QMCTLS: Quasi-Monte-Carlo Texture Likelihood Sampling for Despeckling of Complex Polarimetric SAR Images

“Monte Carlo methods and sampling for computing” course, A.A. 2022/2023

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This presentation introduces a work by Li *et al.:*

F. Li, L. Xu, A. Wong and D. A. Clausi, "QMCTLS: Quasi Monte Carlo Texture Likelihood Sampling for Despeckling of Complex Polarimetric SAR Images," in *IEEE Geoscience and Remote Sensing Letters*, vol. 12, no. 7, pp. 1566-1570, July 2015, doi: 10.1109/LGRS.2015.2413299.

Despeckling complex SAR images

Denoising Synthetic Aperture Radar images

- SAR **speckle noise** is caused by the constructive and destructive interference of electromagnetic waves in SAR imaging. It creates high-frequency noise patterns that obscure details, reduce contrast, and impact image analysis accuracy.

- **Despeckling** enhances SAR image quality for target detection, land cover classification, and change detection. Method selection depends on application requirements and the balance between noise reduction and preserving image details.

Complex polarimetric SAR noise model

Degradation model of multilook polarimetric SAR images

$$\boldsymbol{k} = \left[S_{HH}, S_{HV}, S_{VV}\right]^T$$ ⟵ zero-mean multidimensional gaussian pdf

$$\boldsymbol{C} = E\{\boldsymbol{k}\boldsymbol{k}^H\} = \begin{bmatrix} E\{S_{HH}S_{HH}^*\} & E\{S_{HH}S_{HV}^*\} & E\{S_{HH}S_{VV}^*\} \\ E\{S_{HV}S_{HH}^*\} & E\{S_{HV}S_{HV}^*\} & E\{S_{HV}S_{VV}^*\} \\ E\{S_{VV}S_{HH}^*\} & E\{S_{VV}S_{HV}^*\} & E\{S_{VV}S_{VV}^*\} \end{bmatrix}$$

$$\boldsymbol{Z} = \boldsymbol{C} + \boldsymbol{N_m} + \boldsymbol{N_a}$$

$$\boldsymbol{k}_p = \frac{1}{\sqrt{2}}\left[S_{HH} + S_{VV}, S_{HH} - S_{VV}, 2S_{HV}\right]^T$$

Liu, Xu, et al. "PolSF: PolSAR Image Datasets on San Francisco." *Intelligence Science IV: 5th IFIP TC 12 International Conference, ICIS 2022, Xi'an, China, October 28–31, 2022, Proceedings*. Cham: Springer International Publishing, 2022.

Estimation problem formulation

Bayesian Least Square optimization problem

$$\hat{\boldsymbol{C}} = \arg\min_{\boldsymbol{C}} \left[E\left\{ \left(\boldsymbol{C} - \hat{\boldsymbol{C}} \right)^2 \middle| \boldsymbol{Z} \right\} \right] = \arg\min_{\boldsymbol{C}} \left[\int \left(\boldsymbol{C} - \hat{\boldsymbol{C}} \right)^2 p\left(\boldsymbol{C} | \boldsymbol{Z} \right) d\boldsymbol{C} \right]$$

$$\frac{\partial}{\partial \hat{\boldsymbol{C}}} \int \left(\boldsymbol{C} - \hat{\boldsymbol{C}} \right)^2 p\left(\boldsymbol{C} | \boldsymbol{Z} \right) d\boldsymbol{C} = 2 \int \left(\boldsymbol{C} - \hat{\boldsymbol{C}} \right) p\left(\boldsymbol{C} | \boldsymbol{Z} \right) d\boldsymbol{C} = 0$$

$$\hat{\boldsymbol{C}} \int p\left(\boldsymbol{C} | \boldsymbol{Z} \right) d\boldsymbol{C} = \int \boldsymbol{C} p\left(\boldsymbol{C} | \boldsymbol{Z} \right) d\boldsymbol{C} = E\left\{ \boldsymbol{C} | \boldsymbol{Z} \right\} = \hat{\boldsymbol{C}}$$

we need to compute that pdf

Region-based texture similarity likelihood

Log-likelihood-ratio test statistic

$$\ln Q = n\left(6\ln 2 + \ln|\mathbf{Z}_0| + \ln|\mathbf{Z}_k| - 2\ln|\mathbf{Z}_0 + \mathbf{Z}_k|\right)$$

$$\alpha\left(R_k|R_0\right) = \left\{\prod_j P\left(-2\rho\ln Q_j \geq z\right)\right\}^{\frac{1}{\beta}}$$

