Simulation of the L 98-59 System

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1. Introduction

One of the greatest challenges for astronomers has been the discovery of exoplanets. Viewing them directly with a telescope is no easy feat considering the glare of the star about which they orbit. A recent breakthrough has made this detection of exoplanets much easier through the analysis of the change of the light emitted by stars when an exoplanet passes between the star and the detector, a process spearheaded by the data collected by the Transient Exoplanet Survey Satellite (TESS). To begin, I will be illustrating their method in determining an exoplanet's size and orbital period based on this change in light flux using the built-in functions from Astropy. In the second part, I will find further conditions including the orbital radii and velocities of these exoplanets through implementation of Kepler's third law from which I can simulate the trajectory of the planets within the L 98-59 system.

2. Methods

2a. Analyzing the Data

To begin, I uploaded the data from TESS for L 98-59. I then read the data into variables that I could work with including time, flux, the flux error, and data quality. Data with bad quality was then filtered out. The data is now ready to be plotted. Next, using the BoxLeastSquares toolkit, I analyzed the data which gave me important information on the period, the power, and the transit time. Once the information for the first exoplanet is extracted, we can then search for other exoplanets using the transit_mask method to clear off the main signal from the original data. Transit_mask uses parameters time, period, duration, and transit time to identify this main signal. This process may be repeated until we are unable to identify any more exoplanets.

2b. Determining the initial conditions

We can then move to determining the initial conditions that we will use in 2c. First, we need to find the orbital radii of the identified exoplanets. We can relate the period of the planet with its orbital radius using Kepler's third law:

$$p^2 = \frac{4\pi^2}{GM_{star}} a^3, \quad (1)$$

where p is the period, G is the gravitational constant, and a is the average distance between the star and planet or orbital radius. Rewriting this equation in terms of a yields:

$$a = \sqrt[3]{\frac{GM_{star} p^2}{4\pi^2}} \quad (2)$$

Given the orbital radius and the period, we are also able to find the magnitude of the velocity of the planets using the following equation:

$$v = \frac{2\pi a}{p} \quad (3)$$

We now have found all the necessary values to simulate the L 98-59 system.

2c. Simulating the L 95-58 system with three planets

Simulation of the L 95-58 system requires that I find the trajectory of the planets. Neglecting the attractive and repellent forces that exist between the planets themselves, we can use Newton's second law to say:

$$M_{planet} \frac{d^2\vec{r}}{dt^2} = -\left(\frac{GM_{star}M_{planet}}{r^2}\right)\frac{\vec{r}}{r} \quad (4)$$

where r is the position and $\frac{d^2\vec{r}}{dt^2}$ can be interpreted as the acceleration of the planet. Canceling the M_{planet} on both sides of the equations and separating the equation into its x and y components (we assume the planets do not move in the z-axis) where $r = \sqrt{x^2 + y^2}$, yields the following two second-order differential equations:

$$\frac{d^2x}{dt^2} = -GM_{star}\frac{x}{r^3} \qquad \frac{d^2y}{dt^2} = -GM_{star}\frac{y}{r^3}$$

(5,6)

I then used both the Euler method and the fourth-order Runge-Kutta method to solve these equations to find the trajectory.

Using the Euler method, we can determine the value of the velocity at some time $t + h$ according to equation 7.

$$v_{x,y}(t + h) = \frac{d(x,y)}{d(t+h)} = \frac{d(x,y)}{dt} + h\frac{d^2(x,y)}{dt^2}$$

(7)

Applying the method once more, we are able to find the trajectory in the form of its x- and y-components.

$$(x, y)(t + h) = (x, y)(t) + h\frac{d(x,y)}{dt}$$

(8)

Using the Runge-Kutta method, finding the velocity at the next time step is a bit more complicated:

$$\frac{d(x,y)}{d(t+h)} = \frac{d(x,y)}{dt} + \frac{1}{6}(k1 + 2k2 + 2k3 + k4)$$

(9)

where variables k1-k4 are defined by the following equations:

$$k1 = h\frac{df_{x,y}}{dt} \quad (10)$$

$$k2 = h\frac{df_{x+\frac{k1}{2},y+\frac{k1}{2}}}{d(t+\frac{h}{2})} \quad (11)$$

$$k3 = h\frac{df_{x+\frac{k2}{2},y+\frac{k2}{2}}}{d(t+\frac{h}{2})} \quad (12)$$

$$k4 = h\frac{df_{x+k3,y+k3}}{d(t+h)} \quad (13)$$

From here, we can determine the x and y positions using equation 8. For both cases, we need to know some initial values for the position and velocity. Fortunately, these values may be pulled from the previous section. We will start along the positive x-axis where the value for the x position is equal to the orbital radius. The initial velocity in the x direction will then be 0 as it has reached its maximum position in the x direction, and the initial velocity in the y direction may be pulled from the calculated velocity based on period and orbital radius. Plotting the y position against the x position over time will then generate a trajectory; repeat the process using the initial values for the other exoplanets.

