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1) $[2x\log x - xy]\,dy + 2y\,dx = 0$

$2y\,dx + [2x\log x - xy]\,dy = 0$

$M = 2y \qquad N = 2x\log x - xy$

$\therefore \frac{\partial M}{\partial y} = 2 \qquad \frac{\partial N}{\partial x} = 2(\log x) + 2 - y$

$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

$\therefore$ D.E. is not exact

Now, $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial y} = \cancel{2}\, 2 - [2\log x + 2 - y]$

$= 2 - 2\log x - 2 + y$

$= -2\log x + y$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial y}}{N} = \frac{y - 2\log x}{2x\log x - xy} = \frac{y - 2\log x}{x(2\log x - y)}$$

$= \frac{-1}{x} = f(x)$

$\text{I.F.} = e^{\int -\frac{1}{x}dx}$

$= e^{-\log x} = e^{\log\frac{1}{x}}$

$\frac{2y}{x}\,dx + \left[\left(\frac{2x}{x}\log x - \frac{xy}{x}\right)\right]dy = 0$

$\downarrow$ M $\qquad \downarrow$ N

$\int M\,dx + \int N\,dy = C$

$\int \frac{2y}{x}\,dx + \int -y\,dy = C$

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2) $[y(1+\frac{1}{x}) + \cos y]\,dx + (x + \log x - x \cdot \sin y)\,dy = 0$

$M = y(1+\frac{1}{x}) + \cos y$

$N = x + \log x - x \sin y$

$\frac{\partial M}{\partial y} = 1 + \frac{1}{x} - \sin y$

$\frac{\partial N}{\partial x} = 1 + \frac{1}{x} - \sin y$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Eqⁿ is exact.

$\int M\,dx + \int N\,dy = c$

$\int (y + \frac{y}{x} + \cos y)\,dx + \int \cancel{(x + \log x - x\sin y)}\ 0\,dy$

$+ x\cos y = c$

$y \cancel{(} \cancel{x} \log x) \cancel{+ c} = \cancel{0}\ c$

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3) $xe^{x}(dx-dy)+e^{x}dx+ye^{y}dy=0$

$xe^{x}dx-xe^{x}dy+e^{x}dx+ye^{y}dy=0$

$e^{x}(x+1)dx+(ye^{y}-xe^{x})dy=0$

$M=e^{x}(x+1) \qquad N=(ye^{y}-xe^{x})$

$\frac{\partial M}{\partial y}=0 \qquad \frac{\partial N}{\partial x}=0-(xe^{x}+e^{x})$

$=-e^{x}(x+1)$

$\frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x}$

$\dfrac{\frac{\partial N}{\partial y}-\frac{\partial M}{\partial y}}{M}=\frac{1}{e^{x}(x+1)}[-e^{x}(x+1)-0]$

$=-1=f(y)$

I.F. $=e^{\int f(y)dy}$

$=e^{\int -1\,dy}=e^{-y}$

$e^{x}(x+1)dx+(ye^{y}+xe^{x})dy=0$

$e^{x}.e^{-y}(x+1)dy+e^{-y}(ye^{y}+xe^{x})dy=0$

G.S. $\int M dx+\int N(\neq x)dy\}=c$

$\int e^{x}.e^{-y}(x+1)dx+\int e^{-y}y\,e^{y}dy=c$

$e^{-y}\int e^{x}(x+1)dx+\int y\,dy\}=c$

$$\int e^x(x+1)\,dx = (x+1)e^x - \int 1 \cdot e^x\,dx$$
$$= (x+1)e^x - e^x = xe^x$$
$$e^{-y} \cdot xe^x + \frac{y^2}{2} = c$$

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4) $\left[y\sin(xy)+xy^2\cos(xy)\right]dx+\left[x\sin(xy)+x^2y\cos(xy)\right]dy=0$

$M=\left[y\sin(xy)+xy^2\cos(xy)\right]$
$N=\left[x\sin(xy)+x^2y\cos(xy)\right]$

$\frac{\partial M}{\partial y}=xy\cdot\cos xy+\sin xy+xy^2(-\sin xy)+\cos(xy)\,2xy$
$=3xy\cos(xy)+\sin xy - x^2y^2(\sin xy)$

$\frac{\partial N}{\partial x}=xy\cos(xy)+\sin xy - x^2y\sin(xy)+\cos(xy)\,2xy$

$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ $\therefore$ Eqn is exact.

