1) A basketball player makes a free throw with probability 0.7. Find the probability of the following events:

a) **(1 point)** A : the player makes his first free throw only on the 4^{th} shot;

b) **(2 points)** B : the player makes the first at least 10 consecutive free throws.

a) $k = 3$

$p = 0.7$

$q = 0.3$

$P(3) = 0.7 \cdot 0.3^3$

You should define the success trial

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a) **(1 point)** A : the player makes his first free throw only on the 4^{th} shot;

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b) $P(X \geq 10) = 1 - P(X < 10)$

$$= 1 - \sum_{k=0}^{9} 0{,}3 \cdot 0{,}7^{k}$$

→ success : doesn't makes a throw

success is now when the players misses the hit, so we search the probability to succes in the first 10 trials

2) (2 points) Let $X \in N(0,1)$. Find the pdf of $Y = X^2$. What type of distribution is it?

Normal distribution $N(\mu, \sigma), \mu \in \mathbb{R}, \sigma > 0$: pdf $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, x \in \mathbb{R}$.

$g(x) = x^2$ $\quad \mu = 0$

$g'(x) = 2x$ $\quad \sigma = 1$

$f(x) = \frac{1}{\sqrt{2\pi}} e$

Function $Y = g(X)$: X r.v., $g : \mathbb{R} \to \mathbb{R}$ differentiable with $g' \neq 0$, strictly monotone

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$f_Y(y) = \frac{f_X(g^{-1}(y))}{|g'(g^{-1}(y))|}, \; y \in g(\mathbb{R})$

$P(Y \leq x) = P(X^2 \leq z) = P(|x| \leq z) =$

$= P(-\sqrt{z} \leq X \leq \sqrt{z}) =$

$= \int_{-\sqrt{z}}^{\sqrt{z}} f_X(x)\,dx$

3) Let $X_1, X_2, ..., X_n$ be a random sample drawn from a distribution with pdf $f(x;\theta) = \frac{1}{\theta}$, for $0 < x < \theta$, with $\theta > 0$ unknown.

a) **(2 points)** Find the method of moments estimator, $\hat{\theta}$, for θ.

b) **(2 points)** Is $\hat{\theta}$ an absolutely correct estimator? Explain.

$$E(X) = \bar{X}$$

$$E(X) = \int_{-\infty}^{\infty} x f(x;\theta)\,dx = \int_0^\theta \frac{x}{\theta}\,dx = \frac{1}{\theta}\int_0^\theta x\,dx$$

$$= \frac{\theta}{2}, \quad \frac{\theta}{2} = \bar{X} \Rightarrow \bar{\theta} = 2\bar{X}$$

b) $E(\bar{\theta}) = \theta$, $E(\bar{\theta}) = E(2\bar{X}) = 2E(\bar{X}) = 2E(X)$

$$= \theta$$

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$$\lim_{n\to\infty} V(\bar{\theta}) \overset{?}{=} 0, \quad V(\bar{\theta}) = \frac{V(\theta)}{n} = \frac{E(x^2) - (E(x))^2}{n}$$

$$E(x^2) = \int_0^\theta \frac{x^2}{\theta}\,dx = \frac{1}{\theta}\int_0^\theta x^2\,dx = \frac{\theta^2}{3}$$

$$(E(x))^2 = \frac{\theta^2}{4} \Rightarrow V(\bar{\theta}) = \frac{\theta^2}{12n} \xrightarrow{n\to\infty} 0$$

$\Rightarrow$ absolutely correct estimator