EE1060
Differential Equations And Transform Techniques
Quiz 03

Submission by Team 08

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Question

Compute the response of a series RL circuit to a square wave input as shown in Fig. 1, both numerically and analytically. For the numerical approach, use appropriate numerical techniques to solve the differential equation governing the RL circuit's behaviour. For the analytical approach, utilise Fourier Series to derive the response of the circuit. The square wave is characterised by a duty ratio, denoted by the factor α. You are free to choose the values of resistance R and inductance L for your analysis.

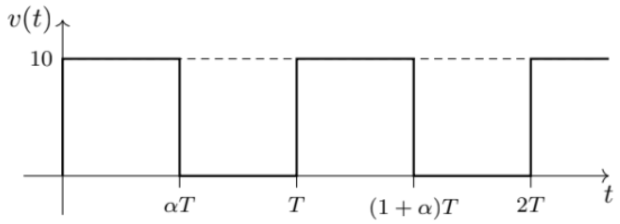

Figure 1: Square wave with a duty ratio α

Analyse the circuit's behaviour obtained from both numerical and analytical methods. In your analysis, apply the relevant concepts from the course related to circuits and differential equations to explore the response and draw comparisons between the two approaches.

Contents

1 Decomposing the Input Wave Into Fourier Series

To analyse the circuit response, we express the input signal using Fourier Series. This can be done in two ways.

1.1 Complex Exponential Form

The Fourier series of a periodic function $f(x)$ with period T can be expressed as:

$$f(x) = \sum_{k=-\infty}^{\infty} C_k e^{j\frac{2\pi k}{T}x} \tag{1}$$

where the Fourier coefficients C_k are given by:

$$C_k = \frac{1}{T}\int_0^T f(x)e^{-j\frac{2\pi k}{T}x}\,dx \tag{2}$$

1.2 Trigonometric Form

The Fourier series can also be written (for real signals) in terms of sinusoidal terms as:

$$\begin{aligned} f(x) = a_0 + \sum_{n=k=1}^{\infty}\left[a_n \cos\left(\frac{2\pi k}{T}x\right)\right. \\ \left. + \sum_{n=k=1}^{\infty}\left[b_n \sin\left(\frac{2\pi k}{T}x\right)\right]\right. \end{aligned} \tag{3}$$

where the coefficients are given by:

$$a_0 = \frac{1}{T}\int_0^T f(x)\,dx \tag{4}$$

$$a_n = C_k + C_{-k} = \frac{2}{T}\int_0^T f(x)\cos\left(\frac{2\pi k}{T}x\right)\,dx \tag{5}$$

$$\text{for } n = k \tag{6}$$

$$b_n = j(C_k - C_{-k}) = \frac{2}{T}\int_0^T f(x)\sin\left(\frac{2\pi k}{T}x\right)\,dx \tag{7}$$

$$\text{for } n = k \tag{8}$$

We will use the notation ω_0 to represent $\frac{2\pi}{T}$ going forward.

1.3 Orthogonality Property of Terms

Considering the complex form of Fourier series, the complex coefficients can be computed as:

$$C_k = \frac{1}{T}\int_0^T f(x)e^{-j\frac{2\pi k}{T}x}\,dx \tag{9}$$

In the complex Fourier series, the set of exponential functions:

$$e^{j\frac{2\pi k}{T}x}, \quad k \in \mathbb{Z}$$

forms an orthogonal set over the interval $[0, T]$ with respect to the inner product:

$$\langle f, g\rangle = \int_0^T f(x)\overline{g(x)}\,dx.$$

For any integers m and n, the orthogonality property is given by:

$$\int_0^T e^{j\frac{2\pi m}{T}x}\cdot e^{-j\frac{2\pi n}{T}x}\,dx = \int_0^T e^{j\frac{2\pi(m-n)}{T}x}\,dx$$

which evaluates to:

$$\begin{cases} T, & \text{if } m = n \\ 0, & \text{if } m \neq n \end{cases}$$

Thus, the complex exponentials $e^{j\frac{2\pi k}{T}x}$ form an **orthonormal basis** in the space of periodic functions.