$\int M\,dx+\int N\,dy=c$

$\int\left[y\sin(xy)+xy^2\cos(xy)\right]dx+\int 0\,dy=c$

$y\int\left[\sin(xy)+xy\cos(xy)\right]dx=c$

Let $v=xy$

Differentiating v w.r.t. x, we get $\frac{dv}{dx}=y$

$dx=dv/y$

$y\int(\sin v+v\cos v)\frac{dv}{y}=c$

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$\int \sin v\, dv + \int v \cos v\, dv = c$

$-\cos v + v\cos v - \int \sin v\, dv = c$

$-\cos v + \sin v + \cos v = c$

$\sin v = c$

$\sin(xy) = c$

5) $y(x+y)\,dx - x(y-x)\,dy = 0$

$M = y(x+y)$

$N = -x(y-x)$

$\frac{\partial M}{\partial y} = x+2y \qquad \frac{\partial N}{\partial x} = -y+2x$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ ∴ Eqⁿ is not exact

$\cancel{\frac{1}{2xy^2}}$ $Mx - Ny = x^2y + xy^2 + xy^2 - x^2y$

$= 2xy^2$

$\neq 0$

~~Mul~~ I.F. $= \frac{1}{2xy^2}$

Multiply it by the eqⁿ,

$\frac{1}{2xy^2}\left[(xy + y^2)\,dx - (xy - x^2)\,dy\right] = 0$

$\underbrace{\left(\frac{1}{2y} + \frac{1}{2x}\right)}_{M} dx - \underbrace{\left(\frac{1}{2y} - \frac{x}{2y^2}\right)}_{N} dy = 0$

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$$\int M\,dx + \int N\,dy = 0$$

$$\int\left(\frac{1}{2y}+\frac{1}{2x}\right)dx + \int\frac{1}{2y}\,dy = c$$

$$\frac{1}{2y} + \frac{\log x}{2} + \frac{1}{2}\log y = c$$

$$\frac{1}{2}\left(\frac{1}{y} + \log x + \log y\right) = c$$

Q Find the I.F. of

1) $(xy^2 - e^{1/x^3})dx - x^2 y\,dy = 0$

$M = xy^2 - e^{1/x^3}$ $\quad N = -x^2 y$

$\frac{\partial M}{\partial y} = 2xy \cancel{- e^{1/x^3}}$ $\quad \frac{\partial N}{\partial x} = -2xy$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ $\therefore$ eqn is not exact.

$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{2xy + 2xy}{-x^2 y} = \frac{4xy}{-x^2 y}$

$= \frac{4}{-x^2}$

$\therefore$ I.F. $= e^{\int f(x)\,dx} = e^{\int \frac{4}{-x} dx}$

$= e^{-4\log x} = e^{\log(1/x^4)} = \frac{1}{x^4}$

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2) $[3x^2y^4 + 2xy]dx + [2x^3y^3 - x^2]dy = 0$

$M = 3x^2y^4 + 2xy \qquad N = 2x^3y^3 - x^2$

$\frac{\partial M}{\partial y} = 12x^2y^3 + 2x \qquad \frac{\partial N}{\partial x} = 6x^2y^3 - 2x$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ $\therefore$ eq^n is not exact.

eq^n is of form $y f_1(x, y)dx + x f_2(x, y)dy = 0$

$Mx - Ny \neq 0$

$\therefore 3x^3y^4 + 2x^2y - 2x^3y^4 + x^2y \neq 0$

$x^3y^4 + 3x^2y \neq 0$

$\therefore I.F. = \frac{1}{x^3y^4 + 3x^2y}$

3) $\{y - xy^2\}dx - \{x + x^2y\}dy = 0$

$y\{1 - xy\}dx - x\{1 + xy\}dy = 0$

eq^n is of the form $y f_1(x, y)dx + x f_2(x, y)dy = 0$

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$$Mx - Ny \neq 0$$

$$xy - x^2y^2 + xy + x^2y^2 \neq 0$$

$$xy \neq 0$$

$$\therefore I.F. = \frac{1}{xy}$$

4) $y[x+y]dx - x[y-x]dy = 0$

~~eq^n is of form $yf_1(xy)dy + xf_2(xy)dx = 0$~~

~~$Mx + Ny \neq 0$~~

~~$x^2y + xy^2 \neq xy^2 + x^2y^2 \neq 0$~~

~~$x^2y + x^2y^2$~~

It is homogeneous eq^n

and $Mx + Ny \neq 0$

$$x^2y + xy^2 - xy^2 + x^2y \neq 0$$

$$2x^2y \neq 0$$

$$I.F. = \frac{1}{2x^2y}$$

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5) $$\frac{dy}{dx} = \frac{x^2y^3 + 2y}{2x - 2x^3y^2}$$

$(2x - 2x^3y^2)\,dy = (x^2y^3 + 2y)\,dx$

$(x^2y^3 + 2y)\,d\cancel{x} - (2x - 2x^3y^2) = 0$

This eqⁿ is of form $y f_1(x,y)\,dy + x f_2(x\,y\,y)\,\cancel{f}\,\cancel{d}x = 0$

$Mx - Ny \neq 0$

$x^3y^3 + 2xy + 2xy - 2x^3y^3 \neq 0$

$4xy - x^3y^3 \neq 0$

$$\therefore I.F. = \frac{1}{4xy - x^3y^3}$$