QMCTLS: Quasi Monte Carlo Texture Likelihood Sampling for Despeckling of Complex Polarimetric SAR Images

“Monte Carlo methods and sampling for computing” course, A.A. 2022/2023

Pisa, 21/06/2023

Francesco Mancuso

This presentation introduces a work by Li *et al.:*

F. Li, L. Xu, A. Wong and D. A. Clausi, "QMCTLS: Quasi Monte Carlo Texture Likelihood Sampling for Despeckling of Complex Polarimetric SAR Images," in *IEEE Geoscience and Remote Sensing Letters*, vol. 12, no. 7, pp. 1566-1570, July 2015, doi: 10.1109/LGRS.2015.2413299.

Despeckling complex SAR images

Denoising Synthetic Aperture Radar images

- SAR **speckle noise** is caused by the constructive and destructive interference of electromagnetic waves in SAR imaging.

[1]Shrey Dabhi, Kartavya Soni, Utkarsh Patel, Priyanka Sharma, Manojkumar Parmar, May 29, 2020, "Virtual SAR: A Synthetic Dataset for Deep Learning based Speckle Noise Reduction Algorithms", IEEE Dataport, doi: *https://dx.doi.org/10.21227/asth-ra98*

Despeckling complex SAR images

Denoising Synthetic Aperture Radar images

- SAR **speckle noise** is caused by the constructive and destructive interference of electromagnetic waves in SAR imaging.

- **Despeckling** enhances SAR image quality for target detection, land cover classification, and change detection.

[1]Shrey Dabhi, Kartavya Soni, Utkarsh Patel, Priyanka Sharma, Manojkumar Parmar, May 29, 2020, "Virtual SAR: A Synthetic Dataset for Deep Learning based Speckle Noise Reduction Algorithms", IEEE Dataport, doi: *https://dx.doi.org/10.21227/asth-ra98*

Complex polarimetric SAR noise model

Degradation model of multilook polarimetric SAR images

$$\boldsymbol{k} = \left[S_{HH}, S_{HV}, S_{VV}\right]^T \longleftarrow$$

zero-mean multidimensional gaussian pdf

$$\boldsymbol{C} = E\{\boldsymbol{k}\boldsymbol{k}^H\} = \begin{bmatrix} E\{S_{HH}S_{HH}^*\} & E\{S_{HH}S_{HV}^*\} & E\{S_{HH}S_{VV}^*\} \\ E\{S_{HV}S_{HH}^*\} & E\{S_{HV}S_{HV}^*\} & E\{S_{HV}S_{VV}^*\} \\ E\{S_{VV}S_{HH}^*\} & E\{S_{VV}S_{HV}^*\} & E\{S_{VV}S_{VV}^*\} \end{bmatrix}$$

$$\boldsymbol{Z} = \boldsymbol{C} + \boldsymbol{N_m} + \boldsymbol{N_a}$$

additive noise

multiplicative noise

$$\boldsymbol{k}_p = \frac{1}{\sqrt{2}}\left[S_{HH} + S_{VV}, S_{HH} - S_{VV}, 2S_{HV}\right]^T$$

[2]Liu, Xu, et al. "PolSF: PolSAR Image Datasets on San Francisco." *Intelligence Science IV: 5th IFIP TC 12 International Conference, ICIS 2022, Xi'an, China, October 28–31, 2022, Proceedings*. Cham: Springer International Publishing, 2022.

Estimation problem formulation

Bayesian Least Square optimization problem

$$\hat{\boldsymbol{C}} = \arg\min_{C} \left[E\left\{ \left(\boldsymbol{C} - \hat{\boldsymbol{C}}\right)^2 \middle| \boldsymbol{Z} \right\} \right] = \arg\min_{C} \left[\int \left(\boldsymbol{C} - \hat{\boldsymbol{C}}\right)^2 p\left(\boldsymbol{C} | \boldsymbol{Z}\right) d\boldsymbol{C} \right]$$

