

# **QMCTLs: Quasi-Monte-Carlo Texture Likelihood Sampling for Despeckling of Complex Polarimetric SAR Images**

“Monte Carlo methods and sampling for computing” course, A.A. 2022/2023



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This presentation introduces a work by Li et al.:

F. Li, L. Xu, A. Wong and D. A. Clausi, "QMCTLs: Quasi Monte Carlo Texture Likelihood Sampling for Despeckling of Complex Polarimetric SAR Images," in *IEEE Geoscience and Remote Sensing Letters*, vol. 12, no. 7, pp. 1566-1570, July 2015, doi: [10.1109/LGRS.2015.2413299](https://doi.org/10.1109/LGRS.2015.2413299).

# Despeckling complex SAR images

## Denoising Synthetic Aperture Radar images

- SAR **speckle noise** is caused by the constructive and destructive interference of electromagnetic waves in SAR imaging. It creates high-frequency noise patterns that obscure details, reduce contrast, and impact image analysis accuracy.
- **Despeckling** enhances SAR image quality for target detection, land cover classification, and change detection. Method selection depends on application requirements and the balance between noise reduction and preserving image details.



# Complex polarimetric SAR noise model

Degradation model of multilook polarimetric SAR images

$$\mathbf{k} = [S_{HH}, S_{HV}, S_{VV}]^T \quad \longleftarrow \quad \text{zero-mean multidimensional gaussian pdf}$$

$$\mathbf{C} = E\{\mathbf{k}\mathbf{k}^H\} = \begin{bmatrix} E\{S_{HH}S_{HH}^*\} & E\{S_{HH}S_{HV}^*\} & E\{S_{HH}S_{VV}^*\} \\ E\{S_{HV}S_{HH}^*\} & E\{S_{HV}S_{HV}^*\} & E\{S_{HV}S_{VV}^*\} \\ E\{S_{VV}S_{HH}^*\} & E\{S_{VV}S_{HV}^*\} & E\{S_{VV}S_{VV}^*\} \end{bmatrix}$$

$$\mathbf{Z} = \mathbf{C} + \underbrace{N_m + N_a}_{\text{multiplicative noise}} \longrightarrow \text{additive noise}$$



$$\mathbf{k}_p = \frac{1}{\sqrt{2}} [S_{HH} + S_{VV}, S_{HH} - S_{VV}, 2S_{HV}]^T$$

Liu, Xu, et al. "PolSF: PolSAR Image Datasets on San Francisco." *Intelligence Science IV: 5th IFIP TC 12 International Conference, ICIS 2022, Xi'an, China, October 28–31, 2022, Proceedings*. Cham: Springer International Publishing, 2022.

# Estimation problem formulation

Bayesian Least Square optimization problem

$$\hat{C} = \arg \min_C \left[ E \left\{ (C - \hat{C})^2 | Z \right\} \right] = \arg \min_C \left[ \int (C - \hat{C})^2 p(C|Z) dC \right]$$

