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9. $\frac{dy}{dx} = \frac{y^3}{e^{2x}+y^2}$

$\Rightarrow$ Given D.E is, $\frac{dx}{dy} = \frac{e^{2x}}{y^3} + \frac{1}{y}$

Multiplying both sides by e^{-2x}

$$e^{-2x}\cdot\frac{dx}{dy} = \frac{1}{y^3} + \frac{1}{y}e^{-2x}$$

$$e^{-2x}\frac{dx}{dy} + \left(\frac{-1}{y}\right)e^{-2x} = \frac{1}{y^3}$$

Put $e^{-2x} = t \Rightarrow -2\cdot e^{-2x}\cdot\frac{dx}{dy} = \frac{dt}{dy}$

$$\therefore \left(-\frac{1}{2}\right)\frac{dt}{dy} + \left(-\frac{1}{y}\right)t = \frac{1}{y^3}$$

$$\frac{dt}{dy} + \left(\frac{2}{y}\right)t = (-2)\cdot\frac{1}{y^3} \quad \cdots\cdots \text{LDE in } t, y$$

$P = \frac{2}{y}, Q = -\frac{2}{y^3}$

$$I.F. = e^{\int P dy} = e^{\int 2\frac{1}{y}dy} = e^{2\log y} = y^2$$

$\therefore$ General soln is,

$$t(I.F.) = \int (Q.I.F)dy + C$$

$$t\cdot y^2 = \int -\frac{2}{y^3}\cdot y^2 dy + C$$

$$t\cdot y^2 = -2\int \frac{1}{y}dy + C$$

$$t\cdot y^2 = -2\log y + C$$

$$e^{-2x}\cdot y^2 = -2\log y + C$$

$$\boxed{e^{-2x}\cdot y^2 + \log y^2 = C}$$