Covid-19 Epidemic Model with Constraints and Feedback

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"All models are wrong, but some are useful."
George Box

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Problem Statement

What?

- Build a simple epidemic model able to capture the lockdown-relaxation, and the lack of immunity effects.

Why?

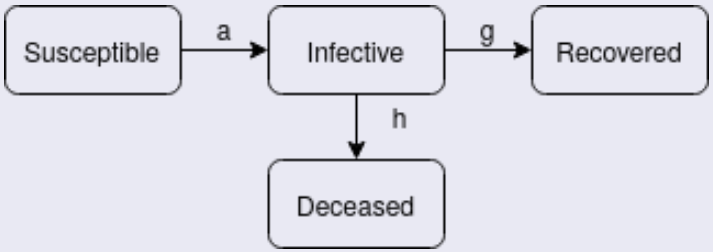
- Relaxing the lockdown constraints could flare up the epidemic.
- Lack of immunity could perpetuate the epidemic for a long time.
- The current models cannot capture correctly the lockdown-relaxation, and the lack of immunity effects.

How?

- SIRD population model: Susceptible, Infective, Recovered, Deceased.
- Constraints: lockdown-relaxation.
- Feedback: lack of immunity.

SIRD

- Standard differential equations model, currently used by everybody:



$$\frac{dS}{dt} = -aSI$$
$$\frac{dI}{dt} = aSI - gI - hI$$
$$\frac{dR}{dt} = gI$$
$$\frac{dD}{dt} = hI$$

- Cannot capture lockdown-relaxation!
- Cannot capture lack of immunity!

SIRD Scaling Properties

- Scaling doesn't change the shape of the solution.
- Assume that this is the SIRD model before scaling:

$$\frac{dS}{dt} = -\bar{a}SI$$
$$\frac{dI}{dt} = \bar{a}SI - \bar{g}I - \bar{h}I$$
$$\frac{dR}{dt} = \bar{g}I$$
$$\frac{dD}{dt} = \bar{h}I$$

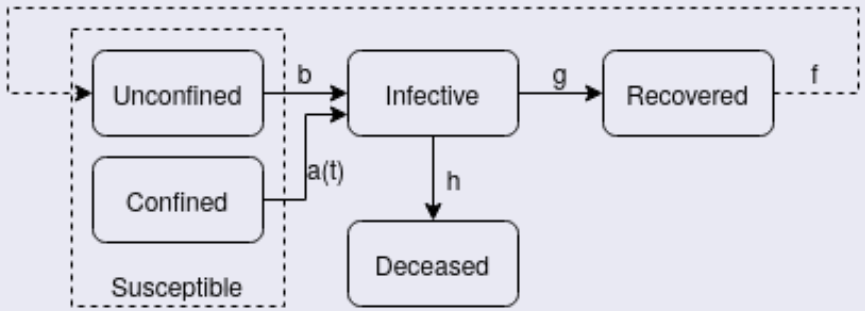
- One can conveniently scale the model such that:

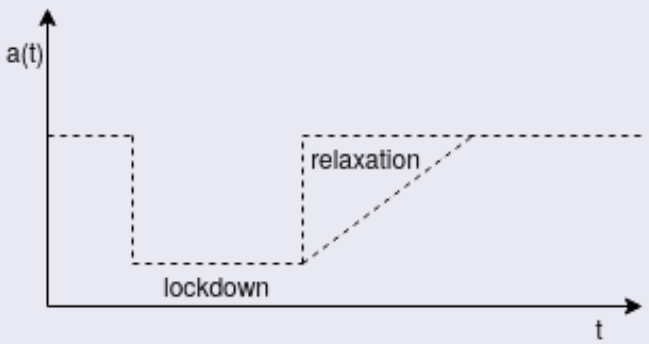
$$g + h = 1$$
$$g = \bar{g}/(\bar{g} + \bar{h})$$
$$h = \bar{h}/(\bar{g} + \bar{h})$$
$$a = \bar{a}/(\bar{g} + \bar{h})$$

- That is, we divide the system parameters by $\bar{g} + \bar{h}$.

The Model: Lack of Immunity and Lockdown-Relaxation

SIRD with constraints and feedback





$$\frac{dC}{dt} = -aCI \longleftarrow \text{Confined, 80\% (models lockdown)}$$

$$\frac{dU}{dt} = -bUI + fR \longleftarrow \text{Unconfined, 20\% (models lockdown)}$$