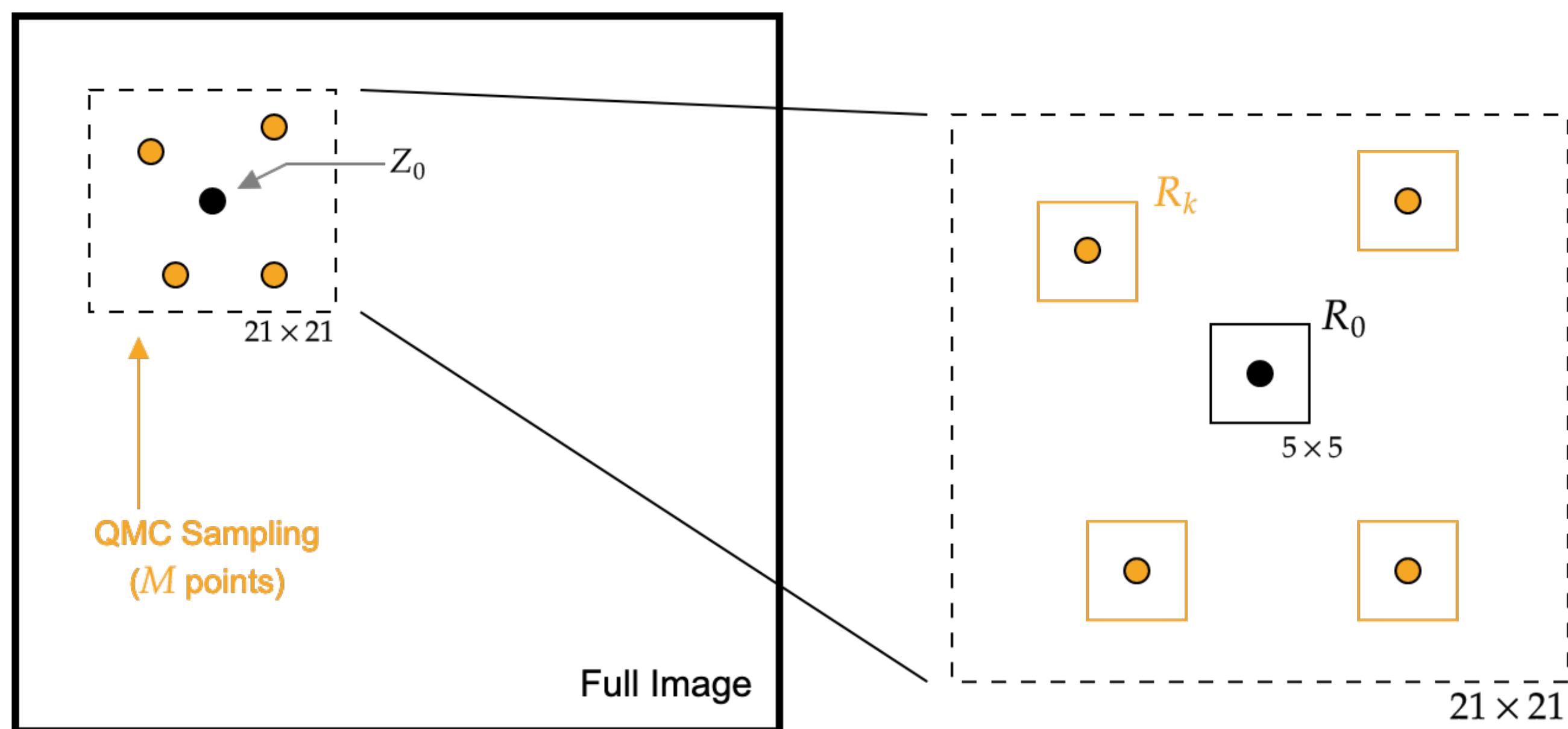
$$\frac{\partial}{\partial \hat{C}} \int (C - \hat{C})^2 p(C|Z) dC = 2 \int (C - \hat{C}) p(C|Z) dC = 0$$

$$\hat{C} \int p(C|Z) dC = \int C p(C|Z) dC = E\{C|Z\} = \hat{C}$$

we need to compute that pdf

# Posterior estimation using QMCS

## And summary of the QMCTLs filter



The search area is set to be  $21 \times 21$ , with approximately 50 % of the pixels in the search area sampled via QMCS, which is a tradeoff between computation cost and filtering performance. The region is defined as a  $5 \times 5$  square window.

- all the pixels belonging to that region
- 1  $\alpha(R_k | R_0) = \left\{ \prod_j P(-2\rho \ln Q_j \geq z) \right\}^{\frac{1}{\beta}}$
  - 2  $u \in U(0,1)$
  - 3  $u \leq \alpha(R_k | R_0) \longrightarrow k \in \Omega$

