# Modeling Drug Diffusion into Blood and Tissue and Interaction with Organs, as a Singular Input and as Periodic Inputs, Including a Control

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***Abstract*—The behavior of drugs as they diffuse into blood is examined here; first the diffusion of a drug when administered, depending on the method of administration, is modeled using pharmacokinetic models. For further analysis, intravenous administration is focused on to examine how the presence of tissue affects the dynamics at hand. In both cases, the drug concentration spikes up and decreases to approach its equilibrium concentration of zero, but the tissue impedes the spike. Regular dosing, rather than a single dose, is explored to see when doses interact versus when they don't, and how overdosing depends on decay of drug concentration. A possible control is then designed, and its effect on the dynamics is explored to show how it prevents overdose by minimizing interaction between doses, and flushing out a drug from the bloodstream so it doesn't compound like in the case of an unimpeded overdose.**

## I. Organ-Drug Interaction

The equations in this section model the rate at which the total drug concentration decreases as it is absorbed into the system. These are fairly simplified, and models with more parameters will be explored in further sections. For now, these models will simply serve as a mathematical groundwork for the later sections, as well as an introduction into the topic of pharmacokinetic models in terms of their intent, basic looks into their derivation, and the logic behind the modeling (i.e. assumptions made).

### A. Creation of Rudimentary Pharmacokinetic Models

These rudimentary models will serve to illustrate the effect of a single drug dose on a system as time goes on and involve the progress of the drug through parts of a system. To begin, we explore a basic oral drug administration model as outlined in Eq. 1 and 2, where $c_s$ is the concentration in the stomach, $c_b$ is the concentration in the bloodstream, and $k_i$ are constants.

$$\frac{dc_s}{dt} = -k_s c_s. \tag{1}$$

$$\frac{dc_b}{dt} = k_s c_s - k_e c_b. \tag{2}$$

To be noted here is the unilateral reliance of the drug diffusion rate in the blood upon the rate in the stomach.

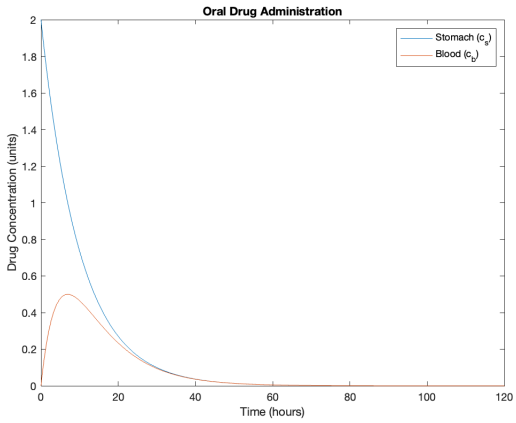


Fig. 1. Simulation of the diffusion of drug administered orally, where, $k_s = 0.1, k_e = 0.2$, and $C_0 = 2$.

Because Eq. 1 is independent of any other system and models the intake of the drug, the solution is dependent only on itself, and so the equation can be analytically solved

$$c_s(t) = C_0 e^{-k_s t} \tag{3}$$

where $C_0$ represents the initial concentration of the drug dosage. Eq. 2 can be modeled as an independent input into a system, and using the principle of superposition, it can be rewritten as

$$\dot{c_b} + k_e c_b = k_s c_s(t) = k_s(C_0 e^{-k_s t}) \tag{4}$$

using this logic, Eq. 2 can be solved

$$c_b(t) = \frac{C_0 k_s}{(k_s - k_e)}(e^{-k_e t} - e^{-k_s t}) \tag{5}$$

We see that the concentration of drug in the blood is consistently lesser than that of the stomach with the summation of their values at any $t$ being necessarily less than their sum at $t + 1$. These ideas represent some of the limitations of this model as it assumes a single input at $t = 0$ and then

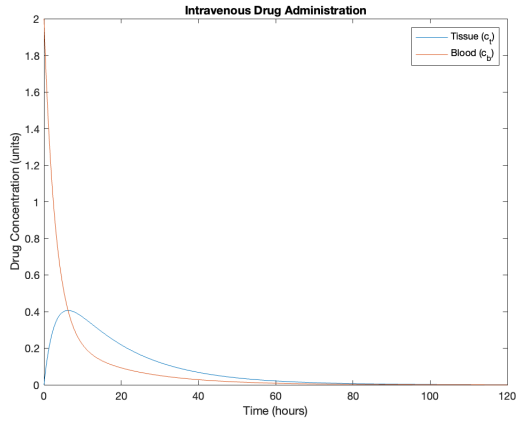


Fig. 2. Simulation of the diffusion of drug administered intravenously, where, $k_s = 0.1, k_b = 0.1, k_e = 0.2$, and $C_0 = 2$.

no other extraneous input. A more realistic model, as will be explored further in this paper, will seek to incorporate a periodic, extraneous input of one or multiple drugs. This model is extremely limited in its use by itself, which is why its pairing with Eq. 2 allows for more depth in the analysis of its behavior throughout a system. However, its usefulness cannot be denied in its generalization of the more complicated methods to come and insight into the nature of exponential functions. Drug administration using a system of two variables dependent upon the each other is seen as drug administration through a non-oral method. For the purposes of this paper, we have delineated the analysis of an intravenous drug administration method.

