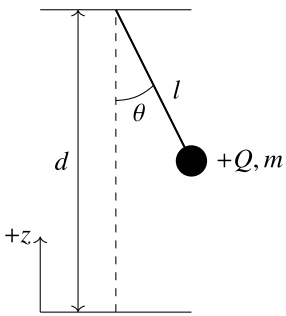
GATE (2021) PH(40-52)

EE24BTECH11040 - Mandara Hosur

Multiple Choice Question (MCQ), carry TWO mark each (for each wrong answer: $-\frac{2}{3}$)

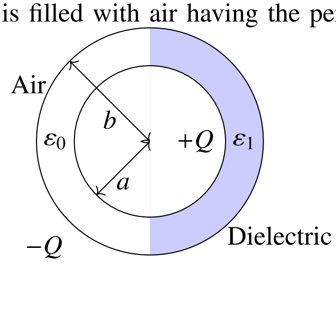
1) Consider a point charge $+Q$ of mass m suspended by a massless, inextensible string of length l in free space (permittivity ε_0) as shown in the figure. It is placed at a height d ($d > l$) over an infinitely large, grounded conducting plane. The gravitational potential energy is assumed to be zero at the position of the conducting plane and is positive above the plane.



If θ represents the angular position and p_θ its corresponding canonical momentum, then the correct Hamiltonian of the system is

a) $\frac{p_\theta^2}{2ml^2} - \frac{Q^2}{16\pi\varepsilon_0(d-l\cos\theta)} - mg(d - l\cos\theta)$
b) $\frac{p_\theta^2}{2ml^2} - \frac{Q^2}{8\pi\varepsilon_0(d-l\cos\theta)} + mg(d - l\cos\theta)$
c) $\frac{p_\theta^2}{2ml^2} - \frac{Q^2}{8\pi\varepsilon_0(d-l\cos\theta)} - mg(d - l\cos\theta)$
d) $\frac{p_\theta^2}{2ml^2} - \frac{Q^2}{16\pi\varepsilon_0(d-l\cos\theta)} + mg(d - l\cos\theta)$

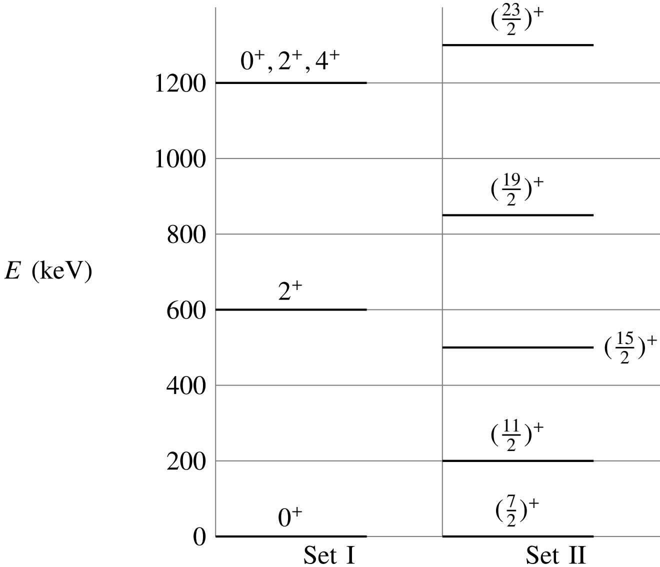
2) Consider two concentric conducting spherical shells as shown in the figure. The inner shell has a radius a and carries a charge $+Q$. The outer shell has a radius b and carries a charge $-Q$. The empty space between them is half-filled by a hemispherical shell of a dielectric having permittivity ε_1. The remaining space between the shells is filled with air having the permittivity ε_0.



The electric field at a radial distance r from the center and between the shells ($a < r < b$) is

a) $\frac{Q}{2\pi(\varepsilon_0+\varepsilon_1)}\frac{\hat{r}}{r^2}$ everywhere
b) $\frac{Q}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2}$ on the air side and $\frac{Q}{4\pi\varepsilon_1}\frac{\hat{r}}{r^2}$ on the dielectric side
c) $\frac{Q}{2\pi\varepsilon_0}\frac{\hat{r}}{r^2}$ on the air side and $\frac{Q}{2\pi\varepsilon_1}\frac{\hat{r}}{r^2}$ on the dielectric side
d) $\frac{Q}{4\pi(\varepsilon_0+\varepsilon_1)}\frac{\hat{r}}{r^2}$ everywhere

3) For the given sets of energy levels of nuclei X and Y whose mass numbers are odd and even, respectively, choose the best suited interpretation.



