Physics 214/224, Practice Quiz 2

Primary instructor: Yurii Maravin

WID:________________________________ NAME:____

Instructions. Print and sign your name on this quiz and on your scantron card. In doing so, you are acknowledging the KSU Honor Code: "On my honor as a student I have neither given nor received unauthorized aid on this academic work."

For two quick points, circle your studio instructor:

Lohman (TU 7:30) Lohman (TU 9:30) Weaver (TU 9:30)

Magrakvelidze (TU 11:30) Maravin (TU 1:30) Weaver (TU 3:30)

Magrakvelidze (WF 7:30) Magrakvelidze (WF 9:30)

Golin (WF 11:30)

Work alone and answer all questions. Part I questions must be answered on the Scantron cards. Put your name on your card. Color in the correct box completely for every answer with a pencil. If you make a mistake, erase thoroughly. **Don't forget to color in the boxes for your WID. If we have to correct this by hand we may take off five points from your score!** Part II must be answered in the space provided on the test. The last page is a detachable equation sheet. You may use a calculator. You may ask the proctors questions of clarification. Show your work in a clear and organized manner. You may receive partial credit for partial solutions. Solutions lacking supporting calculations will receive no points.

Part I: All questions to be answered on your Scantron card (48 points total)

1. (3 points) An electron has charge $-e$ and mass m_e. A proton has charge e and mass $1840m_e$. A "proton volt" equal to

   (a) 1 eV
   (b) $\sqrt{1840}$ eV
   (c) $1/\sqrt{1840}$ eV
   (d) 1840 eV
   (e) 1/1840 eV

2. (3 points) The fact that we can define electric potential energy means that:

   (a) the electric force is conservative
   (b) there is a point where the electric potential energy is exactly zero
   (c) it takes work for the electric force to move from some point a to some other point b and back again
   (d) the electric force is nonconservative
   (e) the work done on a charged particle depends on the path it takes

3. (3 points) During a lightning discharge, a 30 C of charge move through a potential difference of 1.0×10^8 V in 2.0×10^{-2} s. The energy released by this lightning bolt is:

   (a) 3.0×10^9 J
   (b) 3.3×10^6 J
   (c) 1500 J
   (d) 1.5×10^{11} J
   (e) 6.0×10^7 J

4. (3 points) A parallel-plate capacitor C has a charge Q. The actual charges on its plates are

   (a) Q, Q
   (b) $Q/2$, $Q/2$
   (c) Q, $-Q$
   (d) Q, 0
   (e) $Q/2$, $Q/2$

5. (3 points) The plate areas and plate separations of five parallel plate capacitors are the following

Capacitor number	Area	Separation	
1	A_0	d_0	C
2	$2A_0$	$2d_0$	C
3	$2A_0$	$d_0/2$	4C
4	$A_0/2$	$2d_0$	C/4
5	A_0	$d_0/2$	2C

$C = \frac{\varepsilon_0 A}{d}$

Rank these according to their capacitances, least to greatest

(a) 1, 2, 3, 4, 5
(b) 5, then 3 and 4 tie, then 1, then 2
(c) 4, then 1 and 2 tie, then 5, then 3
(d) 3, then 5, then 1 and 2 tie, then 4
(e) 5, 4, 3, 2, 1

6. (3 points) The capacitance of a single isolated spherical conductor with radius R is proportional to:

(a) R
(b) R^2
(c) $1/R$
(d) $1/R^2$
(e) none of these

$C = 4\pi\varepsilon_0 R$

7. (3 points) A car battery is rated at 80 A · h. An ampere-hour is a unit of:

(a) power
(b) charge
(c) current
(d) force
(e) energy

$A = \frac{1C}{1s} \Rightarrow A \cdot s = C$

8. (3 points) In schematic diagrams, currents are indicated using arrows. What do the arrows indicate?

(a) the direction of motion of the electrons
(b) the direction that positive charge carriers would move
(c) the direction of motion of the charge carriers
(d) nothing, they are just a convenient drawing tool
(e) the direction of the current vector

9. (3 points) A wire with a length of 150 m and a radius of 0.15 mm carries a current with a uniform current density of 2.8×10^7 A/m^2. The current is:

   (a) 300 A
   (b) 5.9 A
   (c) 0.63 A
   (d) 26000 A
   (e) 2.0 A

$$I = J\cdot A = 2.8\cdot 10^7 \frac{A}{m^2}\cdot \pi(0.15\cdot 10^{-3} m)^2 \simeq 1.98A$$

10. (3 points) Five cylindrical wires are made of the same material. Their lengths and radii are the following.

