MGM's College of Engineering and Technology Kamothe, Navi Mumbai
Department of Computer Engineering
Assignment- 3

Course Code: CSC302 **Course Name: Discrete Structure and Graph Theory**
Class: BE/III **AY: 2021-2022**
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Q.No	Question
Q1. Fill in the blanks	
a)	A subgroup has the properties of ___________ a) Closure, associative b) Closure, associative, Identity, Inverse
b)	A group (M,*) is said to be abelian if _________ a) (x+y)=(y+x) b) (x*y)=(y*x)
c)	________is the multiplicative identity of natural numbers. a) 0 b) -1 c) 1 d) 2
d)	Consider the recurrence relation a1=4, an=5n+an-1. The value of a64 is___ a) 10399 b) 23760 c) 75100 d) 53700
e)	If a * b = a such that a ∗ (b ∗ c) = a ∗ b = a and (a * b) * c = a * b = a then ____ a) *is associative b) * is commutative
Q2. Choose Correct Options	
a)	A non empty set A is termed as an algebraic structure _____________ a) with respect to binary operation * b) with respect to ternary operation ? c) with respect to binary operation + d) with respect to unary operation –
b)	A monoid is called a group if _____________ a) (a*a)=a=(a+c) b) (a*c)=(a+c) c) (a+c)=a d) (a*c)=(c*a)=e
c)	Which statement is false? a) The set of rational integers is an abelian group under addition b) The set of rational numbers form an abelian group under multiplication c) The set of rational numbers is an abelian group under addition

	d) None of these
d)	What is the identity element In the group G = {2, 4, 6, 8) under multiplication modulo 10? a) 5 b) 9 c) 6 d) 12
e)	The set of all real numbers under the usual multiplication operation is not a group since a) multiplication is not a binary operation b) multiplication is not associative c) identity element does not exist d) zero has no inverse

Q3. State whether the following statements are true or false (Give Reasons)

a)	A group is a monoid in which every element is invertible. (True/False)
b)	A group is called abelian if it is commutative (True/False)
c)	A simple graph is called a multigraph .(True/False)

Q4. Name the following or define or design the following

a) Define group, monoid, semigroup.

Ans: A group is a monoid such that each a ∈ G has an inverse a −1 ∈ G. A monoid is a semigroup with an identity.

A semigroup is a nonempty set G with an associative binary operation.

b) Prove that the set G = (1, 2, 3, 4, 5, 6) is an abelian group under multiplication modulo 7.

Ans: Since set is finite, we prepare the following multiplication table to examine the group axioms.

$\times_7$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

(G1)(G1) All the entries in the table are elements of G. Therefore G is closed with respect to multiplication modulo 7.

(G2)(G2) Multiplication modulo 7 is associative.

(G3)(G3) Since first row of the is identical to the row of elements of G in the horizontal border, the element to the left of first row in vertical border is identity element i.e., 1 is identity element in G with respect to multiplication mod 7.

(G4)(G4) From the table it is obvious that inverses of 1,2,3,4,5,6 are 1,4,5,2,3 and 6 respectively. Hence inverse of each element in G exists.

(G5)(G5) The composition is commutative because the elements equidistant from principal diagonal are

c) Solve the recurrence relation an-7an-1 +10an-2 =6+8n with a0 =13 and a1=29.

Soln: The Characteristic equation is

~~$\alpha^2 - 7a_{n-1} + 10$~~

$\alpha^2 - 7\alpha + 10 = 0$; and we get the 2 roots: $\alpha = 2, 5$.

General Homog. Solution $S_H = A_1(2)^n + A_2(5)^n$

let APS, $a_n = P_1 n + P_0$

Substitute this APS in the given INhomog. LRR; Get P_i's;

You will get $P_1 = 2$ and $P_0 = 8$.

Hence the general solution is

$a_n = A_1(2)^n + A_2(5)^n + 2n + 8$.

Now, use the initial conditions to find A_1 and A_2

$A_1 = 2$, $A_2 = 3$.

Hence the solution is.

$a_n = 2(2)^n + 3(5)^n + 2n + 8$.