3. Results and Discussion

3a. Analyzing the Data

Plotting the flux as a function of time for the original data, we can see clear periodic signals where the flux decreases quickly and significantly, occurring approximately every 3.7 days (Fig.1).

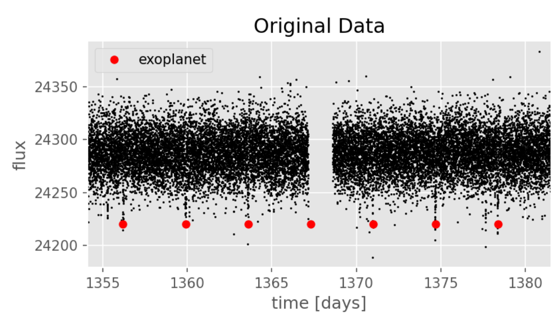


Figure 1: Light flux of original data plotted against the time in days with exoplanet's period identified

This indicates the existence of an exoplanet which revolves around the star in a periodic manner. I then plotted the power against the period using the values in variable "results" from the BoxLeastSquares method (Fig.2).

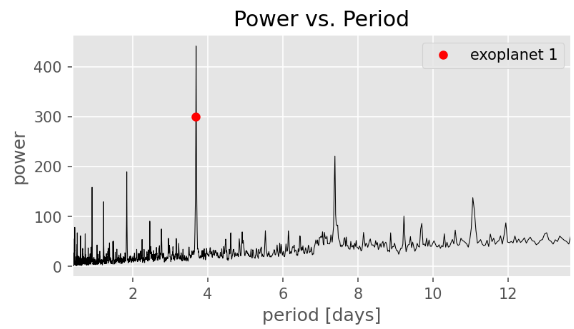


Figure 2: Graph plotting the power of the original data against the period with exoplanet 1's period identified

When power is plotted against the period, we can determine the period of the exoplanet as the value of the period when the power reaches a maximum point. This value can be extracted exactly using the "np.argmax" function on the variable "power" to find the index when the maximum power value is reached and indexing into "period," yielding the period of this exoplanet. In this case, the planet's period is found to be 3.6917 days with frequency 0.2709 days^-1. Plotting the power against the frequency is also useful as it allows us to visualize the harmonic and periodic fluctuation of the power (Fig.3).

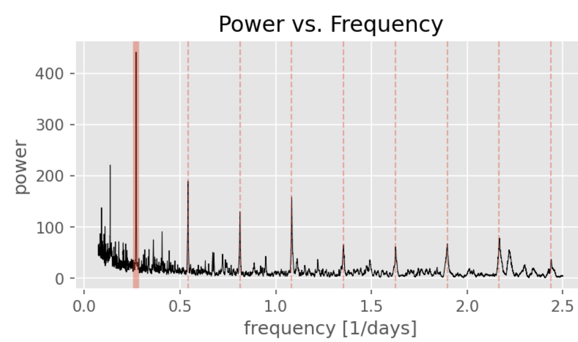


Figure 3: Graph plotting the power of the original data against frequency with harmonic peaks highlighted

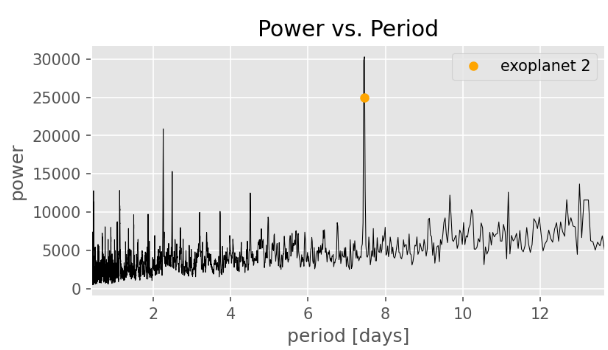


Figure 4: Graph plotting the power of the data, with the first exoplanet's signal removed, against the period with exoplanet 2's period identified

