Using Ratios to 8.9 Describe Size Changes

Objective To explore the use of ratios to describe size changes.



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Ongoing Learning & Practice

Teaching the Lesson

Key Concepts and Skills

- Multiply fractions by whole-number scalars. [Operations and Computation Goal 4]
- Use ratios to solve size-change problems. [Operations and Computation Goal 6]
- Express ratios in words, with colons, as fractions, or as proportions. [Operations and Computation Goal 6]
- · Measure line segments to the nearest $\frac{1}{16}$ inch and millimeter.

[Measurement and Reference Frames Goal 1]

· Recognize properties of similar figures. [Geometry Goal 2]

Key Activities

Students explore the use of ratios to describe size changes for geometric figures, scale models, and maps. They practice using notations to show the size-change factor.



Ongoing Assessment:

Informing Instruction See page 745.



Ongoing Assessment: Recognizing Student Achievement

Use journal page 310.

[Measurement and Reference Frames Goal 1]

Key Vocabulary

size-change factor (scale factor or scale) • n-to-1 ratio ◆ enlargement ◆ reduction

Materials

Math Journal 2, pp. 310 and 311 Student Reference Book, pp. 121-124

Math Masters, p. 264; p. 408 (optional) calculator • inch ruler • Geometry Template

Math Boxes 8.9

Math Journal 2, p. 312 Students practice and maintain skills through Math Box problems.



Study Link 8-9

Math Masters, p. 265 centimeter ruler

Students practice and maintain skills through Study Link activities.

READINESS

Considering What Size Change Means

Differentiation Options

Math Masters, p. 266

centimeter ruler

Students explore the effect of increasing or reducing a measurement or an event by a factor of 10.

EXTRA PRACTICE

Finding Dimensions of Objects Based on Scale Models

Math Masters, p. 267

Students use scale drawings to find the dimensions of objects. They examine whether size-change factors apply to area.

EXTRA PRACTICE

Investigating Perimeter and Size-Change Factor

Math Masters, p. 268 centimeter ruler

Students investigate the relationship between the size-change factors in polygons and their perimeters.

ELL SUPPORT

Illustrating Terms

posterboard • markers Students make posters illustrating enlargements and reductions.

Study Link 8+8

(optional)

Advance Preparation

For the Math Message, make one copy of Math Masters, page 264 for every two students.



Teacher's Reference Manual, Grades 4-6 pp. 65, 66, 199

Getting Started



Mathematical Practices
SMP1, SMP2, SMP3, SMP4, SMP5, SMP6
Content Standards

6.RP.1, 6.RP.2, 6.RP.3, 6.RP.3d

Mental Math and Reflexes

Students use proportional reasoning to convert between units of measurement. *Suggestions:*

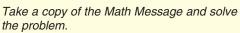
• 00 150 minutes =
$$\frac{2\frac{1}{2}}{2}$$
 hours 360 months = $\frac{30}{2}$ years

••• 3.6 kilometers =
$$\frac{360,000}{0.782}$$
 centimeters 782 milliliters = $\frac{0.782}{0.782}$ liters

••• 3.7 square decimeters =
$$370$$
 square centimeters 7 square feet = $1,008$ square inches



Math Message

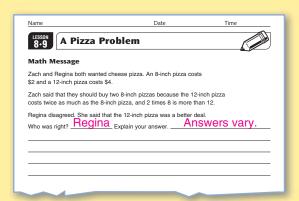




Study Link 8-8 Follow-Up



Remind students they must complete *Math Masters*, pages 260 and 261 within the next two or three days.