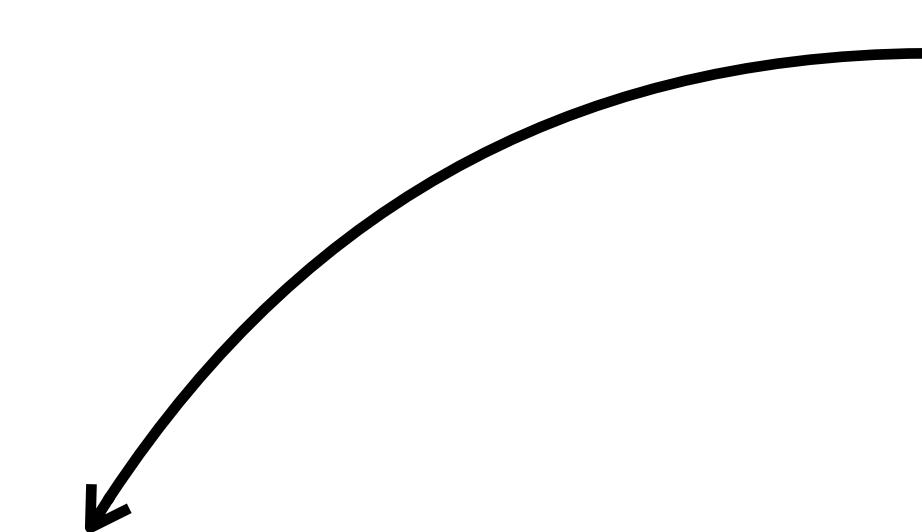
$$\hat{p}(C | Z_0) = \frac{\sum_{k \in \Omega} \alpha(R_k | R_0) \delta(C - Z_k)}{N}$$

# Test Statistic for Complex Wishart Distribution

Application to change detection in polarimetric SAR data

$$X \in W_C(p, n, \Sigma_X) \quad Y \in W_C(p, m, \Sigma_Y)$$

$$Q = \frac{L(\hat{\Sigma})}{L_x(\hat{\Sigma}_x)L_y(\hat{\Sigma}_y)} = \frac{(n+m)^{p(n+m)}}{n^{pn}m^{pm}} \cdot \frac{|X|^n|Y|^m}{|X+Y|^{n+m}}$$



