Problem Set 2
6.S091: Causality
IAP 2023

Problem 1: Identifying direction by non-Gaussianity [2 points]

(a) $\widehat{\beta}_{12} \approx 2.0033$. See Figure 1.

(b) $\widehat{\beta}_{12} \approx 0.4854$. See Figure 1.

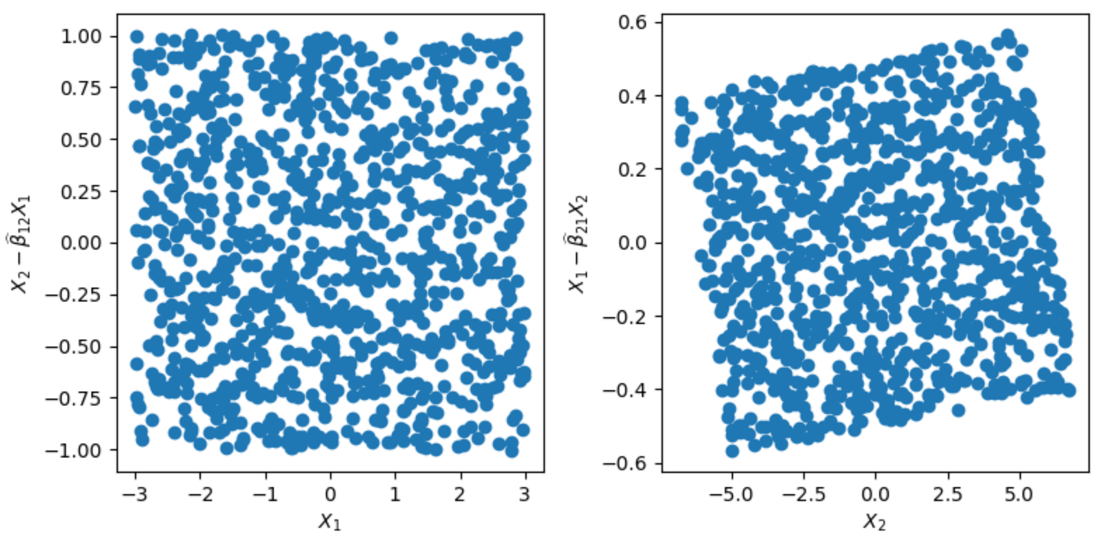

Figure 1: Residuals

(c) The data is more likely to be generated from M_a. If M_a is the true model, then $X_2 - \widehat{\beta}_{21}X_1$ is an empirical version of ε_2. The left plot reflects that $\varepsilon_2 \perp\!\!\!\perp X_1$.

On the other hand, if M_b is the true model, then $X_1 - \widehat{\beta}_{12}X_2$ is an empirical version of ε_1. However, we see in the right plot that ε_1 and X_2 are not independent. For example, when $X_2 = -5.0$, the support of ε_1 is $[-0.6, 0.4]$, whereas when $X_2 = +5.0$, the support of ε_1 if $[-0.4, 0.6]$. Thus, we would reject a hypothesis that M_b is the correct model. *Note: $X_1 - \widehat{\beta}_{12}X_2$ and X_2 are **uncorrelated** by the definition of the least-squares regression coefficient. Although there appears to be a positive slope, this is cancelled by the positive values of the residual when $X_2 < -5.0$ and the negative values of the residuals when $X_1 > +5.0$. Thus, you should say that the residuals and X_2 are **dependent**, not that they are **uncorrelated** or **not orthogonal**.*

Problem 2: The PC Algorithm [8 points]

(a)

$$\widehat{\rho}(X_1, X_4, \varnothing) \approx 0.18515$$

(b)

$$\widehat{\rho}(X_1, X_4, \{X_2, X_3\}) \approx 0.00933$$

Fisher's z-transformation [1 point]

(c)

$$\widehat{z}(X_1, X_4, \{X_2, X_3\}) \approx 0.9332$$

p-values [1 points]

(d)

`compute_pvalue(pcalg_samples, 1, 4, [2, 3])` ≈ 0.3507

Skeleton phase [2 points]

(e) When $\alpha = 0.2$, the estimated skeleton has **11** edges.

(f) When $\alpha = 0.001$, the estimated skeleton has **9** edges.

Orientation phase [2 points]

(g) The only unshielded collider is $X_1 \to X_3 \leftarrow X_2$

(h) By Meek rule 1, we can orient $X_3 \to X_k$ for $k \in \mathcal{K} := \{4, 5, 6, 7\}$, since $X_1 \to X_3$ but X_1 is not adjacent to any X_k for $k \in \mathcal{K}$. The remaining unoriented edges cannot be oriented as none of the Meek rules apply.