Math Masters, page 264

1 Teaching the Lesson

► Math Message Follow-Up



(Math Masters, p. 264)

Ask students to share their solutions and explanations. Address the following points in the discussion:

 \triangleright The diameter of a 12-inch pizza is 50% greater than the diameter of an 8-inch pizza (8 * 1.5 = 12).

$$\frac{(\text{diameter of larger pizza})}{(\text{diameter of smaller pizza})} \, \frac{12 \text{ inches}}{8 \text{ inches}} = \frac{1.5}{1}$$

- \triangleright Considering only the ratio of the diameters $(\frac{1.5}{1})$, it appears that a 12-inch pizza is not worth twice the price of an 8-inch pizza. This is what misled Zach.
- ▶ The area of a pizza is a better measure of its value than the diameter.

Have students use their calculators to find the areas of an 8-inch and a 12-inch pizza.

Area of 8-inch pizza: $\pi * (4 \text{ inches})^2 \text{ is about } 50 \text{ in}^2$

Area of 12-inch pizza: $\pi * (6 \text{ inches})^2$ is about 113 in²

Because the area of a 12-inch pizza is more than twice as much as the area of an 8-inch pizza, it is a better buy than two 8-inch pizzas.

Adjusting the Activity

Have students follow these steps to extend the investigation comparing the areas of the two pizzas.

- Step 1 Use the Geometry Template to make scale drawings of the pizzas as concentric circles using the scale: 1 centimeter represents 1 inch. The inner circle will represent the 8-inch pizza.
- Step 2 Cut off the part of the larger circle that is not covered by the smaller one. Then cut this rim into pieces and place them inside the smaller circle.

Students will find that the pieces of the rim will more than cover the smaller circle, proving that the 12-inch pizza has more than twice as much area as the 8-inch pizza.

AUDITORY KINESTHETIC ◆ TACTILE VISUAL

Using Ratios to **Describe Size Changes**



(Student Reference Book, pp. 121-123)

Algebraic Thinking Read and discuss the essay on *Student* Reference Book, pages 121–123 as a class. Emphasize the following ideas in the discussion:

 \triangleright The **size-change factor** is really an **n-to-1 ratio**: a ratio of some number to 1. It tells the amount of enlargement or reduction that occurs in a size-change situation. For example, a size-change factor of 3X describes an enlargement in which each length is three times the size of the corresponding length in the original object. That is, the ratio of the **enlargement** to the original is 3 to 1. To support English language learners, discuss the different meanings of the word *factor*, including its meaning in this context. Provide some examples of enlargements.

> (enlarged length) (original length)

The same size-change factor applies to every length in the original figure. A size-change factor that is less than 1, such as 0.4X, describes a **reduction** of the original figure. To support English language learners, discuss the meaning of reduction, using examples.

> (enlarged length) 0.4 (original length)

▶ The size-change factor applies only to lengths, not to areas, volumes, or angle sizes. This is what misled Zach in the Math Message. The size-change factor of the diameter from the 8-inch pizza to the 12-inch pizza is 1.5. Zach interpreted this ratio to mean that a 12-inch pizza is only 1.5 times larger than an 8-inch pizza, which is less than the increase in price. But the area changes by a factor of 2.25 (1.5 * 1.5 = 2.25), which is more than the increase in price and considerably more than the 1.5 size-change factor.

Rates, Ratios, and Proportions

Using Ratios to Describe Size Changes

Many situations produce a **size change**. For example, a magnifying glass, a microscope, and an overhead projector al produce size changes that enlarge the original images. Most copying machines can create a variety of size changes—both enlargements and reductions of the original document.

Similar figures are figures that have the same shape but not necessarily the same size. Enlargements or reductions are similar to the originals; that is, they have the same shape as the originals.

the originals. The size-change factor is a number that tells the amount of enlargement or reduction that takes place. For example, if you use a copy machine to make a 2X change in size, then every length in the copy is twice the size of the original. The size-change factor is 2, or 200%. If you make a 0.5X change in size, then every length in the copy is half the size of the original. The size-change factor is $\frac{1}{2}$, or 0.5, or 50%.