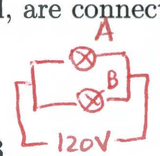
Wire number	Length	Radius	
1	ℓ	r	R
2	$3\ell/2$	$r/2$	$\frac{3}{2}\cdot 4R = 6R$
3	$\ell/2$	$r/2$	$\frac{1}{2}\cdot 4R = 2R$
4	ℓ	$r/2$	$4R$
5	5ℓ	$r/2$	$5\cdot 4R = 20R$

$$R = \rho\frac{L}{A} = \frac{\rho\ell}{\pi r^2}$$

Rank the wires according to their resistances, least to greatest.

   (a) 1, 2, 3, 4, 5
   (b) 1 and 2 tie, then 5, 3, 4
   (c) 5, 4, 3, 2, 1
   (d) 1, 3, 4, 2, 5
   (e) 1, 2, 3, 4, 5

11. (3 points) Lightbulb A, a 60W model, and lightbulb B, a 40W model, are connected in parallel across a 120V power supply. Which statement below is *false*



$I_A = \frac{60W}{120V} = 0.5A$

$I_B = \frac{40W}{120V} = \frac{1}{3}A$

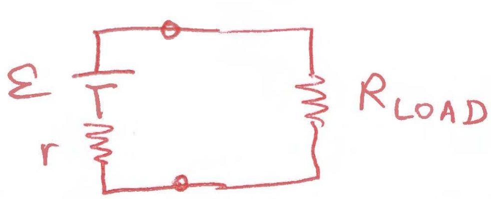
   (a) The voltage difference across lightbulb B is 120V
   (b) More current flows through the battery than through lightbulb B
   (c) Lightbulb B has a higher resistance than lightbulb A
   (d) Less current flows through lightbulb A than lightbulb B

$R_A = \frac{(120V)^2}{60W} = 240\Omega$

$R_B = \frac{(120V)^2}{40W} = 360\Omega$

12. (3 points) The internal resistance in a real battery causes what to happen when the battery is connected to a resistive "load"? (Pick the best answer)

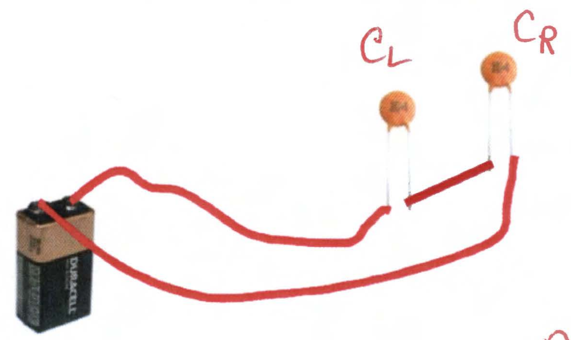
   (a) The voltage across the battery terminals decreases when the battery is connected to low resistance loads.
   (b) The battery warms when connected to low resistance loads.
   (c) The current supplied by the battery falls to less than the nominal battery voltage divided by the load resistance
   (d) All of the above

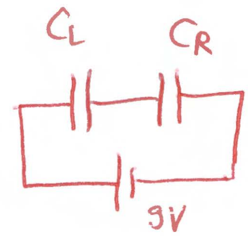


$V_{LOAD} = I\cdot R_{LOAD} = \frac{\mathcal{E}}{r+R_{LOAD}} R_{LOAD}$

$V_{LOAD} \sim \mathcal{E}$ if $r \ll R_{LOAD}$

The next four questions refer to the following figure below. The solid lines in the figure above represent high conductivity copper wire. The left capacitor has $C_L = 0.1\ \mu$F, the right capacitor has $C_R = 0.22\ \mu$F, and the battery establishes a voltage difference $V = 9$ V between its terminals.





$$\frac{Q}{C_L} + \frac{Q}{C_R} = 9V \Rightarrow$$

$$Q = \frac{9V \cdot C_L C_R}{C_L + C_R} \approx 0.62 \mu C$$

13. (3 points) Which statement below is most correct?

   (a) The two capacitors store the same charge
   (b) The two capacitors have the same voltage between their leads
   (c) The two capacitors have the same ratio of charge to voltage across their leads
   (d) The two capacitors store the same energy

14. (3 points) What is the absolute voltage difference between the left lead of the 0.1 μF capacitor C_L and the right lead of the 0.22 μF capacitor C_R?

