CG

Assignment 2

Q1. Choose Correct Options / Fill in the blanks

a. The aliasing effect can be minimised by?

Ans: iii increasing the resolution of the raster display.

b. Pixel intensity is one of the antialiasing techniques.

c. Which of the following is the correct representation to define 2D point using homogeneous point Coordinate [Hint :- (Xw, Yw, w)]

Ans: i) (0, 0, 0)

d. If the scaling factors values of $S_x = 1$ and $S_y = 1$ then

Ans: i Size of an object remains same

e. The negative values of 'θ' gives 90° rotation.

Q2. Choose Correct Options / Fill in the blanks

a. A circle is drawn at (30, 30) with radius = 10. Its mirror image cannot be obtained by?

Ans: iii) Translation by $T_x = 60$ and $T_y = 0$

b. A conceptual line is drawn starting from the particular point and extending to a distance point outside the coordinate extends of the object in direction of X-axis, the line intersects twice with the polygon edges and once with the polygon vertex. Then according to inside outside test, the point lies?

Ans: i Outside the polygon.

c. Which of the following input is accepted only by Boundary Fill method and not by Flood fill method?

Ans: iv) Seed pixel

d. To convert a square into a parallelogram, which transformation is used?

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Ans: i Scaling

e. First reflect a point about x-axis, then perform a counter clock-wise rotation of 90°, this is equivalent to Reflection about a line x.

Q3. Answer the following questions in brief (20 to 30 words)

a. Explain homogeneous coordinates in computer graphics.

Ans: Homogeneous coordinates are ubiquitous in computer graphics because they allow common vector operations such as translation, rotation, scaling and perspective projection to be represented as a matrix by which the vector is multiplied. By the chain rule, any sequence of such operations can be multiplied out into a single matrix, allowing simple and efficient processing. By contrast, using Cartesian coordinates, translations & perspective projection cannot be expressed as matrix.

b. What is aliasing effect? Discuss any one antialiasing technique.

Ans: Aliasing effect is the appearance of jagged edges or "jaggies" in a rasterized image.

• Using high-resolution display:

One way to reduce aliasing effect and increase sampling rate is to simply display objects at a higher resolution. Using high-resolution, the jaggies become so small that they become indistinguishable by the human eye. Hence, jagged edges get blurred out & edges appear smooth.

c.

Flood fill algorithm	Boundary fill algorithm
• It can process the image containing more than one boundary colours.	It can only process the image containing single boundary colour.
• It is comparatively slower than the Boundary fill algorithm	It is faster than the flood-fill algorithm.
• In Flood-fill algorithm a random colour can be used to	In Boundary fill algorithm interior points are painted by continuously

paint the interior portion then the old one is replaced with a new one.	searching for the boundary colour.
• It requires huge amount of memory.	Memory consumption is relatively low in Boundary fill algorithm
• Flood fill algorithms are simple & efficient.	The complexity of Boundary fill algorithm is high.

Q4. Answer the following questions in brief (50 to 70 words)

a. Scale the square ABCD with coordinates A(0,0), B(5,0), C(5,5), D(0,5) by 3 units in x direction and 4 units in y direction.

Ans: We can represent the given square, in matrix form, using homogeneous coordinates of vertices as:

$$\begin{array}{c} A \\ B \\ C \\ D \end{array}\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \\ x_4 & y_4 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 5 & 0 & 1 \\ 5 & 5 & 1 \\ 0 & 5 & 1 \end{bmatrix}$$

Translation factors are, $t_x = 3$, $t_y = 4$

The transformation matrix for translation:

$$T_V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix}$$

New object point coordinates are

$[A' \; B' \; C' \; D'] = [A \; B \; C \; D] \cdot T_V$

$$\begin{array}{c} A' \\ B' \\ C' \\ D' \end{array}\begin{bmatrix} x_1' & y_1' & 1 \\ x_2' & y_2' & 1 \\ x_3' & y_3' & 1 \\ x_4' & y_4' & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 5 & 0 & 1 \\ 5 & 5 & 1 \\ 0 & 5 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 4 & 1 \\ 8 & 4 & 1 \\ 8 & 9 & 1 \\ 3 & 9 & 1 \end{bmatrix}$$