To model intravenous drug administration, Eq. 3 and 4 introduce a more complex model where $c_t$ is the concentration in organ tissue because the blood and tissue interact with each other.

$$\frac{dc_b}{dt} = -(k_b + k_e)c_b + k_t c_t. \tag{6}$$

$$\frac{dc_t}{dt} = k_b c_b - k_t c_t. \tag{7}$$

This idea is shown through how no equation is independent of any other, as the drug concentration in the blood is increasing as it transfers from the tissue to the bloodstream, and vice-versa. To solve these equations, we found the eigenvalues of the system

$$\begin{bmatrix} \dot{c_t} \\ \dot{c_b} \end{bmatrix} = \begin{bmatrix} -k_t & k_b \\ k_t & -(k_b + k_e) \end{bmatrix} \begin{bmatrix} c_t \\ c_b \end{bmatrix} \tag{8}$$

which were

$$(s_1, s_2) = -\frac{1}{2}[(k_b + k_e + k_t) \pm \sqrt{(k_b + k_e + k_t)^2 - 4k_e k_t}] \tag{9}$$

yielding the following general solutions with initial blood concentration $C_0$

$$c_t(t) = \frac{C_0 k_b}{s_1 - s_2}(e^{s_1 t} - e^{s_2 t}) \tag{10}$$

$$c_b(t) = \frac{C_0}{s_1 - s_2}(e^{s_1 t}(k_t + s_1) - e^{s_2 t}(k_t + s_2)) \tag{11}$$

We see a similar structure reflected in Figure 2 as in the first graph. This makes sense analytically as the solution to both drug diffusion models can be generalized as a sum of exponentials. The incorporation of a dependency between the rate of diffusion in the blood and in the tissue which was absent from the oral administration model can be verified graphically in that the concentration in the blood does not reach as great a maximum and that both concentrations take longer to stabilize at the equilibrium point of zero. The interaction of the drug concentration between the blood and the tissue cause less overall of the drug to reach the intended location of the tissue as the diffusion of the drug from the blood does not entirely and directly contribute to the rate of diffusion in the tissue as in the oral administration model. This interplay further prevents the drug from being eliminated at as quick a rate, hence the slower stabilization. Interestingly, we also see that this interdependence has introduced a critical point of 0.4 units of drug concentration at around 10 hours after administration. After this point, the tissue drug concentration remains greater than that of the blood. Analytically, this can be verified by setting both equations equal to each other and solving for time. On the other hand, we now see that oral administration could never have this intersection point because its dynamics lack the interplay that we saw in the intravenous model.

### *B. Applications*

These models represent simplified expressions of the progression of a drug through a system and can be used to rudimentarily explore biological interaction between a drug and a body. With the stomach-bloodstream model, the parameters can be set to be anything with certain logical caveats, such as $1 > k_e, k_x \geq 0$ and $C_0 \geq 0$, because a negative or $\geq 1$ factor of proportionality would allow for the derivative to be greater than zero or less than the total amount of drug, meaning the model could achieve a drug concentration greater than its original amount $C_0$ or move into the negative domain (i.e. $C_0 \leq c_s(t), c_b(t) > 0$). A $C_0 \leq 0$ would have a similar effect, as it would make the value of the derivative positive, not to mention the logical caveat, as it would be difficult to administer a negative amount of drug into a system.

These graphs with various values for each parameter tell of patterns in the model, for example, each graph in Eq. 5 exhibits patterns in how a positive exponential is overpowered by a negative exponential as $t \to \infty$, so $c_b(t) \to 0$ as $t \to \infty$. As the solution of Eq. 3 has one exponential as its only term, its behavior is just that of an exponential and varying its coefficient $C_0$ would only serve to scale it while varying $k_s$ would only change the rate it which it decreased.

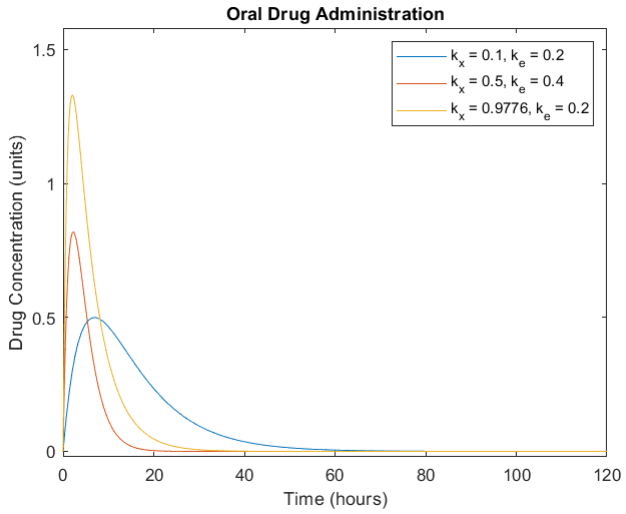


Fig. 3. Simulation of the diffusion of drug administered orally in the bloodstream, only plotting Eq. 5 of the pair Eq. 5 and 3, where the legend gives according values for each free parameter but $C_0$, which is always two.

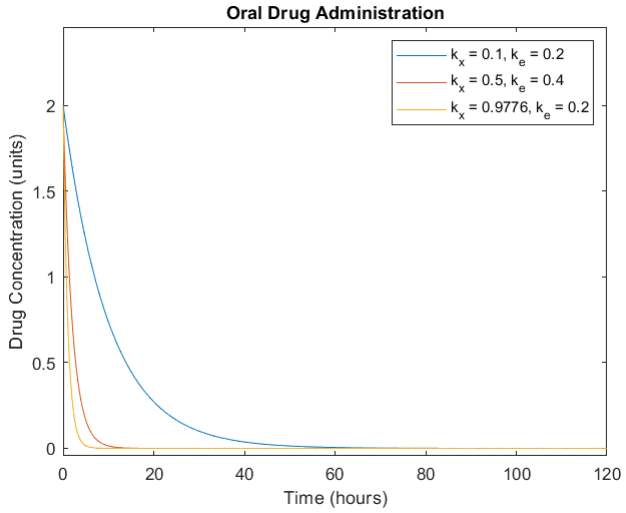


Fig. 4. Simulation of the diffusion of drug administered orally, this time plotting Eq. 3, where the legend gives according values for each free parameter but $C_0$, which is always two.