$$\frac{dI}{dt} = aCI + bUI - gI - hI$$

$$\frac{dR}{dt} = gI - fR$$

$$\frac{dD}{dt} = hI$$

$$a = a(t) \longleftarrow \text{(models lockdown-relaxation)}$$

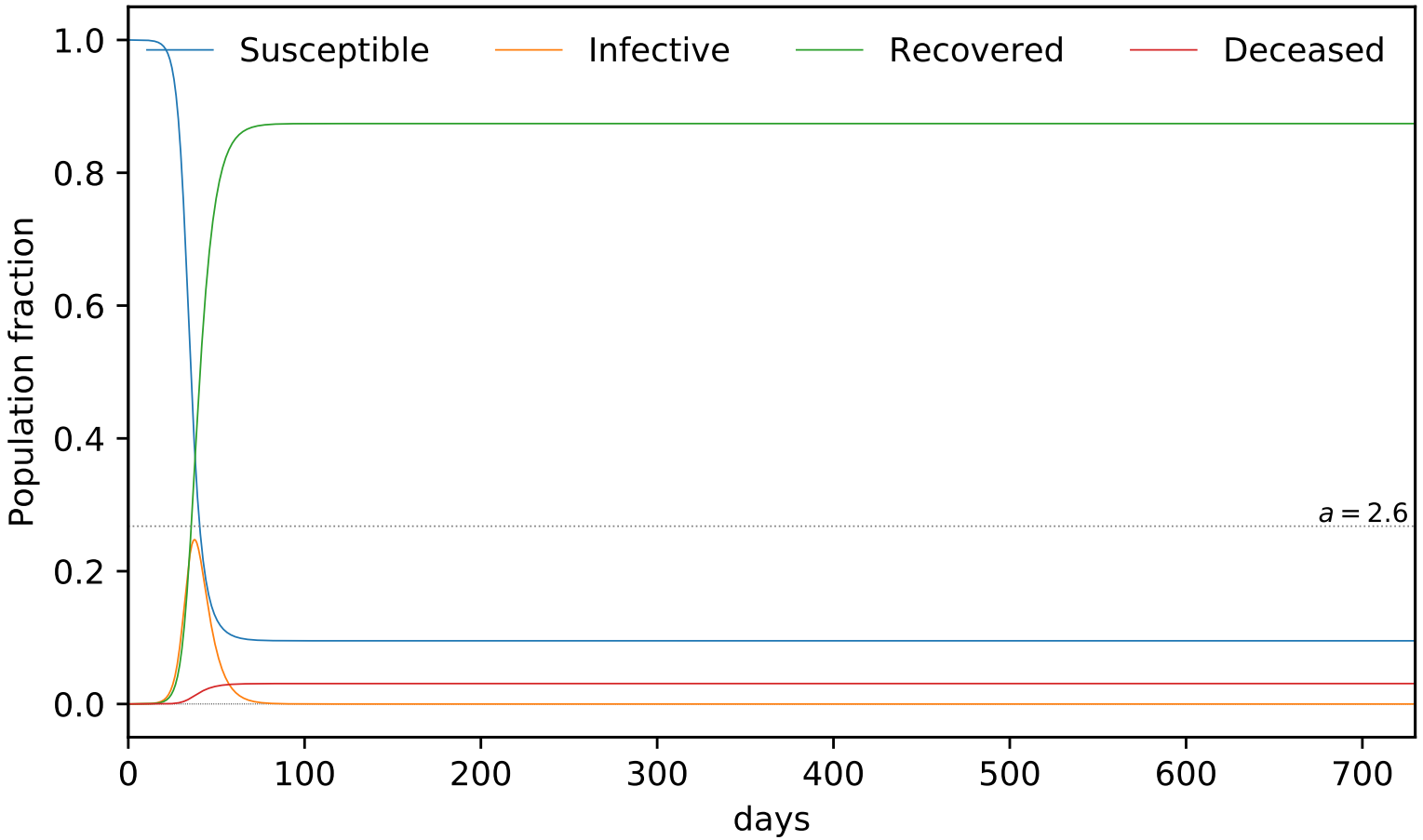
$$f > 0 \longleftarrow \text{(models lack of immunity)}$$

The Model: Lack of Immunity and Lockdown-Relaxation

Main Assumptions

- Susceptible = Confined (80%) + Unconfined (20%)
- Initial fraction of infective: $I(0) = 10^{-5}$
- Integration step: $\delta = 10^{-3}$
- Time scaling: $1000\delta = 5$ days (an infected can be infective for 5 days).
- Reproduction number for Confined: $a = 1$. This is the number of infections resulting form a single infection in the Confined population.
- Reproduction number for Unconfined: $b = 2.6$. This is the number of infections resulting form a single infection in the Unconfined population.
- Rate of Recovered: $g = 0.966$
- Rate of Lack of Immunity: $f = 0.05$
- Rate of Death: $h = 0.034$
- Lockdown starts after: 20 days
- Lockdown duration: 40 days
- Relaxation time: variable from 0 days to 80 days
- Simulation time: 730 days (2 years)

