## Godel's way: exploits into an undecidable world / Gregory Chaitin, Newton da Costa & Francisco Antonio Doria. – Leiden, cop. 2012

## Spis treści

Prologue	xiii xvii	
Acknowledgments  About the Authors		
	xix	
A Caveat	xxi	
1. Gödel, Turing	1	
Gödel: logic and time	2	
A short biography	4	
The incompleteness theorems, I	5	
Kleene's version of the first incompleteness theorem	6	
An immediate consequence of Kleene's proof	7	
The incompleteness theorems II: consistency cannot be proved		
within the system	8	
A weird formal system	9	
Can we prove the consistency of arithmetic?	10	
Chaitin's incompleteness theorem	11	
Berry's Paradox	12	
Rice's theorem	13	
More work by Gödel: the constructive universe of sets	14	
A concluding note: Gödel on time machines	17	
Alan Turing and his mathematical machines	20	
What is a computation?	20	
Turing machines, I	21	
Turing machines, II	22	
The universal machine	23	
The halting problem	23	
Gödel 's first incompleteness theorem revisited	24	
The Church-Turing thesis	24	
Diophantine equations; Hilbert's 10th problem	25	
Undecidable issues	27	
Function F <sub>s</sub> and the Busy Beaver function	27	
Busy-Beaver-like functions	29	
Turing 1939: progressions of theories	30	
The halting problem revisited	30	
Reyond the Gödel phenomenon	31	

2. Complexity, Randomness	32
Weyl, Leibniz, complexity and the principle of sufficient reason	33
What is a scientific theory?	33
Finding elegant programs	34
What is a formal axiomatic theory?	34
Why can't you prove that a program is elegant?	35
Farewell to reason: The halting probability $\Omega$	35
Adding new axioms: Quasi-empirical mathematics	36
Defining information content and conceptual complexity	37
Why theories? Subadditivity and mutual information	39
Combining theories and making conjectures	41
Examples of randomness in real mathematics	42
Universal Diophantine equations	42
Does a Diophantine equation have finitely or infinitely many solutions?	44
Does a Diophantine equation have an even/odd number of solutions?	44
The word problem for semi-groups	45
How real are the real numbers? Borel 1927, 1952 and Turing 1936	
revisited	46
Turing, 1936: There are more uncomputable reals than computable reals	46
Uncomputable reals have probability one, computable reals have	
probability zero	46
Borel 1952: Un-nameable reals have probability one	47
Borel's 1927 oracle number: Nth bit answers the Nth yes/no question	47
"Borel-Turing" oracle number: Nth bit tells us if the Nth program halts	47
First N bits of $\Omega$ tell us which $\leq$ N bit programs halt	48
H (First N bits of $\Omega$ ) > N — c, and $\Omega$ is irreducible	49
Mathematics, biology and metabiology	49
Post Scriptum	53
3. A List of Problems	54
Mr. Contradiction: Newton da Costa	54
An aside: on paraconsistent logics	55
A list of outlandish problems	58
Generic universes in gravitation theory	58
Is stability decidable? Is chaos decidable?	60
Stable or unstable?	61
Generic economies, generic social structures	61
Beyond the Shannon theorems	62
Does $P = NP$ ?	63
Hypercomputation	64
Outlandish stuff in general relativity	64
Paul Cohen and forcing	65
General relativity and its axiomatics	67
Suppes predicates	68

A Suppes predicate for general relativity	69
General relativity and forcing models	70
More on forcing-dependent universes	71
Valentine Bargmann steps in	72
4. The Halting Function and its Avatars	73
Chaos is undecidable	73
Alea iacta est	74
Undecidability and incompleteness of chaos theory	75
The halting function	76
A few technicalities	77
Rice's theorem and beyond	78
Back to chaos	78
Classical mechanics is undecidable and incomplete	79
Arnold's problems	79
The halting function, the halting problem and the $\Omega$ number	81
Economics and the social sciences	82
Nash games	83
Competitive markets and fixed-point theorems	84
The Tsuji result	85
Alain Lewis, Vela Velupillai	85
The meaning of the Lewis-Tsuji result	88
Can we predict the future of historical events?	88
Forcing, the halting function and $\Omega$	89
$\Omega$ and $\theta$	90
5. Entropy, P vs. NP	91
Entropy, random sequences, the Shannon theorems	91
The Shannon Coding Theorem: a closer look at it	91
The Shannon-McMillan-Breiman theorem	92
The equipartition property	93
The Shannon Coding Theorem: final steps	94
Randomness in a game of heads in tails	95
P vs. NP	97
A brief history of problems in the NP class	97
Polynomial Turing machines and related fauna	98
The NP class of problems and the $P = NP$ conjecture	99
Shared thoughts The crucial intuition?	99 100
The main result, I: the counterexample function to $P = NP$ grows too fast	100
The main result, II: if $P < NP$ is true then it cannot be proved by	102
reasonable axiomatic systems	103
If P < NP is independent of S, then it is true of the standard integers	103
Another discussion	103
But do we really have independence?	104
but do we really have independence:	104

Still more conjectures on the counterexample function	105
6. Forays into Uncharted Landscapes	107
Is the world built out of information?	108
Hypercomputation: or where are the limits of software?	111
On hypercomputation	113
Analog computers as ideal hypercomputers	114
A possible hypercomputer	114
Prototype for a hypercomputer	115
The hypercomputer and true arithmetic	116
More on the theory of hypercomputation	116
Recent research	117
Spacetimes: exotic variations on that theme	117
Exoticisms	118
Let there be light!	118
Exotic spacetimes	119
Some consequences for physics	120
On time	121
Cosmic time, the Big Bang	123
Convoluted time structures	123
Anything works	125
Heisenberg's Fourth Uncertainty Relation	125
From classical to quantum	125
Wiener integrals, Feynman integrals and the Multiverse	126
Envoi: On Eternity and Beyond	128
References	130

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