1.4 Fourier Series Representation of the Given Square Input Wave

Substituting the value of $f(x)$ in (2) and computing C_k, we get:

$$C_k = \frac{1}{T}\int_0^T f(t)e^{-jk\omega_0 t}dt \tag{10}$$

$$= \frac{1}{T}\left(\int_0^{\alpha T} 10\cdot e^{-jk\omega_0 t}dt + 0\right) \tag{11}$$

$$= \frac{10}{T}\left(\frac{e^{-jk\omega_0\alpha T}-1}{-jk\omega_0}\right) \tag{12}$$

$$C_k = \frac{10j}{2\pi k}[\cos 2\pi k\alpha - j\sin 2\pi k\alpha - 1] \tag{13}$$

$$= \frac{5\sin(2\pi k\alpha)}{\pi k} + j\left[\frac{5}{\pi k}(\cos 2\pi k\alpha - 1)\right] \tag{14}$$

From (4), (5) and (7), we can compute the trigonometric coefficients of the Fourier Series.

$$a_k = C_k + C_{-k} \tag{15}$$

$$= \frac{10\sin(2\pi k\alpha)}{\pi k} \tag{16}$$

$$b_k = j\{C_k - C_{-k}\} \tag{17}$$

$$= j\,(2j)\,\frac{5}{\pi k}[\cos 2\pi k\alpha - 1] \tag{18}$$

$$= \left(\frac{10}{\pi k}\right)\{1-\cos 2\pi k\alpha\} \tag{19}$$

$$a_0 = C_0 = \frac{1}{T}\int_0^T f(t)dt \tag{20}$$

$$= \frac{1}{T}\int_0^{\alpha T} 10dt \tag{21}$$

$$= \frac{10\alpha T}{T} = 10\alpha \tag{22}$$

Therefore, the Fourier Series representation of the given input wave is given by:

$$f(x) = 10\alpha + \sum_{k=1}^{\infty}\left[\frac{10\sin(2\pi k\alpha)}{\pi k}\cos\left(\frac{2\pi k}{T}x\right) + \frac{10}{\pi k}(1-\cos(2\pi k\alpha))\sin\left(\frac{2\pi k}{T}x\right)\right] \tag{23}$$

1.5 Verification

To verify that the square wave was decomposed correctly into its Fourier Series, we will write a Python program to plot the calculated expression using the first 10000 terms.

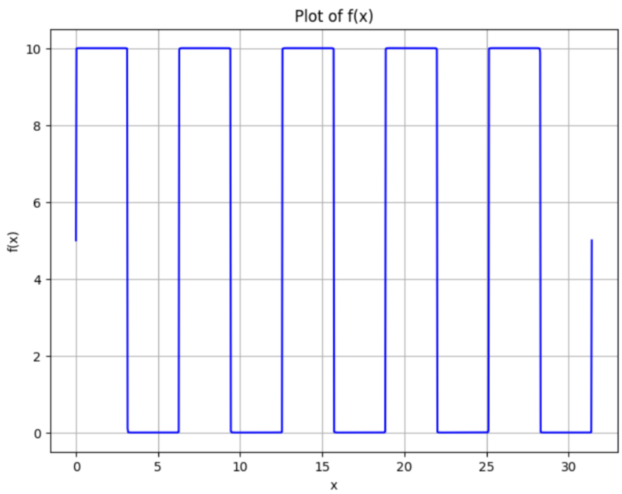

Figure 2: Plot Generated Using Calculated Fourier Series Expression

1.6 Magnitude Spectrum of C_k

The magnitude spectrum is obtained by computing $|C_k|$ for different values of k. It provides insight into how much of each frequency component is present in the signal:

$$|C_k| = \sqrt{\mathrm{Re}(C_k)^2 + \mathrm{Im}(C_k)^2}. \tag{24}$$

The magnitude spectrum plots $|C_k|$ against k.

