Mandavi
5
Moulay

Assignment 3

Q1) Match the following:

(a) $\beta(p, q)$ — (2) $\int_0^\infty \frac{y^{p-1}}{(1+y)^{p+q}} dy$

(b) $\frac{\Gamma m \Gamma n}{\Gamma m+n}$ — (3) $\beta(m, n)$

(c) $\sqrt{\pi}$ — (1) $\Gamma 1/2$

(d) $\frac{\pi}{\sin p\pi}$ — (5) $\Gamma p \, \Gamma 1-p$

Q2) (a) The value of $\int_0^\infty \sqrt{y}\, e^{-y^3} dy$ is

Ans: ii) $\sqrt{\pi}/3$

(b) If $B(n, 2) = 1/6$, and n is a positive integer, then the value of n is

Ans: iii) 2

(c) The value of $\int_0^\infty \frac{t^2}{1+t^4} dt$ is

Ans: d) $\frac{\sqrt{\pi}}{2}$ $\frac{\pi}{2\sqrt{2}}$

mandavi
5
Dubey

Date ________
Page ________

Q3) State true or false

1) ~~False~~ True 2) ~~True~~ False 3) ~~True~~ False

Q4) 1) Show that $\int_0^\infty (x+1)^2 e^{-x^3}\,dx = \frac{1}{3}\left[1+\Gamma\frac{1}{3}+2\Gamma\frac{2}{3}\right]$

Solution $LHS = \int_0^\infty (x+1)^2 e^{-x^3}\,dx = \int_0^\infty (x^2+1+2x)e^{-x^3}dx$

= Let $x^3 = t$

$x = t^{1/3}$

$dx = \frac{1}{3} t^{-2/3}\,dt$

$= \int_0^\infty (t^{2/3}+1+2t^{1/3})e^{-t}\frac{1}{3}t^{-2/3}dt$

$= \frac{1}{3}\int_0^\infty (t^{-2/3}+1+2t^{-1/3})e^{-t}\,dt$

$= \frac{1}{3}\left[1+\Gamma\frac{1}{3}+2\Gamma\frac{2}{3}\right] = RHS$

2) State and prove Duplication formula $\int_0^{\pi/6} \sin^2 6\theta \cos^6 3\theta\, d\theta = \frac{7\pi}{384}$

Soln Duplication formula:

$$\Gamma m\,\Gamma\frac{m+1}{2} = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma 2m$$

mandavi
5
Duley

Proof:

$$\int_0^{\pi/2} \cos^m \theta \sin^m \theta = \frac{\Gamma\frac{m+1}{2}\Gamma\frac{n+1}{2}}{2\Gamma\frac{m+n+2}{2}}$$

$$\int_0^{\pi/2} \sin^{2m-1} \theta \cos^{2n-1} \theta = \frac{\Gamma\frac{2m-1+1}{2}\Gamma\frac{2n-1+1}{2}}{2\Gamma\frac{2m-1+2n-1+2}{2}}$$

$$= \frac{\Gamma m \Gamma n}{\Gamma m+n}$$

$2n-1=0$

$$\boxed{n=\frac{1}{2}}$$

$$\int_0^{\pi/2} \sin^{2m-1} \theta \cos^{2n-1} \theta \, d\theta = \frac{\Gamma m \Gamma n}{2\Gamma m+n}$$

$$\int_0^{\pi/2} \sin^{2m-1} \theta \, d\theta = \frac{\Gamma m \Gamma\frac{1}{2}}{2\Gamma\frac{m+1}{2}} = \frac{\Gamma m \sqrt{\pi}}{2\Gamma\frac{m+1}{2}} \quad -(1)$$

$m=n$ eqn (1)

$$\int_0^{\pi/2} \sin^{2m-1} \theta \cos^{2m-1} \theta \, d\theta = \frac{\Gamma m \Gamma m}{2\Gamma m+m} = \frac{(\Gamma m)^2}{2\Gamma 2m}$$

