Generador de números aleatorios Gamma

Tarea 4 simulación estocástica

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1.

Implementar un algoritmo para generar $X \sim Ga(x|\alpha, \beta)$ usando $G1, G2, G3$.

```
G <- R6Class(classname = "G",
             public = list(
               a = NULL, # alfa
               b = NULL, # beta
               initialize = function(a=1,b=1){
                 self$a <- a
                 self$b <- b
               },
               # alfa entero
               G1 = function(a = self$a, b = self$b){
                 us <- runif(n = a)# n = alfa u's
                 X <- (-1/b) * sum(log(us)) # X ~ Ga(x|n=a,1)
                 return(X/b) # X/b ~ Ga(x|a,b)
               },
               # 1 < alfa
               G2 = function(){
                 repeat{
                   #i.
                   a <- round(self$a,0)
                   u <- runif(1)
                   y <- self$G1(a,a/self$a)
                   #ii.
                   if( u <= ( (a/y)^(self$a - a) * exp( (self$a - a)*(self$a - y)/self$a ) ) ){
                     X <- y / self$b
                     break
                   }
                 }
                 return(X)
               },

               indicadora = function(x,a,b){#regresa 1 si x está en (a,b), 0 de otro modo
                 if(a < x & x < b){
                   return(1)
                 } else {
                   return(0)
                 }
               },
               #alfa < 1
               G3 = function(){
                 repeat{
                   #i.
```

```
                    c <- exp(1) / (exp(1) + self$a)
                    u1 <- runif(1)
                    u2 <- runif(1)
                    #ii.
                    if(u1 < c){
                      y <- (u1 / c)^(1/self$a)
                    } else {
                      y <- -log( (1-u1)/(self$a*c) )
                    }
                    #iii.
                    if(u2 <= (exp(-y) * self$indicadora(u1,0,c) +
                              y^(self$a - 1) * self$indicadora(u1,c,1)) ){
                      X <- y/self$b
                      break
                    }
                  }
                  return(X)
                },
                get_ram = function(){
                  #Si a es entero G1
                  if(is.integer(self$a)){
                    self$G1()
                  } else if(1 < self$a){ #si 1 < alfa usar G2
                    self$G2()
                  } else if(self$a < 1){ #si alfa < 1 usar G3
                    self$G3()
                  }
                }
              ))
```

2.

Generar $x_1, \ldots, x_{5000} \sim Ga(x|\alpha, \beta)$ para

i) $(\alpha, \beta) = (0.3, 7.9)$

```
set.seed(96)
g <- G$new(a = 0.3, b = 7.9)
xs.i <- replicate(n = 5000,expr = g$get_ram())
head(xs.i)
```

```
## [1] 1.549004e-01 2.690635e-01 1.433920e-01 1.197519e-02 4.158036e-05
## [6] 3.423514e-02
```

ii) $(\alpha, \beta) = (3.1415, 2.71)$

```
set.seed(73)
g <- G$new(a = 3.1415, b = 2.71)
```

```r
xs.ii <- replicate(n = 5000,expr = g$get_ram())
head(xs.ii)
```

