MATHEMATICS IV MINI PROJECT PRESENTATION

on

Application of Residue Theorem to evaluate real integrations.

Project Guide : **Prof. Latika Chandramohan**

Team Members:
MANDAVI DUBEY (23)
SAKSHI DUBRY (24)
RITESH MISHRA (58)
NIDHI PAWAR (77)
SUJIT RAJBHAR (81)
SAAD SHAIKH ZAKIR (86)

Department of Computer Engineering
Mahatma Gandhi Mission's College of Engineering & Technology Kamothe, Navi Mumbai–400 209.
University of Mumbai

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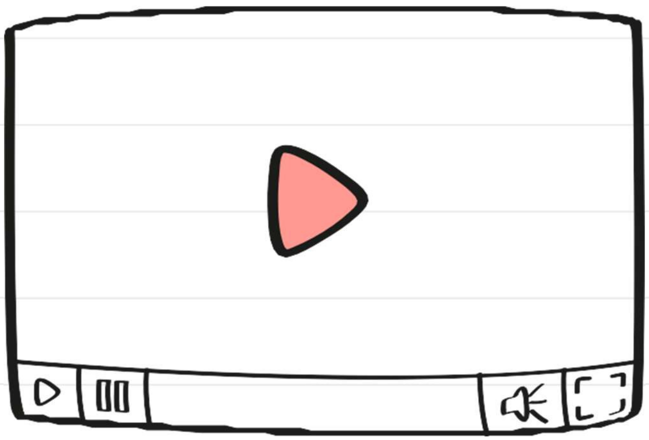
Abstract



1. It is common to encounter an integral that seems impossible to evaluate.

2. The Residue Theorems introduce techniques that may make the impossible, possible.

3. In this capstone I introduce Residue Theorems and apply techniques to integrals of certain forms that cannot be computed using methods in calculus of real variables.

4. Not only do the Residue Theorems make it possible to reach a closed solution, they can make evaluating integrals much easier than before.

INTRODUCTION

1. Have you ever wondered if it was possible to evaluate integrals like R ∞ 0 sin x/x dx with ease?

2. This capstone will give some positive answers to such a question by exploring the various theorems and techniques.

3. To accomplish this, it is necessary to transform these integrals into the complex plane.

4. A complex number is considered to be made up of a real part and imaginary part, where the imaginary part is a real number multiplied by √ −1 (denoted by i).

Residue Theorem

Suppose that C is a simple contour contained in the interior of a simply connected domain D. If F is an analytic everywhere on and within C except possiblity of an finite number of isolated similarities inside C, then:

$$\oint_C f(z)\,dz = 2\pi i \sum_{k=1}^{n} \operatorname{Res}(f(z), z_k).$$

Sum 1

Using complex integration method, evaluate $\int_0^{2\pi} \frac{cos2\theta}{5+4cos\theta} d\theta$.

Solution. Let I =Real part of $\int_0^{2\pi} \frac{e^{2i\theta}}{5+2\left(e^{i\theta}+e^{-i\theta}\right)} d\theta$

=Real part of $\oint_C \frac{z^2}{5+2\left(z+\frac{1}{z}\right)} \left(\frac{dz}{iz}\right)$ =Real part of $\frac{1}{i} \oint_C \frac{z^2}{2z^2+5z+2} dz$

Singularities are given by $2z^2 + 5z + 2 = 0, z = -\frac{1}{2}, -2$ $z = -\frac{1}{2}$ is the only pole which lies inside the unit circle $C = |z| = 1$

Residue of $f(z)$ at $\left(z = -\frac{1}{2}\right)$ is

$R = limit_{z \to -\frac{1}{2}} \left(z + \frac{1}{2}\right) \cdot \frac{z^2}{i(2z+1)(z+2)} = \frac{1}{12i}$

Hence by Cauchy's Residue Theorem,

$I = \oint_c f(z)\, dz = 2\pi i \left(\frac{1}{12i}\right) = \frac{\pi}{6}$.

Sum 2

Evaluate $\int_{-\infty}^{\infty} \frac{x^2+x+2}{x^4+10x^2+9} dx$ using contour integration.