## II. Introduction of Period Inputs

A situation might involve periodic input, such as required medication. This model makes several assumptions, first of all that the patient takes the drugs at exact intervals, just after the drug's effect has petered out. This means both that the drug cannot be overdosed in this model, and that the patient, regardless of the interval, will always take the drug at one exact predetermined interval. The model also assumes that taking a drug while its effect is still active is additive, and can be modeled as such:

$$f(t) \approx \sum_{i=0}^{\infty} f(t - T * i) \tag{12}$$

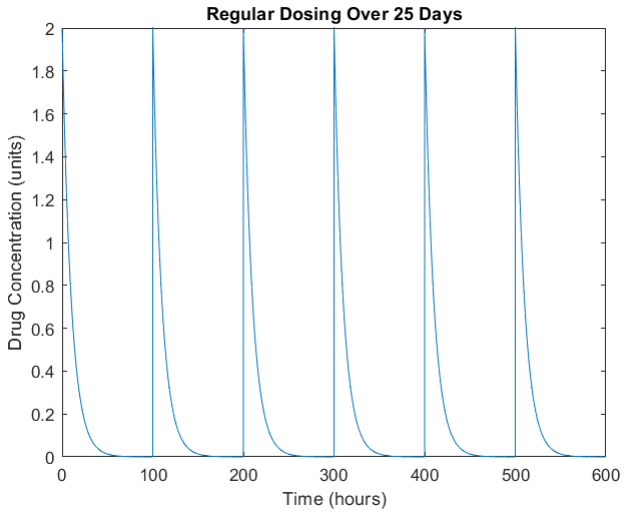


Fig. 5. Simulation of regular dosing using the oral administration model of the stomach, with interval = 100 hours, $k_s = 0.1$, and $C_0 = 2$. It can be noted there is extremely limited interaction between each dose.

where $T$ represents the interval between drug dosages and $t - T * i \geq 0$. It is vital to note that the symbol $\approx$ will be used to define the model, as the model's implementation is purely numerical and as such cannot be well defined as a purely periodic function, but as $t \to \infty$, so does the equivalence between $f(t - T)$ and $f(t)$, as will be shown in Figures 5-11 and explored in the same section. It must also be noted the importance that $t - T * i \geq 0$, as without this caveat, the sinusoidal form of the function would break and the function would simply collapse into a summation of functions $f(t)$ delayed by some $T$ as $t \to \infty$, which is not the goal of the equation. With this rule, each new iteration of $f(t - T * i)$ will only have an impact on the previous $f(t - T(i-1))$ $(i - 1 \geq 0)$ at the point at which the values of the graph become positive, and so do not break the logic behind the administration of a drug.

### A. Application to Oral Administration Model

Applying this to Eq. 3 from our specific model, the equation looks as such:

$$f(t) \approx \sum_{i=0}^{k} e^{-k_s(t - T * i)} \tag{13}$$

where $T$ is the drug dose interval, $k$ is the number of doses, and $t - T * i \geq 0$ or $t * i$. While this equation is not well analytically generalized to a model to be handled by graphing software, it can be implemented numerically through code, and an example is seen in Figure 5.

This graph shows the effects of a drug taken as the effect peters out over an interval of 100 hours over 25 days. This graph exhibits minimal interaction between the drug doses and is an ideal drug concentration for medicines which should not be taken too regularly or haphazardly as their effects are first washed out of the system before the drug is ingested again. A

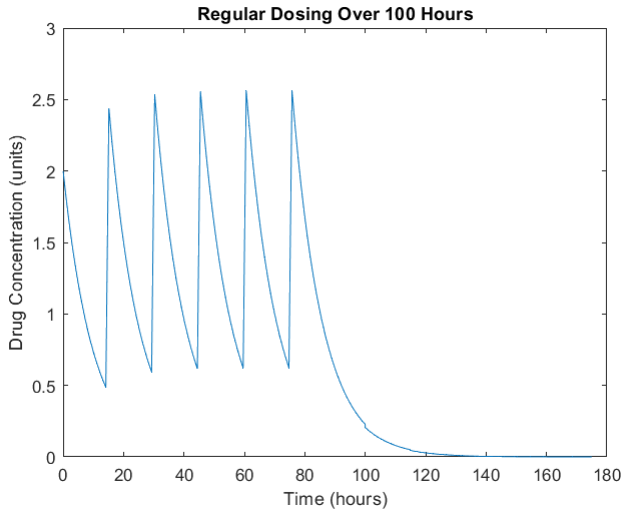


Fig. 6. Simulation of overdosing using the oral administration model of the stomach, with interval = 15 hours, $k_s = 0.1$, and $C_0 = 2$. It can be noted there is much interaction between drug doses and that peaks range upwards of 2.5 units of drug concentration.

