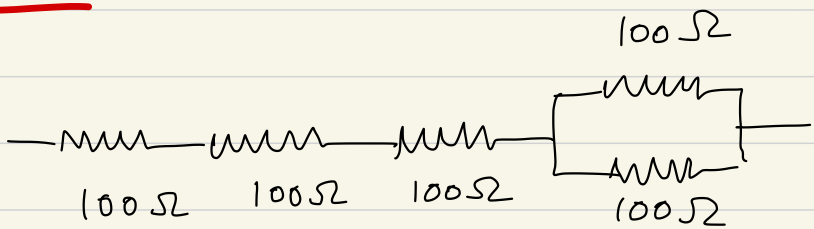
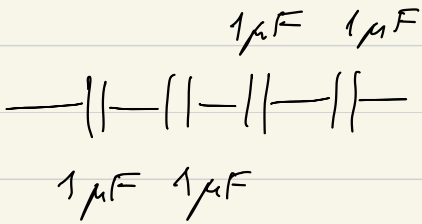
Problem 17

(a)

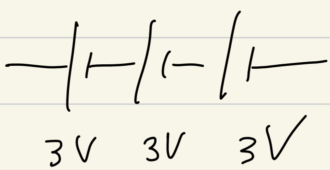


attempt (+1)
correct answer (+3)
} (+4)

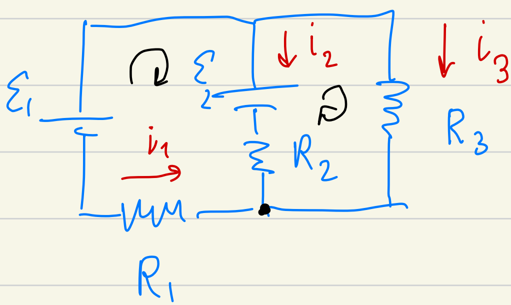
(b)



(c)



Problem 18



(a)

$$\begin{cases} i_1 + i_2 + i_3 = 0 \\ \mathcal{E}_1 - \mathcal{E}_2 - i_2 R_2 + i_1 R_1 = 0 \\ i_2 R_2 + \mathcal{E}_2 - i_3 R_3 = 0 \end{cases}$$

(B) & (C)

Exact solution : $i_3 = -i_1 - i_2 \Rightarrow$

$$\begin{cases} \mathcal{E}_1 - \mathcal{E}_2 - i_2 R_2 + i_1 R_1 = 0 & (1) \\ i_2 R_2 + \mathcal{E}_2 + (i_1 + i_2) R_3 = 0 & (2) \end{cases}$$

$$\mathcal{E}_1 - \mathcal{E}_2 - i_2 R_2 = -i_1 R_1 \Rightarrow$$

$$i_1 = \frac{i_2 R_2 + \mathcal{E}_2 - \mathcal{E}_1}{R_1}$$, plug into (2)

$$i_2 R_2 + \mathcal{E}_2 + i_2 R_3 + \frac{i_2 R_2 + \mathcal{E}_2 - \mathcal{E}_1}{R_1} R_3 = 0$$

$$i_2 \left(R_2 + R_3 + \frac{R_2 R_3}{R_1} \right) + \mathcal{E}_2 + \frac{\mathcal{E}_2 - \mathcal{E}_1}{R_1} R_3 = 0$$

$$i_2 (R_2 R_1 + R_3 R_1 + R_2 R_3) + \mathcal{E}_2 R_1 + \mathcal{E}_2 R_3 - R_3 \mathcal{E}_1 = 0$$

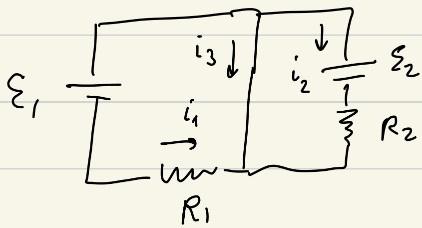
$$\boxed{i_2 = \frac{R_3 \mathcal{E}_1 - \mathcal{E}_2 R_3 - \mathcal{E}_2 R_1}{R_1 R_2 + R_1 R_3 + R_2 R_3}}$$