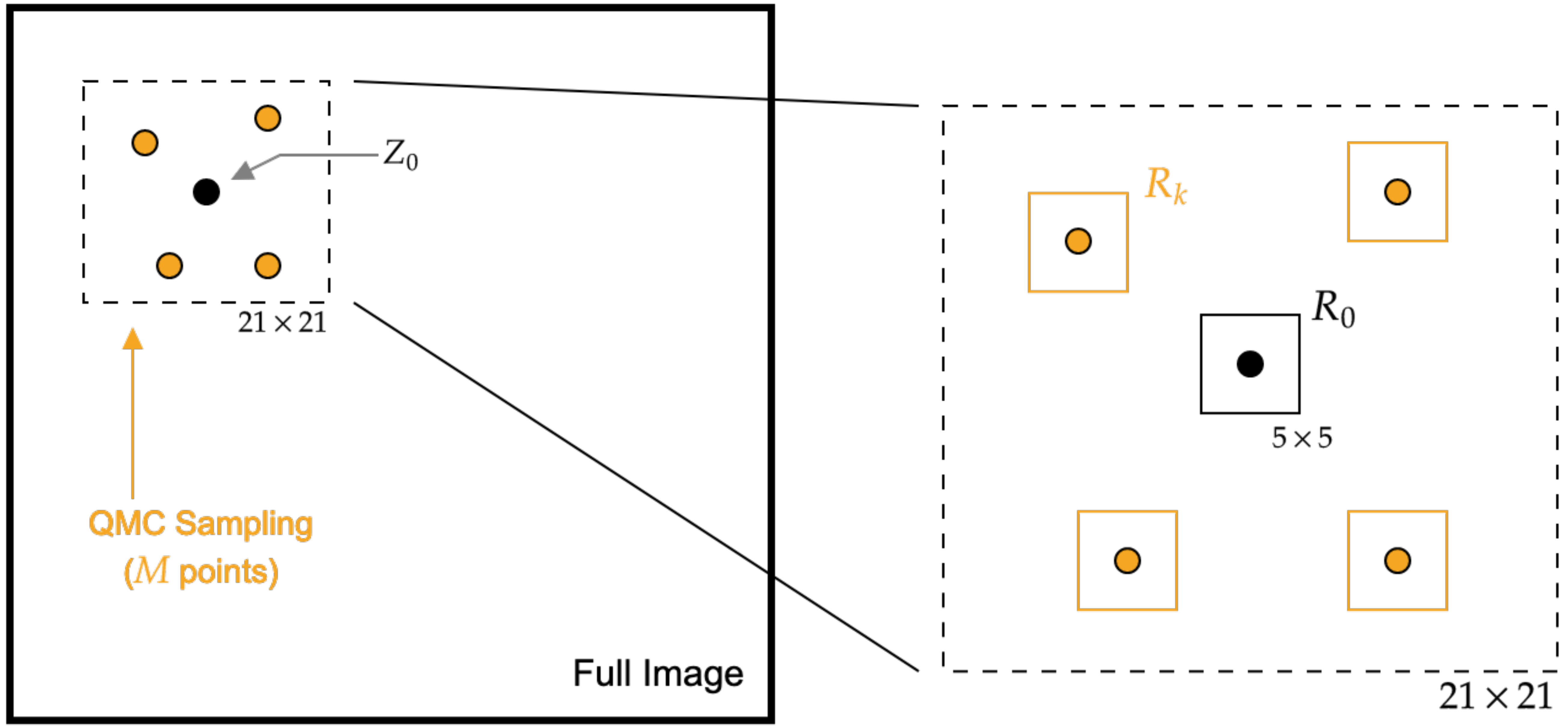
$$\frac{\partial}{\partial \hat{\boldsymbol{C}}} \int \left(\boldsymbol{C} - \hat{\boldsymbol{C}}\right)^2 p\left(\boldsymbol{C} | \boldsymbol{Z}\right) d\boldsymbol{C} = 2 \int \left(\boldsymbol{C} - \hat{\boldsymbol{C}}\right) p\left(\boldsymbol{C} | \boldsymbol{Z}\right) d\boldsymbol{C} = 0$$

$$\hat{\boldsymbol{C}} \int p\left(\boldsymbol{C} | \boldsymbol{Z}\right) d\boldsymbol{C} = \int \boldsymbol{C} p\left(\boldsymbol{C} | \boldsymbol{Z}\right) d\boldsymbol{C} = E\left\{\boldsymbol{C} | \boldsymbol{Z}\right\} = \hat{\boldsymbol{C}}$$

we need to compute that pdf

Posterior estimation using QMCS

And summary of the QMCTLS filter



all the pixels belonging to that region

1. $\alpha\left(R_k | R_0\right) = \left\{\prod_j P\left(-2\rho \ln Q_j \geq z\right)\right\}^{\frac{1}{\beta}}$
2. $u \in U(0,1)$
3. $u \leq \alpha\left(R_k | R_0\right) \longrightarrow k \in \Omega$

