Tutorial 01

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Question 1

1. Implemented on python

```
def func(x):
    return sin(1/x)*pow(e,pow(-x,2))
```

Midpoint Rule

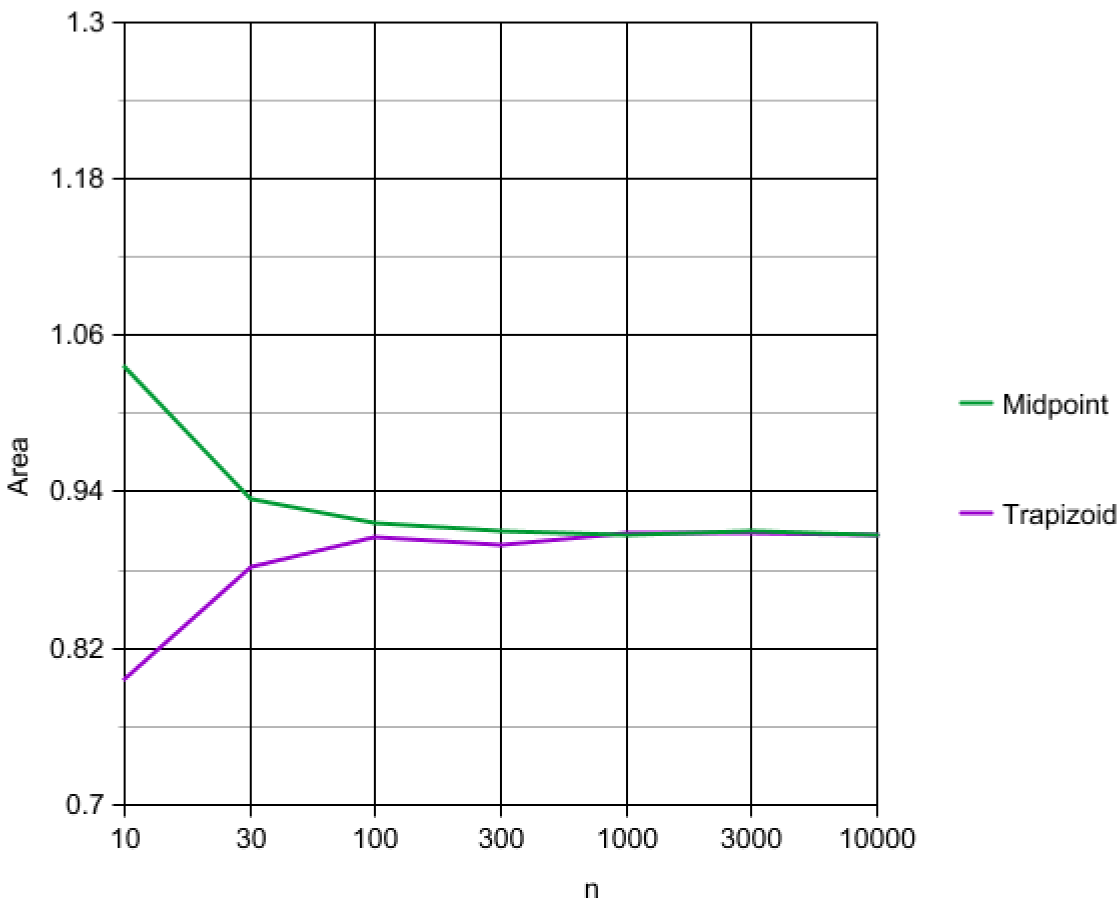
```
def areaMid(a,b,n):
    dx = (b-a)/n
    p = a
    ans=0
    for i in range(1,n+1):
        ans = ans + func((p+p+dx)/2)*dx
        p = p + dx
    return ans
```

Trapezoid Rule

```
def areaTrapizoid(a,b,n):
    dx = (b-a)/n
    ans=0
    p = a
    for i in range(1,n+1):
        ans = ans + (dx/2)*(func(p)+func(p+dx))
        p = p +dx
    return ans
```