Plugging values: $i_2 = -0.012\,A$

$i_1 = -0.010\,A$

$i_3 = 0.022A$

Fast way: R_3 is essentially a short, so we can re-draw the circuit as



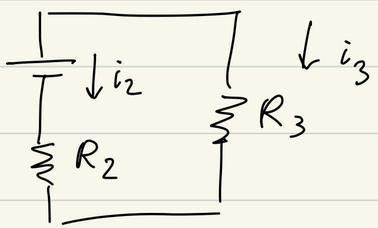
$$i_1 = -\frac{\varepsilon_1}{R_1} = -\frac{10V}{1000\Omega} = -0{,}010A$$

$$i_2 = -\frac{\varepsilon_2}{R_2} = -\frac{12V}{1000\Omega} = -0.012A$$

and $i_3 = -i_1 - i_2 = 0.022A$

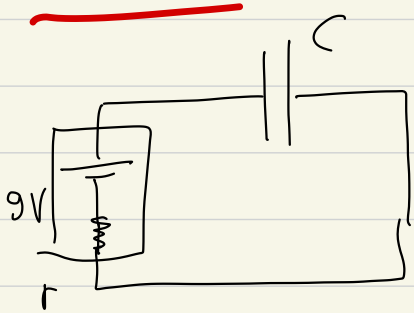
Other values: $i_2 = -0.6A$, $i_1 = -4\cdot 10^{-6}A$, $i_3 = 0.6A$

Fast way: R_3 is so big - it is a break, so: $i_1 = 0$



$$i_3 = -i_2 = \frac{\varepsilon_2}{R_2 + R_3} = \frac{12V}{20\Omega} = 0.6A$$

Problem 19



r – internal resistance

(b) Before C is charged, the current in the circuit is $\frac{\mathcal{E}}{r} = \frac{3V}{r} = 1A \Rightarrow r = 3\Omega$ (bad battery!)

(c) $C = \frac{\varepsilon_0 A}{d}$ where A – surface area of lids, d – separation:

$$C = \frac{8{,}85 \cdot 10^{-12} \frac{F}{m} \cdot \pi \left(\frac{0.30\,m}{2}\right)^2}{0{,}013\,m} = 48.1 \cdot 10^{-12} F =$$

$$= 48.1\,pF$$

(d) $$\frac{CV^2}{2} = \frac{48.1 \cdot 10^{-12} F \cdot (3V)^2}{2} = 2.17 \cdot 10^{-10} J$$

Experiment 2

(e) Since the battery is disconnected Q - stays the same! It was $3V \cdot 48.1\,pF = 1{,}44 \cdot 10^{-10}\,C$, so it stays the same.

Explanation +1
Correct answer +1

(f) C and V changed!

$$C = \frac{\varepsilon_0 A}{d}, \text{ as } d \text{ increases by } \times 2 \Rightarrow$$

$$C_{new} = \frac{1}{2} C_{old}.$$

$$V = Ed = \frac{\sigma}{\varepsilon_0} d \Rightarrow d \text{ increases by } \times 2,$$

$$V_{new} = 2 V_{old}$$

$$PE = \frac{CV^2}{2} \Rightarrow PE_{new} = \frac{C_{new} V_{new}^2}{2} =$$

$$= \frac{1}{2} \frac{C_{old} \cdot 4 V_{old}^2}{2} =$$

$$2 \cdot \frac{C_{old} \cdot V_{old}^2}{2} = 2 PE_{old}$$

So $PE_{new} = 2 \cdot 2.17 \cdot 10^{-10}\,J = 4.34 \cdot 10^{-10}\,J$

(g) The work goes into doubling the energy so

$$PE_{old} + Work = PE_{new} = 2PE_{old} \Rightarrow$$

$$Work = PE_{old} = 2.17 \cdot 10^{-10}\,J$$

Problem 20

(a) The graph is difficult to read but it takes about 0.6 s for the voltage on the capacitor to increase to ~ 66% of the final value (1V),

so $\tau_{RC} = 0.6\,s$ (0.5 – 1 s is ok)

(b) $\tau_{RC} = RC = 0.6\,s$

$$C = \frac{0.6\,s}{200\,\Omega} = 3\,mF$$

(c) The capacitor is discharging, so

$$i = \frac{V}{R} e^{-\frac{t}{RC}} = \frac{1\,V}{200\,\Omega} e^{-\frac{0{,}5\,s}{0{,}6\,s}} \simeq 2.17\,mA$$