Assignment -1

Q1. Fill in the blanks

a) A contingency is a proposition that is neither a tautology nor a contradiction.

b) A compound proposition that is always false is called a contradiction.

c) Obtain DNF of $p \wedge (p \rightarrow q)$
~~$p \wedge (q \leftrightarrow p)$~~. $(p \wedge \neg p) \vee (p \wedge q)$

d) The universal quantification of $P(x)$ is denoted by $\forall x P(x)$.

e) Negation of Quantified statement $\exists x\, p(x) = \forall x\, q(x)$.

Q2 Choose Correct Options

a) Let $P(x)$ denote the statement "$x > 7$". Which of these have truth value true?

Ans: d) $P(9)$

b) Let P : I am in Delhi; then $q \wedge \neg(p$ and $q)$ is:

Ans: a) Delhi is clean and I am in Delhi.

c) If A is any statement, then which of the following is not a contradiction?

Ans: b) $A \vee F$

d) Determine the truth value of $\forall n\,(n+1 > n)$ if the domain consists of all real numbers.

Ans: a) True

e) The premises $(p \wedge q) \vee r$ and $r \Rightarrow s$ implies which of the conclusion?

Ans: b) $p \vee s$

Q3) State whether the following statements are true or false (give reasons).

Ans: a) False. $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$ argument is 'Hypothetical syllogism'.

b) ~~True~~ False. There is a particular domain.

c) True

PAGE No.

DATE / /

Q4. Name the following or define or design the following

a) Prove $(A \vee B) \wedge [(-A) \wedge (-B)]$ is a contradiction.

Sol^n:

A	B	$A \vee B$	$-A \vee -B$	$(A \vee B) \wedge [-A \wedge -B]$
0	0	0	0	0
0	1	1	1	1
1	0	1	1	1
1	1	1	1	1

b) Find the negation of the proposition

$p : -5 < x < \cancel{x<} = 0.$

Ans: $5 > x$ $\quad$ $x \geq 0$

c) Express the statements using quantifiers : "Every student in your school has a computer or has a friend who has a computer".

Ans: $\forall x \, ((C(y) \wedge F(x, y)))$

Q5) Answer the following questions in brief (20 to 30 words)

a) Prove by mathematical Induction $3^n < n^2$ for $n=1$, $n=2$.

Basic Step: Assume true for $n = k$ $(k = 1, 2)$

$3^k \geq k^3$

Prove true for $n = k+1$

$3^{k+1} \geq (k+1)^3$

PAGE No.
DATE / /

Consider:

$\cancel{2}\, 3^{k+1} - (k+1)^3 = \cancel{3}\,3 \cdot 3^k - (k^3 + 3k^2 + 3k + 1)$

$= 3^k + 3^k - (k^3 + 3k^2 + 3k + 1)$

By assumption,

$3^{k+1} - (k+1)^2 \geq 3^k + k^3 - (k^3 + 3k^2 + 3k + 1)$

$\geq 3^k - (3k^2 + 3k + 1)$

But $3^k \geq 3k + 1$

$3^{k+1} - (k+1)^3 \geq 0$

$3^{k+1} \geq (k+1)^2$

∴ statement is true for $n = k+1$

b) Let P: You are good in Mathematics.
Q: You are good in Logic.

Then write converse, inverse, contrapositive.

Solⁿ Converse: If You are good in logic then You are good in Mathematics.

PAGE No.
DATE / /

Inverse - If you are not good in Mathematics then you are not good in Logic.

Contrapositive - If you are not good in Logic then you are not good in Mathematics.

c) Construct the truth table for the statement $(p \to q) \equiv (\sim q \to \sim p)$. It is a tautology.

Solⁿ

p	q	$(p \to q)$	$\sim p$	$\sim q$	$\sim q \to \sim p$
0	0	1	1	1	1
0	1	1	1	0	0
1	0	0	0	1	1
1	1	1	0	0	1

Q6) Answer the following questions in brief (50 to 70 words)

a) Show that following conditional statement is a tautology without using truth tables.
$(p \wedge q) \to (p \to q)$

Ans: $-(p \wedge q) \vee (p \to q)$

$= -(p \vee q) \vee (-p \vee q)$
$= (p \vee p) \quad \vee (q \vee q)$
$= T \quad \vee T$
$= T$

b) Explain all rules of inference with examples

Ans: 1. Modus Ponens:

It is one of the most important rules of inference and it states that if P and $P \rightarrow Q$ is true, then we can infer that Q will be true. It can be represented as:

Notation for Modus ponens: $\frac{P \rightarrow Q, P}{\therefore Q}$

Eg: Statement 1: "If I am sleepy then I go to bed" $\Rightarrow P \rightarrow Q$
Statement 2: "I am sleepy" $\Rightarrow P$
Conclusion: "I go to bed." $\Rightarrow Q$

Hence, we can say that, if $P \rightarrow Q$ is true & P is true then Q will be true.

2. Modus Tollens:

The Modus Tollens rule state that if $P \rightarrow Q$ is true and $-Q$ is true, then $-P$ will also be true.

