Tutorial

1. Solve the following LPP by simplex method

maximize $z = 4x_1 + x_2 + x_2 + 3x_3 + 5x_4$

sub. to $-4x_1 + 6x_2 + 5x_3 + 4x_4 \leq 20$

$-3x_1 - 2x_2 + 4x_3 + x_4 \leq 10$

$-8x_1 - 3x_2 + 3x_3 + 2x_4 \leq 20$

$x_1, x_2, x_3, x_4 \geq 0$

Solⁿ We first express the given problem in standard form.

$z - 4x_1 - x_2 - 3x_3 - 5x_4 + 0s_1 + 0s_2 + 0s_3 = 0$

$-4x_1 + 6x_2 + 5x_3 + 4x_4 + s_1 + 0s_2 + 0s_3 = 20$

$-3x_1 - 2x_2 + 4x_3 + x_4 + 0s_1 + s_2 + 0s_3 = 10$

$-8x_1 - 3x_2 + 3x_3 + 2x_4 + 0s_1 + 0s_2 + s_3 = 20$

We put this information in tabular form as follows:

Simplex Table

Iteration Number	Basic variables	x_1	x_2	x_3	x_4	s_1	s_2	s_3	R.H.S. Solution	Ratio
0	z	-4	-1	-3	-5	0	0	0	0	
s_1 leaves	s_1	-4	6	5	4^*	1	0	0	20	5 ←
x_4 enters	s_2	-3	-2	4	1	0	1	0	10	10
	s_3	-8	-3	3	2	0	0	1	20	10
1	z	-9	13/2	13/4	0	5/4	0	0	25	
	x_1	-1	3/2	5/4	1	1/4	0	0	5	-5
	s_2	-2	-7/2	-1/4	0	-1/4	1	0	5	-5/2
	s_3	-6	-6	1/2	0	-1/2	0	1	10	-5/3

∵ all enteries in the ratio column are negative the problem has unbounded solⁿ.

2. Use penalty method to solve the L.P.P.

Minimise $z = 2x_1 + x_2$

sub. to $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 3$

Sol^n. We have

maximize $z' = -z = -2x_1 - x_2 - 0s_2 - 0s_3 - MA_1 - MA_2$ — (1)

Sub. to $3x_1 + x_2 + 0s_2 + 0s_3 + A_1 + 0A_2 = 3$ — (2)

$4x_1 + 3x_2 - s_2 + 0s_3 + 0A_1 + A_2 = 6$ — (3)

$x_1 + 2x_2 + 0s_2 + s_3 + 0A_1 + 0A_2 = 3$ — (4)

Multiply (2) & (3) by M & to (1).

$\therefore$ Maximise $z' = (-2 + 7M)x_1 + (-1 + 4M)x_2 - Ms_2 + 0s_3 - A_1 - 0A_2 - 9M$

$\therefore z' + (2 - 7M)x_1 + (1 - 4M)x_2 + Ms_2 + 0s_3 + 0A_1 + 0A_2 = -9M$

Simplex Table

Iteration Number	Basic Var.	x_1	x_2	s_2	s_3	A_1	A_2	R.H.S. Sol.	Ratio
0	z'	$2-7M$	$1-4M$	M	0	0	0	$-9M$	
A_1 leaves	A_1	3^*	1	0	0	1	0	3	1 ←
x_1 enters	A_2	4	3	-1	0	0	1	6	1.5
	s_3	1	2	0	1	0	0	3	3
		↑							
1	z'	0	$\frac{1}{3} - \frac{5M}{3}$	M	0		0	$-2-2M$	
A_2 leaves	x_1	1	$1/3$	0	0		0	1	3
x_2 enters	A_2	0	$5/3^*$	-1	0		1	2	6/5 ←
	s_3	0	$5/3$	0	1		0	2	6/5
			↑						
2	z'	0	0	$1/5$	0			$-12/5$	
	x_1	1	0	$1/5$	0			$3/5$	
	x_2	0	1	$-3/5$	0			$6/5$	
	s_3	0	0	1	1			0	

$\therefore x_1 = \frac{3}{5}$, $x_2 = \frac{6}{5}$, $z'_{max} = -\frac{12}{5}$ $\therefore z_{min} = \frac{12}{5}$

3. Minimize $z = 4x_1 + x_2$

sub. to $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 4$

$x_1, x_2 \leq 0$

Sol^n We first write the problem in the standard form

maximise $z' = -z = -4x_1 - x_2$

subject to $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 4$

$\therefore z' = -4x_1 - x_2 - 0s_1 - 0s_2 - MA_1 - MA_2$ — (1)

