1/1.5/36

EE24BTECH11040 - Mandara Hosur

Question:

Point $\mathbf{P}(x, 4)$ lies on the line segment joining the points $\mathbf{A}(-5, 8)$ and $\mathbf{B}(4, -10)$. Find the ratio in which point $\mathbf{P}$ divides the line segment AB. Also, find the value of x.

Solution:

Given Points	Description
$\mathbf{P}(x, 4)$	Point that cuts line segment AB internally
$\mathbf{A}(-5, 8)$	One end of line segment AB
$\mathbf{B}(4, -10)$	Other end of line segment AB

TABLE 0: Given Information

Formula	Description
$\mathbf{A} - \mathbf{B} = r(\mathbf{A} - \mathbf{P})$	r is some constant of proportionality
$\mathbf{P} = \frac{1}{1+k}(\mathbf{A} + k\mathbf{B})$	Section formula

TABLE 0: Formulae Used

To find x:

$$\mathbf{A} - \mathbf{B} = r(\mathbf{A} - \mathbf{P})$$

$$\begin{pmatrix} -5 \\ 8 \end{pmatrix} - \begin{pmatrix} 4 \\ -10 \end{pmatrix} = r\left(\begin{pmatrix} -5 \\ 8 \end{pmatrix} - \begin{pmatrix} x \\ 4 \end{pmatrix}\right)$$

$$\implies \begin{pmatrix} -9 \\ 18 \end{pmatrix} = r\begin{pmatrix} -5 - x \\ 4 \end{pmatrix}$$

$$\implies 18 = 4r$$

$$\implies r = \frac{18}{4} = \frac{9}{2}$$

Also,

$$9 = r(5 + x)$$

$$\implies 9 = \frac{9}{2}(5 + x)$$

$$\text{or, } x = -3$$

To find k:

$$\mathbf{P} = \frac{1}{1+k}(\mathbf{A} + k\mathbf{B})$$

Here, k denotes the ratio in which the line segment AB has been divided by $\mathbf{P}$.

$$\begin{pmatrix} -3 \\ 4 \end{pmatrix} = \frac{1}{1+k}\left(\begin{pmatrix} -5 \\ 8 \end{pmatrix} + k\begin{pmatrix} 4 \\ -10 \end{pmatrix}\right)$$

$$\implies \begin{pmatrix} -3 \\ 4 \end{pmatrix} = \frac{1}{1+k}\begin{pmatrix} 4k-5 \\ 8-10k \end{pmatrix}$$

$$\text{or, } k = \frac{2}{7}$$

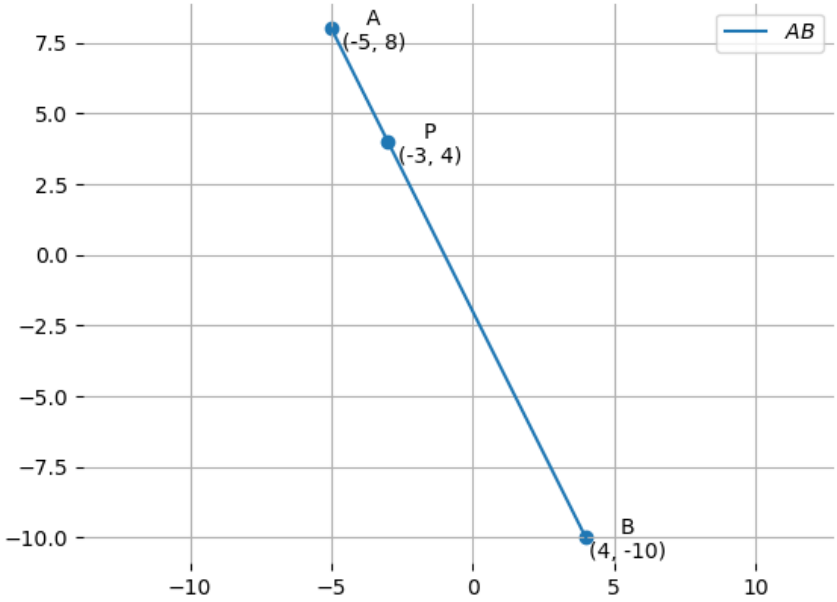

Fig. 0.1: Plot of line segment AB along with point $\mathbf{P}$