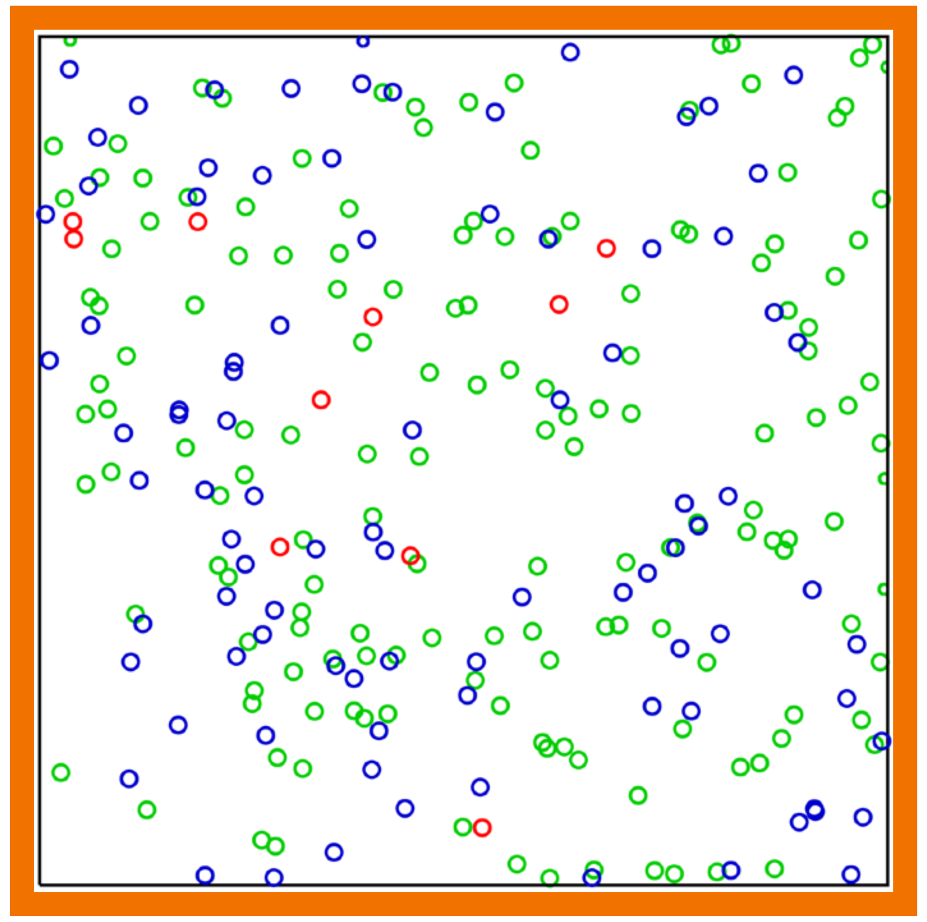
$$\hat{p}\left(C | Z_0\right) = \frac{\sum_{k \in \Omega} \alpha\left(R_k | R_0\right) \delta\left(C - Z_k\right)}{N}$$