2.

n	Midpoint	Trapezoid
10	1.0350754694325164	0.7969348339334276
30	0.9349674279461242	0.8819626737172445
100	0.9159507593760488	0.9046130956214312
300	0.9104721020507185	0.8987967320384952
1000	0.9075935320707456	0.9085743775542424
3000	0.9095481250457645	0.9081498337010934
10000	0.9070860659685501	0.9070263002476655



3. This function returns the **n value** which has the minimum difference (**Ɛ**) from an array of elements of n.

```
def bestN(a,b,error,array):
    for i in array:
        mid=areaMid(a,b,i)
        trap=areaTrapizoid(a,b,i)
        if(abs(mid-trap)<=error):
            break
    return i
```

Question 2.

Points $(a, A)\ (b, B)\ (c, C)$

General equation $y = px^2 + qx + r$

Lets plug this into a matrix

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} Pa^2 & \cancel{+}\ Qa & r \\ Pb^2 & Qb & r \\ Pc^2 & Qc & r \end{bmatrix} \Rightarrow \begin{bmatrix} A-B \\ B \\ C \end{bmatrix} = \begin{bmatrix} P(a^2-b^2) & Q(a-b) & \\ P(b^2)\ \cancel{+} & Qb\ \cancel{+} & r \\ PC^2 & Qc & r \end{bmatrix}$$

$$\begin{bmatrix} A-B \\ B-C \\ C \end{bmatrix} \begin{bmatrix} P(a^2-b^2) & Q(a-b) & \\ P(b^2-c^2) & Q(b-c) & \\ PC^2 & Qc & r \end{bmatrix}$$

$\cancel{(A-B)(b-c)}\ \cancel{P(a^2-b^2)(b-c)}\ \cancel{Q(a-b)(b-c)}$

$\Rightarrow \cancel{(B-C)(a-b)}$

C

$$\begin{pmatrix} (A-B)(b-c) \\ (B-C)(a-b) \\ C \end{pmatrix} \begin{pmatrix} P(a^2-b^2)(b-c) & Q(a-b)(b-c) & 0 \\ P(b^2-c^2)(a-b) & Q(b-c)(a-b) & 0 \\ PC^2 & Qc & r \end{pmatrix}$$

$\cancel{(A-B)(b-c)(}$

$$\begin{matrix} (A-B)(b-c)-(B-C)(a-b) & P\{(a^2-b^2)(b-c)-(b^2-c^2)(a-b)\} & 0 & 0 \\ (B-C)(a-b) & P(b^2-c^2)(a-b) & Q(b-c)(a-b) & 0 \\ C & PC^2 & Qc & r \end{matrix}$$

$$P = \frac{(A-B)(b-c)-(B-C)(a-b)}{(a^2-b^2)(b-c)-(b^2-c^2)(a-b)}$$

$$Q = \frac{(B-C)(a-b)}{(b-c)(a-b)} - \left[\frac{(A-B)(b-c)-(B-C)(a-b)}{(a+b)-(b+c)}\right](b+c) \times \frac{1}{(b-c)(a-b)}$$

$$Q = \left(\frac{B-C}{b-c}\right) - \frac{(A-B)(b+c)}{(a+b)(a-b)-(b+c)(a-b)} + \frac{(B-C)(b+c)}{(a+b)(b-c)-(b+c)(b-c)}$$

$$\cancel{C = (A-B)(b-c) - (B-C)(a-b)}$$

$$C = \frac{(A-B)(b-c) - (B-C)(a-b)}{(a-b)(b-c)(a-c)} c^2$$

$$+ \frac{(B-C)(a^2-b^2) - (A-B)(b^2-c^2)}{(a-b)(b-c)(a-c)} c + r$$

$$\cancel{x = (a-b)(b-c)(a-c)}$$

$$r = C + \frac{\{(B-C)(a-b) - (A-B)(b-c)\} c^2 - \{(B-C)(a^2-b^2) - (A-B)(b^2-c^2)\} c}{(a-b)(b-c)(a-c)}$$