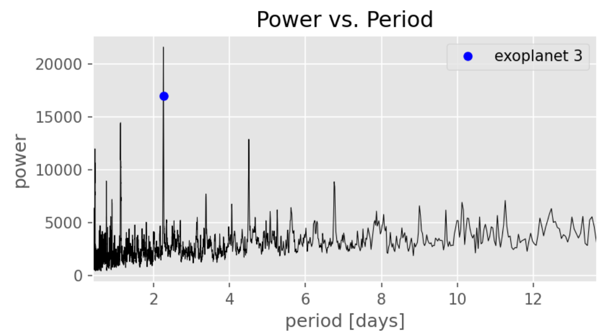


Figure 5: Graph plotting the power of the data, with the first and second exoplanet's signals removed, against the period with exoplanet 3's period identified

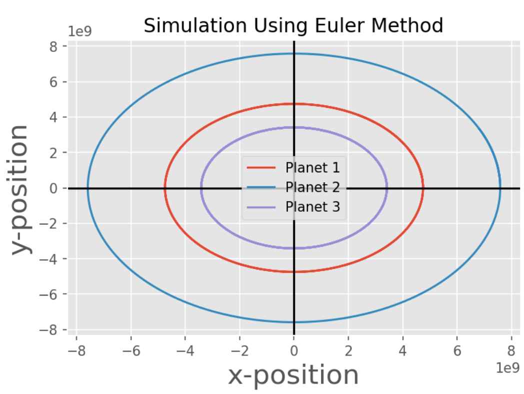
After implementing the transit_mask method, I then plotted the power vs. period for exoplanets 2 and 3 (Fig.4,5). The period of these exoplanets was found to be 7.4598 days and 2.2540 days respectively.

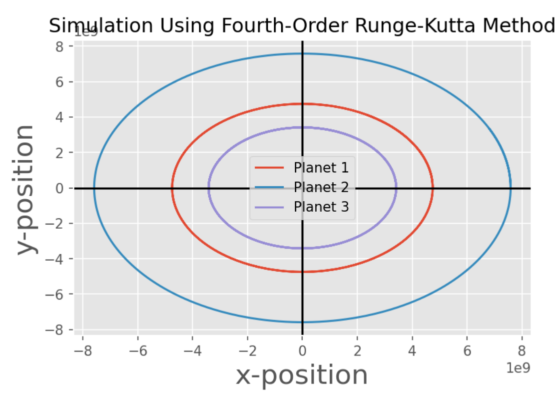
3b. Determining the initial conditions

Now that we have identified the periods of the three exoplanets we are able to find the initial conditions necessary for simulation. Implementing equation (2) in my code, I found that the orbital radii of the three exoplanets were 4.75×10^9 meters, 7.59×10^9 meters, and 3.42×10^9 meters respectively. The values for the mass of the star was pulled from a study by Kostov et al. [3]. Using these values with the known periods, I was also able to estimate the magnitude of the velocity of planet 1 (93,514 m/s), the magnitude of the velocity of planet 2 (73,968 m/s), and the magnitude of the velocity of planet 3 (110,230 m/s).

3c. Simulating the L 95-58 system with three planets

Using these values as constants and applying the equations in part 2c, I was able to produce the following two plots that simulate the trajectory of the three exoplanets with L 95-58 assumed to be at position (0,0) (Fig.6,7).





Figures 6 and 7: Simulated trajectories of the three exoplanets of L 95-58 using the Euler method and the fourth-order Runge-Kutta method

I plotted the trajectory over the time period of 8 days because the exoplanet with the greatest period was exoplanet 2 (7.5 days), and I wanted to show a full revolution of all planets. We can see that the exoplanets with shorter periods revolve closer to the star and move faster as well. Another important takeaway, although the fourth-order Runge-Kutta method has a much lower value of error than does the Euler method, because the numbers we are dealing with are so large, the trajectories do not seem to differ much based on the method used to calculate them. Taking a look into the actual x- and y-values, even after 100 time steps, the values only differed by less than a tenth of a millimeter. I was quite honestly surprised by the similar results produced by both methods. When accuracy is prioritized, I thus feel that there is still value in using the Runge-Kutta method as these small differences will only continue to grow over time. However, for short time periods, the Euler method works well enough.

References

[1] Brennan, Pat. *Cosmic Milestone: NASA Confirms 5000 Exoplanets*. JPL Article, March 2022.

[2] *TESS Exoplanet Mission*. The transient method.

[3] Kostov et al. *The L 95-58 System: The Three Transiting, Terrestrial-size Planets Orbiting a Nearby M Dwarf*. 2019 AJ 158 32.