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Tutorial-2

Examples of Bernoulli's Equation

Q. Solve the following

1. Solve $y^4 dx = (x^{-3/4} - y^3 x) dy$

Sol^n $\frac{dx}{dy} + \frac{x}{y} = \frac{x^{3/4}}{y^4}$

~~$\frac{dx}{dy} = \frac{1}{y^4}(x^{-3/4}) - \frac{x}{y}$~~

~~$\frac{dx}{dy} + \frac{x}{y} =$~~ $P = \frac{1}{y}$ $Q = \frac{1}{y^4}$

Multiplying by $x^{+3/4}$

$x^{3/4}\frac{dx}{dy} + \left(\frac{1}{y}\right)x^{7/4} = \frac{1}{y^4}$

Put $x^{7/4} = t \Rightarrow \frac{7}{4}x^{3/4}\frac{dx}{dy} = \frac{dt}{dy}$

$x^{3/4}\frac{dx}{dy} = \frac{4}{7}\frac{dt}{dy}$

$\therefore$ D.E. $\Rightarrow \frac{4}{7}\frac{dt}{dy} + \left(\frac{1}{y}\right)t = \frac{1}{y^4}$

$\frac{dt}{dy} + \left(\frac{7}{4}\frac{1}{y}\right)t = \frac{7}{4}\frac{1}{y^4}$

L.D.E. in t

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where $P = \frac{7}{4}\frac{1}{y}$, $Q = \frac{7}{4}\frac{1}{y^4}$

$I.F. = e^{\int P\,dy} = e^{\int \frac{7}{4}\frac{1}{y}dy} = e^{7/4 \log y} = y^{7/4}$

$\therefore$ General solution for given D.E. is,

$t\,y^{7/4} = \int y^{7/4}\,\frac{7}{4}\,\frac{1}{y^4}\,dy + c$

$ty^{7/4} = \frac{7}{4}\int y^{-9/4}\,dy + c$

$ty^{7/4} = \frac{7}{4}\,\frac{y^{-5/4}}{-5/4} + c$

$$\boxed{x^{7/4}\,y^{7/4} = -\frac{7}{5}\,\frac{1}{y^{5/4}} + c}$$

2) Solve $\frac{dy}{dx} + y = y^3(\cos x - \sin x)$

Sol^n: Given D.E. is, $\frac{dy}{dx} + (1) = \frac{(\cos x \cancel{-} \sin x)}{y^3}$

--- Bernoulli's D.E.

where $P = 1$, $Q = (\cos x - \sin x)$

Divide by y^3,

$\frac{1}{y^3}\frac{dy}{dx} + (1)\frac{1}{y^2} = (\cos x - \sin x)$

Put $\frac{1}{y^2} = t \Rightarrow -2y^{-3}\frac{dy}{dx} = \frac{dt}{dx}$

$\frac{1}{y^3}\frac{dy}{dx} = \left(\frac{-1}{2}\right)\frac{dt}{dx}$

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$\therefore \left(-\frac{1}{2}\right)\frac{dt}{dx} + (1)\,t = (\cos x - \sin x)$

$\therefore \frac{dt}{dx} + (-2)\,t = -2(\cos x - \sin x)$

$\therefore$ L.D.E in t, x

$P = -2 \qquad Q = -2(\cos x - \sin x)$

I.F. $= e^{\int(-2)dx} = e^{-2x}$

G.S. is $t\,x^{-2x} = \int e^{-2x}\,2(\sin x - \cos x)\,dx + c$

$\therefore t\,e^{-2x} = 2\left[\int e^{-2x}\sin x\,dx - \int e^{-2x}\cos x\,dx\right] + C$

$t \cdot e^{-2x} = 2 \cdot \frac{e^{-2x}}{4+1}\left\{[-2\sin x - \cos x] - [-2\cos x + \sin x]\right\} + c$

$t = \frac{2}{5}\left\{-2\sin x - \cos x + 2\cos x - \sin x\right\} + c \cdot e^{2x}$

$t = \frac{2}{5}\left\{\cos x - 3\sin x\right\} + c \cdot e^{2x}$

$$\boxed{\frac{1}{y^2} = \frac{2}{5}(\cos x - 3\sin x) + c\,e^{2x}}$$

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3) Solve $x\frac{dy}{dx} + y\log y = xye^x$

Solⁿ Given D.E. is

$$\frac{dy}{dx} + \left(\frac{1}{x}\right) y \log y = ye^x$$

$$\frac{dy}{dx} + (-e^x) y = \left(\frac{-1}{x}\right) y \log y$$

Dividing by 'y' also

$$\frac{1}{y}\frac{dy}{dx} + (-e^x) = \left(\frac{-1}{x}\right) \log y$$

$$\frac{1}{y}\frac{dy}{dx} + \left(\frac{1}{x}\right) \log y = e^x$$

Put $\log y = t \Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{dt}{dx}$

$\therefore \frac{dt}{dx} + \left(\frac{1}{x}\right) t = e^x$ ... LDE in t, x.

