Implementation of a Multi-Band Equalizer using Basic RC Filters to Process Modified Audio Signals

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***Abstract*—Signal processing is an extremely relevant study, and so a method of filtering using different techniques in parallel by creating a signal equalizer was studied. To this end, Fourier analysis techniques through Fourier transforms at intervals of time using spectrograms and using an overall Fourier transform were employed. It was found signal processing can be dependent on the signal to be processed, and that overlapping frequencies and/or large differences in relative signal strength can make isolation of frequency bands or tuning of certain frequencies over others more difficult, and that presets and Fourier analysis techniques made the actual processing of signals a much more intuitive process.**

I. Background

Enhancing an audio signal has many different meanings, and what it means to enhance an audio signal varies case by case. To this end, a generalizable method of enhancing a signal by isolating and either dampening or enhancing certain bands of frequency is desirable, so as to abstract away some of the specifics of signal enhancement, and reduce the problem to a range of frequencies to enhance or reduce.

To this end, an equalizer made up of frequency selective filters was the chosen method of processing audio signals. By allowing filters to filter over a specific, modifiable range with a modifiable multiplicative gain, it became possible to alter any range of frequencies. A combination of lowpass and highpass filters were employed to create each filter, and the ranges for each filter were selected through Fourier analysis.

II. Methods

A. Equalizer Design

Although the human hearing spectrum ranges between 20 Hz and 20 kHz, much of the audio we encounter in day-to-day life often lies in the lower areas of this spectrum, and thus the frequency-selective filters employed within the equalizer were designed with this in mind. A generally simple approach was taken within these constraints, with each type of filter, lowpass, highpass, and bandpass, being implemented using variations of the rudimentary RC circuit. The RC circuit with resistive load has an impulse response as follows:

$$h(t) = \frac{1}{f_c} e^{-\frac{1}{f_c}t} u(t)$$

where f_c is the cutoff frequency and is equivalent to $\frac{1}{RC}$ with R, the equivalent resistance, and C, the equivalent capacitance. It's nature as a rapidly decaying exponential made it effective in selectively passing lower frequencies. The RC circuit with capacitative load has a similar impulse response but flipped vertically:

$$h(t) = \delta(t) - \frac{1}{f_c} e^{-\frac{1}{f_c}t} u(t)$$

allowing lower frequencies instead to be attenuated while isolating the higher frequencies. By applying the two RC filters in succession, what is equivalent to convolution of the two impulse responses, an impulse response as follows results:

$$h(t) = \left(\frac{\frac{1}{f_{c1}}^{2}}{\frac{1}{f_{c1}} - \frac{1}{f_{c2}}} e^{-\frac{1}{f_{c1}}t} - \frac{\frac{1}{f_{c1} f_{c2}}}{\frac{1}{f_{c1}} - \frac{1}{f_{c2}}} e^{-\frac{1}{f_{c2}}t} \right) u(t)$$

isolating a range of frequencies, specifically between the two cutoff frequencies, f_{c1} for the low pass and f_{c2} for the high pass, to be passed and manipulated. Utilizing the three filtering methods, the equalizer (Fig. 1) was constructed. The original input signal was processed in parallel through the six filters before applying a variable amplification to each result based on the aim, bass-boosting, treble-boosting, etc. By then recombining these individual, filtered signals, an equalized output was obtained. Specifically, gains were applied by multiplying each filtered signal channel by a static $\alpha = 10^{G/20}$ where G is the desired gain for the specific frequency range in decibels.

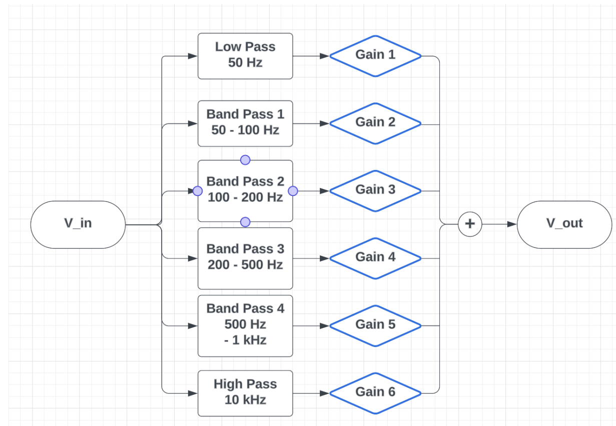

Fig. 1. Equalization process for signals with specification of individual filter frequency ranges