For the given input wave, $|C_k|$ is

$$|C_k| = \frac{5}{\pi k}\left(\sqrt{2(1-cos(2\pi k\alpha)}\right) \tag{25}$$

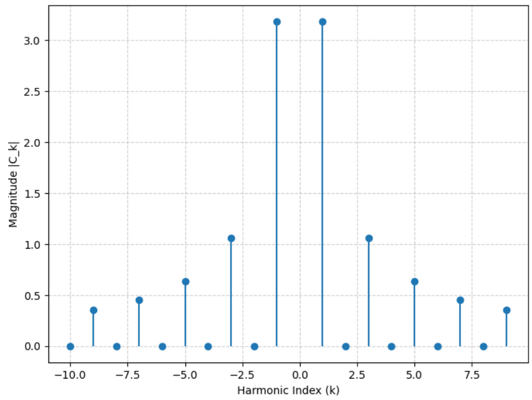

Figure 3: For alpha=0.5

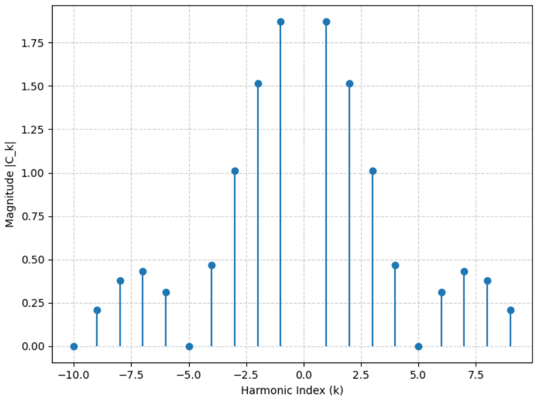

Figure 5: For alpha=0.8

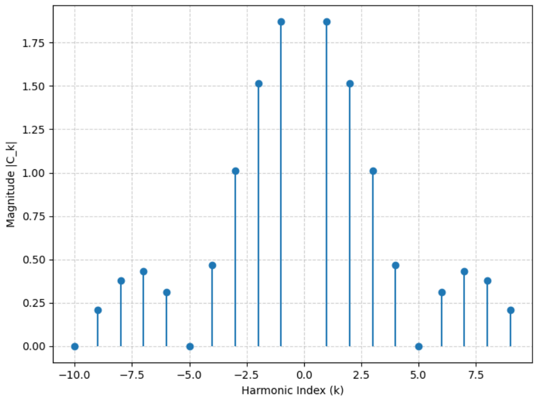

Figure 4: For alpha=0.2

2 Deducing the Differential Equation

2.1 Defining the Square Wave

The square wave $v(t)$ alternates between two values, V_0 and 0, with a period T. The duty ratio α determines the fraction of the period during which the voltage is V_0. Specifically:

- For $0 \leq t < \alpha T$, $v(t) = V_0$.
- For $\alpha T \leq t < T$, $v(t) = 0$.

2.2 The Differential Equation for a Series RL Circuit

The voltage across a series RL circuit is given by Kirchhoff's Loop Law as:

$$v(t) = L\frac{di(t)}{dt} + Ri(t)$$

where $i(t)$ is the current through the circuit.

2.3 Solving the Differential Equation

We need to solve this equation for two intervals:

1. When $v(t) = V_0 = 10$ (for $0 \leq t < \alpha T$):

$$V_0 = L\frac{di(t)}{dt} + Ri(t) \tag{26}$$

$$\implies \frac{di(t)}{dt} + \frac{R}{L}i(t) = \frac{10}{L} \tag{27}$$

Integrating Factor $= e^{\int \frac{R}{L}dt} \implies e^{\frac{Rt}{L}}$
Multiply both sides by the integrating factor and integrate.

$$\int d(e^{\frac{Rt}{L}}i(t)) = \int e^{\frac{Rt}{L}}\frac{10}{L}dt \tag{28}$$

$$e^{\frac{Rt}{L}}i(t) = \frac{10}{L}\frac{e^{\frac{Rt}{L}}}{\frac{R}{L}} + C \tag{29}$$

Divide both sides with $e^{\frac{Rt}{L}}$

$$i(t) = \frac{10}{R} + Ce^{-\frac{Rt}{L}} \tag{30}$$

Apply initial condition $i(0)$ at $t = 0$

$$i(0) = \frac{10}{R} + C \tag{31}$$

$$C = i(0) - \frac{10}{R} \tag{32}$$

equation (32) in (30),

$$i(t) = \frac{10}{R}\left(1 - e^{-\frac{R}{L}t}\right) + i(0)e^{-\frac{R}{L}t} \tag{33}$$

since $i(0) = 0$ (34)

$$i(t) = \frac{10}{R}\left(1 - e^{-\frac{R}{L}t}\right) \tag{35}$$

2. When $v(t) = 0$ (for $\alpha T \leq t < T$):

$$0 = L\frac{di(t)}{dt} + Ri(t) \tag{36}$$

The solution to this DE is:

$$i(t) = i(\alpha T)e^{-\frac{R}{L}(t-\alpha T)} \tag{37}$$

where $i(\alpha T)$ is the current at $t = \alpha T$.