You can think of the size-change factor as a ratio. For a 2X size change, the ratio of a length in the copy to the corresponding length in the original is 2 to 1.

size-change factor 2: copy size original size

For a 0.5X size change, the ratio of a length in the copy to a corresponding length in the original is 0.5 to 1.

size-change factor 0.5: $\frac{\text{copy size}}{\text{original size}} = \frac{0.5}{1}$

If the size-change factor is greater than 1, then the copy is an enlargement of the original. If it is less than 1, then the copy is a reduction of the original.

a A reduction of ue original.

A photographer uses an enlarger to make prints from negatives If the size of the image on the negative is 2" by 2" and the size of the image on the print is 6" by 6", then the size-change factor is 3. Binoculars that are 8X, or "8 power," magnify all the lengths you see with the naked eye to 8 times their actual size.









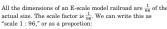
Student Reference Book, page 121

Student Page

A model that is a careful, reduced copy of an actual object is called a **scale model**. You have probably seen scale models of cars, trains, and airplanes. The size-change factor in scale models is usually called the scale factor.

Dollhouses often have a scale factor of $\frac{1}{12}.$ You can write this as " $\frac{1}{12}$ of actual size," "scale 1:12," " $\frac{u-1}{12}$ scale," or as a proportion:

 $\frac{\text{dollhouse length}}{\text{real house length}} = \frac{1 \text{ inch}}{12 \text{ inches}}$



 $\frac{\text{model railroad length}}{\text{real railroad length}} = \frac{1 \text{ inch}}{96 \text{ inches}}$

We can also write this as "scale: $\frac{1}{8}$ inch represents 1 foot," or s "scale: 0.125 inch represents 12 inches." To see this, write

$$\frac{\frac{1}{8}}{12} = \frac{\frac{1}{8} * 8}{12 * 8} = \frac{1}{96}$$

1 inch: 12 inches is the same as 1 inch: 96 inches.

Scale Drawings
The size-change factor for scale drawings is usually called the scale. If an architect's scale drawing shows "scale $\frac{1}{4}$ inch : 1 foot or "scale: $\frac{1}{4}$ inch represents 1 foot," then $\frac{1}{4}$ inch on the drawing represents 1 foot of actual length.

$$\frac{\text{drawing length}}{\text{real length}} = \frac{\frac{1}{4} \text{inch}}{1 \text{ foot}}$$

Since 1 foot = 12 inches, we can rename

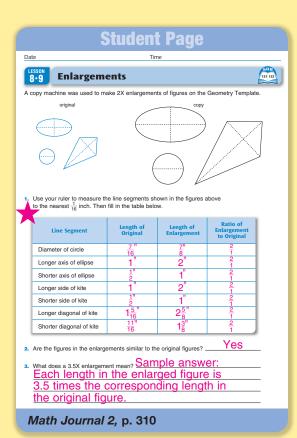
$$\frac{\frac{1}{4} \text{ inch}}{1 \text{ foot}}$$
 as $\frac{\frac{1}{4} \text{ inch}}{12 \text{ inches}}$

Multiply by 4 to change this to an easier fraction

$$\frac{1}{4}$$
 inch \circ 4 _ 1 inch

 $\frac{\frac{1}{4} \operatorname{inch} \circ 4}{12 \operatorname{inches} \circ 4} = \frac{1 \operatorname{inch}}{48 \operatorname{inches}}$ The drawing is $\frac{1}{48}$ of the actual size

Student Reference Book, page 122



Caution students that at times they may see scales written in this way:

scale: 1 inch = 1 foot

The two measures are obviously not equal. The equal sign is being used as a symbol for the word *represents*. To support English language learners, also discuss the meaning of the word *scale* in this context.

Size Change	Size-change Factor Changed length original length
8X	<u>8</u> 1
scale 1:12	<u>1</u> 12
scale 1:96	<u>1</u> 96
scale 1:48	<u>1</u> 48
scale 1:24,000	<u>1</u> 24,000

Adjusting the Activity

Have students draw a 5×5 square on a sheet of grid paper (*Math Masters*, p. 408). Then have them draw and label a 2X enlargement of the square (a 10×10 square). Ask students to measure the angles, find the side lengths, and calculate the area of each square. Discuss their findings.