Ans:

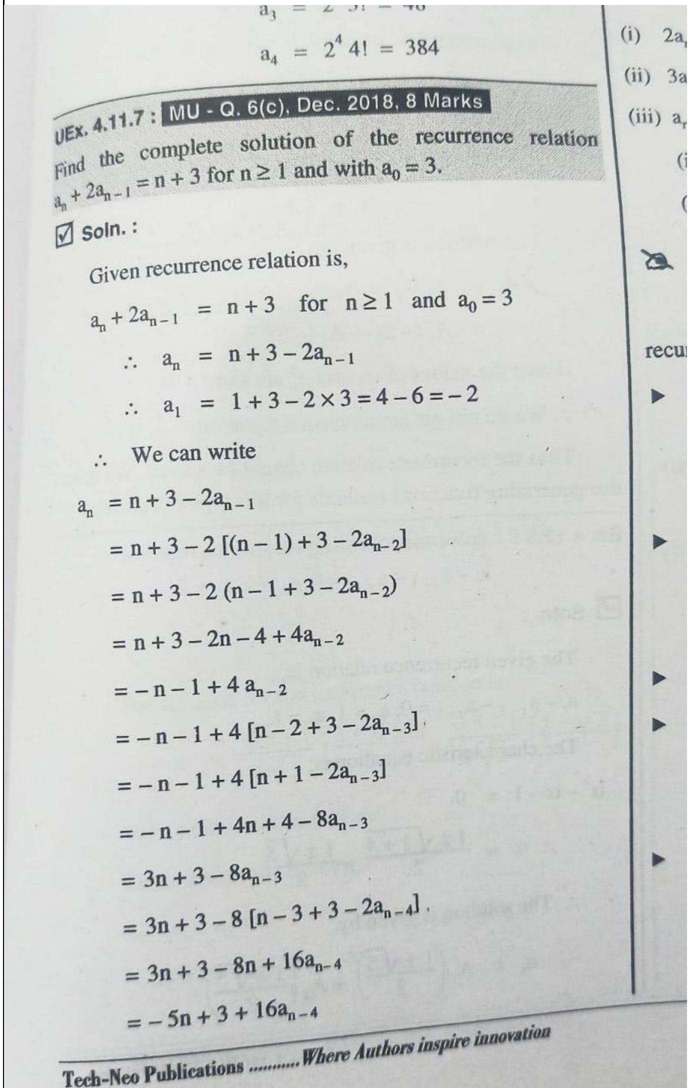
Q5. Answer the following questions in brief (20 to 30 words)

a) How many bit strings of length eight either start with a 1 bit or end with the two bits 00?

Ans: Number of bit strings of length 8 that start with 1: 27 = 128. Number of bit strings of length 8 that end with 00: 26 = 64. Number of bit strings of length 8 that start with 1 and end with 00: 2 5 = 32. Applying the subtraction rule, the number is 128 + 64 − 32 = 160.

b) Find the complete solution of an+2an-1= n+3for n≥1 with a0=3

Ans:



$a_4 = 2^4\, 4! = 384$

UEx. 4.11.7 : MU - Q. 6(c), Dec. 2018, 8 Marks

Find the complete solution of the recurrence relation $a_n + 2a_{n-1} = n + 3$ for $n \geq 1$ and with $a_0 = 3$.

☑ Soln. :

Given recurrence relation is,

$$a_n + 2a_{n-1} = n + 3 \quad \text{for} \quad n \geq 1 \quad \text{and} \quad a_0 = 3$$

$$\therefore \quad a_n = n + 3 - 2a_{n-1}$$

$$\therefore \quad a_1 = 1 + 3 - 2 \times 3 = 4 - 6 = -2$$

$\therefore$ We can write

$$\begin{aligned} a_n &= n + 3 - 2a_{n-1} \\ &= n + 3 - 2\,[(n-1) + 3 - 2a_{n-2}] \\ &= n + 3 - 2\,(n - 1 + 3 - 2a_{n-2}) \\ &= n + 3 - 2n - 4 + 4a_{n-2} \\ &= -n - 1 + 4\,a_{n-2} \\ &= -n - 1 + 4\,[n - 2 + 3 - 2a_{n-3}] \\ &= -n - 1 + 4\,[n + 1 - 2a_{n-3}] \\ &= -n - 1 + 4n + 4 - 8a_{n-3} \\ &= 3n + 3 - 8a_{n-3} \\ &= 3n + 3 - 8\,[n - 3 + 3 - 2a_{n-4}] \\ &= 3n + 3 - 8n + 16a_{n-4} \\ &= -5n + 3 + 16a_{n-4} \end{aligned}$$

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c) How many bit strings of length seven are there?