$P = \frac{1}{x}$, $Q = e^x$

$I.F. = e^{\int P dx} = e^{\int \frac{1}{x} dx} = e^{\log x} = x$

General solⁿ is, $t(I.F.) = \int Q(I.F.) dx + c$

$$t x = \int (e^x \cdot x) dx + c$$

$$t x = x e^x - e^x + c$$

$$\boxed{x(\log y) = e^x (x-1) + c}$$

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4) Solve $\frac{dy}{dx} = \frac{y^3}{e^{2x}+y^2}$

Sol^n: Given D.E. is, $\frac{dx}{dy} = \frac{e^{2x}}{y^3} + \frac{1}{y}$

Multiplying both sides by e^{-2x}

$e^{-2x}\frac{dx}{dy} = \frac{1}{y^3} + \frac{1}{y}e^{-2x}$

$e^{-2x}\frac{dx}{dy} + \left(\frac{1}{y}\right)e^{-2x} = \frac{1}{y^3}$

~~Put $e^{-2x} = t$~~ $e^{-2x}\frac{dx}{dy} + \left(\frac{-1}{y}\right)e^{-2x} = \frac{1}{y^3}$

Put $e^{-2x} = t \Rightarrow -2e^{-2x}\cdot\frac{dx}{dy} = \frac{dt}{dy}$

$\therefore \left(\frac{-1}{2}\right)\frac{dt}{dy} + \left(\frac{-1}{y}\right)t = \frac{1}{y^3}$

$\frac{dt}{dy} + \left(\frac{2}{y}\right)t = (-2)\frac{1}{y^3}$ ... LDE in t, y.

$P = \frac{2}{y}$, $Q = \frac{-2}{y^3}$

I.F. $= e^{\int P dy} = e^{\int 2/y\, dy} = e^{2\log y} = y^2$

$\therefore$ G.S. is, $t(\text{I.F.}) = \int Q(\text{I.F.})\,dy + c$

$t y^2 = \int \frac{-2}{y^3} y^2\, dy + c$

$ty^2 = -2\int \frac{1}{y}\, dy + c$

$ty^2 = -2\log y + c$

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$$e^{-2x} y^2 = -2\log y + C$$

$$\boxed{e^{-2x} y^2 + \log y^2 = c}$$

5) $\frac{dy}{dx} + x(x+y) = x^3(x+y)^3 - 1$

Solⁿ

$$x + y = t$$

$$\frac{dt}{dx} = 1 + \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{dt}{dx} - 1$$

$$\frac{dt}{dx} - 1 + xt = x^3t^3 - 1$$

$$\frac{dt}{dx} + xt = x^3t^3$$

~~$P = x$~~ ~~$Q = x^3$~~ Dividing by t^3

~~I.F. = e =~~ $\frac{1}{t^3}\frac{dt}{dx} + \frac{x}{t^2} = x^3$

~~Putting~~ Put $\frac{1}{t^2} = V$

$$\frac{-2}{t^3}\frac{dt}{dx} = \frac{dv}{dx}$$

$$\frac{1}{t^3}\frac{dt}{dx} = -\frac{1}{2}\frac{dv}{dx}$$

$$-1/2\, dv/dx + Vx = x^3$$

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$\frac{dv}{dx} + 2vx = -2x^3$

$\frac{dv}{dx} - 2vx = -2x^3$

$\frac{dv}{dx} + (-2x)v = -2x^3$

$P = -2x \quad Q = -2x^3$

$e^{\int P dx} = e^{\int -2x dx} = e^{-x^2}$

$\therefore$ The solⁿ is $ve^{-x^2} = \int -2x^3 (e^{-x^2}) dx + c$

Put $x^2 = t$

$2x dx = dt$

$x dx = dt/2$

$\therefore ve^{-x^2} = \int -t(e^{-t}) dt + c$

$= te^{-t} + e^{-t} + c$

$x^2 e^{-x^2} + e^{-x^2} + c$ [Put $t = x^2$]