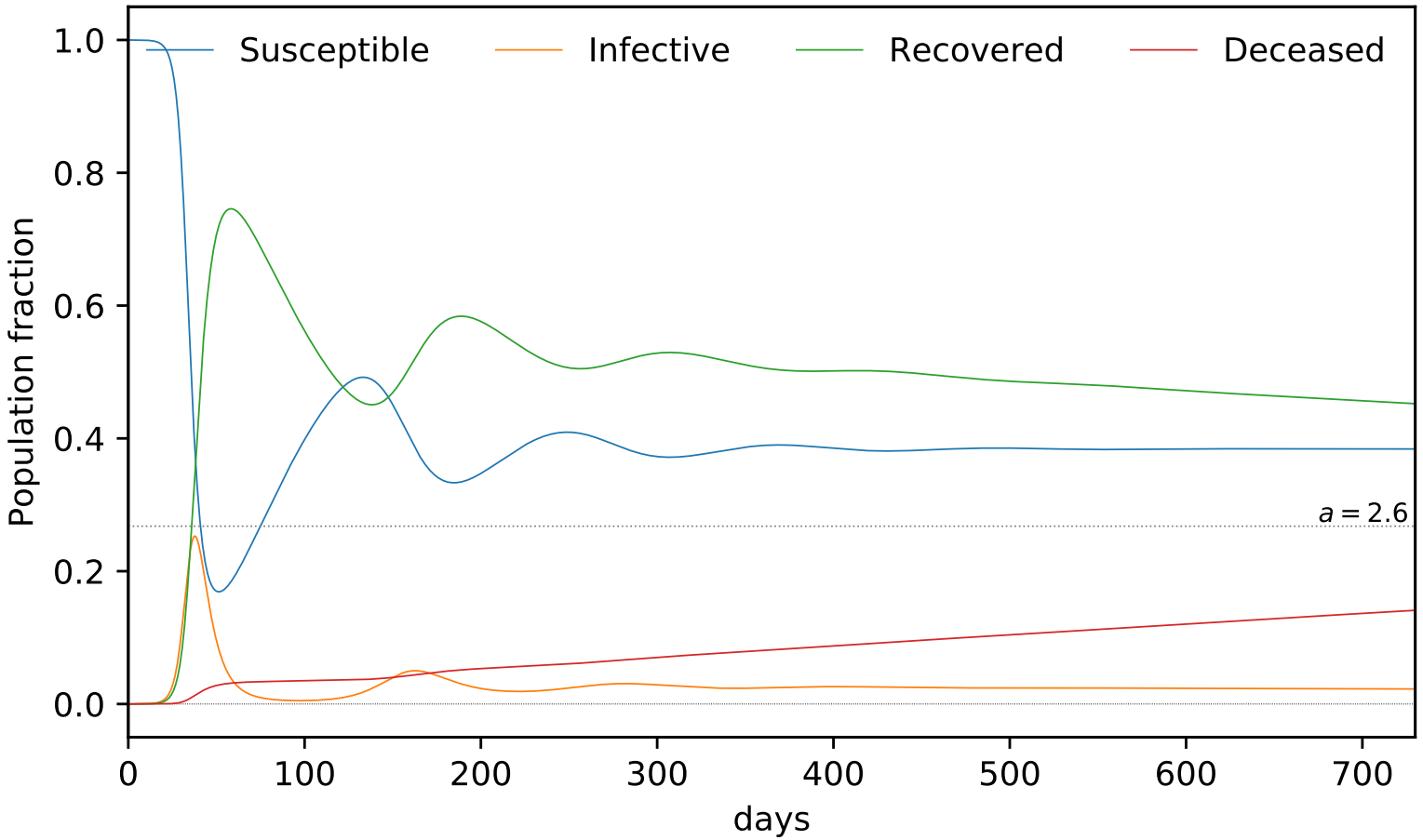
SIRD - pure model



SIRD - pure model

- $a = b = 2.6,\ f = 0,\ g = 0.966,\ h = 0.034$
- This is what would have happened without any lockdown measures and with 100% immunity. Short and painful effect.

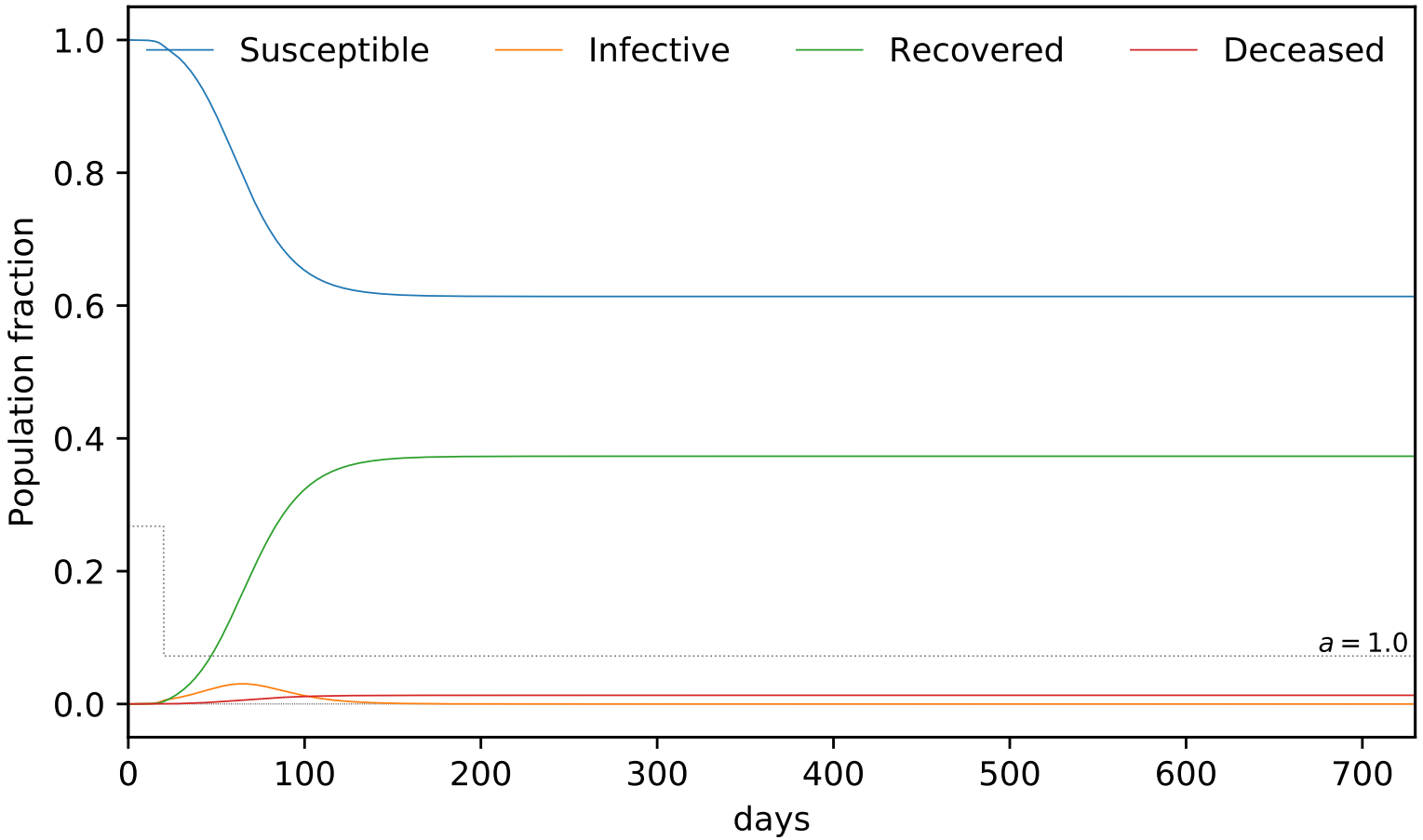
SIRD with lack of immunity



SIRD with lack of immunity

- $a = b = 2.6,\ f = 0.05,\ g = 0.966,\ h = 0.034$
- No lockdown, but 5% of Recovered don't gain immunity. Waves of epidemics, number of deaths increases substantially.

