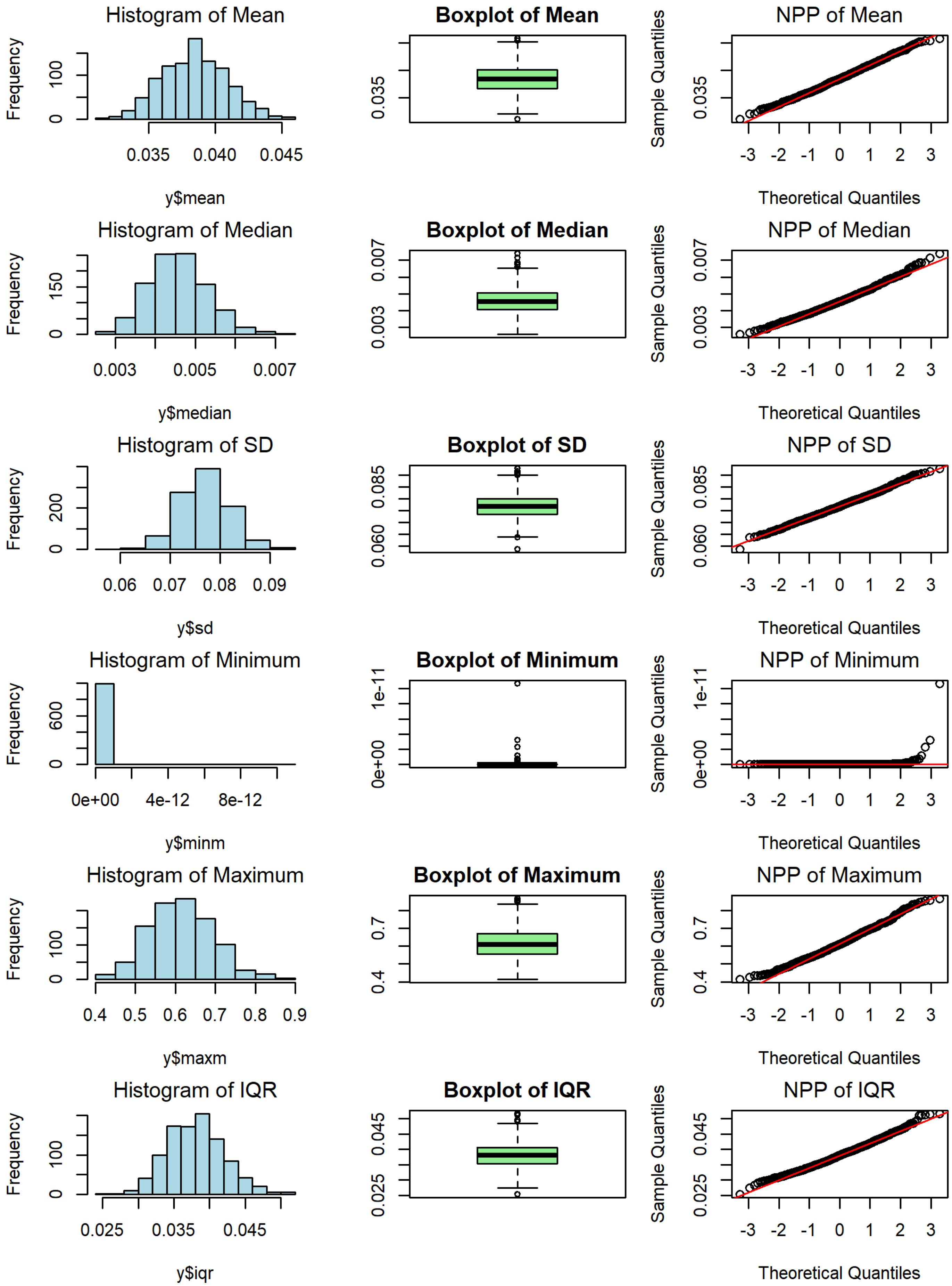


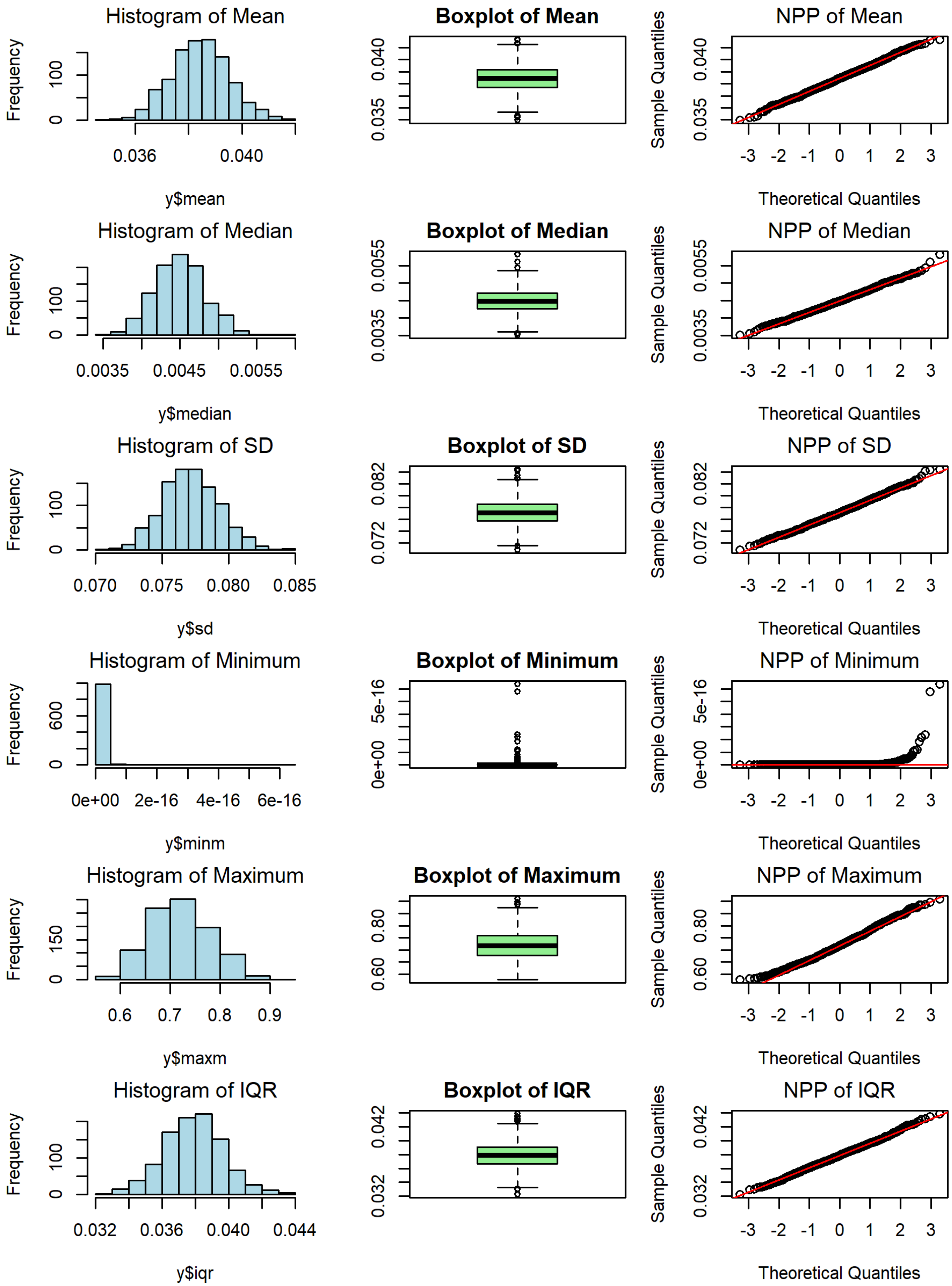
BETA DISTRIBUTION PLOT

(n=500, nn=1000, $\alpha=0.2$, $\beta=5$)



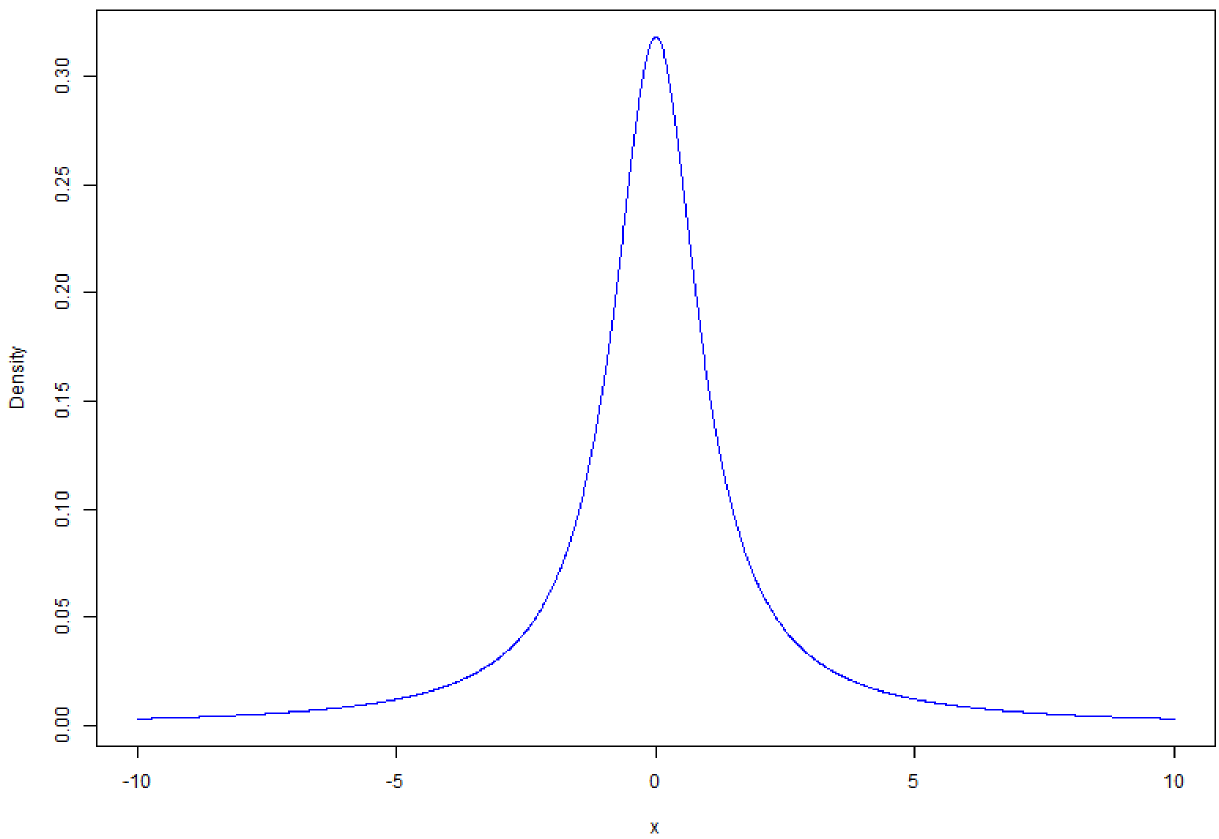
BETA DISTRIBUTION PLOT
(n=1000, nn=1000, α=0.2, β=5)



