Ellipse Question

EE24BTECH11040 - Mandara Hosur

Question:
For the differential equation given below, find a particular solution that satisfies y=1 when x=0:

$$\left(x^3 + x^2 + x + 1\right)\frac{dy}{dx} = 2x^2 + x$$

Solution:
The required particular solution can be found using the method of finite differences.

$$\frac{dy}{dx} = \frac{y(x+h) - y(x)}{h} \tag{0.1}$$

$$\implies y(x+h) = y(x) + h \cdot \frac{dy}{dx} \tag{0.2}$$

As can be seen from the question above,

$$\frac{dy}{dx} = \frac{2x^2 + x}{(x^3 + x^2 + x + 1)} \tag{0.3}$$

$$\implies y(x+h) = y(x) + h \cdot \frac{2x^2 + x}{(x^3 + x^2 + x + 1)} \tag{0.4}$$

Let $x_0 = 0$ and $y_0 = 1$ (as per the given condition)
Let some $x_1 = x_0 + h$. Then

$$y_1 = y_0 + h \cdot \frac{2x_0^2 + x_0}{\left(x_0^3 + x_0^2 + x_0 + 1\right)} \tag{0.5}$$

Iterating through the above-mentioned process to generate y_2, y_3, y_4 and so on generalises equation (0.5) to

$$y_{n+1} = y_n + h \cdot \frac{2x_n^2 + x_n}{\left(x_n^3 + x_n^2 + x_n + 1\right)} \tag{0.6}$$

The smaller the value of h, the more accurate the curve is.

The equation of the curve is found by manual methods is

$$y = \frac{1}{4}\ln\left((x+1)^2\left(x^2+1\right)^3\right) - \frac{1}{2}\tan^{-1} x + 1 \tag{0.7}$$

The curve generated using the method of finite differences for the given question, taking $h = 0.1$ and running iterations 100 times is given below.

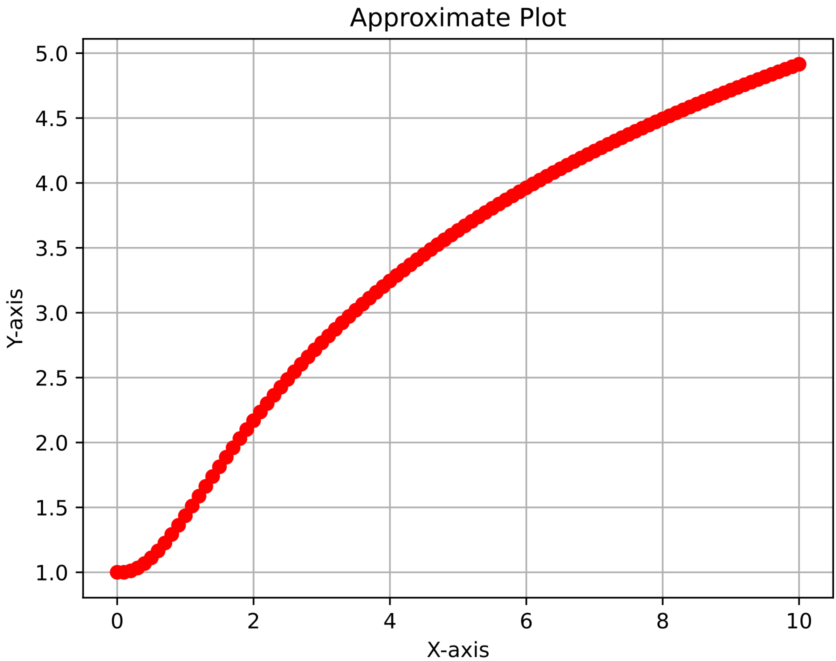

Fig. 0.1: Solution of given DE