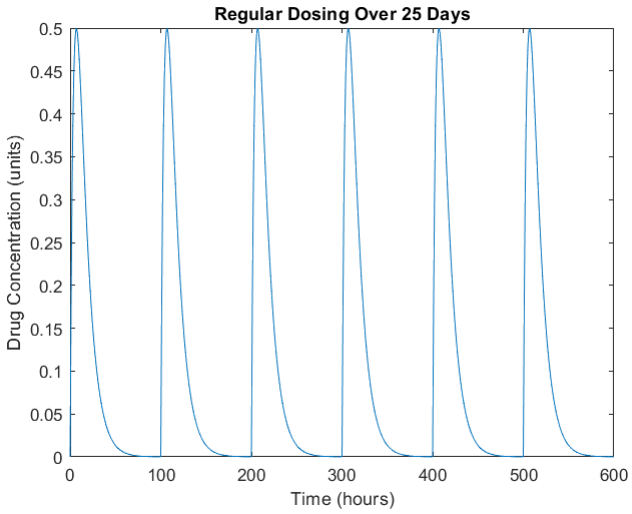


Fig. 7. Simulation of regular dosing using the oral administration model of the bloodstream, with interval = 100 hours, $k_s = 0.1$, $k_e = 0.2$, and $C_0 = 2$. It can be noted there is little interaction between drug doses, and that the drug concentration in the bloodstream's peaks are up to a fourth of the peaks for the stomach's.

second example, one with much shorter drug intervals over a shorter period of time is seen in Figure 6.

This graph shows the effects of overdosing, which forces the drug concentration upwards of 25% greater than an initial dose, which means the dose from a rudimentary standpoint, would show an efficacy of 25% above the norm for a single dose. Until the dosing is over, the graph also never settles around zero, meaning the patient would always be under some effect of the drug as the concentration is still relatively high even at minimum points for the graph.

This is a model of the concentration of drug while the drug remains in the stomach, and to see the concentration of the drug in the bloodstream, Eq. 5 would have to be put through the same periodic treatment, where the function can be expressed as:

$$f(t) \approx \sum_{i=0}^{k} \frac{C_0 k_s}{(k_s - k_e)} (e^{-k_s(t-T*i)} - e^{-k_e(t-T*i)}) \qquad (14)$$

where $T$ is the drug interval, $k$ is the number of doses, and $t * i$. Using the logic behind this equation to express this graphically through numeric means, the result is seen in Figure 7. The graph, with equal $C_0$ and $T$ to its counterpart in Figure 5, can be seen to exhibit similar behavior to it, its drug doses having extremely minimal interaction. This idea also applies to Figures 6 and 8, which show signs of similar trends, where their patterns are similar in how additional dosing increases the total drug concentration. Because the decay of the drug is slower in the bloodstream, while the stomach's drug concentration experienced an increase of about 25%, the bloodstream experienced about a 60% increase, which shows how dangerous overdosing can be.

### *B. Application to IV Administration Model*

This process in part A can be generalized now to our more complicated intravenous drug administration model yielding the following periodic input functions.

$$f_t(t) \approx \frac{C_0 k_b}{s_1 - s_2} (e^{s_1(t-T*i)} - e^{s_2(t-T)}) \qquad (15)$$

$$f_b(t) \approx \frac{C_0}{s_1 - s_2} (e^{s_1(t-T*i)}(k_t + s_1) - e^{s_2(t-T)}(k_t + s_2)) \qquad (16)$$

where $f_t(t)$ and $f_b(t)$ are functions for the tissue drug concentration and blood concentration respectively. The simulation of regular dosage for both are shown in Figures 10 and 12, and the simulation of overdosing are shown in Figures 11 and 13.

These figures largely mimic the patterns that we saw in the oral administration simulations with more jagged maxima for the bloodstream drug concentration, where the drug is initially administered, and rounder, smoother dynamics for the tissue drug concentration. The incorporation of an interdependence between the bloodstream and tissue concentration does birth some notable differences between the two models, however. Figure 10 is an important graph to analyze, as it displays the highest increase from regular dosing to overdosing. This is telling of the behavior of the graph, as it shows that the tissue's drug concentration decreases the slowest, as well as the idea that dosing using higher intervals is the most effective for IV methods, specifically in the tissue. Further, the interactive factor between the bloodstream and tissue causes the drug concentration in the blood to peter out much more quickly than in the oral administration model (Figure 11). This phenomenon makes sense intuitively also, as the therapeutic site is the tissue

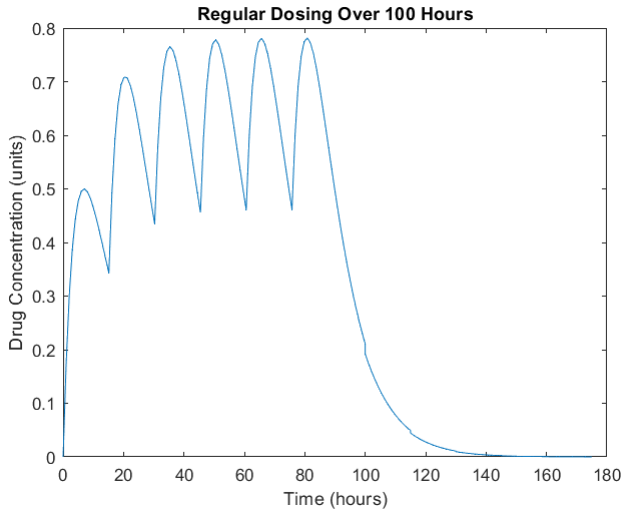


Fig. 8. Simulation of regular overdosing using the oral administration model of the bloodstream, with interval = 15 hours, $k_s = 0.1$, $k_e = 0.2$, and $C_0 = 2$. It can be noted there is much interaction between doses, and the drug concentration level reaches 0.8 compared to the previous figure's 0.5, which is a 60% increase.

