Sequential Search with Flexible Information [*]

Pavel Ilinov[†]
École Polytechnique Fédérale de Lausanne

Andrei Matveenko[‡]
University of Mannheim

Salil Sharma[§]
Princeton University

Elias Tsakas[¶]
Maastricht University

Mark Voorneveld[‖]
Stockholm School of Economics

November 10 2024, latest version here

Abstract

We consider a model of sequential search in which an agent (the employer) has to choose one alternative (a candidate) from a finite set. A key feature of our model is that the employer is not restricted to specific forms of information acquisition, i.e., she is free to endogenously choose any interview for each candidate that arrives. Our main characterization result shows that the employer's unique optimal strategy is to offer a gradually easier interview to later candidates. Further, we study whether the candidates are treated equally in terms of the probability of being hired; we show that the discrimination created by the order of consideration depends crucially on the functional form of the learning cost. Finally, we characterize a wide range of cases where the employer prefers to start by interviewing ex-ante worse candidates.

JEL-codes: D81, D83, D91, J71.
Keywords: sequential search, rational inattention, discrete choice.

[*]We are grateful to Nageeb Ali, Georgios Angelis, Yair Antler, Sarah Auster, Luigi Butera, Francesc Dilmé, Pierre Fleckinger, Duarte Gonçalves, Pavel Kocourek, Eugen Kovac, Daniel Krähmer, Jimmy Martinez-Correa, Filip Matějka, Fabio Michelucci, José L. Moraga, Marek Pycia, David Ronayne, Rastislav Rehák, Max Riegel, Nicolas Schutz, Alexander Sebald, Rava da Silveira, Yiman Sun, Karol Szwagrzak, Yiannis Vailakis, Mark Whitmeyer and numerous participants at various conferences and seminars for helpful comments and valuable feedback. Funding by the German Research Foundation (DFG) through CRC TR 224 (project B03) is gratefully acknowledged.

[†]Transport and Mobility Laboratory, Ecole Polytechnique Fédérale de Lausanne, Switzerland (pawa.ilinov@gmail.com)

[‡]Department of Economics, University of Mannheim, Germany (matveenko@uni-mannheim.de)

[§]Department of Politics, Princeton University, USA (ss5924@princeton.edu)

[¶]Department of Microeconomics and Public Economics, Maastricht University, The Netherlands (e.tsakas@maastrichtuniversity.nl)

[‖]Department of Economics, Stockholm School of Economics, Sweden (mark.voorneveld@hhs.se)

1 Introduction

Since the seminal work of Stigler (1961), search models have had a prominent place both within the microeconomic and macroeconomic literature (e.g., see Anderson and Renault, 2018; Chade et al., 2017; Mortensen and Pissarides, 1999). Multiple applications can be studied within this framework, e.g., a consumer's search through different alternatives before purchasing one, an employer's search through different candidates before hiring one, an employee's search through different jobs before accepting one, etc. The common denominator in all of these situations is that search involves some kind of friction, which, at the end of the day, is what makes the search problem non-trivial.

Within this literature, sequential search models are considered a benchmark setting for studying such problems (McCall, 1965; Weitzman, 1979). In this basic environment, an agent goes through a number of options, acquiring information about them sequentially, before stopping at some point and selecting one. While this model describes a rather stylized search procedure, it is still parsimonious enough to accommodate most applications of interest, and thus allows us to study most interesting features of search problems. In this setting, frictions usually arise as the cost of information acquisition or as waiting costs.

In this paper, we focus on sequential search, where the agent can flexibly differentiate the information she collects about each option. The running application that we use throughout the paper is one where an employer sequentially interviews candidates. The reason why we are interested specifically in the role of flexible information acquisition is twofold: first, flexibility in the interviewing process is quite common in practice, as employers often keep calibrating their questions throughout the interviewing process (Forbes, 2023); second, by allowing full flexibility, we can study the employer's best case scenario, thus obtaining an upper bound on what the employer can achieve in terms of efficiency. The latter will allow us to address questions regarding the optimal order of interviewing candidates (Weitzman, 1979) or regarding fairness concerns and biases that stem from the interviewing process (Bertrand and Duflo, 2017; Neumark, 2018; Schlag and Zapechelnyuk, 2024).

Our formal model is a variant of the Pandora's boxes (Weitzman, 1979) with a rationally inattentive employer (Sims, 2003). In the benchmark case, the type of each candidate is drawn independently from the same (Bernoulli) distribution. Of course, the actual types remain unobservable to the employer. The candidates arrive for interviews sequentially at a fixed order. A candidate's interview takes the form of a usual Bayesian signal which is chosen by the employer, and crucially may depend on the realizations of the earlier interviews. Aligned with the rational inattention literature, the cost of each interview is posterior-separable (Caplin et al., 2022; Denti, 2022): in this sense, our model covers as special cases all the commonly-used cost specifications, including entropic costs. Upon observing the realized signal of an interview, the employer may either reject the candidate and proceed to the next interview or hire him right away. In this sense, our model maintains the no-recall assumption, which is common in many papers in the literature on committed Pandora's boxes (Beyhaghi and Cai, 2023, Section 2.2), on the secretary's problem (Correa et al., 2024, and references therein), as well as in the more extensive search literature within labor economics (Mortensen and Pissarides, 1999). Besides being common in previous work, this assumption is naturally justified in settings where the candidates have outside options and/or big egos that do not allow them to consider employers that previously rejected them. In this sense, it fits well in job markets of highly-skilled candidates, as well as labor markets with a thick supply side and risk-averse candidates.

We start by asking a general question: what is the employer's optimal interviewing strategy? We characterize the optimal strategy by reducing the dynamic information acquisition problem into a static one (Theorem 1). This characterization uncovers a special feature of the employer's behavior: we show that candidates who are interviewed earlier face a "more difficult interview. " That is, the optimal interviews are ranked with respect to a novel order – difficulty. Remarkably, this order turns out to have striking similarities with the likelihood ratio dominance order for lotteries (Shaked and Shanthikumar, 2007, and references therein), the information bias order (Gentzkow et al., 2014), and a toughness measure in the context of product testing (Gill and Sgroi, 2012).

The implications of the previous result are twofold. First, the employer has the luxury of overshooting for very high expected quality in early candidates, as there are still plenty of candidates to come. Second, by having a large expected quality of a failed early candidate, the employer makes the interview of this early candidate relatively cheap, thus balancing the risk that she undertakes (via overshooting) and the information acquisition cost that she has to incur.

Our paper continues with another question that has interesting practical implications for recruiters, candidates, regulators, etc. We ask whether sequential search with flexible information acquisition is discriminative, in the sense of ex-ante identical candidates having non-equal chances of being hired (Bertrand and Duflo, 2017; Neumark, 2018). Opinions among practitioners on this subject wildly differ, typically based on psychological arguments, i.e., some hiring managers suggest that it is better to be interviewed first because of primacy bias, whereas others suggest that it is better to go last because of recency bias (Forbes, 2019, 2023). Here, we approach this problem from a completely different angle, focusing on the fact that the optimal interview induces a tradeoff, viz., early candidates have a higher probability of being interviewed in the first place, but at the same time, they have a lower probability of being hired conditional on being interviewed because of the more difficult interview they face. Thus, it is not ex-ante clear which of the two effects will dominate the other.

We study the aforementioned question in the context of the two most common cost functions (namely, quadratic and entropic information costs). Except for some extreme cases, the candidates will be treated differently in terms of the total probability of being hired, although, ex-ante, they only differ in the order of being interviewed. In particular, the only case where they will not be differentiated is when there are two candidates, and the costs are quadratic. A similar conclusion is reached in a very different model by Schlag and Zapechelnyuk (2024), who show that only very specific interviewing protocols guarantee fair treatment of the candidates.

Back to our context, if we instead look at the case of two candidates and entropic costs, the way discrimination enters the picture is determined by the prior, i.e., the first candidate is favored for large priors, while the second candidate is favored for small priors (Theorem ??). This means that the answer to our question is very sensitive to parameters that are difficult to observe and test directly, which in turn may explain why practitioners are not settled on a clear conclusion.

But then, something quite unexpected happens. If we increase the number of candidates, the first one seems to be the one who is clearly favored, both in the quadratic and the entropic case (Theorem ??). In this sense, we get a rather strong prediction, which —unlike the folk explanation that circulates among practitioners— does not rely on any behavioral bias. Instead, it is simply the consequence of the employer's rational strategy.

In the final main part of the paper, we relax the assumption that candidates are ex-ante identical. Within this more general framework, we first naturally ask whether our results about the employer's optimal interview carry; and second, we ask whether the employer prefers to start by interviewing better or worse candidates first. Once again, similarly to the question about discrimination, this is a question to which practitioners do not agree (Selby, 2023). Starting with the first of these two last questions, we show that our previous result about the difficulty of the interview still holds, i.e., earlier interviews are easier than later ones. Then, turning to the second question in a setting with two candidates, we show that under quadratic costs, the order does not matter. However, with entropic costs, for large priors, it is often better to start with the better candidate, whereas for small priors, it is often better to start with the worse candidate.

Finally, we consider a simple extension of our model. Namely, we discuss a naive policy that forbids discrimination in the interview process: we force the employer to choose the same interview for all candidates. We show that in this case, the optimal interview that is used for all candidates is easier than the interview for the first candidate in the unrestricted case but more difficult than the interview for the second-to-last candidate. The consequence is immediate: the probability of hiring the first candidate in the restricted case is higher than the same probability in the unrestricted case. That is, in many cases, the outcome of the naive policy is the opposite of the desired one: if the first candidate had preferential treatment without restrictions, under the restricted interview policy, this candidate is treated even better.

Our results are important for several strands of literature. First, our work should be primarily seen as part of the literature on Pandora's boxes (Weitzman, 1979), and in particular on the case with no-recall (Salop, 1973). The main difference to our setting is that in Pandora's boxes, information acquisition boils down to an all-or-nothing decision, i.e., in our language, the employer will either learn the type of candidate with certainty (perfectly informative interview) or will not learn anything at all (completely uninformative interview). In addition, the early papers did not allow a candidate to be hired without having been interviewed first, something which was later permitted by Doval (2018). For an excellent recent overview of this literature, we refer to Beyhaghi and Cai (2023).