The angles of each square measure 90° . The side lengths of the enlargement are twice as great as the side lengths of the original. The enlargement's area is 4 times the area of the original.

AUDITORY + KINESTHETIC + TACTILE + VISUAL

▶ The size-change factor is often identified by other names, such as **scale factor** or **scale**. Many different notations indicate this ratio. Use the following examples to show a variety of notations:

aX scale 1:a $\frac{1}{a}$ actual size 0.aX $\frac{1}{a}$ scale a power a unit 1=b unit 2

In practice, any of these notations will serve. However, more explicit notations are easier for students to understand. For this reason, use $\frac{(\text{changed size})}{(\text{original size})} = \frac{a}{1}$ whenever possible, even though it is not the most common notation.

Each size-change factor has two possible forms, because a ratio may be expressed in either order. Context will always dictate which size-change factor is being used, and students are unlikely to be confused in this regard. For example, for 8X binoculars, the size-change factor 8 clearly refers to the view of the object through the binoculars compared to the view of the object without binoculars. The size-change factor $\frac{1}{8}$ refers to the size of the object when viewed without binoculars compared to the size of the object when viewed through the binoculars.

- ▶ Make a table on the board or a transparency and record each new notation (from *Student Reference Book*, pp. 122 and 123) as it arises. Then write the size-change factor for this notation and its reciprocal. Refer to the table as you read about the binoculars, dollhouse, model railroad examples, and map scales.
- ▶ In cases where two different units convey size-change factor information, caution students to convert one of the quantities, so both have the same unit, before making a ratio calculation. For example, for the architect's scale, ½ inch represents 1 foot, converting all units to inches means that 1 inch represents 48 inches, which yields the scale of 1:48.

▶ Solving Size-Change Problems

(*Math Journal 2*, pp. 310 and 311; *Student Reference Book*, pp. 121–124)



PARTNER

Assign the journal pages to partnerships. Suggest that students keep the *Student Reference Book* open to pages 121–124 so they can refer to vocabulary and examples of notation. For Problem 4 on journal page 311, students may need to look up the number of feet in a mile in the back of their journals or in the *Student Reference Book*.



Ongoing Assessment: Informing Instruction

Watch for students who are unsure of how to calculate the number of inches in a mile. Suggest that they apply their knowledge of proportions to set up the following comparison:

$$\frac{\text{(in.)}}{\text{(ft)}} \frac{12}{1} = \frac{x}{5,280}$$



Ongoing Assessment: Recognizing Student Achievement

Journal
Page 310
Problem 1

Use **journal page 310, Problem 1** to assess students' ability to measure line segments to the nearest $\frac{1}{16}$ inch. Students are making adequate progress if they can complete the table in Problem 1.

[Measurement and Reference Frames Goal 1]

2

Ongoing Learning & Practice

► Math Boxes 8·9



(Math Journal 2, p. 312)

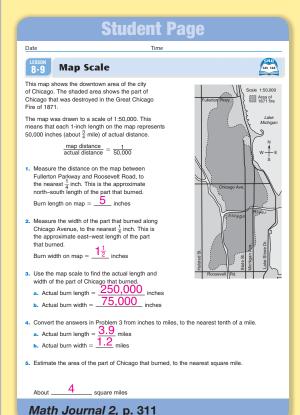


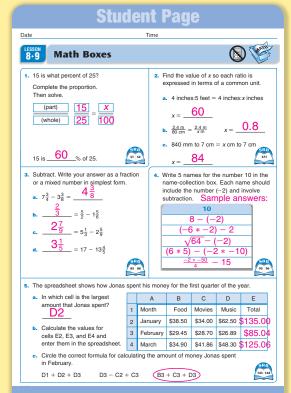
Mixed Practice Math Boxes in this lesson are paired with Math Boxes in Lesson 8-11. The skill in Problem 5 previews Unit 9 content.