Ans: 47-bit strings of length seven are there.

Q6. Answer the following questions in brief (50 to 70 words)

a) Suppose statement labels in a programming language can be either a single letter or a letter followed by a digit. Find the number of possible labels.

Ans: If A1, A2, … , Am are finite sets, then the number of elements in the Cartesian product of these sets is the product of the number of elements of each set. ● The task of choosing an element in the Cartesian product A1 ⨉ A2 ⨉ ⋯ ⨉ Am is done by choosing an element in A1, an element in A2 , ..., and an element in Am. ● By the product rule, it follows that: |A1 ⨉ A2 ⨉ ⋯ ⨉ Am |= |A1| ⋅ |A2| ⋅ ⋯ ⋅ |Am|.

Use the product rule. 26 + 26 ⋅ 10 = 286

b) How many different license plates can be made if each plate contains a sequence of three uppercase English letters followed by three digits?

Ans: By the product rule, there are 26 ⋅ 26 ⋅ 26 ⋅ 10 ⋅ 10 ⋅ 10 = **17,576,000** different possible license plates.

c) Show that, the set of all integers is a group with respect to addition

Ans: Let Z = set of all integers. Let a, b, c are any three elements of Z.

1. Closure property : We know that, Sum of two integers is again an integer. i.e., a + b ∈ Z for all a,b ∈ Z
2. Associativity: We know that addition of integers is associative. i.e., (a+b)+c = a+(b+c) for all a,b,c ∈ Z.
3. Identity : We have 0 ∈ Z and a + 0 = a for all a ∈ Z . ∴ Identity element exists, and 0 is the identity element.
4. Inverse: To each a ∈ Z , we have – a ∈ Z such that a + (– a) = 0 Each element in Z has an inverse.
5. Commutativity: We know that addition of integers is commutative. i.e., a + b = b +a for all a,b ∈ Z. Hence, (Z , +) is a group.

Q7. Think and Answer

a)	How many cards must be selected from a standard deck of 52 cards to guarantee that at least three cards of the same suit are chosen? How many must be selected to guarantee that at least three hearts are selected? **Ans:** Suppose that for each suite, we have a box that contains cards of that suit. The number of boxes is 4, by the generalized pigeonhole principle, to have at least 3 (= N/4) cards at the same box, the total number of the cards must be at least N = 2 . 4 + 1 =9. The worst case, we may selects all the clubs, diamonds, and spades (39 cards) before any hearts. So, to guarantee that at least three hearts are selected, 39+3=42 cards should be selected.
b)	The mathematics department must choose either a student or a faculty member as a representative for a university committee. How many choices are there for this representative if there are 37 members of the mathematics faculty and 83 mathematics majors and no one is both a faculty member and a student. **Ans:** There are 37 ways to choose a member of the mathematics faculty and there are 83 ways to choose a student who is a mathematics major. Choosing a member of the mathematics faculty is never the same as choosing a student who is a mathematics major because no one is both a faculty member and a student. By the sum rule it follows that there are 37+83=120 possible ways to pick this representative.
Q8. My Ideas	
a)	Among any group of 367 people, there must be at least two with the same birthday, because there are only 366 possible birthdays. **Ans:** As we know, there are 366 possible days in a year (including leap year) and a college usually has more than 367 students. In the most extreme condition when each of the first 366 students have their birthdays on different days from January 1st to December 31th, the birthday of the 367th person must be a repeat of one of those days. Thus, there are definitely two of the students who have their birthday falling on the same day.
b)	Show that for every integer n there is a multiple of n that has only 0s and 1s in its decimal expansion. **Ans:** Let nn be a positive integer. Consider the n+1n+1 integers 1,11,1,11, 111,...,1111,...111,...,1111,... (where the last integer in this list is the integer with n+1n+1 1s 1s in its decimal expansion). Note that there are nn possible remainders when an integer is divided by nn. Because there are n+1n+1 integers in this list, by the *pigeonhole principle* there must be two with the same remainder when divided by nn. The larger of these integers less the smaller one is a multiple of nn, which has a decimal expansion consisting entirely of 0s0s and 1s1s.