SARDAR VALLABHBHAI PATEL INSTITUTE OF TECHNOLOGY

Electrical Department

TOPIC: TRACING OF CURVE (CARTESIAN AND POLAR)

PREPARED BY:

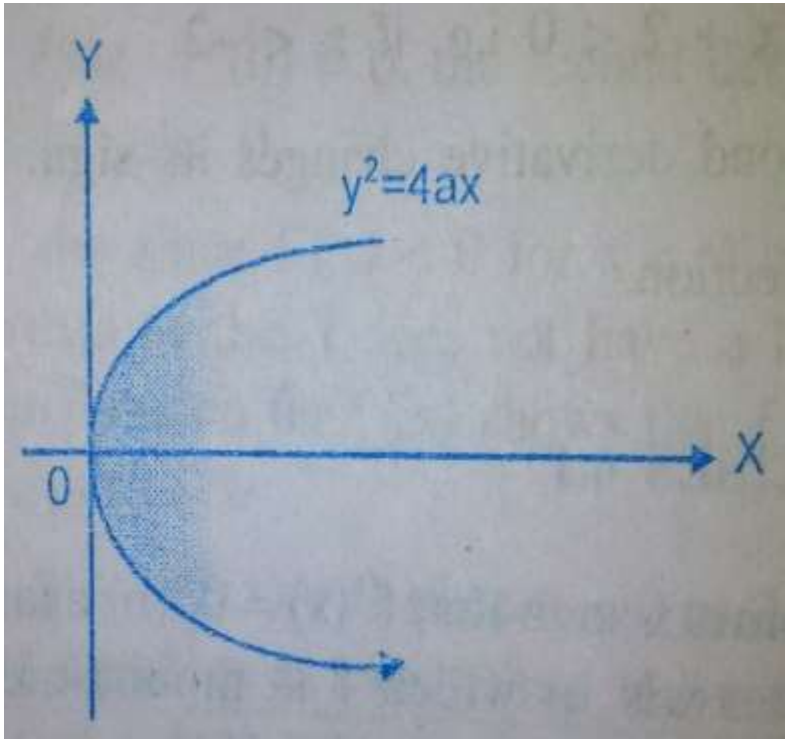
NAME	ID.NO.
ZAHEER KHAN	- 109
RONIT PARMAR	-110
RUSHABH SUKLA	-111
SMIT SHAH	-112
RISHABH ZALA	-113

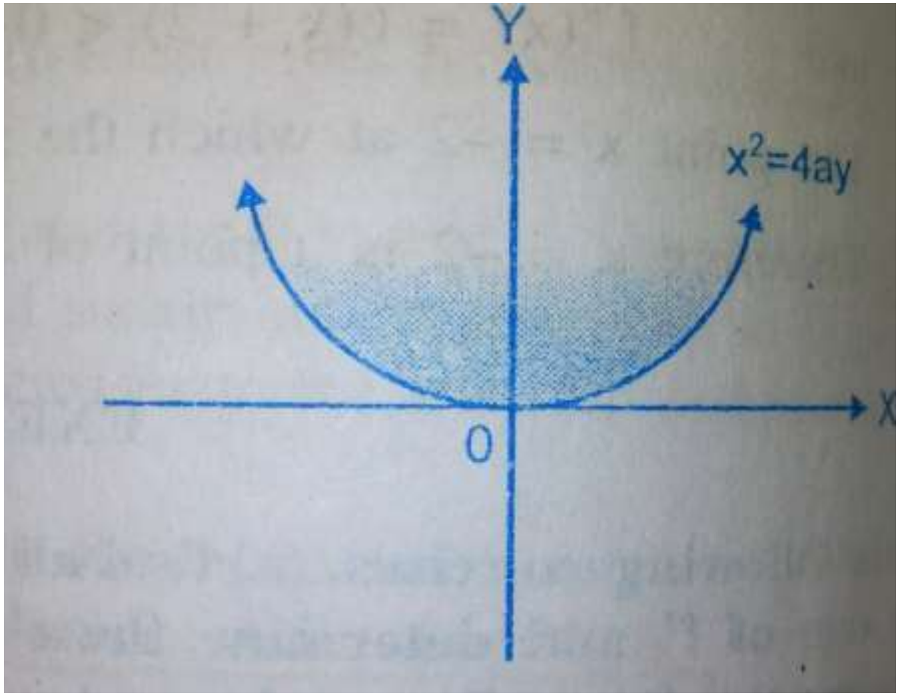
Step 1. Symmetry

- Find out whether the curve is symmetric about any line or a point. The various kinds of symmetry arising from the form of the equation are as follows:

- ***i) <u>symmetric about the y-axis</u>***
- If the equation of the curve remain unaltered when x is replace by –x and the curve is an even function of x.