$$u \in U(0,1) \qquad\qquad u \leq \alpha\left(R_k|R_0\right)$$

Test Statistic for Complex Wishart Distribution

Application to change detection in polarimetric SAR data

$$\hat{\boldsymbol{\Sigma}}_x = \frac{1}{n}\boldsymbol{X}$$

$$\hat{\boldsymbol{\Sigma}}_y = \frac{1}{m}\boldsymbol{Y}$$

$$\hat{\boldsymbol{\Sigma}} = \frac{1}{n+m}(\boldsymbol{X}+\boldsymbol{Y})$$

scale matrices ML estimate

$$\boldsymbol{X} \in W_C\left(p, n, \boldsymbol{\Sigma}_X\right) \qquad \boldsymbol{Y} \in W_C\left(p, m, \boldsymbol{\Sigma}_Y\right)$$

$$Q = \frac{L\left(\hat{\boldsymbol{\Sigma}}\right)}{L_x\left(\hat{\boldsymbol{\Sigma}}_x\right) L_y\left(\hat{\boldsymbol{\Sigma}}_y\right)} = \frac{(n+m)^{p(n+m)}}{n^{pn}m^{pm}} \cdot \frac{|\boldsymbol{X}|^n |\boldsymbol{Y}|^m}{|\boldsymbol{X}+\boldsymbol{Y}|^{n+m}}$$

$$n = m \;: \qquad \ln Q = n\left(2p\ln 2 + \ln|\boldsymbol{X}| + \ln|\boldsymbol{Y}| - 2\ln|\boldsymbol{X}+\boldsymbol{Y}|\right)$$

K. Conradsen, A. A. Nielsen, J. Schou and H. Skriver, "A test statistic in the complex Wishart distribution and its application to change detection in polarimetric SAR data," in IEEE Transactions on Geoscience and Remote Sensing, vol. 41, no. 1, pp. 4-19, Jan. 2003, doi: 10.1109/TGRS.2002.808066.

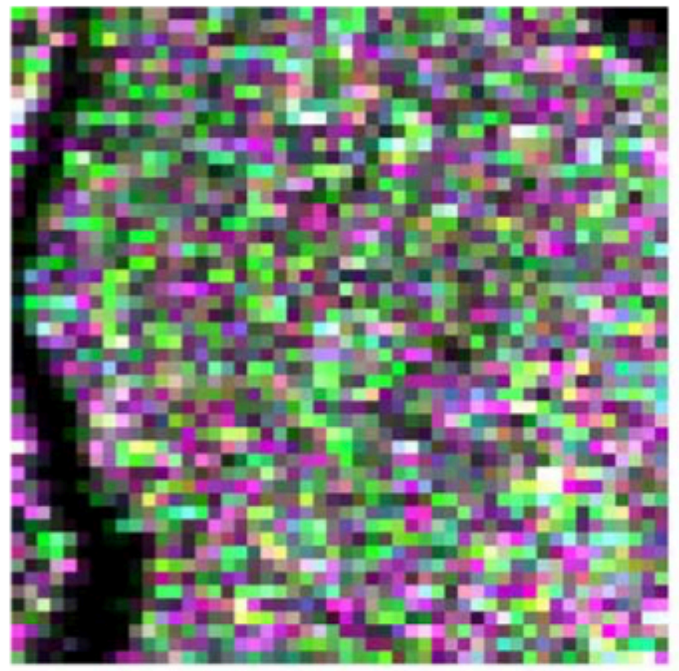
Posterior estimation using QMCS

And summary of the QMCTLS filter

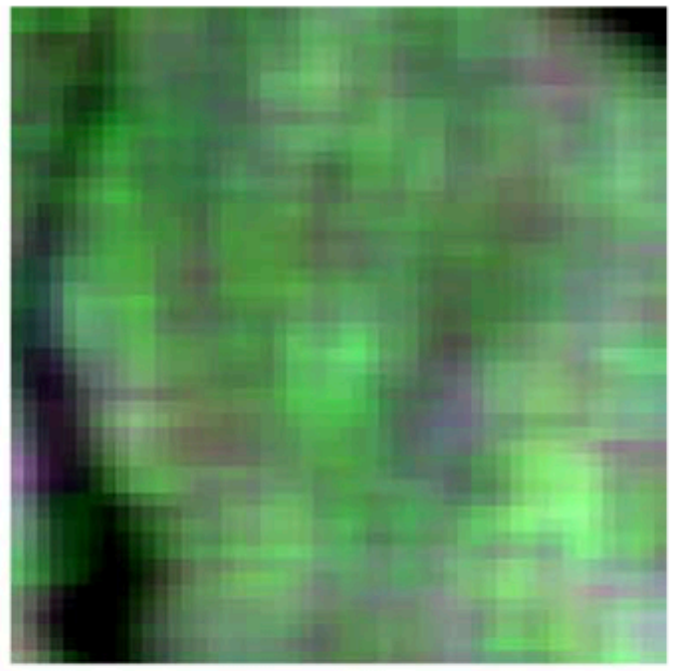
$$\hat{p}\left(\boldsymbol{C} \,|\, \boldsymbol{Z}_0\right) = \frac{\sum_{k \in \Omega} \alpha\left(R_k \,|\, R_0\right) \delta\left(\boldsymbol{C} - \boldsymbol{Z}_k\right)}{N}$$