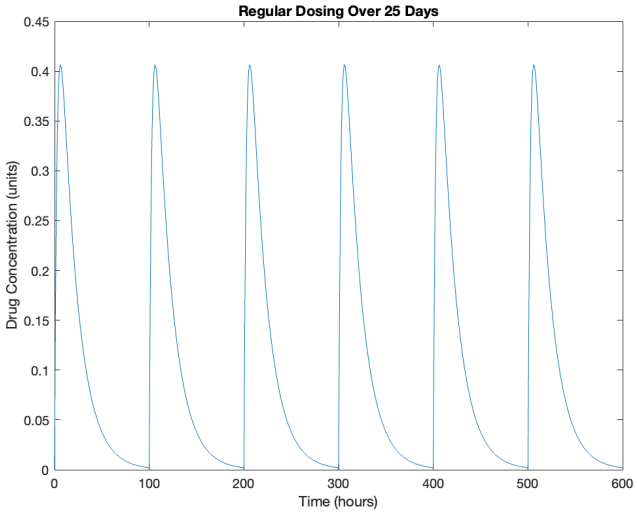


Fig. 9. Simulation of regular dosing using the IV administration model of the tissue, with interval = 100 hours, $k_b = 0.1$, $k_t = 0.1$, $k_e = 0.2$, and $C_0 = 2$. It should be noted there is limited interaction between the doses.

and a common goal of drug administration is to get as much drug to the site as quickly as possible.

### C. Discussion

This periodic model is limited in three major ways: first, the amount of drug taken cannot vary between doses. This means the model does not account for an administration of lesser amounts of drug due to its effectiveness, and so the input is assumed to be constant every period $T$. The constant period $T$ is another limitation of the model, as the system is only able to receive input through an unchanging period, so it is limited in its inflexibility in this respect. The model

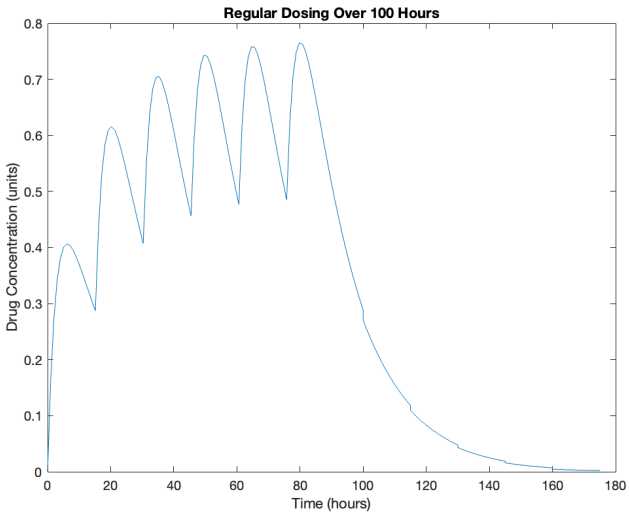


Fig. 10. Simulation of regular overdosing using the IV administration model of the tissue, with interval = 15 hours, $k_b = 0.1$, $k_s = 0.1$, $k_e = 0.2$, and $C_0 = 2$. It should be noted the percentage increase from a regular dosing interval to overdosing is much higher for tissue, displaying an increase from about 0.4 units to 0.77 units of drug concentration, a 92.5% increase.

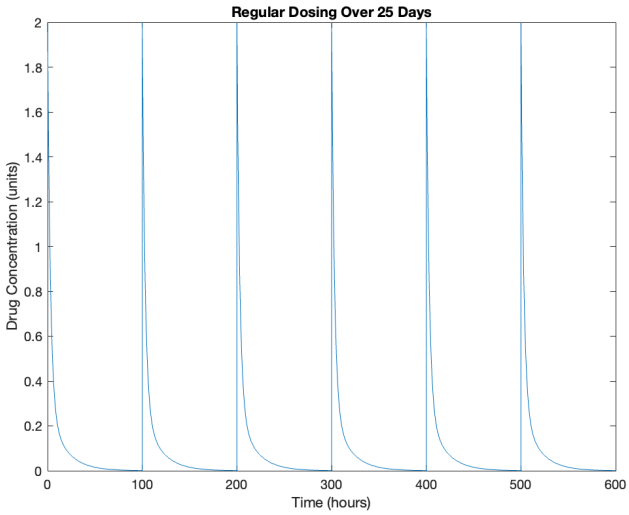


Fig. 11. Simulation of regular dosing using the IV administration model of the blood, with interval = 100 hours, $k_b = 0.1$, $k_t = 0.1$, $k_e = 0.2$, and $C_0 = 2$. It should be noted there is minimal interaction between the drug dosages.

inability to adapt makes it weak to drugs which would require a consistent target amount of drug concentration, but effective in dealing with drugs which would benefit from a model which can deliver swift and strong input periodically.

This model of periodic inputs assumes different doses interact additively, which is not true. It might be more accurate to use a piecewise function along the lines of:

$$f(t) = \begin{cases} f(t-T) & f(t-2T) \leq 0 \\ C_1 f(t-T) & f(t-2T) > 0 \end{cases}$$

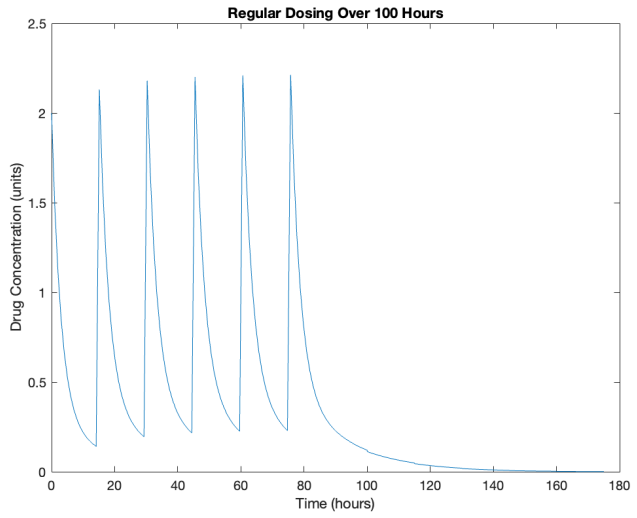


Fig. 12. Simulation of overdosing using the IV administration model of the blood, with interval = 15 hours, $k_b = 0.1$, $k_s = 0.1$, $k_e = 0.2$, and $C_0 = 2$. It should be noted the model reaches a peak drug concentration of about 2.3 units, which is an increase of about 1.15 times the initial dosage, or a 15% increase.

where $t \geq 0$

This piecewise would scale the value of the addition of two function, so the interaction of two doses would not be purely additive, and the dose, until the time $t$ when the next dose is administered, will not be affected by the factor of proportionality which allows for a proportionally additive relationship between the doses, or at any time $t, C = \beta \sum_{i=1}^{k} D_i$, where $D_i$ is the current drug concentration for any dose $i$, $k$ is the maximum number of doses, C is the total drug concentration and $\beta$ is the constant of proportionality, so $0 < \beta \leq 1$.

