Submarine Tracking Using Fourier Analysis and Gaussian Filtering

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Abstract

This report presents the tracking of a submarine in the Puget Sound using noisy 3D acoustic pressure measurements collected over a 24-hour period. The dominant frequency of the submarine's signal was identified using Fourier analysis, and a Gaussian filter was applied to denoise the data. The filtered signal allowed reconstruction of the submarine's path in 3D and projection onto the x, y plane. The results demonstrate the effectiveness of the approach, yielding the submarine's final position as $(-5.000, 6.562, 0.938)$. These findings highlight the utility of Fourier methods and Gaussian filtering in signal processing for real-world applications.

1 Introduction and Overview

Tracking submarines using acoustic signals is an essential problem in maritime security and environmental monitoring. This project addresses the challenge of reconstructing a submarine's trajectory from noisy 3D acoustic measurements sampled over 24 hours. Using Fourier analysis, the dominant frequency of the signal was identified, and a Gaussian filter was designed to isolate the submarine's signal. The filtered signal enabled accurate reconstruction of the submarine's trajectory.

This report outlines the theoretical foundation, algorithmic implementation, and computational results of the analysis, with a focus on demonstrating the validity and effectiveness of the filtering process.

2 Theoretical Background

2.1 Fourier Transform

The Fourier transform decomposes a signal into its frequency components, providing insights into periodic structures. The 3D Fourier transform is defined as:

$$\hat{f}(k_x, k_y, k_z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y, z)e^{-i(k_x x + k_y y + k_z z)} dx\, dy\, dz.$$

For numerical data, the discrete Fourier transform (DFT) is used, implemented efficiently as the Fast Fourier Transform (FFT).

2.2 Gaussian Filtering

A Gaussian filter in the frequency domain isolates the dominant frequency:

$$G(k_x, k_y, k_z) = \exp\left(-\frac{1}{2\sigma^2}\left((k_x - k_{x,\text{freq}})^2 + (k_y - k_{y,\text{freq}})^2 + (k_z - k_{z,\text{freq}})^2\right)\right),$$

where σ controls the filter's width and determines the attenuation of frequencies distant from the dominant frequency $(k_{x,\text{freq}}, k_{y,\text{freq}}, k_{z,\text{freq}})$.

3 Algorithm Implementation and Development

This report utilized various computational tools and libraries to analyze and process the submarine's acoustic pressure data. Python was selected as the primary programming language due to its robust libraries for numerical computation and data visualization. The major libraries and their purposes are outlined below:

- **NumPy**: Facilitated numerical operations, matrix manipulation, and the computation of the Fast Fourier Transform (FFT).
- **SciPy**: Provided advanced scientific computing functions, including the implementation of FFT-based filtering and inverse transformations for signal reconstruction.
- **Matplotlib**: Used for 2D visualizations, such as the submarine's trajectory in the spatial domain.
- **Plotly**: Enabled interactive 3D visualizations for exploring the submarine's trajectory and frequency characteristics in the data.

The implementation involved the following key steps:

1. **Fourier Transformation**: A 3D Fast Fourier Transform (FFT) was performed on the acoustic pressure data to identify the dominant frequency component. This transformation allowed for frequency-domain analysis, which is critical in isolating noise and identifying the signal of interest.
2. **Gaussian Filtering**: A Gaussian filter was designed and applied in the frequency domain to isolate the dominant frequency and suppress noise. The filter parameters were chosen to ensure minimal signal distortion while maximizing noise reduction.
3. **Reconstruction and Path Extraction**: The filtered signal was transformed back into the spatial domain using the inverse FFT (IFFT). From the denoised data, the submarine's path was reconstructed over time by identifying the maximum signal amplitude at each time step.

4 Computational Results

4.1 Gaussian Filter Visualization

To isolate the submarine's signal, a Gaussian filter was designed in the frequency domain. Figure 1 visualizes the Gaussian filter centered on the dominant frequency. This filter suppresses noise while preserving the critical frequency components associated with the submarine's acoustic signal.