Notation: $\frac{P \rightarrow Q, \sim Q}{\sim P}$

S1: "If I am sleepy then I go to bed" $\Rightarrow P \rightarrow Q$
S2: "I do not go to the bed" $\Rightarrow \sim Q$
S3: Which infers that "I am not sleepy" $\Rightarrow \sim P$

3. Hypothetical Syllogism:

It states that if $P \rightarrow R$ is true whenever $P \rightarrow Q$ is true, and $Q \rightarrow R$ is true. ~~It can be represented as the fo~~

Eg: S1: If you have my home key then you can unlock my home. $P \rightarrow Q$
S2: If you can unlock my home then you can take my money. $Q \rightarrow R$
Conclusion: If you have my home key then you can take my money $P \rightarrow R$

PAGE No. DATE / /

4) Disjunctive Syllogism

It states that if $P \vee Q$ is true and $-P$ is true then Q will be true.

Notation: $\dfrac{P \vee Q, \ -P}{Q}$

Eg: S1: Today is Sunday or Monday $\Longrightarrow P \vee Q$

S2: Today is not Sunday. $\Longrightarrow -P$

Conclusion: Today is Monday $\Longrightarrow Q$

5) Addition:

It is one of the common interface rule, and it states that if P is true, then $P \vee Q$ will be true.

$\dfrac{P}{P \vee Q}$

Eg: S1: I have a vanilla ice-cream $\Longrightarrow P$

S2: I have chocolate ice-cream

Conclusion: I have vanilla or chocolate ice-cream $\Longrightarrow P \vee Q$

6. Simplification

If $P \wedge Q$ is true, then Q or P will also be true.

Notation: $\dfrac{P \wedge Q}{Q}$ or $\dfrac{P \wedge Q}{P}$

7. Resolution:

If $P \vee Q$ and $-P \wedge R$ is true, then $Q \vee R$ will also be true.

It can be represented as

$\dfrac{P \vee Q, \ -P \wedge R}{Q \vee R}$

c) Find the conjunctive normal form of the proposition $(p \wedge -q) \vee r$.

Sol^n: $(p \vee -q) \wedge (-q \vee r)$

Q7) Think and Answer

a) Write a logically equivalent statement form for "$(q \Leftrightarrow p) \wedge (\sim p \rightarrow q)$" without using "$\rightarrow$" or "$\Leftrightarrow$". Then find a logically equivalent statement to your answer that is as simple as possible. Back up your statements with authorities.

Sol^n: If it is sunny, it is hot and if it is not sunny it is hot.

b) Use mathematical induction to prove that $1+2+3+\dots+n = n(n+1)/2$ for all positive integers n.

Soln: S1) Let $P(n): 1+2+3+\dots+n = \frac{n(n+1)}{2}$

S2) Prove for $n=1$

for n = 1,

LHS = 1

$RHS = \frac{n(n+1)}{2} = \frac{1(1+1)}{2} = \frac{1 \times 2}{2} = 1$

LHS = RHS

$\therefore P(n)$ is true for $n=1$.

S3) Assume $P(k)$ to be true & then prove $P(k+1)$ is true)

Let $P(k): 1+2+3+\dots+k = \frac{k(k+1)}{2}$

be true.

We will prove $P(k+1)$ is true

① — $1+2+3+\dots+(k+1) = \frac{(k+1)(k+1)+1}{2}$

② — $1+2+3+\dots+k+(k+1) = \frac{(k+1)(k+2)}{2}$

From ①,

$1+2+3+\dots+k = \frac{k(k+1)}{2}$

Adding $k+1$ both sides

$1+2+3+\dots+k+(k+1) = \frac{k(k+1)}{2} + (k+1)$

$1+2+3+\dots+k+(k+1) = \frac{k(k+1)+2(k+1)}{2}$

$1+2+3+\dots+k+(k+1) = \frac{(k+1)(k+2)}{2}$

which is the same as $P(k+1)$

$\therefore P(k+1)$ is true when $P(k)$ is true.

Q8) My Ideas

a) q = "My friends are free"

p = "I get my Christmas bonus"

r = "I will take a road trip with my friends"

s = "My friends will find a job after Christmas"

Premises:

$(p \land q) \rightarrow r$

$\neg s \rightarrow q$

$\left.\begin{matrix} p \\ s \end{matrix}\right\} p \land s$

Conclusion: r

1) $\neg s \rightarrow q$ Premises.

2) $\neg s$ Premises

3) q modus Ponens (using 1,2)

4) p Premises

PAGE No.
DATE / /

5) $p \wedge q$ Conjunction
6) $(p \wedge q) \rightarrow$ Premises
7) r Modus Ponens (using 5, 6)

b) We introduced the set U of all surveyed people anf its ~~subtitles~~ subsets N, T, F who reads news week, time, fortune.
the condition of problem can be written as

$n(U) = 60$
$n(N) = 25$
$n(T) = 26$
$n(F) = 25$
$n(N \cap F) = 9$
$n(N \cap T) = 11$
$n(F \cap T) = 8$

(i) $n(N \cap T \cap F) = 8$

(ii) $n(N \cup T \cup F) = n(N) + n(T) + n(F) - n(N \cap F) - n(N \cap T) - n(F \cap T) + n(N \cap T \cap F) \quad - (1)$

$n(N \cap T \cap F) + n(N \cup T \cup F) = 60$
$n(N \cup T \cup F) = 60 - 8 = 52$

$n(N \cap T \cap F) = 4$

iii) Exactly one magazine $= 9 + 11 + 12$
$= 32$.