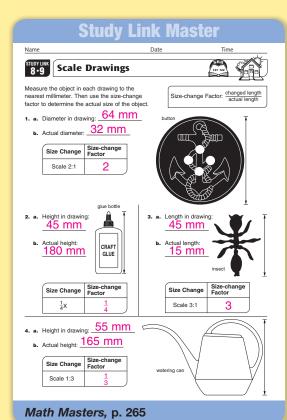


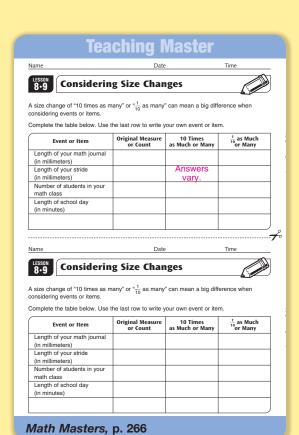
Writing/Reasoning Have students write a response to the following: *Explain the method you used to find the difference in Problem 3b.* Sample answer: Convert $1\frac{5}{6}$ to

an improper fraction $(\frac{11}{6})$. Use the LCM to rename each fraction $(\frac{15}{6} - \frac{11}{6})$. Subtract and simplify $(\frac{4}{6} = \frac{2}{3})$.









► Study Link 8.9

(Math Masters, p. 265)





Home Connection Students calculate the original size of objects shown in scale drawings.

Differentiation Options

READINESS

PARTNER

Considering What Size **Change Means**

5-15 Min

(Math Masters, p. 266)

Students explore the effect of increasing or reducing a measurement or an event by a factor of 10. They complete a table of "10 times as much" and " $\frac{1}{10}$ as much" for everyday items or events. Have students share the item or the event they wrote in the last row. This is especially beneficial for English language learners.

EXTRA PRACTICE

PARTNER ACTIVITY

15-30 Min

▶ Finding Dimensions of **Objects Based on Scale Models**

(Math Masters, p. 267)

Students study scale drawings of objects and then calculate the actual dimensions. They discover that a size-change factor, which applies to lengths, does not apply to areas.

EXTRA PRACTICE

INDEPENDENT **ACTIVITY**

▶ Investigating Perimeter and Size-Change Factor

5-15 Min

(Math Masters, p. 268)

Students measure the dimensions of polygons and determine the size-change factors. This activity is a review of scale factors from Fifth Grade Everyday Mathematics.

ELL SUPPORT



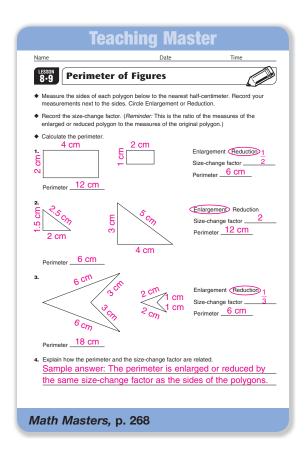


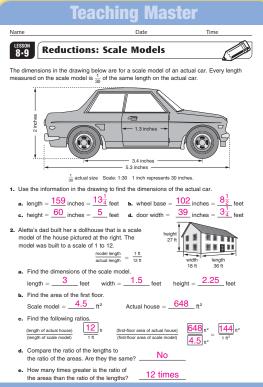


Students make posters illustrating the meaning of *enlargement* and *reduction*. The posters should include labels. After students present their posters to the class, display the posters in the classroom to facilitate vocabulary development.

Planning Ahead

Students will use pattern blocks in Lesson 8-10. Each partnership will need at least 16 triangles and 10 trapezoids. Other shapes are useful but not required. If you do not have these materials, you might be able to borrow them from another teacher.





Math Masters, p. 267