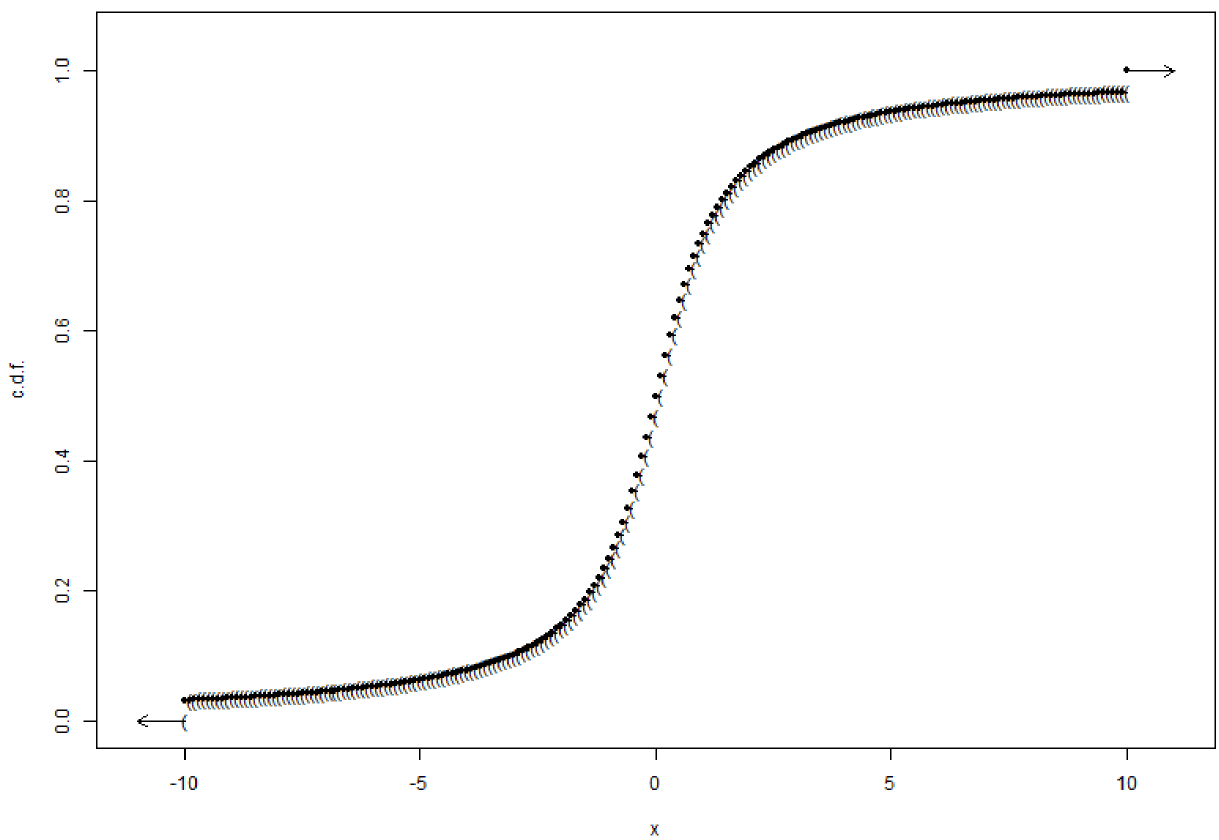

Histogram of Mean
Frequency
0
100
0.036
0.040
y$mean
Boxplot of Mean
0.035
0.040
NPP of Mean
Sample Quantiles
0.035
0.040
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of Median
Frequency
0
100
0.0035
0.0045
0.0055
y$median
Boxplot of Median
0.0035
0.0055
NPP of Median
Sample Quantiles
0.0035
0.0055
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of SD
Frequency
0
100
0.070
0.075
0.080
0.085
y$sd
Boxplot of SD
0.072
0.082
NPP of SD
Sample Quantiles
0.072
0.082
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of Minimum
Frequency
0
600
0e+00
2e-16
4e-16
6e-16
y$minm
Boxplot of Minimum
0e+00
5e-16
NPP of Minimum
Sample Quantiles
0e+00
5e-16
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of Maximum
Frequency
0
150
0.6
0.7
0.8
0.9
y$maxm
Boxplot of Maximum
0.60
0.80
NPP of Maximum
Sample Quantiles
0.60
0.80
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of IQR
Frequency
0
100
0.032
0.036
0.040
0.044
y$iqr
Boxplot of IQR
0.032
0.042
NPP of IQR
Sample Quantiles
0.032
0.042
-3 -2 -1 0 1 2 3
Theoretical Quantiles

CAUCHY DISTRIBUTION (0,1)

PDF of Cauchy(0, 1)



CDF of Cauchy(0,1)



CAUCHY DISTRIBUTION

	Values of n to achieve normality (nn=1000, loc=0, scale=1)							
Statistic	n=10	n=50	n=100	n=500	n=1000	n=5000	Any n	Min(n)
Mean	No	No	No	No	No	No	No	NA
Median	No	Yes	Yes	Yes	Yes	Yes	Yes	50
Std Dev	No	No	No	No	No	No	No	NA
Min	No	No	No	No	No	No	No	NA
Max	No	No	No	No	No	No	No	NA
IQR	No	No	No	Yes	Yes	Yes	Yes	500

Conclusion for Cauchy Distribution

Normality Achieved:

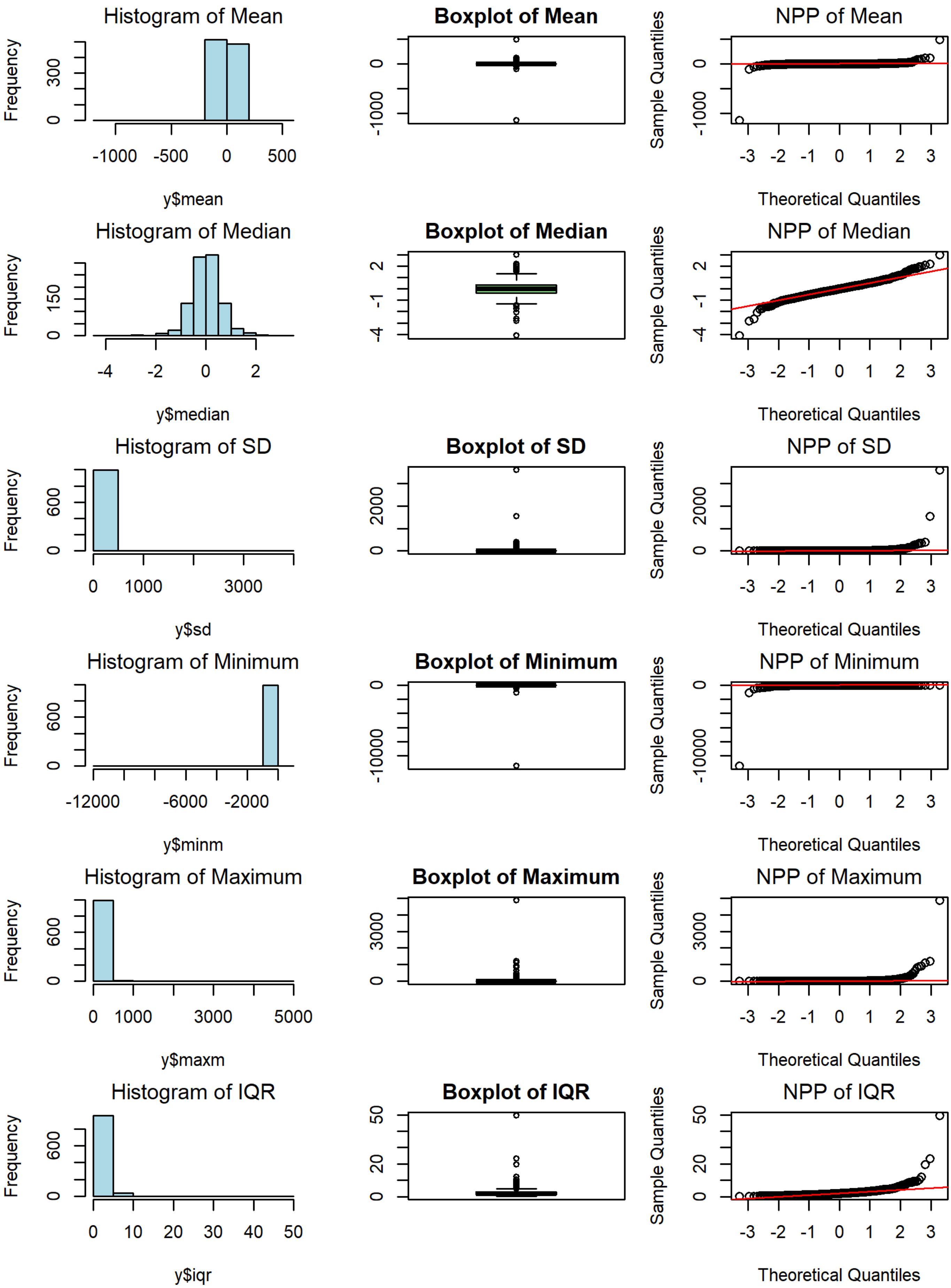
- **Median**: Achieves normality for n≥50, converging relatively quickly despite the heavy tails of the Cauchy distribution.
- **IQR**: Achieves normality for n≥500, requiring larger sample sizes due to the variability introduced by the distribution's heavy tails.