## III. Designing a Possible Control

To reduce the effect of the drug concentration over time, it is not sufficient to simply subtract by a constant, nor is it enough to subtract by a negative version of the graph, as that would assume both drugs are administered at the same time and through the same method. It would be more realistic to show a control of a different method of drug administration some time delay $n$ after the initial drug has been administered. To this effect, the IV and oral models were combined to create a model of the drug concentration over time based on the idea one drug directly counteracts another drug's concentration. This over a single instance is illustrated in the Subsection A while the latter Subsection treats this idea over several doses.

### A. Counteracting Drug Effects Through a Model Combination

With $(s1, s2)$ as seen in Eq. 9, it is possible to employ both

$$c_b(t) = \frac{C_0}{s_1 - s_2}(e^{s_1 t}(k_t + s_1) - e^{s_2 t}(k_t + s_2)) \tag{17}$$

and

$$c_b(t) = \frac{C_0 k_s}{(k_s - k_e)}(e^{-k_e t} - e^{-k_s t}) \tag{18}$$

to treat the drug concentration of the blood, though through different means. So, under the assumption one drug, the orally adminstered drug in this case, directly counteracts the IV-administered drug concentration through an additive effect, the equation then becomes

$$c_b(t) = C_0 \frac{1}{s_1 - s_2}(e^{s_1 t}(k_t + s_1) - e^{s_2 t}) - (\frac{k_s}{(k_s - k_e)}(e^{-k_e t} - e^{-k_s t})) \tag{19}$$

This equation represents the work an orally administrated drug of the same initial concentration, $C_0$, would do against the intravenously administrated drug over time as a kind of control. This equation can be worked around to have the intravenous portion of it be negative instead, to work against the orally administrated drug, but the free parameters must also follow certain rules for the equation to hold up.

One example is seen in the relationship between $s_1 - s_2$ and $\frac{k_s}{k_s - k_e}$. This is because, assuming both terms of the equation share the same drug concentration, $C_0$ and the orally administered drug are working against the intravenously administered drug and if

$$s_1 - s_2 \leq \frac{k_s}{k_s - k_e} \tag{20}$$

then the graph would break and it would be possible to reach negative values of drug concentration. If the intravenous model takes place at $f_{IV}(t)$, a waiting period of $n$ may be implemented in the oral model $f_oral(t - n)$ so the maxima of the drugs line up better, which corresponds to the waiting period patients should take between one drug to another, or the ideal time to take a dose of a counter-effective drug.

Here, in Figure 14, the effect of the control is apparent; the drug concentration still spikes, as predicted, but the concentration is nearly immediately brought back down to a much smaller level. The intended effect of such a control is to control how long the drug is present in the bloodstream, so though it might spike like it would without the control, a high amount doesn't remain in the blood for nearly as long, preventing potential major damage a drug might do. This is what we see in the drastic decrease. A key feature of the control is that the doses are controlled so they don't interact, preventing overdose like a drug such as Naloxone might. Because the dynamics of the blood drug concentration are not dependent on a single model, we cannot expect a smooth exponential model that was seen in the oral and intravenous models in Sections 2 and 3. Indeed, graphically, we see there is a slight increase before the concentration fully approaches zero which occurs as a result of the dynamics lining up in an adverse way, showing a side effect of using a drug to control the effects and spread of another drug. Thus, the interaction between the drugs in question is shown, as well as the ultimate result.

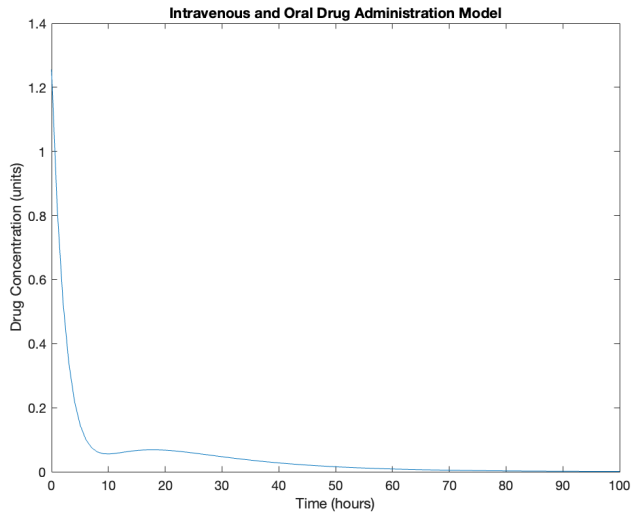


Fig. 13. Simulation of the diffusion of drug administrated intravenously and another counteractive drug administered orally, where, $k_b = 0.1$, $k_e = 0.2$, $k_t = 0.1$, $k_s = 0.3$, and $C_0 = 2$. To note, the drop-off occurs at a noticeably quicker rate than a singular drug administration model, and a small spike occurs at around t = 19 hours.