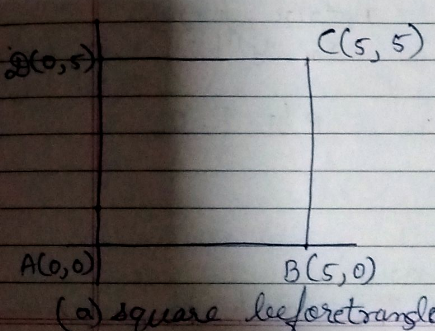
Thus, $A'(x_1', y_1') = (3, 4)$

$B'(x_2', y_2') = (8, 4)$

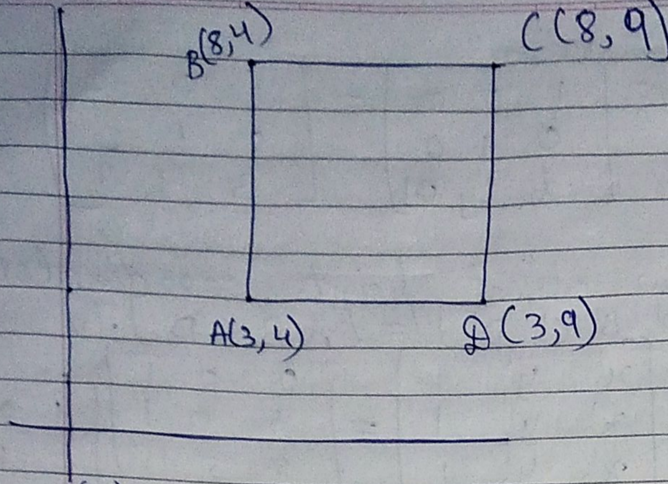
$C'(x_3', y_3') = (8, 9)$

$D'(x_4', y_4') = (3, 9)$

The graphical representation is given below:



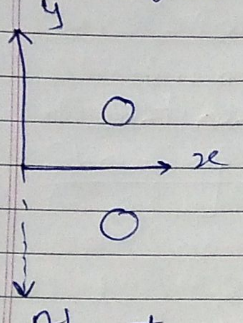
(a) square before translation



(b) Square after translation.

ii) Derive 2-D composite transformation matrix to reflect the point (x,y) about the fixed point (X_p, Y_p) (point other than the origin).

Ans: About x-axis:
If P(x,y) is the point on x-y plane then P'(x', y') is the reflection about x-axis given as $x' = x$; $y' = -y$



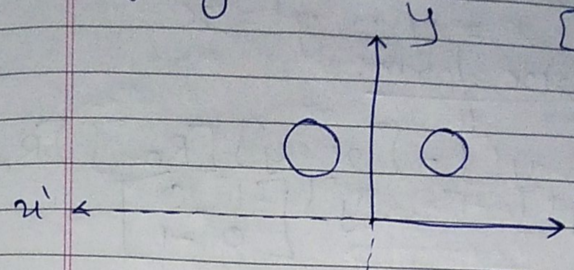
matrix form:

$$[x' \ \ y'] = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

$$P' = P \cdot R_x$$

About y-axis:
If p(x, y) is the point on x-y plane

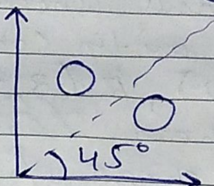
then P'(x', y') is the reflection about y-axis given as $x' = -x$; $y' = y$



$$[x' \ \ y'] = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$P' = P \cdot R_y$$

About x=y line:



Imposing the line clockwise (−45°) imposing it on the x-axis we have

$$R_{\theta-} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(-45) & \sin(-45) \\ -\sin(-45) & \cos(-45) \end{bmatrix}$$

We know,
$\cos(-\theta) = \cos\theta$ & $\sin(-\theta) = -\sin\theta$

$$R_{\theta-} = \begin{bmatrix} \cos(45) & -\sin(45) \\ \sin(45) & \cos(45) \end{bmatrix}$$

Now perform reflection along x axis.

$$R_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Now rotate the line back 45° in an anticlockwise direction

$$R_{\theta+} = \begin{bmatrix} \cos(45) & \sin(45) \\ -\sin(45) & \cos(45) \end{bmatrix}$$

Now if P(x, y) is the point on x-y plane then P'(x', y') is the reflection about x=y line given as $x' = y$; $y' = x$

Matrix form:

$[x' \ y'] = [x \ y][R_{\theta-}][R_x][R_{\theta+}]$

$[x' \ y'] = [x \ y]\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

c. Apply Xshear and Yshear transform to the square with coordinates P(0, 0), Q(3, 0), R(3, −3) and S(0, −3), xshear parameter value and yshear parameter value is 2.

