Q. Solve $\frac{dy}{dx}+y=y^3(\cos x-\sin x)$

Sol^n. Given D.E. is, $\frac{dy}{dx}+(1)y=(\cos x-\sin x)y^3$ ---- Bernoulli's D.E

where $P=(1)$, $Q=(\cos x-\sin x)$

Divide by y^3

$$\frac{1}{y^3}\frac{dy}{dx}+(1)\frac{1}{y^2}=(\cos x-\sin x)$$

Put $\frac{1}{y^2}=t \Rightarrow -2y^3\frac{dy}{dx}=\frac{dt}{dx}$

$$\frac{1}{y^3}\frac{dy}{dx}=\left(-\frac{1}{2}\right)\frac{dt}{dx}$$

$\therefore \left(-\frac{1}{2}\right)\frac{dt}{dx}+(1)t=(\cos x-\sin x)$

$\therefore \frac{dt}{dx}+(-2)t=-2(\cos x-\sin x)$ ..... LDE in t, x

$P=-2$, $Q=-2(\cos x-\sin x)$

I.F. $=e^{\int(-2)dx}=e^{-2x}$

General solution is, $t.e^{-2x}=\int e^{-2x}2(\sin x-\cos x)dx+c$

$\therefore t.e^{-2x}=2\left[\int e^{-2x}\sin(1)x\,dx-\int e^{-2x}\cos(1)x\,dx\right]+c$

$t.e^{-2x}=2\cdot\frac{e^{-2x}}{4+1}\left\{[-2\sin x-(1)\cos x]-[-2\cos x+(1)\sin x]\right\}+c$

$t=\frac{2}{5}\{-2\sin x-\cos x+2\cos x-\sin x\}+c.e^{2x}$

$t=\frac{2}{5}\{\cos x-3\sin x\}+c.e^{2x}$

$$\boxed{\frac{1}{y^2}=\frac{2}{5}(\cos x-3\cdot\sin x)+c.e^{2x}}$$