3 The Analytical Solution

As discussed in the previous section, the given voltage input can also be expressed as a linear combination of DC and AC voltage sources, with the help of Fourier Series. Thus we can interpret the differential equations of the system as $L\frac{di}{dt} + Ri = f(x)$ where

$$f(x) = 10\alpha + \sum_{n=1}^{\infty} \left[\frac{10\sin(2\pi n\alpha)}{\pi n} \cos\left(\frac{2\pi n}{T}x\right) + \frac{10}{\pi n}(1-\cos(2\pi n\alpha))\sin\left(\frac{2\pi n}{T}x\right) \right] \tag{38}$$

3.1 Current Response of the Circuit for Individual Inputs

3.1.1 For DC Voltage Input

Assume the current produced in the circuit by the DC source to be i_0 The differential equation for this system is:

$$\frac{di_0}{dt} + \frac{R}{L}i_0 = \frac{a_0}{L} \tag{39}$$

Multiplying with Integrating Factor $e^{\int \frac{R}{L}dt} = e^{\frac{R}{L}t}$ and integrating on both sides with limits $t = 0$ to $t = t$, and considering $i_0(0) = 0$,

$$i_0(t) = \frac{a_0}{R}\left(1 - e^{\frac{-R}{L}t}\right) \tag{40}$$

where $a_0 = 10\alpha$.

3.1.2 For AC Voltage Input

Assume the current produced in the circuit by the AC source $a_n cos(n\omega_0 t) + b_n sin(n\omega_0 t)$ to be i_n.

The differential equation for this system is:

$$\frac{di_n}{dt} + \frac{R}{L}i_n = \frac{a_n}{L}cos(n\omega_0 t) + \frac{b_n}{L}sin(n\omega_0 t) \tag{41}$$

Multiplying with Integrating Factor $e^{\int \frac{R}{L}dt = e^{\frac{R}{L}t}}$ and integrating on both sides with limits $t = 0$ to $t = t$, and considering $i_n(0) = 0$,

$$e^{\frac{R}{L}t}i_n(t) = \int_0^t e^{\frac{R}{L}t}\left(\frac{a_n}{L}\cos(n\omega_0 t)\right)dt + \int_0^t e^{\frac{R}{L}t}\left(\frac{b_n}{L}\sin(n\omega_0 t)\right)dt \tag{42}$$

Rewriting $a_n cos(n\omega_0 t) + b_n sin(n\omega_0 t)$ on RHS as

$$\begin{aligned}
&= a_n cos(n\omega_0 t) + b_n sin(n\omega_0 t) \\
&= \sqrt{a_n^2 + b_n^2}\left(\frac{a_n}{\sqrt{a_n^2+b_n^2}}\cos(n\omega_0 t) + \frac{b_n}{\sqrt{a_n^2+b_n^2}}\sin(n\omega_0 t)\right) \\
&= \sqrt{a_n^2 + b_n^2}\left(cos(n\omega_0 t - \phi)\right) \\
&= \sqrt{a_n^2 + b_n^2}\left(\frac{e^{j(n\omega_0 t - \phi)} + e^{j(\phi - n\omega_0 t)}}{2}\right)
\end{aligned}$$

where $\phi = tan^{-1}\left(\frac{b_n}{a_n}\right)$ and substituting it back in (42), on RHS side we get

$$= \frac{\sqrt{a_n^2 + b_n^2}}{2L}\int_0^t \left(e^{\frac{R}{L}t + j(n\omega_0 t - \phi)} + e^{\frac{R}{L}t + j(\phi - n\omega_0 t)}\right) \tag{43}$$

After integrating, we get

$$e^{\frac{R}{L}t}i_n(t) = \frac{\sqrt{a_n^2 + b_n^2}}{2L}\left(\frac{e^{-j\phi}\left(e^{(\frac{R}{L}+jn\omega_0)t} - 1\right)}{\frac{R}{L} + jn\omega_0}\right. \tag{44}$$

$$\left. + \frac{e^{j\phi}\left(e^{(\frac{R}{L}-jn\omega_0)t} - 1\right)}{\frac{R}{L} - jn\omega_0}\right) \tag{45}$$

By rearranging the terms and simplifying, we get

$$i_n(t) = \frac{\sqrt{a_n^2 + b_n^2}}{\sqrt{R^2 + (n\omega_0 L)^2}} \cos(n\omega_0 t - \phi - \theta) \tag{46}$$

$$- e^{\frac{-R}{L}t} \frac{\cos(\phi + \theta)}{\sqrt{R^2 + (n\omega_0 L)^2}} \tag{47}$$

where

$$\theta = tan^{-1}\left(\frac{n\omega_0 L}{R}\right)$$
$$a_n = \frac{10 \sin 2\pi n\alpha}{\pi n}$$
$$b_n = \left(\frac{10}{\pi n}\right)(1 - \cos 2\pi n\alpha)$$

