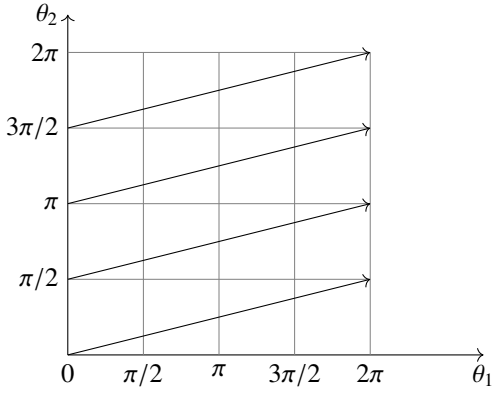
GATE (2022) PH(53-65)

EE24BTECH11040 - Mandara Hosur

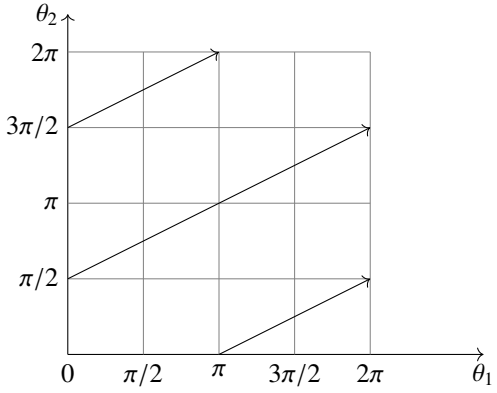
Carry TWO marks Each

1) A parallel plate capacitor with spacing d and area of cross-section A is connected to a source of voltage V. If the plates are pulled apart quasistatically to a spacing of $2d$, then which of the following statements are correct?
   a) The force between the plates at spacing $2d$ is $\frac{1}{8}\left(\frac{\epsilon_0 A V^2}{d^2}\right)$
   b) The work done in moving the plates is $\frac{1}{8}\left(\frac{\epsilon_0 A V^2}{d}\right)$
   c) The energy transferred to the voltage source is $\frac{1}{2}\left(\frac{\epsilon_0 A V^2}{d}\right)$
   d) The energy of the capacitor reduces by $\frac{1}{4}\left(\frac{\epsilon_0 A V^2}{d}\right)$
2) A system with time independent Hamiltonian $H(q, p)$ has two constants of motion $f(q, p)$ and $g(q, p)$. Then which of the following Poisson brackets are always zero?
   a) $\{H, f + g\}$
   b) $\{H, f, g\}$
   c) $\{H + f, g\}$
   d) $\{H, H + fg\}$
3) In the action-angle variables $(I_1, I_2, \theta_1, \theta_2)$, consider the Hamiltonian $H = 4I_1 I_2$ and $0 \leq \theta_1, \theta_2 < 2\pi$. Let $\frac{I_1}{I_2} = \frac{1}{2}$. Which of the following are possible plots of the trajectories with different initial conditions in $\theta_1 - \theta_2$ plane?

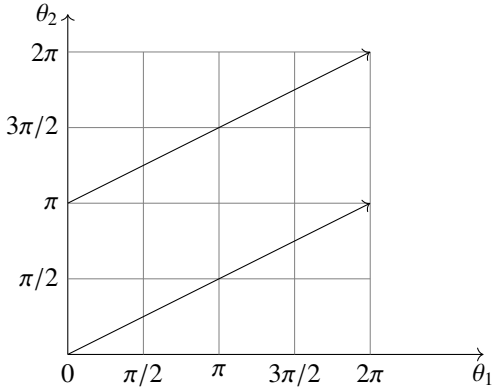
a)



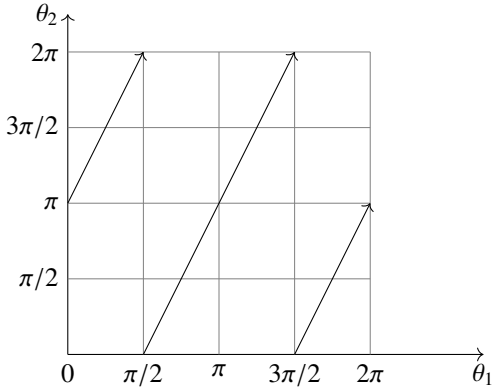
b)



c)



d)



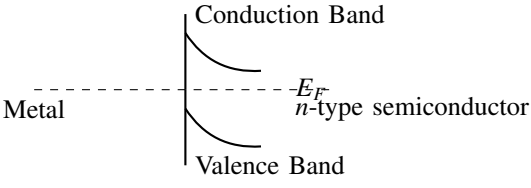
4) A particle of mass m in the $x-y$ plane is confined in an infinite two-dimensional well with vertices at $(0,0)$, $(0,L)$, (L,L), $(L,0)$. The eigenfunctions of this particle are $\psi_{n_x,n_y} = \frac{2}{L}\sin\left(\frac{n_x\pi x}{L}\right)\sin\left(\frac{n_y\pi y}{L}\right)$. If perturbation of the form $V = Cxy$, where C is a real constant, is applied, then which of the following statements are correct for the first excited state?

a) The unperturbed energy is $\frac{3\pi^2\hbar^2}{2mL^2}$

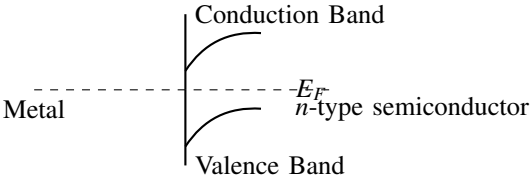
b) The unperturbed energy is $\frac{5\pi^2\hbar^2}{2mL^2}$
c) First order energy shift due to the applied perturbation is zero
d) The shift (δ) in energy due to the applied perturbation is determined by an equation of the form $\begin{vmatrix} a-\delta & b \\ b & a-\delta \end{vmatrix} = 0$, where a and b are real, non-zero constants

5) A junction is formed between a metal on the left and an n-type semiconductor on the right. Before forming the junction, the Fermi level E_F of the metal lies below that of the semiconductor. Then which of the following schematics are correct for the bands and the $I - V$ characteristics of the junction?