$$\hat{\Sigma}_x = \frac{1}{n} X$$

$$\hat{\Sigma}_y = \frac{1}{m} Y$$

$$\hat{\Sigma} = \frac{1}{n+m} (X + Y)$$

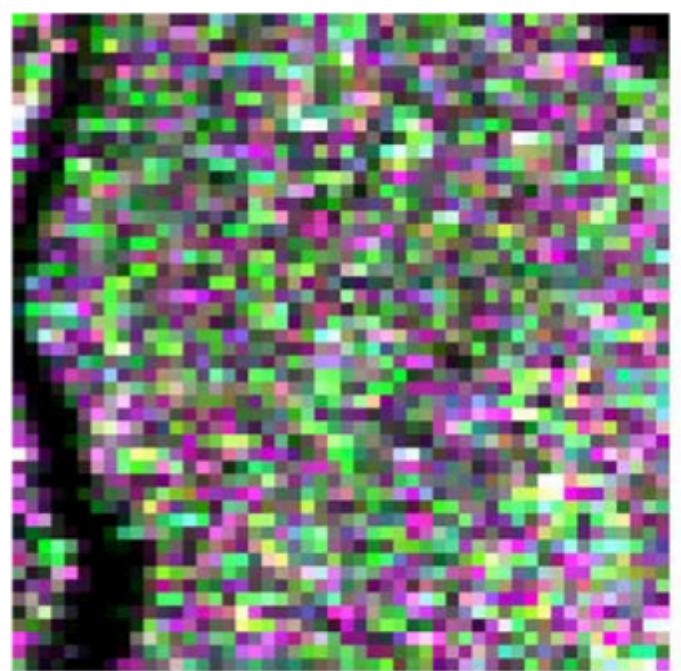
scale matrices ML estimate

$$n = m : \quad \ln Q = n (2p \ln 2 + \ln |X| + \ln |Y| - 2 \ln |X+Y|)$$

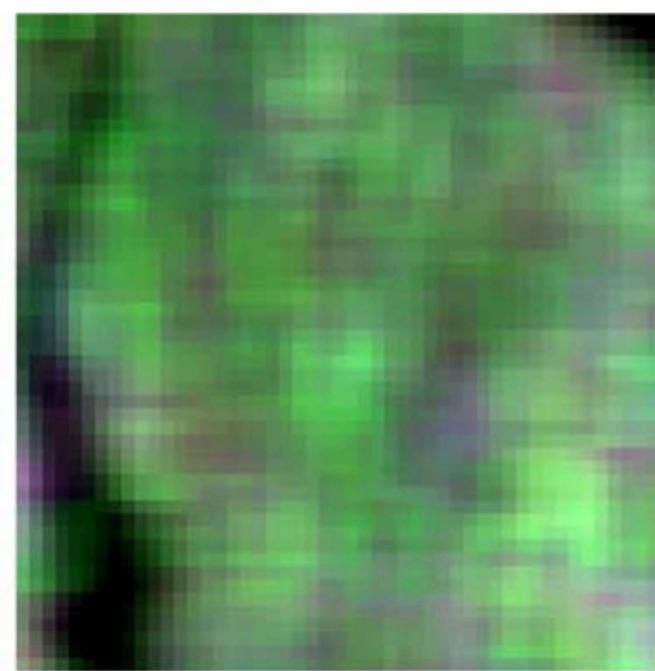
K. Conradsen, A. A. Nielsen, J. Schou and H. Skriver, "A test statistic in the complex Wishart distribution and its application to change detection in polarimetric SAR data," in IEEE Transactions on Geoscience and Remote Sensing, vol. 41, no. 1, pp. 4-19, Jan. 2003, doi: [10.1109/TGRS.2002.808066](https://doi.org/10.1109/TGRS.2002.808066).