3.2 Combined Current Response (Exploiting Linearity Property)

Since the first-order circuit is linear, we have by the principle of superposition:

$$i(t) = i_0(t) + \sum_{n=1}^{\infty} i_n(t) \tag{48}$$

$$i(t) = \frac{a_0}{R}\left(1 - e^{\frac{-R}{L}t}\right) \tag{49}$$

$$+ \sum_{n=1}^{\infty}\left(\frac{\sqrt{a_n^2 + b_n^2}}{\sqrt{R^2 + (n\omega_0 L)^2}} \cos(n\omega_0 t - \phi - \theta)\right. \tag{50}$$

$$\left. - e^{\frac{-R}{L}t} \frac{\cos(\phi + \theta)}{\sqrt{R^2 + (n\omega_0 L)^2}}\right) \tag{51}$$

where

$$a_0 = 10\alpha$$
$$\theta = tan^{-1}\left(\frac{n\omega_0 L}{R}\right)$$
$$a_n = \frac{10 \sin 2\pi n\alpha}{\pi n}$$
$$b_n = \left(\frac{10}{\pi n}\right)(1 - \cos 2\pi n\alpha)$$

3.3 Visual Representation of the Response

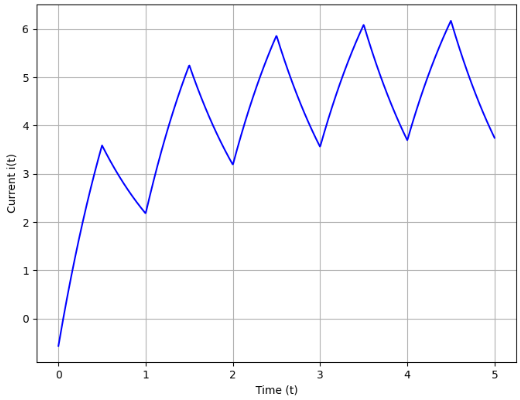

Figure 6: Transient Response

4 The Numerical Solution

4.1 Trapezoidal Rule

For a differential equation

$$\frac{dy}{dx} = f(x, y)$$

whose solution is $y(x)$, the update equation for trapezoidal method is given by:

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y_{n+1})\right] \tag{52}$$

4.2 Derivation of Update Equation

To solve the differential equation numerically using the trapezoidal method, we discretize it as:

$$L\frac{I_{n+1} - I_n}{\Delta t} + R\frac{I_{n+1} + I_n}{2} = \frac{V_{n+1} + V_n}{2} \tag{53}$$

Rearranging to isolate I_{n+1}:

$$L\frac{I_{n+1} - I_n}{\Delta t} = \frac{V_{n+1} + V_n}{2} - R\frac{I_{n+1} + I_n}{2} \tag{54}$$

$$I_{n+1}\left(\frac{L}{\Delta t} + \frac{R}{2}\right) = I_n\left(\frac{L}{\Delta t} - \frac{R}{2}\right) + \frac{V_{n+1} + V_n}{2} \tag{55}$$

Defining:

$$A = \frac{2L - R\Delta t}{2L + R\Delta t} \tag{56}$$

$$B = \frac{\Delta t}{2L + R\Delta t} \tag{57}$$

The iterative formula becomes:

$$I_{n+1} = AI_n + B(V_{n+1} + V_n) \tag{58}$$

4.3 Why Trapezoidal?

1. Trapezoidal Rule is a popular method that is most used to solve real-world problems.
2. It is an A-stable numerical method.
3. Forward Euler and Backward Euler methods are first-order accurate, whereas Trapezoidal Rule is second-order accurate.