- ***ii) <u>symmetric about x-axis</u>***
- If the equation of the curve remains unaltered when y is replaced by –y and the curve is an even function of y.


Y
$y^2=4ax$
X
0

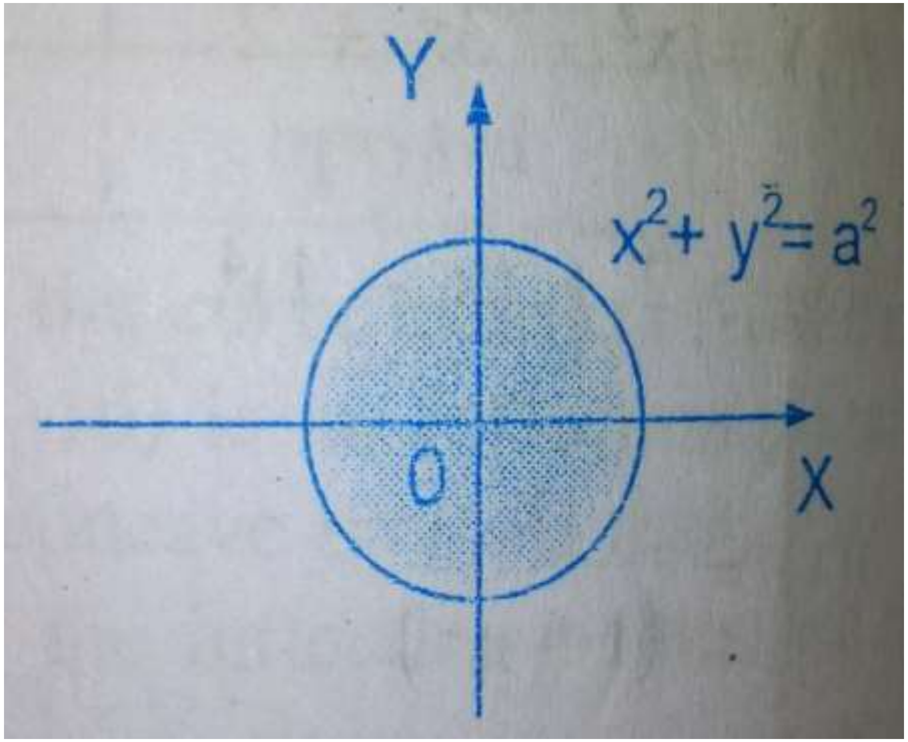

Y
$x^2=4ay$
X
0

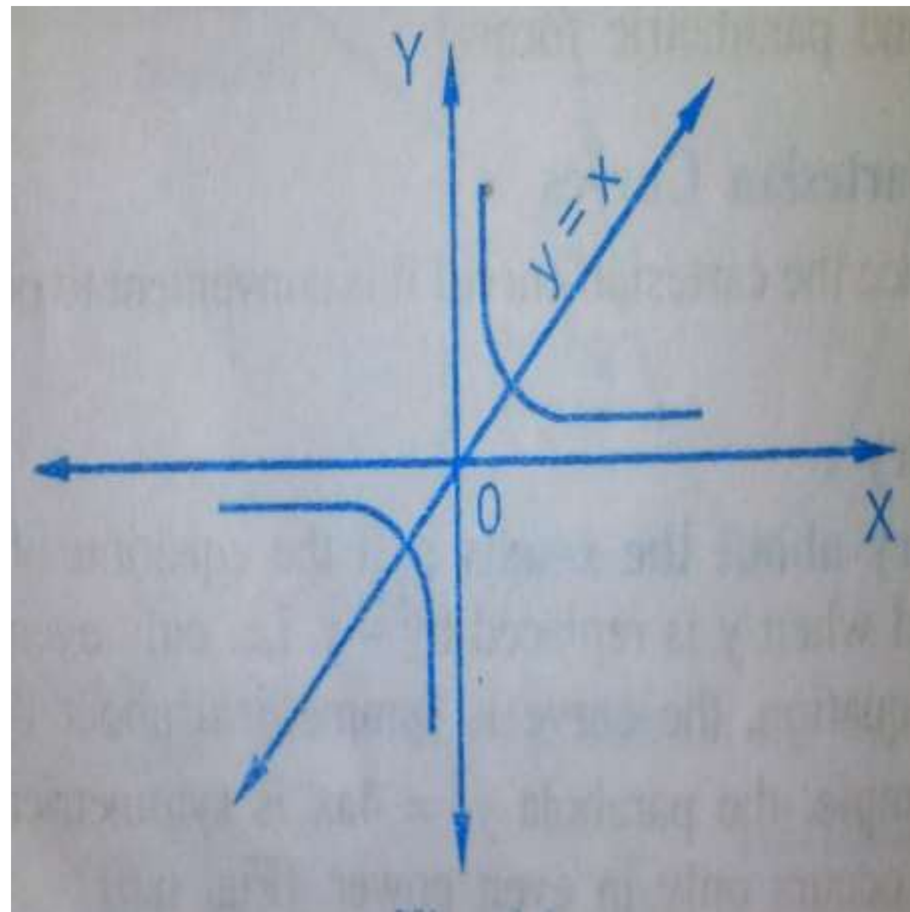
- ***iii) symmetric about both x and y axes***
- If the equations of the curve is such that the powers of x and y both are even everywhere then the curve is symmetrical about both the axes. for example, the circle..