Normality Not Achieved:

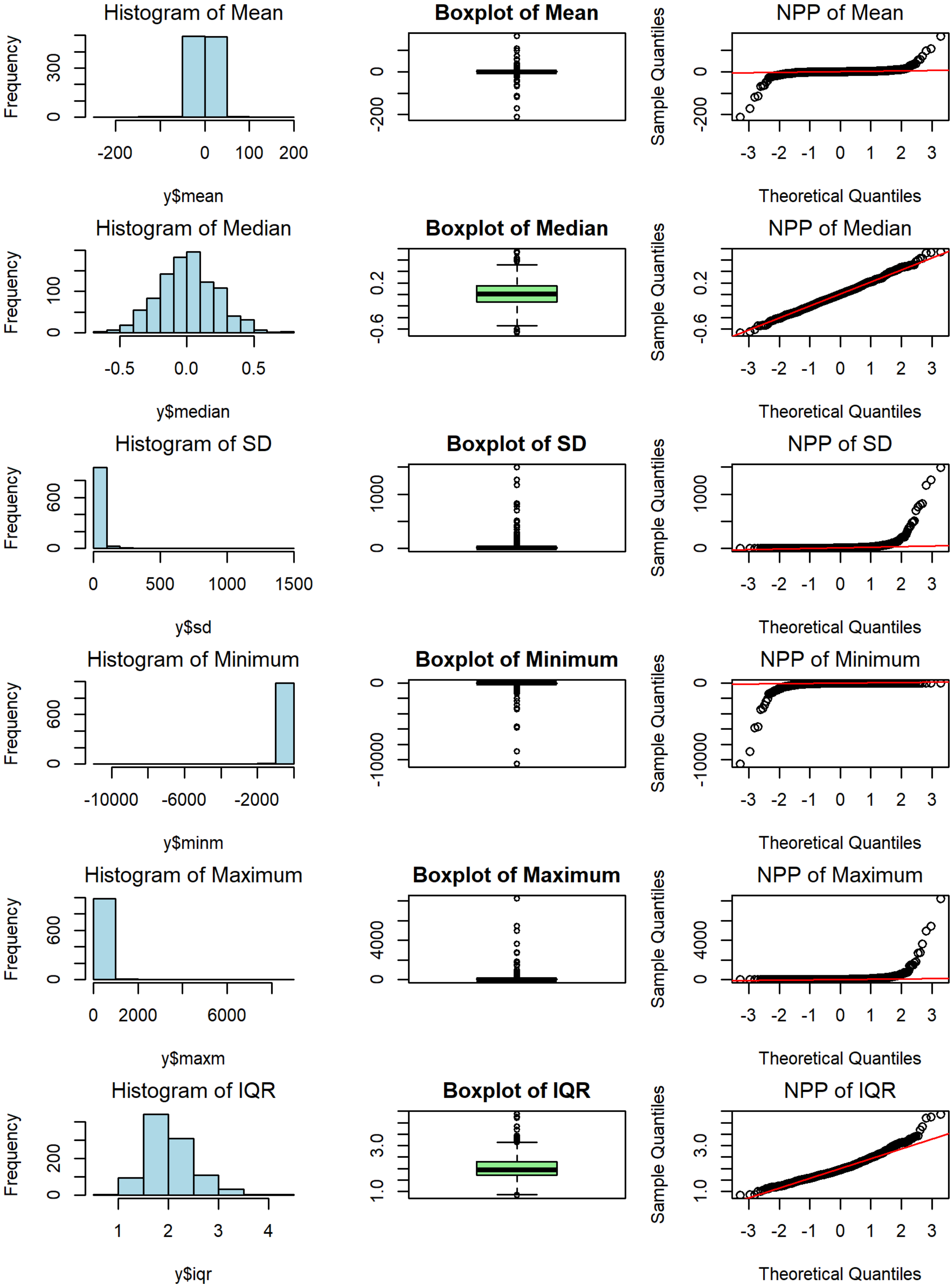
- **Mean**: Does not achieve normality for any n, as the Cauchy distribution lacks a finite mean, making the mean highly unstable.
- **Standard Deviation (SD)**: Does not achieve normality for any n, reflecting the undefined variance of the Cauchy distribution.
- **Minimum and Maximum**: Do not achieve normality for any n, as they are heavily influenced by the extreme values inherent in the heavy-tailed nature of the Cauchy distribution.

Overall:
The median and IQR are the only statistics to achieve normality, with the median converging faster (n≥50). The mean, standard deviation, minimum, and maximum fail to achieve normality due to the Cauchy distribution's lack of finite moments and heavy-tailed properties.

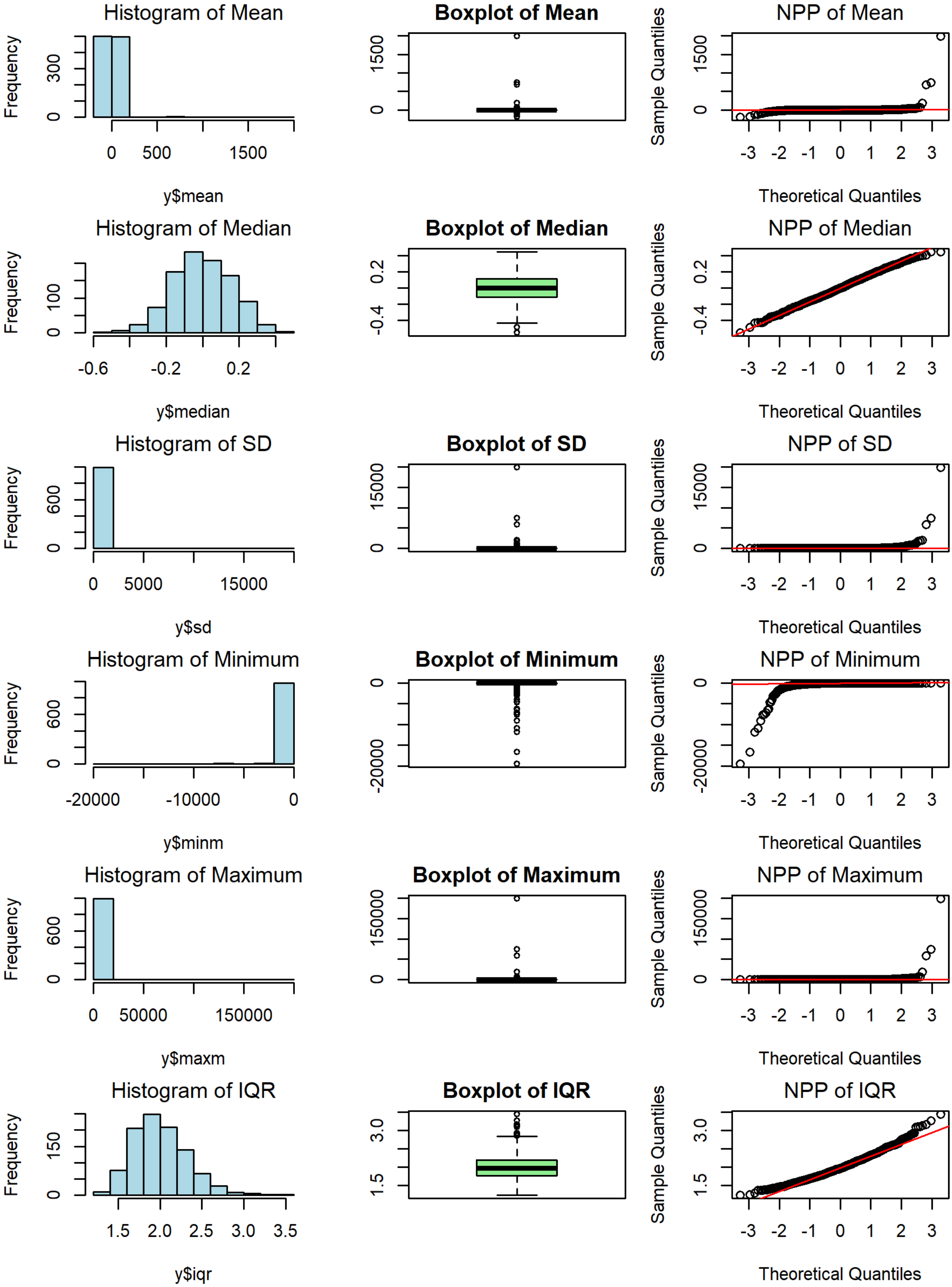
CAUCHY DISTRIBUTION PLOT
(n=10, nn=1000, loc=0, scale=1)