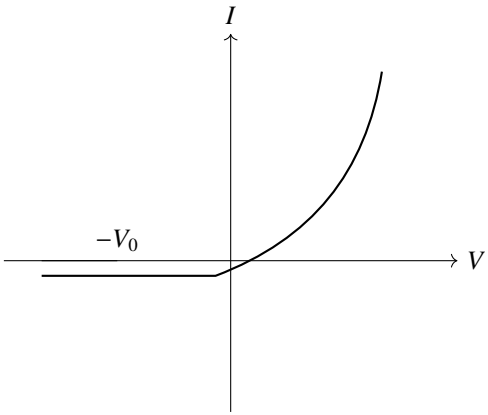
a)

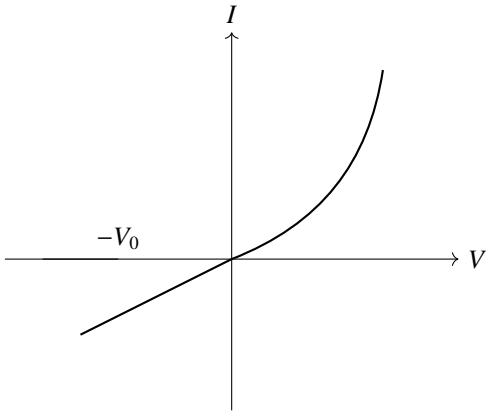


b)



c)





d)

6) A plane polarized electromagnetic wave propagating in $y-z$ plane is incident at the interface of two media at Brewster's angle. Taking $z = 0$ as the boundary between the two media, the electric field of the reflected wave is given by

$$\mathbf{E_R} = A_R \cos\left[k_0\left\{\frac{\sqrt{3}}{2}y - \frac{1}{2}z\right\} - \omega t\right]\hat{x}$$

then which among the following statements are correct?

a) The angle of refraction is $\frac{\pi}{6}$
b) Ratio of permittivity of the medium of refraction (ϵ_2) with respect to the medium on incidence (ϵ_1), $\frac{\epsilon_2}{\epsilon_1} = 3$
c) The incident wave can have components of its electric field in $y-z$ plane
d) The angle of reflection is $\frac{\pi}{6}$

7) The minimum number of two-input NAND gates required to implement the following Boolean expression is _

$$Y = \left[A\bar{B}\,(C + BD) + \bar{A}\bar{B}\right]C$$

8) In a nucleus, the interaction $V_{so}\mathbf{l}\cdot\mathbf{s}$ is responsible for creating spin-orbit doublets. The energy difference between $p_{\frac{1}{2}}$ and $p_{\frac{3}{2}}$ states in units of $V_{so}\frac{\hbar^2}{2}$ is _ (round off to the nearest integer)

9) Two identical particles of rest mass m_0 approach each other with equal and opposite velocity $v = 0.5c$, where c is the speed of light. The total energy of one particle as measured in the rest frame of the other is $E = \alpha m_0 c^2$. The value of α is _ (Round off to two decimal places)

10) In an X-Ray diffraction experiment on a solid with FCC structure, five diffraction peaks corresponding to (111), (200), (220), (311) and (222) planes are observed using 1.54 ÅX-rays. On using 3 ÅX-rays on the same solid, the number of observed peaks will be _

11) For 1 mole of Nitrogen gas, the ratio $\left(\frac{\Delta S_I}{\Delta S_{II}}\right)$ of entropy change of the gas in processes (I) and (II) mentioned below is _ (Round off to one decimal place)
(I) The gas is held at 1 atm and is cooled from 300 K to 77 K.
(II) The gas is liquified at 77 K.

(Take C_p = 7.0 cal mol^{-1}K^{-1}, Latent heat L = 1293.6 cal mol^{-1})

12) Frequency bandwidth $\Delta\nu$ of a gas laser of frequency ν Hz is

$$\Delta\nu = \frac{2\nu}{c}\sqrt{\frac{\alpha}{A}}$$

where $\alpha = 3.44 \times 10^6 \text{m}^2\text{s}^{-2}$ at room temperature and A is the atomic mass of the lasing atom. For ^4He - ^{20}Ne laser (wavelength = 633 nm), $\Delta\nu = n \times 10^9$ Hz. The value of n is _ (Round off to one decimal place)

13) A current of 1 A is flowing through a very long solenoid made of winding density 3000 turns/m. As shown in the figure, a parallel plate capacitor, with plates oriented parallel to the solenoid axis and carrying surface charge density $6\epsilon_0 C\text{m}^{-2}$, is placed at the middle of the solenoid. The momentum density of the electromagnetic field at the midpoint X of the capacitor is $n \times 10^{13}$ N s m^{-3}. The value of n is _ (Round off to the nearest integer).

(speed of light $c = 3 \times 10^8 \text{ms}^{-1}$)