$$y = \frac{(A-B)(b-c) - (B-C)(a-b)}{(a-b)(a-c)(b-c)} x^2 + \frac{(B-C)(a^2-b^2) - (A-B)(b^2-c^2)}{(a-b)(b-c)(a-c)} x$$

$$+ C + \frac{\{(B-C)(a-b) - (A-B)(b-c)\} c^2 - c\{(B-C)(a^2-b^2) - (A-B)(b^2-c^2)\}}{(a-b)(b-c)(a-c)}$$

Assume $L = (a-b)(b-c)(a-c)$

$M = (A-B)(b-c) - (B-C)(a-b)$

$N = (A-B)(b^2-c^2) - (B-C)(a^2-b^2)$

$$y = \frac{M}{L}x^2 + \frac{N}{L}x + C + \frac{M}{L}c^2 + \frac{N}{L}e$$

$$L = (a-b)\left(\frac{b-a}{2}\right)\left(\frac{a-b}{2}\right)$$

$$L = \frac{1}{4}(b-a)^3$$

$$M = \{f(A)-f(B)\}\left(\frac{b-a}{2}\right) - \{f(B)-f(C)\}(a-b)$$

$$M = \frac{(b-a)}{2}\{f(A) - \cancel{f(B)} + \cancel{2}f(B) - 2f(C)\}$$

$$M = \frac{(b-a)}{2}\{f(A) + f(B) - 2f(C)\}$$

$$N = \{f(A)-f(B)\}\left(\frac{b-a}{2}\right)\left(\frac{3b+a}{2}\right) - \{f(B)-f(C)\}(a-b)(a+b)$$

$$N = \frac{(b-a)}{4}\{(f(A)-f(B))(3b+a) + 4(f(B)-f(C))(a+b)\}$$

$$N = \frac{b-a}{4}\{3bA + aA - 3bB - Ba + 4Ba - 4Ca + 4Bb - 4Cb\}$$

$$N = \frac{b-a}{4}\{3bA + aA + bB + 3Ba - 4Ca - 4Cb\}$$

$$P = \frac{2\left\{f(a) - 2f\left(\frac{a+b}{2}\right) + f(b)\right\}}{(a-b)^2}$$

$$Q = \frac{a f(a) - 4af\left(\frac{a+b}{2}\right) + 3a f(b) + 3b f(a) - 4bf\left(\frac{a+b}{2}\right) + bf(b)}{(a-b)^2}$$

$$r = \frac{a^2 f(b) + abf(b) - 4abf\left(\frac{a+b}{2}\right) + abf(a) + b^2 f(a)}{(a-b)^2}$$

$$I \approx \int_a^b f(x)dx \to \int_a^b (px^2 + qx + r)\,dx$$

$$= p\left[\frac{x^3}{3}\right]_a^b + q\left[\frac{x^2}{2}\right]_a^b + r\left[x\right]_a^b$$

$$= p\left(\frac{b^3-a^3}{3}\right) + q\left(\frac{b^2-a^2}{2}\right) + r(b-a)$$

Substituting values from P Q R to the above we get

$$\int_a^b f(x)\,dx = \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

2. The function to calculate the area from Simpson's Rule given multiple parts

```
def sym(a,b,n):
    ans = f(a)+f(b)
    p=a
    dx=(b-a)/n
    for i in range(1,n):
        if(i%2==0):
            ans=ans+2*f(p+dx)
            p=p+dx
        else:
            ans=ans+4*f(p+dx)
            print(p+dx)
            p=p+dx
    return ((b-a)/(n*3))*ans
```

For **n=10,000** answer is 0.9069772969497373