B. Custom Audio Presets

With this equalizer design, we were then able to design specific presets that could be used to modify any arbitrary signal. Firstly, an analysis of the simplest filter, the Unity filter. Its purpose is to preserve the state of the signal from before it was passed through the system, so the ideal frequency response would approximate some constant, such that a gain would be applied to even the signal out to its original state. By first applying zero decibel gain to each band and analyzing its effect on the output signal with the respect to the original, it was found that, due to the emphasis put on the lower frequencies, these ranges experienced more greatly the effect of filtering and had diminished magnitude while the higher frequency tones remained relatively unchanged after filtering. Thus in order to offset this imbalance, a positive gain was needed to counteract the loss in magnitude of the lower frequencies while a negative gain was needed to create an overall constant frequency response as desired.

Next came the bass- and treble-boosting presets. As the frequency ranges used in the design isolated more clearly the lower frequencies, the bass-boosting preset was designed to apply positive gains over the lowest frequency band filters while keeping the higher frequencies relatively constant by applying zero or a slightly negative gain so the lower frequencies' magnification was more clear. It was found that treble boosting was slightly more difficult as less control was left over the higher frequencies due to the emphasis put on the lower ones, however, the same principle applied: the signal through the highpass filter was greatly amplified while the magnitude of the other other filtered parallel signals was diminished in order to more clearly reflect the treble-boosting in the combined output.

C. Removal of Noise

While boosting and unity filters dealt with wide ranges of frequencies the removal of specific noise and sounds within an audio signal required more specific ranges to be isolated and magnified or weakened. In order to clearly isolate the noise and its frequency range throughout a signal, spectrograms were used to visually identify the bands requiring modification. In cases where this noise oscillated, yielding a wide range to be modified, or the noise to be filtered was relatively similar in frequency to the frequencies to be passed, full removal through the basic equalizer employed in this case study was shown to be relatively difficult. Thus an ideal case was also explored where an approach of equalizing the audio multiple times was used in order to create a greater rift between the wanted and unwanted frequency ranges and tones, so that filtering could work much more successfully.

III. Results

A. Total Frequency Responses

To assess the performance of the equalizer, the frequency and impulse responses of the individual filters as well as the total summed responses of the equalizer system for the three presets were examined. To begin, an analysis of the summed frequency responses would allow for analysis of individual frequency responses as well as impulse responses.

The Unity filter in the equalizer shown in Fig. 2 has about an equal spread of steady state values, with none approaching

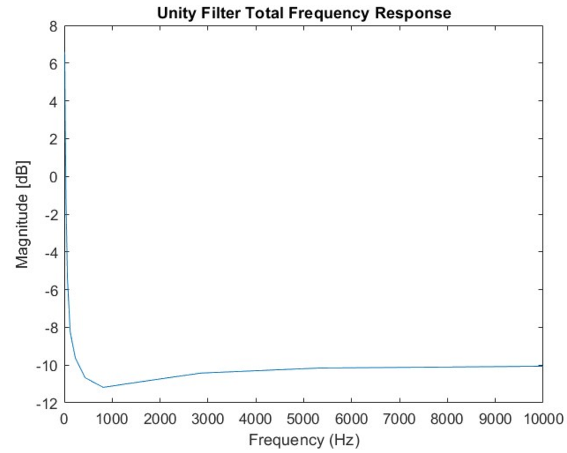

Fig. 2. Frequency response of overall system for Unity preset. Note the system quickly stabilizes and keeps an approximately constant value for relevant frequencies.