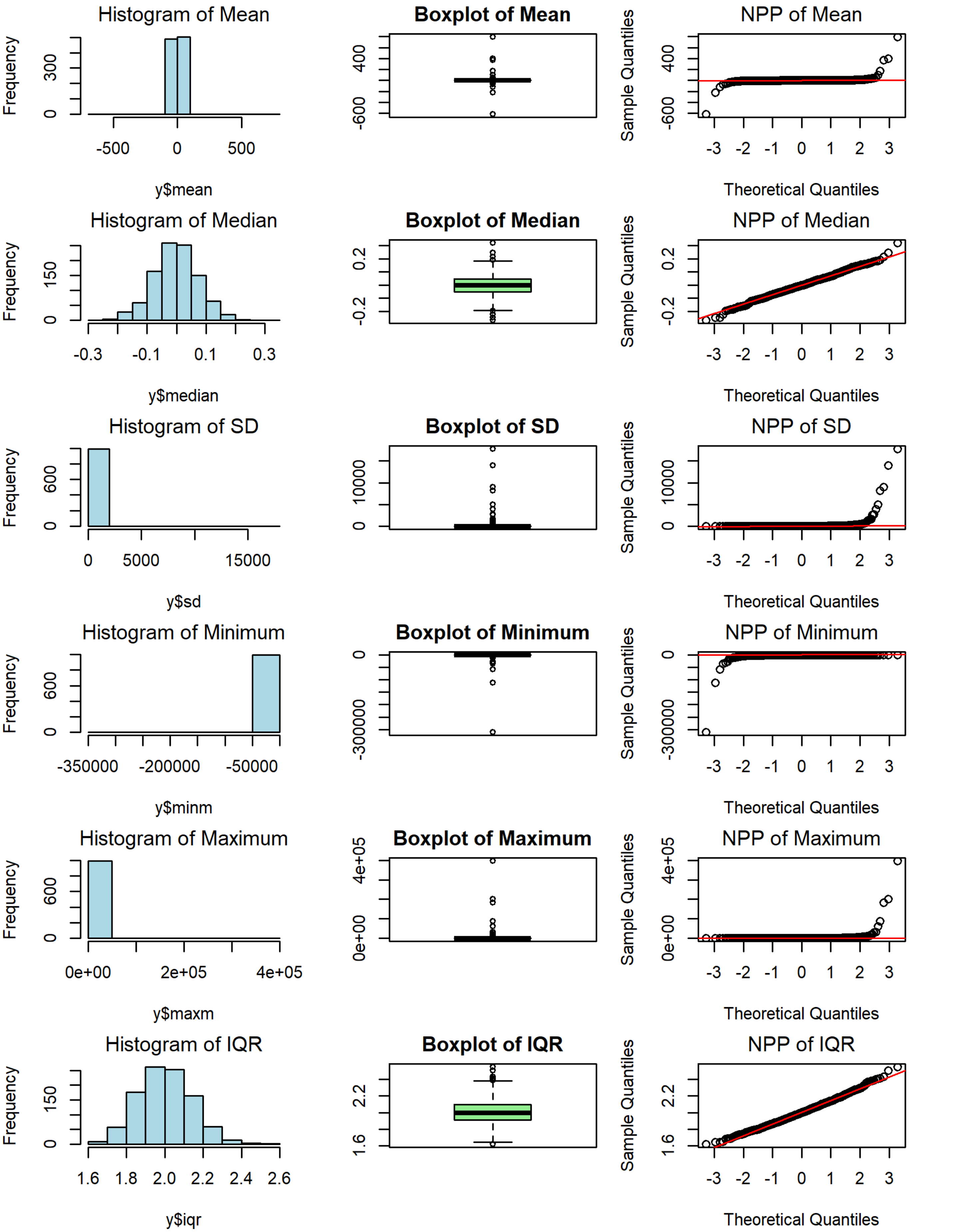


CAUCHY DISTRIBUTION PLOT
(n=50, nn=1000, loc=0, scale=1)

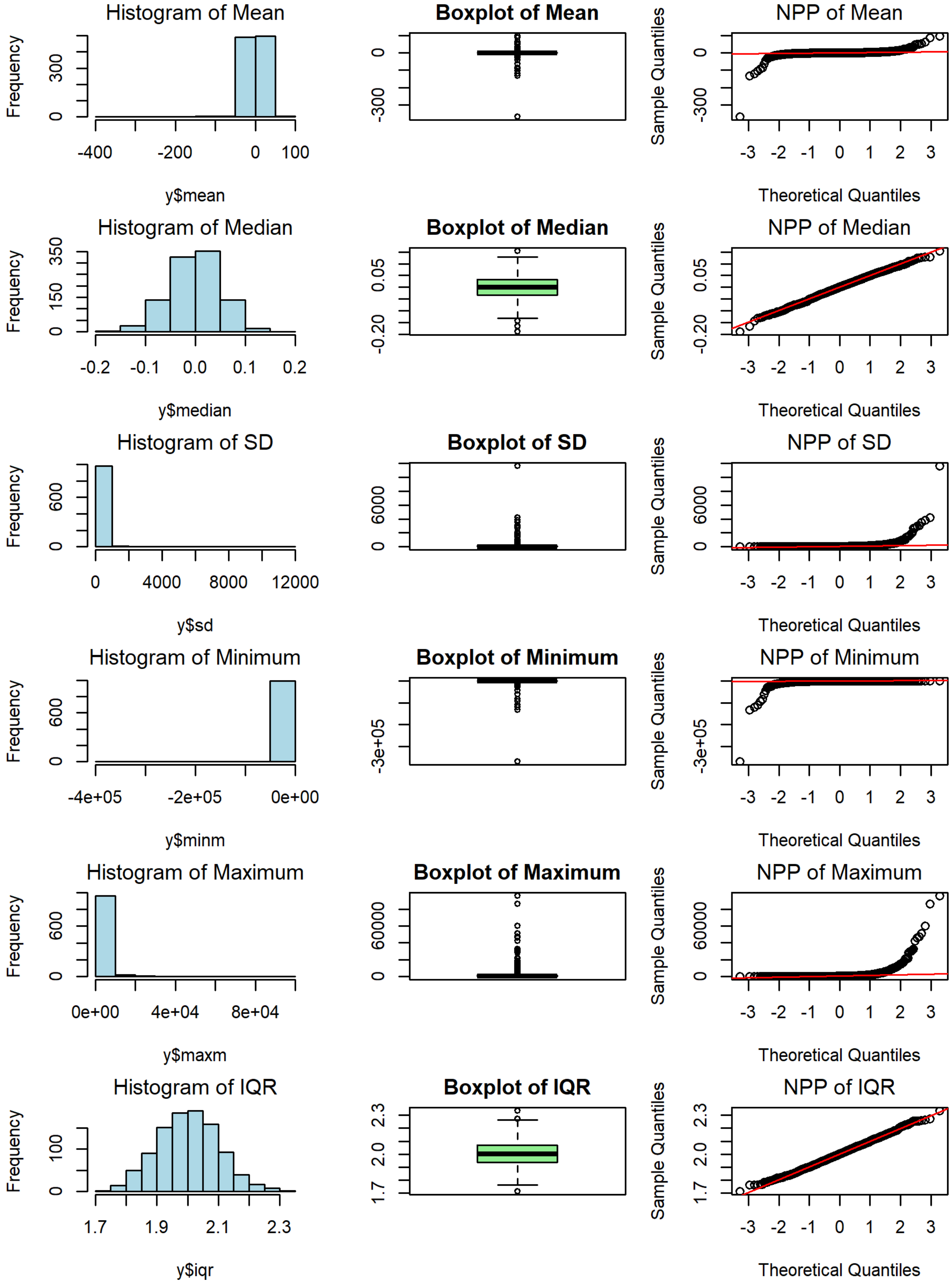


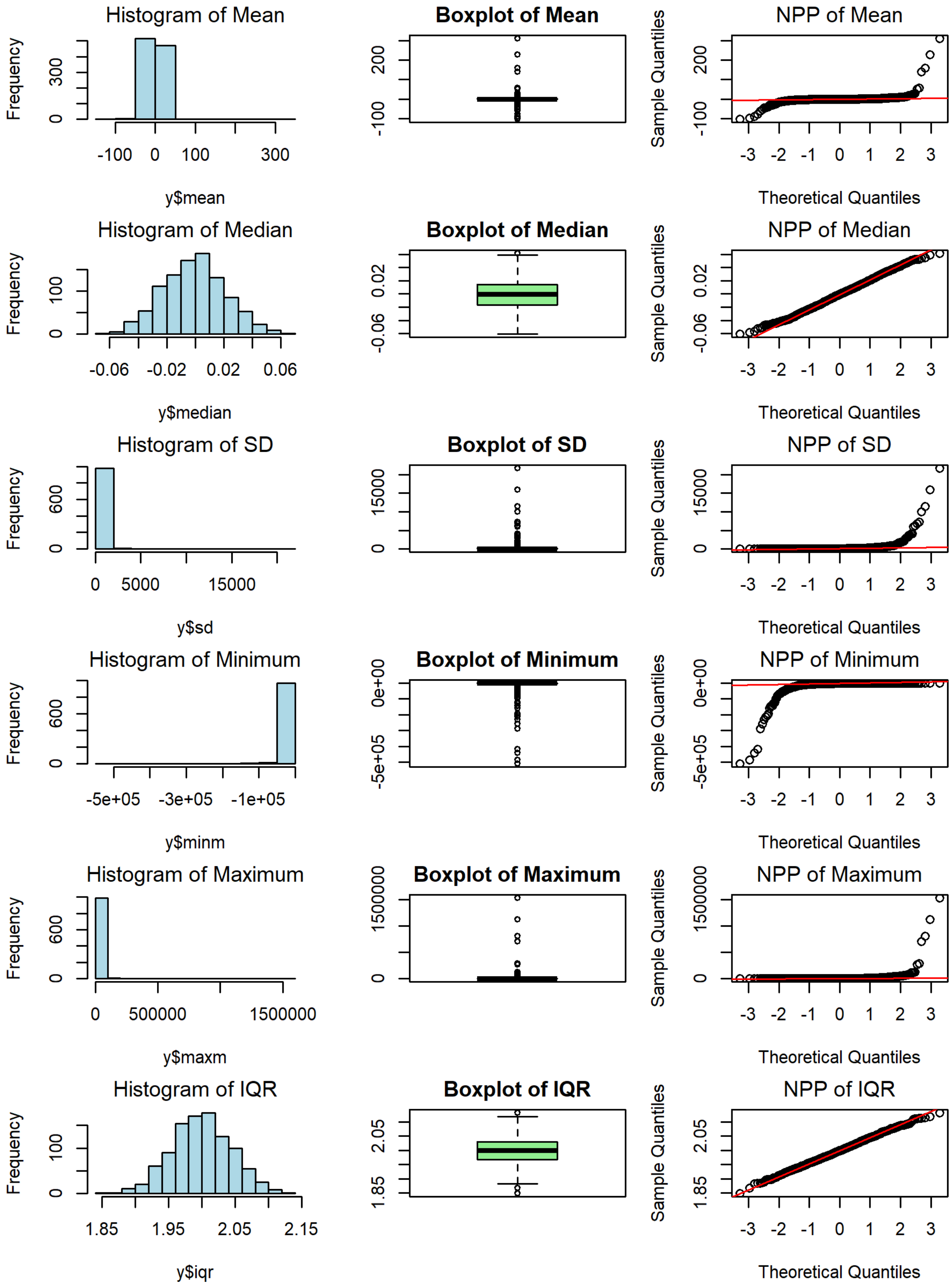
CAUCHY DISTRIBUTION PLOT
(n=100, nn=1000, loc=0, scale=1)