- ***iv) symmetry in the opposite quadrants***

- If the equation of the curve remains unaltered when x is replaced by –x and y is replaced by –y simultaneously, the curve is symmetrical in opposite quadrant. for example, the hyperbola…

- ***v) symmetrical about the line y = x***
- If the equation of the curve remains unaltered when x and y are interchanged, the curve is symmetrical about the line y=x


Y
$x^2+y^2=a^2$
0
X


Y
$y=x$
0
X

Step 2. Origin:

- ***(A)Tangents at the origin***
- The equations of the tangents to the curve at the origin is obtained by equating the lowest degree terms in x and y in the given equation to zero, provided the curve passes through the origin.

- ***(B)Curve through the origin***

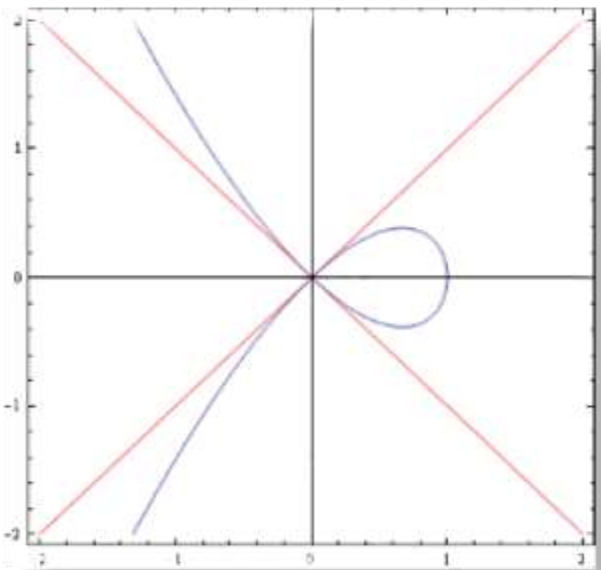
- If the equations of the curve does not contain any constant term,the curve passes through the origin.it will pass through the origin if the equation is satisfied by (0,0) .

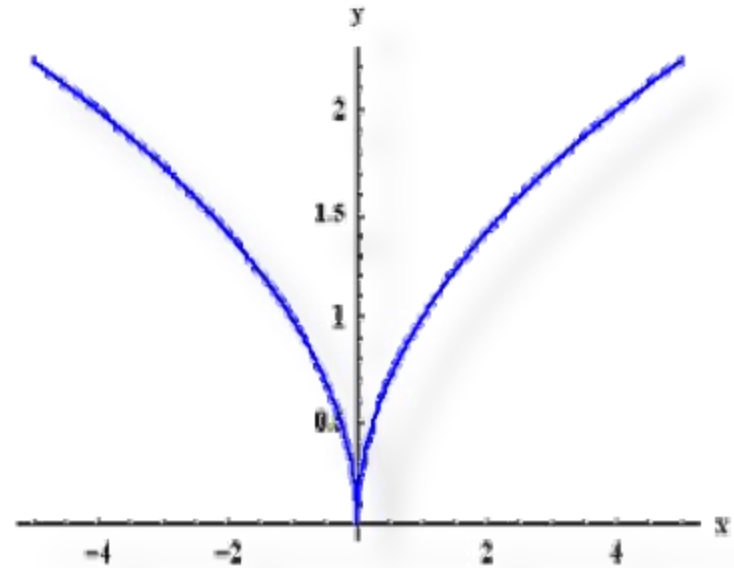
Step:3
Study of special points on the curve

(a) Cusp, Nodes and Conjugate points

A point is called a double point if two branches of the curve passes through it. The double point may be classified as (i) Node, (ii) Cusp, (iii) conjugate point. At such a point the curve has two tangents, one for each branch.

