Q. Solve $y^4\,dx = (x^{-3/4} - y^3 x)\,dy$

Sol^n:- Give D.iff equation is,

$$y^4 \frac{dx}{dy} = x^{-3/4} - y^3 x$$

$$\frac{dx}{dy} = \frac{1}{y^4} \cdot x^{-3/4} - \frac{1}{y} x$$

$$\frac{dx}{dy} + \left(\frac{1}{y}\right) x = \left(\frac{1}{y^4}\right) x^{-3/4} \quad \text{.... Bernoulli's D.E.}$$

where $P = \frac{1}{y}$, $Q = \frac{1}{y^4}$

Multiplying by $x^{3/4}$

$$x^{3/4} \cdot \frac{dx}{dy} + \left(\frac{1}{y}\right) x^{\frac{7}{4}} = \frac{1}{y^4}$$

Put $x^{7/4} = t \Rightarrow \frac{7}{4} x^{3/4} \cdot \frac{dx}{dy} = \frac{dt}{dy}$

$$\Rightarrow x^{3/4} \frac{dx}{dy} = \frac{4}{7} \frac{dt}{dy}$$

$$\therefore \text{D.E} \Rightarrow \frac{4}{7} \frac{dt}{dy} + \left(\frac{1}{y}\right) t = \frac{1}{y^4}$$

$$\frac{dt}{dy} + \left(\frac{7}{4} \frac{1}{y}\right) t = \frac{7}{4} \frac{1}{y^4} \quad \text{.... LDE in } t, y$$

where $P = \frac{7}{4} \frac{1}{y}$, $Q = \frac{7}{4} \frac{1}{y^4}$

$$\text{I.F.} = e^{\int P dy} = e^{\int \frac{7}{4} \frac{1}{y} dy} = e^{\frac{7}{4} \log y} = y^{7/4}$$

$\therefore$ General solution for given D.E. is,

$$t \cdot y^{7/4} = \int y^{7/4} \cdot \frac{7}{4} \frac{1}{y^4} dy + C$$

$$t \cdot y^{7/4} = \frac{7}{4} \int y^{-9/4} dy + C$$

$$t \cdot y^{7/4} = \frac{7}{4} \frac{y^{-5/4}}{-5/4} + C$$

$$\boxed{x^{7/4} y^{7/4} = -\frac{7}{5} \frac{1}{y^{5/4}} + C}$$