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Tutorial-4

1) Solve $(D^2+D)y = e^{4x}$

Sol^n The A.E. is $m^2+m=0$

$$m(m+1)=0$$

$$m=0 \text{ or } m=-1$$

C.F. is $Ae^{0x}+Be^{-x}$

$$P.I. = \frac{1}{D^2+D}e^{4x} = \frac{e^{4x}}{16+4} = \frac{e^{4x}}{20}$$

so the general sol^n is $y = C.F. + P.I.$

$$y = A + Be^{-x} + \frac{e^{4x}}{20}$$

2) Solve $(D^3-4D)y = 2\cosh x \cosh 2x$

Sol^n: A.E. is $m^3-4m=0$

$$m(m^2-4)=0$$

$$m=0 \text{ or } m=2,-2$$

$$C.F. = C_1e^0 + C_2e^{2x} + C_3e^{-2x}$$

$$P.I. = \frac{1}{D^3-4D} 2\cosh x \cosh 2x$$

$$= \frac{1}{D^3-4D}\, 2\left(\frac{e^{2x}+e^{-2x}}{2}\right)^2$$

$$= \frac{1}{2}\,\frac{1}{D^3-4D}\left(e^{4x}+2+e^{-4x}\right]$$

$$= \frac{1}{2}\left[\frac{1}{D^3-4D}e^{4x} + \frac{2}{D(D^2-4)}e^{0x} + \frac{1}{D^3-4D}e^{-4x}\right]$$

$$= \frac{1}{2}\left[\frac{1}{48}e^{4x} - \frac{x}{2} - \frac{1}{48}e^{-4x}\right]$$

$$PI = -\frac{x}{2} + \frac{1}{48}\left(\frac{e^{4x}-e^{-4x}}{2}\right)$$

$$= -\frac{x}{4} + \frac{1}{4x}\sinh 4x.$$

$$y = C_1 + C_2 e^{2x} + C_3 e^{-2x} - \frac{x}{4} + \frac{1}{48}\sinh 4x.$$

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3) Solve $(D-1)^2(D^2+1)y = e^x + \sin^2\left(\frac{x}{2}\right)$

Sol^n AE: $(D-1)^2(D^2+1) = 0$

$(D-1)^2 \cancel{(D+1)} (D^2+1) = 0$

$D = 1, +1, i, -i$

$C.F. = (C_1 + C_2 x)e^x + (C_3 \cos x + C_4 \sin x)$

$PI = \frac{1}{(D-1)^2(D^2+1)}\left[e^x + \sin^2\frac{x}{2}\right]$

$\frac{1}{(D-1)^2(D^2+1)} e^x = \frac{x^2}{2!} \cdot \frac{1}{2} e^x$

$\frac{1}{(D-1)^2(D^2+1)} \sin^2\frac{x}{2} = \frac{1}{(D-1)^2(D^2+1)}\left[\frac{1-\cos x}{2}\right]$

$= \frac{1}{(D-1)^2(D^2+1)}\left(\frac{1}{2}e^{0x}\right) = \frac{1}{(D-1)^2(D^2+1)}\left(\frac{1}{2}\cos x\right)$

$= \frac{1}{(-1)^2(1)} \frac{1}{2} - \frac{1}{(D^2-2D+1)(D^2+1)}\left(\frac{1}{2}\cos x\right)$

$= \frac{1}{2} - \frac{1}{-2D} \frac{1}{(D^2+1)}\left(\frac{\cos x}{2}\right) = \frac{1}{2} - \frac{1}{(D^2+1)} \cdot \frac{D}{(-2D^2)}$

$\left(\cos\frac{x}{2}\right) = \frac{1}{2} + \frac{1}{4} \frac{1}{(D^2+1)} \frac{1}{(-1)}(-\sin x)$

$= \frac{1}{2} + \frac{1}{4} \frac{1}{(D^2+1)} \sin x = \frac{1}{2} + \frac{1}{4} \frac{x}{2D} \sin x$

$= \frac{1}{2} + \frac{x}{\cancel{2}} \int \sin x \, dx = \frac{1}{2} - \frac{x}{8} \cos x$

$y = (C_1 + C_2 x)e^x + C_3 \cos x + C_4 \sin x +$
$\frac{1}{4} x^2 e^x + \frac{1}{2} - \frac{x}{8} \cos x.$

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4) Solve: $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2 + e^x + \cos 2x$

Sol^n

$D^2 - 4D + 4 = 0$

$m^2 - 4m + 4 = 0$

$m = 2, 2$

C.F. $= (C_1 + C_2 x)e^{2x}$

P.I. $= \frac{x^2}{D^2 - 4D + 4} + \frac{e^x}{D^2 - 4D + 4} + \frac{\cos 2x}{D^2 - 4D + 4}$