# Results and comparison

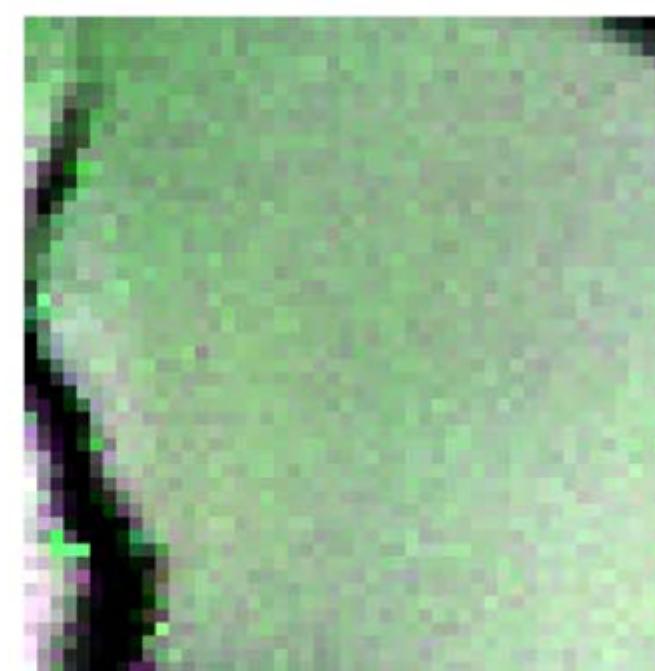
## Pauli RGB of despeckling results



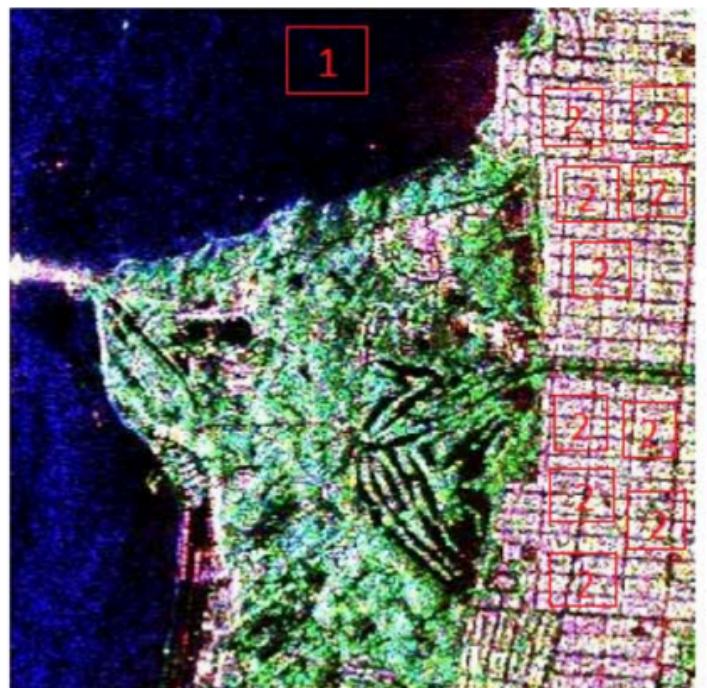
Original



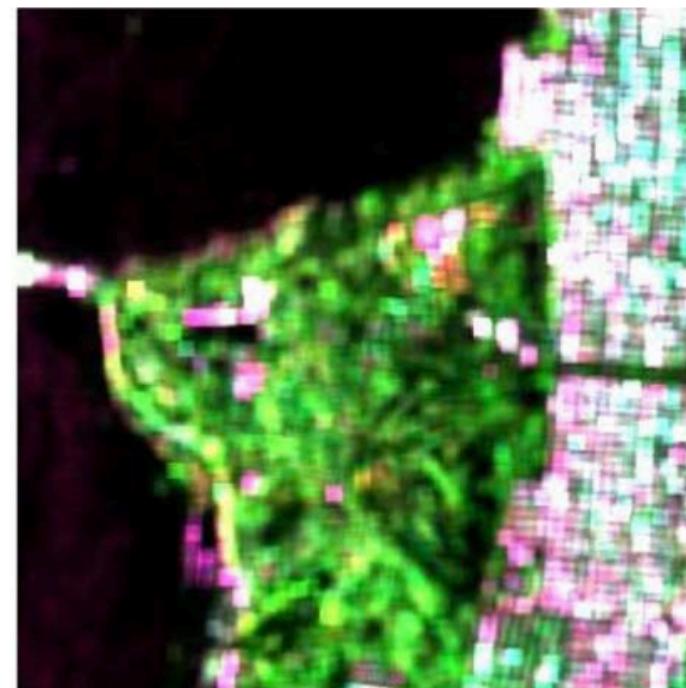
Boxcar



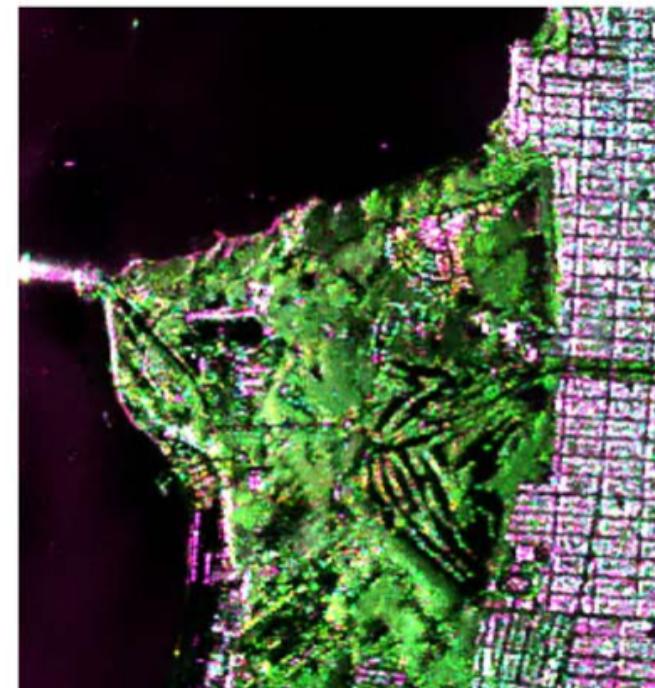
QMCTLs



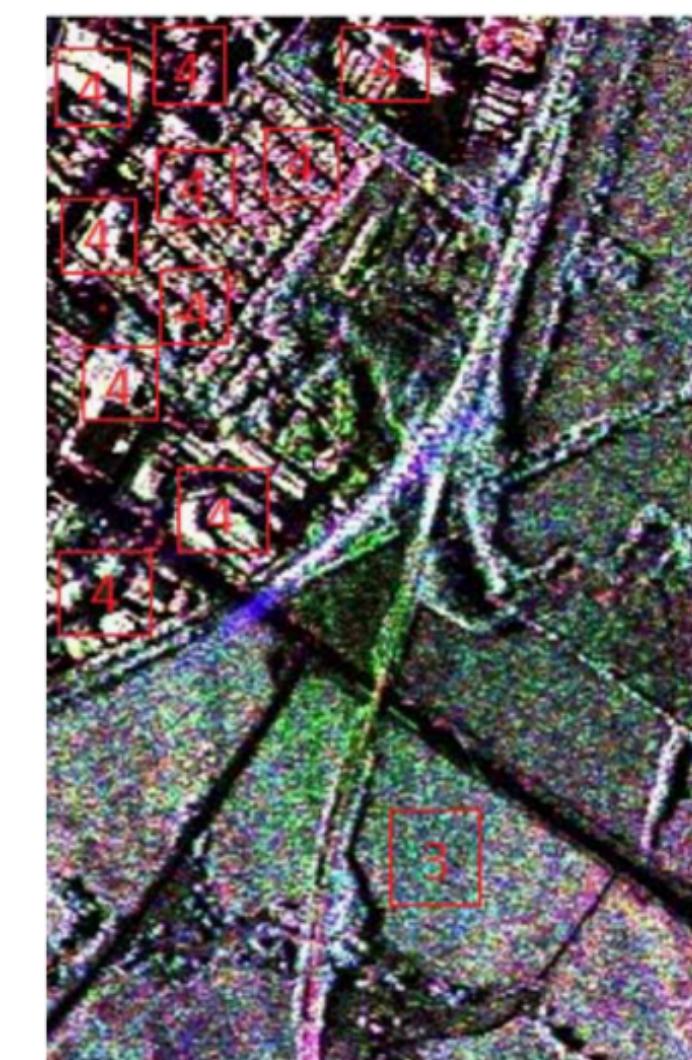
Original



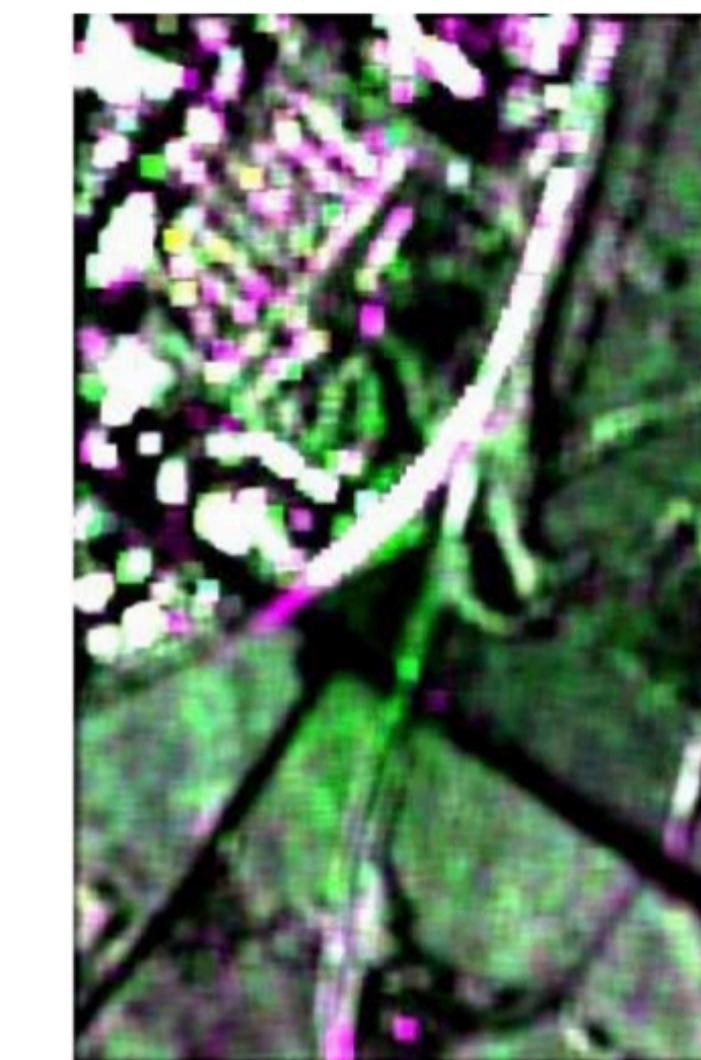
Boxcar



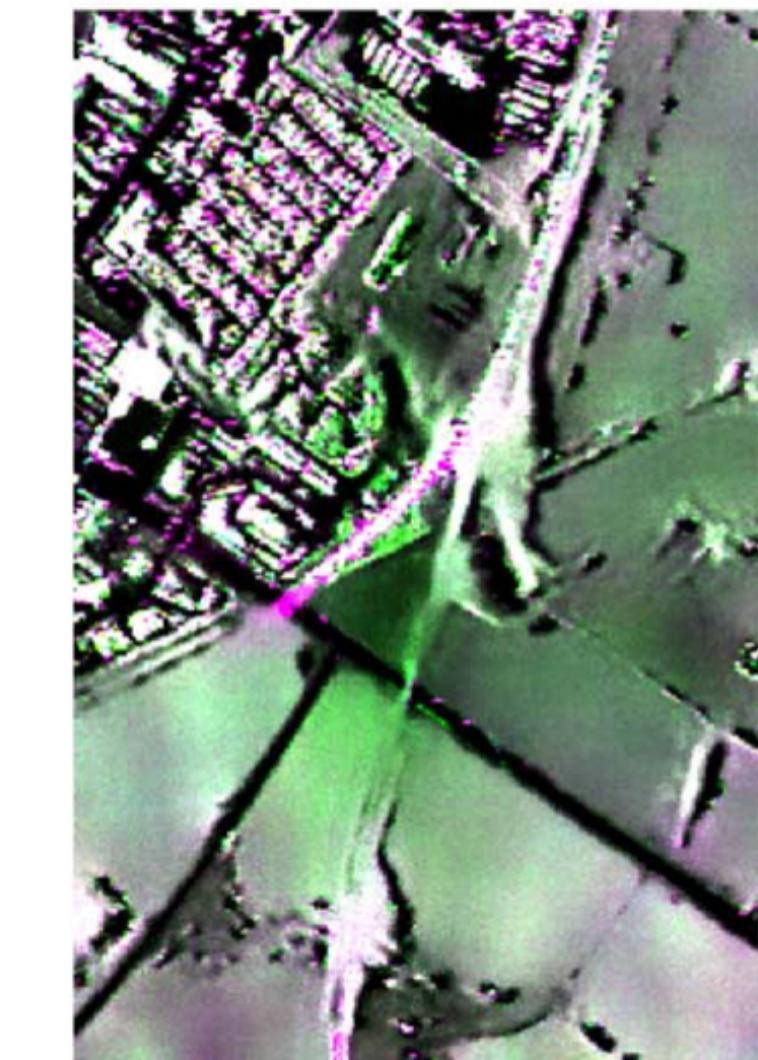
QMCTLs



Original



Boxcar



QMCTLs

# Conclusion

## And discussion

- Compared with the traditional Monte Carlo sampling, **QMCS** has a **faster convergence rate** and thus requires fewer samples to achieve an accurate estimation of the posterior distribution;
- Compared with the **pixel-based likelihood**, the **region-based likelihood** is capable of capturing the local textual patterns and is therefore more robust to speckle noise;
- The proposed QMCTL filter **achieve better balance** between noise removal and detail preservation, compared to the classical boxcar filtering;
- When the window size increases, the acceptance probability is more reliable by considering more pixels, but the chance of finding similar samples decreases, and the computation cost will increase as well.