Mandavi

5

Dubey

Date

Page

$$\frac{1}{2^{2m-1}}\int_0^{\pi/2} 2^{2m-1} \sin^{2m-1}\theta \cos^{2m-1}\theta \, d\theta = \frac{(\Gamma m)^2}{2\Gamma 2m}$$

$$\frac{2}{2^{2m}}\int_0^{\pi/2} (\sin 2\theta)^{2m-1} \, d\theta = \frac{(\Gamma m)^2}{2\Gamma 2m}$$

$$2\theta = \phi \qquad 2 \times \frac{\pi}{2} = \phi$$

$$2\,d\theta = d\phi \qquad \phi = \pi$$

$$2 \times 0 = \phi \Rightarrow \phi = 0$$

$$\frac{1}{2^{2m}}\int_0^{\pi} (\sin\phi)^{2m-1} \, d\phi = \frac{(\Gamma m)^2}{2\Gamma 2m}$$

$$\frac{2}{2^{2m}}\int_0^{\pi/2} (\sin\phi)^{2m-1} \, d\phi = \frac{(\Gamma m)^2}{2\Gamma 2m}$$

$$\int_0^{\pi/2} (\sin\phi)^{2m-1} \, d\phi = \frac{2^{2m}(\Gamma m)^2}{2 \cdot 2\Gamma 2m}$$

$$\int_0^{\pi/2} (\sin\phi)^{2m-1} \, d\phi = \frac{2^{2m-1}(\Gamma m)^2}{2\Gamma 2m}$$

from eq^n 2 & 3

$$\frac{2^{2m-1}(\Gamma m)^2}{2\Gamma 2m} = \frac{\Gamma m \Gamma\pi}{2\Gamma \frac{m+1}{2}}$$

$$\boxed{\Gamma m \, \Gamma \frac{m+1}{2} = \frac{\sqrt{\pi}}{2^{2m-1}} \Gamma 2m}$$

Mandavi
5
Dubey

Date
Page

3) Show that $\int_0^{\pi/6} \sin^2 6\theta \cos^6 3\theta \, d\theta$
$= 7\pi / 384$

Sol^n Let $I = \int_0^{\pi/6} \sin^2 6\theta \cos^6 3\theta \, d\theta$

$= 4 \int_0^{\pi/6} (2 \sin 3\theta \cos 3\theta)^2 \cos^6 3\theta \, d\theta$

$= 4 \int_0^{\pi/6} \sin^2 3\theta \cos^8 3\theta \, d\theta$

Let $3\theta = x \Rightarrow \theta = x/3 \Rightarrow d\theta = dx/3$

θ	0	$\pi/6$
x	0	$\pi/2$

$I = 4 \int_0^{\pi/2} \sin^2 x \cos^8 x \frac{dx}{3} = \frac{2}{3} \left[2 \int_0^{\pi/2} \sin^2 x \cos^8 x \, dx \right]$

$= \frac{2}{3} B\left(\frac{3}{2}, \frac{9}{2}\right) = \frac{2}{3} \frac{\Gamma\frac{3}{2} \Gamma\frac{9}{2}}{\Gamma 6}$

$= \frac{\frac{2}{3} \times \frac{1}{2} \times \sqrt{\pi} \times \frac{7}{2} \times \frac{5}{2} \times \frac{3}{2} \times \frac{1}{2} \times \sqrt{\pi}}{5!}$

$\boxed{\therefore \sin^2 6\theta \cos^6 3\theta = \frac{7\pi}{384}}$

Mandavi
5
Dubey

Date
Page

Q5) i) Assuming the validity of differentiation under integral sign prove that

$$\int_0^{\pi/2} \frac{\log(1+\cos\alpha\cos x)}{\cos x}\,dx = \frac{\pi^2-4\alpha^2}{8}$$

Sol^n Let $I(\alpha)$ be the given integral.

