Assignment 2

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Q1. Fill in the blanks

a) Let f and g be the function from the set of integers to itself, defined by $f(x) = 2x+1$ and $g(x) = 3x+4$. Then the composition of f and g is $\underline{6x+9}$.

b) $\underline{8}$ is the Cardinality of the Power set of the set $\{0, 1, 2\}$.

c) A function is said to be <u>one to one</u> if and only if $f(a) = f(b)$ implies that $a = b$ for all a and b in the domain of f.

d) A relation with transitive, reflexive and symmetric is <u>equivalence relation</u>.

e) The intersection of the sets $\{1, 2, 5\}$ and $\{1, 2, 6\}$ is the set $\{1, 2\}$

Q2. Correct Options

a) A c) <u>Set</u> is an ordered collection of objects.

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b) The relation {(1,2)(1,3), (3,1), (1,1), (3,3), (3,2), (1,4), (4,2), (3,4)} is

Ans: d) Asymmetric

c) Which of the following two sets are equal?

Ans: c) $A = \{1, 2, 3\}$ and $B = \{2, 1, 3\}$

d) In a Venn Diagram, the overlap between two circles represents:

Ans: b) the intersection of two sets.

e) Two sets are called disjoint if there c) intersection is the empty set.

Q3) State whether the following statements are true or false (give reasons)

a) True. $X \subset X$, etc. we can say, an empty set is considered as a subset of every set.

b) True. It is bijective because every possible element is mapped to by exactly one argument.

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c) The function $f(x) = x+1$ from the set of integers to itself is onto. True All elements in B are used if $f(a) = b$

Q4) Name the following or define or design the following.

a) Define Reflexive Closure, Symmetric Closure, Transitive Closure with the help of example.

Ans: Reflexive closure - $\Delta = \{(a,a) \mid a \in A\}$ is the diagonal relation on set A. The reflexive closure of R on set A is $R \cup \Delta$

Symmetric closure - Let R be a relation on set A, and let R^{-1} be the inverse of R. The symmetric closure of relation R on set A is $R \cup R^{-1}$

Transitive Closure - Let R be a relation on set A. The Connectivity relation is defined as - $R^* = \bigcup_{n=1}^{\infty} R^n$. The transitive closure of R is R^*.

Eg: Let R be a relation on set {1, 2, 3, 4} with R = {(1,1), (1,4), (2,3), (3,1), (3,4)}

Find the reflexive symmetric & transitive closure of R.

Sol^n For the given set, $\Delta = \{(1,1),(2,2),(3,3),(4,4)\}$. So the reflexive closure of R is $R \cup \Delta = \{(1,1),(1,4),(2,2),(2,3),(3,1),(3,3),(3,4),(4,4)\}$

For the symmetric closure we need the inverse of R, which is $R^{-1} = \{(1,1),(1,3),(3,2),(4,1),(4,3)\}$

The symmetric closure of R is —
$\{(1,1),(1,3),(1,4),(2,3),(3,1),(3,2),(3,4),(4,1),(4,3)\}$

For the transitive closure, we need to find R^*.

$\therefore$ we need to find $R^1, R^2, \ldots$ until $R^n = R^{n-1}$. We stop when this condition is achieved since finding higher powers of R would be the same.

$R^1 = \{(1,1),(1,4),(2,3),(3,1),(3,4)\}$

$R^2 = \{(1,1),(1,4),(2,1),(2,4),(3,1),(3,4)\}$

$R^3 = \{(1,1),(1,4),(2,1),(2,4),(3,1),(3,4)\}$

$\because R^2 = R^3$ we stop the process.

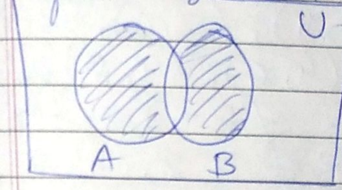
Transitive closure, $R^\infty = R^1 \cup R^2$ — $\{(1,1),(1,4),(2,1),(2,3),(2,4),(3,1),(3,4)\}$

b) Define symmetric difference of two sets A and B and explain it with Venn-diagrams.

Sol^n The symmetric difference using Venn diagram of two subsets A and B is a subset of U, denoted by $A \Delta B$ and is defined by $A \Delta B = (A-B) \cup (B-A)$

Let A and B ~~= (A−B) ∪ (B−A)~~ are two sets. The symmetric difference of two sets A and B is the set $(A-B) \cup (B-A)$ and is denoted by $A \Delta B$.

Thus, $A \Delta B = (A-B) \cup (B-A) = \{x : x \notin A \cap B\}$. or, $A \Delta B = \{x : [x \in A \text{ and } x \notin B] \text{ or } [x \in B \text{ and } x \notin A]\}$. The shaded part of the given Venn diagram represents $A \Delta B$.