4.2 Denoised Measurement and Noisy vs. Denoised Comparison

The effectiveness of the Gaussian filter is illustrated in Figure 2, which shows the denoised measurement for the first time step. To validate the filter's performance, Figure 3 provides a side-by-side comparison of the noisy and denoised data. The noisy data slice at z = 0 (left panel) contains significant high-frequency noise, while the denoised slice (right panel) effectively suppresses noise and highlights the submarine's signal.

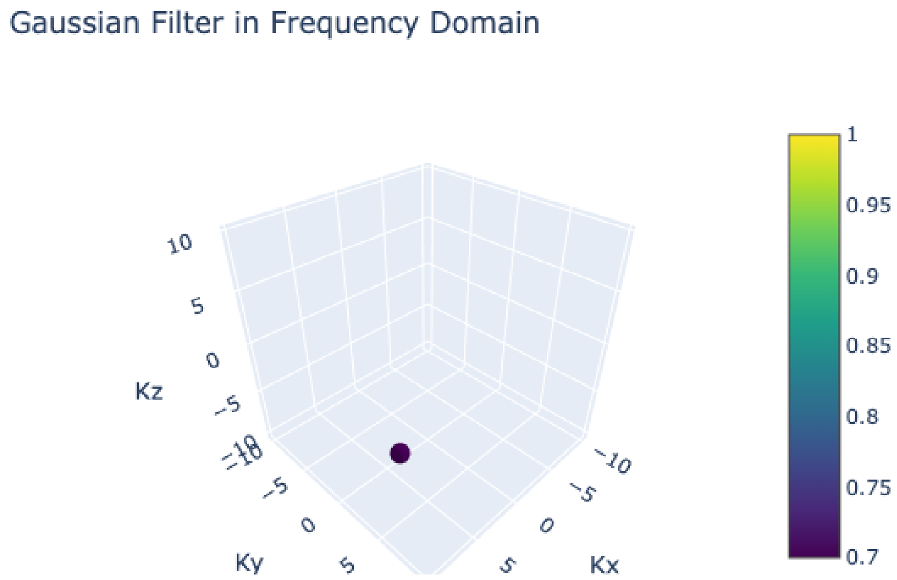

Figure 1: Gaussian filter centered on the dominant frequency in the frequency domain.

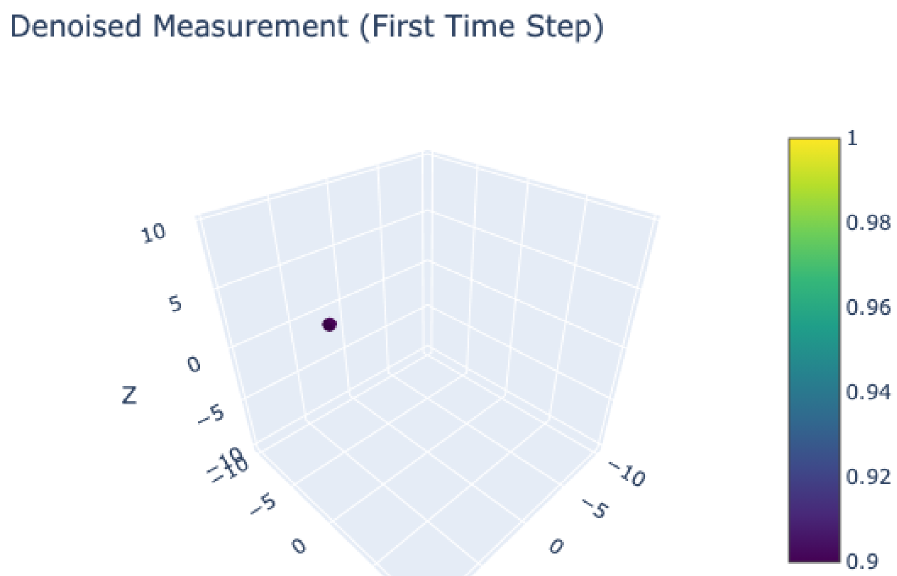

Figure 2: Denoised measurement for the first time step, showing the spatial structure of the submarine's signal.