4.4 The Code

```python
import numpy as np
import matplotlib.pyplot as plt

def square_wave(t, T, alpha):
    return np.array([10 if (time % T
        ) < (alpha * T) else 0 for
        time in t])

def rl_circuit_response(R, L, T,
    alpha, t_end, dt=0.00005):
    t = np.arange(0, t_end, dt)
    V = square_wave(t, T, alpha)
    I = np.zeros(len(t))
    A = (2 * L - R * dt) / (2 * L +
        R * dt)
    B = dt / (2 * L + R * dt)
    for i in range(1, len(t)):
        I[i] = A * I[i - 1] + B * (V
            [i] + V[i - 1])
    plt.figure(figsize=(10, 5))
    plt.plot(t, I, label="Current (I
        )")
    plt.xlabel("Time (s)")
    plt.ylabel("Current (A)")
    plt.grid(True)
    plt.legend()
    plt.show()

R = 1
L = 1
T = 1
alpha = 0.5
t_end = 5
dt = 0.00005

rl_circuit_response(R, L, T, alpha,
    t_end, dt)
```

Listing 1: Code for Numerical Analysis

The output generated by the above code is:

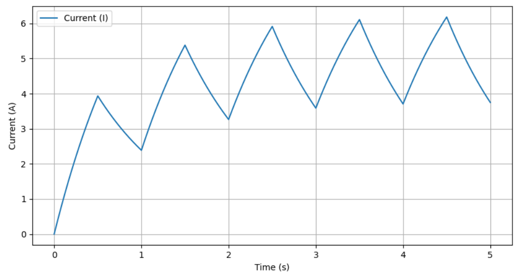

Figure 7: Current Response for $R = 1\Omega$, $L = 1H$, $T = 1s$, and $\alpha = 0.5$

5 Comparison of Solutions

1. The response of the series RL circuit to a square wave input was computed using both numerical and analytical methods.
2. The numerical solution was obtained by solving the differential equation using the Trapezoidal Rule, while the analytical solution was derived using the Fourier series representation of the input signal.
3. Upon plotting the results from both methods, it was observed that the numerical and analytical solutions produced identical waveforms.

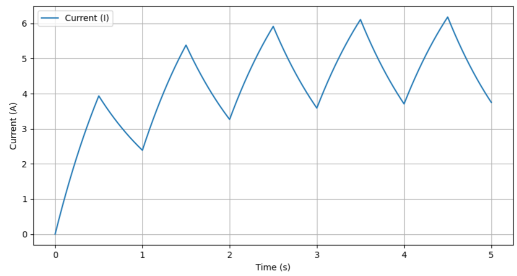

Figure 8: Response Obtained via Numerical Method

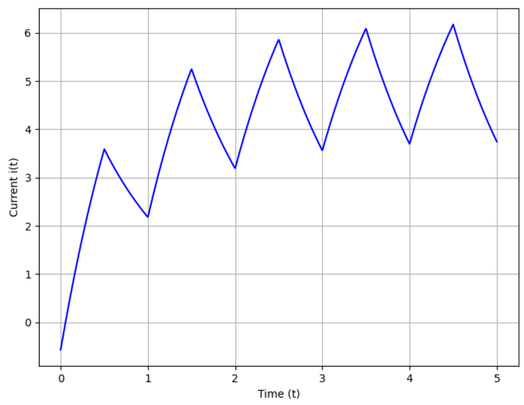

Figure 9: Response Obtained via Analytical Method

This indicates that the numerical integration method accurately approximates the theoretical response of the circuit.

6 Analysis of the Response

Following is an analysis that details how the current response of the series RL circuit to the given input wave varies as R, L, and α change.

6.1 Case I: $T = \tau$

1. We can note moderate transient effects.
2. The current tries to reach to its steady-state value each time the voltage is high ($10V$).
3. When the voltage is low ($0V$), these attempts cease and resume only when the high voltage is achieved again.
4. As α increases, we can note that the variations in current decrease, owing to the fact that the 'ON' period of the input signal is longer, and thus aids in approaching the steady-state.

All these can be realised in the plot given below:

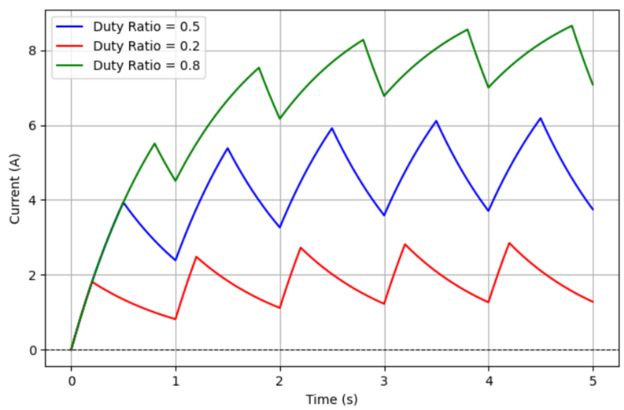

Figure 10: Current Response for $R = 1\Omega$, $L = 1H$, and $T = 1s$

6.2 Case II: $T > \tau$

1. We can note fast transient effects.
2. The resistor dominates the circuit, and affects its behaviour accordingly.
3. The current response closely resembles the input voltage wave (regardless of α). This is due to the low time constant, which enables the RL circuit to attain steady state current fast.