Solution:

1. Consider the contour consisting of a large semi-circle with centre at the origin, in the upper half of the plane and its diameter on the real axis.
2. Now, $zf(z) = \frac{z^3+z^2+2z}{z^4+10z^2+9} \to 0$ as $|z| \to \infty$
3. Further $z^4 + 10z^2 + 9 = 0$ i.e. $(z^2+1)(z^2+9) = 0$ gives $z = +i, -i, +3i, -3i$. The poles lying in the upper half are $z = i, 3i$
4. Residue (at $z = i$)= $lim_{z \to i} \frac{(z-i)(z^2+z+2)}{(z-i)(z+i)(z^2+9)} = \frac{1+i}{16i}$

   Residue (at $z = 3i$)= $lim_{z \to 3i} \frac{(z-3i)(z^2+z+2)}{(z^2+1)(z-3i)(z+3i)} = \frac{7-3i}{48i}$
5. $\int_{-\infty}^{\infty} \frac{x^2+x+2}{x^4+10x^2+9} dx = 2\pi i \left[\frac{1+i}{16i} + \frac{7-3i}{48i}\right] = \frac{5\pi}{12}.$

Sum 3

Evaluate $\int\limits_0^\infty \frac{sinx}{x} dx$

Solution: Consider $\int_c \frac{e^{iz}}{z} dz = \int_c f(z)\, dz$ where is the contour consisting of

1. the real axis from r to R where r is small and R is large.
2. the upper half of the large semi circle $C_1, |z| = R$
3. the real axis from -R to -r

Since there is no singularity within the contour C,by Cauchy's Theorem,we have

$$\int_c f(z)\, dz = \int\limits_r^R f(x)\, dx + \int_{C_1} f(z)\, dz + \int\limits_{-R}^{-r} f(x)\, dx + \int_{C_2} f(z) = 0.....(1)$$

by Jordan's lemma $\int_{C_1} f(z)\, dz = 0$

Contd.

Putting $z = re^{i\theta}$ and $dz = rie^{i\theta}d\theta$, we get $\int_{C_2} f(z)\,dz = -\pi i$

Hence as $r \to 0$ and $R \to \infty$, we get from (1),

$$\int_0^\infty f(x)\,dx + \int_{-\infty}^0 f(x)\,dx + (-\pi i) = 0$$

$$\int_{-\infty}^\infty f(x)\,dx = \pi i$$

$$\int_{-\infty}^\infty \frac{e^{ix}}{x}\,dx = \pi i$$

Equating imaginary parts on both sides.

$$\int_{-\infty}^\infty \frac{sinx}{x}\,dx = \pi$$

$$\int_0^\infty \frac{sinx}{x}\,dx = \frac{\pi}{2}.$$

Application

1. The design of aerofoil sections for aircraft is an area where the theory was developed using complex variable techniques.

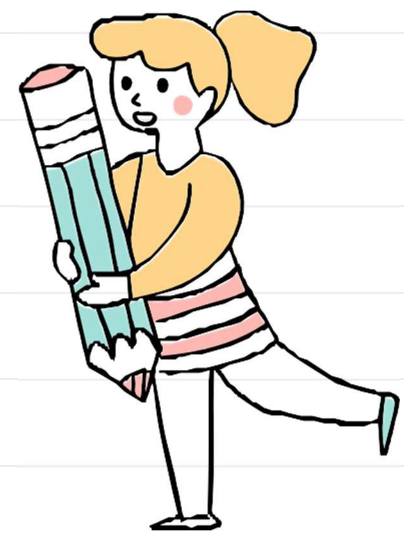
2. Throughout engineering, transforms defined as complex one form or another play a major role in analysis and design.

3. The use of complex variable techniwues allos us to develop criteria for the stability of systems.

Conclusion

1. This capstone has focused on how to use complex analysis to evaluate various definite integrals in the real plane.

2. Aside from evaluating integrals in the real plane, the amazing result of the Residue Theorem is the ability to evaluate contour integrals such that non-analytic points lie inside the closed contour.



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THANK YOU