$= PI_1 + PI_2 + PI_3$

$$PI_1 = \frac{x^2}{D^2-4D+4} = \frac{x^2}{4\left[\frac{1-4D-D^2}{4}\right]} = \frac{1}{4}\left[1-\left(\frac{4D-D^2}{4}\right)\right]^{-1}x^2$$

$$= \frac{1}{4}\left[1+\left(\frac{4D-D}{4}\right)^2+D^2\right]x^2$$

$$= \frac{1}{4}\left[x^2+\frac{1}{4}(8x-2)+2\right] = \frac{1}{4}\left[x^2+2x+\frac{3}{2}\right]$$

$$PI_2 = \frac{e^{x}}{D^2-4D+4} \quad (D \to 2) \quad (\because Dr = 0)$$

$$= \frac{\cancel{x}\, e^{x}}{\cancel{2}}$$

$$PI_3 = \frac{\cos 2x}{D^2-4D+4} = \frac{\cos 2x}{-4D} \times \frac{D}{D} = \frac{-\sin 2x \cdot 2}{4D^2}$$

$$= \frac{-\sin 2x}{8}$$

$$y = CF + PI$$

$$= (c_1 + c_2 x)\, e^{2x} + \frac{\cancel{x}\, e^{x}}{\cancel{2}} - \frac{\sin 2x}{8}$$

$$\cancel{PI_1} = + \frac{1}{4}\left[x^2 + 2x + \frac{3}{2}\right]$$

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5) Solve $\frac{d^2y}{dx^2} - y = x\sin 3x$

Sol^n $(D^2 - 1)y = x\sin 3x$

A.E., $D^2 - 1 = 0$

$D = 1$

$C.F. = (C_1 + C_2x)e^{kx} = (C_1 + C_2x)e^{x}$

$P.I. = u_1(x)y_1 + u_2(x)y_2$

$u_1'(x) = -y_2\left(\frac{X}{W^*}\right)$

$U_2'(x) = y_1\left(\frac{X}{W^*}\right)$

W^* $P.I. = \frac{1}{\phi(D)} e^{\alpha x}\int e^{-\alpha x}\phi\, dx$

$y = (C_1 + C_2X)e^{x} + \frac{1}{\phi D} e^{x}\int e^{-x} x\sin 3x$

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6) Apply method of variation of parameters to solve

$$(D^3 - 6D^2 + 12D - 8)Y = \frac{e^{2x}}{x}$$

Sol^n

$$m^3 - 6\cancel{D}m^2 + 12m - 8 = 0$$

$$\cancel{m(m^2 - 6m + 4) = 0}$$

$m = 2, 2, 2.$ $\cancel{m = 0}$

The C.F. is $c_1 e^{2x} + c_2 x e^{2x} + c_3 x^2 e^{2x}$

$$W = \begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{vmatrix} = \begin{vmatrix} e^{2x} & xe^{2x} & x^2e^{2x} \\ 2e^{2x} & (2x+1)e^{2x} & (2x^2+2x)e^{2x} \\ 4e^{2x} & 4(x+1)e^{2x} & (4x^2+8x+2)e^{2x} \end{vmatrix}$$

$$= e^{2x} \begin{vmatrix} 1 & x & x^2 \\ 2 & 2x+1 & 2x^2+2x \\ 4 & 4(x+1) & 4x^2+8x+2 \end{vmatrix}$$

$$= 2e^{6x} \begin{vmatrix} 1 & x & x^2 \\ 2 & 2x+1 & 2x^2+2x \\ 2 & 2x+2 & 2x^2+4x+1 \end{vmatrix}$$

By $R_2 - 2R_1$ & $R_3 - R_2$

$$W = 2e^{6x} \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & 2x \\ 0 & 1 & 2x+1 \end{vmatrix}$$

$$= 2e^{6x}(2x+1-2x) = 2e^{6x}$$

$$PI = uy_1 + vy_2 + wy_3$$

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Notes

$$u = \int \frac{\left[x e^{2x}(2x^2+2x)e^{2x} - x^2 e^{2x}(2x+1)e^{2x}\right]\frac{e^{2x}}{x}}{2e^{6x}}\,dx$$

$$= \int \frac{x}{2}\,dx = \frac{x^2}{4}$$

$$v = \int \frac{(y_3 y_1' - y_1 y_3')}{W}\,dx$$

$$= \int \frac{\left[x^2 e^{2x}\, 2e^{2x} - e^{2x}(2x^2+2x)e^{2x}\right]\frac{e^{2x}}{x}}{2e^{6x}}\,dx$$

$$= \int -\,dx = -x$$

$$w = \int \frac{(y_1 y_2' - y_2 y_1')}{W}\,x\,dx$$

$$= \int \frac{e^{2x}(2x+1)e^{2x} - x e^{2x}\, 2e^{2x}}{2e^{6x}}\,\frac{e^{2x}}{x}\,dx$$

$$= \int \frac{1}{2x}\,dx = \frac{1}{2}\log x$$

$$PI = uy_1 + vy_2 + wy_3$$

$$= \frac{x^2}{4}e^{2x} - x\cdot x e^{2x} + \frac{1}{2}\log x \cdot \frac{x^2}{e^{2x}}$$

$$y = (c_1 + c_2 x + c_3 x^2)e^{2x} - \frac{3x^2}{4}e^{2x} - \frac{x^2}{2}\log x\, e^{2x}$$