Results and comparison

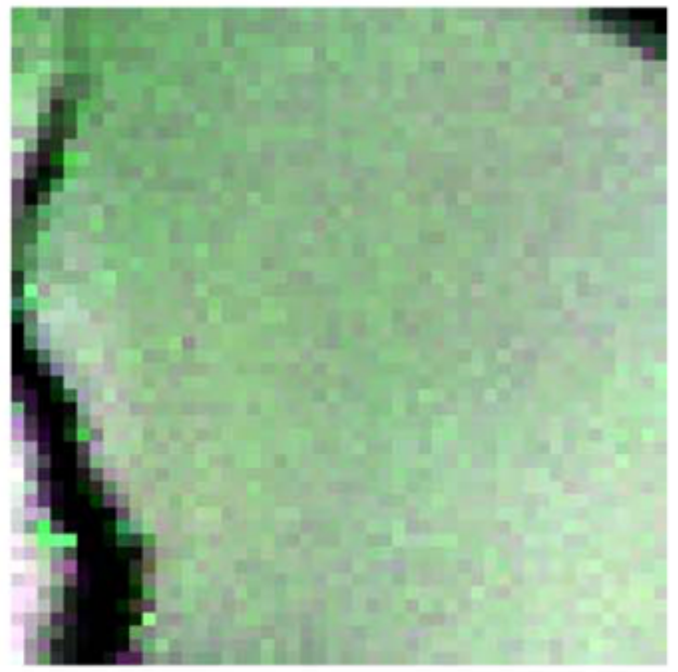
Pauli RGB of despeckling results



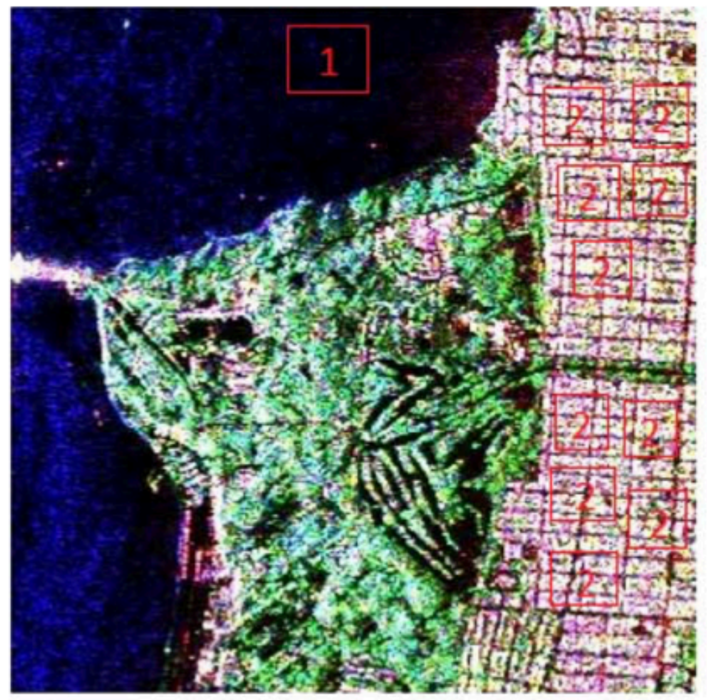
Original