$$dI/d\alpha = \int_0^{\pi} \frac{\partial}{\partial\alpha}\,dx = \int_0^{\pi/2} \frac{-\sin\alpha\cos x}{(1+\cos\alpha\cos x)\cos x}$$

$$= \int_0^{\pi/2} \frac{-\sin x}{1+\cos\alpha\cos x}\,dx \qquad \text{Put } t = \frac{\tan x}{2}$$

$$\therefore dx = 2\,dt/1+t^2 \quad \& \quad \cos x = 1-t^2/1+t^2$$

when $x=0$, $t=0$. when $x=\pi/2$, $t=1$

$$\therefore dI/d\alpha = \int_0^1 \frac{-\sin\alpha}{1+\cos\alpha\left(\frac{1-t^2}{1+t^2}\right)} \cdot \frac{2\,dt}{1+t^2}$$

$$= -\sin\alpha\int_0^1 \frac{2\,dt}{(1+t^2)+\cos\alpha(1-t^2)}$$

$$= -\sin\alpha\int_0^1 \frac{2\,dt}{(1+\cos\alpha)+(1-\cos\alpha)t^2}$$

$$= \frac{-2\sin\alpha}{1-\cos\alpha}\int_0^1 \frac{dt}{\left[(1+\cos\alpha)/(1-\cos\alpha)\right]+t^2}$$

$$= \frac{-2\sin\alpha}{1-\cos x}\,\frac{\sqrt{1-\cos x}}{\sqrt{1+\cos x}}\left[\tan^{-1}\left(\frac{\sqrt{1-\cos\alpha}}{\sqrt{1+\cos\alpha}}\right)t\right]_0^1$$

$$= \frac{-2\sin\alpha}{\sqrt{1-\cos^2\alpha}}\left[\tan^{-1}\sqrt{\frac{2\sin^2(\alpha/2)}{2\cos^2(\alpha/2)}}\right. = -2\tan^{-1}\tan\frac{\alpha}{2}$$

$$= \frac{-2\alpha}{2} = -\alpha$$

5) i) Integrating both sides w.r.t α,

$$I = -\frac{\alpha^2}{2} + C \quad \text{--- (1)}$$

$$\therefore I\left(\frac{\pi}{2}\right) = -\frac{\pi^2}{8} + C$$

Putting $\alpha = \pi/2$

$$I\left(\frac{\pi}{2}\right) = \int_0^{\pi/2} 0\,dx$$

$$\boxed{C = \frac{\pi^2}{8}}$$

from (1), $I = -\frac{\alpha^2}{2} + \frac{\pi^2}{8} = \frac{\pi^2 - 4\alpha^2}{8}$

Nandavi
S
Dubey

2) Evaluate $\int_0^\infty \frac{e^{-x}}{x}(1-e^{-ax})dx \quad a>-1$

Soln $\frac{d\phi(a)}{da} = \int_0^\infty \frac{\partial}{\partial a} \frac{e^{-x}}{x}(1-e^{-ax})dx$

$= \int_0^\infty \frac{e^{-x}}{x} dx \frac{\partial}{\partial a}(1-e^{-ax})$

$= \int_0^\infty \frac{e^{-x}}{x} dx (e^{-ax})(-x)$

$= \int_0^\infty \cancel{+}-e^{-x} e^{-ax} dx$

$= \int_0^\infty -e^{-x(a+1)}dx$

$= \int_0^\infty -\left[\frac{e^{-(a+1)x}}{a+1}\right]_0^a$

$= -\left[-\frac{1}{a+1}\right] = \frac{1}{a+1}$

3) Show that $\int_0^\infty \frac{(\tan^{-x/a} - \tan^{-x/b})}{x} dx$

$= \frac{\pi}{2}\log\left(\frac{b}{a}\right)$ where $a>0, b>0$

Solution Let $I(a) = \int_0^\infty \frac{\tan^{-1}\left(\frac{x}{a}\right) - \tan^{-1}\frac{x}{b}}{x} dx$

mamdavi
5
Mulley

Date
Page

Applying DUIS Rule,

$$I(a) = \int_0^\infty \frac{\partial}{\partial a}\left[\frac{\tan^{-1}\left(\frac{x}{a}\right) - \tan^{-1}\left(\frac{x}{b}\right)}{x}\right] dx$$

$$\frac{d}{da} I(a) = \int_0^\infty \frac{1}{x} \frac{\partial}{\partial a}\left[\tan^{-1}\left(\frac{x}{a}\right) - \tan^{-1}\left(\frac{x}{b}\right)\right] dx$$

$$\frac{d}{dI} I(a) = \int_0^\infty \frac{1}{x}\left[\frac{1}{1+\left(\frac{x}{a}\right)^2} \frac{\partial}{\partial a}\left(\frac{x}{a}\right) - 0\right] dx$$