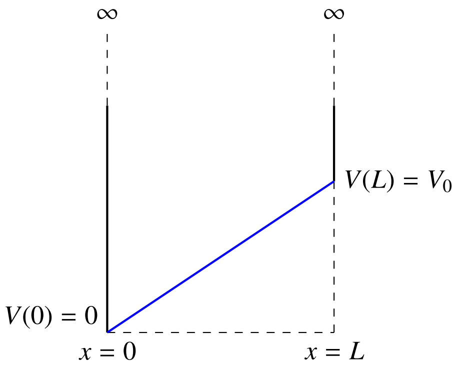
a) Set I: Rotational band of X
Set II: Vibrational band of Y
b) Set I: Rotational band of Y
Set II: Vibrational band of X
c) Set I: Vibrational band of X
Set II: Rotational band of Y
d) Set I: Vibrational band of Y
Set II: Rotational band of X

4) Consider a system of three distinguishable particles, each having spin $S = \frac{1}{2}$ such that $S_z = \pm\frac{1}{2}$ with corresponding magnetic moments $\mu_z = \pm\mu$.When the system is placed in an external magnetic field H pointing along the z-axis, the total energy of the system is μH. Let x be the state where the first spin has $S_z = \frac{1}{2}$. The probability of having the state x and the mean magnetic moment (in the $+z$ direction) of the system in state x are

a) $\frac{1}{3}, -\frac{1}{3}\mu$

b) $\frac{1}{3}$, $\frac{2}{3}\mu$
c) $\frac{2}{3}$, $-\frac{2}{3}\mu$
d) $\frac{2}{3}$, $\frac{1}{3}\mu$

5) Consider a particle in a one-dimensional infinite potential well with its walls at $x = 0$ and $x = L$. The system is perturbed as shown in the figure.



The first order correction to the energy eigenvalue is
a) $\frac{V_0}{4}$
b) $\frac{V_0}{3}$
c) $\frac{V_0}{2}$
d) $\frac{V_0}{5}$

6) Consider a state described by $\psi(x,t) = \psi_2(x,t)+\psi_4(x,t)$, where $\psi_2(x,t)$ and $\psi_4(x,t)$ are respectively the second and fourth normalized harmonic oscillator wave functions and ω is the angular frequency of the harmonic oscillator. The wave function $\psi(x, t = 0)$ will be orthogonal to $\psi(x,t)$ at time t equal to
a) $\frac{\pi}{2\omega}$
b) $\frac{\pi}{\omega}$
c) $\frac{\pi}{4\omega}$
d) $\frac{\pi}{6\omega}$

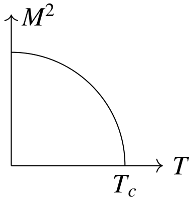
7) Consider a single one-dimensional harmonic oscillator of angular frequency ω, in equilibrium at temperature $T = (k_B\beta)^{-1}$. The states of the harmonic oscillator are all non-degenerate having energy $E_n = \left(n + \frac{1}{2}\right)\bar{h}\omega$ with equal probability, where n is the quantum number. The Helmholtz free energy of the oscillator is
a) $\frac{\bar{h}\omega}{2} + \beta^{-1} \ln \left[1 - e^{\beta\bar{h}\omega}\right]$
b) $\frac{\bar{h}\omega}{2} + \beta^{-1} \ln \left[1 - e^{-\beta\bar{h}\omega}\right]$
c) $\frac{\bar{h}\omega}{2} + \beta^{-1} \ln \left[1 + e^{-\beta\bar{h}\omega}\right]$
d) $\beta^{-1} \ln \left[1 - e^{-\beta\bar{h}\omega}\right]$

8) A system of two atoms can be in three quantum states having energies 0, ϵ and 2ϵ. The system is in equilibrium at temperature $T = (k_B\beta)^{-1}$. Match the following Statistics with the Partition function.

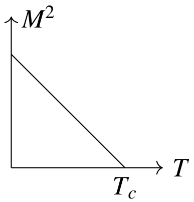
Statistics	Partition function
CD: Classical (distinguishable particles)	Z1: $e^{-\beta\epsilon} + e^{-2\beta\epsilon} + e^{-3\beta\epsilon}$
CI: Classical (indistinguishable particles)	Z2: $1 + e^{-\beta\epsilon} + 2e^{-2\beta\epsilon} + e^{-3\beta\epsilon} + e^{-4\beta\epsilon}$
FD: Fermi-Dirac	Z3: $1 + 2e^{-\beta\epsilon} + 3e^{-2\beta\epsilon} + 2e^{-3\beta\epsilon} + e^{-4\beta\epsilon}$
BE: Bose-Einstein	Z4: $\frac{1}{2} + e^{-\beta\epsilon} + \frac{3}{2}e^{-2\beta\epsilon} + e^{-3\beta\epsilon} + \frac{1}{2}e^{-4\beta\epsilon}$

a) CD:Z1, CI:Z2, FD:Z3, BE:Z4
b) CD:Z2, CI:Z3, FD:Z4, BE:Z1
c) CD:Z3, CI:Z4, FD:Z1, BE:Z2
d) CD:Z4, CI:Z1, FD:Z2, BE:Z3

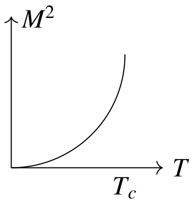
9) The free energy of a ferromagnet is given by $F = F_0 + a_0(T - T_C)M^2 + bM^4$, where F_0, a_0, and b are positive constants, M is magnetization, T is the temperature, and T_C is the Curie temperature. The relation between M^2 and T is best depicted by



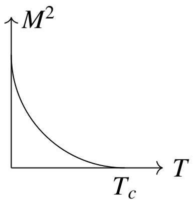
a)



b)

