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Assignment No.1

1. (a) (i) Fourth order, first degree.

(b) (iii) $2xy\,dx + (2+x^2)\,dy = 0$

(c) (a) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(d) (i) Non-exact

2. (i) $\left[\frac{dx}{dy}\right] + Px = Qx^n$ is (c) Linear D.E.

(ii) For $Mdx + Ndy = 0$ is factor $\frac{\partial M}{\partial x} - \frac{\partial N}{\partial y} = 0$
(a) Exact D.E.

(iii) $\left[\frac{dx}{dy}\right] + Px = Q$ is (d) Bernoulli's D.E

(iv) For D.E. $Mx + Ny \neq 0$ then $\frac{1}{Mx+Ny}$
(b) Integrating

3. i) $y(I.F.) = \int Q(I.F.)\,dx + C$

(ii) (iv) none of the above

4. i) The degree of a differential equation is the degree of the highest ordered derivative which occurs in

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it provided the equation has been made free of the radical signs and fractional powers as far as the derivatives are concerned.

(ii) The order of a differential equation is the order of the highest ordered derivative occuring in the differential equation.

(iii) Let $M(x, y)dx + N(x, y)dy = 0$ be a first degree and first order differential equation where M and N are real valued functions for some x, y. Then the equation $Mdx + Ndy = 0$ is said to be an exact differential equation if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$;

(iv) A non-linear differential equation of the first order that has the general form $\frac{dy}{dx} + f(x)y = g(x)y^n$ and that can be put in linear form by dividing through by y^n and

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making the change of variable
$Y = y - x + 1$.

5. 1. True. 2. True 3. True.

6. a) $\left[y\left(1+\frac{1}{x}\right)+\cos y\right]dx + (x + \log x - x$
$\sin y)\, dy = 0$

Solution $M = y\left(1+\frac{1}{x}\right) + \cos y$

$N = x + \log x - x \sin y$

$\frac{\partial M}{\partial y} = 1 + \frac{1}{x} - \sin y$

$\frac{\partial N}{\partial x} = 1 + \frac{1}{x} - \sin y$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ $\therefore$ eq^n is exact

$\int M dx + \int N dx = c$

$\int \left(y + \frac{y}{x} + \cos y\right) dx + \int 0\, dy = c$

$y(x \log x) + x \cos y = c$

b) Solve $[2y^2 - 4x + 5]\, dx = [y - 2y^2$
$- 4xy]\, dy$.

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Solution $[2y^2 - 4x + 5]dx - [y - 2y^2 - 4xy]dy = 0$

$\frac{\partial M}{\partial y} = 4y \quad \frac{\partial N}{\partial x} = 4y$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \therefore$ D.E. is exact.

$\int M dx + \int N dy = c$

$\int (2y^2 - 4x + 5) dx + \int (y - 2y^2) dy = c$

$2y^2 - 2x^2 + 5x + \frac{y^2}{2} - \frac{2}{3}y^3 = c$

$\frac{5y^2}{2} - 2x^2 + 5x - \frac{2y^3}{3} = c.$

c) Solve $[x^4 + y^4] dx - xy^3 dy = 0$

Soln $M = [x^4 + y^4] \quad N = -xy^3$

$\frac{\partial M}{\partial y} = 4y^3 \quad \frac{\partial N}{\partial x} = -y^3$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \quad \therefore$ D.E. is not exact

$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{4y^3 + y^3}{-xy^3} = \frac{5y^3}{-xy^3} = \frac{5}{-x}$

$= f(x)$

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$\therefore I.F. = e^{\int f(x)\,dx} = e^{\int \frac{5}{-x}\,dx} = e^{-5\log x}$

$= -5x$

Multiplying I.F. to eq^n,

$-5x[x^4+y^4]dx + 5x^2y^3\,dy = 0$

$\int M\,dx + \int N\,dy = c$

$\int(-5x^5 - 5xy^4)\,dx + \int 0\,dy = c$

$-25x^4 - 5y^4 = c$

$-5(5x^4 - y^4) = c$

7) a) Solve $xy^4\,dx = (x^{-3/4} - y^3 x)\,dy$

Sol^n: $\cancel{\frac{dx}{dy} + \frac{x}{y} = \frac{x^{3/4}}{y^4}}$

$\frac{\partial M}{\partial y} = 4y^3 \qquad \frac{\partial N}{\partial x} = \frac{7}{4}x^{-7/4} - y^3$

$\cancel{5y}\ \cancel{\frac{\partial M}{\partial y}}$ $Mx - Ny \neq 0$

$xy^4 + x^{-3/4}y - y^4x \neq 0$

$\frac{y}{x^{3/4}} \neq 0$

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$\therefore$ I.F. $= \frac{x^{3/4}}{y}$

Multiplying I.F. to eqn.

$x^{3/4} y^3 dx - (y - y^4 x^{1/4}) dy = c$

$\int M dx - \int N dy = c$

$\int (x^{3/4} y^3) dx - \int y \, dy = c$

$\frac{4}{7} x^{7/4} y^3 - \frac{y^2}{2} = c$

$y^2 \left(\frac{4}{7} x^{7/4} y - \frac{1}{2} \right) = c$

b) Solve $\frac{dy}{dx} = -\frac{x^2 y^3 + 2y}{2x - 2x^3 y^2}$

$(x^2 y^3 + 2y) dx + (2x - 2x^3 y^2) dy = 0$

This eqn is of form $y f_1(x, y) dy + x f_2(x, y) dx = 0$

$Mx - Ny \neq 0$

$x^3 y^3 + 2xy - 2xy + 2x^3 y^3 \neq 0$

$3x^3 y^3 \neq 0$

I.F. $= \frac{1}{3x^3 y^3}$

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~~Soln~~

Multiply I. F. ~~by~~ to eqn

$$\left(\frac{x^2y^3}{3x^3y^3}+\frac{2y}{3x^3y^3}\right)dx+\left(\frac{2x}{3x^3y^3}-\frac{2x^3y^2}{3x^3y^3}\right)dy=0$$