$$\frac{d}{da} I(a) = \int_0^\infty \frac{1}{x}\left[\frac{1}{1+\frac{x^2}{a^2}} \, x\left(\frac{-1}{a^2}\right)\right] dx$$

$$\frac{d}{da} I(a) = \int_0^\infty \frac{1}{x}\left[\frac{a^2}{a^2+x^2} \cdot \frac{-x}{a^2}\right] dx$$

$$\frac{d}{da} I(a) = \int_0^\infty -\frac{1}{x^2+a^2}\, dx$$

$$\frac{d}{da} I(a) = -\int_0^\infty \frac{1}{x^2+a^2}\, dx$$

$$\frac{d}{da} I(a) = -\left[\frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)\right]_0^\infty$$

$$\frac{d}{da} I(a) = -\frac{1}{a}\left[\frac{\pi}{2} - 0\right]$$

$\because \tan^{-1}(\infty) = \pi/2$
$\tan^{-1}(0) = 0$

mandavi
5
~~Dubey~~

$$\frac{d}{da} I(a) = \frac{-\pi}{2a}$$

Integrating ~~on~~ both sides w.r.t. 'a':

$$I(a) = \int \frac{-\pi}{2a} da$$

$$I(a) = -\frac{\pi}{2} \int \frac{1}{a} da$$

$$I(a) = -\frac{\pi}{2} \log a + C \quad -(2)$$

Now to find 'c' sub. a=b in eqn (2)

$$I(b) = -\frac{\pi}{2} \log b + C$$

But $I(b) = 0$

$$0 = -\frac{\pi}{2} \log b + C$$

$$\therefore \boxed{\frac{\pi}{2} \log b = C}$$

4) Evaluate $\int_0^{\pi/2} \frac{dx}{a+b\cos x}$, $a>0$, $b>0$, and deduce that $\int_0^{\pi} \frac{dx}{(a+b\cos x)^2} = \frac{\pi b}{(a^2-b^2)^{3/2}}$

Mamdavi
5
Quley

Sol and $\int_0^{\pi} \frac{\cos x\, dx}{(a+b\cos x)^2} = -\frac{\pi b}{(a^2-b^2)^{3/2}}$

Sol $I = \int_0^{\pi} \frac{dx}{a+b\cos x}, \quad a, b > 0$

Put $\tan\left(\frac{x}{2}\right) = t, \quad dx = \frac{dt}{1+t^2}$

$\cos x = \frac{1-t^2}{1+t^2}$

when $x = 0, \ t = 0 \ \& \ x = \pi, \ t = \infty$

$I = \int_0^{\infty} \frac{dt/1+t^2}{a+b\left[\frac{1-t^2}{1+t^2}\right]}, \ a, b > 0$

$I = \int_0^{\infty} \frac{dt}{a(1+t^2)+b(1-t^2)} = \int_0^{\infty} \frac{dt}{(a+b)+(a-b)t^2}$

$= \frac{1}{a-b} \int_0^{\infty} \frac{dt}{(a+b)/(a-b)+t^2},$

$I = \frac{1}{\sqrt{\frac{a+b}{a-b}}\,(a-b)} \left[\tan^{-1} \frac{x}{\sqrt{\frac{a+b}{a-b}}}\right]_0^{\infty}$

$= \frac{\pi}{2\sqrt{a^2-b^2}}$

$I = \int_0^{\pi} \frac{dx}{a+b\cos x}, \quad a, b > 0$

$I = \int \frac{\pi}{2\sqrt{a^2-b^2}}$

mandavi
5
Marks

Date
Page

$$\int_0^{\pi} \frac{dx}{a+b\cos x} = \frac{\pi}{2\sqrt{a^2-b^2}}$$

Differentiating w.r.t. b

$$= \int_0^{\pi} \frac{\partial}{\partial b} \frac{dx}{} \left[\frac{\pi}{2\sqrt{a^2-b^2}}\right]$$

$$= \int_0^{\pi} \frac{(-1)\,dx}{(a+b\cos x)^2} \cos x$$

$$\frac{d}{db}\left[\frac{\pi}{2\sqrt{a^2-b^2}}\right] = \frac{\pi}{2}\left(\frac{-1}{2}\right)(a^2-b^2)^{-3/2}(-2b)$$

$$= \frac{\pi b}{2(a^2-b^2)^{3/2}}$$

$$\int_0^{\pi} \frac{(-1)\,dx}{(a+b\cos x)^2} \cos x = \frac{\pi b}{2(a^2-b^2)^{3/2}}$$

$$\int_0^{\pi} \frac{\cos x\,dx}{(a+b\cos x)^2} = -\frac{\pi b}{2(a^2-b^2)^{3/2}}$$