$A \Delta B$

$A \Delta B$ is the set of all those elements which belongs either to A or to B but not to both. $A \Delta B$ is also expressed by $(A \cup B) - (B \cap A)$. It follows that $A \Delta \phi = A$ for all subset A, $A \Delta A = \phi$ for all subset A.

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c) If $A = \{1, 2, 3\}$. Determine the power set of A.

Soln. Subset of A are: ϕ, $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{1, 3\}$, $\{2, 3\}$ and $\{1, 2, 3\}$. Hence, total no. of subset are $2^3 = 8$.
Hence, $P(A) = \{\phi, \{1\} \{2\} \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$.

Q5) Answer the following questions in brief (20 to 30 words)

a) If $A = \{1, 2, 3, 4\}$ and $B = \{2, 4, 6, 8, 10\}$, then find a relation from A to B, defined as
$xRy \Rightarrow x + y$ is an even number. Also find the domain and range of R.

Ans. $R = \{(2, 2), (2, 4), (2, 6), (2, 8), (4, 2), (4, 4), (4, 6), (4, 8), (2, 10), (4, 10)\}$
Hence Domain of $R = \{2, 4\}$
Range of $R = \{2, 4, 6, 8\}$

b) The relation $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3), (4, 4), (5, 5)\}$ is an equivalence relation on set $A = \{1, 2, 3, 4, 5\}$. Find the equivalence class of each element of A.

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Sol. Equivalence class $= \{\{1, 2\}, \{2, 1\}, \{3, 4\}, \{4, 3\}, \{5\}\}$

c) Write the Cartesian product $A \times B$ where $A = \{a, b, c\}$ & $B = \{1, 2, 3\}$

Soln. ~~$A \times B = \{(a, 2),$~~
$A \times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3), (c, 1), (c, 2), (c, 3)\}$

Q6. Answer the following questions in brief (50 to 70 words)

a) Function f, g, h are defined on a set, $X = \{1, 2, 3\}$ as $f = \{(1, 2), (2, 3), (3, 1)\}$ $g = \{(1, 2), (2, 1), (3, 3)\}$ $h = \{(1, 1) (2, 2), (3, 1)\}$

(i) Find fog, gof are they equal)

Ans: $fog(1) = f(g(1)) = f(2) = 3$
$fog(2) = f(g(2)) = f(1) = 2$
$fog(3) = f(g(3)) = f(3) = 1$
$fog \{(1, 3), (3, 1), (2, 2)\}$
$gof(1) = g(f(1)) = g(2) = 1$
$gof(2) = g(f(2)) = g(3) = 3$
$gof(3) = g(f(3)) = g(1) = 2$
$gof = \{(1, 1), (2, 3), (3, 2)\}$

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$\therefore$ from above $f\circ g \neq g\circ f$.

(ii) find $fogoh$ & $fohog$

Ans: $fogoh(1) = f(g(h(1))) = f(g(1)) = f(2) = 3$

$fogoh(2) = f(g(h(2))) = f(g(2)) = f(1) = 2$

$fogoh(3) = f(g(h(3))) = f(g(1)) = f(2) = 3$

So, $fogoh = \{(1,3),(2,2),(3,3)\}$

$fohog(1) = f(h(g(1))) = f(h(2)) = f(2) = 3$

$fohog(2) = f(h(g(2))) = f(h(1)) = f(1) = 2$

$fohog(3) = f(h(g(3))) = f(h(3)) = f(1) = 2$

So, $fohog = \{(1,3),(2,2),(3,2)\}$

b) i) if $R = \{(1,1),(1,2),(2,3),(3,1),(3,2)\}$ is a relation on the set $A = \{1,2,3\}$, then find the reflexive closure of R.

Ans: $R \cup \Delta = \{(1,1),(1,2),\cancel{(1,3)},\cancel{(2,1)},(2,2),(2,3),(3,1),\cancel{(3,2)},(3,3)\}$

(ii) Find the symmetric closure of the relation on $R = \{(a,b),(a,c),(b,a),(b,d),(c,d),(d,c),(d,d)\}$ on the set $A = \{a,b,c,d\}$

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Soln $R_S = R \cup R^{-1} = \{(a,b),(b,a),(a,c),(c,a),(b,d),(d,a),(c,d),(d,c)\}$

c) If $A = \{a,b,c,d\}$ and $f = \{(a,b),(b,d),(c,a),(d,c)\}$, show that f is one-one onto from A to A. Find f^{-1}.

Ans: f is one-one since each element of A is assigned to distinct element of the set A. Also, f is onto since $f(A) = A$. Moreover, $f^{-1} = \{(b,a),(d,b),(a,c),(c,d)\}$.

Q7) Think & Answer

a) Prove that a function $f: R \to R$ defined by $f(x) = 2x-3$ is a bijective function.