Boxcar



QMCTLS



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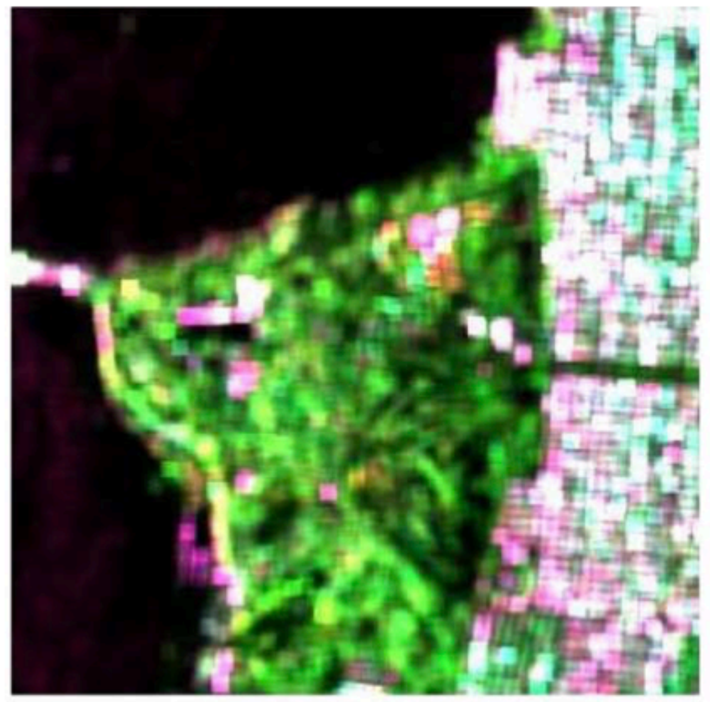
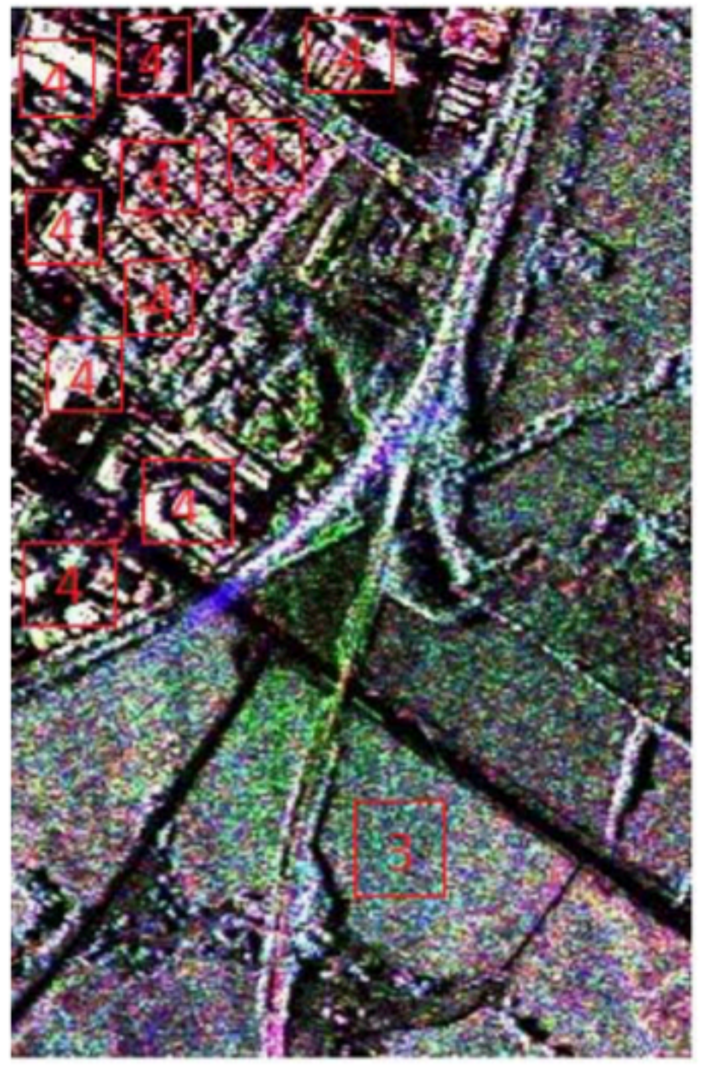