(i) If the tangents are real and distinct, the double point is called a NODE





(ii) If the tangents are real and coincident the double point is called a Cusp.

(iii) A double point is called a cinjugate point or an isolated point if it is neither a node nor a cusp, i.e. the two tangents at the point are all imaginary.
Such a point cannot be shown in figure.

(b) The points of intersection with the co-ordinate axis

The points of intersection, if any with the x and y-axis can respectively obtained by putting y=o and x=0.

For example, the circle $x^2 + y^2 = r^2$ meets the x-axis, where y=0,

i.e. $x^2 = a^2$ or $x = \pm a$. It meets the y-axis, where x=0, i.e. $y^2 = a^2$ or $y = \pm a$. Thus, the circle meets the x-axis at $(\pm a, 0)$ and the y-axis at $(0, \pm a)$.

(c) Points, where tangents are parallel to the co-ordinate axes

Points, where the tangents are parallel to the x-axis are given by $dy/dx=0$ and the points, where the tangents are parallel to the y-axis are given by $dy/dx=\pm\infty$

Asymptotes

Asymptotes are the tangents to the curve at infinity. We shall consider separately the cases which arises when an asymptote is (a) parallel to either co-ordinate axis or (b) an oblique asymptote.

(a) Asymptotes parallel to co-ordinate axis:

(i) To find the asymptotes parallel to x-axis, equal to zero the coefficient of the highest degree terms in x.

(ii) To find the asymptotes parallel to y-axis, equal to zero the coefficient of the highest degree terms in y.

(b) Oblique asymptotes :

(1) Let $y = mx + c$ be the equation of the asymptote to the curve
(2) Form an nth degree polynomial of m by putting $x = 1$, $y = m$ in the given equation to the curve
(3) $\emptyset_n(m)$ and $\emptyset_{n-1}(m)$ be polynomials of terms of degree n and $(n-1)$
(4) Solve $\emptyset_n(m) = 0$ to determine m.
(5) Find 'c' by the formula $c = -\emptyset_{n-1}(m)/\emptyset'_n(m)$
(6) Substitute the values of m and c in $y = mx + c$ in turn.

EXAMPLE : find the asymptotes of the curve

$y^3 - x^2 (6 - a) = 0$

Solution : it has no asymptotes parallel to the axis. Let $y = mx + c$ be an oblique asymptote to the curve.

Putting $x=1$ and $y=m$ in the third degree terms, We find $\emptyset_3 (m) = m^3 + 1$

Also putting $x = 1$ and $y = m$ in the second degree term, we find $\emptyset_2 (m) = -6$.

Now $\emptyset_3(m) = 0 \rightarrow m^3 + 1$

or $(m + 1)(m^2 - m + 1) = 0$

or $m = -1$ remaining roots are imaginary.

$\emptyset'_3(m) = 3m^2$

$c = -\emptyset_2(m)/\emptyset'_3(m) = -(-6)/3m^2 = 2/m^2$

$c = 2$ when $m = -1$

Hence the asymptote is $y = -x + 2$

i.e. $x + y = 2$

Step 5 : Regions Where no Part of the Curve Lies :

(a) If it is possible to express the equation as $y = f(x)$ and if y becomes imaginary for some value of $x > a$ (say), then no part of the curve exists beyond $x = a$.

(b) Similarly, if it is possible to express the equation as $x = f(y)$ and if x becomes imaginary for some value of $y > b$(say), then no part of the curve exists beyond $y = b$.

EXAMPLE 1 **Cissoid of Diocle**

Trace the curve $y^2(2a - x) = x^3$.

SOLUTION The equation of the curve can be written as

$$y^2 = \frac{x^3}{2a - x} \quad \text{...(1)}$$

(i) **Symmetry :** The equation contains only even powers of y, therefore it is symmetrical about x–axis.