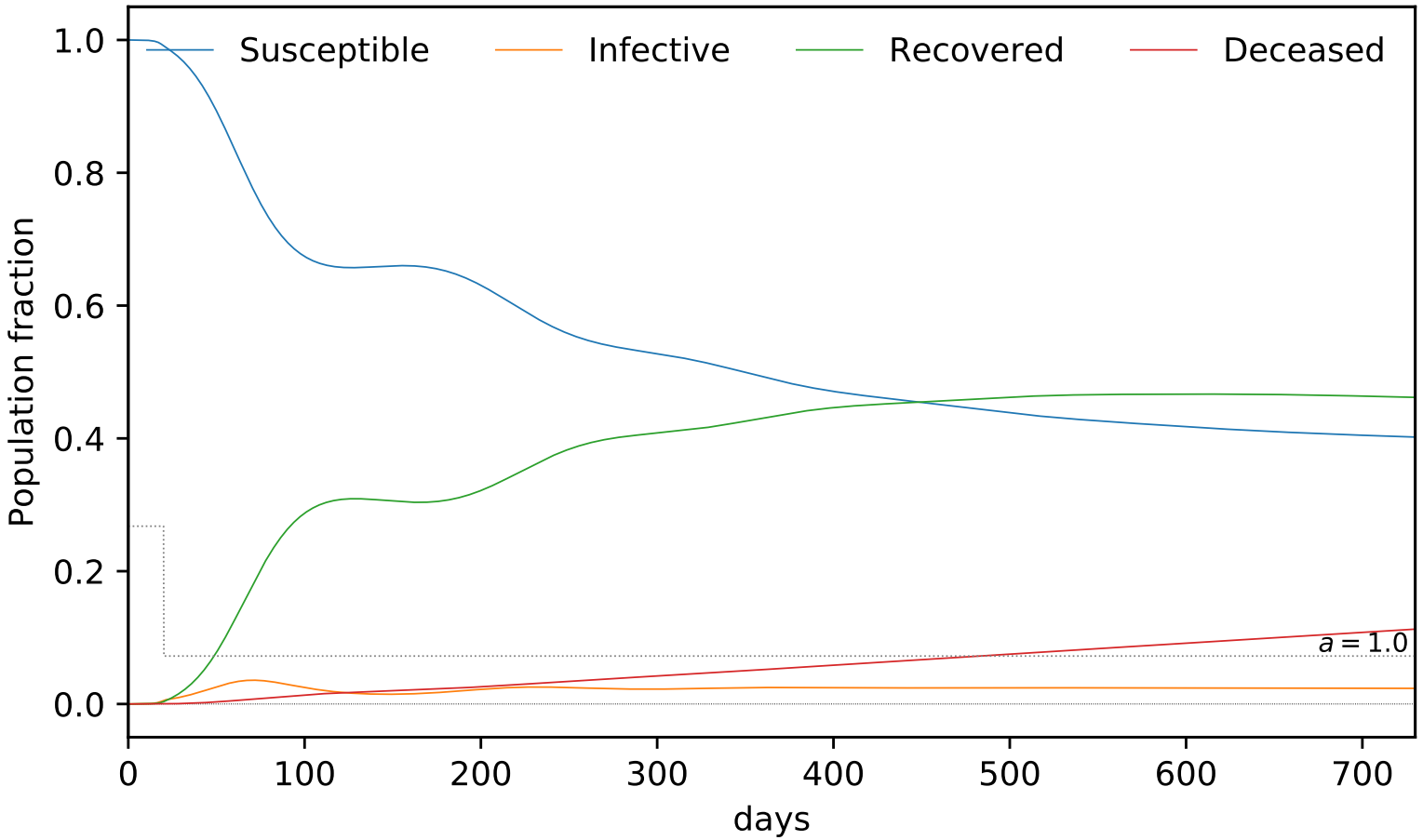
SIRD with perfect lockdown



SIRD with perfect lockdown

- $a = 1$ if $t \geq 20$, $a = b$ if $t < 20$, $b = 2.6$, $f = 0$, $g = 0.966$, $h = 0.034$
- 80% of population in continuous lockdown after 20 days from the onset of epidemic. Small number of deaths, no epidemic waves.

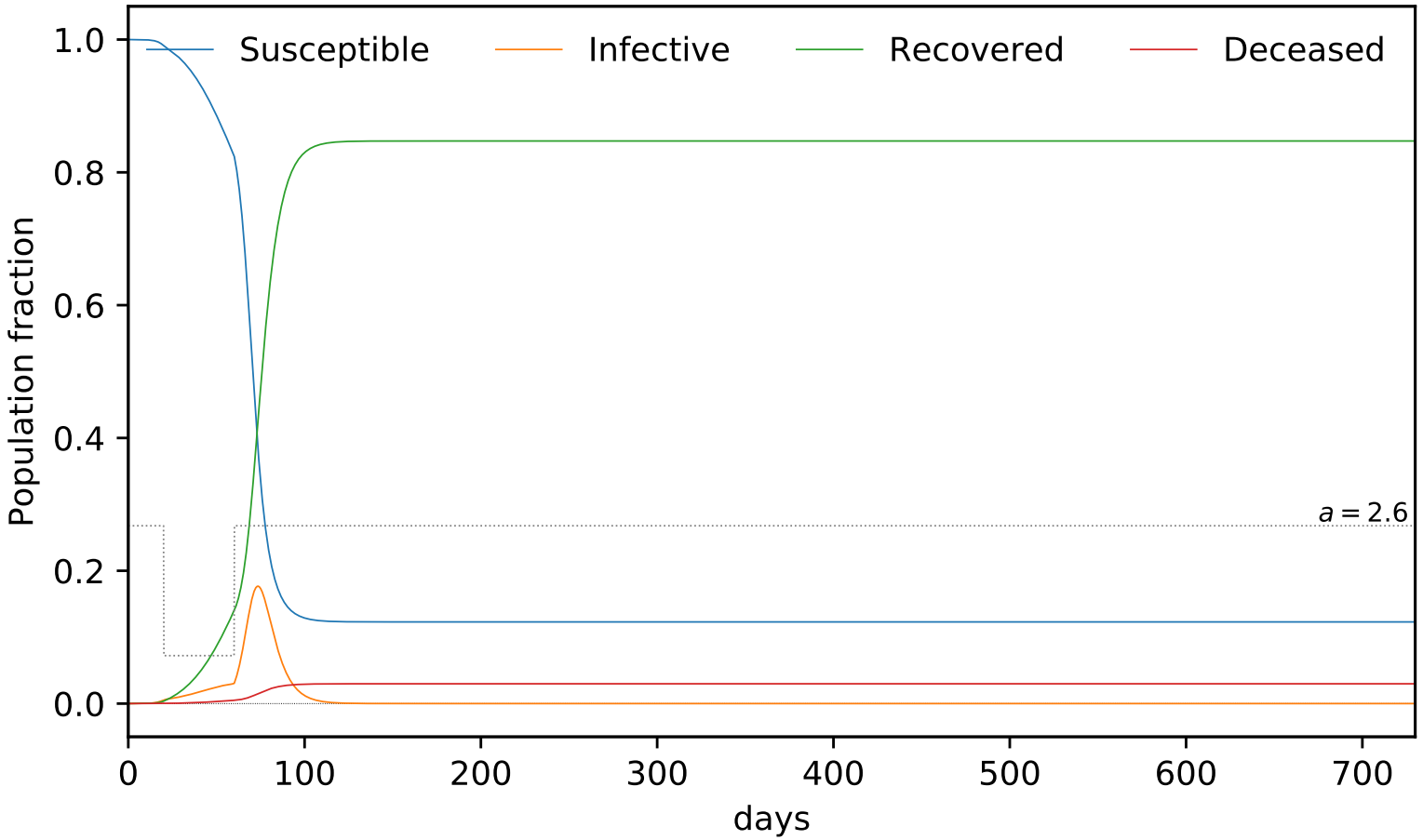
SIRD with perfect lockdown and lack of immunity