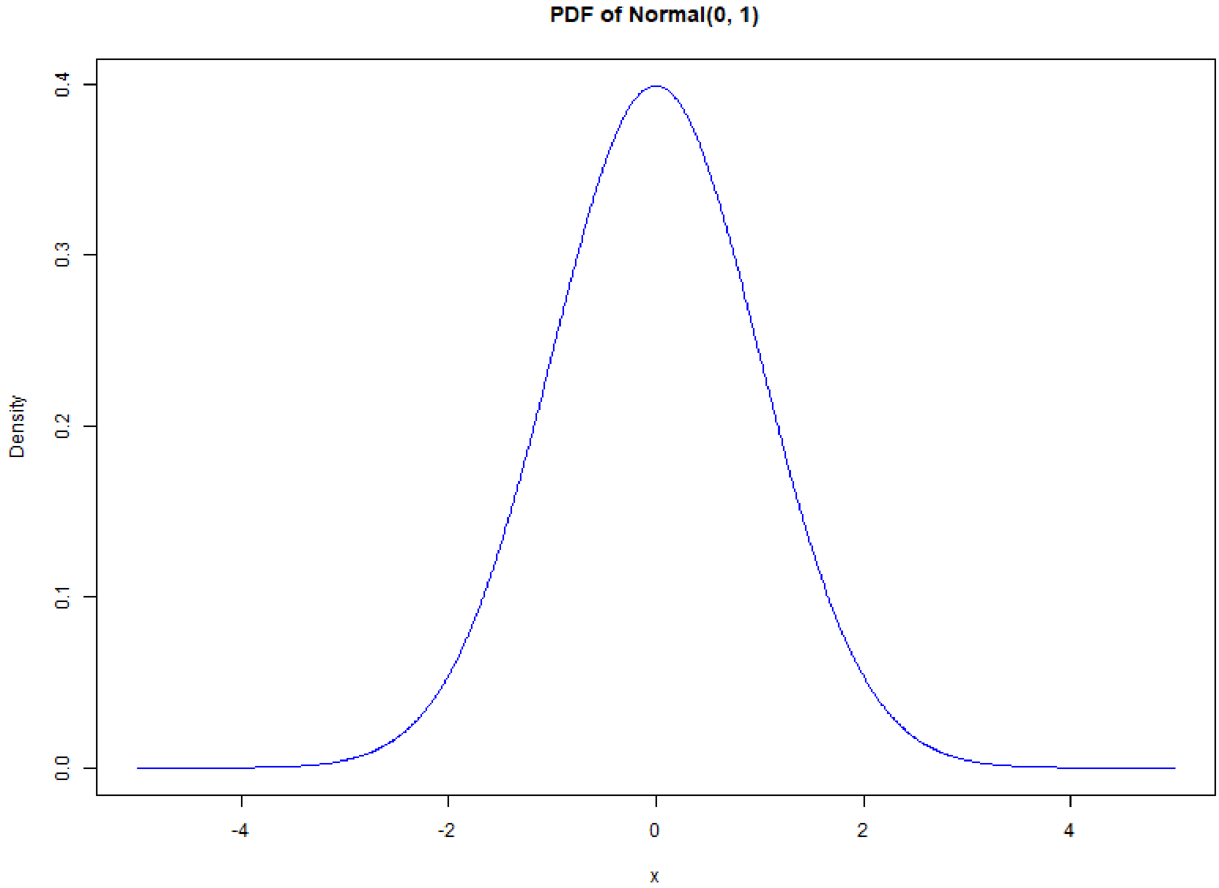

Histogram of Mean
Frequency
0
300
-500
0
500
y$mean
Boxplot of Mean
-600
400
NPP of Mean
Sample Quantiles
-600
400
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of Median
Frequency
0
150
-0.3
-0.1
0.1
0.3
y$median
Boxplot of Median
-0.2
0.2
NPP of Median
Sample Quantiles
-0.2
0.2
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of SD
Frequency
0
600
0
5000
15000
y$sd
Boxplot of SD
0
10000
NPP of SD
Sample Quantiles
0
10000
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of Minimum
Frequency
0
600
-350000
-200000
-50000
y$minm
Boxplot of Minimum
-300000
0
NPP of Minimum
Sample Quantiles
-300000
0
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of Maximum
Frequency
0
600
0e+00
2e+05
4e+05
y$maxm
Boxplot of Maximum
0e+00
4e+05
NPP of Maximum
Sample Quantiles
0e+00
4e+05
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of IQR
Frequency
0
150
1.6
1.8
2.0
2.2
2.4
2.6
y$iqr
Boxplot of IQR
1.6
2.2
NPP of IQR
Sample Quantiles
1.6
2.2
-3 -2 -1 0 1 2 3
Theoretical Quantiles

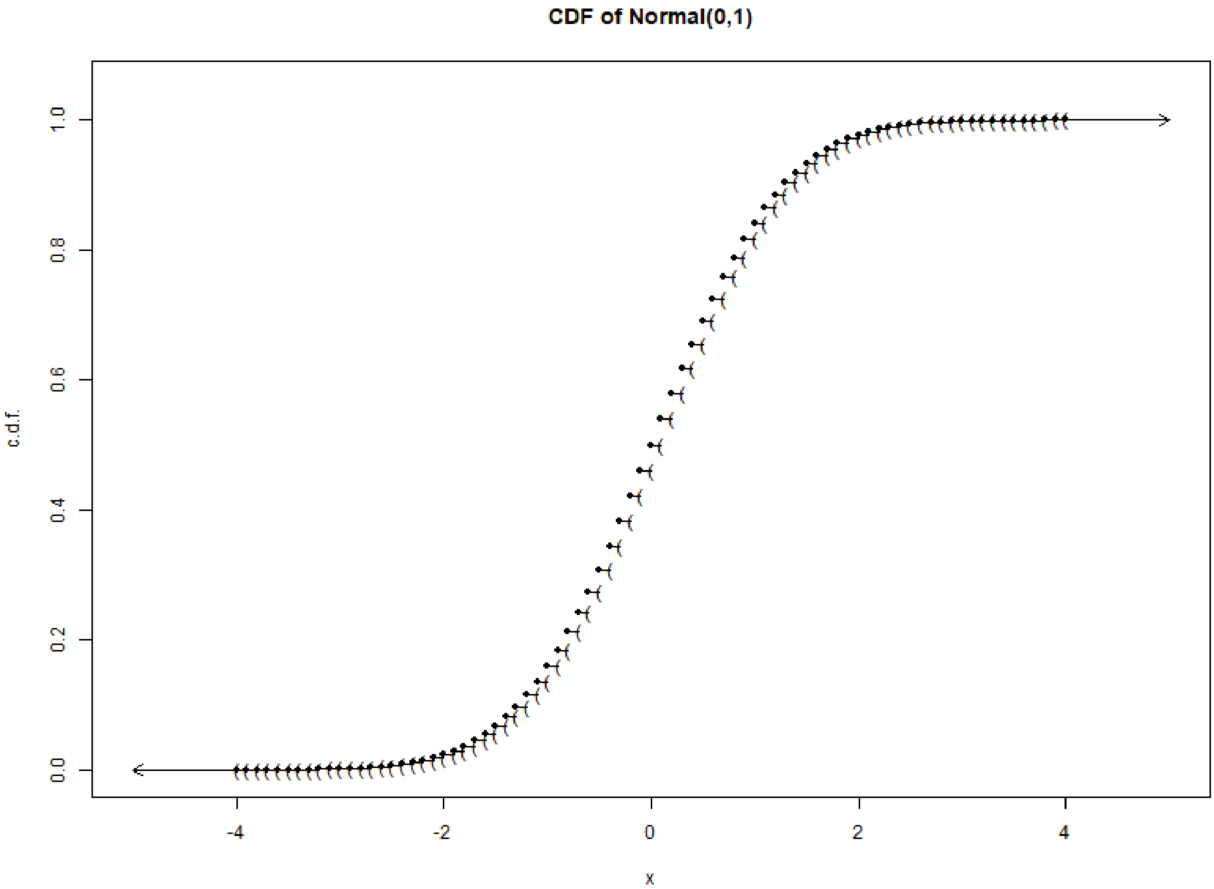
CAUCHY DISTRIBUTION PLOT
(n=1000, nn=1000, loc=0, scale=1)