Ans: $F: A \to B$ is a bijection only if F is

- injective: $F(x) = F(y) \Rightarrow x = y$, &
- surjective: $\forall b \in B$ there is some $a \in A$ such that $F(a) = b$

Take $x, y \in R$ & assume that $f(x) = f(y)$

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$\therefore 2x-3=2y-3$. We can cancel out 3 & divide by 2, then we get $x=y$.
This is a trivial case $\because$ the 2 sets of the f^n are the set of real nos. Every $x \in \mathbb{R}$ is bound to a value defined by $2x-3$.
$\therefore F$ is bijective.

b) Let R be the set of real nos & $f: R \to R$ be the f^n defined by $f(x) = 4x+5$. Show that f is invertible & find f^{-1}.

Ans: Here the f^n $f: R \to R$ is defined as $f(x) = 4x+5 = y$ (say).
Then $4x = y-5$ or $x = \frac{y-5}{4}$

This leads to a f^n $g: R \to R$ defined as $g(y) = \frac{y-5}{4}$

$\therefore gof(x) = g(f(x)) = g(4x+5)$

$= \frac{4x+5-5}{4} = x$ or $gof = I_g$

Similarly $fog(y) = f(g(y))$

$= f\left(\frac{y-5}{4}\right) = 4\left(\frac{y-5}{4}\right)+5 = y$

or $fog = I_f$

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Hence f is invertible & $f^{-1} = g$ which is given by $f^{-1}(x) = \frac{x-5}{4}$.

Q8) My Idea

a) Let R be a relation on set $A = \{1, 2, 3, 4\}$, given as $R = \{(1,1), (1,4), (2,2), (2,3), (3,2), (3,3), (4,1), (4,4)\}$. Find transitive closure using Warshall's Algorithm.

Sol^n Let M_R denote the matrix representation of R. Take $W_0 = M_R$, we have
$W_0 = M_R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ and $n=4$ (as M_R is a 4×4 matrix) We compute W_4 by using Warshall's algorithm. For $k=1$.
In column 1 of W_0, '1' is at position 1, 4. Hence $p_1 = 1$, $p_2 = 4$. In row 1 of W_0 '1' is at position 1, 4. Hence $q_1 = 1$, $q_2 = 4$. $\therefore$ to obtain W_1, we put '1' at the position:
$\{(p_1, q_1), (p_1, q_2), (p_2, q_1), (p_2, q_2)\} = \{(1,1), (1,4), (4,1), (4,4)\}$.
Thus, $W_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

For $k=2$. In column 2 of W_1, '1' is at position 2, 3. Hence $p_1 = 2$, $p_2 = 3$. In row 2 of W_1, '1' is at position 2, 3. Hence $q_1 = 2$, $q_2 = 3$. ∴ to obtain W_2, we ~~get~~ put '1' at the position $\{(p_1, q_1), (p_1, q_2), (p_2, q_1), (p_2, q_2)$
$= (2,2), (2,3), (3,2), (3,3)\}$

Thus, $W_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

For $k=3$. In column 3 of W_2, '1' is at position 2, 3. Hence, $p_1 = 2$, $p_2 = 3$. In row 3 of W_2 '1' is at position 2, 3. Hence $q_1 = 2$, $q_2 = 3$. ∴ to obtain W_3, we put '1' at the position:
$\{(p_1, q_1), (p_1, q_2), (p_2, q_1), (p_2, q_2)$
$= (2,2), (2,3), (3,2), (3,3)\}$.

Thus, $W_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

For $k=4$. In column 4 of W_3, '1' is at position 1, 4. Hence $p_1 = 1$, $p_2 = 4$. In row 4 of W_3 '1' is at position 1, 4.

Hence $q_1 = 1$, $q_2 = 4$. ∴ to obtain W_3, we put '1' at the position $\{(p_1, q_1), (p_1, q_2), (p_2, q_1), (p_2, q_2)$
$= (1,1), (1,4), (4,1), (4,4)\}$.

Thus, $W_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Thus, the transitive closure of R is given as

$R = \{(1,1), (1,4), (2,2), (2,3), (3,2), (3,3), (4,1), (4,4)\}$

b) Let I be the set of all integers. Let R be a relation on a set I, defined as "xRy iff $x-y$ is divisible by 4". Then prove that R is an equivalence relation on I.

Ans: Let $R = \{(x, y) : a, b \in I$ and $(x-y)$ is divisible by $4\}$. Show that R is an equivalence relation on I.
Given:
$R = \{(x, y) : x, y \in I$ & $(x-y)$ is divisible by $4\}$.
$R = (x, y)$
$(x-y)$ is divisible by 4.

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Reflexive

$(x, x) \Longrightarrow (x - x)$ is divisible by 4.

Symmetric

$(x, y) \Leftrightarrow (y, x)$

$(x - y)$ is divisible by 4.

$(y - x)$ is divisible by 4.

Transitive

$(x, y), (y, z) \Leftrightarrow (x, z)$

$(x - y)$ is divisible by 4

$(y - z)$ is divisible by 4.

$(x - z)$ is divisible by 4.

R is an equivalent relation on $\underline{I}$.