SIRD with perfect lockdown and lack of immunity

- $a = 1$ if $t \geq 20$, $a = b$ if $t < 20$, $b = 2.6$, $f = 0.05$, $g = 0.966$, $h = 0.034$
- 80% of population in continuous lockdown after 20 days, 5% of Recovered without immunity. Smaller number of deaths and epidemic waves.

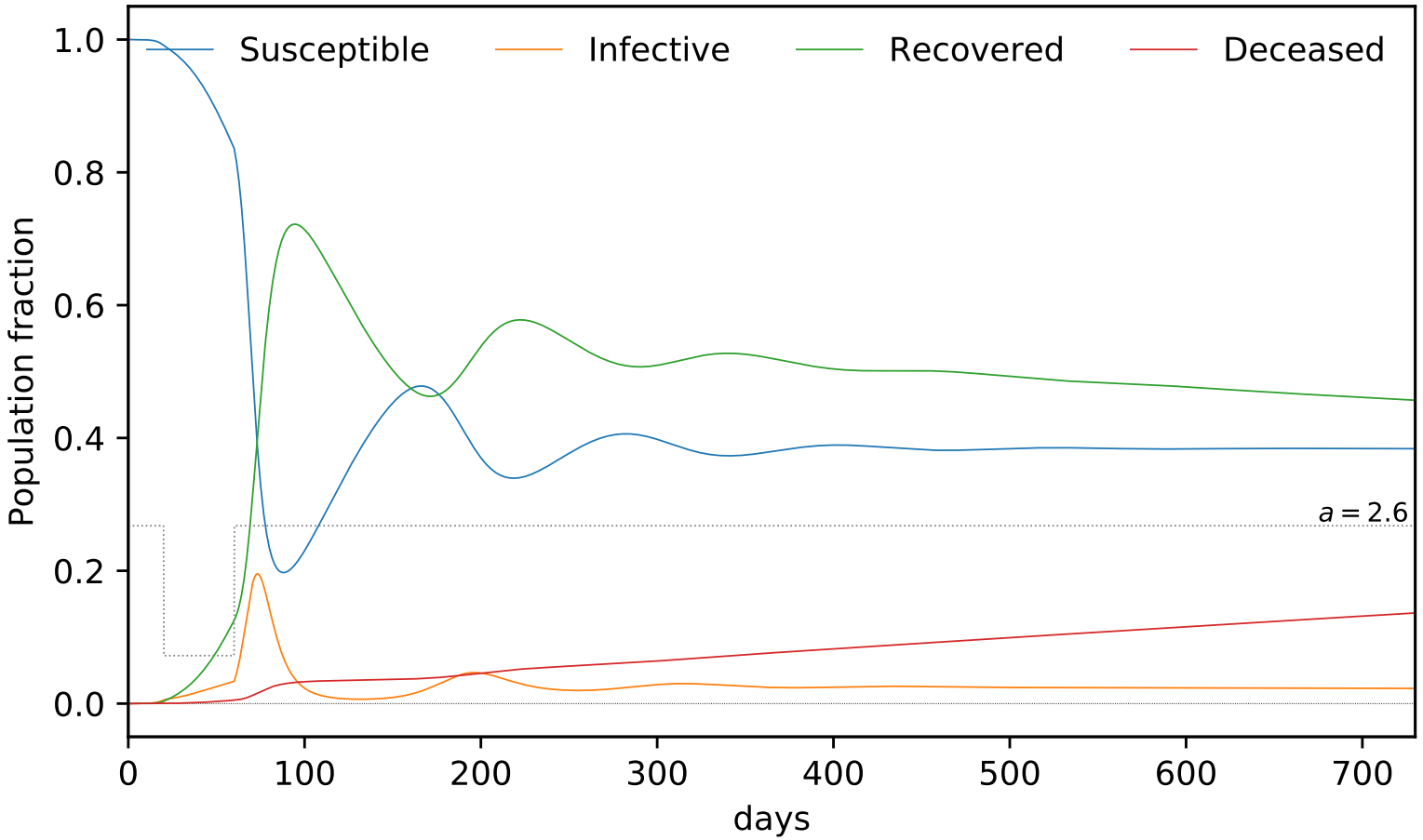
SIRD with step lockdown-relaxation



SIRD with step lockdown-relaxation

- $a = 1$ if $20 \leq t < 60$, $a = b$ otherwise, $b = 2.6$, $f = 0$, $g = 0.966$, $h = 0.034$. 80% of population in lockdown for 40 days, perfect immunity. Big infective peak after relaxing the lockdown.