(ii) **Origin :** Equation does not contain any constant, therefore it passes through the origin.

From (1) we have

$$y^2(2a - x) = x^3 \text{ i.e. } 2ay^2 - xy^2 - x^3 = 0$$

In order to find tangents at the origin, equating to zero the lowest degree terms, we have.

$$2ay^2 = 0 \Rightarrow y^2 = 0,\ y = 0,\ 0 \text{ is a double point}$$

$\therefore$ x–axis is tangent at origin.

(iii) **Special point :** Since the two tangents at the origin are coincident, therefore origin is a **Cusp**.

For intersection with x–axis, we put $y = 0$

$$\therefore \quad \frac{x^3}{2a - x} = 0 \Rightarrow x = 0$$

and for intersection with y–axis, we put $x = 0$

$\therefore \quad 2ay^2 = 0$

$\therefore \quad y^2 = 0 \Rightarrow y = 0$

Thus curve meets the coordinate axes only at (0, 0).

(iv) **Asymptotes :** $y^2 = \dfrac{x^3}{2a - x}$

As $x \to 2a$, $y \to \infty$, hence the only asymptote parallel to y–axis is $x = 2a$.

(v) **Regions :** From the equation we observe that for $x < 0$ and $x > 2a$, y^2 becomes negative hence y becomes imaginary, therefore the curve does not exist for $x < 0$ and $x > 2a$.

A rough sketch of the curve is shown in Fig. 6.12.

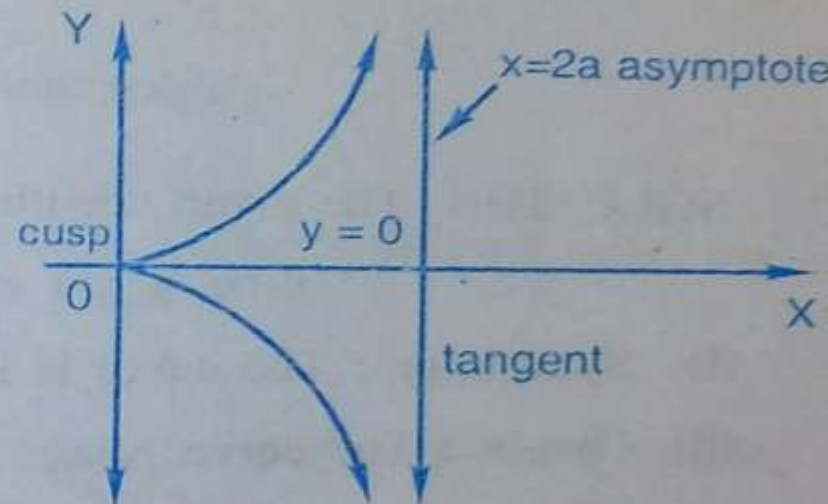


Fig. 6.12

II. Tracing of Polar Curves

To trace a polar curve $r = f(\theta)$ or $g(r, \theta) = c$, a constant, we use the following procedure.

1. Symmetry

i) if the equation of the curve is an even function of θ, then the curve is symmetrical about the initial line.

ii) If the equation is an even function of r, the curve is symmetric about the origin.

iii) If the equation remains unaltered when θ is replaced by $-\theta$ and r is replace by -r then curve is symmetric about the line through the pole and perpendicular to the initial line…

iv) If the equation remains unaltered when r is replace by –r then curve is symmetric about the pole.In such a case only even power of r will occur in the equation.

2. Region:

Determine the region for θ for which r is defined and real .

3. Tabulation:

For selected values of θ determine the values of r and tabulate them.

4. The angle ϕ :

Find the value of ϕ the angle between the radius vector and tangent to the curve defined by

$$\phi = \tan^{-1}\left(\frac{r}{(dr/d\theta)}\right)$$

Then the angle ψ made by the tangent to the curve with the initial line is given by

$$\psi = \theta + \phi$$

The tangents to the curve at different points can be determined by the angle ψ.

5. Asymptotes: Find out the asymptotes of the curve, if any.

<u>Examples :</u>

1) Trace the curve $r = a(1+\cos\theta)$, $a > 0$.

- The equation is an even function of $\theta \Rightarrow$ the curve is <u>symmetric about the initial line</u>. It is also a periodic function of θ with period 2π.
- Therefore it is sufficient to trace the curve for $\theta \in (0,2\pi)$. By symmetry it is sufficient to trace the curve for $\theta \in (0, \pi)$.
- Curve is defined (r is real) for $\theta \in (0, \pi)$.

