

Exa. Solve  $\frac{dy}{dx} = \frac{4x^3y^2 + y \cdot \cos(xy)}{2x^4y + x \cdot \cos(xy)}$

Solution: Given D.E. is,  $\frac{dy}{dx} = - \left[ \frac{4x^3y^2 + y \cdot \cos(xy)}{2x^4y + x \cdot \cos(xy)} \right]$

$$(2x^4y + x \cdot \cos(xy))dy = -(4x^3y^2 + y \cdot \cos(xy))dx$$

$$(4x^3y^2 + y \cdot \cos(xy))dx + (2x^4y + x \cdot \cos(xy))dy = 0$$

Hence,  $M = 4x^3y^2 + y \cdot \cos(xy)$ ,  $N = 2x^4y + x \cdot \cos(xy)$

$$M_y = 8x^3y + \cos(xy) - y \sin(xy) \cdot x$$

$$N_x = 8x^3y + \cos(xy) - x \sin(xy) \cdot y$$

$$\therefore M_y = N_x$$

$\therefore$  Given D.E. is exact. D.E.  
General solution for given D.E. is,

$$\int M dx + \int (\text{Terms in 'N' does not contain } x) dy = C$$

$$\int [4x^3y^2 + y \cdot \cos(xy)] dx + \int 0 dy = C$$

$$4 \frac{x^4}{4} y^2 + y \cdot \frac{\sin(xy)}{y} = C$$

$$\boxed{x^4 y^2 + \sin(xy) = C}$$