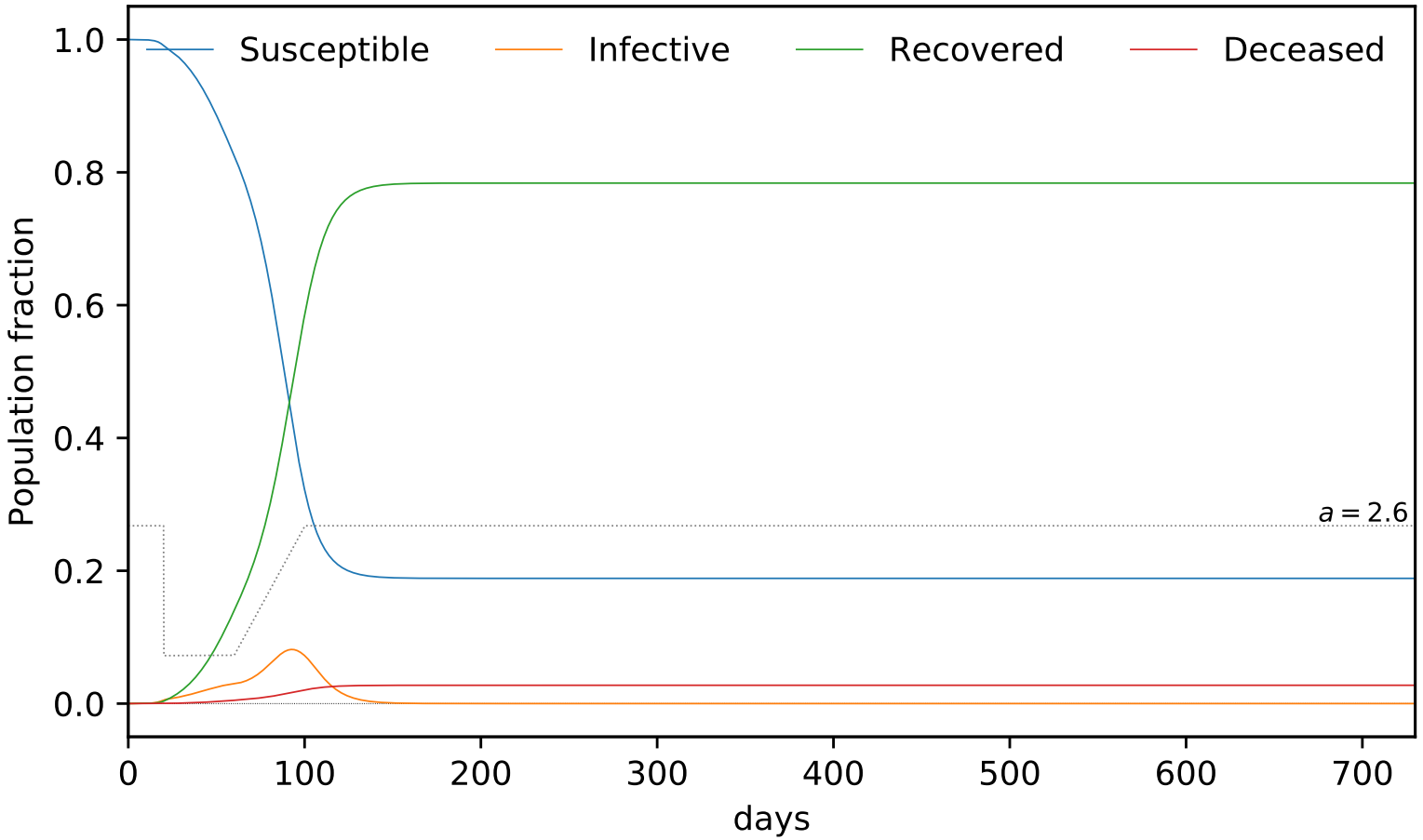
SIRD with step lockdown-relaxation and lack of immunity



SIRD with step lockdown-relaxation and lack of immunity

- $a = 1$ if $20 \leq t < 60$, $a = b$ otherwise, $b = 2.6$, $f = 0.05$, $g = 0.966$, $h = 0.034$. 80% of population in lockdown for 40 days, 5% lack of immunity. Big infective peak after relaxing the lockdown.