Within this literature, particularly relevant to our work are the recent papers of Schlag and Zapechelnyuk (2024), where the problem of fair sequential interviews is studied, and the one of Ursu et al. (2020), which allows for flexible information acquisition but restricts attention to a specific information acquisition technology.

Related to this literature is the one on the secretary's problem (Correa et al., 2024, and references therein). This literature can be seen as a stream of sequential search with unknown distribution of types. The common feature to our work is that they also typically assume no recall.

Second, our work can be seen as part of a broad stream within the dynamic rational inattention literature that focuses on the timing of information acquisition (Steiner et al., 2017; Morris and Strack, 2019; Zhong, 2022; Hebert and Woodford, 2023). There is variation in the underlying assumptions that they impose, e.g., some allowing for flexible information acquisition, some assumming discounting, some considering continuous time. At the same time, all these papers differ from ours in that they allow for information acquisition about the entire state space at any point in time. In our context this would imply that the employer can potentially interview multiple candidates at any point in time, and may also invite back candidates for follow-up interviews. While this assumption certainly makes sense for the applications that these authors have in mind, it seems less appealing in the job market setting that we have in mind as our main application.

Third, our paper is incidentally related to the literature on ordering Bayesian signals, by introducing our difficulty order, which is equivalent to the toughness order that Gill and Sgroi (2012) use in a different context. A similar order has also been used by Gentzkow et al. (2014) and Charness et al. (2021) in an attempt to model information biases. All of these orders share striking similarities with the likelihood ratio dominance order (Shaked and Shanthikumar, 2007).

Finally, there is a large literature on ordered consumer search, studying dynamic information acquisition about different consumption choices before an eventual purchase decision is made (for an overview, see Armstrong, 2017). The main difference to our work is that the focus of this literature is on the role of different asymmetries across choices, e.g., which product does the consumer inspect first when the products differ in price or differ in inspection costs? Similarly to the literature on Pandora's Boxes, most of the work on ordered consumer search imposes strict exogenous assumptions on the information acquisition technology, which at the outset seems quite natural in the context of the corresponding applications. To the best of our knowledge, the one exception is the paper of Jain and Whitmeyer (2021) where the effect of flexible information acquisition on market outcomes is studied.

2 Model

We study a (female) employer who considers an ordered set of a priori identical (male) candidates $I = \{1, \dots, T\}$. Each candidate $i \in I$ is associated with a type

$$\theta_i \in \Theta := \{\text{Good}, \text{ Bad}\} = \{G, B\},$$

which is independently drawn from the same distribution that assigns probability $\mu \in (0, 1)$ to the good type G.[1]

[1]The binary type assumption is not essential for our analysis. Our results hold in a more general setting, more specifically, when the utility of the employer only depends on the beliefs

The employer must choose one candidate, and there is no outside option. Before making a decision, she may acquire information about the candidates' types. Information acquisition is sequential, following the candidates' order. That is, at stage i, the employer selects a Blackwell experiment $\sigma_i : \Theta \to \Delta(S_i)$ for candidate i. Upon observing a signal realization $s \in S_i$, she forms a posterior distribution on Θ, that we identify with her posterior belief about the state G

$$p^s := \frac{\mu\sigma_i(s|G)}{\mu\sigma_i(s|G) + (1-\mu)\sigma_i(s|B)}.$$

It follows from the work of Kamenica and Gentzkow (2011) that each experiment is identified by a mean-preserving distribution of posteriors, i.e., by some $\pi_i \in \Delta([0,1])$ such that $\mathbb{E}_{\pi_i}(p) = \mu$. A fully uninformative signal is one that puts probability 1 to the prior μ.

After having updated to belief p_i^s about candidate i, the employer either hires i or proceeds to interview the next candidate $i+1$. We assume no recall, i.e., if a candidate is not hired right after an interview, he is no longer available to the employer. There can be several rationales for such assumption: For instance, it can be driven by psychological factors of the rejected agent (e.g., pride, frustration, etc.), or by conditions on the labor market (e.g., there is excess labor demand and the rejected candidate is hired immediately by another firm), or because of institutional rules (e.g., HR rules dictate that every candidate is only interviewed once). Thus, the employer chooses an action

$$a_i \in A_i := \{0, 1\}$$

following the realization of an interview for candidate i, where action 0 corresponds to not hiring a candidate and 1 to hiring. Hiring a good candidate brings utility 1 and hiring a bad candidate 0.

Formally, a non-terminal history at round $i \in \{1, 2, \ldots, T\}$ is identified by the set of realized posteriors for all candidates $j \in \{1, \ldots, i-1\}$, i.e.,

$$\mathcal{H}_i := [0,1]^{i-1}.$$

The employer's action at every $h \in \mathcal{H}_i$, consists of a signal, that leads to a posteriors π_i^h and a mapping $\alpha_i^h : \text{supp}(\pi_i^h) \to A_i$. Whenever, $p \in \text{supp}(\pi_i^h)$ is realized and $\alpha_i^h(p) = 1$ is chosen, a terminal history is reached, and candidate i is hired. In case round T is reached, the last candidate will be hired regardless of the realization of the respective signal. A typical strategy of the employer is henceforth denoted by $(\boldsymbol{\pi}, \boldsymbol{\alpha})$.

Information acquisition is costly. In line with the rational inattention literature we consider posterior separable costs (Caplin et al., 2022; Denti, 2022): signal π_i costs

$$C(\pi_i) = \lambda\bigg(\mathbb{E}_{\pi_i}[c(p)]\bigg), \tag{1}$$

about posterior means as is typically assumed in the literature of the information design or costly information acquisition, for example, see Arieli et al. (2023) and Mensch and Malik (2023) for the recent references. We hold the binary type assumption mainly for the simplicity of the exposition.

where $\lambda \in \mathbb{R}_{++}$ is the marginal cost of information and $c : [0, 1] \to \mathbb{R}$ is continuous, strictly convex and smooth on the interior of the unit interval.

Throughout the paper, we impose the following assumption:

$$\textsc{Boundary condition} \quad : \quad \lim_{p \to 0^+} c'(p) + \frac{1}{\lambda} < c'(\mu) < \lim_{p \to 1^-} c'(p) - \frac{1}{\lambda}. \tag{2}$$

Intuitively this puts a lower bound on the marginal cost at the boundaries, i.e., learning the true state with certainty is expensive. Throughout the paper, we will denote the set of priors that satisfy the boundary condition by $M \subseteq (0, 1)$.[2] It is not difficult to verify that M will be an open interval.

The two most common special cases are the quadratic and the entropic costs, which we will use in most applications throughout the paper.

Example 1. We say that the cost function is ***quadratic*** whenever

$$c(p) = \lambda \Big(p - \mu \Big)^2.$$

◁

Example 2. We say that the cost function is ***entropic*** whenever

$$c(p) = \lambda \Big(-\mu \log \mu - (1 - \mu) \log(1 - \mu) - (-p \log p - (1 - p) \log(1 - p)) \Big).$$

Recall that $-p \log p - (1 - p) \log(1 - p)$ is known as Shannon entropy. ◁

If the employer chooses the action (π_i^h, α_i^h) at history $h \in \mathcal{H}_i$, the expected payoff that she will want to maximize is equal to

$$\mathbb{E}_{\pi_i^h} \Big[\alpha_i^h(p)p + (1 - \alpha_i^h(p))V_i - \lambda c(p) \Big],$$

where V_i denotes her maximum net expected payoff in case she continues and interviews candidate $i + 1$. Note that V_i depends only on the number of remaining candidates, as the types of the different candidates are drawn independently from the same probability distribution, and there is no recall possibility. Hence, without loss of generality, we can restrict attention to $\mathcal{H}_i$-measurable strategies, i.e., to strategies such that $(\pi_i^h, \alpha_i^h) = (\pi_i, \alpha_i)$ for all $h \in \mathcal{H}_i$. This means that the employer's expected payoff is simplified to

$$\mathbb{E}_{\pi_i} \Big[\alpha_i(p)p + (1 - \alpha_i(p))V_i - \lambda c(p) \Big]. \tag{3}$$

Definition 1. *The employer's* ***dynamic optimization problem*** *is*

$$\max_{(\pi_i, \alpha_i)} \mathbb{E}_{\pi_i} \Big[\alpha_i(p)p + (1 - \alpha_i(p))V_i - \lambda c(p) \Big], \tag{4}$$

subject to

$$V_i = \max_{(\pi_{i+1}, \alpha_{i+1})} \mathbb{E}_{\pi_{i+1}} \Big[\alpha_{i+1}(p)p + (1 - \alpha_{i+1}(p))V_{i+1} - \lambda c(p) \Big], \tag{5}$$

$$V_T = 0. \tag{6}$$

[2] The boundary condition will be trivially satisfied for every $\mu \in (0, 1)$ if and only if c does not become not infinitely steep at the boundaries of the unit interval, i.e., formally, $M = (0, 1)$ if and only if c is not subdifferentiable at both 0 and 1.

Constraint (5) ensures dynamic consistency, that is DM behaves optimally in every history. Constraint (6) captures the intuition that if the final candidate is indeed reached, it implies that the DM has rejected all candidates before. In that case DM rejects all candidates and ends up with zero payoff.

3 Optimal interviewing strategy

The optimal strategy $(\boldsymbol{\pi}, \boldsymbol{\alpha})$ in problem (4) is the collection of the optimal actions (π_i, α_i) for all $i \in I$. Note that the interview design problems at some stages i, j differ only by their continuation values V_i, V_j. Therefore, in the dynamic problem, the employer behaves as if she solves a collection of static problems with different continuation values. These continuation values are determined from the future behavior of the employer, and the value is *exogenous* at stage i. Thus, at stage i, a continuation value V_i serves the role of an outside option to the employer. Therefore, we conclude that at each stage i, the employer solves a static problem with an exogenous outside option. A static problem with an exogenous outside option is a building block for the dynamic problem, and we discuss a static problem in great detail in this section.[3]

Additionally, we observe that at stage i given the realized value p the employer simply selects candidate i if $p \geq V_i$ and continues search otherwise, and therefore in our previously-stated optimization problem we can replace $\alpha_i(p)p+(1-\alpha_i(p))V_i$ with $\max\{p, V_i\}$. Thus, the optimization problem at stage i boils down to the following (static) optimization problem with parameter $V := V_i$.