$$\left(\frac{1}{3x}+\frac{2}{3x^3y^2}\right)dx+\left(\frac{2}{3x^2y^3}-\frac{2}{3y}\right)dy=0$$

$$\int M\,dx+\int N\,dy=c$$

$$\cancel{\frac{\log x}{3}+}$$

$$\int\left(\frac{1}{3x}+\frac{2}{3x^3y^2}\right)dx+\int\left(-\frac{2}{3y}\right)dy=c$$

$$\frac{\log x}{3}+\frac{x^4\log x^3}{6y^2}-\frac{2}{3}\log y=c$$

$$\frac{1}{3}\left(\log x+\frac{x^4\log x^3}{2y^2}-2\log y\right)=c$$

(c) Solve $y(x+y)dx-x(y-x)dy=0$ (d)

Soln $M=y(x+y)$ $\quad N=-x(y-x)$

$$\frac{\partial M}{\partial y}=x+2y \qquad \frac{\partial N}{\partial x}=-y+2x$$

$$\frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x}$$

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$\therefore$ eq^n is not exact

$$Mx - Ny = x^2y + xy^2 + xy^2 - x^2y$$
$$= 2xy^2$$

y=0

$$I.F. = \frac{1}{2xy^2}$$

Multiply I.F. to eq^n,

$$\frac{1}{2xy^2}\left[(yx + y^2)\,dx - (xy - x^2)\,dy\right] = 0$$

$$\int M\,dx + \int N\,dy = c$$

$$\int\left(\frac{1}{2y} + \frac{1}{2x}\right)dx + \int\left(\frac{1}{2y}\right)dy = c$$

$$\frac{1}{2y} + \frac{\log x}{2} + \frac{1}{2}\log y = c$$

$$\frac{1}{2}\left(\frac{1}{y} + \log x + \log y\right) = c$$

(d) Solve $\left[\log(x^2+y^2) + \frac{2x^2}{x^2+y^2}\right]dx +$

$$\left(\frac{2xy}{x^2+y^2}\right)dy = 0$$

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Soln

$$M = \log(x^2+y^2) + \frac{2x^2}{x^2+y^2} \qquad N = \frac{2xy}{x^2+y^2}$$

$$\frac{\partial M}{\partial y} = \frac{1}{x^2+y^2}(2y) + 2x^2\left(-\frac{1}{(x^2+y^2)^2}\right)(2y)$$

$$\frac{\partial M}{\partial y} = \frac{2y}{x^2+y^2}\left(1-\frac{2x^2}{x^2+y^2}\right)$$

$$\frac{\partial N}{\partial x} = 2y\left[x\left(-\frac{1}{(x^2+y^2)^2}\,2x + \frac{1}{x^2+y^2}\right)\right]$$

$$\frac{\partial N}{\partial x} = \frac{2y}{x^2+y^2}\left(1-\frac{2x^2}{x^2+y^2}\right)$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \therefore \text{DE is exact.}$$

$$\int M\,dx + \int N\,dy = c$$

$$\int\left[\log(x^2+y^2) + \frac{2x^2}{x^2+y^2}\right]dx + \int 0\,dy = c$$

$$\int\left[\log(x^2+y^2) + \frac{x}{x^2+y^2}\,2x\right]dx = c'$$

$$\int\left[\log(x^2+y^2) + \frac{d}{dx}\log(x^2+y^2)\right]dx = c'$$

$$\int\frac{d}{dx}\left[x\log(x^2+y^2)\right]dx = c'$$

$$\therefore x\log(x^2+y^2) = c'$$

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e) Solve $xy(1+xy^2)\frac{dy}{dx}=1$

Sol^n

$\frac{dy}{dx}-xy=x^3y^3$

$\frac{1}{x^2}\frac{dx}{dy}-\frac{y}{x}=y^3$

Put $-\frac{1}{x}=v$

$\frac{1}{x^2}dx/dy=dv/dy$

$dv/dy+vy=y^3$

Compare with $dv/dy+P'v=Q'$

here $P'=y$ & $Q'=y^3$

$I.F.=e^{\int P'dy}=e^{\int y\,dy}=e^{y^2/2}$

Sol^n is $v(I.F.)=\int(I.F.)\,Q'dy+c$

$ve^{y^2/2}=\int e^{y^2/2}y^3dy+c$

Put $y^2/2=t$

$\frac{1}{2}2y\,dy=dt$

$y\,dy=dt$

$-1/x\,e^{y^2/2}=\int e^t\,2t\,dt+c$

$=2t(t-1)+c$

$=2e^{y^2/2}\left(\frac{y^2}{2}-1\right)+c$

$-\frac{1}{x}=y^2-2+ce^{-y^2/2}$

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~~Maths~~

8) One of the most basic examples of differential equations is the Malthusian law of population growth $\frac{dp}{dt} = rp$ shows that how population (p) changes w.r.t time. The constant r will change depending upon the species ~~Malthu~~ More complicated D.E. can be used to model the relationship between predators and prey. For eg, as predators increase then prey decreases as more get eaten. But then the predators will have less to eat and start to die out, which allows more prey to survive. The interactions between the 2 populations are connected by differential equations.

- In medicines for modelling cancer growth or spread of disease.
- In engineering for describing the movement of electricity.
- In chemistry for modelling chemical rxns.