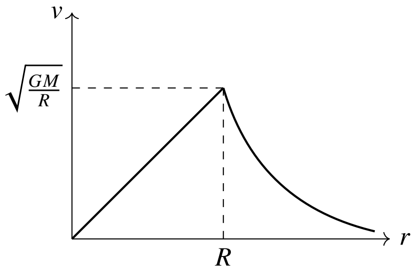


c)

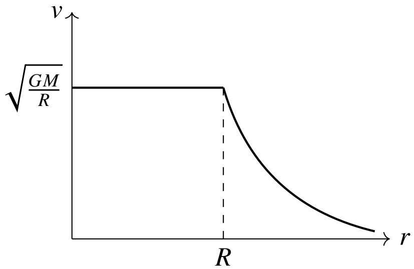


d)

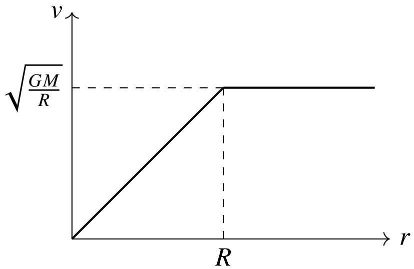
10) Consider a spherical galaxy of total mass M and radius R, having a uniform matter distribution. In this idealized situation, the orbital speed v of a star of mass m ($m << M$) as a function of the distance r from the galactic center is best described by (G is the universal gravitational constant)



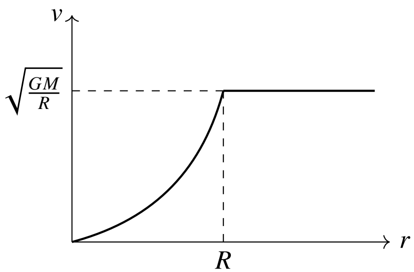
a)



b)



c)



d)

11) Consider the potential $U(r)$ defined as

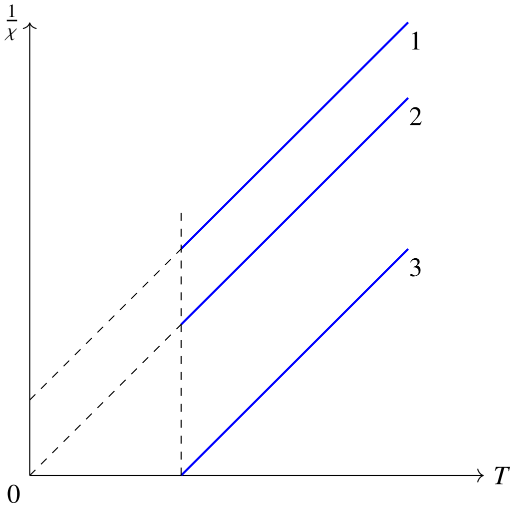
$$U(r) = -U_0 \frac{e^{-\alpha r}}{r}$$

where α and U_0 are real constants of appropriate dimensions. According to the first Born approximation, the elastic scattering amplitude calculated with $U(r)$ for a (wave-vector) momentum transfer q and $\alpha \to 0$, is proportional to
(Useful integral: $\int_0^\infty \sin(qr) e^{-\alpha r} dr = \frac{q}{\alpha^2+q^2}$)

a) q^{-2}

b) q^{-1}
c) q
d) q^2

12) As shown in the figure, inverse magnetic susceptibility ($\frac{1}{\chi}$) is plotted as a function of temperature (T) for three different materials in paramagnetic states.



(Curie temperature of ferromagnetic material = T_C
Néel temperature of antiferromagnetic material = T_N)
Choose the correct statement from the following

a) Material 1 is antiferromagnetic ($T < T_N$), 2 is paramagnetic, and 3 is ferromagnetic ($T < T_C$).
b) Material 1 is paramagnetic, 2 is antiferromagnetic ($T < T_N$), and 3 is ferromagnetic ($T < T_C$).
c) Material 1 is ferromagnetic ($T < T_C$), 2 is antiferromagnetic ($T < T_N$), and 3 is paramagnetic.
d) Material 1 is ferromagnetic ($T < T_c$), 2 is paramagnetic, and 3 is antiferromagnetic ($T < T_N$).

13) A function $f(t)$ is defined only for $t \geq 0$. The Laplace transform of $f(t)$ is

$$\mathcal{L}(f;s) = \int_0^\infty e^{-st} f(t) dt$$

whereas the Fourier transform of $f(t)$ is

$$\tilde{f}(\omega) = \int_0^\infty f(t) e^{-i\omega t} dt$$

The correct statement(s) is (are)

a) The variable s is always real.
b) The variable s can be complex.
c) $\mathcal{L}(f;s) = \int_0^\infty e^{-st} f(t) dt$ and $\tilde{f}(\omega) = \int_0^\infty f(t) e^{-i\omega t} dt$ can never be made connected.

d) $\mathcal{L}(f;s) = \int_0^\infty e^{-st} f(t)dt$ and $\tilde{f}(\omega) = \int_0^\infty f(t)e^{-i\omega t}dt$ can be made connected.