Definition 2. *The* ***static optimization problem*** *(or the problem with exogenous outside option) is*

$$\max_{\pi} \mathbb{E}_{\pi}\Big[\underbrace{\max\{p, V\} - \lambda c(p)}_{\phi(p,V,\lambda)}\Big]. \tag{7}$$

We discuss properties of the static problem using two technical lemmas. The first lemma characterizes the solution exploiting the convexity and differentiability of the function $c(p)$.

Lemma 1. *The following statements hold:*

1. *The solution to problem (7) exists, and it is unique and interior.*

2. *There exist two thresholds $V_L, V_H \in (0,1)$ with $V_L < \mu < V_H$, such that the optimal signal π_V satisfies:*

$$\begin{aligned} V \leq V_L \quad &\Longrightarrow \quad \operatorname{supp}(\pi_V) = \{\mu\}, \\ V_L < V < V_H \quad &\Longrightarrow \quad \operatorname{supp}(\pi_V) = \{p_V^L, p_V^H\}, \\ V \geq V_H \quad &\Longrightarrow \quad \operatorname{supp}(\pi_V) = \{\mu\}. \end{aligned}$$

[3]Such a problem is a variant of the problem of a rationally inattentive agent with an exogenous outside option, see, e.g., Matějka and McKay (2015), Caplin and Dean (2013) for the reference.

3. The optimal hiring decision is given by the following:

$$
\begin{array}{llllll}
V \leq V_L & & & \Longrightarrow & \alpha(\mu) = 1, \\
V_L < V < V_H & \textit{and} & p = p_V^H & \Longrightarrow & \alpha(p) = 1, \\
V_L < V < V_H & \textit{and} & p = p_V^L & \Longrightarrow & \alpha(p) = 0, \\
V \geq V_H & & & \Longrightarrow & \alpha(\mu) = 0.
\end{array}
$$

The previous lemma is illustrated in Figure 1 below. The idea is that $\phi(p, V, \lambda)$ consists of two strictly concave parts, with a kink at V. This induces the two posteriors p_V^L and p_V^H, and as the prior lies between these two the employer will acquire an informative signal that distributes its probability to these two posteriors; otherwise, she will pick the completely uninformative signal. These posteriors are obtained, for example, using the concavification technique as in Caplin and Dean (2013).[4]

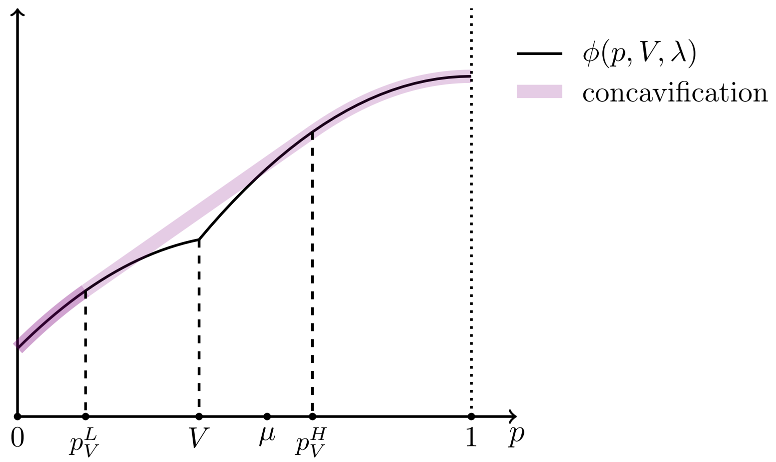

Figure 1: The employer's static payoff function and its concavification when $V \in (V_L, V_H)$.

The key observation is that both p_V^L and p_V^H are in general continuously increasing in V (see Lemma B4 in the Appendix for a formal proof). Moreover, we have

$$\lim_{V \to 0^+} p_V^L = \lim_{V \to 0^+} p_V^H = 0 \text{ and } \lim_{V \to 1^-} p_V^L = \lim_{V \to 1^-} p_V^H = 1.$$

Hence, for sufficiently large V, the whole interval $[p_V^L, p_V^H]$ will lie to the right of μ. Likewise for sufficiently small V, the interval will lie to the left of μ. Thus, we can define the two thresholds:

$$
\begin{aligned}
V_H &:= \min\{V \in [0, 1] : p_V^L \geq \mu\}, \\
V_L &:= \max\{V \in [0, 1] : p_V^H \leq \mu\}.
\end{aligned}
$$

[4]For recent use of concavification to the related rationally inattentive problems, see, e.g., Jain and Whitmeyer (2021), Kim et al. (2022).

Note that by the prior $\mu \in (0,1)$ being full-support, we obtain both $V_L \in (0,1)$ and $V_H \in (0,1)$.

In the dynamic problem of the employer, an outside option at stage i equals the value of the problem at stage $i+1$. In its turn, this value equals the maximal attained value in a static problem (7) for a particular value of outside option V. To study the maximal attained value in static problem we define a value function $g : [0,1] \to [0,1]$ such that

$$g(V) = \max_{\pi} \mathbb{E}_{\pi}\Big[\phi(p, V, \lambda)\Big].$$

Function $g(V)$ is clearly non-decreasing by construction, as $\phi(p, V, \lambda)$ is weakly increasing in V for every p. We note that function $g(V)$ is linear on $[0, V_L]$ and $[V_H, 1]$, namely, we have $g(V) = \mu$ if $V \leq V_L$ and $g(V) = V$ if $V \geq V_H$, as in either of these two regions the employer does not incur any costs for acquiring information, and makes a hiring decision straight away. The following lemma completes the analysis for the entire unit interval.

Lemma 2. *The function g is strictly increasing, convex and differentiable everywhere in $[0,1]$.*

We will now proceed to characterize the solution to the dynamic problem. To do so, we first define a specific sequence of static problems, by means of a sequence of outside options, namely, for each $i \in \{1, \ldots, T\}$, we have

$$V_i := g(V_{i+1}), \tag{8}$$

with $V_T = 0$.

Lemma 3. *For the sequence $(V_i)_{i=1}^T$ defined in (8), the following statements hold:*

1. *The continuation value V_i is strictly decreasing in i.*

2. *For every $i \in \{1, \ldots, T-1\}$, we have $\mu \leq V_i < V_H$.*

We illustrate Lemma 3 graphically (Figure 2). First of all, by combining convexity and differentiability of g with the fact that $g(V) = V$ for all $V \geq V_H$, it follows that $g(V) > V$ for all $V < V_H$. Hence, V_i keeps shrinking as the employer moves to later candidates. This is not surprising, as there are fewer candidates left to interview. The fact that for all candidates except the last one the outside option lies always in the learning region (V_L, V_H) holds similarly.

Theorem 1. *The solution to the dynamic problem of Definition 1 is as follows:*

1. *At every round $i \in \{1, \ldots, T-1\}$, the employer draws the signal π_{V_i} which is optimal in the static problem (according to Lemma 1), with the outside option V_i that we defined in (8). Moreover, we have:*

   (a) If $p^H_{V_i}$ is realized, the search stops and candidate i is hired.

   (b) If $p^L_{V_i}$ is realized, the search continues to candidate $i+1$.

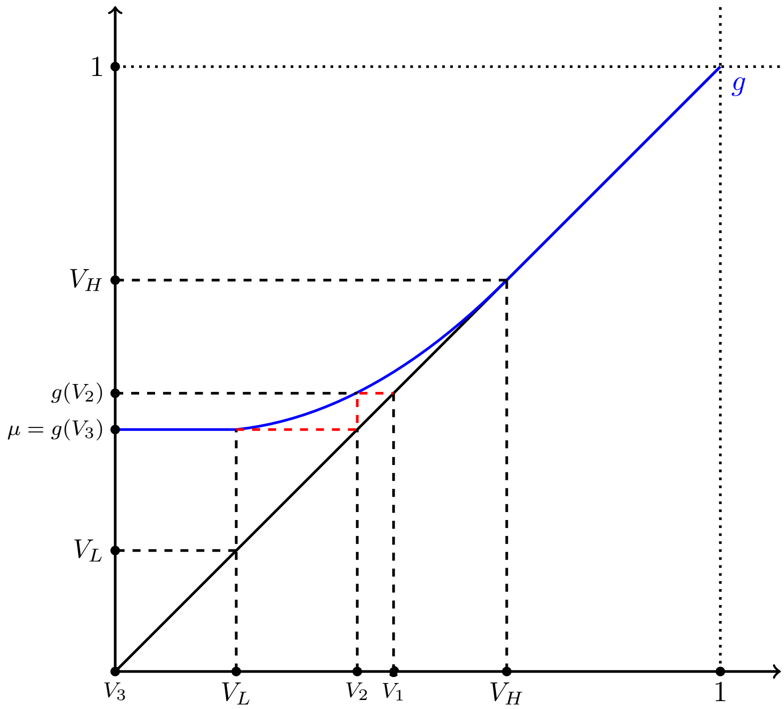

Figure 2: The sequence of outside options with three candidates.

2. *At round T, the employer does not acquire information and hires the candidate right away.*

The optimal interview design of the employer in the dynamic problem is very straightforward: she continues the search until she receives a high signal about the quality of a candidate. If only low signals have been realized during the first $T-1$ interviews, she simply chooses the last candidate with the fully uninformative signal. We employ the natural interpretation of an optimal interview at stage i as a binary test. The employer offers a test to a candidate i. If a candidate passes, he is hired; if a candidate fails, he is discarded. We state our main result about the characterization of the optimal tests in the next section.

4 Difficulty of interviews

To compare the different tests that the employer (optimally) chooses for the different candidates, we introduce the following partial order.