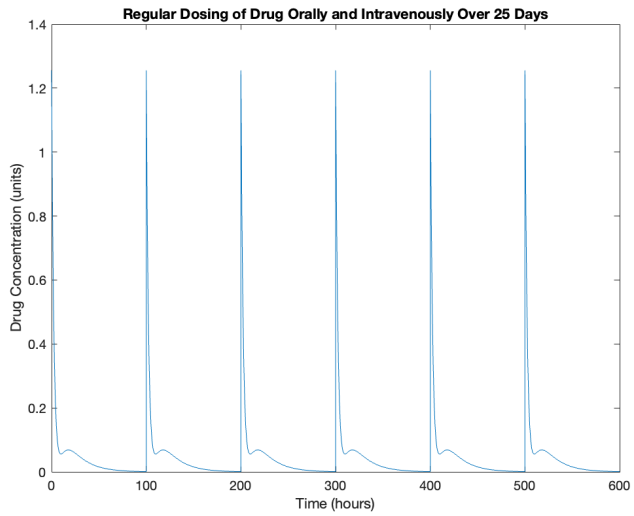


Fig. 14. Simulation of the counteractive drug model on blood drug concentration levels, with interval = 100 hours, $k_b = 0.1$, $k_e = 0.2$, $k_t = 0.1$, $k_s = 0.3$, and $C_0 = 2$. Notice again, the quicker drop-off rate and the accentuated hump that occurs after the initial drug administration.

### *B. Periodic Doses*

To further inspect the effect the control, the model for a counter-agent were applied to a periodic treatment as with Section II, where

$$f(t) \approx \sum_{i=0}^{\infty} f(t - T * i) \tag{21}$$

The difference comes in how the graphs react, as it is dependent on the solutions of two independent differential equations,

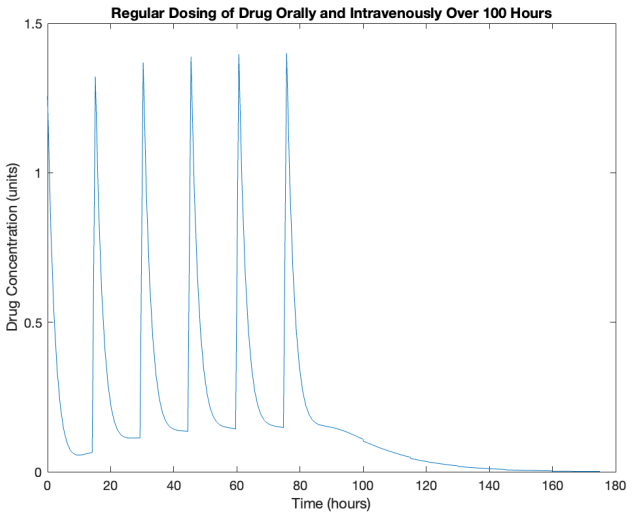


Fig. 15. Simulation of regular dosing using the counteractive drug model, with interval = 15 hours, $k_b = 0.1$, $k_e = 0.2$, $k_t = 0.1$, $k_s = 0.3$, and $C_0 = 2$. It should be noted the model reaches a peak drug concentration of about 1.4 units, which is an increase of about 1.12 times the initial dosage, or a 12% increase.

so it can be further simplified to

$$f(t) \approx \sum_{i=0}^{\infty} (f(t - T * i) - \beta g(t - T * i)) \tag{22}$$

where $f(t)$ represents the system with the drug input, $g(t)$ represents the control, and $\beta$ represents a proportional factor, or a free parameter. This is left as 1 for the purposes of this section, and henceforth will be left out of further expressions but can be used if deemed necessary past adjusting the drug dosage to allow for an application in a real-world context.

To show how the new function as defined in Eq. 19 would behave under a periodic repetition (i.e. a new dose of both drugs every $T$ interval $k$ times), the function is then redefined as

$$\begin{aligned} c_b(t) = C_0 \sum_{i=0}^{k} \Big( \frac{1}{s_1 - s_2} \big( e^{s_1(t - T*i)} (k_t + s_1) \\ - e^{s_2(t - T*i)} \big) - \big( \frac{k_s}{(k_s - k_e)} \big( e^{-k_e(t - (T*i) - n)} \\ - e^{-k_s(t - T*i - n)} \big) \big) \Big) \end{aligned} \tag{23}$$

where $n$ is the interval offset (i.e. the time between when each IV dose is taken and the time when each oral dose is taken) and $t \geq T * i$.

An application of this equation can be seen in Fig. 14 and 15, where $n = 7$, or the time between each of the IV doses and of the oral doses is 7 hours (note $n$ and $T$ both mathematically represent a delay, but are conceptually entirely separate).

Even when the doses are too close together for the control to fully minimize their interaction, as in Figure 15, the increase isn't nearly as dramatic as it is without the control, such as

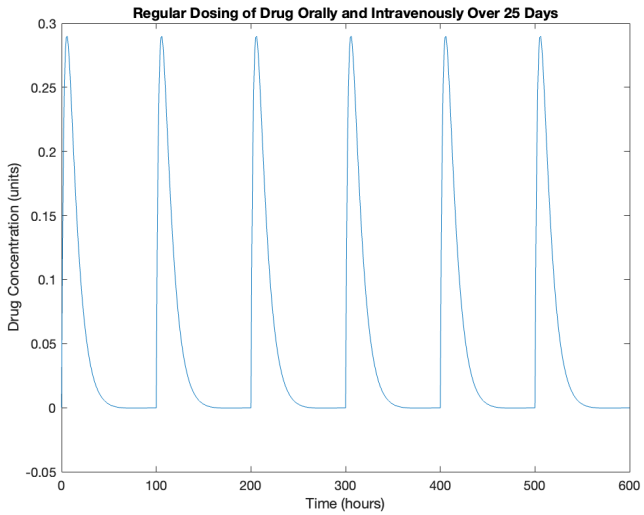


Fig. 16. Simulation of regular dosing using the counteractive drug model in which control is administered intravenously, with interval = 100 hours, $k_b = 0.1$, $k_e = 0.2$, $k_t = 0.1$, $k_s = 0.1$, $C_{s,0} = 2$, $C_{b,0} = 0.5$. It should be noted that the initial concentrations of drug had to be differentiated for the control method to realistically work which caused significantly lower maxima and fluctuations.