   (a) 0.0 V
   (b) 2.8 V
   (c) 6.2 V
   (d) 9.0 V

15. (3 points) How much charge do the two capacitors together store?

   (a) 0.62 μC
   (b) 0.90 μC
   (c) 1.98 μC
   (d) 2.88 μC

16. (3 points) What is the absolute voltage difference between the two leads of the 0.1 μF capacitor C_L?

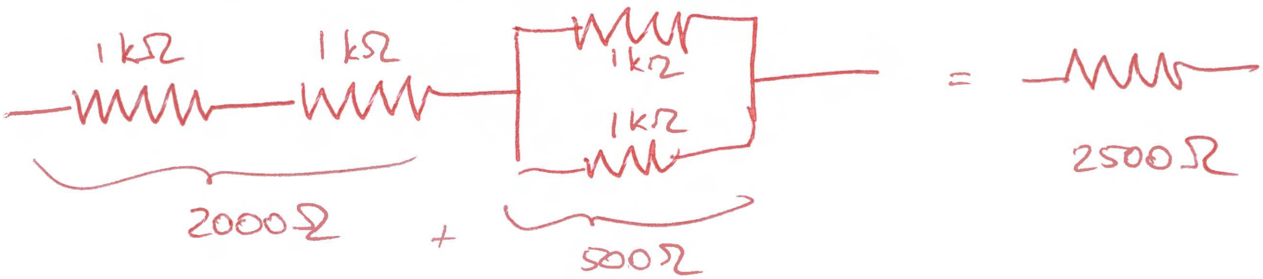
   (a) 0.0 V
   (b) 2.8 V
   (c) 6.2 V
   (d) 9.0 V

Part II: Work out this part in the space provided. Show your work! (50 points total)

17. (9 points) You have the following at your disposal: a box of twelve 9V batteries, a box of forty 1000 Ω resistors, and a box of forty 1 μF capacitors. Draw clear diagrams to indicate how you would connect parts together to make the following (you need not use all the available parts):

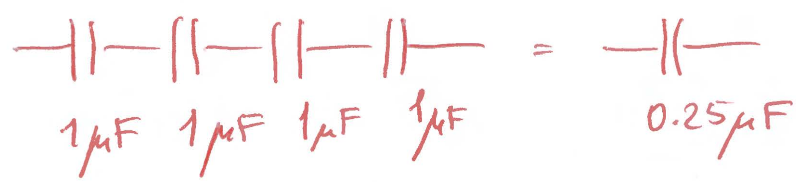
(a) (3 points) A 2500 Ω resistor

To make $2000\,\Omega$ connect two $1000\,\Omega$ in series, to make $500\,\Omega$ connect two $1000\,\Omega$ in parallel, then connect them both in series:

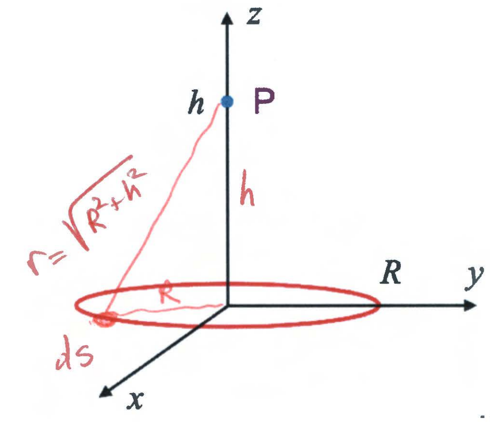


(b) (3 points) A 0.25 μF capacitor

$0.25\,\mu F = \frac{1\,\mu F}{4}$ – connect 4 capacitors in series:



(c) (3 points) A 27 V battery



18. (15 points) A charge Q is evenly spread along the circumference of a circle of radius R.

(a) (5 points) Show that the voltage at a point P located a hight h above the center of the circle is $V = \frac{Q}{4\pi\epsilon_0\sqrt{R^2+h^2}}$.