Histogram of Mean
Frequency
0
300
-100
0
100
300
y$mean
Boxplot of Mean
-100
200
NPP of Mean
Sample Quantiles
-100
200
-3
-2
-1
0
1
2
3
Theoretical Quantiles
Histogram of Median
Frequency
0
100
-0.06
-0.02
0.02
0.06
y$median
Boxplot of Median
-0.06
0.02
NPP of Median
Sample Quantiles
-0.06
0.02
-3
-2
-1
0
1
2
3
Theoretical Quantiles
Histogram of SD
Frequency
0
600
0
5000
15000
y$sd
Boxplot of SD
0
15000
NPP of SD
Sample Quantiles
0
15000
-3
-2
-1
0
1
2
3
Theoretical Quantiles
Histogram of Minimum
Frequency
0
600
-5e+05
-3e+05
-1e+05
y$minm
Boxplot of Minimum
-5e+05
0e+00
NPP of Minimum
Sample Quantiles
-5e+05
0e+00
-3
-2
-1
0
1
2
3
Theoretical Quantiles
Histogram of Maximum
Frequency
0
600
0
500000
1500000
y$maxm
Boxplot of Maximum
0
1500000
NPP of Maximum
Sample Quantiles
0
1500000
-3
-2
-1
0
1
2
3
Theoretical Quantiles
Histogram of IQR
Frequency
0
100
1.85
1.95
2.05
2.15
y$iqr
Boxplot of IQR
1.85
2.05
NPP of IQR
Sample Quantiles
1.85
2.05
-3
-2
-1
0
1
2
3
Theoretical Quantiles

NORMAL DISTRIBUTION (0,1)





NORMAL DISTRIBUTION

	Values of n to achieve normality (nn=1000, μ=0, σ=1)							
Statistic	n=10	n=50	n=100	n=500	n=1000	n=5000	Any n	Min(n)
Mean	Yes	Yes	Yes	Yes	Yes	Yes	Yes	10
Median	Yes	Yes	Yes	Yes	Yes	Yes	Yes	10
Std Dev	Yes	Yes	Yes	Yes	Yes	Yes	Yes	10
Min	No	No	No	No	No	No	No	NA
Max	No	No	Yes	Yes	No	No	Yes	100
IQR	No	No	No	Yes	Yes	Yes	Yes	500

Conclusion for Normal Distribution

Normality Achieved:

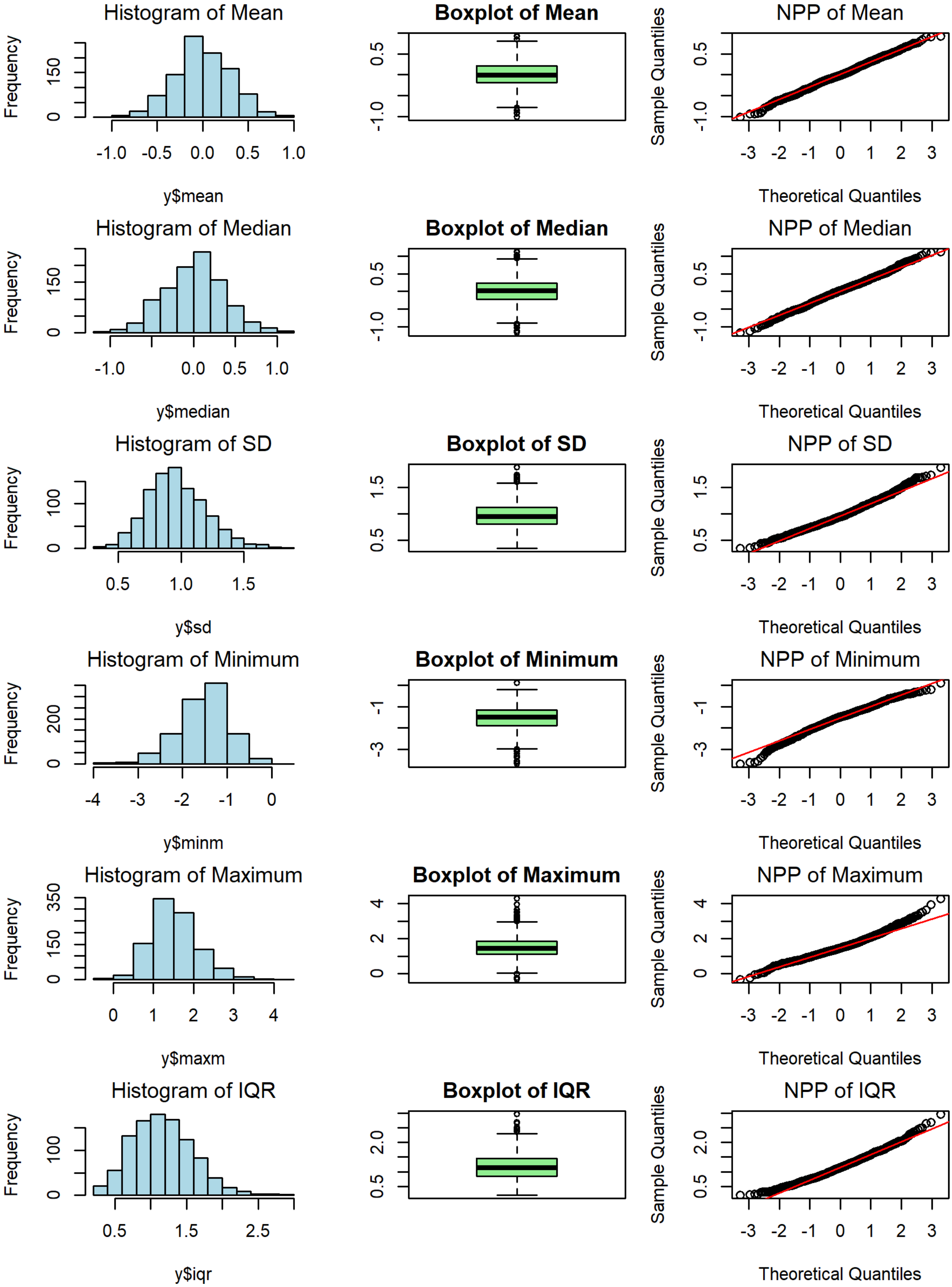
- **Mean**: Achieves normality for n≥10, reflecting rapid convergence due to the Central Limit Theorem (CLT).
- **Median**: Achieves normality for n≥10, showing a faster convergence compared to skew-sensitive statistics.
- **Standard Deviation (SD)**: Achieves normality for n≥10, demonstrating consistent behaviour across sample sizes.
- **IQR**: Achieves normality for n≥500, indicating moderate convergence influenced by the parent distribution's skewness.

Normality Not Achieved:

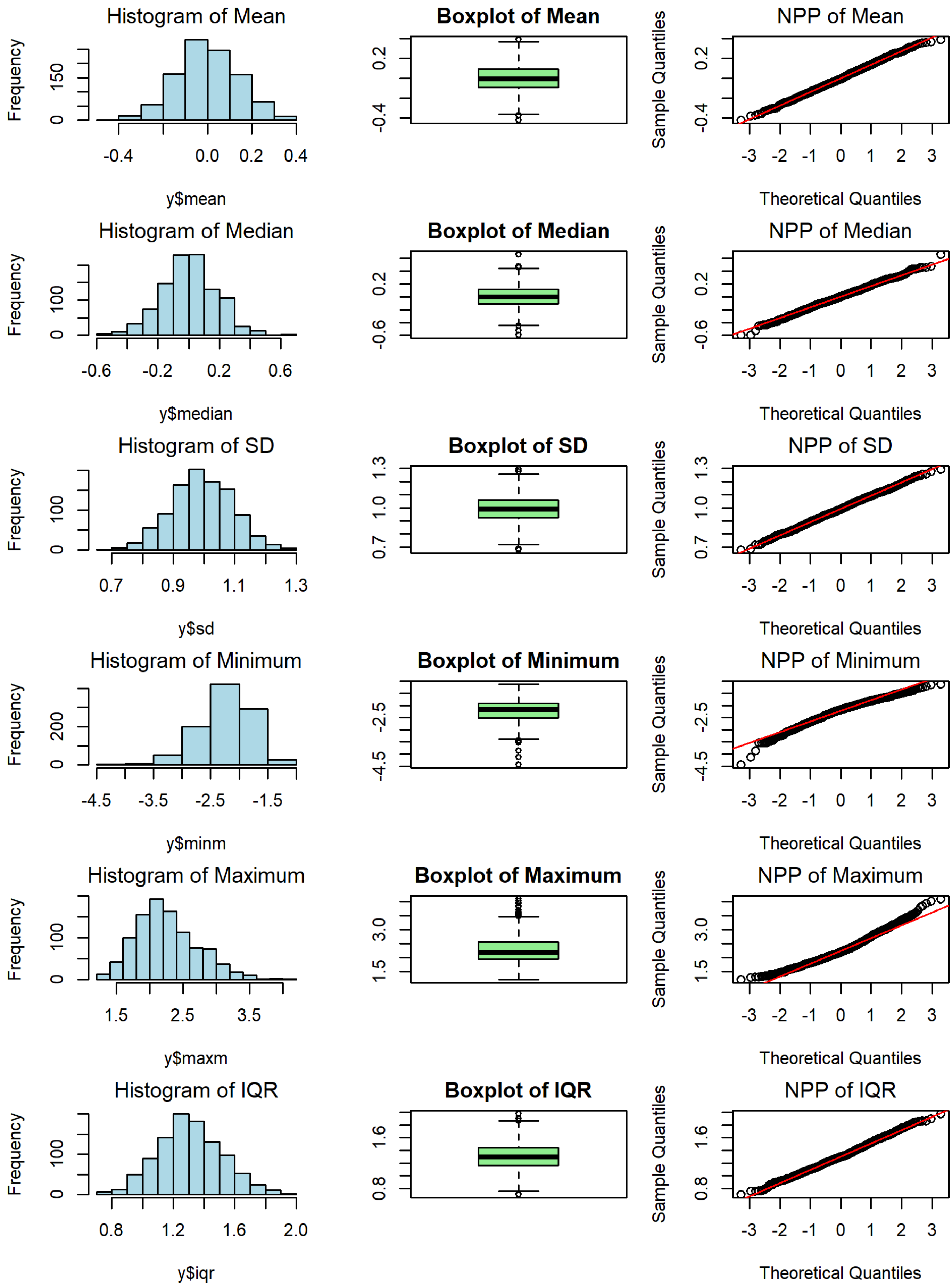
- **Minimum**: Does not achieve normality for any n, as it is highly sensitive to extreme values and the asymmetry of the Normal distribution.
- **Maximum**: Achieves normality only for n=100 and 500, but fails for larger sample sizes, likely due to variability in extreme values.