Definition 3. *Let π_i and and π_j be two binary tests, with $p_i^L < p_i^H$ and $p_j^L < p_j^H$ being the posteriors beliefs in the respective supports. We say that π_i is* ***more difficult than*** *π_j if*

$$p_i^L > p_j^L \text{ and } p_i^H > p_j^H.$$

The condition above has a simple interpretation. The two candidates are ex ante identical from the point of view of the employer, and they are offered one test each such that, whenever there is a tie, i is deemed better than j, i.e., in particular,

(a) if both of them pass their respective tests, the expected quality of i is higher than the expected quality of j, and

(b) if both of them fail their respective tests, the expected quality of i is higher than the expected quality of j.

The concept of test difficulty can also be demonstrated by means of a simple example. Consider two following interview tasks, the first being "Formulate and prove the Hahn-Banach theorem" and another one "Compute the value of $2+2$." The first task is clearly difficult, and if the candidate succeeds, the posterior belief about the candidate becomes relatively high, while if he fails, the belief does not go down by much. While for the second task, which is clearly easy, the beliefs dynamics are the opposite.

Let us provide some further justification. For calling π_i more difficult than π_j. Recall that the two signals π_i and π_j are respectively characterized by the underlying experiments σ_i and σ_j (see Section 2).

Definition 4. *We say that experiment σ_i likelihood-ratio dominates experiment σ_j whenever, for every signal realization $s \in \{H, L\}$,*

$$\frac{\sigma_i(s|G)}{\sigma_i(s|B)} > \frac{\sigma_j(s|G)}{\sigma_j(s|B)}.$$

The underlying idea is as follows: conditional on every test result, the relative evidence for the good type is stronger under i's interview than under j's interview. A similar relation has been used in the literature on biased information sources (Gentzkow et al., 2014; Charness et al., 2021) and in the literature on product testing (Gill and Sgroi, 2012). All these orders bear striking similarities with the likelihood ratio dominance, which is often used in the literature to compare lotteries (Shaked and Shanthikumar, 2007).

Proposition 1. *The following are equivalent:*

(i) Signal π_i is more difficult than signal π_j.

(ii) Experiment σ_i likelihood-ratio dominates experiment σ_j.

(iii) For every prior μ, the passing probability under σ_i is lower than the passing probability under σ_j, i.e.,

$$\underbrace{\mu\sigma_i(H|G) + (1-\mu)\sigma_i(H|B)}_{\text{passing probability under } \sigma_i} < \underbrace{\mu\sigma_j(H|G) + (1-\mu)\sigma_j(H|B)}_{\text{passing probability under } \sigma_j}.$$

From the previous result it follows directly that our notion of more difficult interview does not depend on the prior. This is a desirable property, satisfied by other well-known orders over the set of Bayesian experiments. The idea is that difficulty is a property of the test alone, defined independently of the candidate who takes the test.

Furthermore, if the two candidates actually share the same prior, i's interview is more difficult than j's interview if and only if the probability to pass the test is lower for i than it is for j.

Let us now state our main result which says that, in the optimal interviewing strategy, the interviews decrease in difficulty as the employer proceeds to later candidates.

Theorem 2. *In the optimal strategy from Theorem 1, the following hold:*

1. *Difficulty is decreasing with respect to the order of being interviewed, i.e., for all $i \in \{1, \ldots, T-2\}$, signal π_{V_i} is more difficult than signal $\pi_{V_{i+1}}$.*

2. *As the number of candidates grows large, we obtain:*

$$\lim_{T\to\infty} p^L_{V_1} = \mu \text{ and } \lim_{T\to\infty} p^H_{V_1} = V_H.$$

Hence, the following hold:

   (a) The probability of the first candidate being hired converges to 0.

   (b) The probability of hiring a good candidate is bounded away from 1.

In Figure 3, we show an example of an optimal learning strategy. On the vertical axis we have the number of remaining candidates, besides the one currently interviewed. So for instance, if there are ten candidates in total, we depict the optimal interviews for the first nine, recalling that the last one will be hired anyway without an interview.

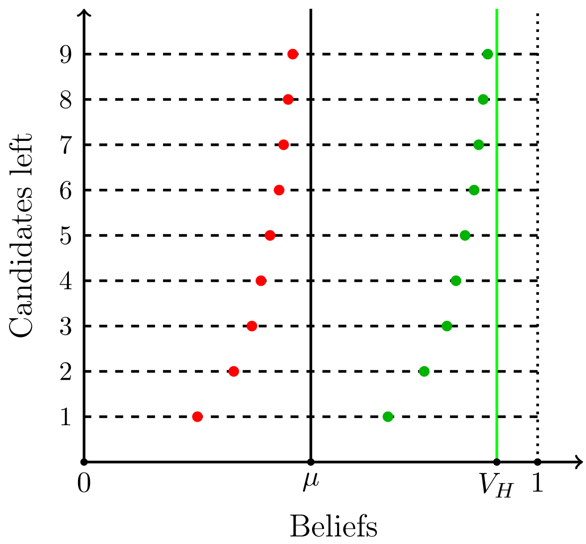

Figure 3: Optimal interviews as a function of the number of remaining candidates.

Decreasing the high posterior realizations $p^H_{V_i}$ is intuitive. If a posterior $p^H_{V_i}$ is realized on the interview i, the employer stops the search and chooses candidate i. Thus, it makes sense that in order for the employer to stop the search early, she needs to be sufficiently certain that the candidate she hires is good, as she is foregoing the chance to interview many other potential candidates.

Decreasing the low posterior realizations $p^L_{V_i}$ is less intuitive. By using such a strategy, the employer optimally procrastinates: instead of acquiring the most information during the first interviews, she wants to spread expected information acquisition towards all interviews. Intuitively, during the first interviews, she offers hard tests for the applicants because she has some applicants that are left. The employer wants to bear the risk and try to "catch a big fish" at the beginning. The fewer candidates are left, the safer the strategy used by the employer is.

Additionally, we describe the dynamics of the optimal interviews in terms of the statistical errors that the employer makes. We consider type I and type II errors as the probability of hiring a bad candidate, and as the probability of rejecting a good candidate, respectively. Combining results from Theorem 2 and Proposition 1, we obtain that in the optimum, the sequence of type I errors decreases in i, and the sequence of type II errors increases in i. At the first stages, the employer bears the risks, offers the hardest tests, and tolerates the false negatives, whereas in the later stages she plays safer, decreasing the probability of false negatives and increasing the probability of false positives.

It is remarkable that *even with an arbitrarily large number of candidates*, the employer will not be certain that a good candidate will be hired in the end. This is because, even the first candidate's test (which will be very difficult due to the vast number of candidates that are still to follow) will not fully rule out the possibility of a false positive. The reason is that the marginal cost of information close to the boundary grows arbitrarily large, i.e., it becomes too expensive to try and split hairs at the top end of test results.

5 Discrimination

From Theorem 2, we know that the optimal interviews are more difficult for earlier candidates: *conditional* on being interviewed, later candidates get an easier test than earlier ones. However, from Theorem 1, we know that the probability that a candidate is interviewed at all is lower than that of his predecessors, since it depends on those candidates failing their interviews. The aggregate effect is unclear. In this section, we ask which effect dominates, that is, what can we say about the *unconditional* probability of a candidate being hired?

Denote by q^T_i the unconditional probability of candidate i being hired from a pool of T candidates. Whenever T is obvious from the context, with slight abuse of notation, we will omit it and simply write q_i. An interview design is *discriminatory* if these choice probabilities are not uniform across the T candidates. We use the term discriminatory because all candidates are a priori identical and they only differ in their relative position in the interview. In what follows, we will ask whether there is systematic discrimination. Interestingly, the presence and the nature of discrimination depends crucially on the cost of learning function. We first analyze the problem in the case of two candidates.

5.1 Two candidates

In order to make the problem tractable, we make certain assumptions on the cost function. We first define symmetric cost functions.

Definition 5. *c is said to be symmetric about a point $z \in (0,1)$ if for any pair of posteriors $p, r \in [0,1]$*

$$|z-p| = |z-r| \implies c(p) = c(r)$$

Note here that the point of symmetry, z may depend on the prior as in quadratic functions or may be independent of the prior as is the case of entropy. For example, for the quadratic cost function $c(p) = (p-\mu)^2$ the axis of symmetry is $z = \mu$, for the entropic cost $c(p) = -p \log p - (1-p)\log(1-p)$ the axis of symmetry is $z = 1/2$. We denote the subset of posterior separable and symmetric cost functions by $\mathcal{C}_s$. Moreover, we assume that:

Definition 6. *c belongs to either of the two following families:*

$$\mathcal{C}_1 = \left\{ c \in \mathcal{C}_s \;\middle|\; c' \textit{ is concave on } (0,z) \right\},$$

$$\mathcal{C}_2 = \left\{ c \in \mathcal{C}_s \;\middle|\; c' \textit{ is convex on } (0,z) \right\}.$$

We say that candidate 1 is *favored* if the unconditional probability of hiring him weakly exceeds 0.5.

Proposition 2. *If the cost of learning belongs to the class $\mathcal{C}_1(\mathcal{C}_2)$, the first (second) candidate is favored if and only if the prior about this candidate being a good type is at least as large as z.*

Proposition 2 suggests that the probability of choice depends on the parameters of the model and properties of the cost function in a very nontrivial way, even when $T = 2$. In particular, it depends on a combination of the curvature of the first derivative of c and the prior belief. To get some intuition for the result, we point out that if $\mu = z$, then the rates of change of the posteriors, p_μ^H and p_μ^L, with respect to μ are identical. Consequently, the comparison between q_{12} and q_{22} depends on the absolute value of the derivative of the high posterior p_μ^H. In its turn, the latter depends on the sign of the third derivative of c.

5.2 Many candidates

The complexity of the discrimination pattern increases if there is a third candidate. For instance, as shown in Figure 4 below, the probabilities of the different candidates being drawn for prior $\mu = 0.15$ and different values of the parameter λ, we see that the results are a very mixed bag.