Boxcar

QMCTLS



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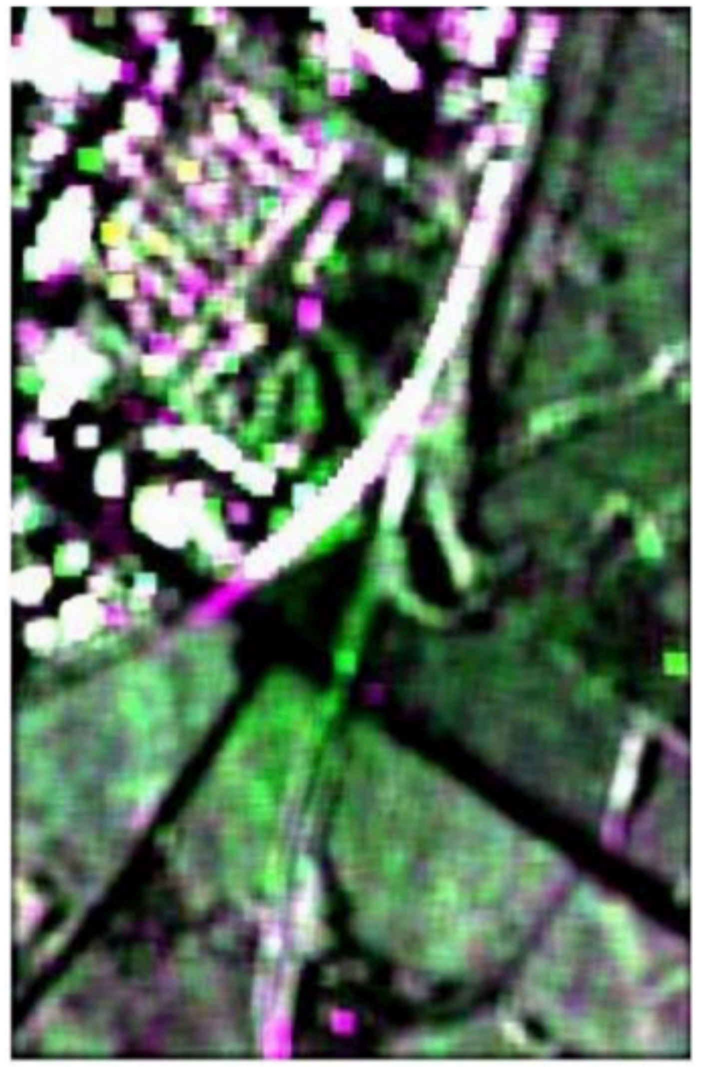

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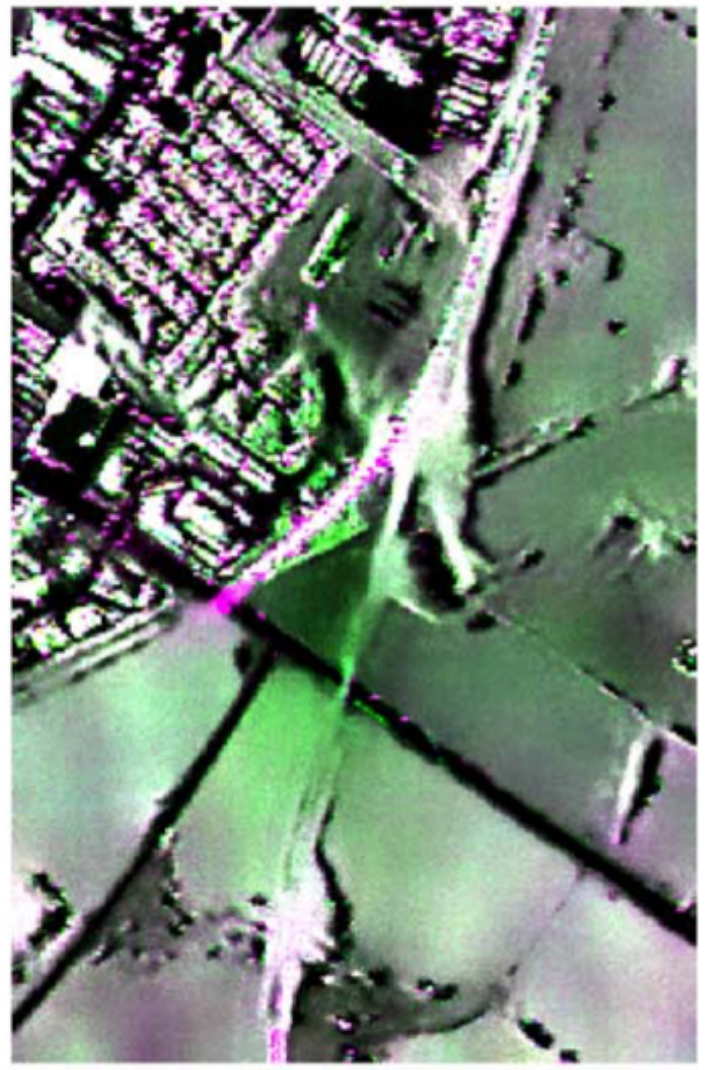

QMCTLS