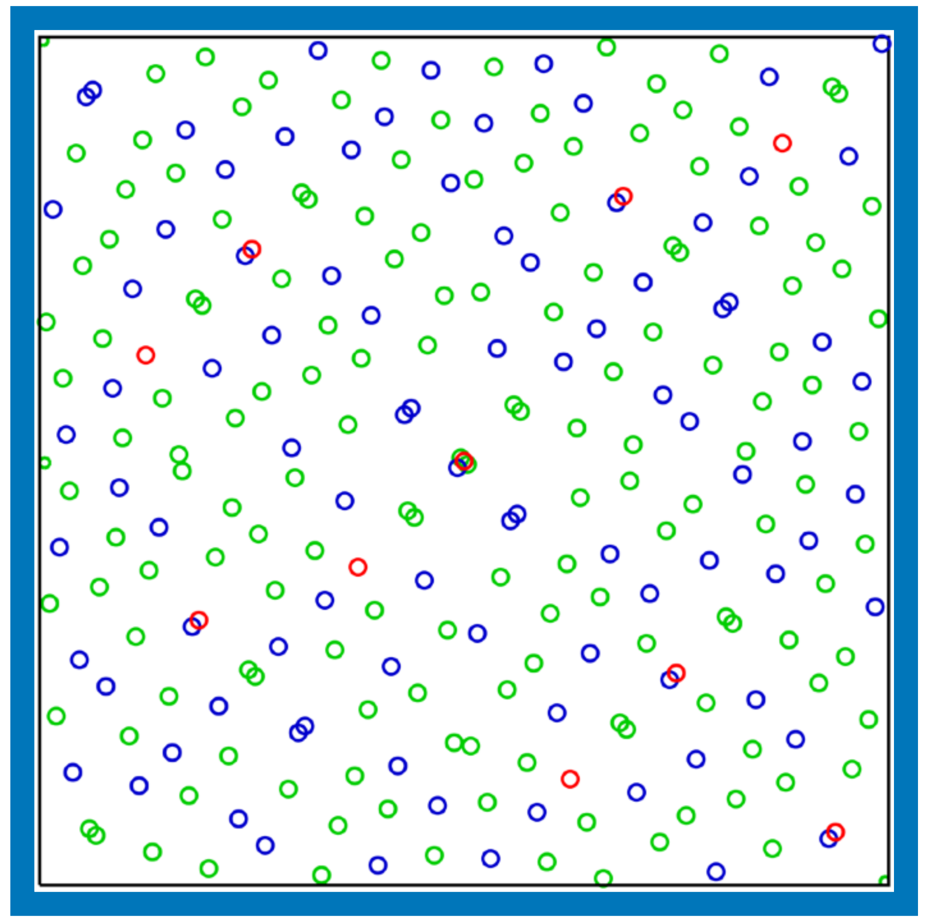
Low-discrepancy sequences

Or quasirandom sequences

- **Low-discrepancy sequences** have points that are **more uniformly distributed** across a space compared to **pseudorandom sequences**;
- Given an interval in d dimensions, the **local discrepancy** of n samples $\boldsymbol{x}_i$ is defined as:

$$\delta(\boldsymbol{a}) = \frac{1}{n}\sum_{i=1}^{n} 1_{\boldsymbol{x}_i \in [\boldsymbol{0}, \boldsymbol{a})} - \prod_{j=1}^{d} a_j$$





[3] https://en.wikipedia.org/wiki/Sobol_sequence

[4] Banterle F., "Monte Carlo methods and sampling for computing," June 2023, http://www.banterle.com/francesco/courses/2023/mc/

Test Statistic for Complex Wishart Distribution

Application to change detection in polarimetric SAR data

$$\hat{\mathbf{\Sigma}}_x = \frac{1}{n}\boldsymbol{X}$$

$$\hat{\mathbf{\Sigma}}_y = \frac{1}{m}\boldsymbol{Y}$$

$$\hat{\mathbf{\Sigma}} = \frac{1}{n+m}(\boldsymbol{X}+\boldsymbol{Y})$$

scale matrices ML estimate

$$\boldsymbol{X} \in W_C\left(p, n, \mathbf{\Sigma}_X\right) \qquad \boldsymbol{Y} \in W_C\left(p, m, \mathbf{\Sigma}_Y\right)$$