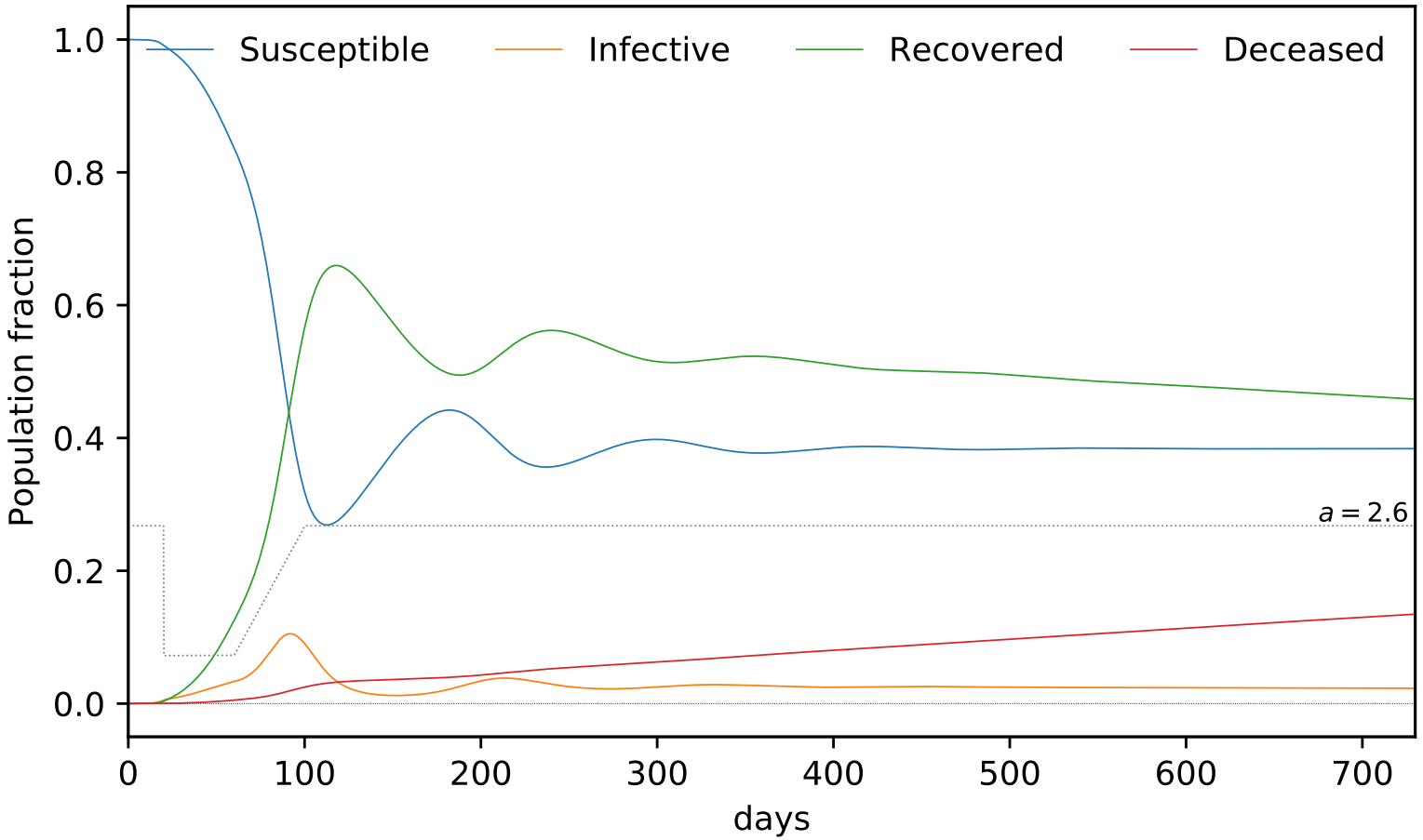
SIRD with step-linear lockdown-relaxation



SIRD with step-linear lockdown-relaxation

- $a = 1$ if $20 \leq t < 60$, $a = a(t), 60 \leq t < 100$, $b = 2.6$, $f = 0$, $g = 0.966$, $h = 0.034$. 80% of population in lockdown for 40 days, perfect immunity. Linear relaxation 40 days. Medium infection peak during the relaxation.

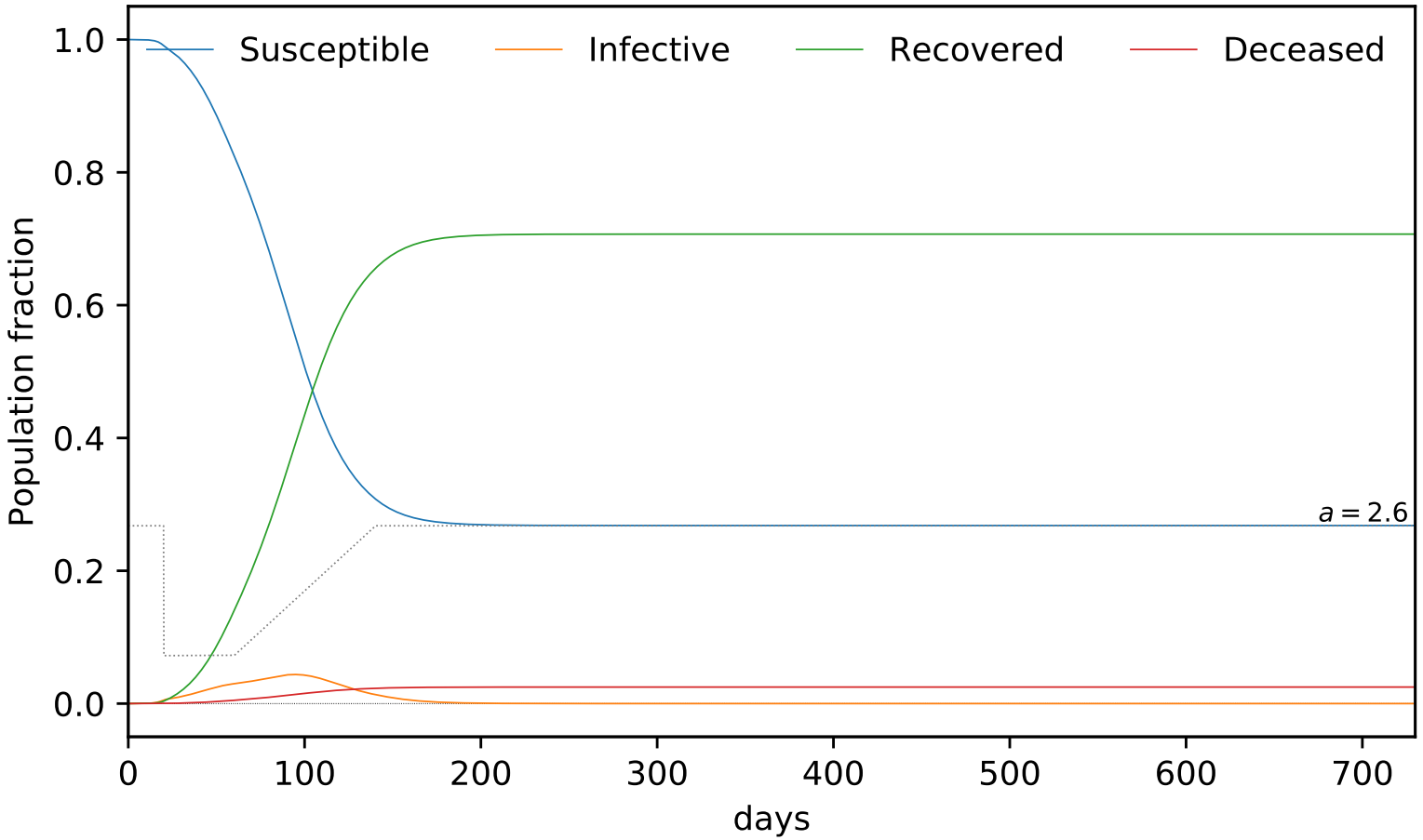
SIRD with step-linear lockdown-relaxation and lack of immunity



SIRD with step-linear lockdown-relaxation and lack of immunity

- $a = 1$ if $20 \leq t < 60$, $a = a(t), 60 \leq t < 100$, $b = 2.6$, $f = 0.05$, $g = 0.966$, $h = 0.034$. 80% of population in lockdown for 40 days, 5% lack immunity. Linear relaxation 40 days. Medium infection peak and waves.