in Figure 10. While the concentration that it decreases to between doses is still larger than when the doses don't interact, it's much smaller than without the control. This means the control is effective at flushing out the drug and keeping it from remaining in the blood stream for a long time to do major damage. Furthermore, the decreases are much sharper than without the control, which means a high concentration of the drug in question is present in the bloodstream for much less time, so there's less potential for major damage to occur. In essence, comparing Figure 10 and 15 shows that the control leads to a much smaller increase in drug concentration and for much less time. Therefore, the control is still effective in preventing overdose and the effects of overdose.

### C. Discussion

Comparing Fig. 15 and Fig. 11, as 11 is the IV model of drug concentration in the blood periodically without a control and 15 has a control, helps emphasize the clear difference in the equations, as the drug's strength is diminished from a peak of 2 to a peak of less than 1.4, however the drug remains in the system at a low level for longer than wihtout the control, which from a real-world standpoint could be interpreted as a side effect of the counteracting drug, or could be part of the drug's intended purpose, to retain some of the drug inside of the recipient's system for an extended period of time.

Looking at both Fig. 12 and 15, the overdosing graphs, it can be seen that the control continues to limit the peaks of the graphs, although Fig. 12 sees a 15% increase while 15 sees a 12%, so the control does not limit the effectiveness of overdosing. The control does, however, smooth the minima of the graphs, and so the drug could help with retaining some target level of the drug in the system at all times, if it was desired. This could be considered a side effect of the drug, or part of its intended purpose as many drugs, such as insulin, must be maintained to a certain degree, and so the IV drug could be considered an IV form of insulin, while the orally administered drug could be considered a sort of calming drug which would lower the maxima of the IV insulin while maintaining a higher minima for the value of the insulin drug concentration, effects which both benefit the patient.

Figure 16 shows the effects of switching the drug to be administered intravenously rather than orally. Because the drug concentration in the blood is equivalent to the initial drug concentration administered compared to the slower diffusion of drug from the stomach into the bloodstream, diffusion dynamics and constants had to be tweaked in a way that is not exactly realistic.

## IV. Conclusion

The behavior of drugs in the human body was modeled with increasing complexity, starting with just a singular dose administered either intravenously or orally, with no control mechanism, finding that the body naturally flushed it out of the bloodstream over time, as expected. Introducing the interaction between tissue and the bloodstream saw the introduction of an interdependence between the two, and the peak that the drug concentration seemed to reach was dampened.

Periodic dosing was then introduced, seeing how the drug concentration consistently peaked to the same amount when the drug would be fully flushed out by the body. When the doses were too close together, the concentration would compound, causing overdose. In particular, the oral administration model of the stomach was particularly deadly, as the drug concentration would increase rapidly but decrease slowly, so doses would compound dramatically. IV administration still had potential to overdose, but less so.

Understanding the dynamics of an overdose made the dynamics with the control in play clearer. As expected, the controlled doses still peaked like before, but would decrease rapidly, correlating to a situation where a drug is flushed out even quicker than normal and therefore can't cause as much damage. There was generally less interaction between doses, so they wouldn't compound and cause overdose. There was a slight bump present in the trajectory of the drug concentration, which was interpreted as a side effect of the control. As such, the interaction between the control and the drug was shown. When doses were rapid and would interact, the compounding present in an overdose was minimized, showing that the control was effective in flushing out the drug.

All of the periodic models suffered greatly from a lack of a clear mathematical definition, and so a numerical solution was necessitated to illustrate the model graphically. While the model would adjust to definable expression as the graph's time increased, this would limit our analysis of the graphs to much deeper in time, after the patient had administered several doses of drugs, and would be more or less simply removing part of the graphs near a crucial point, as when $t \ll T * k$ (the

period multiplied by the total amount of doses) represents the adjustment period of the patient.

On the topic of mathematical generalization, generalizing to a form where the oral drug is where the main drug is administered and the intravenous drug is to control its effect was a goal of Part III, but because of the nature of the oral administration graphs, and the exponential nature of the intravenous solution, it was difficult to create a such model without reducing the exponential graph by some proportion to near insignificant values, or without drastically changing the values of the free parameters to some unreasonable number. This meant the designed control only worked one way, and so the model was limited by this factor.

Furthermore, the applicability of the models at hand to real situations is limited by the fact that they only consider one drug, and in some situations a control. As such, they're not as applicable to situations where medications/drugs are mixed, which is not an uncommon occurrence. This was mainly done because mixing drugs gives rise to interactions and complexity that the models might not have been able to easily account for, but the applicability to such situations suffers. Further exploration could explore the introduction of multiple drugs being taken at one time.

The generalization of the model also doesn't account for differences across people. For example, a drug might spike much more quickly in a smaller person than in a larger person, which might cause a control to be less effective. If it were a product being sold, this might cause a fatal mistake, or false advertising as to how effective the control mechanism is for a person. Further exploration could also examine differences across age, gender, weight, and height. In doing so, there would be a greater understanding of how such dynamics are different in different bodies; or in essence, how the environment the dynamics take place play a role in the dynamics.

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