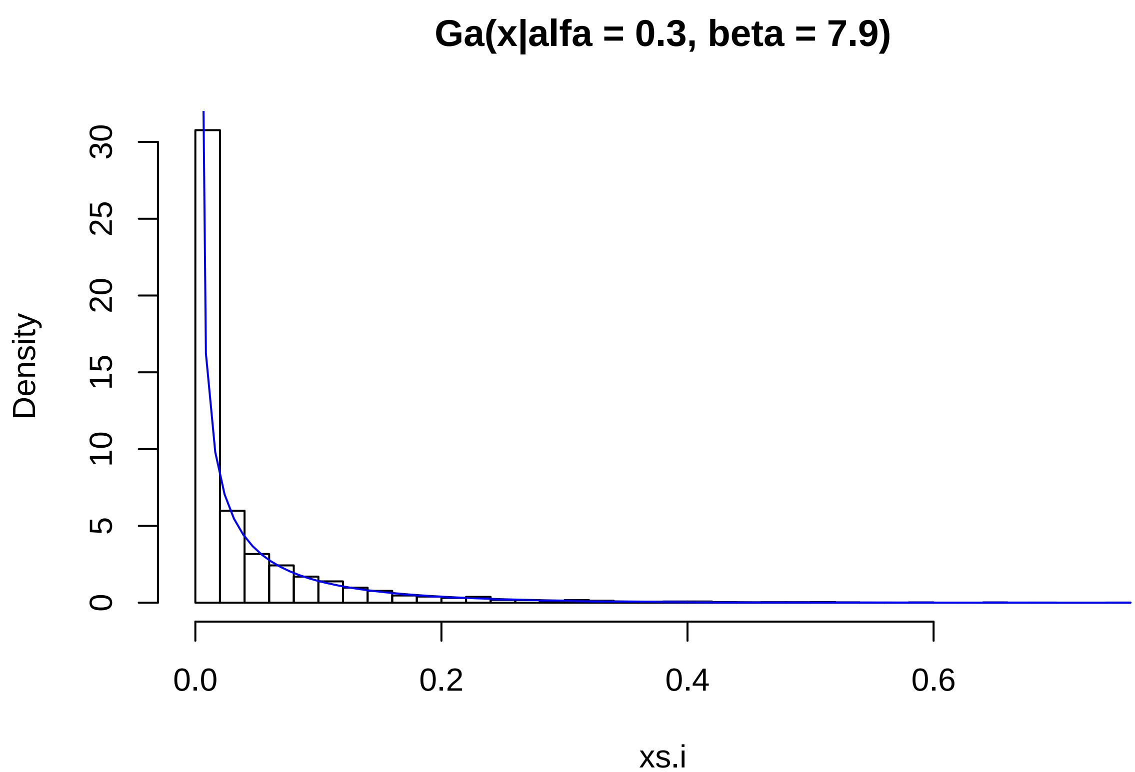
```
## [1] 1.3951281 1.1314239 0.4702894 0.1471525 1.6136538 0.5095676
```

3.