Any small point on a circle of ~~ds~~ size ds has a charge $dq = \frac{ds}{2\pi R} Q$. It's contribution to the potential at P is $dV = \frac{k\,dq}{r} = \frac{k\,dq}{\sqrt{R^2+h^2}} = \frac{kQ\,ds}{2\pi R\sqrt{R^2+h^2}}$

$$V = \oint_{circle} dV = \oint_{circle} \frac{kQ\,ds}{2\pi R\sqrt{R^2+h^2}} = \frac{kQ}{2\pi R\sqrt{R^2+h^2}} \oint_{circle} ds = \frac{kQ}{\cancel{2\pi R}\sqrt{R^2+h^2}} \cdot \cancel{2\pi R} = \frac{kQ}{\sqrt{R^2+h^2}}$$

(b) (5 points) Find the electrical force acting on a small particle at the point P carrying charge q and mass m in the vertical, or z direction. *Hint: the relationship between the z-component of $\vec{E}$ is $E_z = -\frac{\partial V}{\partial z}$.*

$$V = \frac{kQ}{\sqrt{R^2+z^2}} \qquad -\frac{\partial V}{\partial z} = +\frac{1}{2}\frac{kQ}{(R^2+z^2)^{3/2}} 2z = +\frac{kQz}{(R^2+z^2)^{3/2}}$$

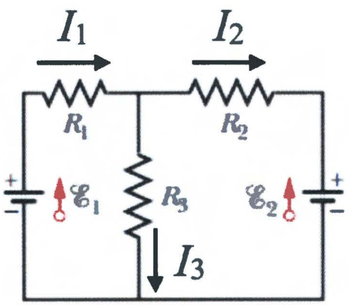
$$\vec{E}_z = \frac{kQz}{(R^2+z^2)^{3/2}}$$

(c) (5 points) Determine the radius R that would make the *total force* acting on this particle vanish. Assume $g = 9.8$ m/s^2, and the direction of the gravitational field is downward. Assume $Qq > 0$.

$$F_{NET} = \underbrace{-mg\hat{k}}_{\text{down}} + \frac{kQz}{(R^2+z^2)^{3/2}} \cdot q \cdot \underbrace{\hat{k}}_{\text{up}} = 0 \Rightarrow$$

$$mg = \frac{kQzq}{(R^2+z^2)^{3/2}}, \quad \text{solve for } R: \quad R^2 + h^2 = \left(\frac{kqQh}{mg}\right)^{\frac{2}{3}}$$

$$R = \sqrt{\left(\frac{kqQh}{mg}\right)^{2/3} - h^2}$$



19. (14 points) Call I_1, I_2, and I_3 the current flowing through the resistors labeled R_1, R_2, and R_3, respectively. To help the graders out, use the directions of the currents shown in the figure.

(a) (8 points) Deduce three independent equations for three unknown currents. Use the symbolic values for resistances and voltages.

$$\begin{cases} I_1 = I_2 + I_3 & (1) \\ \mathcal{E}_1 - I_1R_1 - I_3R_3 = 0 & (2) \\ -I_2R_2 - \mathcal{E}_2 + I_3R_3 = 0 & (3) \end{cases}$$

Exact solution: plug I_1 into (2): $\mathcal{E}_1 - I_2R_1 - I_3(R_3 + R_1) = 0$
multiply that by R_2 and add to (3), multiplied by $-R_1$:

$$+\begin{cases} R_2\mathcal{E}_1 - I_2R_1R_2 - I_3(R_2R_3 + R_1R_2) = 0 \\ I_2R_1R_2 + \mathcal{E}_2R_1 - I_3R_3R_1 = 0 \end{cases}$$

$$\mathcal{E}_1R_2 + \mathcal{E}_2R_1 - I_3(R_1R_2 + R_1R_3 + R_2R_3) = 0 \Rightarrow$$

$$I_3 = \frac{\mathcal{E}_1R_2 + \mathcal{E}_2R_1}{R_1R_2 + R_1R_3 + R_2R_3}; \quad I_1 = \frac{\mathcal{E}_1 - I_3R_3}{R_1} \text{ (from (2))}; \quad I_2 = I_1 - I_3 \text{ (from (1))}$$

(b) (3 points) Take $\mathcal{E}_1 = 6$ V, $\mathcal{E}_2 = 12$ V, $R_1 = R_3 = 1000\ \Omega$, $R_2 = 1\ \Omega$. Solve for the currents. You can solve your equations from part (a) for this, but if you can see a simpler path to a very good estimate of the exact answer, use it, explaining whatever approximation you make in one sentence. *Hint: Wow, R_2 is small; what is the voltage difference across R_3?*

If $R_2 = 1\Omega$, then most of $\mathcal{E}_2$ is on R_3, and $I_3 = \frac{\mathcal{E}_2}{R_3} = \frac{12V}{1000\Omega} = 12m$