All these can be realised in the plot given below:

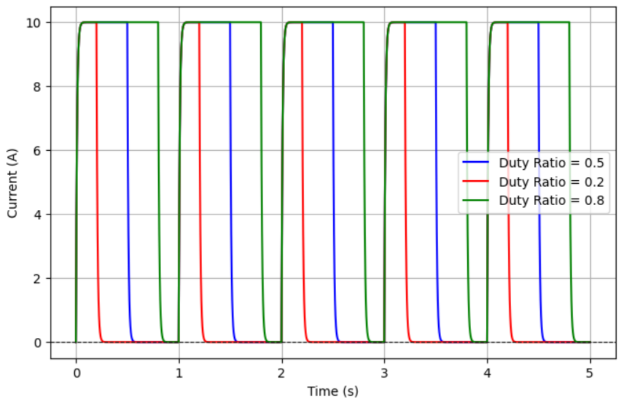

Figure 11: Current Response for $R = 1\Omega$, $L = 0.01H$, and $T = 1s$

6.3 Case III: $T < \tau$

1. We can note a slow transient response.
2. The inductor dominates the circuit, and affects its behaviour accordingly.
3. The current struggles to rise at each high voltage ($10V$) due to the inductive nature of the circuit.
4. As α increases, the current ramps up faster each time the input voltage is

high. The current vs. time graph tends to a straight line.

All these can be realised in the plot given below:

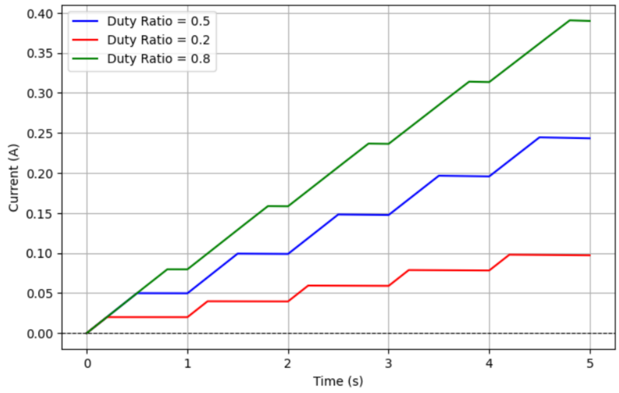

Figure 12: Current Response for $R = 1\Omega$, $L = 100H$, and $T = 1s$

7 Fourier Analysis of Current

7.1 Fourier Transform and Frequency Spectrum

After calculating the current $I(t)$, we apply the Discrete Fourier Transform (DFT) using the Fast Fourier Transform (FFT) algorithm:

$$I_{\text{FFT}}(k) = \sum_{n=0}^{N-1} I_n e^{-j\frac{2\pi kn}{N}} \tag{59}$$

The magnitude spectrum is computed to analyze the frequency components. The frequency axis is determined as:

$$f_k = \frac{k}{N\Delta t} \tag{60}$$

7.2 Dominant Frequency

The dominant frequency is the frequency with the highest magnitude in the Fourier spectrum, excluding the DC component at 0 Hz. Ideally, for a square wave with period T, the dominant frequency should be:

$$f_{\text{dominant}} \approx \frac{1}{T} \tag{61}$$

7.3 Impact of RL Circuit as Low-Pass Filter

An RL circuit behaves as a **low-pass filter**, allowing low-frequency signals to pass while attenuating higher-frequency components. The cutoff frequency is given by:

$$f_c = \frac{R}{2\pi L} \tag{62}$$

For the given parameters:

$$R = 1\,\Omega, \quad L = 1\,\text{H} \implies f_c \approx 0.01\,\text{Hz} \tag{63}$$

When the cutoff frequency is much lower than the fundamental frequency of the square wave ($f_{\text{fundamental}} = \frac{1}{T}$), the higher harmonics of the square wave are heavily attenuated.