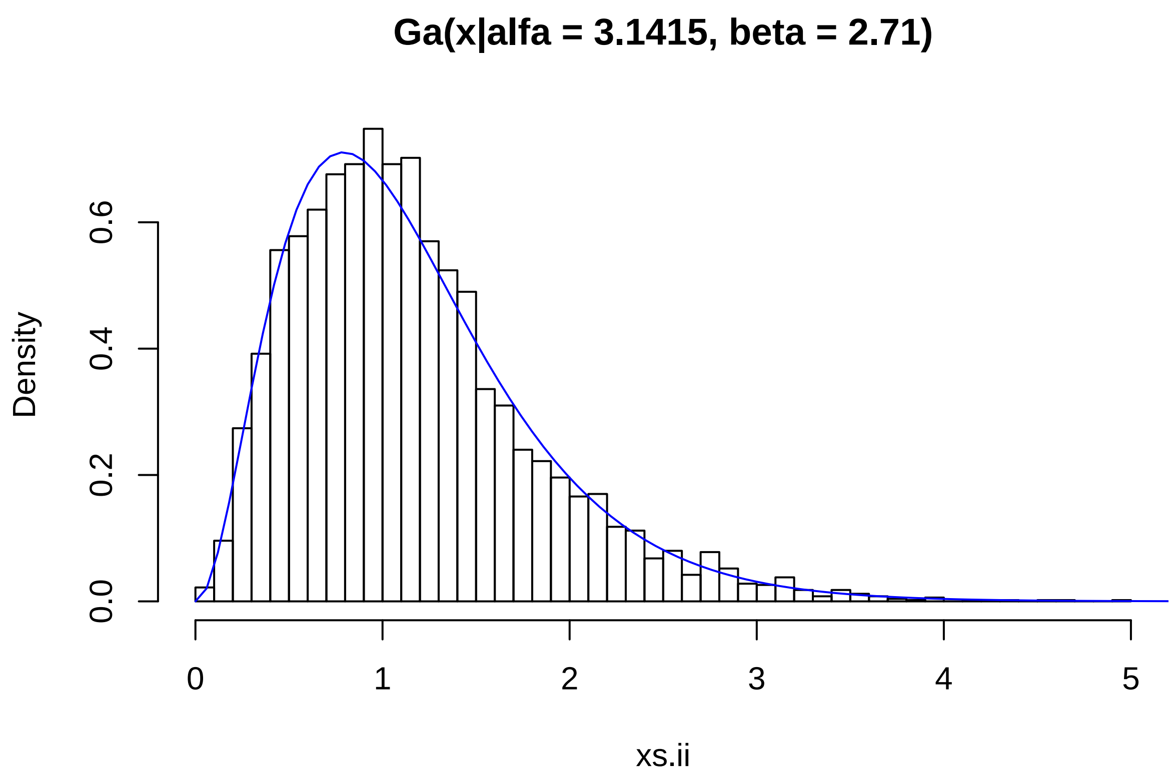
Comparar

i. Histograma vs distribución

```r
#Ga(x|alfa = 0.3, beta = 7.9)
hist(xs.i, breaks = 50, freq = F, main = "Ga(x|alfa = 0.3, beta = 7.9)")
curve(dgamma(x,shape=0.3,rate=7.9),from = 0.001,add = T,col="blue")
```




```r
hist(xs.ii,breaks = 50, freq = F, main = "Ga(x|alfa = 3.1415, beta = 2.71)")
curve(dgamma(x,shape=3.1415,rate=2.71),0,6,add = T, col = "blue")
```



ii. $E[X] = \alpha/\beta$ **vs** $\bar{x}$**,** $V(X) = \alpha/\beta^2$ **vs** S^2

```
#X ~ Ga(x|a = 0.3, b = 7.9)
#E[X],
0.3/7.9
```

```
## [1] 0.03797468
```

```
#barX
mean(xs.i)
```

```
## [1] 0.03860332
```

```
#V(X)
0.3/7.9^2
```

```
## [1] 0.004806922
```

```
#S^2
var(xs.i)
```

```
## [1] 0.005021824
```

```
#X ~ Ga(x|a = 3.1415, b = 2.71)
#E[X]
3.1415 / 2.71
```

```
## [1] 1.159225
```

```r
#barX
mean(xs.ii)
```

```
## [1] 1.157306
```

```r
#V(X)
3.1415 / 2.71^2
```

```
## [1] 0.4277583
```

```r
#S^2
var(xs.ii)
```

```
## [1] 0.4148998
```

iii. $\{Q_{0.75}, Q_{0.50}, Q_{0.25}\}$ vs $\{W_{0.75}, W_{0.50}, W_{0.25}\}$

X ~ Ga(x|a = 0.3, b = 7.9)

```r
#X ~ Ga(x|a = 0.3, b = 7.9)
  #Q
  reales <- qgamma(c(0.75,0.5,0.25),0.3,7.9)
  #W
  muestrales <- quantile(xs.i,c(0.75,0.50,0.25))
  rbind(muestrales,reales)
```

```
##                   75%         50%          25%
## muestrales 0.04397261 0.008773355 0.0008122645
## reales     0.04340500 0.009257106 0.0008733927
```

X ~ Ga(x|a = 3.1415, b = 2.71)

```r
#X ~ Ga(x|a = 3.1415, b = 2.71)
  #Q
  reales <- qgamma(c(0.75,0.5,0.25),3.1415,2.71)
  #W
  muestrales <- quantile(xs.ii,c(0.75,0.50,0.25))
  rbind(muestrales,reales)
```

```
##                 75%      50%       25%
## muestrales 1.470286 1.045893 0.6955649
## reales     1.509421 1.038817 0.6788302
```