Since $-1 \le \cos\theta \le 1$, we have $0 \le r \le 2a$.
Therefore the curve lies within the circle $r = 2a$.

The value of ϕ: Let ϕ be the angle made by the tangent at (r, θ) with the radius vector, then

$$\tan\phi = \frac{r}{(dr/d\theta)} = \frac{a(1+\cos\theta)}{-a\sin\theta} = \frac{2\cos^2\theta/2}{-2(\sin\theta/2)(\cos\theta/2)}$$

$$= -\cot\theta/2 = \tan(\frac{\pi}{2}+\frac{\theta}{2})$$

$$\phi = \frac{\pi}{2}+\frac{\theta}{2}$$

$$\textit{Therefore}\quad \psi = \theta+\phi = \frac{\pi}{2}+\frac{\theta}{2}+\theta = \frac{\pi}{2}+\frac{3\theta}{2}$$

Now r = 0 $\Leftrightarrow$ a(1+cos θ) = 0

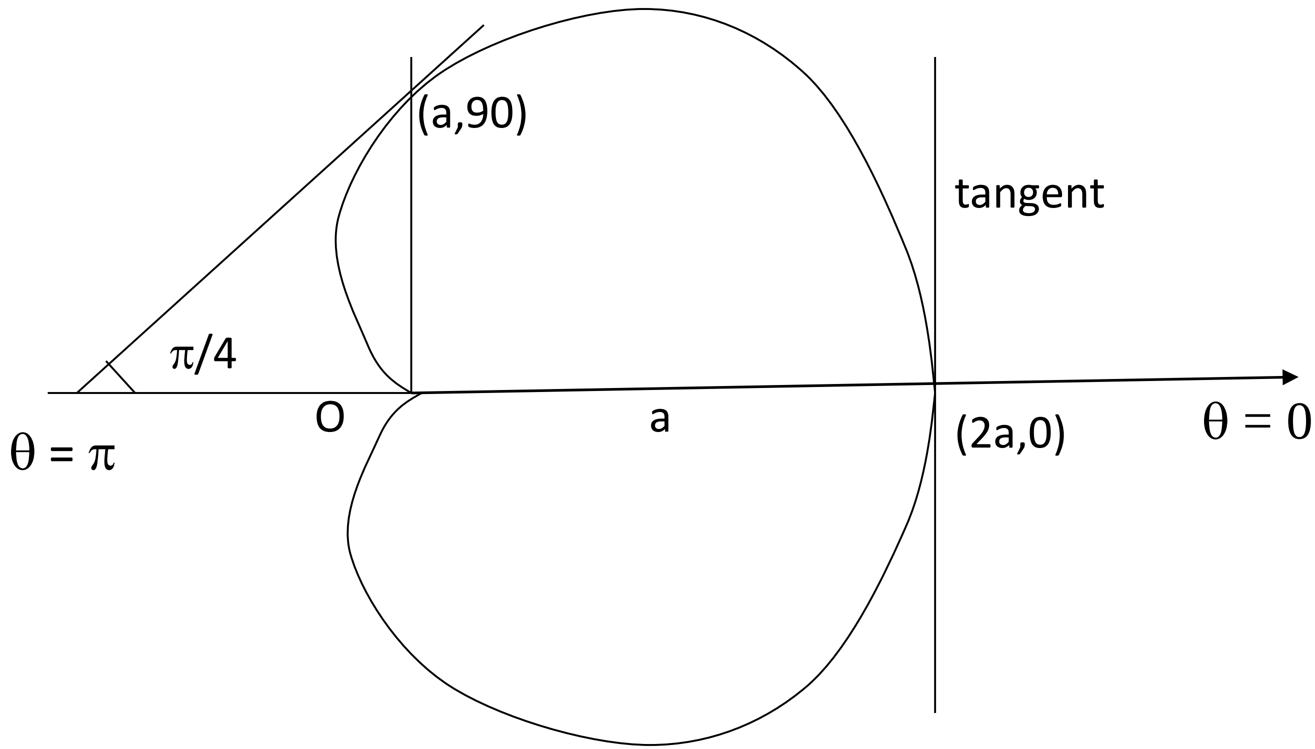
$\Leftrightarrow$ cos θ = -1 ($0 \le \theta \le 2\pi$) .