These results may explain why there is no consensus as to whether it is beneficial for a candidate to be interviewed early or late. In particular, the folk views among practitioners are split, some arguing that it is better to be interviewed early on, some arguing that it is better to be interviewed late. The typical explanation that is

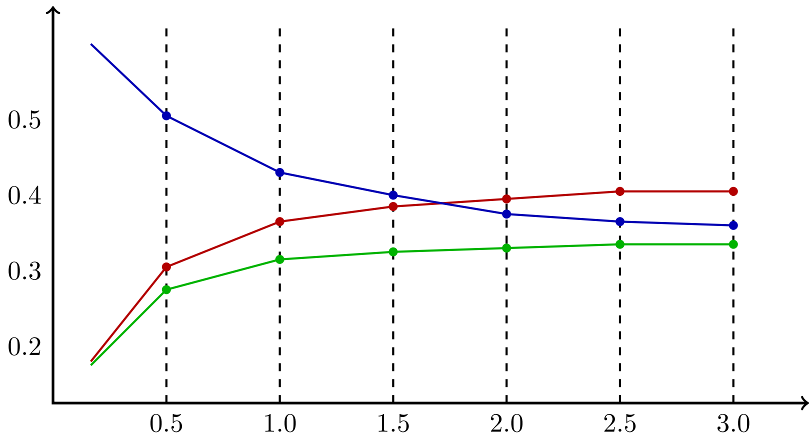

Figure 4: Non-monotonicity with three candidates and entropic costs.

used to support either of these arguments is that employers are biased, suffering from primacy and recency bias, respectively. However, our previous analysis suggests that all the reason why there is such plurality of views among practitioners can very well be attributed to the sensitivity of the discrimination to unobservable parameters, but can be nonetheless still explained through the lens of a rational model.

But quite remarkably, the story does not end here. If we switch our focus to the case of quadratic costs, we can make a clear prediction. Namely, the first candidate starts emerging as the one who is consistently favored.

Proposition 3. *Let $c(p) = (p - \mu)^2$. Then for any fixed T unconditional probability to choose candidate t, q_{iT} is strictly decreasing in t for all $t < T-1$ with $q_{T-1T} = q_{TT}$.*

6 Different productivities

Throughout the text, we assumed that all candidates were ex ante identical. Although that may seem a substantial simplifying assumption, it allows us to investigate the effect of the sequential structure on the employer's incentives to acquire information, without needing to worry about confounding effects due to differential prior information. In this section, we will relax this assumption, thus allowing the manager to have different priors about the different candidates, e.g., due to different education, work experience, recommendations from past employers, etc.

The first question that naturally arises then is whether our earlier results on the employer's optimal interview will still carry. It is not difficult to verify that the basic structure remains the same, i.e., the outside option V_i will still be decreasing across candidates, in the sense that removing an early candidate can only be (weakly) detrimental for the employer. The sequence of outside options $(V_i)_{i=1}^{T}$ is still obtained by Lemma 3, with the only difference that the function g is now prior dependent and therefore becomes

$$g_i(V) = \max_{\pi} \mathbb{E}_\pi \Big[\phi(p, V, \lambda) \Big],$$

where π is chosen from the distributions with mean μ_i instead of μ.

As a result, we can simplify the employer's optimization problem by first dropping some candidates. There are two types of candidates that we can drop:

1. The candidates that are preceded by someone who is hired without an interview, i.e., every $j = 2, \ldots, T$ such that $\mu_i \geq p^H_{V_i}$ for some $i < j$.

2. The candidates that are skipped without an interview, i.e., every i such that $\mu_i \leq p^L_{V_i}$.

These candidates do not play any role in the optimization problem, as it is known ex ante that they will never be optimally hired, either because they follow someone who is much better than them, or because they are followed by someone who is much better than them, respectively.

The remaining candidates can be seen as the ones that have been shortlisted for an interview. Notice that the way a shortlist is obtained is dependent on the order we plan to conduct the interviews, i.e., a different order may very well lead to a different shortlist. Once a shortlist has been obtained, the problem looks very similar to the one we previously solved, and the main conclusions of Theorem 2 still hold.

Proposition 4. *Suppose that $\tilde{I} = \{1, \ldots, \tilde{T}\}$ is the ordered set of shortlisted candidates with $\tilde{T} \geq 2$. Then, the employer's optimal strategy satisfies the following:*

1. *Difficulty is decreasing with the order of being interviewed, i.e., for every $i = 1, \ldots, \tilde{T} - 2$, it is the case that $p^H_{V_i} > p^H_{V_{i+1}}$ and $p^L_{V_i} > p^L_{V_{i+1}}$.*

2. *As the number of shortlisted candidates grows large, we obtain:*

   (a) The probability of the first candidate being hired converges to 0.

   (b) The probability of hiring a good candidate is bounded away from 1.

However, the really interesting question within this framework is whether the employer should prefer to interview the best or the worst candidate first. This question is discussed among practitioners (Selby, 2023), but the issue remains still unsettled.

Given the complexity of the question, we will focus on the simplest possible context, i.e., a setting with only two candidates, and the employer's cost function coming from one of the common specifications. Let μ_1 and μ_2 be the priors of the two candidates respectively. Moreover, denote by U_1 and U_2 the employer's indirect net utility when candidate 1 is interviewed first and when candidate 2 is interviewed first, respectively. Obviously, if $\mu_1 = \mu_2$, the two utilities are identical, and the order does not matter.

Theorem 3. *Suppose that the are only two candidates, i.e., we have $T = \{1, 2\}$. Then, the following hold:*

(a) QUADRATIC COST: *The employer does not have a preference on the order of interviewing the candidates, i.e., $U_1 = U_2$ for any $\mu_1, \mu_2 \in M$.*

(b) Entropic cost: *For any large prior there is an even larger prior with which the employer would rather start; likewise, for any small prior there is a larger prior with which the employer would rather follow, i.e., there exists some* $\mu_0 \in (0,1)$ *such that:*

1) For every $\mu_1 > \mu_0$*, there exists some* $\mu_2 > \mu_1$ *such that* $U_1 < U_2$*.*

2) For every $\mu_1 < \mu_0$*, there exists some* $\mu_2 > \mu_1$ *such that* $U_1 > U_2$*.*

The previous result suggests that the employer's preferences regarding the order of interviewing the candidates is highly sensitive with respect to the prior beliefs, meaning that unless we exogenously impose very strong assumptions on the primitive parameters of the model, it will not be possible to make strong predictions about the employer's optimal order of interviewing candidates. This conclusion reinforces the view that general results about the optimal interviewing order can only be obtained under strong structural assumptions Doval (2018). This is in contrast to the impression that many people had regarding the generality of the corresponding result of (Weitzman, 1979).

7 Extensions

7.1 Restricted interview design

At the optimum, the employer in our model fully leverages the flexibility of the interview design. She constructs different interviews depending on the serial position of a candidate. Utilizing the dynamic structure of the problem, the employer offers more difficult tests to the candidates who are arriving early. Such a difference in the treatment can create discrimination in the hiring outcomes, as discussed in Sections 5 and 6, and may be seen as normatively unfair. In this Section, we instead consider the scenario in which the employer is exogenously restricted to use interviews of the same difficulty for all candidates, including the last one.

We will maintain some features of our main model for comparison purposes. In particular, we will still consider a common prior and only binary interviews. Moreover, passing an interview will always lead to a candidate being hired, whereas failing the interview will always lead to a candidate being rejected, including the last one.

Suppose that all candidates face exactly the same interview. We refer to this interpretation as the ***structured interview*** in the hiring process. In their guide to conducting a fair selection process, the National Institute of Health (the primary agency of the US government) suggests that using a structured interview reduces bias in the hiring procedure.[5] Formally, this means that the posteriors will be constant across candidates (conditional on being interviewed), i.e., for every i, we will have

$$p_i^H = p^H \text{ and } p_i^L = p^L,$$

[5]See Manager's Fair Selection Toolkit. Office of Equity, Diversity, and Inclusion, National Institutes of Health. https://www.edi.nih.gov/sites/default/files/public/EDI_Public_files/guidance/toolkits/managers/manager-fair-selection-toolkit01.pdf

and a fortiori, the conditional probability of i passing the interview is equal to

$$\pi_H := \frac{\mu - p^L}{p^H - p^L}.$$

Then, it is not difficult to show the following:

Proposition 5. *The optimal structured interview satisfies the following:*

1) *Early candidates are favored, i.e., for each i the probability of being hired is*

$$q_i := (1 - \pi_H)^{i-1} \pi_H.$$

2) *The probability of hiring a good candidate is bounded away from 1, even when T becomes arbitrarily large.*

The previous result suggests that the fairness requirement (that we initially set out to satisfy) cannot be achieved in a sequential setting through structured interviews. Moreover, full efficiency cannot be fully restored, even if the pool of candidates is arbitrarily large. In this sense, structured interviews are not really suitable if the designer wants to achieve fairness of full efficiency.

One additional observation is that the optimal structured interview will not yield a sequentially rational strategy. The reason is quite simple: since the outside option is strictly decreasing in i, the stationary interview that yields the pair of posteriors (p^L, p^H) cannot be rational for more than one candidates. As a result, even if the designer requires a structured interview, it will not be incentive compatible for the employer to follow this recommendation.

A structured interview might be undesirable for an additional —quite different— reason, viz., structured interviews are usually understood as a series of predetermined questions, which unfortunately may become publicly known after having been used in early interviews. As a result, fairness will be compromised in the opposite direction. Still, it is entirely unclear whether this type of late candidate's advantage will offset the early candidate's advantage that we describe in the previous proposition.

8 Conclusion

As documented in the economic literature, see, e.g., Bertheau et al. (2023), hiring is difficult for firms, and one of the reasons is that the firms face time constraints while hiring candidates. This means that firms do not learn the potential workers' productivities perfectly (since it will take too long time) but instead acquire noisy information about those. In this paper, we model the process of sequential search with costly but flexible learning in each stage.