Q6) 1) Find total length of loop of curve
$9y^2 = (x+7)(x+4)^2$

Soln If $y=0$, $x=-7$ or $x=-4$ the loop intersects at $x=-7$ and at $x=-4$

If S is the total length of the

mandavi
5
Mulae

loop.

$$S = 2\int_{-7}^{-4} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$9y^2 = (x+7)(x+4)^2$$

$$18y\frac{dy}{dx} = (x+7)2(x+4)+(x+4)^2$$

$$= (x+4)(2x+14+x+4)$$

$$= (x+4)(3x+18) = (x+4)3(x+6)$$

$$\frac{dy}{dx} = \frac{(x+4)(x+6)}{6y}$$

$$1+\left(\frac{dy}{dx}\right)^2 = 1+\frac{(x+4)^2(x+6)^2}{36y^2}$$

$$1+\frac{(x+4)^2\cdot(x+6)^2}{4(x+7)(x+4)^2} = \frac{4x+28+x^2+12x+36}{4(x+7)}$$

$$\frac{(x+8)^2}{4(x+7)}$$

$$S = 2\int_{-1}^{-4} \frac{x+8}{2\sqrt{x+7}}$$

Put $x+7 = t^2$

$dx = 2t\,dt$

$x = -7, \quad t = 0$

$x = -4, \quad t = \sqrt{3}$

$$S = 2\int_0^{\sqrt{3}} \frac{t^2+1}{2t}\,2t\,dt$$

$$S = 2\left[\frac{t^3}{3}+t\right]_0^{\sqrt{3}} = 4\sqrt{3}$$

Mandavi
5
Dubey

2) Show that the length of the parabola $y^2 = 4ax$ from the vertex to the end of the latus rectum is a $[\sqrt{2} + \log(1+\sqrt{2})]$. Hence prove that length of the arc cut off by the line $3y = 8x$ is a $\left[\log 2 + \frac{15}{16}\right]$.

Sol^n length of the parabola from the vertex to the end of the latus rectum is

$$S = \int_{y=0}^{2a} \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy \quad \text{--- (1)}$$

$$x = \frac{y^2}{4a}$$

$$\frac{dx}{dy} = \frac{2y}{4a} = \frac{y}{2a}$$

$$\therefore 1 + \left(\frac{dx}{dy}\right)^2 = 1 + \frac{y^2}{4a^2}$$

$$= \frac{4a^2+y^2}{4a^2}$$

$$\therefore \sqrt{1+\left(\frac{dx}{dy}\right)^2} = \frac{\sqrt{4a^2+y^2}}{2a}$$

$\therefore$ from (1)

$$S = \int_{y=0}^{2a} \frac{\sqrt{4a^2+y^2}}{2a}\, dy \quad \text{--- (2)}$$

Mandavi

5

Aruley

$$= \frac{1}{2a}\left[\frac{y}{2}\sqrt{y^2+4a^2}+\frac{4a^2}{2}\log\left(y+\sqrt{y^2+4a^2}\right)\right]_0^{2a}$$

$$= \frac{1}{2a}\left\{\left[\frac{2a}{2}\sqrt{4a^2+4a^2}+\frac{4a^2}{2}\log\left(2a+\sqrt{4a^2+4a^2}\right)\right]\right.$$

$$\left. - \left[0+\frac{4a^2}{2}\log\left(0+\sqrt{0+4a^2}\right)\right]\right\}$$

$$= \frac{1}{2a}\left\{a2\sqrt{2}\cdot a+2a^2\log\left(2a+2\sqrt{2}a\right)\right.$$