Therefore θ = π and so the curve passes through

the origin once and $\theta = \pi$ is the tangent to the curve at the pole.

Different Points on the curve:

θ	0	60	90	120	180
r	2a	3a/2	a	a/2	0


(a,90)
tangent
π/4
O
a
θ = π
(2a,0)
θ = 0

2. Example: Trace the curve $r^2 = a^2 \cos 2\theta$

- The equation is an even function of θ and r.

  Therefore the curve is symmetric about the initial line and the origin.

- The equation is periodic function of θ with period π. Therefore it is sufficient to trace the curve in $(0, \pi)$.
- Also if θ is replaced by $\pi - \theta$, the equation remains unaltered . Therefore the curve is symmetric about the ray $\theta = \pi/2$.

- Region: r is real if $\cos 2\theta \geq 0$

$\Rightarrow\ 0 \leq \theta < \pi/2$ or $3\pi/2 < 2\theta < 2\pi$. Therefore the curve exists for $0 < \theta < \pi/4$, $3\pi/4 < \theta < \pi$,

- For $0 \leq \theta \leq \pi$, r is finite, the curve has no asymptotes.

θ	0	$\pi/4$	$3\pi/4$	π
r	a	0	0	a

- Taking log both sides

$2 \log r = 2 \log a + \log \cos 2\theta$

Differentiate with respect to θ

$2 (1/r) (dr/d\theta) = 0 = 2 \sin 2\theta / \cos 2\theta$

Therefore $\tan \phi = - \cos 2\theta / \sin 2\theta = -\cot 2\theta = \tan(\pi/2+2\theta)$

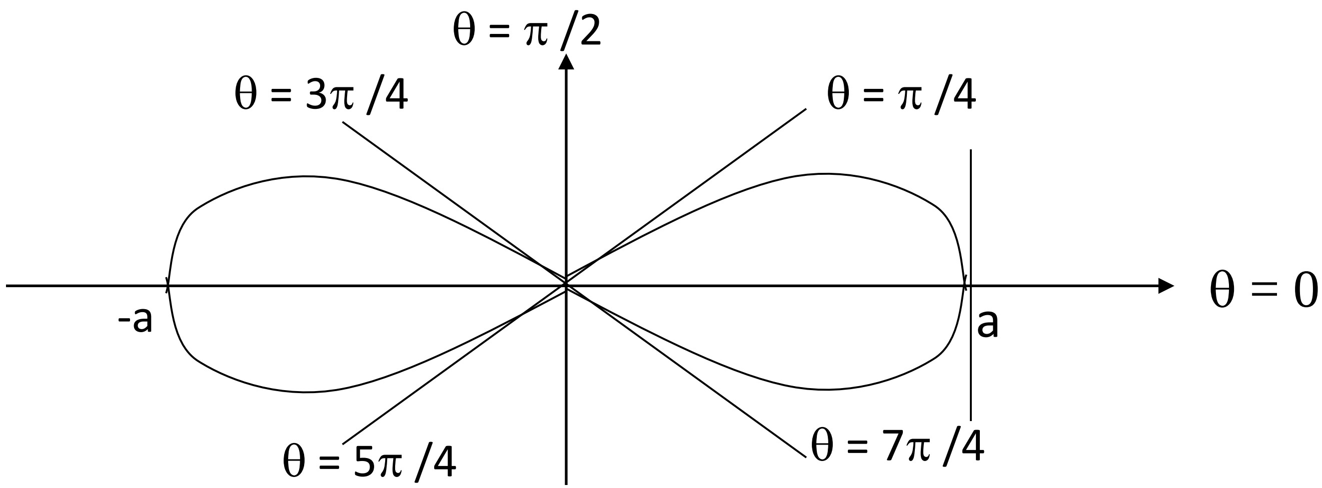
Therefore $\tan\phi = -\cos 2\theta/\sin 2\theta = -\cot 2\theta$

$= \tan(\pi/2 + 2\theta)$ and so $\phi = \pi/2 + 2\theta$.

Thus $\psi = \theta + \phi = \pi/2 + 3\theta$

At $\theta = 0$, $\psi = \pi/2$. Therefore the tangent is perpendicular to the initial line.

At $\theta = \pi/4$, $\psi = \pi/2 + 3\pi/4 = \pi+\pi/4$


θ = π /2
θ = 3π /4
θ = π /4
θ = 0
-a
a
θ = 5π /4
θ = 7π /4

THANK YOU