13/A/E/6-21

EE24BTECH11040 - Mandara Hosur

E. Subjective Problems

1) a) PQ is a vertical tower. P is the foot and Q is the top of the tower. A, B, C are three points in the horizontal plane through P. The angles of elevation of Q from A, B, C are equal, and each is equal to θ. The sides of the triangle ABC are a, b, c; and the area of the triangle ABC is Δ. Show that the height of the tower is $\frac{abc\tan\theta}{4\Delta}$.

   b) AB is a vertical pole. The end A is on the level ground. C is the middle point of AB. P is a point on the level ground. The portion CB subtends an angle β at P. If $AP = nAB$ then show that $\tan\beta = \frac{n}{2n^2+1}$.

(1980)

2) Let the angles A, B, C of a triangle ABC be in A.P. and let $b : c = \sqrt{3} : \sqrt{2}$. Find the angle A.

$(1981 - 2Marks)$

3) A vertical pole stands at a point Q on a horizontal ground. A and B are points on the ground, d meters apart. The pole subtends angles α and β at A and B respectively. AB subtends an angle γ at Q. Find the height of the pole.

$(1982 - 3Marks)$

4) Four ships A, B, C and D are at sea in the following relative positions: B is on the straight line segment AC, B is due North of D and D is due west of C. The distance between B and D is 2 km. $\angle BDA = 40°$, $\angle BCD = 25°$. What is the distance between A and D? [Take $\sin 25° = 0.423$]

$(1983 - 3Marks)$

5) The ex-radii r_1, r_2, r_3 of ΔABC are in H.P. Show that its sides a, b, c are in A.P.

$(1983 - 3Marks)$

6) For a triangle ABC it is given that $\cos A + \cos B + \cos C = \frac{3}{2}$. Prove that the triangle is equilateral.

$(1984 - 4Marks)$

7) With usual notation, if in a triangle ABC; $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$ then prove that $\frac{\cos A}{7} = \frac{\cos B}{19} = \frac{\cos C}{25}$.

$(1984 - 4Marks)$

8) A ladder rests against a wall at an angle α to the horizontal. Its foot is pulled away from the wall through a distance a, so that it slides a distance b down the wall making an angle β with the horizontal. Show that $a = b\tan\frac{1}{2}(\alpha+\beta)$.

$(1985 - 5Marks)$

9) In a triangle ABC, the median to the side BC is of length $\frac{1}{\sqrt{11-6\sqrt{3}}}$ and it divides the angle A into angles 30° and 45°. Find the length of the side BC.

$(1985 - 5Marks)$

10) If in a triangle ABC, $\cos A \cos B + \sin A \sin B \sin C = 1$, show that $a : b : c = 1 : 1 : \sqrt{2}$.

$(1986 - 5Marks)$

11) A sign-post in the form of an isosceles triangle ABC is mounted on a pole of height h fixed to the ground. The base BC of the triangle is parallel to the ground. A man standing on the ground at a distance d from the sign-post finds that the top vertex A of the triangle subtends an angle β and either of the other two vertices subtends the same angle α at his feet. Find the area of the triangle.

$(1988 - 5Marks)$

12) ABC is a triangular park with $AB = AC = 100$m. A television tower stands at the midpoint of BC. The angles of elevation of the top of the tower at A, B, C are 45°, 60°, 60°, respectively. Find the height of the tower.

$(1989 - 5Marks)$

13) A vertical tower PQ stands at a point P. Points A and B are located to the South and East of P respectively. M is the mid point of AB. PAM is an equilateral triangle; and N is the foot of the perpendicular from P on AB. Let $AN = 20$ metres and the angle of elevation of the top of the tower at N is $\tan^{-1} 2$. Determine the height of the tower and the angles of elevation of the top of the tower at A and B.

$(1990 - 4Marks)$

14) The sides of a triangle are three consecutive natural numbers and its largest angle is twice the smallest one. Determine the sides of the triangle.

$(1991 - 4Marks)$

15) In a triangle of base a the ratio of the other two sides is $r\,(< 1)$. Show that the altitude of the triangle is less than or equal to $\frac{ar}{1-r^2}$.

$(1991 - 4Marks)$

16) A man notices two objects in a straight line due west. After walking a distance c due north he observes that the objects subtend an angle α at his eye; and, after a further distance $2c$ due north, and angle β. Show that the distance between the objects is $\frac{8c}{3\cot\beta - \cot\alpha}$; the height of the man is being ignored.

$(1991 - 4Marks)$