Ans: Given: old corner coordinates of a square = P(0, 0), Q(3, 0), R(3, −3) and S(0, −3)

Shearing parameter towards X direction (Sh_x) = 2

Shearing parameter towards Y direction (Sh_y) = 2

Shearing in X-axis:

For coordinates P(0, 0)

Let the new coordinates of corner A after shearing = (X_{new}, Y_{new})

Applying the shearing equations, we have-

- $X_{new} = X_{old} + Sh_x \times Y_{old} = 0 + 2 \times 0 = 0$
- $Y_{new} = Y_{old} = 0$

Thus, new coordinates of corner P after shearing = (0, 0)

For coordinates Q(3, 0)

Let the new coordinates of corner Q after shearing = (X_{new}, Y_{new})

Applying the shearing equations, we have-

- $X_{new} = X_{old} + Sh_x \times Y_{old} = 3 + 2 \times 0 = 3$
- $Y_{new} = Y_{old} = 0$

Thus, new coordinates of corner Q after shearing = (3, 0)

For coordinates R(3, −3)

Let the new coordinates of corner R after shearing = (X_{new}, Y_{new})

Applying the shearing equations, we have-

- $X_{new} = X_{old} + Sh_x \times Y_{old} = 3 + 2 \times (-3) = -3$
- $Y_{new} = Y_{old} = -3$

Thus, new coordinates of corner R after

shearing ~~us~~ = (-3, -3)

For coordinates S(0, -3)

Let the new coordinates of corner S after shearing = (X_{new}, Y_{new})
Applying the shearing equations, we have -

- $X_{new} = X_{old} + Sh_x \times Y_{old} = 0 + 2 \times (-3) = -6$
- $Y_{new} = Y_{old} = -3$

Thus, new coordinates of corner S after shearing = (-6, -3)

Thus, new coordinates of square after shearing in X axis = P(0, 0), Q(3, 0), R(-3, -3) and S(-6, -3)

Shearing in Y-axis

For coordinates P(0, 0) ~~Q(3~~

Let the new coordinates of P after shearing = (X_{new}, Y_{new})
Applying the shearing equations, we have -

- $X_{new} = X_{old} = 0$
- $Y_{new} = Y_{old} + Sh_y \times X_{old} = 0 + 2 \times 0 = 0$

Thus, new coordinates ~~after~~ of P after shearing are = (0, 0)

For coordinates of Q (3, 0)

Let the new coordinates of Q after shearing = (X_{new}, Y_{new})
Applying the shearing equations, we have -

- $X_{new} = X_{old} = 3$
- $Y_{new} = Y_{old} + Sh_y \times X_{old} = 0 + 2 \times 3 = 6$

Thus, new coordinates of Q after shearing are = (3, 6)

For coordinates R(3, -3)

Let the new coordinates of R after shearing = (X_{new}, Y_{new})
Applying the shearing equations, we have -

- $X_{new} = X_{old} = 3$
- $Y_{new} = Y_{old} + Sh_y \times X_{old} = -3 + 2 \times 3 = 3$

Thus, new coordinates of Q = (3, 3)

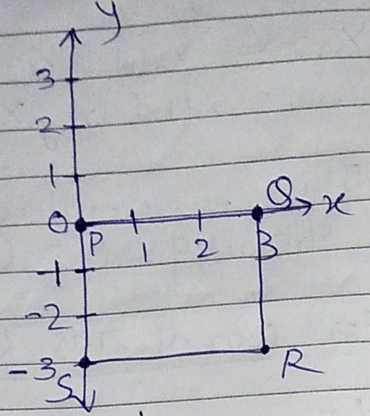
For coordinates S (0, -3)