7.4 Why Dominant Frequency is Lower for High-Frequency Input

When the input square wave has a high frequency, most of its harmonic components lie above the cutoff frequency and are significantly attenuated. As a result, the observed dominant frequency in the current response is much lower than the fundamental frequency of the input square wave:

$$f_{\text{dominant}} \ll \frac{1}{T} \quad \text{when } f_{\text{fundamental}} \gg f_c \tag{64}$$

7.5 Example: High-Frequency Square Wave Attenuation

Consider an example where:

- Period of the square wave: $T = 0.01\,\text{s} \implies f_{\text{fundamental}} = 100\,\text{Hz}$
- Cutoff frequency of the RL circuit: $f_c \approx 0.01\,\text{Hz}$

Since the square wave's fundamental frequency is much higher than the cutoff frequency, the RL circuit strongly attenuates the harmonics. As a result, the dominant frequency observed in the current is not 100 Hz but a much lower value, approximately equal to the cutoff frequency:

$$f_{\text{dominant}} \approx 0.01\,\text{Hz} \tag{65}$$

This confirms that the RL circuit acts as a low-pass filter, suppressing higher frequencies and causing the dominant frequency to be much lower than expected.

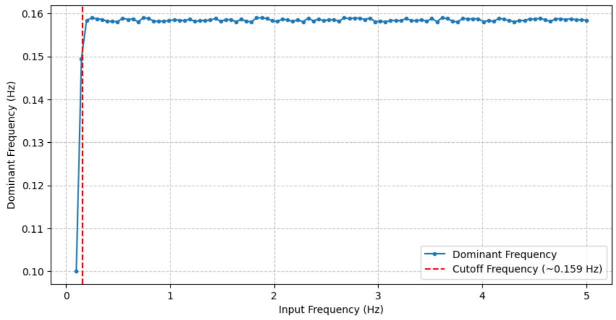

Figure 13: Frequency vs Dominant Frequency of $R = 1\Omega$, $L = 1H$

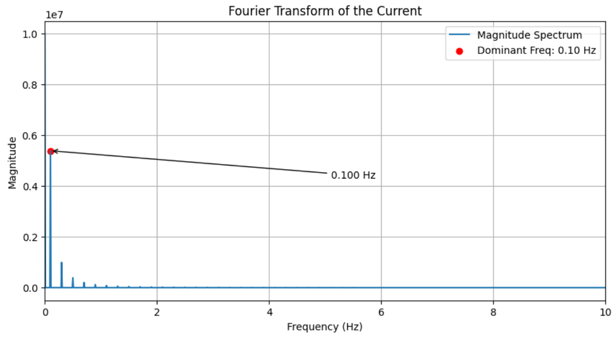

Figure 14: Plot of Fourier coefficients of current function when $R = 1\Omega$, $L = 1H$, and $T = 10s$

8 Summary and Conclusion

1. The given input wave has been successfully decomposed into its Fourier Series expression.

$$v(t) = 10\alpha + \sum_{n=1}^{\infty} \left[\frac{10\sin(2\pi n\alpha)}{\pi n} \cos\left(\frac{2\pi n}{T}t\right) + \frac{10}{\pi n}(1-\cos(2\pi n\alpha))\sin\left(\frac{2\pi n}{T}t\right) \right]$$

2. The differential equation was deduced, and it was used to solve for the current response of the RL circuit analytically and via a numerical method (trapezoidal rule).

$$i(t) = \frac{a_0}{R}\left(1 - e^{\frac{-R}{L}t}\right) + \sum_{n=1}^{\infty} \left(\frac{\sqrt{a_n^2 + b_n^2}}{\sqrt{R^2 + (n\omega_0 L)^2}} \cos(n\omega_0 t - \phi - \theta) - e^{\frac{-R}{L}t} \frac{\cos(\phi + \theta)}{\sqrt{R^2 + (n\omega_0 L)^2}} \right)$$

where

$$\begin{aligned} a_0 &= 10\alpha \\ \phi &= \arctan \frac{b_n}{a_n} \\ \theta &= tan^{-1}\left(\frac{n\omega_0 L}{R}\right) \\ a_n &= \frac{10\sin 2\pi n\alpha}{\pi n} \\ b_n &= \left(\frac{10}{\pi n}\right)(1 - \cos 2\pi n\alpha) \end{aligned}$$

3. Responses obtained through both methods were compared, and furthur analysis was done to understand the nature of the response and how it varies with R, L, and α.
4. Finally, a Fourier analysis was done on the current response, and other related characteristics were explored.
5. Codes for all generated plots can be found at this GitHub repository.

References

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