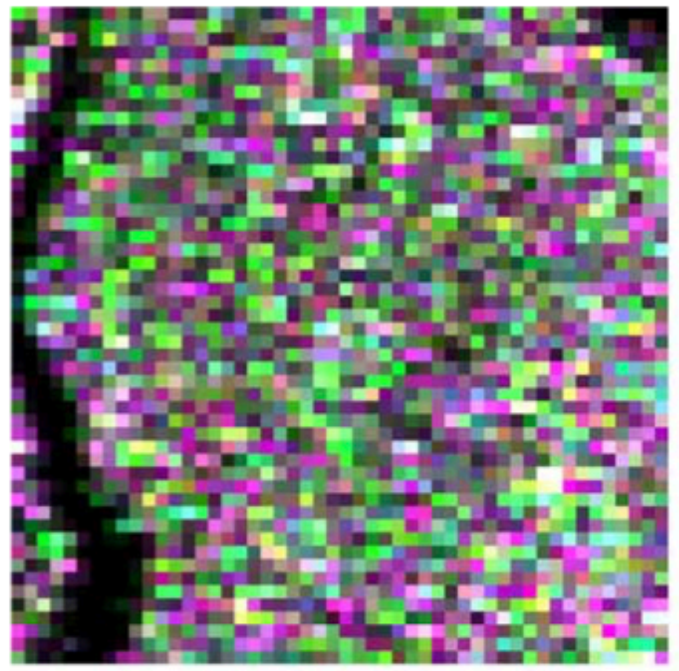
$$Q = \frac{L\left(\hat{\mathbf{\Sigma}}\right)}{L_x\left(\hat{\mathbf{\Sigma}}_x\right)L_y\left(\hat{\mathbf{\Sigma}}_y\right)} = \frac{(n+m)^{p(n+m)}}{n^{pn}m^{pm}} \cdot \frac{|\boldsymbol{X}|^n\,|\boldsymbol{Y}|^m}{|\boldsymbol{X}+\boldsymbol{Y}|^{n+m}}$$

$$n = m \;: \qquad \ln Q = n\left(2p\ln 2 + \ln|\boldsymbol{X}| + \ln|\boldsymbol{Y}| - 2\ln|\boldsymbol{X}+\boldsymbol{Y}|\right)$$

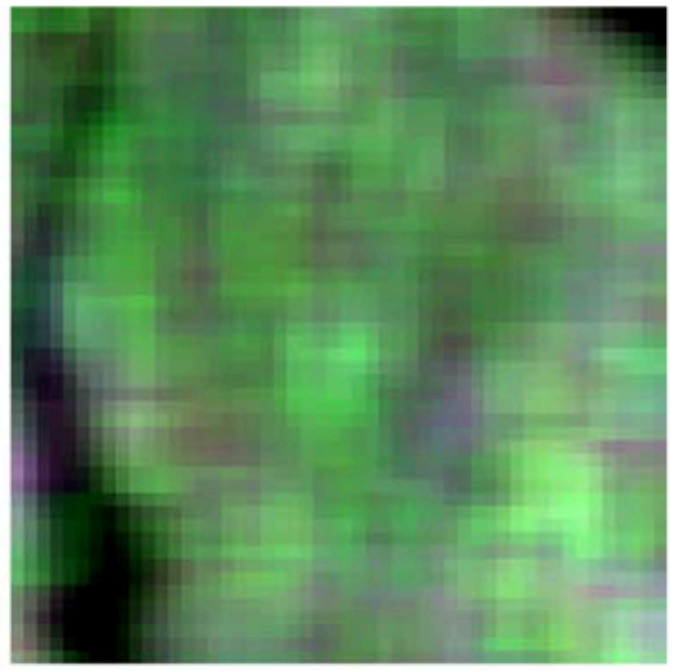
[5] K. Conradsen, A. A. Nielsen, J. Schou and H. Skriver, "A test statistic in the complex Wishart distribution and its application to change detection in polarimetric SAR data," in IEEE Transactions on Geoscience and Remote Sensing, vol. 41, no. 1, pp. 4-19, Jan. 2003, doi: 10.1109/TGRS.2002.808066.