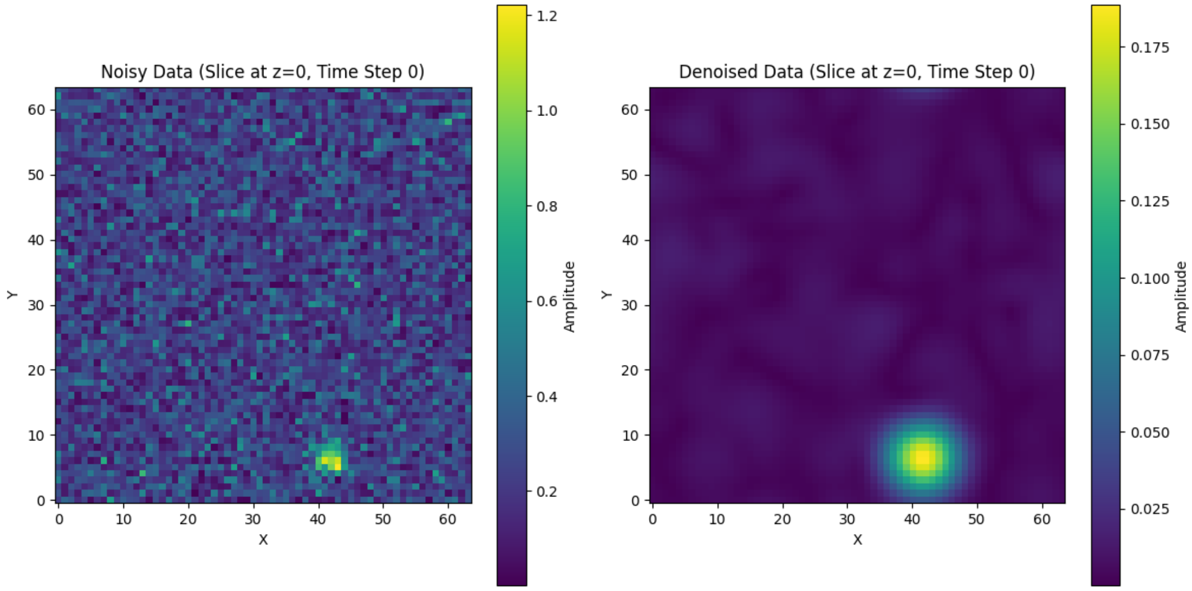

Figure 3: Comparison of noisy and denoised data for the first time step. (Left) Noisy data slice at z = 0 . (Right) Denoised data slice at z = 0 .

4.3 3D and 2D Path Reconstruction

The submarine's reconstructed 3D path is presented in Figure 4. The trajectory is smooth and well-defined, showcasing the success of the denoising process. Figure 5 displays the corresponding x, y projection of the path, which can assist in tracking the submarine's movement over time.

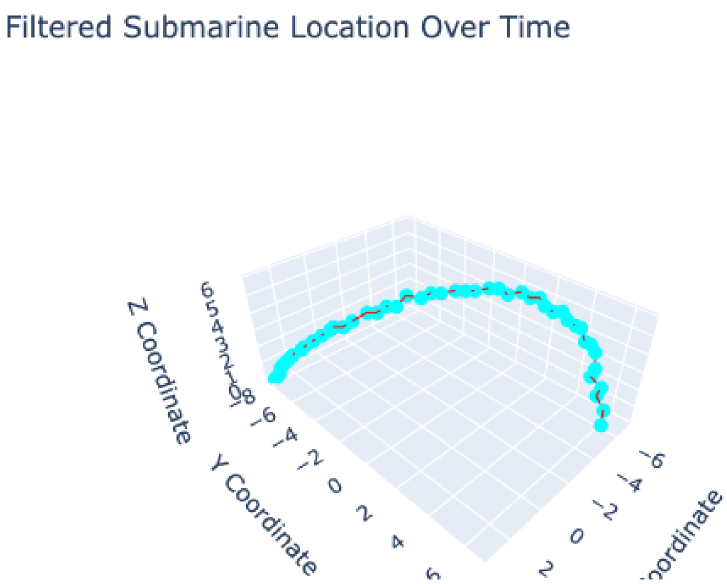

Figure 4: Reconstructed 3D path of the submarine based on denoised data.