The hiring firm observes several candidates who arrive sequentially and can design interviews for each candidate individually. We show that the optimal learning strategy has a simple feature – the later the candidate appears (the higher the serial number she has), the easier questions she will be facing. That is, the optimal interviews are decreasing in their difficulty in time. However, it does not mean that the workers should try to be interviewed in the end since the probability of being hired as a function of time of arrival is not necessarily increasing.

Our paper is the first step in studying sequential search with flexible and endogenous information acquisition. Therefore, many research questions are left for the future. For instance, we study only the situation when the distribution of productivities is known to the employer. The problem of studying a similar problem with extra layer of learning about the workers' productivities is interesting and intriguing.

Another suggestion for future research is to consider a model similar to ours but with an opportunity for recall. We suspect that the decreasing difficulty property will remain present in this class of problems.

A Reformulation of the static problem

Instead of relying on the concavification technique to solve the static problem (2), we will introduce the unconditional probability q of the the interview having a good outcome explicitly as a choice variable. This is in contrast, in the concavification technique, the optimal unconditional probability was derived as a function of the two posterior beliefs that satisfy Bayesian consistency.

For every $V \in [0, 1]$, define the function

$$\begin{aligned} U(q, V) \quad := \quad & \max_{x,y} \Big[q\phi(x, V) + (1-q)\phi(y, V) \Big], \\ & \text{subject to} \\ & \mu \leq x \leq 1, \\ & 0 \leq y \leq \mu, \\ & qx + (1-q)y = \mu. \end{aligned}$$

Note that for notation simplicity, we have omitted λ from ϕ. Then, we can reformulate the static optimization problem as follows:

Lemma A1. *For every $V \in [0, 1]$, the optimization problem*

$$\max_{q \in [0,1]} U(q, V) \tag{A.1}$$

has a unique solution, henceforth denoted by q_V. Moreover, we have

$$g(V) = U(q_V, V). \tag{A.2}$$

Additionally, problem (A.1) is concave in q, with unique solution, and the interior solution of (A.1) is decreasing in V.

Proof. We first analyze the case in which inequality $p^H > \mu > p^L$ holds. That is, when $V \in (V_L, V_H)$. Using the fact that support of the optimal posterior distribution has no more than two points we rewrite the static problem (2) as

$$\begin{aligned} \max_{(q,p^H,p^L) \in [0,1]^3} & \{q(p^H - \lambda c(p^H)) + (1-q)(V - \lambda c(p^L))\} \\ \text{s.t.} \quad & qp^H + (1-q)p^L = \mu, \\ & p^H \geq p^L. \end{aligned} \tag{A.3}$$

We denote the objective in the above problem as $\tilde{U}(q, p^H, p^L)$. Using the identity from the multivariable calculus we can write $\max_{q,p^H,p^L} \tilde{U}(q, p^H, p^L) = \max_q \max_{p^H,p^L} \tilde{U}(q, p^H, p^L)$. Therefore, problem (A.3) can always be solved sequentially finding optimal p^L, p^H given q and then optimize over q. To show the equivalence between problems, we need to show first that for given q, there is only one pair of optimal (p^L, p^H) and second that there is a unique optimal q.

To show that there exists a unique pair of optimal (p^L, p^H) given q we observe that the interior solution to the static problem (2) should satisfy necessary optimality conditions in problem (A.3). In particular, for given q optimal (p^L, p^H) should satisfy the system

$$\begin{cases} -\lambda c'(p^L) = 1 - \lambda c'(p^H) \\ qp^H + (1-q)p^L = \mu. \end{cases}$$

We show that given $q \in (0,1)$ the system has a unique solution (p^L, p^H). We rewrite the first equation as $p^L = (c')^{-1}(c'(p^H) - \frac{1}{\lambda})$. This expression defines a function $p^L(p^H)$. Indeed, $c'(p^H)$ is increasing in p^H, therefore mapping $p^L(p^H)$ is also increasing and the mapping defines unique p^L for any p^H.

We rewrite the second equation as $q = (\mu - p^L)/(p^H - p^L)$. Simple algebra shows that the derivative of the right-hand side with respect to p^H is positive and, therefore, the right-hand side is increasing in p^H. Therefore, for any given q, there exists unique p^H and, thus, for any given q, there exists unique pair (p^L, p^H) that solves the system above. Further, we write $p^L(q), p^H(q)$ to emphasize the dependence of optimal posteriors from the marginal distribution.

Following Fosgerau et al. (2023) we show that problem (A.1) is concave. Without abuse of notation, we omit V in the argument and denote the objective function as U. The derivative equals to

$$U'(q) = -p^H(q) + \lambda c(p^H(q)) + (V - \lambda c(p^L(q)))+$$
$$+(1-q)((p^H(q))' - \lambda c'(p^H(q))(p^H(q))') - (q)\lambda c'(p^L(q))(p^L(q))',$$

where $(p^H(q))'$ and $(p^L(q))'$ are the derivatives of the posteriors with respect to q. Using optimality condition $\lambda c'(p^H) - \lambda c'(p^L) = 1$ and differentiating the Bayesian consistency constraint to get $(1-q)(p^H(q))' + q(p^L(q))' = p^L(q) - p^H(q)$ we obtain

$$U'(q) = p^H(q) - \lambda c(p^H(q)) - (V - \lambda c(p^L(q))) + \lambda c'(p^L(q))(p^H(q) - p^L(q)). \quad \text{(A.4)}$$

Differentiating the expression with respect to q one more time and using optimality condition for posteriors result in

$$U''(q) = \lambda c''(p^L(q))(p^H(q) - p^L(q))(p^L(q))'.$$

We show that inequality $(p^L(q))' < 0$ holds. Combining optimality condition for posteriors and differentiable Bayesian consistency condition we obtain that equality $(p^L(q))'\left(q\frac{c''(p^L(q))}{c''(p^H(q))} + 1 - q\right) = p^L(q) - p^H(q)$ holds. Thus $(p^L(q))' < 0$ holds and $U''(q) < 0$ holds and problem (A.1) is concave. Therefore, problem (A.1) has a unique solution that is determined from the first-order condition or on the boundary. However, because the inequality $p^H > \mu > p^L$ holds, the optimal q is interior and uniquely determined from the first-order condition.

If $V \notin (V_L, V_H)$ then DM chooses degenerate distribution, optimal q equals to 0 if $V \leq V_L$ and equals to 1 if $V \geq V_H$.

To get comparative statics of the optimal q with respect to V we employ standard supermodularity argument: optimal interior q is decreasing in V, because the mixed derivative of $\frac{\partial^2 U(q,V)}{\partial q \partial V} = -1$ is negative. $\square$

B Proofs of Section 3

B.1 Intermediate results

Lemma B2. $p_V^L, p_V^H \in (0,1)$.

Proof. By a standard concavification argument, the following inequality holds:

$$1 - \lambda c'(p_V^H) > -\lambda c'(p_V^L).$$

Now, assume the contrary to what we want to prove, i.e., for a given V the employer chooses the optimal signal π with the pair of posterior beliefs p_V^L, p_V^H such that at least one of them belongs to the boundary $\{0,1\}$.

If $p_V^L = 0$, by (2), we have

$$1 - \lambda c'(p_V^H) > -\lambda \lim_{p \to 0^+} c'(p) > 1 - \lambda c'(\mu),$$

and therefore, because c is convex, it is the case that $\mu > p_V^H$. Likewise, if $p_V^H = 1$, again by (2), we obtain

$$-\lambda c'(p_V^L) < 1 - \lambda \lim_{p \to 1^-} c'(p) < -\lambda c'(\mu)$$

and therefore by convexity of c, it is the case that $\mu < p_V^L$. In either case $p_V^L < \mu < p_V^H$ is violated, and the proof is complete. $\square$

Lemma B3. *Both p_V^L and p_V^H are differentiable with respect to V in (V_L, V_H).*

Proof. Under Lemma 1, the concave closure of ϕ as defined in the static problem (2) for $p \in [p_V^L, p_V^H]$ is a straight line that is tangent to ϕ at p_V^L and p_V^H. This tangent is characterized by the following equality for $p \in [p_V^L, p_V^H]$

$$V - \lambda c(p_V^L) - \lambda c'(p_V^L)(p - p_V^L) = p_V^H - \lambda c(p_V^H) - [\lambda c'(p_V^H) - 1](p - p_V^H), \tag{B.1}$$

such that

$$\lambda c'(p_V^L) = \lambda c'(p_V^H) - 1. \tag{B.2}$$

By virtue of strict convexity of c and (B.2), we can implicitly define p_V^H as a continuously differentiable function of p_V^L. Using this in (B.1), we have that

$$V = p_V^H - \lambda c(p_V^H) - [\lambda c'(p_V^H) - 1](p - p_V^H) - [-\lambda c(p_V^L) - \lambda c'(p_V^L)(p - p_V^L)]. \tag{B.3}$$

Using (B.2) yields:

$$0 = -V + p_V^H - \lambda c(p_V^H) + \lambda c(p_V^L) + \lambda c'(p_V^L)(p_V^H - p_V^L) \tag{B.4}$$

Next, note that the RHS in (B.4) is a continuously differentiable function of p_V^L. Moreover its derivative with respect to p_V^L is given by

$$\begin{aligned}(1 - \lambda c'(p_V^H)) \cdot (p_V^H)' + \lambda c'(p_V^L) + \lambda c'(p_V^L) \cdot (p_V^H)' - \lambda c'(p_V^L) + \lambda c''(p_V^L)(p_V^H - p_V^L) \\ = \lambda c''(p_V^L)(p_V^H - p_V^L) > 0,\end{aligned}$$

where the last equality comes from (B.2). The implicit function theorem implies that p_V^L is a continuously differentiable function of V and consequently so is p_V^H. $\square$

Lemma B4. *Both p_V^L and p_V^H are increasing with respect to V in (V_L, V_H).*

Proof. Differentiating tangent optimality conditions (B.2) and (B.4) with respect to V gives the system

$$\frac{\partial p_V^L}{\partial V} = \frac{1}{\lambda c''(p_V^L)(p_V^H - p_V^L)}, \tag{B.5}$$

$$\frac{\partial p_V^H}{\partial V} = \frac{1}{\lambda c''(p_V^H)(p_V^H - p_V^L)}. \tag{B.6}$$

By convexity of c, together with inequality $p_V^H > p_V^L$, both derivative are positive. □

B.2 Proof of Lemma 2

For every $V \in (V_L, V_H)$, the optimal signal π_V assigns to the two respective posteriors, p_V^L and p_V^H, probability

$$\pi_V(p_V^L) = \frac{p_V^H - \mu}{p_V^H - p_V^L} \text{ and } \pi_V(p_V^H) = \frac{\mu - p_V^L}{p_V^H - p_V^L},$$

and the employer's indirect expected utility in (V_L, V_H) is

$$g(V) = \pi_V(p_V^H)\big(p_V^H - \lambda c(p_V^H)\big) + \pi_V(p_V^L)\big(V - \lambda c(p_V^L)\big).$$

Since p_V^L and p_V^H are differentiable in V, so is g. By the Envelope Theorem, we have

$$g'(V) = \pi_V(p_V^L) > 0.$$

Thus, g is strictly increasing in (V_L, V_H). Then, simple algebra yields

$$\frac{\partial \pi_V(p_V^L)}{\partial V} = \frac{\partial p_V^H}{\partial V}(\mu - p_V^L) + \frac{\partial p_V^L}{\partial V}(p_V^H - \mu),$$

which, by Lemma B4, is non-negative. Therefore, g is convex.