Results and comparison

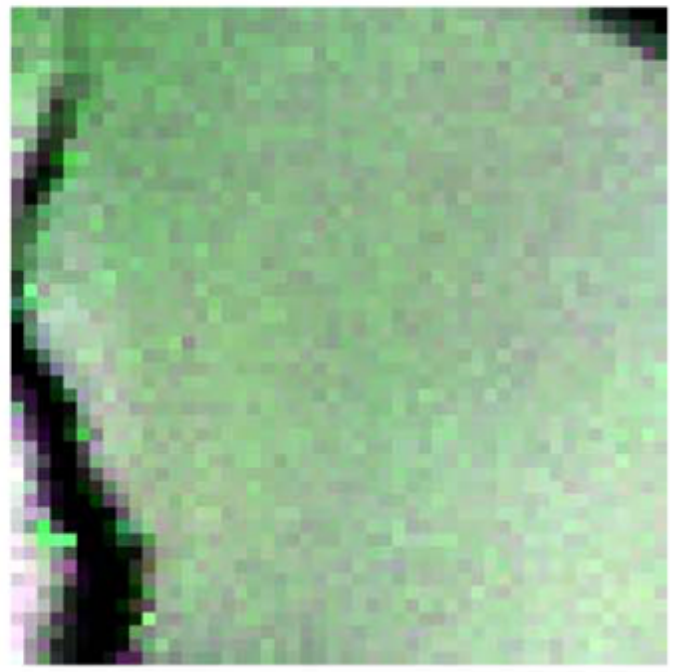
Pauli RGB of despeckling results



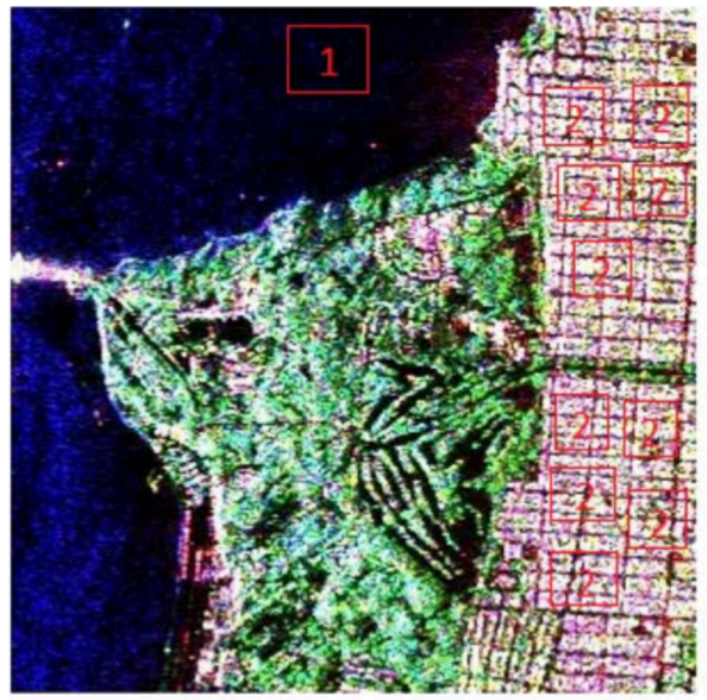
Original



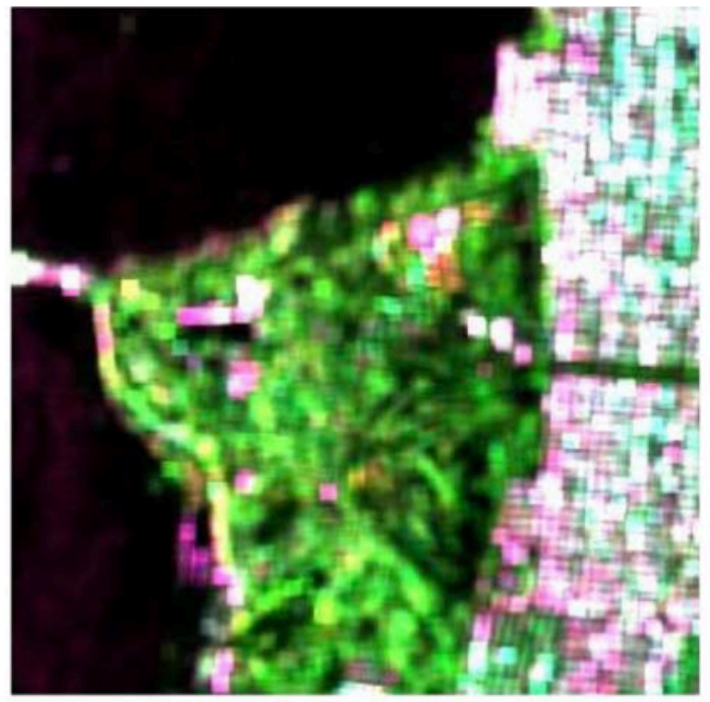
Boxcar



QMCTLS

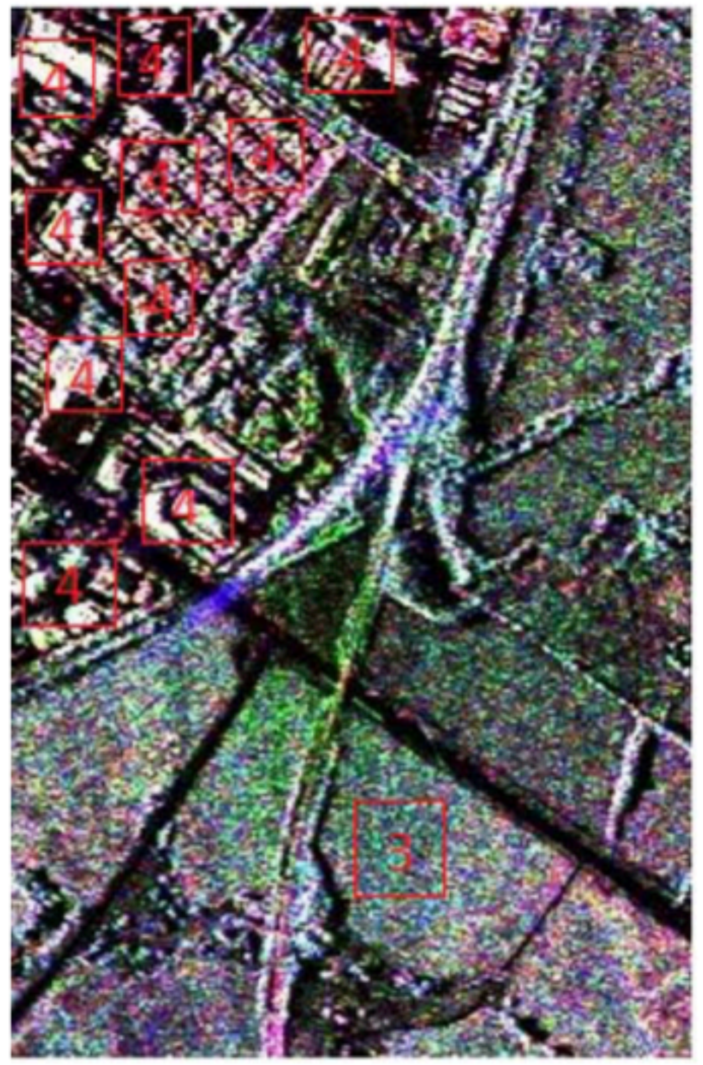


Original