Overall:
The mean, median, and standard deviation converge to normality quickly, making them reliable statistics for smaller sample sizes. However, skew-sensitive statistics like the minimum and maximum are less consistent, with the minimum failing to achieve normality entirely. The IQR requires larger sample sizes to approximate normality.

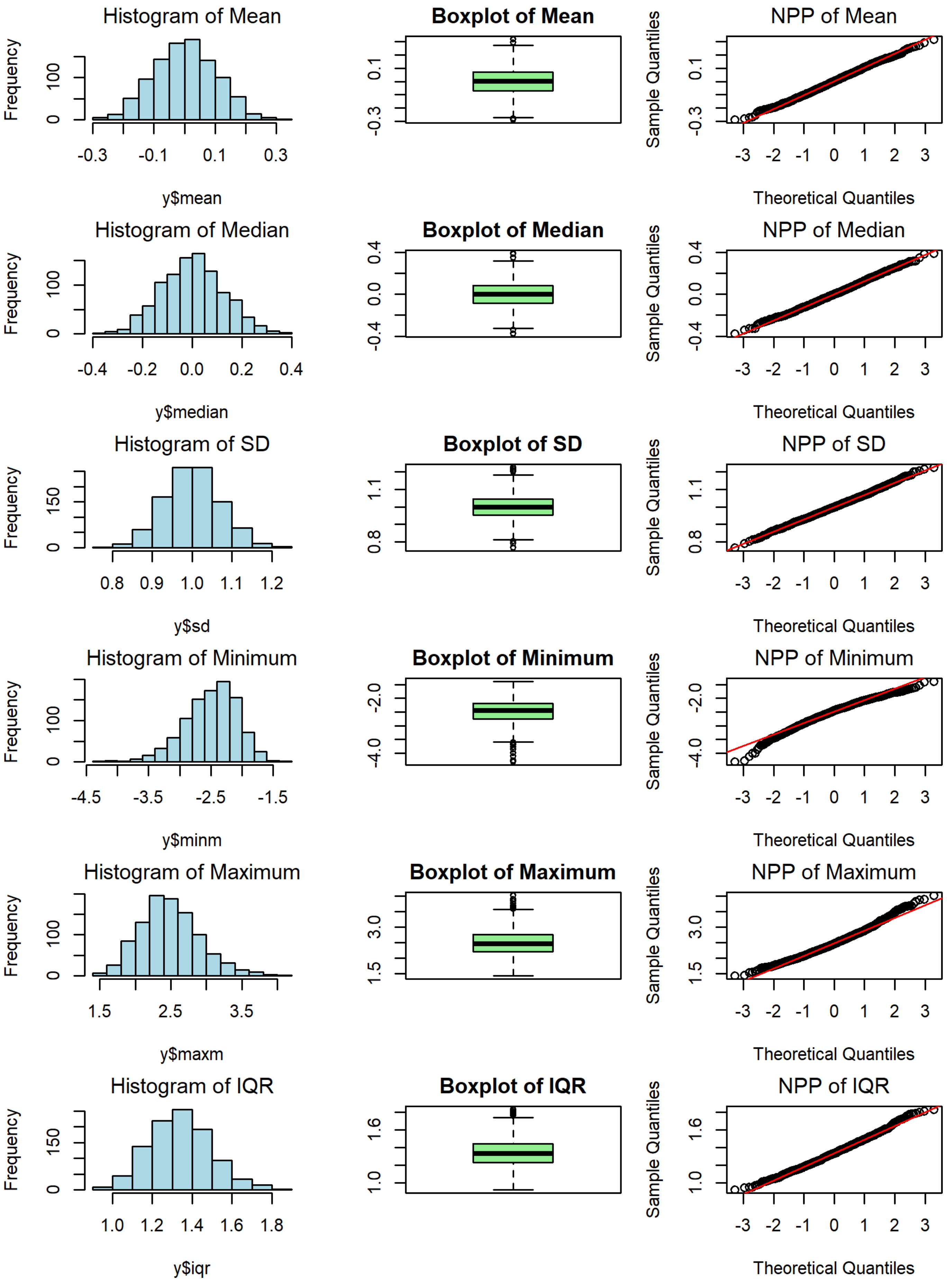
NORMAL DISTRIBUTION PLOT
(n=10, nn=1000, μ=0, σ=1)

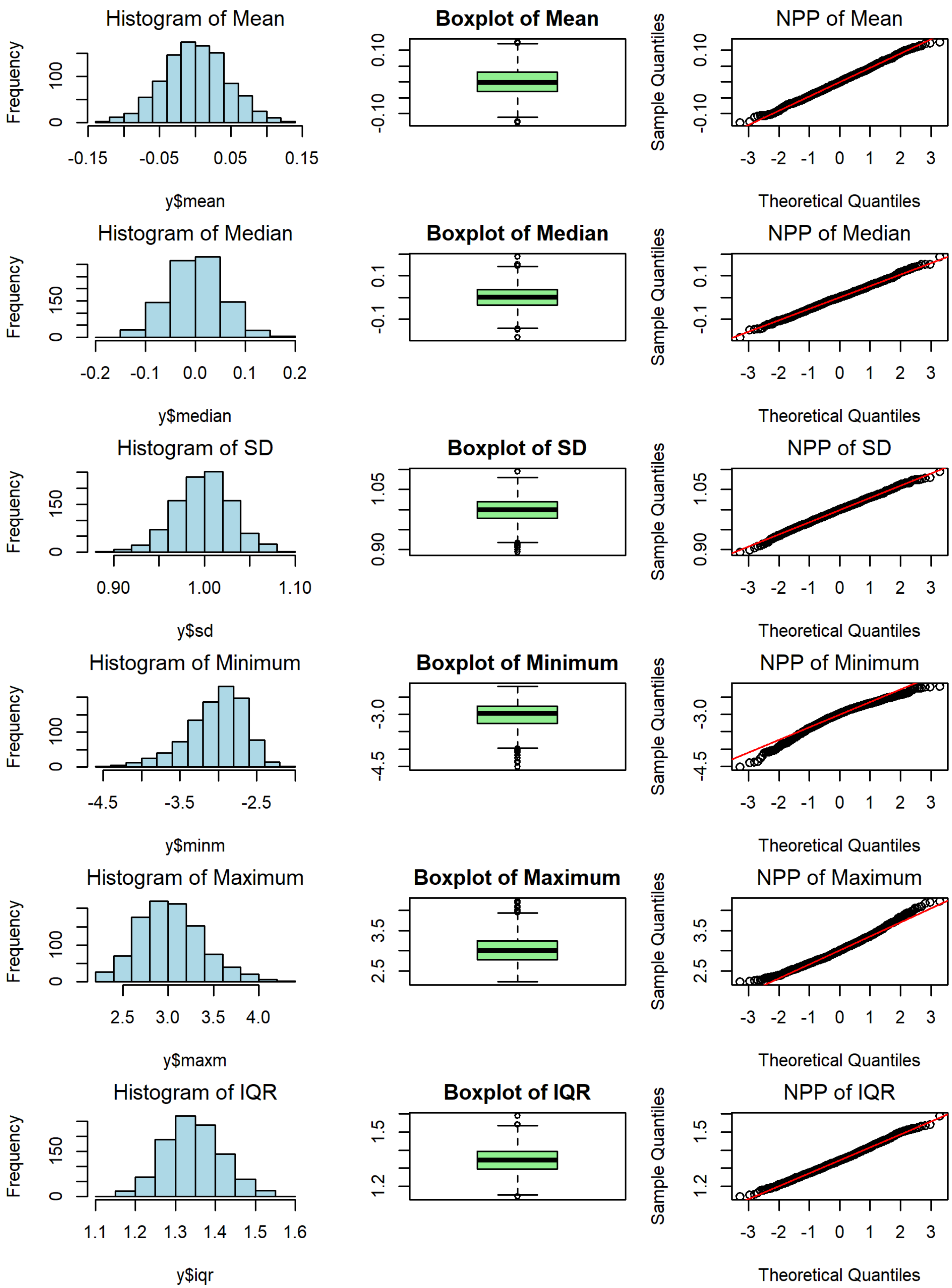


NORMAL DISTRIBUTION PLOT
(n=50, nn=1000, μ=0, σ=1)



