1. Evaluate $\int_C \frac{z+2}{(z-3)(z+4)}\,dz$, where C is the circle $|z|=1$

Here the points are $z=3$, $z=-4$, which lie outside the circle $|z|=1$

$\therefore$ By Cauchy's theorem,

$$\int_C \frac{z+2}{(z-3)(z+4)}\,dz = 0$$

2. Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-2)(z-3)}\,dz$, where C is the circle $|z|=4$

Here $z=2$, $z=3$ lie inside the circle $|z|=4$.

$$\frac{1}{(z-2)(z-3)} = \frac{1}{z-3} - \frac{1}{z-2}$$

$$I = \int_C \frac{\sin \pi z^2 + \cos \pi z^2}{z-3}\,dz - \int_C \frac{\sin \pi z^2 + \cos \pi z^2}{z-2}\,dz$$

$$= 2\pi i\, f(3) - 2\pi i\, f(2).$$

$$= 2\pi i \{(\sin 9\pi + \cos 9\pi) - (\sin 4\pi + \cos 4\pi)\}$$

$$= 2\pi i \{(0 + -1) - (0+1)\}$$

$$= 2\pi i \{-2\}$$

$$= -4\pi i$$

(3) Evaluate $\int_c \frac{4z-1}{z^2-3z-4}\,dz$, where c is the ellipse $x^2+4y^2=4$

$\frac{x^2}{4}+\frac{y^2}{1}=1$, has the centre at the origin and major axis 2 and minor axis 1

$z^2-3z-4=0$

$(z-4)(z+1)=0$

$z=4,\ z=-1$

$z=-1$ lies inside c and $z=4$ lies outside c

$$\int_c \frac{4z-1}{z^2-3z-4}\,dz = \int_c \frac{(4z-1)/(z-4)}{(z+1)(z-4)/(z-4)}\,dz$$

$$= \int_c \frac{(4z-1)/z-4}{z+1}\,dz \qquad f(z)=\frac{4z-1}{z-4}, \quad z_0=-1$$

$$= 2\pi i\, f(z_0) \qquad f(-1)=\frac{-4-1}{-1-4} = 1$$

$$= 2\pi i\, f(-1)$$

$$= 2\pi i\,(1)$$

$$= \underline{\underline{2\pi i}}$$

(4)