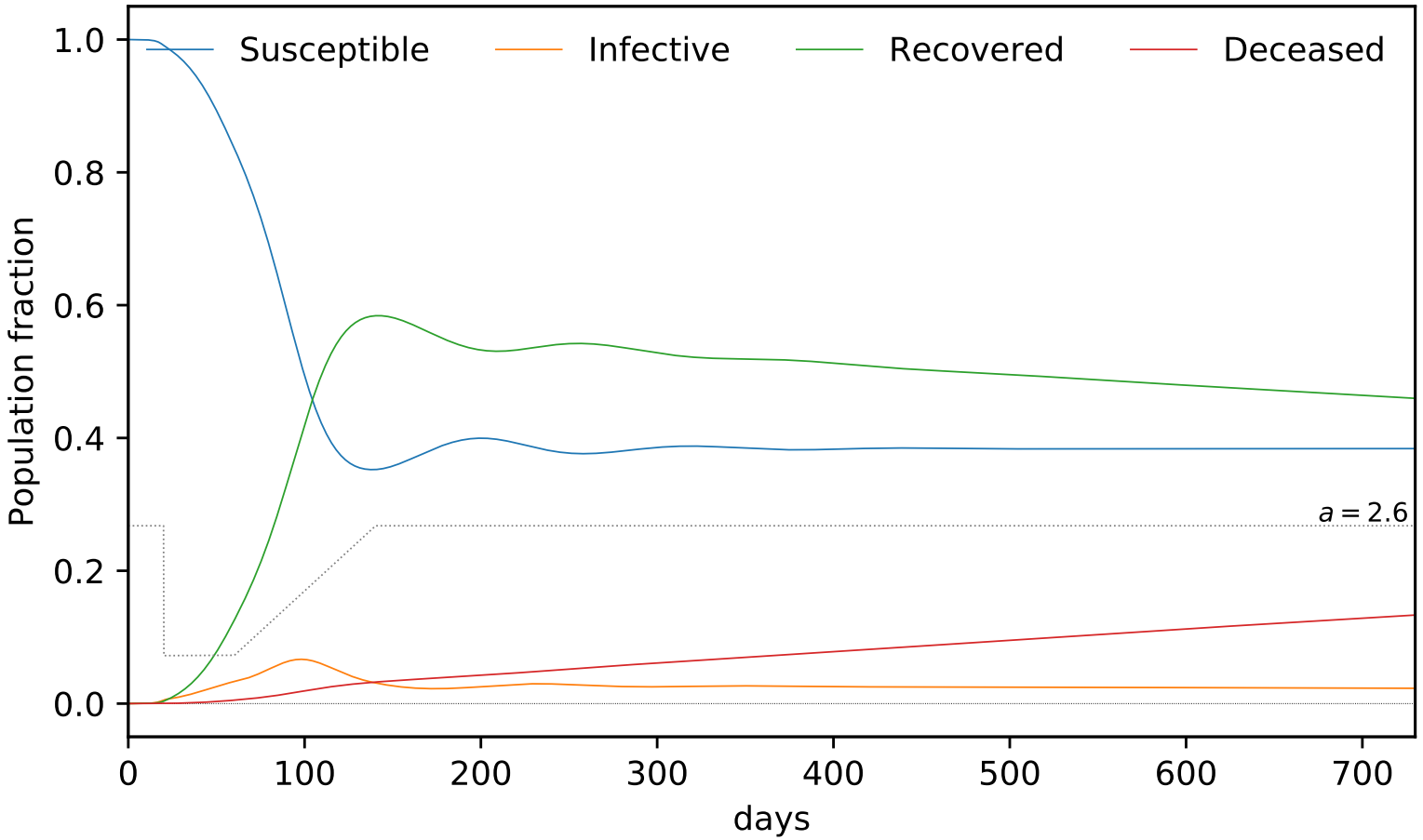
SIRD with step-linear lockdown-relaxation



SIRD with step-linear lockdown-relaxation

- $a = 1$ if $20 \leq t < 60$, $a = a(t), 60 \leq t < 140$, $b = 2.6$, $f = 0$, $g = 0.966$, $h = 0.034$. 80% of population in lockdown for 40 days, perfect immunity. Linear relaxation for 80 days. Smaller infection peak during the relaxation.

SIRD with step-linear lockdown-relaxation and lack of immunity



SIRD with step-linear lockdown-relaxation and lack of immunity

- $a = 1$ if $20 \leq t < 60$, $a = a(t), 60 \leq t < 140$, $b = 2.6$, $f = 0.05$, $g = 0.966$, $h = 0.034$. 80% of population in lockdown for 40 days, 5% lack immunity. Linear relaxation for 80 days. Smaller infection peak and waves.

Conclusions

- The step lockdown-relaxation cannot flatten the peak of possible epidemic flare up.
- The step-linear lockdown-relaxation requires long time to flatten the peak of possible epidemic flare up.
- The lack of immunity is particularly worrying because it can create waves of epidemics and it can keep the virus active for long time.
- The number of deaths can be very high if the death rate is 3.4% (current WHO estimate), and the immunity only reaches 95%, with 5% of Recovered becoming Susceptible again.

Thank You!