$$\left. - 2a^2\log 2a\right\}$$

$$= \frac{1}{2a}\left\{a2\sqrt{2}a+2a^2\log\left(2a+2\sqrt{2}a\right)\right.$$

$$\left. - 2a^2\log 2a\right\}$$

$$= \frac{2a^2}{2a}\left\{\sqrt{2}+\log 2a\left(1+\sqrt{2}\right)-\log 2a\right\}$$

$$= a\left\{\sqrt{2}+\log 2a+\log\left(1+\sqrt{2}\right)-\log 2a\right\}$$

$$\boxed{S = a\left\{\sqrt{2}+\log\left(1+\sqrt{2}\right)\right\}}$$

If the line $3y=8$ intersects, then we have

$x = \frac{3y}{8}$ $\therefore$ $y^2 = 4ax$

$$y^2 = \frac{4a3y}{8} = \frac{3a}{2}$$

$\therefore$ y co-ordinate of A is $\frac{3a}{2}$.

mandavi

5

~~Duley~~

Date

Page

$\therefore$ ~~arc O~~ arc OA $= \frac{1}{2}\int_0^{3a/2} \sqrt{y^2+4a^2}\, dy$

--- from ②

$$= \frac{1}{2a}\left[\frac{y}{2}\sqrt{y^2+4a^2} + \frac{4a^2}{2}\log\left(y+\sqrt{y^2+4a^2}\right)\right]_0^{3a/2}$$

$$= \frac{1}{2a}\left\{\left[\frac{3a}{2}\sqrt{\frac{9a^2}{4}+4a^2} + 2a^2\log\left(\frac{3a}{2}+\sqrt{\frac{9a^2}{4}+4a^2}\right)\right] - \left[0+2a^2\log\left(0+\sqrt{0+4a^2}\right]\right\}$$

$$= \frac{1}{2a}\left\{\frac{3a}{4}\left(\frac{5a}{2}\right) + 2a^2\log\left(\frac{3a}{2}+\frac{5a}{2}\right) - 2a^2\log 2a\right\}$$

$$= \frac{1}{2a}\left\{\frac{15a^2}{8} + 2a^2\log 4a - 2a^2\log 2a\right\}$$

$$= \frac{1}{2a}\left\{\frac{15a^2}{8} + 2a^2\log 2\right\}$$

$$= \frac{2a^2}{2a}\left\{\frac{15}{16} + \log 2\right\}$$

$$\text{arc } OA = a\left\{\frac{15}{16} + \log 2\right\}$$

mandavi
5
Nauleey

3. Find the length of cardiode $r = a\cos\theta$ lying inside the circle $r = a(1-\cos\theta)$

Sol. The circle and the cardiode are shown in the figure. They intersect where,

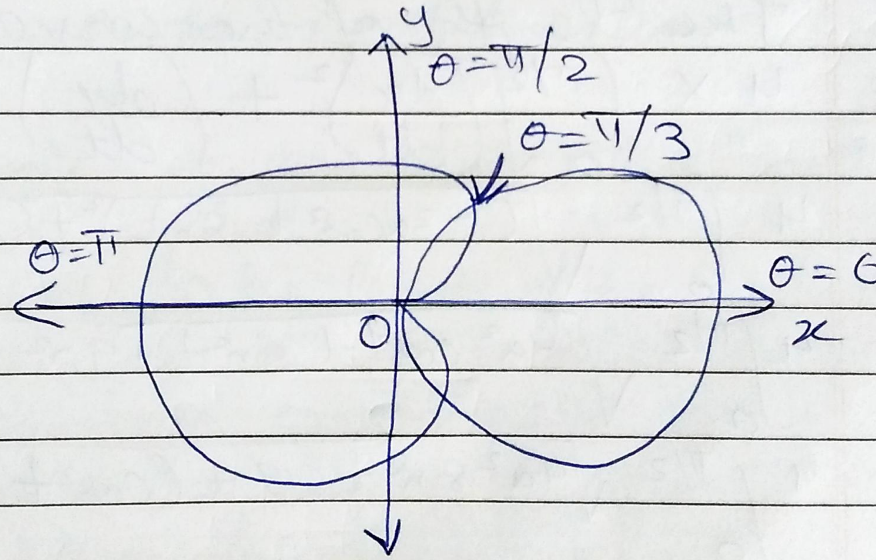
$a(1-\cos\theta) = a\cos\theta$

$1 - \cos\theta = \cos\theta$

$1 = 2\cos\theta$

$\cos\theta = 1/2$

$\theta = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$



The length of the cardiode lying in the circle is $S = 2\int_0^{\pi/3} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}$ --- (i)