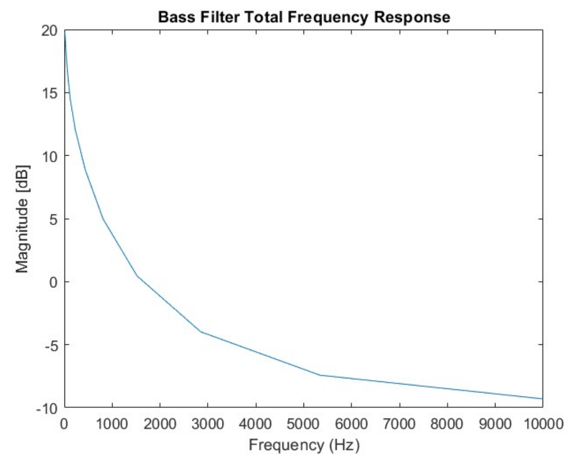

Fig. 3. Frequency response of overall system for Bass preset. Note the magnitude decreases for higher frequencies, while maintaining a positive magnitude for lower frequencies.

zero, so a constant gain accounts for the lack of magnitude and the frequencies are largely preserved.

As for the bass boosting preset, its frequency response's magnitude is above zero for all frequencies below 1800 Hz, and below zero for all other frequencies, which is characteristic of a bass booster. The treble preset, essentially the bass booster's counterpart, is almost exactly the opposite, and has a magnitude below zero for frequencies below about 800 Hz. These frequency responses are seen in Fig. 3 and 4 for the bass and treble presets, respectively.

B. Impulse and Individual Responses

Analyzing the frequency response of each filter individually, each of the three presets approach a steady state, and the unity preset approaches its steady state (SS) quickly, while the bass preset takes longer and settles at a largely negative magnitude, and the treble preset begins with a large band of varying values, the sum of which increases over time.

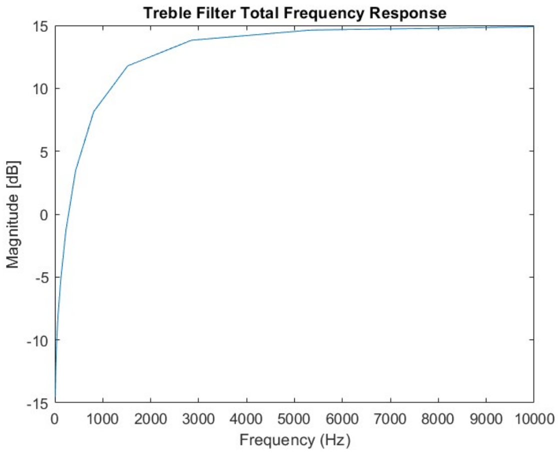

Fig. 4. Frequency response of overall system for Treble preset. Note the magnitude decreases for lower frequencies, while maintaining a positive magnitude for higher frequencies.

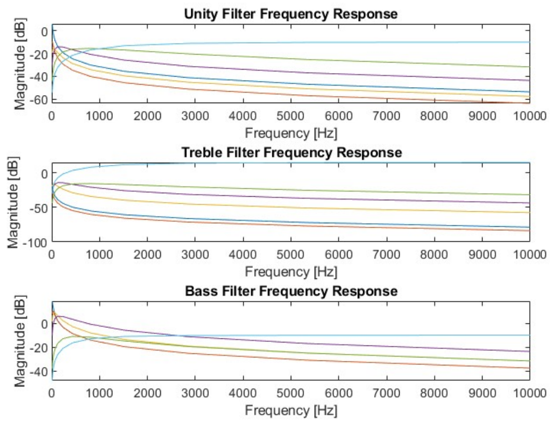

Fig. 6. Frequency responses of each individual band of the equalizer, for each of the three presets.

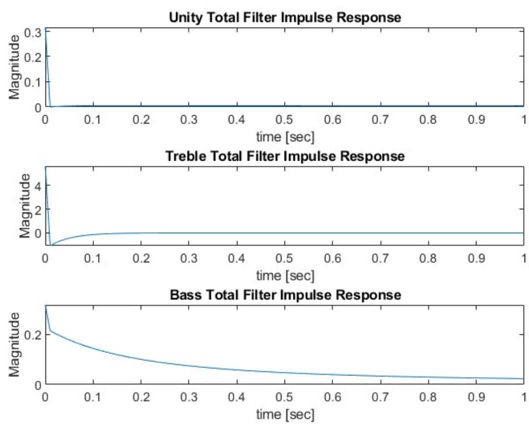

Fig. 5. Impulse response of overall system for Unity, Bass, and Treble presets. Note the system quickly stabilizes and keeps a consistently signed derivative in its steady state.