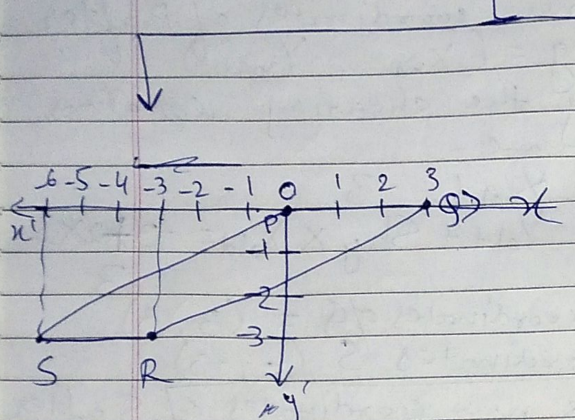
Let the new coordinates of S after shearing = (X_{new}, Y_{new})
Applying the shearing equations, we have -

- $X_{new} = X_{old} = 0$
- $Y_{new} = Y_{old} + Sh_y \times X_{old} = -3 + 2 \times 0 = -3$

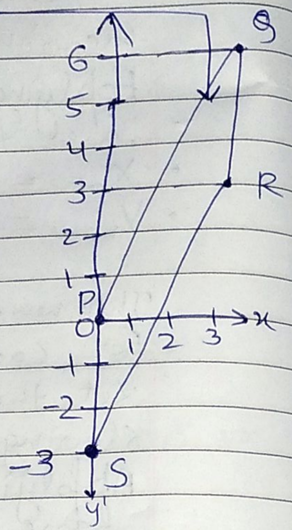
Thus, new coordinates of S after shearing are = (0, -3)

Thus, new coordinates of square after shearing in Y axis = P(0, 0), Q(3, 6), R(3, 3) and S(0, −3)





Shearing in X axis



Shearing in Y axis

Q5) Think & Answer

a. What is homogeneous transformation

matrix for 2D. Write homogeneous transformation matrix for translation, Rotation & Scaling in terms of P' = P * T (where P = Original object matrix, & P' = New object matrix & T = 2D transformation matrix)

Ans. For 2D geometric transformation we can take value of h as any the number so we can get ∞ homogeneous representation for coordinate value (x, y)

Translation

$P' = T(t_x, t_y) \cdot P$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotation

$P' = R(\theta) \cdot P$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scaling

$P' = S(s_x, s_y) \cdot P$ $\begin{bmatrix} x' \\ \end{bmatrix}$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

b) Prove that 2D rotations are additive.

Ans: The rotation matrix R is given as,

$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

we can write rotation matrix $R(\theta_1)$ as

$$R(\theta_1) = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{bmatrix} \text{ and } R(\theta_2) = \begin{bmatrix} \cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{bmatrix}$$

$$R(\theta_1)\cdot R(\theta_2) = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{bmatrix} \times \begin{bmatrix} \cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_1\cdot\cos\theta_2 + \sin\theta_1\cdot(-\sin\theta_2) & \cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_2 \\ -\sin\theta_1\cdot\cos\theta_2 + \cos\theta_1(-\sin\theta_2) & -\sin\theta_1\sin\theta_2 + \cos\theta_1\cos\theta_2 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta_1+\theta_2) & \sin(\theta_1+\theta_2) \\ -\sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) \end{bmatrix}$$

$\because$, $\cos(\theta_1+\theta_2) = \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2$

$\sin(\theta_1+\theta_2) = \cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_2$

Q6) <u>My Ideas</u>

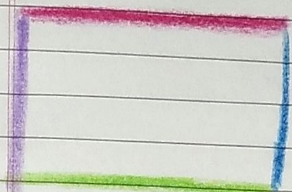
a. Derive the composite matrix to scale an object w.r.t. a fixed point.

Ans: $S_p = T_1 S T_2$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ -h & -k & 1 \end{bmatrix} \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ h & k & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -h & -k & 1 \end{bmatrix} \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ h & k & 1 \end{bmatrix}$$

$$= \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ -S_x h + h & S_y k + k & 1 \end{bmatrix}$$

b.  Develop function / procedure to fill colour into the above polygon using 8 connected approach.

Ans:
```
void floodfill (int x, int y, int old,
    int newcol)
{
    int current;
    current = getpixel (x, y);
    if (current == old)
    {
        delay (5);
        putpixel (x, y, newcol);
```

```
floodfill (x+1, y, old, newcol);
floodfill(x-1, y, old, new col);
floodfill ( x, y+1, old, newcol);
floodfill( x, y-1, old, newcol);
floodfill( x+1, y+1, old, newcol);
floodfill( x-1, y+1, old, newcol);
floodfill ( x+1, y-1, old, newcol);
floodfill( x-1, y-1, old, newcol);
}
}
```