## Indian Statistical Institute

Semester-II 2012-2013

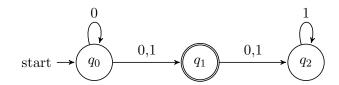
M.Tech.(CS) - First Year

Class Test I (8 February, 2013)

Subject: Automata, Languages and Computation

Total: 20 marks
Solutions

## 1. Suppose the following non-deterministic finite automaton (NFA) is converted to an equivalent deterministic finite automaton (DFA) using the standard algorithm. [4]



Determine whether each of the following statements is true or false.

(a) 
$$\delta(\{q_1\}, 0) = \{q_1, q_2\}.$$
 FALSE

(b) 
$$\delta(\lbrace q_2 \rbrace, 0) = \lbrace \varnothing \rbrace$$
. (This was a typo; it should read  $\delta(\lbrace q_2 \rbrace, 0) = \varnothing$ .)

(c) The state 
$$\{q_0, q_2\}$$
 is unreachable.

(d) The state 
$$\{q_0, q_1, q_2\}$$
 is a final state.

2. Write down the regular expression for hexadecimal numbers in C. [4]

Answer: 
$$0 \times 1 = 0$$
  $1 \times 1$   $1 \times 1$ 

3. The language  $L = \{0^p | p \text{ is prime }\}$  is not regular. If you have to prove this using the Pumping Lemma, how many times should you pump v? Your answer should be in terms of the lengths of u, v, w (u, v, w have their usual significance). [6]

**Answer:** Let  $x = uvw \in L$ . Then  $uv^{|x|+1}w \notin L$ .

(Length of 
$$uv^{|x|+1}w = |uvw| + |x||v| = |x|(1+|v|)$$
, where  $|v| \ge 1$ .)

For just the correct answer (proof missing / incorrect), you get 2 marks.

4. Let  $M_1 = (Q_1, \Sigma, \delta_1, q_0^{(1)}, F_1)$  and  $M_2 = (Q_2, \Sigma, \delta_2, q_0^{(2)}, F_2)$  be two DFAs. Describe DFAs  $M_{\cup}$  and  $M_{\cap}$  that accept, respectively,  $L(M_1) \cup L(M_2)$  and  $L(M_1) \cap L(M_2)$ . [6]

$$M_{\cup}$$
  $M_{\cap}$ 

States (1 mark) 
$$(Q_1 \times Q_2)$$
 for both

Alphabet 
$$\Sigma$$
 for both

Transition (1 mark) 
$$\delta((q_1, q_2), a) = (\delta_1(q_1, a), \ \delta_2(q_2, a) \text{ for both}$$

Initial state (1 mark) 
$$(q_0^{(1)}, q_0^{(2)})$$
 for both

Final states (1.5 marks 
$$\times$$
 2)  $(Q_1 \times F_2) \cup (F_1 \times Q_2)$   $F_1 \times F_2$