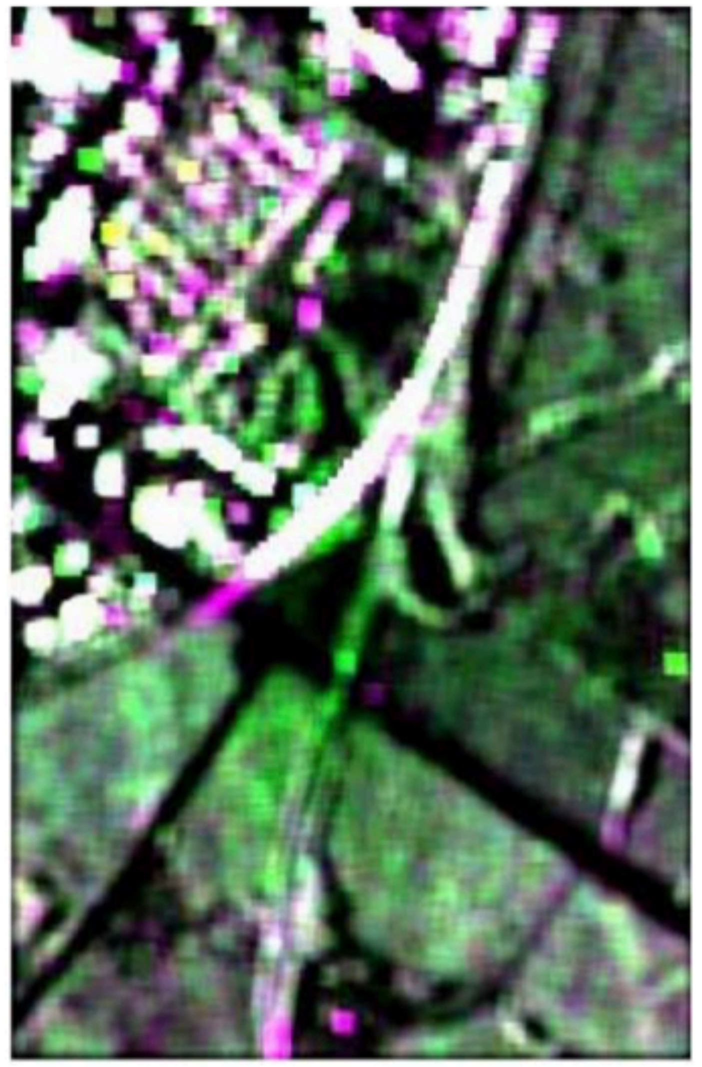


Boxcar

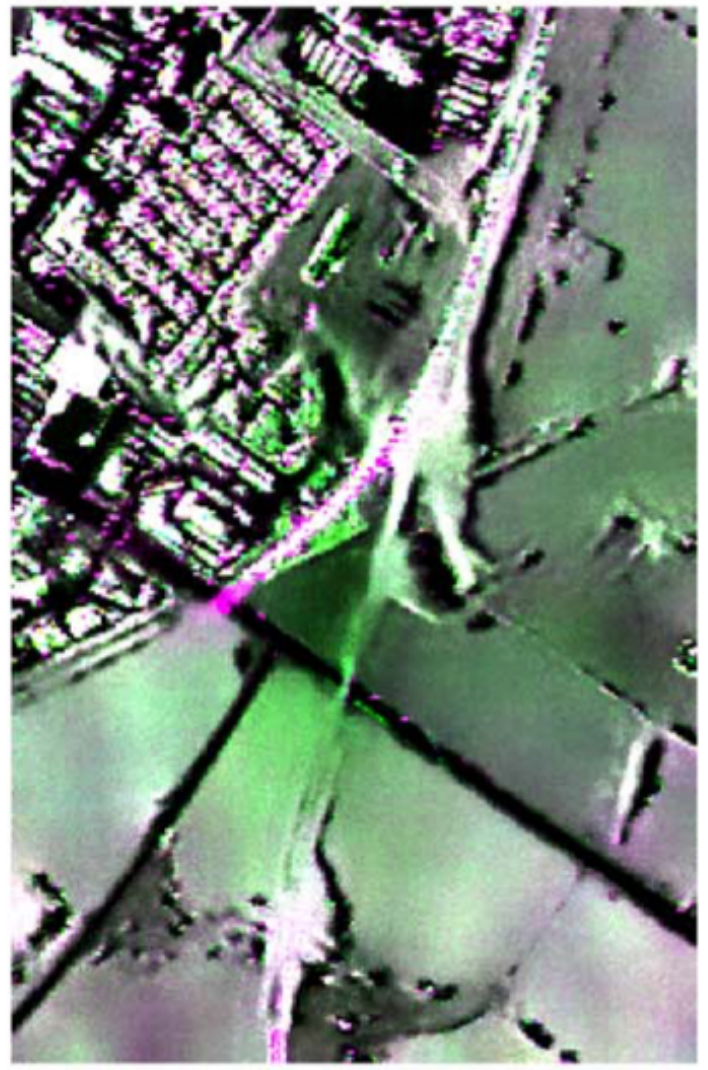
QMCTLS



Original



Boxcar



QMCTLS

Conclusion

And discussion

- **QMCS** has a **faster convergence rate** and thus requires fewer samples to achieve an accurate estimation of the posterior distribution;

- The **region-based likelihood** is capable of capturing the local textual patterns and is therefore more robust to speckle noise;

- The proposed QMCTLS filter **achieve better balance** between noise removal and detail preservation, compared to the classical boxcar filtering.