Put $v = \frac{1}{t^2}$, G.S. is $\frac{1}{t^2} e^{-x^2}$

$= x^2 e^{-x^2} + e^{-x^2} + c$

$\frac{1}{t^2} = x^2 + 1 + ce^{x^2}$

$\frac{1}{(x+y)^2} = x^2 + 1 + ce^{x^2}$

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Q. Find Solutions

1) $(1+\sin y)\,dx = (2y\cos y - x\sec y - x\tan y)\,dy$

Sol^n Rewrite the equation in standard form

$\frac{dx}{dy} + Px = Q$

$\frac{dx}{dy} + \frac{(\sec y + \tan y)}{(1+\sin y)} x = \frac{2y\cos y}{1+\sin y}$

$\frac{dx}{dy} + \frac{1}{\cos y} x = \frac{2y\cos y}{1+\sin y}$ --- LDE

$P = \sec y \qquad Q = \frac{2y\cos y}{1+\sin y}$

$I.F. = e^{\int P\,dy} = e^{\int \sec y\,dy} = e^{\log(\sec y + \tan y)}$

$= \sec y + \tan y$

$I.F. = \frac{1+\sin y}{\cos y}$

General solution is $x e^{\int P\,dy} = \int Q e^{\int P\,dy}\,dy + C$

$x \frac{1+\sin y}{\cos y} = \int \frac{2y\cos y}{1+\sin y} \frac{1+\sin y}{\cos y}\,dy + C$

$x \frac{1+\sin y}{\cos y} = \int \frac{2y}{\cancel{\cos}}\,dy + C = y^2 + C$

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2) $x \log x \frac{dy}{dx} + y = 2 \log x$

Sol^n $\frac{dy}{dx} + \frac{1}{x \log x} y = \frac{2}{x}$

It is ~~th~~ LDE of the form $\frac{dy}{dx} + Py = Q$

So, here $P = \frac{1}{x \log x}$ & $Q = \frac{2}{x}$

$I.F. = e^{\int P dx} = e^{\int \frac{1}{x \log x} dx} = e^{\log|\log x|}$

$= \log x$

Hence, Sol^n of given DE is

$y \times I.F. = \int Q \times I.F. \, dx$

$y \log x = \int \frac{2}{x} \log x \, dx$

$y \log x = 2 \int \frac{1}{x} \log x \, dx = 2 \frac{(\log x)^2}{2} + c$

$y \log x = (\log x)^2 + c$

3) Solve $x^2 \frac{dy}{dx} = 3x^2 - 2xy + 1$

Sol^n Given D.E. can be written as

$\frac{dy}{dx} = 3 + \frac{1 - 2xy}{x^2}$

$\frac{dy}{dx} = 3 + \frac{1}{x^2} - \frac{2y}{x}$

$dy/dx + 2y/x = 3 + 1/x^2$ .... LDE $\frac{dy}{dx} + Py = Q$

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$P = 2/x, \quad Q = 3 + 1/x^2$

$I.F. = e^{\int P dx} = e^{\int \frac{2}{x} dx} = e^{\log x^2} = x^2$

$G.S. \Rightarrow y e^{\int P dx} = \int Q e^{\int P dx} dx + C$

$y x^2 = \int \left(3 + \frac{1}{x^2}\right) x^2 \, dx + C$

$x^2 y = x^3 + x + c$ --- solⁿ.

4) Solve $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right) \frac{dx}{dy} = 1$

<u>Solⁿ</u> $\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}} = \frac{dy}{dx}$

$\frac{dy}{dx} + \frac{y}{\sqrt{x}} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$

Differential Eqⁿ is of the form $dy/dx + Py = Q$

where $P = \frac{1}{\sqrt{x}}$ & $Q = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$

$I.F. = e^{\int P dx} = e^{\int dx/\sqrt{x}} = e^{\int x^{-\frac{1}{2}} dx} = e^{\int \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} dx}$

$= e^{2\sqrt{x}}$

$G.S.$ is $y(IF) = \int (Q \times IF) dx + C$

$y e^{2\sqrt{x}} = \int \left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} \times e^{2\sqrt{x}}\right) dx + C$

$y e^{2\sqrt{x}} = \int \frac{dx}{\sqrt{x}} + C = \int \frac{1}{\sqrt{x}} dx + C = \int x^{-1/2} dx + C$

$= \int \frac{x^{-1/2+1}}{-\frac{1}{2}+1} + C = 2x^{1/2} + C$

$\boxed{y e^{2\sqrt{x}} = 2\sqrt{x} + C}$