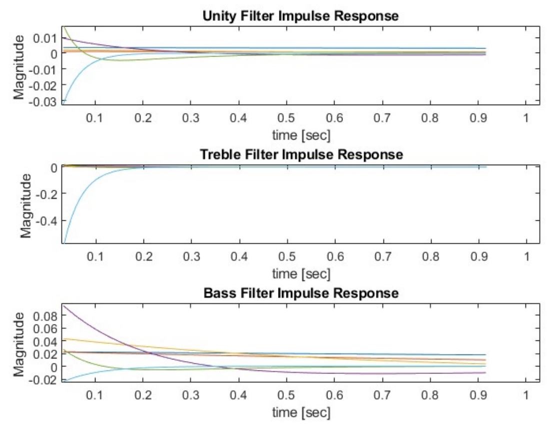

Fig. 7. Impulse responses of each individual band of the equalizer, for each of the three presets.

The impulse responses may be obtained through the Fourier Transforms of their frequency counterparts. Each of the frequency responses is some version of the lowpass and highpass filter frequency responses, which is some graph mimicking the shape of $1/x$, as the bandpasses' frequency response is just the multiplication of the lowpass and highpass frequency responses in the frequency domain, and, due to the linearity of the Fourier Transform, the total system frequency response becomes the sum of the individual frequency responses. This means the total frequency response will approximate f(x) where:

$$f(x) = \frac{\sum_{k=0}^{m} a_k x^k}{\sum_{i=0}^{N} b_i x^i},\ N \geq M \tag{1}$$

Alternatively, the summation can be generalized using partial fraction decomposition to some summation of exponentials in the time domain, as (Note f(x) is as defined above):

$$f(x) \xrightarrow{\mathscr{F}} \sum_{j=0}^{V} c_j e^{t-t_j} \tag{2}$$

This explains the apparent similarity of the impulse response to its Fourier transform (the frequency response), and analyzing the decomposed individual responses, each impulse response can be represented as its Fourier transform, where each individual filter's frequency response can be attributed entirely to a single impulse response of one of the filters.

C. Bass and Treble Applications

Applying the bass and treble presets to audio signals, the results are seen in for the bass and treble presets in Fig. 8 and 9, respectively. The results are shown using a spectrogram for the bass booster and a fft for the treble booster, so as to show both ways of interpreting results used in the study. Finding values for gains which best bass boost and treble boost a signal were performed on a case to case basis, as different signals

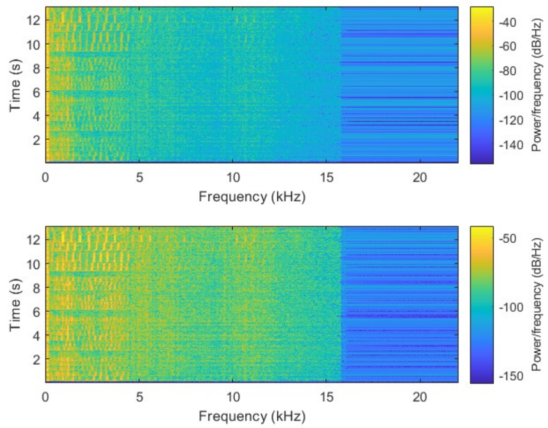

Fig. 8. A spectrogram of a song being bass boosted, the lower frequencies are increased in magnitude while the higher ones are decreased.

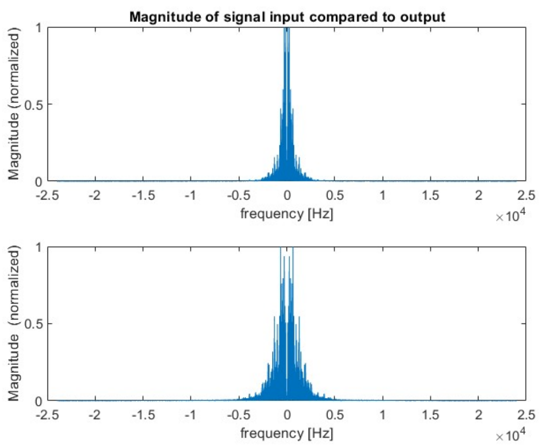

Fig. 9. An fft of a song being treble boosted, the higher frequencies are increased in magnitude while the lower ones are decreased. Note the graph is normalized by the maximum value in the graph.