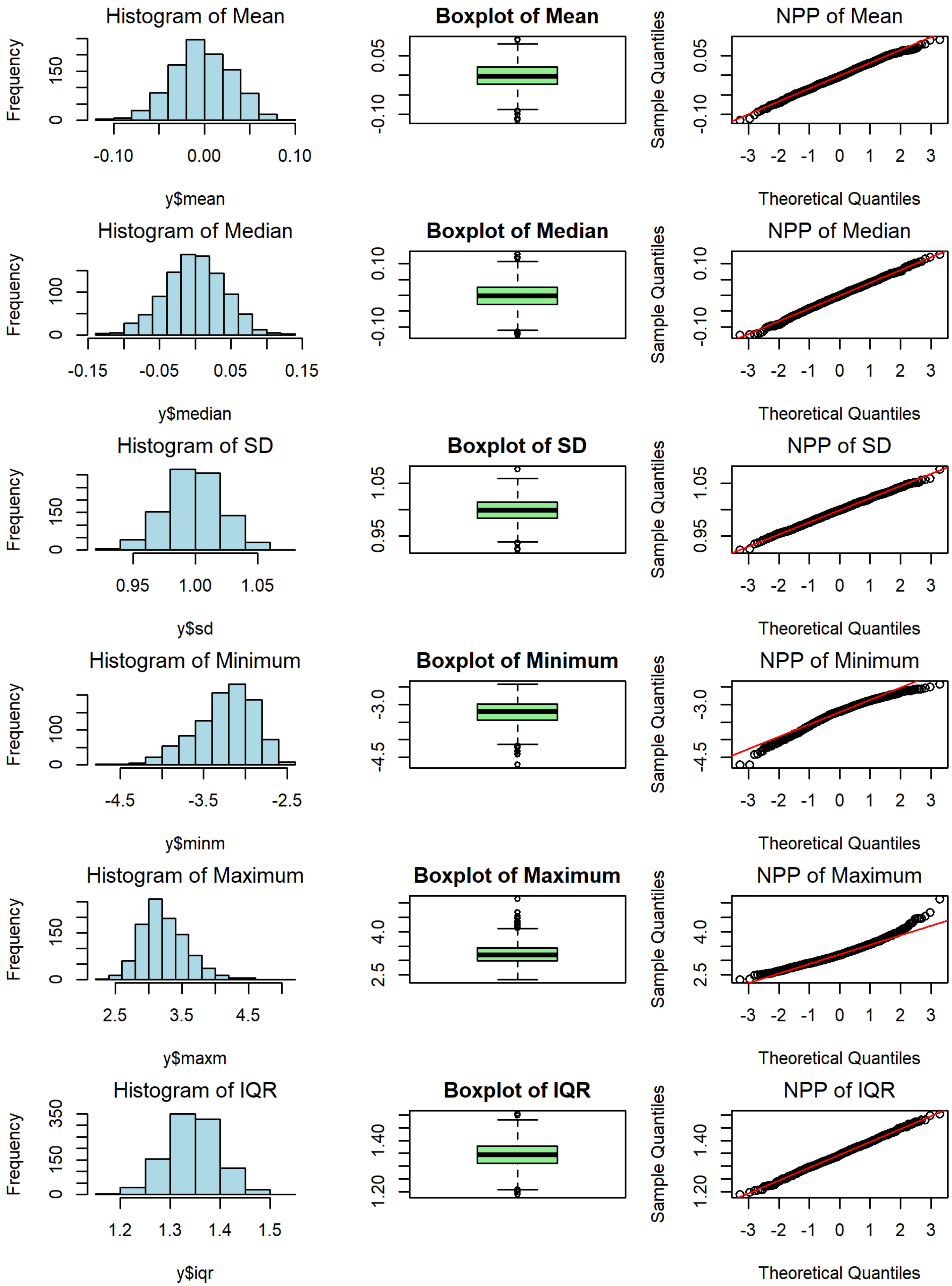
NORMAL DISTRIBUTION PLOT
(n=100, nn=1000, μ=0, σ=1)

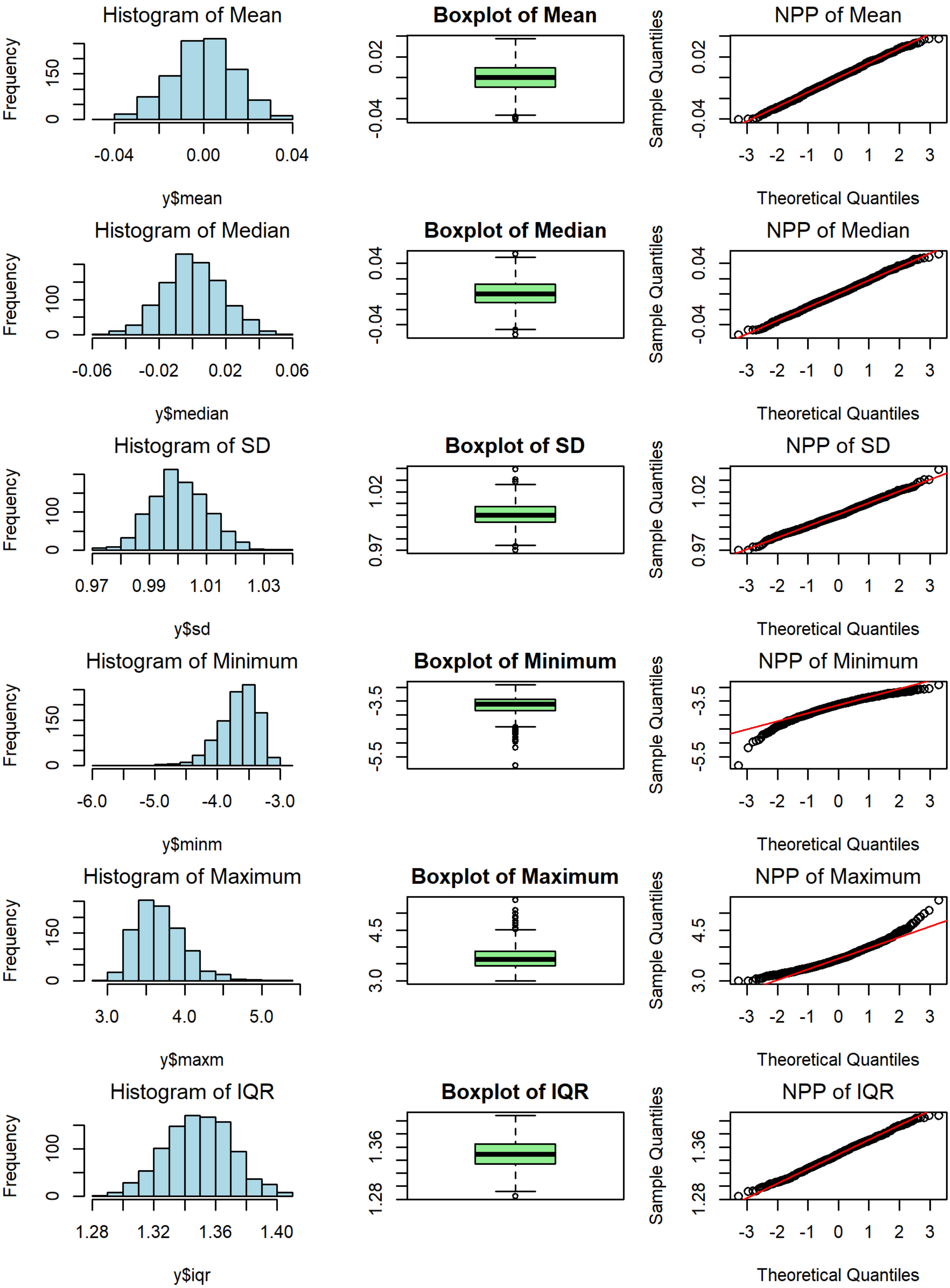




Histogram of Mean
Frequency
0
100
-0.15
-0.05
0.05
0.15
y$mean
Boxplot of Mean
-0.10
0.10
NPP of Mean
Sample Quantiles
Theoretical Quantiles
-3
-2
-1
0
1
2
3
Histogram of Median
150
-0.2
-0.1
0.0
0.1
0.2
y$median
Boxplot of Median
-0.1
0.1
NPP of Median
Histogram of SD
0.90
1.00
1.10
y$sd
Boxplot of SD
0.90
1.05
NPP of SD
Histogram of Minimum
-4.5
-3.5
-2.5
y$minm
Boxplot of Minimum
-4.5
-3.0
NPP of Minimum
Histogram of Maximum
2.5
3.0
3.5
4.0
y$maxm
Boxplot of Maximum
2.5
3.5
NPP of Maximum
Histogram of IQR
1.1
1.2
1.3
1.4
1.5
1.6
y$iqr
Boxplot of IQR
1.2
1.5
NPP of IQR

NORMAL DISTRIBUTION PLOT
(n=1000, nn=1000, μ=0, σ=1)




Histogram of Mean
Frequency
0
150
-0.04
0.00
0.04
y$mean
Boxplot of Mean
-0.04
0.02
NPP of Mean
Sample Quantiles
-0.04
0.02
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of Median
Frequency
0
100
-0.06
-0.02
0.02
0.06
y$median
Boxplot of Median
-0.04
0.04
NPP of Median
Sample Quantiles
-0.04
0.04
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of SD
Frequency
0
100
0.97
0.99
1.01
1.03
y$sd
Boxplot of SD
0.97
1.02
NPP of SD
Sample Quantiles
0.97
1.02
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of Minimum
Frequency
0
150
-6.0
-5.0
-4.0
-3.0
y$minm
Boxplot of Minimum
-5.5
-3.5
NPP of Minimum
Sample Quantiles
-5.5
-3.5
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of Maximum
Frequency
0
150
3.0
4.0
5.0
y$maxm
Boxplot of Maximum
3.0
4.5
NPP of Maximum
Sample Quantiles
3.0
4.5
-3 -2 -1 0 1 2 3
Theoretical Quantiles
Histogram of IQR
Frequency
0
100
1.28
1.32
1.36
1.40
y$iqr
Boxplot of IQR
1.28
1.36
NPP of IQR
Sample Quantiles
1.28
1.36
-3 -2 -1 0 1 2 3
Theoretical Quantiles