$3x_1 + x_2 + 0s_1 + 0s_2 + A_1 + 0A_2 = 3$ — (2)

$4x_1 + 3x_2 - s_1 + 0s_2 + 0A_1 + A_2 = 6$ — (3)

$x_1 + 2x_2 + 0s_1 + s_2 + 0A_1 + 0A_2 = 4$ — (4)

multiply (2) & (3) by M & add to (1)

$\therefore z' = (-4 + 7M)x_1 + (-1 + 4M)x_2 - Ms_1 - 0s_2 - 0A_1 - 0A_2 - 9M$

$\therefore z' + (4 - 7M)x_1 + (1 - 4M)x_2 + Ms_1 + 0s_2 + 0A_1 + 0A_2 = -9M$

Simplex Table

Iteration Number	Basic Var.	x_1	x_2	s_1	s_2	A_1	A_2	R.H.S Sol.	Ratio
0	z'	$4-7M$	$1-4M$	M	0	0	0	$-9M$	
A_1 leaves	A_1	3^*	1	0	0	1	0	3	1
x_1 enters	A_2	4	3	-1	0	0	1	6	1.5
	s_2	1	2	0	1	0	0	4	4
		↑							
1	z'	0	$\frac{1+5M}{3}$	M	0	0	$-(4+2M)$		

A_2 leaves	x_1	1	1/3	0	0	0	1	3
x_2 leaves	A_2	0	5/3*	−1	0	1	2	6/5 ←
	s_2	0	5/3	0	1	0	3	9/5
			↑					

2	z'	0	0	−1/5	0	−18/5	
s_2 leaves	x_1	1	0	1/5	0	3/5	3
s_1 enters	x_2	0	1	−3/5	0	6/5	—
	s_2	0	0	1*	1	1	1 ←
				↑			

3	z'	0	0	0	1/5	−17/5
	x_1	1	0	0	−1/5	2/5
	x_2	0	1	0	3/5	9/5
	s_1	0	0	1	1	1

$\therefore x_1 = 2/5, \quad x_2 = 9/5, \quad z'_{max} = -17/5$

$\therefore z_{min} = \frac{17}{5}$

4. Find the dual to the following LPP

Max. $z = x_1 - 2x_2 + 3x_3$

Sub. to $-2x_1 + x_2 + 3x_2 = 2$

$2x_1 + 3x_2 + 4x_3 = 1$

$x_1, x_2, x_3 \geq 0$

Solⁿ. $\because$ the problem is of maximisation type, the constraints must be expressed in less than or equal to form.

$\therefore -2x_1 + x_2 + 3x_3 \geq 2$ and $(-2x_1 + x_2 + 3x_3) \leq 2$

i.e. $-(-2x_1 + 2x_2 + 3x_3) \leq -2$ i.e. $2x_1 - x_2 - 3x_3 \leq -2$ and $-2x_1 + x_2 + 3x_3 \leq 2$

Also $2x_1 + 3x_2 + 4x_3 \geq 1$ & $2x_1 + 3x_2 + 4x_3 \leq 1$

i.e. $-(2x_1 + 3x_2 + 4x_3) \leq -1$ i.e. $-2x_1 - 3x_2 - 4x_3 \leq -1$ and $2x_1 + 3x_2 + 4x_3 \leq 1$

Hence, the given problem becomes,

maximise $z = x_1 - 2x_2 + 3x_3$

Subject to
$$-2x_1 + x_3 + 3x_3 \le 2$$
$$2x_1 + 3x_2 + 4x_3 \le 1$$
$$-2x_1 - 3x_2 - 4x_3 \le -1$$
$$x_1, x_2, x_3 \ge 0$$

Now, if y_1', y_1^*, y_2', y_2^* are the associated variables then the dual of the given problem is

minimise $w = 2y_1' - 2y_1^* + y_2' - y_2^*$

sub. to
$$-2y_1' + 2y_1^* + 2y_2' - 2y_2^* \ge 1$$
$$y_1' - y_1^* + 3y_2' - 3y_2^* \ge -2$$
$$3y_1' - 3y_1^* + 4y_2' - 4y_2^* \ge 3$$

Putting $y_1' - y_1^* = y_1$ & $y_2' - y_2^* = y_2$ the problem becomes.

Minimise $w = 2y_1 + y_2$

Sub. to
$$-2y_1 + 2y_2 \ge 1$$
$$y_1 + 3y_2 \ge -2$$
$$3y_1 + 4y_2 \ge 3$$