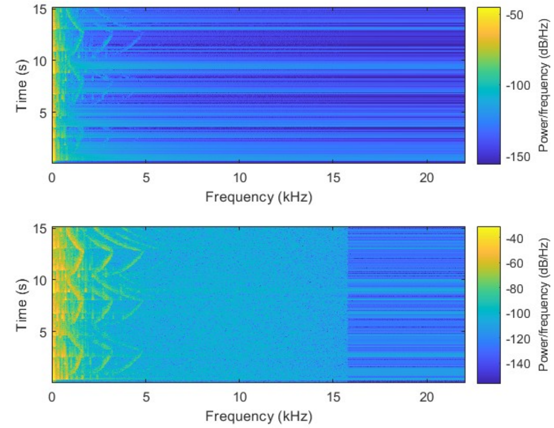

Fig. 10. The audio file after and before it was processed. Note the above graph is missing upper bands of the siren noise, while dampening the noise that remains.

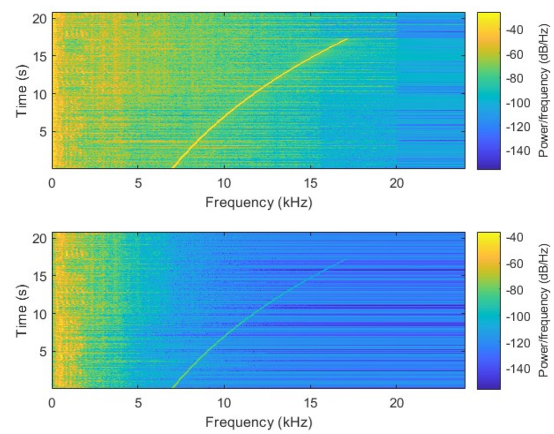

Fig. 11. The audio file before and after it was processed. Note the above graph is missing higher frequency tones, but the band from 5kHz to 10kHz sees the most drastic drop in magnitude.

can have drastically different ranges for what is considered high and low frequency.

D. Filtering Noise from a Signal

The goal of this section was to remove an unwanted siren from a piano recording, but as the siren and piano are similar and overlap in frequency (as seen in Fig. 10), removing the siren entirely was impossible. In Fig. 10, the bottom graph is the signal before filtering, and the triangular shape an its accompanying higher frequency bands were the siren, while the piano sound was the lower frequency yellow stripes at below 1 kHz. The graph above shows the filtered graph, and while losing some higher frequencies in the piano, the siren became much quieter, becoming impossible to hear at the beginning of the audio file, while still leaving the piano mostly intact using a gain.

E. Filtering Oscillating Frequencies from a Signal

To further the study of signal processing and specifically noise filtering, a song was chosen, and an oscillating frequency signal was superposed on top of the song. The graph representing the original signal with the high frequency signal can be seen on the top plot in Fig. 11, where the high frequency signal is the strong yellow logarithmic plot from 6-7 kHz to 16-17kHz. This interloping signal makes up much of the higher frequency ranges still within the audible frequency spectrum for humans.

To filter this audio, several iterations of the Equalizer were needed, and after applying a gain, the graph as it is shown in Fig. 11 illustrates the frequency content at each second of the chosen piece of the song. The high frequency pitch is noticeably quieter, and because higher frequencies are more difficult to hear, this setup of sequential bandreject filters allowed for the filtering of most of the higher frequency content in the signal. A much larger amount of bandreject filters could have been theoretically used to deepen the difference between the lower frequency song and intervening signal, but it would also mean applying gains at or above 100 dB, which was considered to be too much given the context.

IV. Conclusion

Conducting the project resulted in several findings, including the creation of an equalizer with presets to conserve audio through the filter, boost lower frequencies while diminishing higher ones, and vice versa. This idea of reducing signal processing to three subtypes of problems is extremely generalizable and has many applications to fields outside of audio, such as filtering high frequency tones out of stock trading data and comparing that data to the same data with the lower frequency tones filtered out, showing the difference in trading habits for high frequency versus low frequency traders. While this method of filtering can be effective, there do exist better, and more efficient, methods of filtering. Using Fourier analysis, however, was the largest contributor to the success of this study, as its usage in identifying the problem frequencies and seeing the efficacy of any given equalizer was invaluable in filtering signals an interpreting the results.