For the first loop: $\mathcal{E}_1 - I_1R_1 - 12V = 0 \Rightarrow$

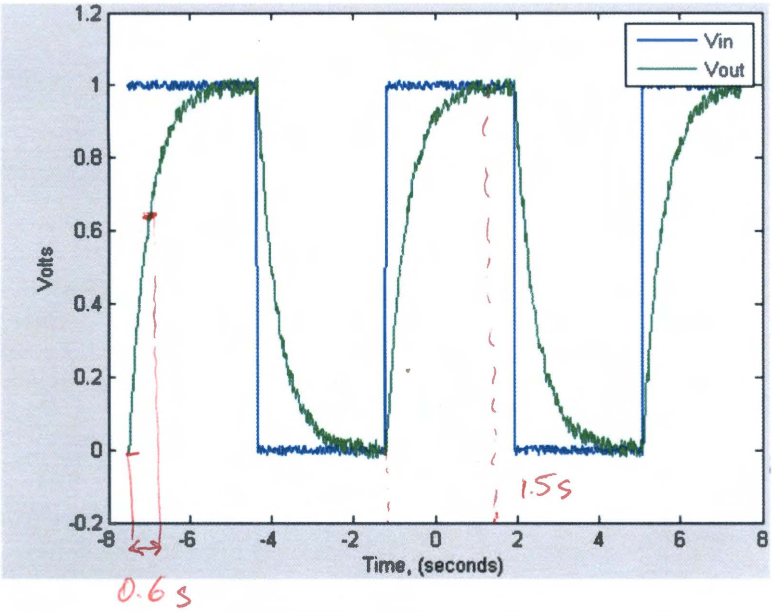
$I_1 = -\frac{6V}{1000\Omega} = -6mA$, and $I_2 = I_1 - I_3 = -6mA - 12mA = -18m$

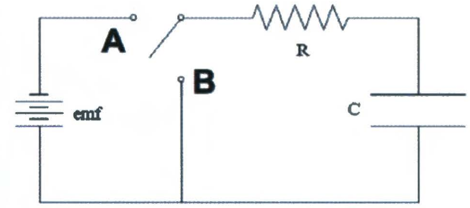
Compare with exact solution: $I_3 = 11.98\ mA$
$I_1 = -5.98\ mA$
$I_2 = -17.96\ mA$ Close!

(c) (3 points) Do the same thing for the case $R_2 = 10^6\ \Omega$, with the other circuits components the same as the previous part. *Hint: this is a very big resistance!*

If R_2 is so big, we can treat it as a break in circuit, so $I_1 = I_3 = \frac{\mathcal{E}_1}{R_1 + R_2} = 3mA$, $I_2 = 0\ mA$.

Compare with the exact solution? $I_1 = I_3 \simeq 3mA$
$I_2 = -9\mu A$.
Very close!





20. (12 points) The graph produced by an oscilloscope, like the one used in studio, shows the voltage across a 0.1 F capacitor (green line), and the voltage across the series combination of capacitor and resistor.

(a) (2 points) What is the RC time constant for this circuit?

In time τ_{RC} the capacitor is charged to ~ 0.63 of its final voltage (charge).

$$\tau_{RC} \simeq 0.6\,s$$

(b) (2 points) What is the resistance of the circuit?

$$\tau_{RC} = RC = R \cdot 0.1F = 0.6\,s \Rightarrow R = 6\Omega$$

(c) (2 points) How much charge is stored in the circuit with the capacitor at its maximum voltage?

$$V = \frac{Q}{C} \Rightarrow Q = V \cdot C = 1V \cdot 0.1F = 0.1\,C$$

(d) (2 points) What is the minimum time, in seconds, that one should wait after disconnecting the circuit to ensure that the capacitor voltage is less than 0.8 V?

$$V = V_0 e^{-\frac{t}{\tau}} \qquad 0.8V = 1V \cdot e^{-\frac{t}{\tau}} \Rightarrow \ln(0.8) = -\frac{t}{\tau} \Rightarrow$$

$$t = -\ln(0.8) \cdot \tau_{RC} = -0.6s \cdot (-0.22) \simeq 0.13\,s$$

(e) (2 points) How much current is flowing into the capacitor at $t = +1.5$ s? *Hint:* $\frac{dQ}{dt}$

at $t = 1.5\,s$ capacitor is almost fully charged, so $\frac{dQ}{dt} = I \approx 0$

To be exact: $I(t) = \frac{EMF}{R} e^{-\frac{t-t_0}{\tau_{RC}}} = \frac{1V}{6\Omega} e^{-\frac{(1.5-(-1))}{0.6}} \simeq 2.58\,mA$

t_0 - time when capacitor starts charging