$r = a(1-\cos\theta) \Rightarrow \frac{dr}{d\theta} = a\sin\theta$

$r^2 + \left(\frac{dr}{d\theta}\right)^2 = a^2 - 2a^2\cos\theta + a^2\cos^2\theta + a^2\sin^2\theta$

mandavi

5

Dubey

Date ______

Page ______

$$= a^2 - 2a^2\cos\theta + a^2$$

$$= 2a^2(1-\cos\theta) = 2a^2\left(2\sin^2\frac{\theta}{2}\right)$$

$$= 4a^2\sin^2\frac{\theta}{2}$$

$$\therefore \sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2} = 2a\sin\frac{\theta}{2}$$

from ①

$$S = 2\int_0^{\pi/3} 2a\sin\frac{\theta}{2}\,d\theta = 4a\left[\frac{-\cos\theta/2}{1/2}\right]_0^{\pi/3}$$

$$= -8a\left[\cos\frac{\pi}{6} - \cos 0\right] = -8a\left[\frac{\sqrt{3}}{2} - 1\right]$$

$$\boxed{\therefore S = 8a\left[1-\frac{\sqrt{3}}{2}\right]}$$

mandavi
5
Nuley

4) Find the total length of the curve
$x^{2/3} + y^{2/3} = a^{2/3}$

Sol^n $x = a\cos^3 t$, $y = a\sin^3 t$

$\frac{dx}{dt} = \frac{d}{dt} a\cos^3 t$

$= a \cdot 3 \cdot \cos^2 t(-\sin t)$

$\frac{dx}{dt} = -3a\cos^2 t \sin t$

$\frac{dy}{dt} = a \cdot 3 \cdot \sin^2 t \cos t = 3a\sin^2 t\cos t$

The length of the curve

$= 4 \times \int_0^{\pi/2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)} \, dt$

$= 4 \int_0^{\pi/2} \sqrt{(-3a\cos^2 t \sin t)^2 + (3a\sin^2 t + \cos t)^2} \, dt$

$= 4 \int_0^{\pi/2} \sqrt{(9a^2\cos^4 t \sin^2 t + 9a^2 \sin^4 t \cos^2 t)} \, dt$

$= 4 \int_0^{\pi/2} \sqrt{9a^2 \sin^2 t \cos^2 t (\cos^2 t + \sin^2 t)} \, dt$

$= 4 \int_0^{\pi/2} \sqrt{9a^2 \sin^2 t \cos^2 t \times 1} \, dt$

$= 4 \int_0^{\pi/2} 3a \sin t \cos t \, dt$

$= 4 \times 3a \int_0^{\pi/2} \frac{2\sin t \cos t}{2} \, dt$

mandavi
5
Nidulay

$= \frac{12a}{2}\int_0^{\pi/2} \sin 2t\, dt$

$= \underline{6a}\left[-\frac{\cos 2t}{2}\right]_0^{\pi/2}$

$= \frac{6a}{2}\left[\left(-\cos 2\frac{\pi}{2}\right)-\left(-\cos 2\times 0\right)\right]$

$= 3a\left[\left(-\cos \pi\right)-\left(-\cos 0\right)\right]$

$= 3a\left[-(-1)-(-1)\right]$

$= 6a$

Q7) Illustrate Rectification

Ans: Rectification means finding the lengths of the curves. We shall develop in the chapter various formulae for the length of the curve, of which the eq^n is given either in cartesian or in polar co-ordinates. It is used to measure the length of curve.

1) Cartesian form : $[x, y]$

$y = f(x)$, then length of arc $= s =$

$\int\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$ or $\int\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$

2) Polar form : $[r, \theta)$

$x = f(y)$, then length of arc $= s =$

$\int\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$

Mandavi

5

Nauleey

Date

Page

3) Parametric form: $x = f_1(t),\ y = f_2(t),$

then length of arc $= \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dx$