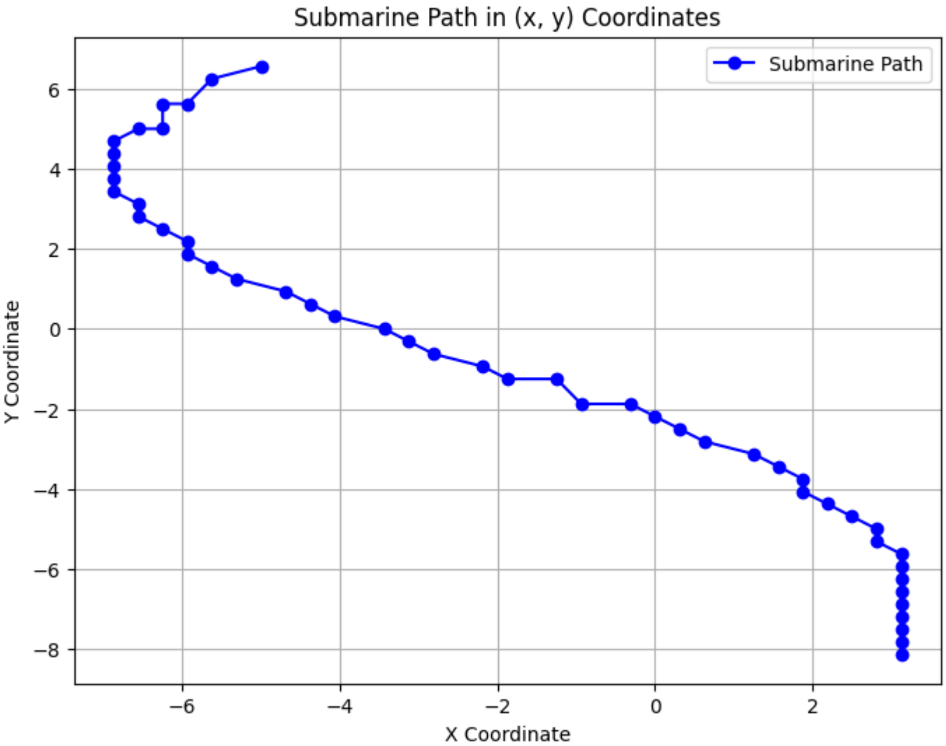

Figure 5: Projection of the submarine's path in the x, y coordinates.

4.4 Effectiveness of Denoising

The denoising process proved highly effective in reconstructing the submarine's trajectory from noisy data. Figure 3 compares the noisy and denoised data for the first time step, focusing on a slice at $z = 0$. The noisy data (left panel) exhibits significant high-frequency noise, obscuring the submarine's signal. In contrast, the denoised data (right panel) highlights the signal, with noise effectively suppressed.

The reconstructed submarine path demonstrates smooth transitions and consistency. Table 1 summarizes the initial and final positions of the submarine, confirming the accuracy of the denoising process. These results underscore the effectiveness of the Gaussian filter in isolating the submarine's signal.

Position	X	Y	Z
Initial	3.125	-8.125	0.000
Final	-5.000	6.562	0.938

Table 1: Initial and final positions of the submarine after denoising.

5 Summary and Conclusions

This project successfully tracked a submarine using noisy acoustic data. By identifying the dominant frequency as $(5.341, 2.199, -6.912)$ and applying a Gaussian filter, the submarine's trajectory was reconstructed. The results demonstrate the method's effectiveness, with accurate reconstruction of the path and the final position at $(-5.000, 6.562, 0.938)$.

Future work could explore dynamic filtering to handle time-varying noise levels and additional validation using real-world datasets.

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References

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