B.3 Proof of Lemma 3

Part 1 follows directly from (8) combined with Lemma 2.

By definition we have $V_T = 0$. Then, Part 2 follows directly from the fact that $g(V) \in (V_L, V_H)$ for all $V \in [0, V_H)$.

B.4 Proof of Theorem 1

The proof follows directly from Lemmas 1 and 3.

In particular, by $V_T = 0$, we get $V_T < V_L$. Hence, $\mathrm{supp}(\pi_{V_T}) = \{\mu\}$ and $\alpha_T(\mu) = 1$.

Moreover, for every $i \in \{1, \ldots, T-1\}$, we have $V_L < V_i < V_H$, and therefore $\mathrm{supp}(\pi_{V_i}) = \{p_{V_i}^L, p_{V_i}^H\}$ with $\alpha_i(p_{V_i}^L) = 0$ and $\alpha_i(p_{V_i}^H) = 1$.

C Proofs of Section 4

C.1 Proof of Proposition 1

(i) $\iff$ (ii): For each $s \in \{H, L\}$, we have:

$$p_i^s = \frac{\mu}{\mu + (1-\mu)\frac{\sigma_i(s|B)}{\sigma_i(s|G)}} > p_i^s = \frac{\mu}{\mu + (1-\mu)\frac{\sigma_j(s|B)}{\sigma_j(s|G)}} \iff \frac{\sigma_i(s \mid G)}{\sigma_i(s \mid B)} > \frac{\sigma_j(s \mid G)}{\sigma_j(s \mid B)}$$

(i) $\Rightarrow$ (iii): By the Bayes rule the passing probability under test k equals to $(\mu - p_k^L)/(p_k^H - p_k^L)$. The required follows from the inequalities

$$\frac{\mu - p_j^L}{p_j^H - p_j^L} > \frac{\mu - p_j^L}{p_i^H - p_j^L} < \frac{\mu - p_i^L}{p_i^H - p_i^L}.$$

(iii) $\Rightarrow$ (i): Let the passing probability under test i is lower than under test j, but signal π_i is not more difficult than π_j. In this case at least one inequality $p_j^H \geq p_i^H, p_j^L \geq p_i^L$ holds. In the first case for a candidate $\mu = p_i^H$ and in the second case for a candidate $\mu = p_j^L$ the passing probability of test i is higher. Thus, the signal π_i has to be more difficult than π_j.

C.2 Proof of Theorem 2

For the first part, it is sufficient to show that both optimal posterior beliefs in the static problem (7) are increasing functions of the outside option. The latter follows from the proof of Lemma 2.

For the second part, note that by Proposition 1 and Lemma 1 continuation value

$$\lim_{T\to\infty} p_{V_1}^H = V_H.$$

In the solution to problem (7) with an outside option V_H, the solution to the first-order conditions from Lemma 1 implies that the lower optimal posterior equals the prior. Therefore, by the continuity

$$\lim_{T\to\infty} p_{V_1}^L = \mu.$$

Finally, by Theorem 1, recall that the probability of the first candidate being hired is $(\mu - p_{V_1}^L)/(p_{v_1}^H - p_{V_1}^L)$, and therefore statements (a) and (b) follow trivially.

D Proofs of Section 5

D.1 Proof of Proposition 2

Recall an optimality condition A.4:

$$p^H(q) - \lambda c(p^H(q)) - (\mu - \lambda c(p^L(q))) + \lambda c'(p^L(q))(p^H(q) - p^L(q)) = 0. \qquad \text{(D.1)}$$

Using $\lambda(p^H(q)-\mu)(c'(p^H(q))-c'(p^L(q)))=p^H(q)-\mu$, we obtain:

$$D(\mu,p^H(q))=D(\mu,p^L(q)) \tag{D.2}$$

where $D:[0,1]^2\to\mathbb{R}_+$ is defined[6] as $D(x,y):=\lambda(c(x)-c(y)-c'(y)(x-y))$. Note that convexity of c implies that D is non-negative and $D(x,y)=0$ if and only if $x=y$. Next, redefine the posteriors in terms of the auxiliary variables (Δ_H,Δ_L) such that $p^H=\mu+\Delta_H$ and $p^L=\mu-\Delta_L$ with $\Delta_H\in[0,1-\mu]$ and $\Delta_L\in[0,\mu]$.[7] The first candidate is favored, i.e., is chosen with probability higher than 0.5, preferred if and only if $\Delta_L\geq\Delta_H$. For an arbitrary $\Delta\in[0,\min\{\mu,1-\mu\}]$, we have that:

$$\frac{\partial D(\mu,\mu+\Delta)}{\partial\Delta}=[\lambda\Delta]\cdot c''(\mu+\Delta)\quad\&\quad\frac{\partial D(\mu,\mu-\Delta)}{\partial\Delta}=[\lambda\Delta]\cdot c''(\mu-\Delta) \tag{D.3}$$

Now, let $c\in\mathcal{C}_1$. Then, for $\mu\leq z$, it is immediate that $c''(\mu+\Delta)\leq c''(\mu-\Delta)$ with equality holding if and only if either $\Delta=0$ or $\mu=z$. $\Delta=0$ implies that the optimal experiment is uninformative, which is ruled out under our earlier assumption. Thus, $c''(\mu+\Delta)\leq c''(\mu-\Delta)$ with strict inequality for all $\mu<z$. This, in turn, implies that for the equality $D(\mu,p^H)=D(\mu,p^L)$, we must have that $\Delta_L<\Delta_H$ which in turn implies that the probability of passing for the first candidate is strictly below half whenever $\mu<z$ and is equal to half at $\mu=z$. For $\mu\geq z$, through similar arguments we have that $c''(\mu+\Delta)\geq c''(\mu-\Delta)$ with strict inequality for all $\mu>z$. This, in turn, implies that for $D(\mu,p^H)=D(\mu,p^L)$, we must have $\Delta_L>\Delta_H$, thereby implying that the probability that the first candidate passes is strictly larger than half. The argument for $c\in\mathcal{C}_2$ is analogous.

D.2 Proof of Proposition 3

Simple algebra shows that the system has unique solution $p_V^L=V-\frac{1}{4\lambda}, p_V^H=V+\frac{1}{4\lambda}$. Substituting the solution gives the value of the problem as $g(V)=V+\lambda(\mu-V+\frac{1}{4\lambda})^2$.

We show that for a given $T>2$ inequality $q_{1T}>q_{2T}$ holds. Using the derived expressions above we get that

$$q_{1T}=2\lambda\Big(\mu-V_1+\frac{1}{4\lambda}\Big);\qquad q_{2T}=4\lambda^2\Big(V_1-\mu+\frac{1}{4\lambda}\Big)\Big(\mu-V_2+\frac{1}{4\lambda}\Big),$$

moreover equality

$$V_1=V_2+\lambda\Big(\mu-V_2+\frac{1}{4\lambda}\Big)^2$$

holds. We denote $t=\mu-V_2+\frac{1}{4\lambda}$, thus,

$$q_{1T}=2\lambda(t-\lambda t^2)$$
$$q_{2T}=4\lambda^2 t\Big(\frac{1}{2\lambda}-t+\lambda t^2\Big)$$

[6]Note that it is the definition of Bregman divergence.

[7]Note that we have suppressed the dependence of the auxiliary variables on the prior for notational convenience.

Therefore, the inequality $q_{1T} > q_{2T}$ is equivalent to the inequality $\frac{1}{2\lambda} > t$. Because inequality $V_2 > \mu - \frac{1}{4\lambda}$ holds for all $T > 2$, inequality $q_{1T} > q_{2T}$ also holds.

By the Bayes rule the inequality $q_{tT} > q_{t+1T}$ is equivalent to the inequality $q_{1T-t+1} > q_{2T-t+1}$, therefore, the inequality holds.

Finally, if $T = 2$ then $q_{12} = 2\lambda(\mu - V_1 + \frac{1}{4\lambda}) = \frac{1}{2}$ because in this case $V_1 = \mu$. Therefore, $q_{T-1T} = q_{TT}$ for all T.

E Proofs of Section 6

E.1 Proof of Theorem 3

QUADRATIC COST: Let us first take any two $\mu_1, \mu_2 \in M$, which are sufficiently close to each other, i.e.,

$$\mu_1 \in \left(p^L_{\mu_2}, p^H_{\mu_2}\right) \quad \text{and} \quad \mu_2 \in \left(p^L_{\mu_1}, p^H_{\mu_1}\right).$$

Then, using the conventional notation $\kappa := 1/4\lambda$, by (??) we obtain

$$\begin{aligned} U_1 &:= \frac{\mu_1 - p^L_{\mu_2}}{p^H_{\mu_2} - p^L_{\mu_2}} p^H_{\mu_2} + \frac{p^H_{\mu_2} - \mu_1}{p^H_{\mu_2} - p^L_{\mu_2}} \mu_2 \\ &= \frac{\mu_1 - \mu_2 + \kappa}{2\kappa}\big(\mu_2 + \kappa\big) + \frac{\mu_2 - \mu_1 + \kappa}{2\kappa}\mu_2 \\ &= \frac{\mu_1 + \mu_2 + \kappa}{2}. \end{aligned}$$

Then, by symmetry, we obviously get $U_1 = U_2$.

Then, suppose that $\mu_1 < \mu_2$ are not sufficiently close to each other, i.e., let $\mu_2 > p^H_{\mu_1}$. Now, let us introduce the convenient notation:

$$\begin{aligned} h_L(V) &:= p^L_V, \\ h_H(V) &:= p^H_V, \end{aligned}$$

and then take the useful composite function

$$h := h_L \circ h_H. \tag{E.1}$$

Then, notice that for every $V \in (0, 1)$, we trivially obtain

$$h(V) = V.$$

Hence, it will necessarily be the case that $\mu_1 < p^L_{\mu_2}$, meaning that regardless who arrives first, candidate 2 will be hired without any of the two being interviewed. Hence, we will have $U_1 = U_2$.

ENTROPIC COST: We will now use the notational convention $\kappa := 1/\lambda$, and we will use again the function h as defined in (E.1). Then, using the formulas for the posterior beliefs for entropic costs,

$$p^L_\mu = \frac{e^{\mu/\lambda} - 1}{e^{1/\lambda} - 1} \qquad\qquad p^H_\mu = \frac{e^{1/\lambda} - e^{(1-\mu)/\lambda}}{e^{1/\lambda} - 1}, \tag{E.2}$$

we obtain

$$h'(V) = \frac{\kappa e^{\kappa h_H(V)}}{e^\kappa - 1} \cdot \frac{\kappa e^{\kappa(1-V)}}{e^\kappa - 1},$$

which is obviously positive. Furthermore, we have

$$h''(V) = \frac{\kappa e^{\kappa(1-V)}}{e^\kappa - 1} - 1.$$

which has a unique root

$$\mu_0 := 1 - \frac{1}{\kappa} \log \frac{e^\kappa - 1}{\kappa} \in (0, 1).$$

In addition, we have $h''(V) > 0$ if and only if $V < \mu_0$. This means that h is strictly convex below μ_0 and strictly concave above μ_0. Therefore, it will be the case that

$$h(V) < V \text{ for all } V < \mu_0, \text{ and } h(V) > V \text{ for all } V > \mu_0.$$

Part 1: Take an arbitrary $\mu_1 < \mu_0$. By h_L being continuously increasing, for each $\mu_2 > p^H_{\mu_1}$ there is some $\varepsilon > 0$ such that

$$p^L_{\mu_2} = h_L(\mu_2) = h_L(p^H_{\mu_1}) + \varepsilon = h(\mu_1) + \varepsilon.$$

By taking μ_2 sufficiently close to $p^H_{\mu_1}$, it will be the case that the corresponding ε will satisfy

$$\varepsilon < \mu_1 - h(\mu_1).$$

By strict convexity of h below μ_0, the righthand side is strictly positive. As a result, we obtain

$$p^L_{\mu_2} < \mu_1.$$

Thus, by $\mu_2 > p^H_{\mu_1}$, if candidate 2 goes first, he will be directly hired without any interview, and therefore we will have $U_2 = \mu_2$. On the other hand, by $p^L_{\mu_2} < \mu_1$, if candidate 1 goes first, there will be an informative interview, meaning that it is not optimal to hire candidate 2 without an interview, and therefore $U_1 > \mu_2$. As a result, we conclude that $U_1 > U_2$.

Part 2: We follow same steps as in the previous part. Take an arbitrary $\mu_1 > \mu_0$. Then, there is some $\mu_2 < p^H_{\mu_1}$ and some $\varepsilon < h(\mu_1) - \mu_1$ such that $p^L_{\mu_2} = h(\mu_1) - \varepsilon$. Hence, it will be the case that

$$p^L_{\mu_2} > \mu_1.$$

Then, by the same argument as above, we have $U_1 = \mu_2$ and $U_2 > \mu_2$, which implies $U_1 < U_2$, thus completing the proof.

F Proofs of Section 7

F.1 Proof of Proposition 5

Part 1 is trivial. So let us focus on Part 2. Notice that for any finite T, the total probability of hiring a good candidate is equal to

$$q^* = \pi_H p^H \sum_{i=0}^{T-1} (1 - p_H)^i = p^H \Big(1 - (1 - \pi_H)^T\Big),$$

with the second equality following directly from the fact that we have a geometric series on the righthand side. This means that for any finite T, we will obtain $q^* < p^H$. However, if we let T go to infinity, q^* will converge to p^H, i.e., $q^* \rightarrow 1$ if and only if $p^H \rightarrow 1$. However, the latter can never be the case due to the boundary condition, which completes the proof.

References

Anderson, S. and Renault, R. (2018). Firm pricing with consumer search. *Handbook of Game Theory and Industrial Organization*, II.

Arieli, I., Babichenko, Y., Smorodinsky, R., and Yamashita, T. (2023). Optimal persuasion via bi-pooling. *Theoretical Economics*, 18(1):15–36.

Armstrong, M. (2017). Ordered consumer search. *Journal of the European Economic Association*, 15(5):989–1024.

Bertheau, A., Larsen, B., and Zhao, Z. (2023). What makes hiring difficult? Evidence from linked survey-administrative data.

Bertrand, M. and Duflo, E. (2017). Field experiments on discrimination. *Handbook of Economic Field Experiments*, I.

Beyhaghi, H. and Cai, L. (2023). Recent developments in pandora box problem: Variants and applications.

Caplin, A. and Dean, M. (2013). Behavioral implications of rational inattention with Shannon entropy.

Caplin, A., Dean, M., and Leahy, J. (2022). Rationally inattentive behavior: Characterizing and generalizing Shannon entropy. *Journal of Political Economy*, 130(6):1676–1715.

Chade, H., Eeckhout, J., and Smith, L. (2017). Sorting through search and matching models in economics. *Journal of Economic Literature*, 55(2):493–544.

Charness, G., Oprea, R., and Yuksel, S. (2021). How do people choose between biased information sources? evidence from a laboratory experiment. *Journal of the European Economic Association*, 19(3).

Correa, J., Cristi, A., Feuilloley, L., and Oosterwijk, Timand Tsigonias-Dimitriadis, A. (2024). The secretary problem with independent sampling. *Management Science*.

Denti, T. (2022). Posterior separable cost of information. *American Economic Review*, 112(10):3215–3259.

Doval, L. (2018). Whether or not to open pandora's box. *Journal of Economic Theory*, 175:127–158.

Forbes (2019). A little biased? interview order does matter. https://www.forbes.com/councils/forbeshumanresourcescouncil/2019/03/22/a-little-biased-interview-order-does-matter/.

Forbes (2023). Understanding the impact of interview scheduling on hiring decisions. https://www.forbes.com/sites/benjaminlaker/2023/12/14/understanding-the-impact-of-interview-scheduling-on-hiring-decisions/.

Fosgerau, M., Sethi, R., and Weibull, J. (2023). Equilibrium screening and categorical inequality. *American Economic Journal: Microeconomics*, 15(3):201–242.

Gentzkow, M., Shapiro, J., and Stone, D. (2014). Media bias in the marketplace: Theory.

Gill, D. and Sgroi, D. (2012). The optimal choice of pre-launch reviewer. *Journal of Economic Theory*, 147:1247–1260.

Hebert, B. and Woodford, M. (2023). Rational inattention when decisions take time. *Jouenal of Economic Theory*, 208:105612.

Jain, V. and Whitmeyer, M. (2021). Search and competition with endogenous investigations. Technical report, Working Paper.

Kamenica, E. and Gentzkow, M. (2011). Bayesian persuasion. *American Economic Review*, 101(6):2590–2615.

Kim, J. H. J., Kim, K., and Pease, M. (2022). Unemployment duration under flexible information acquisition.

Matějka, F. and McKay, A. (2015). Rational inattention to discrete choices: A new foundation for the multinomial logit model. *American Economic Review*, 105(1):272–298.

McCall, J. (1965). The economics of information and optimal stopping rules. *Journal of Business*, 84(1):300–317.

Mensch, J. and Malik, K. (2023). Posterior-mean separable costs of information acquisition. *arXiv preprint arXiv:2311.09496*.

Morris, S. and Strack, P. (2019). The Wald problem and the relation of sequential sampling and ex ante information costs.

Mortensen, D. and Pissarides, C. (1999). New developments on models of search in the labor market. *Handbook of Labor Economics*, III.

Neumark, D. (2018). Experimental research on labor market discrimination. *Journal of Economic Literature*, 56:799–866.

Salop, S. (1973). Systematic job search and unemployment. *Review of Economic Studies*, 40:191–201.

Schlag, K. and Zapechelnyuk, A. (2024). Fair hiring procedures. Technical report, Working Paper.

Selby (2023). Do employers interview the best candidate first? https://www.selbyjennings.com/blog/2023/04/do-employers-interview-best-candidate-first.

Shaked, M. and Shanthikumar, G. (2007). *Stochastic orders*. Springer.

Sims, C. (2003). Implications of rational inattention. *Journal of Monetary Economics*, 50(3):665–690.

Steiner, J., Stewart, C., and Matejka, F. (2017). Rational inattention dynamics: Inertia and delay in decision-making. *Econometrica*, 85(2):521–553.

Stigler, G. (1961). The economics of information. *Journal of Political Economy*, 69(3):213–225.

Ursu, R. M., Wang, Q., and Chintagunta, P. K. (2020). Search duration. *Marketing Science*, 39(5):849–871.

Weitzman, M. L. (1979). Optimal search for the best alternative. *Econometrica*, 47(3):641–654.

Zhong, W. (2022). Optimal dynamic information acquisition. *Econometrica*, 90(4):1537–1582.